Engineering Models for the Gas-Solid Motion and Interaction in the Return Loop of Circulating Fluidized Beds

**Topical Report
January 1992 - June 1992**

**I. Celik
G-Q. Zhang**

August 1992

Work Performed Under Contract No.: DE-FC21-87MC24207

**For
U.S. Department of Energy
Office of Fossil Energy
Morgantown Energy Technology Center
Morgantown, West Virginia**

**By
West Virginia University
Department of Mechanical and Aerospace Engineering
Morgantown, West Virginia**

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ABSTRACT

It is reported on development, testing and verification of engineering models for predicting the pressure drop, the solids flow rate, and the downcoming gas flow rate through an L-valve for a given aeration flow rate. The models are, in particular, applicable for studying the one-dimensional gas-solids motion through the return loop of a circulating fluidized bed. A literature review is presented in a comparative manner. One-dimensional transient equations governing the dense two-phase flows are derived. Those equations are then used to deduce relevant characteristic dimensionless parameters. Experimental data from literature have been analyzed and empirical correlations are suggested. A calculation procedure is proposed for predicting relevant gas and solid flow parameters. The model is based on integrated conservation equations for mass and momentum for both phases. Some experiments of our own have been performed and the data have been analyzed. The model is calibrated against experimental data.

TABLE OF CONTENTS

Section No.	Page No.
1.0 INTRODUCTION	1
1.1 Literature review and background	1
2.0 MATHEMATICAL MODEL	3
2.1 One-Dimensional transient equations	3
2.2 Constitutive equation for solids pressure	5
2.3 Dimensional analysis for L-valve	7
2.3.1 L-valve pressure drop	7
2.3.2 Solids flow rate trough L-valve system	10
2.4 Suggested model	11
2.5 Solution procedure	16
3.0 EXPERIMENTS	17
3.1 Measurement techniques and flow control	17
3.2 Experimental results	19
4.0 CALIBRATION AND TESTING OF THE MODEL	22
5.0 CONCLUSIONS AND SUGGESTIONS	25
6.0 REFERENCES	26
APPENDIX A: Governing equations for two-phase flow	41
APPENDIX B: Description of the computer program	54

LIST OF TABLES

Table		Page No.
Table 1.	Properties of the solids material used in the experiments	19
Table 2.	Experimental results: case 1.	20
Table 3.	Experimental results: case 2.	21
Table 4.	Experimental results: case 3.	21
Table 5.	Experimental results: case 5.	22

LIST OF FIGURES

Figure		Page No.
Figure 1.	Major Components of a Lab-Scale Circulating Fluidized-Bed (after Huang, 1990)	28
Figure 2.	The three most common types on non-mechanical valves (after Geldart, 1986)	28
Figure 3.	Friction Force Between Solids and Wall vs Solid Velocity (Comparison of Geldart Equ. with Dogin-Lebedev, and Y and Eqs.)	29
Figure 4.	L-Valve $\Delta P/L$ Versus Solids Flow Rate for Different Aeration Tap Locations (After Knowlton and Hirsan, 1980)	30
Figure 5.	L-Valve $\Delta P/L$ Versus Solids Flow Rate For Various L-Valve Diameters (After Knowlton and Hirsan, 1980)	31
Figure 6.	Relation of Friction Coefficient of L-valve, C_f , and Particle Reynolds Number, Re	32
Figure 7.	The Linear Relation Between $\ln(C_f)$ and $\ln(Re)$	33
Figure 8.	Comparison of the function of the ratio between solid velocity and gas velocity with the Knowlton's experimental data	34
Figure 9a.	Experimental Systems for the Modified Return Loop	35
Figure 9b.	Schematic of the Cold Flow CFB	36
Figure 10.	Experimental System for Determining the Voidage of Moving Bed	37
Figure 11.	Comparison of experimental data with model calculations: solids flow rate vs. L-valve aeration rate	38
Figure 12.	Comparison of experimental data with model calculations: pressure drop vs. solids flow rate	39

Figure 13. Comparison of experimental data with model calculations: L-valve pressure drop vs. solids flow rate 40

Figure 14. Comparison of experimental data with model calculations: raiser pressure drop vs. solids flow rate 40

Figure b.1. Flow Chart of CFB Loop Model . . Appendix B 57

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EXECUTIVE SUMARY

A mathematical model has been developed for predicting the solids flow rate and the pressure drop through various sections of the return loop of a circulating fluidized bed(CFB). The model is formulated using the equations for conservation of mass and momentum for the solids phase and the gas phase separately. It is formulated in terms of relevant dimensionless parameters , hence it can be applied to a wide range of operation conditions, in particular for scaling up purposes. The empirical constants and/or functions involved in the model have been calibrated using experimental data from the literature as well as our own experiments. The present experiments have been performed mainly for calibration of the mathematical model, but they also provide more insight to the problem of solids flow transfer vis L-Valves which are non-mechanical valves and have many advantages compared to the classical mechanical valves. This report describes the mathematical model, the experimental system and measurment techniques, and the results of the calculations performed using the present model.

1.0 INTRODUCTION

The smooth operation of a circulating fluidized bed reactor (CFBR) primarily depends upon the stable and continuous circulation of solids in the system. The recycled particle flow rates are usually an order of magnitude higher than the fresh particle feeding rate. In case of hot flow units these particles provide process heat. The steady circulation, in turn, depends on the particle separation efficiency of the cyclone and the steady flow of solids through the surge tank and the stand pipe. An example of a CFBR is depicted in Fig. 1 as reference to major components and the terminology used. In this report emphasis is focused on the return loop of a CFBR, i.e., the primary cyclone, the surge tank and the stand pipe as shown in Fig. 1.

Almost all circulating fluid bed reactors utilize non-mechanical valves as part of the stand pipe system to regulate the flow rate of recirculated particles. The non-mechanical valves have no moving parts which are subject to wear and/or seizure, and they can be constructed easily. Unfortunately, the dynamics of gas/solid motion through such valves are not well understood. As mentioned by Geldart (1986) there are no published correlations to predict valve pressure drop and aeration requirements. The limited information available in the literature (e.g. Geldart, 1986) are only for specific cases and can not be generalized. Here a predictive engineering model is developed and verified for calculating the pressure drop and the solids flow rate through conventional non-mechanical valves such as the ones shown in Fig. 2. The model is based on basic principles of conservation of mass and momentum. Dimensionless similarity parameters are derived from the governing differential equations for dense two-phase gas solid flows. Algebraic correlation equations relating these parameters are formulated and verified against experimental data. Since the surge tank is an integral part of the return loop, it is also analyzed such that continuity is maintained between the cyclone and the stand pipe. A good understanding of the hydrodynamics of such a complex system is essential for proper design and operation of CFBR.

Before describing the details of the present model a review of the relevant literature is presented first as well as giving a background on the flow in the return loop of a CFBR.

1.1 Literature Review and Background

The Literature on downflow of solids through standpipes and valves has been reviewed in some detail by Jones and Leung (1985), Knowlton (1986) and recently continued to be briefly reviewed by Mountziaris and Jackson (1991).

A return loop system usually consists of two sections, namely the standpipe and an L-valve system and is used to transfer solids against a pressure gradient (e.g transfer solids from a low pressure region to a high pressure region). We will only consider the steady state situations in this report. In the standpipe, the solids flow usually downwards due to gravitational force. The gas-solids two-phase flow in the standpipe can be divided into two regimes namely, packed bed (or moving bed), and fluidized bed (or suspension). Both of these regimes belong to dense phase conveying and they are usually considered separately.

In packed bed regime, the gravitational force is balanced by the sum of

solids-gas drag force and solids-wall friction force. The variation of void fraction is relatively small. For practical purposes it can be taken as a constant. The Ergun equation (Ergun, 1952) is an important milestone in packed bed history and is widely used for gas-solids and liquid-solids two phase flows. Many investigators have modified the Ergun equation as the gas phase momentum equation was used to determine the pressure drop in the moving bed. A normal stress acting on solids and a solid-wall shear stress were included in the solids momentum equation. The two commonly used correlations to calculate the pressure drop per unit length are the Leung correlation (Leung et al., 1971), and the Wen and Galli correlation (1971). Both of these correlations employ Ergun's equation, but also include other terms to account for the wall friction and the pressure drop needed for the gas to support the solids.

In fluidized bed regime, the solids-wall friction force is negligible and the void fraction changes such that the drag force balances the gravitational force. The Richardson-Zaki (1954) correlation has been added by some investigators to the Ergun equation to extend it to the fluidized regime. The normal stress acting on solids is taken as zero. The particle-wall friction force is neglected.

There are two common types of standpipes: underflow standpipe and overflow standpipe. The underflow standpipe can be operated at both packed bed regime and fluidized bed regime. But it is usually used in the packed bed regime. The overflow standpipe is usually operated in the fluidized bed regime.

Review of the previous literature indicated that the solids-wall friction force is very important for moving bed regime, especially at lower flow rates. However, there is very limited information in the literature about this term. Knowlton (1991) agreed that a solids-wall friction term should be included in the solid momentum equation. At the same time, he indicated that this is usually difficult to determine with any degree of accuracy. We reviewed three relevant solid-wall friction correlations which were suggested by Geldart and Ling (1990), Dogin and Lebedev (1962), and Yang (1977). These are compared with each other in Figure 3. It seems that there are significant differences among these relations.

The L-valve system is under the standpipe and the solid flow rate can be controlled by changing the aeration flow rate. For the L-valve system, the data of Knowlton and Hirsan (1980) seem to be the only source of data for determining the solids flow rate as a function of aeration (or gas flow rate), and the pressure drop as a function of solids flow rate. The range of this data is limited and it is presented in dimensional form. In Knowlton's experiments, no influence of the aeration tap location was observed since the standpipe above the L-valve was operating in the packed bed regime. The total gas flow through the L-valve did not change as the aeration tap location is changed in the lower section of the standpipe. If the L-valve is located at the bottom of an overflow standpipe, the L-valve will run in an automatic operation mode in which the solids flowrate through the standpipe will be automatically adjusted by the automatic L-valve system. It was suggested by Knowlton that the Wen and Galli (1971) correlation can be used for both sections above and below the aeration point in this automatic L-valve system.

The use of non-dimensional parameters is necessary in order that the empirical correlations have some degree of universality. Mountziaris and Jackson (1991) presented rather unique dimensionless parameters. Using the

scaling variables proposed by Chen et al. (1984) they first wrote the continuity and momentum equations for gas and solid phases in dimensionless forms, then taking some assumptions such as Nguyen and Kaza condition (the discharge rate from sufficiently deep hoppers is independent of the head of the material in the hopper) they wrote the steady state model describing the two-phase flow of gas and solid in dimensionless form with appropriate boundary conditions the matching conditions, their steady state model can be integrated to give the gas pressure profile and particle phase normal stress for each section, that is for the feed hopper, the standpipe, and the discharge region.

The review of the literature has shown that many things from the continuity equations about the fundamental mechanism underlying the flow of gas-solid mixtures through tubes. Hence, we studied these equations in somewhat more detail. These equations are presented next.

2.0 MATHEMATICAL MODEL

2.1 One-Dimensional Transient Equations:

The governing equations for the gas-solid two-phase flow considered here are derived in appendix A. Here we present the results only.

Continuity for gas phase:

$$\frac{\partial}{\partial t} (A \epsilon_1 \rho_1) + \frac{\partial}{\partial x} (A \epsilon_1 \rho_1 U_1) = 0. \quad (1)$$

Continuity for solids phase:

$$\frac{\partial}{\partial t} (A \epsilon_2 \rho_2) + \frac{\partial}{\partial x} (A \epsilon_2 \rho_2 U_2) = 0. \quad (2)$$

Momentum for the gas phase:

$$\begin{aligned} \frac{\partial}{\partial t} (A \epsilon_1 \rho_1 U_1) + \frac{\partial}{\partial x} (A \epsilon_1 \rho_1 U_1^2) &= - A \epsilon_1 \frac{\partial P_1}{\partial x} + (A/R_h) \epsilon_1 \tau_{1w} \\ &\quad - A F_D + A \epsilon_1 \rho_1 g \sin \theta \end{aligned} \quad (3)$$

Momentum for the solids phase:

$$\begin{aligned} \frac{\partial}{\partial t} (A \epsilon_2 \rho_2 U_2) + \frac{\partial}{\partial x} (A \epsilon_2 \rho_2 U_2^2) = & -A \epsilon_2 \frac{\partial P_2}{\partial x} + (P_2 - P_1) A \frac{\partial \epsilon_1}{\partial x} - (A / R_h) \epsilon_2 \tau_{2w} \\ & + A F_D + A \epsilon_2 \rho_2 g \sin \theta \end{aligned} \quad (4)$$

Momentum for the mixture:

$$\begin{aligned} \frac{\partial}{\partial t} (A \rho U) + \frac{\partial}{\partial x} (A \rho U^2) = & \frac{\partial}{\partial x} (P A) - (A / R_h) \tau_w \\ & + A \rho g \sin \theta \end{aligned} \quad (5)$$

where subscripts "1" and "2" denote the fluid and the solid phase respectively, $\epsilon_2 = (1 - \epsilon_1) = (1 - \epsilon)$ is the solids fraction by volume, ϵ is the void fraction, A is the cross-sectional area, R_h is the hydraulic radius, and θ is the angle between the conveying tube and the vertical direction.

The wall shear stresses, τ_{1w} , and τ_{2w} , can be calculated via appropriate friction coefficients. Using the usual convention we define the following friction coefficients:

$$\tau_{1w} = \frac{1}{2} \rho_1 U_1^2 f_1 \quad (6)$$

$$\tau_{2w} = \frac{1}{2} \rho_2 U_2^2 f_2 \quad (7)$$

$$\tau_w = \frac{1}{2} \rho U^2 f \quad (8)$$

Since

$$\tau_w = \epsilon_1 \tau_{1w} + \epsilon_2 \tau_{2w} + \hat{\rho} U_{slip}^2 \quad (9)$$

the three friction coefficients defined above are related by

$$\rho U^2 f = \epsilon_1 \rho_1 U_1^2 f_1 + \epsilon_2 \rho_2 U_2^2 f_2 + \hat{\rho} U_{slip}^2 \quad (10)$$

In Equations 9 and 10, $\hat{\rho}$ denotes the harmonic mean mixture density (see Equ. 11 of Appendix A). There are five primary unknowns, namely, ϵ_1 , U_1 , U_2 , P_1 , and P_2 , but we have only four independent equations namely Eqs. 1-4. An additional equation is needed to close the system of equations. This is selected as a constitutive equation for the solids-phase pressure, P_2 , as described below. Provided that the friction coefficients, f_1 and f_2 are calculated from empirical correlations, Eqs. 1-4, along with the constitutive equation given below form a closed set of coupled partial differential equations which can be solved for the five primary unknowns. Any other method would have to make some simplifying assumptions so that the above equations can be reduced to integral or algebraic equations.

2.2 Constitutive Equation for Solids Pressure

A constitutive equation for the solids pressure is formulated in Appendix A in which we present a summary of these empirical relations. The solids pressure P_2^* (the extra pressure in addition to the weight of the particles in the packed bed regime) is related to the modulus of elasticity, G (or particle-to-particle interaction coefficient) via

$$\frac{dP_2^*}{d\epsilon} = G(\epsilon) \quad (11)$$

Here $P_2^* = P_2 - \rho_s g X \exp[-B(\epsilon - \epsilon_0)]$, X is the vertical distance, and B is a large number selected such that $P_2 \rightarrow P_2^*$ as $\epsilon \rightarrow \epsilon_{mf}$ (the minimum fluidization velocity). The empirical relation suggested for $G(\epsilon)$ by Pritchett, et al. (1978), is

$$-G(\epsilon) = \begin{cases} -G_0 - A^2(\epsilon - \epsilon_0) & \epsilon \leq \epsilon_0 \\ \text{Anti Log}_{10} \left[m(\epsilon - \epsilon_0) + a + b \frac{\sigma}{d_m} \right] & \epsilon > \epsilon_0 \end{cases} \quad (12)$$

where $m = -8.87$, $a = 0.209$, $b = 2.30$, d_m = mean particle size,

ϵ_0 = minimum voidage (≈ 0.4), σ is the standard deviation from the mean, and

$$G_o = \text{Anti Log}_{10} \left[a + b \frac{\sigma}{d_m} \right] \quad (13)$$

Eq. 11 together with Eq. 12 can be integrated and the result can be cast into the following convenient form

$$P_2^* - P_2^*(\epsilon_o) = \begin{cases} (\epsilon - \epsilon_o) \left[G_o + \frac{A}{2} (\epsilon - \epsilon_o) \right] & \epsilon \leq \epsilon_o \\ \phi_p \left[\exp(-c(\epsilon - \epsilon_o)) - 1 \right] & \epsilon > \epsilon_o \end{cases} \quad (14)$$

where $C = 0.26$, and ϕ_p is a particle size and/or shape coefficient given by

$$\phi_p = \frac{1}{c} \exp \left(\frac{\sigma}{d_m} + 0.09 \right) \quad (15)$$

For large void fractions, i.e., dilute flow regime, the particle-to-particle interaction should be negligible. This requires $P_2(\epsilon) \rightarrow 0$ as $\epsilon \rightarrow 1$. Using this constraint at the maximum value $\epsilon = 1$ in Eq. 14, we obtain an expression for $P_2(\epsilon_o)$ which is

$$P_2^*(\epsilon_o) = -\phi_p \left[\exp(-c(1 - \epsilon_o)) - 1 \right] \quad (16)$$

Equation (4) can be rearranged to write the pressure gradient terms as

$$\begin{aligned} & -\epsilon_2 A \frac{\partial P_2}{\partial x} + (P_2 - P_1) A \frac{\partial \epsilon_1}{\partial x} \\ & = -\epsilon_2 A \frac{\partial P_1}{\partial x} + A (P_2 - P_1) \frac{\partial G_1}{\partial x} - \epsilon_2 A \frac{\partial}{\partial x} \left[A (P_2 - P_1) \right] \end{aligned} \quad (17)$$

If we let the last two terms on the right hand side of Eq. (17) equal to $AG(\epsilon) \frac{d\epsilon}{dx}$ Equation (4) becomes the same as the one-dimensional version the equations presented by Bouillard et al. (1991). The solids elastic modulus, $G(\epsilon)$, is given by

$$G(\epsilon) = G_0 \exp [-c(\epsilon - \epsilon_x)] \quad (18)$$

where $G_0 = 1.0$ Pa, $c = 600$, and $\epsilon_x = 0.445$

Bouillard et al. (1991) used Eq. (18) to calculate the normal component of the solids coulombic stress using the relation $\Delta P_2 = \frac{\partial P}{\partial \epsilon} \Delta \epsilon = G(\epsilon) \Delta \epsilon$.

Solids-Wall Friction Coefficient

The solids shear stress can be related to the average solids pressure (normal stress) (see e.g., Jones and Leung, 1985) via

$$\tau_{2w} = \mu_{2w} P_2 \frac{(1 - \sin \delta)}{(1 + \sin \delta)} \quad (19)$$

Here μ_{2w} is the static coefficient of friction between the solid particles and wall surfaces and δ is the internal angle of friction; $\mu_{2w} = \tan \delta$ for a cohesionless Coulomb powder, see e.g. Mountziaris and Jackson, (1991). The proportionality coefficient between the shear stress at the wall and the average normal stress is known as the Janssen coefficient.

Using Eq. 7 together with Eqs. 19, we obtain an expression for the dynamic solids-wall friction coefficient f_2 as

$$f_2 = \frac{\mu_{2w}}{\frac{1}{2} \rho_2 U_2^2} \left(\frac{1 - \sin \delta}{1 + \sin \delta} \right) P_2(\epsilon) \quad (20)$$

where P_2 is given by Eq. 14. and Eq. 20 indicates that the solids friction coefficient is a function of minimum voidage ϵ_0 , voidage ϵ , the internal angle of friction, δ , and the particle size, d_p , as it should be expected.

2.3 Dimensional Analysis for L-valve

The dimensionless empirical relations have been developed for calculating the pressure drop and solids flow rate for L-valve system by using the theoretical analysis of gas-solids two phase flow in the return loop. The pressure drop and the solids flow rate for a given aeration amount and a certain granular material and considered separately.

2.3.1 L-valve Pressure Drop

Experiments concerning the L-valve pressure drop are scarce in the literature. The experimental data from Knowlton and Hirsan (1980) seems to be the only source of data for determining the total L-valve pressure drop. Their experimental results of Knowlton and Hirsan are reproduced in Figs. 4

and 5 for convenience. The pressure drop per unit length ($\Delta p/L$) for a 3-inch-diameter, 18-inch-long L-valve was measured as a function of solid flow rate for different aeration tap locations and horizontal valve diameters using a 40x80 mesh (mean size 0.24 mm) Ottawa sand. In order to make use of these experimental results in our study, it is necessary to modify the dimensional relation, $\Delta p = f_1(m_s)$, to obtain a non-dimensional relation, $Cr = f_2(Rep)$. The non-dimensional form can be obtained from the theoretical analysis for a single phase flow in tube by integrating and rearranging Eq. 5 for a steady uniform flow case. This yields

$$\Delta p_L = \frac{L}{D} \frac{1}{2} \rho_m u_m^2 f_m + \rho_m g L \sin\theta \quad (21)$$

$$\begin{aligned} \Delta p_L = & \frac{L}{D} \frac{1}{2} \rho_s u_s^2 \epsilon f_s + \frac{L}{D} \frac{1}{2} \rho_g u_g^2 (1-\epsilon) f_g \\ & + \frac{L}{D} \frac{1}{2} \hat{\rho} U_{slip}^2 + \rho_m g L \sin\theta \end{aligned} \quad (22)$$

Here the subscript "s", "g" and "m" denote solid, gas, and mixture, respectively. If for the highly dense flow considered here we assume that the gas to wall friction is negligible equation (22) reduced to

$$\Delta p_L = \frac{L}{D} \frac{1}{2} \rho_s u_s^2 (1-\epsilon) f_s + \rho_m g L \sin\theta \quad (23)$$

By defining a friction coefficient $Cr = (1-\epsilon) f_s$, and assume that $\rho_s \gg \rho_g$ then $\rho_m \sim \rho_s$ we obtain

$$\begin{aligned} \Delta p_L = & \frac{L}{D} \frac{1}{2} \rho_s u_s^2 f_s + \rho_s g L \sin\theta \\ & \text{[Solids-Wall Friction]} \quad \text{[Gravity force]} \end{aligned} \quad (24)$$

Letting $\Delta p'_L = \Delta p_L - \rho_s g L \sin\theta$, a non-dimensional relation can be written as

$$\Delta p'_L = Cr \frac{L}{D} \frac{1}{2} \rho_s u_s^2 \quad (25)$$

by definition

$$u_s = \frac{m_s}{A \rho_s} = \frac{4 m_s}{\pi D^2 \rho_s} \quad (26)$$

$$Cr = \frac{2 D \Delta p'_L}{L \rho_s u_s^2} = \frac{\pi^2}{8} \frac{D^2}{L} \frac{\Delta p'_L \rho_s}{m_s^2} \quad (27)$$

$$Rep = \frac{\rho_g d_p u_s}{\mu_g} = \frac{4}{\pi} \frac{\rho_g}{\rho_s} \frac{d_p}{D^2 \mu_g} m_s \quad (28)$$

In the above equations, $\Delta p'_L$ is the total pressure drop in excess of the weight of the materials of L-valve, N/m^3 ; Cr is the total friction coefficient for the L-valve, Rep is the Reynold's number; ρ_g is gas density, kg/m^3 ; ρ_s is particle density, kg/m^3 . D is the pipe diameter, m ; L is the L-valve length, m ; m_s is solid flow rate, kg/s ; u_s is average solid velocity, m/s ; μ_g is gas viscosity, $kg/m \cdot s$. The non-dimensional form is obtained by combining equation (27) and equation (28). The friction coefficient of L-valve, Cr , is a function of the particle Reynold's number, Rep . The results are presented in Figure 6 and Figure 7. Figure 7 shows a linear relation as

$$Cr = A Rep^B \quad (29)$$

$$\ln(Cr) = B \ln(Rep) + \ln(A) \quad (30)$$

The constants A and B were determined by using a linear regression analysis as $A = -1.76$, $\ln(A) = -2.55575$, $A = 0.0776$ with a correlativity $r=0.9998$. Hence, the L-valve pressure drop can be calculated from the following equations:

$$\Delta p_{L-total} = Cr \frac{L}{D} \frac{1}{2} \rho_s u_s^2 \quad (31)$$

$$Cr = 0.0776 Rep^{-1.76} \quad (32)$$

$$Rep = \rho_g d_p u_s / \mu_g \quad (33)$$

$$0 < Rep < 1.85$$

A similar expression for the pressure drop, Δp_g can be derived by using the gas-phase momentum equation (3). Neglecting the transient and inertial terms in this equation yields

$$\epsilon \frac{\Delta p_g}{L} = - \frac{2}{D} \rho_g U_g^2 f_g - F_D + \epsilon \rho_g g \sin \theta \quad (34)$$

The gravitational force for the gas-phase can be neglected since it is usually small. The drag force F can be calculated from (see e.g. Syamal and O'Brien, 1988).^D

$$F_D = \frac{3}{4} C_D \frac{\epsilon \rho_g (1 - \epsilon)}{d_p} |U_g - U_s| (U_g - U_s) \quad (35)$$

where $C_D = C_D(Re_p)$, Re_p = particle Reynolds number based on the slip velocity. The gas-phase friction coefficient f_g is usually a function of $Re_g = U_g D / \nu_g$. Hence with Eq. (35) Eq. (34) can be rearranged to obtain

$$\frac{\Delta p_g}{\frac{1}{2} \rho_g U_g^2} = \frac{L}{D} 4 f_g + \frac{L}{D} \frac{3}{4} C_D (1 - \epsilon) \left(\frac{D}{d_p} \right)^3 \left(\frac{Re_p}{Re_g} \right)^3 \quad (36)$$

2.3.2 Solids Flow Rate Through L-valve System

Solids flow rate through L-valve as a function of gas flow rate was also studied experimentally by Knowlton and Hirsan (1977). Their experimental results provided solids mass flow rate as a function of the total gas flow rate in dimensional form. In order to use these results in our study, it is necessary to modify the dimensional relation, $m_s = f(Q_g)$, to obtain a non-dimensional relation, $Re_p = f(Re_g, D/L)$. The non-dimensional form can be obtained by the dimensionless analysis and a relation among these dimensionless parameters can be obtained by least-square analysis. A careful analysis of Knowlton and Hirsan's data indicates that the solids flow does not start until a critical gas Reynold's number, Re_{cr} , is reached and that Re_{cr} is a function of particle size and density. We propose a preliminary dimensionless function for calculating Re_{cr} , that is

$$Re_{cr} = 7 \left(\frac{\rho_s}{\rho_g} \right)^{1.02} \left(\frac{d_p}{D} \right)^{0.65} \quad (37)$$

This equation can be used in combination with the well known Ergun equation to calculate a critical pressure drop Δp_{cr} . If the pressure drop is lower than Δp_{cr} the flow will be in the fixed bed regime where there is no solids flow.

From Eq. (36) it can be inferred that any appropriate dimensionless dependent variable will be a function of the same dimensionless independent

variables as those that appear on the right hand side of this equation. The dimensionless solids flow rate $\dot{m}_s/\dot{m}_g$ can then be written in a functional form as:

$$\frac{\dot{m}_s}{\dot{m}_g} = \text{func} \left(\frac{L}{D}, \frac{D}{d_p}, \text{Re}_g, \text{Re}_p, \text{Re}_{\text{crt}} \right) \quad (38)$$

But since

$$\frac{\dot{m}_s}{\dot{m}_g} = \frac{\rho_s (1-\epsilon) U_s A}{\rho_g \epsilon U_g A} \quad (39)$$

Equation (38) can be written in terms of the solids velocity U_s the gas velocity U_g can be written in the following form:

$$\frac{U_s}{U_g} = \text{func} \left[\frac{\rho_g \epsilon}{\rho_s (1-\epsilon)}, \frac{D}{d_p}, \frac{L}{D}, \text{Re}_g, \text{Re}_{\text{cr}} \right] \quad (40)$$

A least square regression analysis performed using the data of Knowlton and Hirsan gave the following empirical correlation.

$$\frac{u_s}{u_g} = C \left(\frac{\rho_g}{\rho_s} \right)^{1.476} \left(\frac{D}{d_p} \right)^{2.142} \left(\frac{L}{D} \right)^{2.863} (\text{Re}_g - \text{Re}_{\text{cr}}) \left(1 - \frac{a/b - \text{Re}_{\text{cr}}}{\text{Re}_g} \right) \quad (41)$$

where $\text{Re}_g = \rho_g d_p u_g / \mu_g$; ρ_g is gas density, d_p is particle diameter, $u_s = \dot{m}_s / (\rho_s A) = 4 \dot{m}_s / (\pi \rho_s D^2)$; $u_g = Q_g / A = 4 Q_g / (\pi D^2)$; $a = 0.51 \times 10^{-4}$, $b = 0.074 \times 10^{-6}$, $c = 7.02 \times 10^{-6}$, $\text{Re}_{\text{cr}} = 100$ is the Critical Reynold's Number, D is L-valve diameter, L is L-valve length, $\dot{m}_s$ is solids mass flow rate, Q_g is gas volumetric flow rate, A is cross-sectional area for L-valve. Comparison of equation (41) with the Knowlton-Hirsan experimental data is shown in Figure 8. The agreement between this empirical function and the experimental data can be considered satisfactory. However, more testing and validation is necessary against independent experiments to establish confidence.

2.4 Suggested Model

We base our model on the governing equations of motion for both phases, namely, the equations of continuity and momentum. Assuming that the flow is

steady and using Eqs.(6) and (7) we rewrite Eqs. (1) through (4) as follows:

$$\frac{d}{dx} (A \epsilon \rho_g U_g) = \frac{d}{dx} (\dot{m}_g) = 0 \quad (42)$$

$$\text{or } \dot{m}_g = \text{const} = \rho_g \epsilon A U_g$$

$$\frac{d}{dx} (A(1-\epsilon) \rho_s U_s) = \frac{d}{dx} (\dot{m}_s) = 0 \quad (43)$$

$$\text{or } \dot{m}_s = \text{const} = \rho_s (1-\epsilon) A U_s$$

$$\begin{aligned} \frac{d}{dx} (A \rho_g^2 U_g) = & -A \epsilon \frac{dp_g}{dx} - 4 \frac{A}{D} \epsilon \frac{1}{2} \rho_g U_g^2 f_g \\ & -A F_D + A \epsilon \rho_g g \sin \theta \end{aligned} \quad (44)$$

$$\begin{aligned} \frac{d}{dx} (A(1-\epsilon) \rho_s U_s^2) = & -A (1-\epsilon) \frac{dp_s}{dx} + (P_s - P_g) a \frac{d\epsilon}{dx} \\ & - \frac{4A}{D} (1-\epsilon) \frac{1}{2} \rho_s U_s^2 f_s + A F_D + (A(1-\epsilon) \rho_s g \sin \theta \end{aligned} \quad (45)$$

Here we make a bold assumption about the second term on the r.h.s. of Eq. (45), i.e. the buoyancy term in that we assume it is small and not of any significant importance for our problem. This assumption is certainly justified when the axial variations in ϵ is small, or when the solids normal stresses are of the same order of magnitude as the gas phase pressure. The next step is to perform piecewise integration of Eqs. (42-45) with respect to axial direction. As an example the convective (or inertial) term in Eq. (44) is integrated as

$$\begin{aligned} & = \int_1^2 \frac{d}{dx} (\epsilon \rho_g A U_g^2) dx = \left(\epsilon \rho_g A U_g^2 \right)_2 - \left(\epsilon \rho_g A U_g^2 \right)_1 \\ & = \dot{m}_g \left[(U_g)_2 - (U_g)_1 \right] \\ & = \dot{m}_g \left[\frac{1}{(\epsilon \rho_g)_2} - \frac{1}{(\epsilon \rho_g)_1} \right] \end{aligned} \quad (46)$$

The other terms can be integrated in a similar manner. The resulting equations can be simplified greatly if we define an average quantity ($\tilde{\phi}$) over the length of integration, L,

$$\tilde{\phi} = \frac{1}{\tilde{A}_L} \int_0^L \phi \, dx \quad (47)$$

With this notation we define the following quantities

$$\tilde{r}_{ag} = \frac{\dot{m}_g}{\tilde{A}_L} \left[\frac{1}{(A\varepsilon\rho_g)_2} - \frac{1}{(A\varepsilon\rho_g)_1} \right] \quad (48)$$

$$\tilde{F}_{wg} = \frac{1}{\tilde{A}_L} \int_0^L \frac{4A}{D} \varepsilon \frac{1}{2} \rho_g U_g^2 f_g \, dx \cong (1 + \alpha_w) \left(\frac{2}{D} \varepsilon \rho_g U_g^2 f_g \right) \quad (49)$$

$$\tilde{F}_D = \frac{1}{\tilde{A}_L} \int_0^L A F_D \, dx \quad (50)$$

$$\tilde{F}_{gg} = \frac{1}{\tilde{A}_L} \int_0^L A \varepsilon \rho_g g \sin \theta \, dx \quad (51)$$

$$(1 + \alpha_p) \tilde{\varepsilon} \frac{\Delta P}{L} = \frac{1}{\tilde{A}_L} \int_0^L A \varepsilon \frac{dP}{dx} g \, dX \quad (52)$$

where α_w and α_p are correction factors for the integrations; if $A_1 \cong A_2 = A$ and $\varepsilon_2 \cong \varepsilon_1 = \varepsilon$ then $\alpha_p \cong 0$. Introducing similar notation for the solids-phase, and noting that U_g and U_s can be calculated from Eqs. (42) and (43) when $\dot{m}_g$ and $\dot{m}_s$ are given the integrated momentum equations can be written in a compact form as

$$(1 + \alpha_p) \varepsilon \frac{\Delta P}{L} = -F_D - F_{wg} + F_{gg} - F_{ag} = f(\dot{m}_s, \dot{m}_g) \quad (53)$$

$$(1 + \alpha_p)(1 - \varepsilon) \frac{\Delta \sigma}{L} = F_D - F_{ws} + F_{gs} - F_{as} = g(\dot{m}_s, \dot{m}_g) \quad (54)$$

Here we introduced the notation $\sigma = P_s$ for convenience, and dropped the symbols ($\sim$) which denote average quantities. The other symbols used are defined below for convenience.

$$\begin{aligned}
 \dot{m}_s &= \text{solid flow rate (kg/s), } \dot{m}_s = U_s \rho_s (1 - \epsilon_o) A \\
 \dot{m}_g &= \text{gas flow rate (kg/s), } \dot{m}_g = U_g \rho_g \epsilon_o A \\
 U_s &= \text{solid velocity (m/s)} \\
 U_g &= \text{gas velocity (m/s)} \\
 \rho_s &= \text{solid density (kg/m}^3\text{)} \\
 \rho_g &= \text{gas density (kg/m}^3\text{)} \\
 A &= \text{cross-sectional area (m}^2\text{)} \\
 P &= \text{gas pressure (N/m}^2\text{)} \\
 F_D &= \text{drag force (N/m}^2\text{)} \\
 F_{wg} &= \text{gas-wall friction force (N/m}^2\text{)} \\
 F_{ws} &= \text{solid-wall friction force (N/m}^2\text{)} \\
 F_{ag} &= \text{gas gravity force (N/m}^2\text{)} \\
 F_{gs} &= \text{solid gravity force (N/m}^2\text{)} \\
 F_{ag} &= \text{gas acceleration force (N/m}^2\text{)} \\
 F_{as} &= \text{solid acceleration force (N/m}^2\text{)}
 \end{aligned}$$

Equations (53) and (54) can be used to calculate the pressure drop in each specific section of a CFB loop only if the mass flow rates $\dot{m}_s$ and $\dot{m}_g$ are known in addition to friction coefficients f and f , as well as the void fraction ϵ . The primary unknowns are $\dot{m}_s$, $\dot{m}_g$, ΔP , $\Delta\sigma$ and ϵ . Two additional equations can be obtained using the pressure drop constraints for a closed loop, namely

$$\sum_{i=1}^N \Delta P_i = 0 = \sum_{i=1}^N f_i (\dot{m}_s, \dot{m}_g) = F(\dot{m}_s, \dot{m}_g) \quad (55)$$

$$\sum_{i=1}^N \Delta\sigma_i = 0 = \sum_{i=1}^N g_i (\dot{m}_s, \dot{m}_g) = G(\dot{m}_s, \dot{m}_g) \quad (56)$$

where N is the number of sections in the closed loop.

The drag force F_D can be calculated using the empirical relation suggested by Mountziaris and Jackson (1991), which is

$$F_D = \left[(1-w)\beta_1 + w\beta_2 \right] (U_s - U_g) \quad (57)$$

$$\beta_1 = \frac{(1-\epsilon)}{\epsilon d_p} \left[150 \mu_g (1-\epsilon) / \epsilon d_p + 1.75 \rho_g |U_s - U_g| \right]$$

$$\beta_2 = \rho_g (1-\epsilon) g / (U_t \epsilon^{n-1})$$

$$U_t = d_p^2 (\rho_s - \rho_g) g / 18 \mu_g$$

where n is a function of $Re_t = U_t \rho_g D / \mu_g$, and w is a transition function given by

$$w = \exp \left[-w(1-\epsilon) / (\epsilon - \epsilon_0) \right] \quad (58)$$

The gas-phase friction coefficient, f_g , is calculated from

$$f_g = 0.079 Re_g^{-0.25} ; \quad Re_g = \frac{\rho_g D U_g}{\mu_g} \quad (59)$$

The solids-wall friction coefficient is calculated from

$$f_s = \beta \sqrt{gD} / 2 U_s (1-\epsilon) \quad (60)$$

$$\beta = 0.057$$

For the present model the voidage, ϵ , must also be calculated empirically. We suggest the following relations which are a combination of equations suggested by Knowlton (1977) and Richardson-Zaki (1954) on:

$$\epsilon = \begin{cases} \epsilon_0 - (\epsilon_{mf} - \epsilon_0) U_{slip} / U_{mf} & U_{slip} < U_{mf} \\ \epsilon_{mf} + (U_{slip} - U_{mf}) \frac{(\epsilon_{cw} - \epsilon_{mf})}{(U_{cw} - U_{mf})} & U_{mf} < U_{slip} < U_{cw} \\ (U_{slip} / U_t)^{1/n} & U_{cw} < U_{slip} < U_t \end{cases} \quad (61)$$

where $U_{\text{slip}} = |U_s - U_g|$ is the slip velocity, U_t is the terminal particle velocity, ϵ_{mf} is the minimum fluidization voidage, U_{mf} is the gas velocity when $\epsilon = \epsilon_{mf}$, $\epsilon_{cw} = 1.15 \epsilon_{mf}$, and $U_{cw} = \epsilon_{cw}^n U_t$.

2.5 Solution Procedure

Equations (55) and (56) form a non-linear system of equations in the form

$$\vec{F}(\vec{X}) = 0 \quad (62)$$

which can be solved by Newton's method (see e.g., Gerald and Wheatley, 1984) that is

$$\vec{F}(\vec{X}) + \vec{J}(\vec{X}) \Delta \vec{X} = 0 \quad (63)$$

$$\vec{X}_{i+1} = \vec{X}_i + \Delta \vec{X}_i \quad (64)$$

For our case, equation (63) becomes

$$\begin{pmatrix} \frac{\partial m}{\partial \dot{m}_s} & \frac{\partial m}{\partial \dot{m}_g} \\ \frac{\partial F}{\partial \dot{m}_s} & \frac{\partial F}{\partial \dot{m}_g} \end{pmatrix} \begin{pmatrix} \Delta \dot{m}_s \\ \Delta \dot{m}_g \end{pmatrix} = - \begin{pmatrix} G(\dot{m}_s, \dot{m}_g) \\ F(\dot{m}_s, \dot{m}_g) \end{pmatrix} \quad (65)$$

and equation (64) becomes

$$\dot{m}_s(i+1) = \dot{m}_s(i) + \Delta \dot{m}_s \quad (66)$$

$$\dot{m}_g(i+1) = \dot{m}_g(i) + \Delta \dot{m}_g \quad (67)$$

The four partial derivatives on the l.h.s. of equation (65) can be estimated by different quotients when a variable is perturbed by a small amount. Once these derivatives are determined, the linear system of equations (65) can be solved by using the Gaussian elimination method. Then $\Delta \dot{m}_s$ and $\Delta \dot{m}_g$ are substituted into the iteration formulations (66), (67), and the same procedure is repeated until the solution converges.

3.0 EXPERIMENTS

In this section we describe our experiments the results of which were used for calibration and testing of the model described in the preceding section. These experiments were undertaken because there were certain key parameters that we needed in our model but were not available in the experiments reported in the literature.

For this purpose, a readily available circulating fluidized bed (CFB) model (Gautam, 1992) was used with minor modifications. The schematic of the modified cold flow CFB is shown in Figure 9. The dimensions are marked in millimeters on each section. The physical model consists of a raiser of 0.20m in diameter and 4.5m in length, with a distributor plate at the bottom, one primary cyclone, one secondary cyclone, a surge tank (section 1-2), a stand pipe (section 3-4), and an L-valve. The gas distributor plate at the bottom of the raiser column ensures a uniform gas velocity through the lateral area of the raiser pipe. The pipe connecting the secondary cyclone to the surge tank was a perforated pipe so that the gas flow inside it attains a value close to atmospheric pressure. Hence the pressure drop between section 7 and 1 remains nearly constant for all test cases. This value was approximately 1.2 inches of water. The stand pipe and the L-Valve was made of transparent paxiglass in order to observe the solids motion through these sections. The raiser section was also made of transparent paxiglass. Two L-Valves with different diameters, namely, 6.985 cm and 5.08 cm were used to study the effect of tube diameter.

As for solid materials we selected two different sizes of spherical glass microbeads from the Cathode Inc. and one type of regular salt was used as bed materials. The properties of these materials are listed in Table 1 for reference. These selections were made so that the influence of particle size and density, and to a certain extent that of particle shape, could be studied.

3.1 Measurement Techniques and Flow Control

The aeration-gas flow was measured by a rotameter with a range of 0-50 SCFH. The raiser gas flow rate was determined by a pitot-static probe at the primary air entrance and maintained at a constant value of $0.078 \text{ (m}^3\text{/sec)}$ for all of the tests. The pitot probe was installed on the gas supply pipe and it was calibrated against standard curves to measure the raiser gas flow rate. The standard curves show that the gas flow rate or gas velocity is a function of dynamic pressure drop which is the difference between the total pressure and static pressure. The dynamic pressure can be measured by the clinometry manometer with an accuracy of 0.001 inch of water. A set of manometers was used to determine the pressure drops in the experimental system. The pressure tap locations used for our system are shown in Figure 1.

A good control of the gas distributor pressure drop is important to ensure a uniform gas velocity in the cross section of the raiser. Usually a pressure drop greater than twenty percent of the total pressure drop in the bed is required. The pressure drop is directly measured from the pressure difference between the two points separated by the distributor plate. The distributor pressure drop was about 60 inches water at the $0.08 \text{ (m}^3\text{/sec)}$ of gas flow rate.

In order to determine the total solids return flow rate, a control valve is installed at bottom of the pipe between the surge tank and the cyclone #1 (see Figure 9). Under steady running conditions, the solids will be accumulated in the pipe above the surge tank when the control valve is closed. The accumulation of the solid in the pipe is indicated by the changing height, Δh , of solids in the pipe. The solids flow rate, $\dot{m}_s$ (kg/s) is calculated from

$$\dot{m}_s = \frac{\Delta h A}{\rho_k \Delta t} \quad (68)$$

Here A is the cross-sectional area of the tube, ρ_k is the solids bulk density, and Δt is the time period corresponding to Δh . Once the pipe is filled up with the solids, the controlling valve is opened to let the accumulated solids fall into the surge tank. In this way the system runs continuously without interruptions.

The properties of the solids including particle size, density and voidage are the essential parameters for the mathematical model. The particle size is specified by the company and the average particle size, $\bar{d}_p$, is estimated by using a microscope to observe about fifty particles at random. The true

density, ρ_t , and bulk density, ρ_b , are determined by the volume replacement method. The voidage for packed and moving beds is calculated using an empirical function: $\epsilon = 1 - \rho_b / \rho_t$. The experimental system for determining the voidage of moving bed is shown in Figure 10. A vertical pipe filled with solids is controlled and moved up at a set velocity by a controlling box. When solids in the pipe fall down and form a moving bed which is similar to the experimental conditions, the plate valve at the bottom of the pipe is closed, the height (h) and weight (w) of the solids in the closed pipe were measured. The solids bulk density of moving bed is calculated from $\rho_b = w / (A h)$. The solids properties are summarized in Table 1.

Table 1 Properties of the solids material used in the experiments

Material ---->	Glass Beads-1	Glass Beads-2	Salt
Mean Diameter($m\mu$)	116	230	400
True Density(kg/m^3)	2500	2500	2156
Bulk Density(kg/m^3)	1550	1500	1272
Shape Factor, ϕ ,	1.0	1.0	0.90
Packed Bed Voidage, ϵ_p ,	0.35	0.37	0.39
Moving Bed Voidage, ϵ_m ,	0.38	0.40	0.41

3.2 Experimental Results:

Four different cases were considered to evaluate the effects of L-valve diameter, solid particle size and particle density on solids flow rate through the L-valve and pressure drops in the return loop. For each case, the solids mass flow rate through the loop as a function of aeration gas flow rate, and the pressure drops across standpipe, L-valve and the raiser as a function of solids flow rate were measured. The experimental results along with the key parameters are summarized in the Tables 2 - 5. The notation used in these tables are as follows:

QA = Aeration gas flow rate;
Ms = solids mass flow rate;
DPL = pressure drop cross the L-valve;
DPST = pressure drop cross the standpipe;
DPR = pressure drop cross the raiser;
dp = average particle diameter;
SF = particle shape factor;
 ϵ = void fraction;
DPLC = Critical pressure drop of L-valve;

The experimental data corresponding to Tables 2-5 are also presented graphically in Figures 11 through 14. Comparison of Case 1 with Case 2 shows the influence of the L-Valve diameter, comparison of Case 1 with Case 3 shows the influence of the particle size, and comparison of Case 1 with Case 4 shows the influence of particle density and particle shape together. Figure 11 show that as expected the solids flow rate increases as the aeration flow rate is increased. At lower aeration flow rates the solids flow does not start until some critical condition is attained. This was explained via Eq. 37 for Knowlton's experiments. At higher aeration flow rates the experimental data indicates that the solids flow rates attain an asymptotic value. Our experimental set up was not suitable to study what may happen if the aeration rate would be increased further.

Figure 11 also shows the significant influence of L-Valve diameter(Case 1 vs 2), the particle size (Case 1 vs 3), and the particle density (Case 1 vs 4) on the solids flow rate as a function of the aeration flow rate.

Figure 12 show that all of the above mentioned factors have a significant influence on the standpipe pressure drop; The pressure drop variation from Cases 2,3, and 4 show a considerable variation from that of Case 1. The information depicted in Figure 12 where L-Valve pressure drop is plotted against solids flow rate is another way of showing the same influence.

In Figure 14 it is shown that the raiser pressure drop is a linear function of the solids flow rate. This well known relation can be taken as a confidence indicator for the present measurements.

Table 2 Experimental results: Case 1. Glassbeads

$dp=0.116E-3(m)$, $SF=1$, $D=0.06985(m)$, $Density=2500(kg/m^3)$
 $L=0.385(m)$, $\epsilon=0.38$, $DPLC=4.1(inwc)$

	QA (ft ³ /hr)	Ms (Klb/hr)	DPL (inwc)	DPST (inwc)	DPR (inwc)
1	.01	.01	3.50	3.80	.15
2	7.50	.10	4.10	4.80	.20
3	9.50	.48	5.20	5.90	.50
4	11.00	.83	6.00	6.40	.75
5	14.00	1.95	7.80	7.40	1.60
6	14.50	2.41	8.47	7.60	1.90
7	16.20	3.13	9.00	7.90	2.50
8	16.50	3.28	9.23	8.00	2.60
9	18.10	3.98	10.20	8.30	3.15
10	20.00	4.50	10.60	8.40	3.50
11	21.00	5.23	11.60	8.60	4.10
12	21.50	5.27	11.70	8.65	4.15
13	23.50	6.07	11.90	8.80	4.70
14	25.50	6.53	12.00	8.95	5.00

Table 3 Experimental results: Case 2. Glassbeads
 $dp=0.116E-3(m)$, $SF=1$, $D=0.0508(m)$, Density= $2500(kg/m^3)$
 $L=0.385(m)$, $\epsilon=0.38$, $DPLC=4.2(inwc)$

	QA (ft ³ /hr)	Ms (Klb/hr)	DPL (inwc)	DPST (inwc)	DPR (inwc)
1	.01	.01	3.50	3.50	.10
2	5.00	.08	4.20	4.50	.15
3	6.00	.16	5.00	6.25	.25
4	7.00	.48	6.30	7.25	.50
5	8.00	1.01	7.50	8.00	.90
6	9.00	1.48	8.30	8.50	1.23
7	10.00	2.02	9.10	8.90	1.70
8	11.00	2.42	9.70	9.20	1.94
9	12.00	2.75	10.00	9.40	2.20
10	13.00	2.90	10.40	9.50	2.42

Table 4 Experimental results: Case 3. Glassbeads
 $dp=0.23E-3(m)$, $SF=1$, $D=0.06985(m)$, Density= $2500(kg/m^3)$
 $L=0.385(m)$, $\epsilon=0.40$, $DPLC=5.3(inwc)$

	QA (ft ³ /hr)	Ms (Klb/hr)	DPL (inwc)	DPST (inwc)	DPR (inwc)
1	.01	.01	-0.25	3.50	.08
2	5.00	.01	4.20	4.50	.12
3	6.00	.01	5.00	5.25	.17
4	11.00	.10	5.38	6.00	.23
5	13.00	0.39	6.20	6.80	.46
6	15.00	0.89	7.70	7.70	.86
7	17.00	1.66	8.80	8.50	1.47
8	18.00	2.15	9.70	8.80	1.86
9	19.00	2.75	10.60	9.20	2.33
10	20.00	3.26	11.20	9.40	2.75
11	21.80	3.71	11.70	9.60	3.10

Table 5 Experimental results: Case 4. Salt
 $dp=0.4E-3(m)$, $SF=0.898$, $D=0.06985(m)$, $Density=2156(kg/m^3)$
 $L=0.385(m)$, $\epsilon=0.41$, $DPLC=4.0(inwc)$

	QA (ft ³ /hr)	Ms (Klb/hr)	DPL (inwc)	DPST (inwc)	DPR (inwc)
1	.01	.01	-.25	2.50	.14
2	5.00	.01	2.44	3.82	.15
3	10.00	.08	4.00	4.13	.20
4	13.00	.13	4.20	4.82	.24
5	17.00	.47	5.60	6.64	.50
6	19.00	.68	6.00	7.09	.65
7	22.00	1.02	7.00	7.73	.92
8	25.00	1.68	8.20	8.43	1.40
9	29.00	2.35	9.20	8.91	1.93
10	31.00	2.94	10.00	9.22	2.38
11	31.50	3.25	10.50	9.37	2.50
12	33.50	3.36	10.53	9.42	2.70
13	33.70	3.40	10.60	9.43	2.72

4.0 Calibration and Testing of the Model

The mathematical model described in section 2 is applied to the flow loop of our experimental set up (Fig. 9a and 9b). In this section we present the details concerning the present experiments and discuss the results from the present model.

The continuity equations (Eqs. 42 and 43) are implemented in the following manner:

$$u_s = \dot{m}_s / (A \rho_s), \quad U_s = u_s / (1 - \epsilon) \quad (69)$$

$$u_g = \dot{m}_g / (A \rho_g), \quad U_g = u_g / \epsilon \quad (70)$$

The momentum equations (Eqs. 53 and 54) are written as:

$$\Delta p = + FD - FGG + FAG + FWG \quad (71)$$

$$\Delta \sigma = - FD - FGS + FAS + FWS \quad (72)$$

where the pressure differences are given by $\Delta p = p_1 - p_2$ and $\Delta \sigma = \sigma_1 - \sigma_2$

The return loop contains four sections (hopper section, standpipe section, L-valve section and raiser section). We apply Eq. (69) for all sections, Eq. 70 for sections 1, 2, and 3. For sections 4, 5, and 6 we apply Eq (70) with

$$\dot{m}_g(4) = \dot{m}_g + Q_a \rho_g \quad (73)$$

$$\dot{m}_g(6) = \dot{m}_g(4) + Q_m \rho_g \quad (74)$$

Momentum equations for the loop are broken down as follows:

gas phase:

$$p_1 - p_2 = 300 \text{ (=1.2 inches of water)} \quad (75)$$

$$p_2 - p_3 = \Delta p_2 = + FD_2 - FGG_2 + FAG_2 + FWG_2 = f_1(\dot{m}_s, \dot{m}_g) \quad (76)$$

$$p_3 - p_4 = \Delta p_3 = + FD_3 - FGG_3 + FAG_3 + FWG_3 = f_2(\dot{m}_s, \dot{m}_g) \quad (77)$$

$$p_4 - p_6 = \Delta p_4 = + FD_4 - FGG_4 + FAG_4 + FWG_4 = f_3(\dot{m}_s, \dot{m}_g) \quad (78)$$

$$p_6 - p_{11} = \Delta p_6 = + FD_6 + FGG_6 + FAG_6 + FWG_6 = f_4(\dot{m}_s, \dot{m}_g) \quad (79)$$

solids phase:

$$\sigma_1 - \sigma_2 = 0 \quad (80)$$

$$\sigma_2 - \sigma_3 = \Delta \sigma_2 = - FD_2 - FGS_2 + FAS_2 + FWS_2 = g_1(\dot{m}_s, \dot{m}_g) \quad (81)$$

$$\sigma_3 - \sigma_4 = \Delta \sigma_3 = - FD_3 - FGS_3 + FAS_3 + FWS_3 = g_1(\dot{m}_s, \dot{m}_g) \quad (82)$$

$$\sigma_4 - \sigma_6 = \Delta \sigma_4 = - FD_4 - FGS_4 + FAS_4 + FWS_4 = g_1(\dot{m}_s, \dot{m}_g) \quad (83)$$

$$\sigma_6 - \sigma_{11} = \Delta \sigma_6 = - FD_6 + FGS_6 + FAS_6 + FWS_6 = g_1(\dot{m}_s, \dot{m}_g) \quad (84)$$

Boundary Conditions:

$$\Sigma \Delta p = p_1 - p_{11} - 300 = 0 \quad (85)$$

$$\Sigma \Delta \sigma = \sigma_1 - \sigma_{11} = 0 \quad (86)$$

The voidages of moving bed are determined as described in Section 3.2 and the results are given in Table 1.

The voidage for raiser is a function of solids flow rate and determined by experiments:

$$\epsilon_6 = 0.999 - 5.8 \frac{\sqrt{D/g} \ u_s}{A \ L} \quad (87)$$

The drag force for the standpipe is calculated from Eq. 57 as suggested by Mountziaris and Jackson (1991) which is a combination of Ergun Equation and Richardson-Zaki equations) The drag force for the L-Valve is calculated using the following equation which was derived using our own experiemntal results:

$$FD = 3.21 \rho_s (1-\epsilon) g L \left(\frac{dp}{D} \right)^{0.284} U_s^{0.16145} \quad (88)$$

For the raiser a modified version of the equation suggested by Knowlton is used; this equation is given by

$$FD = 2 L C \rho_s U_s^2 \quad (89)$$

where C is a sort of friction coefficient given by

$$C = 5.8 \sqrt{g D} / U_s \quad (90)$$

The inertial forces are calculated via Eq. (46) The gas phase friction coefficient is calculated using Eq. (49) together with the friction coefficient function of Eq. (59).

As discussed above, the solid wall friction force is an important factor for the present applications. Since there is very little information available in the literature concerning the solids -wall friction factor, we calculate this force directly from empirical equations. These equations, of course, can be rearranged to derive relations in terms of friction coefficients.

The solids-wall friction force, FWS, for the hopper and standpipe sections are calculated using a relation suggested by Jones and Leung (1971) which is:

$$FWS = \frac{4 L}{D} \bar{\sigma} \tan(\delta) \frac{1-\sin(\phi)}{1+\sin(\phi)} \quad (91)$$

$$\bar{\sigma}_2 = \sigma_1 + 0.5 \Delta \sigma_2$$

$$\delta_h = 56^\circ, \quad \delta_{st} = 65^\circ$$

$$\phi = 34^\circ$$

For the L-valve we derived our own empirical equation which reads as

$$FWS = 1.6 L \left(\frac{dp}{D} \right)^{0.31} \left(\frac{G_s}{D} \right)^{0.8} / U_g^{1.8} \quad (92)$$

where $G_s = U_s \rho_s (1 - \epsilon)$.

For the raiser section the following relation suggested by Geldart and Ling(..?) is used.

$$FWS = K \left(\frac{G_s}{D} \right)^m \left(\frac{\mu_g}{\rho_g} \right)^{0.4} \frac{L_v}{u_g} \quad (93)$$

$$\text{when } G_s/D < 4.7 \times 10^4 \quad K = 106, \quad m = 0.83$$

$$\text{when } G_s/D > 4.7 \times 10^4 \quad K = 0.83, \quad m = 1.28$$

5.0 Conclusions and Suggestions:

A mathematical model was developed for calculating the solids flow rate and the pressure drops over various sections of the return loop of a circulating fluidized bed. The present model is applicable to isothermal (cold) flow situations only.

The model parameters are determined using our own experimental result as well as data from the literature. The comparison of the calculations with our own experimental data indicates very good agreement for a wide range of flow conditions and solids material properties. This good agreement is mostly due to the fact that the model has been calibrated using our experimental data. The testing of the model against other independent data sets is necessary. However, such data is very scarce in the literature.

The present study also leads to the following conclusions:

(1) The Mountziaris and Jackson equation seems to give good results for calculation of the drag force in case of moving beds.

(2) The pressure drop and voidage in the raiser are linear functions of the solids mass flow rate.

(3) The pressure drops in each section are mainly depended on the drag force.

(4) The solids flow rate through the loop is mainly determined by the drag force, the gravitational force acting on the solids, and the friction force between the solids and wall.

Suggestions for future research:

(1) The model should be verified against independent data sets

(2) The friction force between the solids and wall, and drag force between solids and gas phases should be studied in more detail.

(3) More advanced experimental techniques such as particle imaging, radio active particle tracing, laser-doppler velocity meters, and Jenick shear cell technique should be used to characterize the solids motion in the L-Valve.

(4) The present model should be extended to include the influence of heat and mass transfer in the actual cases with chemical reaction.

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Figure 1. Major Components of a Lab-Scale Circulating Fluidized-Bed (after Huang, 1990)

Figure 2. The three most common types of non-mechanical valves (after Geldart, 1986)

Figure 3. Friction Force Between Solids and Wall VS. Solid Velocity
(Comparison of Geldart Equ. with Dogin-Lebedev, and Yang Eqs.)

Figure 4. L-VALVE $\Delta P/L$ VERSUS SOLIDS FLOW RATE FOR DIFFERENT AERATION TAP LOCATIONS
(After Knowlton and Hirsan, 1980)

Figure 5. L-VALVE $\Delta P/L$ VERSUS SOLIDS FLOW RATE FOR VARIOUS L-VALVE DIAMETERS
 (After Knowlton and Hirsan, 1980)

Fig. 6 Relation of Friction Coefficient of L-valve, C_f , and Particle Reynolds Number, Re .

Fig. 7 The Linear Relation Between $\ln(\text{CF})$ and $\ln(\text{RE})$

Figure 8. Comparison of the function of the ratio between solid velocity and gas velocity with the Knowlton's experimental data

Fig. 10. Experimental System for
Determining the Voidage of
Moving Bed.

Figure 11. Comparison of experimental data with model calculations: solids flow rate v.s. L-valve aeration rate

Figure 12. Comparison of experimental data with model calculations: pressure drop v.s. solids flow rate

Figure 13. Comparison of experimental data with model calculations: L-valve pressure drop v.s. solids flow rate

APPENDIX A:
GOVERNING EQUATIONS FOR TWO-PHASE FLOW

Governing Equations for Two-phase Flow

The equations of motion for multiphase flows using continuum approach have been fairly well established (Anderson and Jackson, 1967; Ahmadi and Ma, 1987; Drew, 1983; Green and Naghdi (1969); O'Brien and Syamlal, 1990.

There remains however, major difficulties with regards to the constitutive equations for stresses and/or interaction terms. The development presented here follows closely that of Drew, 1983 and McTigue, 1983. Both the fluid and the dispersed particulate phases are assumed to behave as continuous media. The fraction of the volume occupied by each of the phases is denoted by ϵ_k where k describes the two phases; $k=1$ is for fluid phase, $k=2$ is for particulate phase. By definition, it follows that

$$\sum_{k=1}^2 \epsilon_k = 1 \quad (1)$$

The bulk densities of the two phases are $\epsilon_k \rho_k$, $k=1, 2$, where ρ_1 is the actual fluid density, and ρ_2 is the solids density, both of which are assumed constant for incompressible fluid and solid. We consider only isothermal flows without reactions so that no interphase mass transfer occurs between the two phases. Hence, the conservation of mass equations are analogous to the single-phase continuity equation are written as

$$\frac{\partial}{\partial t} (\epsilon_k \rho_k) + \nabla \cdot (\epsilon_k \rho_k \mathbf{V}_k) = 0 \quad (2)$$

where $\mathbf{V}_k$ is the velocity(vector) of the k th phase. The Momentum equations

are also similar to the single phase momentum equations(i.e. Navier-Stokes equations), except for the added interphase momentum transfer terms. The fluid-phase momentum equation is written as

$$\frac{\partial}{\partial t}(\epsilon_1 \rho_1 \mathbf{V}_1) + \nabla(\epsilon_1 \rho_1 \mathbf{V}_1 \mathbf{V}_1) = -\nabla(\epsilon_1 p_1) + \nabla \cdot (\alpha_1 \tau_1) + \epsilon_1 \rho_1 \mathbf{g} - \mathbf{F}_I \quad (3)$$

and the particulate-phase momentum equations are written as

$$\frac{\partial}{\partial t}(\epsilon_2 \rho_2 \mathbf{V}_2) + \nabla(\epsilon_2 \rho_2 \mathbf{V}_2 \mathbf{V}_2) = -\nabla(\epsilon_2 p_2) + \nabla \cdot (\alpha_2 \tau_2) + \epsilon_2 \rho_2 \mathbf{g} + \mathbf{F}_I \quad (4)$$

Here τ is the extra stress (or shear-stress) tensor (i.e it does not include the normal stresses), $\mathbf{g}$ is the gravitational vector, p_1 is the fluid pressure, p_2 is a solids pressure, and $\mathbf{F}_I$ includes the interphase momentum transfer (interaction) terms.

It should be pointed out that sometimes the divergence terms containing the fluid or the solids stress tensors are written without multiplying them by the void fraction (see e.g. Homsy, 1983). This seems to be only a matter of definition. We prefer the above form because when the void fraction is zero both equations automatically reduce to the Navier Stokes equations for a single phase.

An equation for the mixture can be obtained by adding Eqs. 3 and 4 and making use of the continuity equations. Denoting the mixture density and the baricentric mixture velocity, by ρ and $\mathbf{V}$, and the effective pressure and effective extra stresses by p , and τ , respectively, the momentum equations

for the mixture can be written as

$$\frac{\partial}{\partial t}(\rho \mathbf{V}) + \nabla(\rho \mathbf{V} \mathbf{V}) = -\nabla(p) + \nabla \cdot (\boldsymbol{\tau}) + \rho \mathbf{g} \quad (5)$$

Here the following definitions are adopted:

$$\rho = \epsilon_1 \rho_1 + \epsilon_2 \rho_2 \quad (6)$$

$$\rho \mathbf{V} = \epsilon_1 \rho_1 \mathbf{V}_1 + \epsilon_2 \rho_2 \mathbf{V}_2 \quad (7)$$

$$p = \epsilon_1 p_1 + \epsilon_2 p_2 \quad (8)$$

$$\boldsymbol{\tau} = \epsilon_1 \boldsymbol{\tau}_1 + \epsilon_2 \boldsymbol{\tau}_2 + \epsilon_1 \rho_1 \mathbf{V}_1 (\mathbf{V} - \mathbf{V}_1) + \epsilon_2 \rho_2 \mathbf{V}_2 (\mathbf{V} - \mathbf{V}_2) \quad (9)$$

In Eq. 9 the last two terms on the right hand side are the momentum exchange due to the relative motion of the two components of the mixture with respect to the bulk mean motion. These can be interpreted also as the inertial coupling resulting from the relative motion of each constituent of the mixture relative to the mean motion, i.e. transport of momentum by diffusion. Some investigators (e.g. Truesdell and Toupin, 1960; Nunziato, 1983; McTigue, 1983)) prefer to include these divergence terms as part of the stress tensor, some others (e.g. Green and Naghdi, 1969; Chao, et.al. 1978) prefer to include it as part of the interaction term. Here we follow the first school of thought. The inertial coupling terms can be simplified further by using the relations

$$\begin{aligned} \epsilon_1 \rho_1 \mathbf{V}_1 (\mathbf{V} - \mathbf{V}_1) + \epsilon_2 \rho_2 \mathbf{V}_2 (\mathbf{V} - \mathbf{V}_2) &= \epsilon_1 \rho_1 (\mathbf{V} - \mathbf{V}_1)^2 + \epsilon_2 \rho_2 (\mathbf{V} - \mathbf{V}_2)^2 \\ &= \epsilon_1 \rho_1 \epsilon_2 \rho_2 (\mathbf{V}_2 - \mathbf{V}_1)^2 / \rho \quad (10) \\ &= \hat{\rho} V_{\text{slip}}^2 \end{aligned}$$

where $\hat{\rho}$ is the harmonic mean mixture density defined by

$$\frac{1}{\hat{\rho}} = \frac{1}{\epsilon_1 \rho_1} + \frac{1}{\epsilon_2 \rho_2} \quad (11)$$

The stresses and the interaction forces are usually assumed to be functions of properly invariant independent variables (see e.g. Drew , 1983; McTigue, 1983; Nunziato, 1983). Here we follow closely the approach suggested by Homsy(1983) and formulate a constitutive equation for $\mathbf{F}_I$. For the case of gas-solid flows with relatively large solids to fluid density ratio, and for brevity we limit the present approach with only the two seemingly dominant forces, namely the buoyancy force and the drag force. Other forces such as the added-mass force, Shaffman Lift force, Magnus force, etc. should also be considered whenever they are found to play a significant role in flow dynamics.

The buoyancy force due to gradient of the void fraction is expressed as

$$\mathbf{F}_b = - p_1 \nabla \epsilon_1 \quad (12)$$

The fluid-particle drag forces are expressed as (Symlal and O'Brien, 19..)

$$\mathbf{F}_D = \frac{3}{4} C_D \frac{\epsilon_2 \rho_1 \epsilon_1}{d_p} |\mathbf{V}_1 - \mathbf{V}_2| (\mathbf{V}_1 - \mathbf{V}_2) \quad (13)$$

One-dimensional Transient Equations:

If we neglect the variations of flow parameter with radial distance, i.e over the cross-section of the conveying tube, the above eqations can be drastically simplified. With this assumption, and futher assuming that the solid phase can be represented by one size and density, the above equations can be integrated to yield:

Continuity for gas phase:

$$\frac{\partial}{\partial t} (A \epsilon_1 \rho_1) + \frac{\partial}{\partial x} (A \epsilon_1 \rho_1 U_1) = 0. \quad (14)$$

Continuity for solids phase:

$$\frac{\partial}{\partial t} (A \epsilon_2 \rho_2) + \frac{\partial}{\partial x} (A \epsilon_2 \rho_2 U_2) = 0. \quad (15)$$

Momentum for the gas phase:

$$\begin{aligned} \frac{\partial}{\partial t} (A \epsilon_1 \rho_1 U_1) + \frac{\partial}{\partial x} (A \epsilon_1 \rho_1 U_1^2) &= \frac{\partial}{\partial x} (\epsilon_1 p_1 A) + A p_1 \frac{\partial \epsilon_1}{\partial x} - (A/R_h) \epsilon_1 \tau_{1w} \\ &\quad - A F_D + A \epsilon_1 \rho_1 g \sin \theta \end{aligned} \quad (16)$$

Momentum for the solids phase:

$$\begin{aligned} \frac{\partial}{\partial t} (A \epsilon_2 \rho_2 U_2) + \frac{\partial}{\partial x} (A \epsilon_2 \rho_2 U_2^2) &= \frac{\partial}{\partial x} (\epsilon_2 p_2 A) - A p_1 \frac{\partial \epsilon_1}{\partial x} - (A/R_h) \epsilon_2 \tau_{2w} \\ &\quad + A F_D + A \epsilon_2 \rho_2 g \sin \theta \end{aligned} \quad (17)$$

Momentum for the mixture:

$$\frac{\partial}{\partial t} (A \rho U) + \frac{\partial}{\partial x} (A \rho U^2) = \frac{\partial}{\partial x} (p A) - (A/R_h) \tau_w + A \rho g \sin \theta \quad (18)$$

where subscripts "1" and "2" denote the fluid and the solid phase respectively, $\epsilon_2 = (1 - \epsilon_1) = (1 - \epsilon)$ is the solids fraction by volume, ϵ is the void fraction, A is the cross-sectional area, R_h is the hydraulic radius, and θ is the angle between the conveying tube and the vertical.

The wall shear stresses, τ_{1w} , and , τ_{2w} , can be calculated via appropriate friction coefficients. Using the usual convention we define the following friction coefficients:

$$\tau_{1w} = \frac{1}{2} \rho_1 U_1^2 f_1 \quad (19)$$

$$\tau_{2w} = \frac{1}{2} \rho_2 U_2^2 f_2 \quad (20)$$

$$\tau_w = \frac{1}{2} \rho U^2 f \quad (21)$$

Since

$$\tau_w = \epsilon_1 \tau_{1w} + \epsilon_2 \tau_{2w} + \hat{\rho} U_{slip}^2 \quad (22)$$

the three friction coefficients defined above are related by

$$\rho U^2 f = \epsilon_1 \rho_1 U_1^2 f_1 + \epsilon_2 \rho_2 U_2^2 f_2 + \hat{\rho} U_{slip}^2 \quad (23)$$

There are five primary unknowns, namely, $\epsilon_1, U_1, U_2, p_1$, and p_2 , but we have only four independent equations namely Eqs. 14 -17. An additional equation is needed to close the system of equations. This is selected as a constitutive equation for the solids-phase pressure p_2 as described below. Provided that the friction coefficients, f_1 and f_2 are calculated from empirical correlations, Eqs.14-17, along with Eq 7 form a closed set of coupled non-linear partial differential equations which can be solved for the four primary unknowns.

Constitutive Equation for Solids Pressure

A constitutive relation for the solids pressure was suggested by Pritchett, et al., (1978) and was later used by Gidaspow (1986). These empirical relations were based on the experimental work of Rietema and Mutsers (1973). In what follows we present these empirical relations on a slightly different form which is appropriate for the one-dimensional model presented above. The extra solids pressure P_2^* (i.e., pressure in addition to the weight of the particles) is related to the modulus of elasticity, G (or particle to particle interaction coefficient) via

$$\frac{dP_2^*}{d\epsilon} = G(\epsilon) \quad (24)$$

Here $P_2^* = P_2 - \rho_2 gZ$, or $\nabla P_2 = \nabla P_2^* - \rho_2^* g$, Z is the vertical distance.

When ϵ approaches to the minimum fluidization velocity the drag force on particles will balance the gravitational force. Hence the definition of ρ^* should be modified such that as $\epsilon \rightarrow \epsilon_{mf}$, $\rho^* \rightarrow \rho_2$. That is, we should probably add an empirical exponential delay function such as

$P_2^* = P_2 - \rho g Z \exp[-B(\epsilon - \epsilon_0)]$ where B is a large number. Another alternative is to calculate P_2 from continuity analogous to the incompressible fluid flow calculations when $\epsilon \ll \epsilon_{mf}$.

The empirical relation suggested for $G(\epsilon)$ by Pritchett, et al., is

$$-G(\epsilon) = \begin{cases} -G_o - A^2(\epsilon - \epsilon_o) & \epsilon \leq \epsilon_o \\ \text{Anti Log}_{10} \left[m(\epsilon - \epsilon_o) + a + b \frac{\sigma}{d_m} \right] & \epsilon > \epsilon_o \end{cases} \quad (25)$$

where $m = -8.87$, $a = 0.209$, $b = 2.30$, d_m = mean particle size, ϵ_o = minimum voidage (≈ 0.4), the maximum solids concentration is $(1 - \epsilon)$, σ is the standard deviation from the mean, and

$$G_o = \text{Anti Log}_{10} \left[a + b \frac{\sigma}{d_m} \right] \quad (26)$$

Eq. (24) together with Eq. 25 can be integrated and the result can be cast into the following convenient form

$$P_2^* - P_2^*(\epsilon_o) = \begin{cases} (\epsilon - \epsilon_o) \left[G_o + \frac{A}{2} (\epsilon - \epsilon_o) \right] & \epsilon \leq \epsilon_o \\ \phi_p \left[\exp(-c(\epsilon - \epsilon_o)) - 1 \right] & \epsilon > \epsilon_o \end{cases} \quad (27)$$

where $C = 0.26$, and ϕ_p is a particle size and/or shape coefficient given by

$$\phi_p = \frac{1}{c} \exp \left(\frac{\sigma}{d} + 0.09 \right) \quad (28)$$

In the above $P_2^*(\epsilon_o)$ is solids pressure at minimum voidage.

Note that the maximum solid pressure is obtained at minimum value of $\epsilon = \epsilon_0$ where $P_2^* = P_2^*(\epsilon_0)$, where $\epsilon \leq \epsilon_0$. Eq. 27 also gives $P_2^* < P_2^*(\epsilon_0)$. For large void fractions, i.e., dilute flow regime, the particle-to-particle interaction should be negligible. This requires $P_2^*(\epsilon) = 0$. Using this condition at the maximum value $\epsilon = 1$ in Eq. 27 we obtain an expression for $P_2^* = 0$ which is

$$P_2^*(\epsilon_0) = -\phi_p \left[\exp(-c(1-\epsilon_0)) - 1 \right] \quad (29)$$

Here it should be noted that the pressure terms on the r.h.s. of Eq. 4 are modeled in a different manner by some other investigators. For example Bouillard and Lyczkowski (1991) uses $[-\epsilon_2 \nabla P_1 - \nabla P_2]$ instead of

$[-\nabla(\epsilon_2 P_2) - P_1 \nabla \epsilon_2]$ then P_2 (the normal component of the solids coulombic stress) is modeled as

$$\nabla P_2 = G(\epsilon) \nabla \epsilon \quad (30)$$

which is the same equation as Eq. (24). Based on the same reference (Pritchett et.al., 1978) they suggest

$$G(\epsilon) = G_0 \exp[-c(\epsilon - \epsilon^*)] \quad (31)$$

where $G_0 = 1.0 P_a$, $c = 600$, and $\epsilon^* = 0.445$. However these constants can change considerably from application to application (see e.g. Gidaspo, 1986). Bouillard et.al., (1991) suggest $c = -20$, $\epsilon^* = 0.6$ for fast bubble simulations in two-dimensional fluidized beds.

Solids-Wall Friction Coefficient

The solids shear stress can be related to the average solids pressure (normal stress) (see e.g., Jones and Leung) (1985) via

$$\tau_{2w} = \mu_{2w} P_2 \frac{(1 - \sin \delta)}{(1 + \sin \delta)} \quad (32)$$

Here μ_{2w} is the static coefficient of friction between the solid particles and wall surfaces and δ is the internal angle of friction; $\mu_{2w} = \tan \delta$ for a cohesionless Coulomb powder, see e.g. Mountziaris and Jackson, (1991). The proportionality coefficient between the shear stress at the wall and the average normal stress is known as the Janssen (1895) coefficient.

Using Eq. 20 together with Eqs. 27, 28, and 29, we obtain an expression for the dynamic solids-wall friction coefficient f_2 as

$$f_2 = \frac{\mu_{2w}}{\frac{1}{2} \rho_2 U_2^2} \left(\frac{1 - \sin \delta}{1 + \sin \delta} \right) P_2(\epsilon) \quad (33)$$

where P_2 is given by Eq. 27. As given by Eq. (31) the solids friction coefficient is a function of minimum voidage ϵ_0 , voidage ϵ , the internal angle of friction, δ , and the particle size as it should be expected.

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APPENDIX B:
Description of The Computer Program

The computer program consists of three sections: (i) main section which includes, input data, output results, and call subroutines (MDLNIN, NLSYST, and FCN); (ii) model section contains the momentum equations, FCN(X,F). Here: X(1) is the gas flow rate through the standpipe, X(2) is the solids flow rate through the loop, F(1) is the sum of pressure drops, and F(2) is the sum of solids normal stresses; (iii) Newton method section includes subroutine MDLNIN for initial data, subroutine NLSYST for solving the FCN(X,F). A flow chart of the computer program is shown in Figure b.1. An a listing is included here for reference.

LISTING OF THE COMPUTER PROGRAM

Fig. b.1 Flow Chart of CFB Loop Model

LISTING OF THE COMPUTER PROGRAM

C234567 ZLOOP.FOR

C THIS PROGRAM IS USED FOR DETERMINING

C THE SOLID AND GAS FLOW RATE THROUGH A LOOP AND COMPARING

C THE EFFECTS OF SOME PARAMETERS CHANGING ON THE LOOP PERFORMS

C X(1):GAS FLOWRATE; X(2):SOLID FLOWRATE

REAL X(2),F(2),DELTA(2),XTOL,FTOL

REAL MS,MG,L1,L2,L22,L3,L4,L5,L6,MS1

INTEGER N,MAXIT,I

DIMENSION DPSDM(4),RPDM(4),DBDM(4),PCDM(4),ECIDM(4),QAMX(4),ANG(4)

COMMON /BLK1/L1,L2,L3,L4,L5,L6,DB1,DB2

COMMON /BLK2/RG,GVIS,USRS

COMMON /BLK3/DP,DP,DPS,ECI,BT,BA,BA1,BAS,BAG,EMF,UMF

COMMON /BLK4/QA,QM,UUG5,FST

COMMON /BLK5/P1,P2,P3,P4,P5,P6,P11,F1,F2,F3,F4,F5,F6,F11

COMMON /BLK6/DF1,DF2,DF3,DF4,DF5,DF6,DP1,DP2,DP3,DP4,DP5,DP6

COMMON /BLK7/US1,UG1,FD1,FWS1,FWG1,FGS1,FGG1,FAS1,FAG1

COMMON /BLK8/US2,UG2,FD2,FWS2,FWG2,FGS2,FGG2,FAS2,FAG2

COMMON /BLK9/US3,UG3,FD3,FWS3,FWG3,FGS3,FGG3,FAS3,FAG3

COMMON /BLK10/US4,UG4,FD4,FWS4,FWG4,FGS4,FGG4,FAS4,FAG4

COMMON /BLK11/US5,UG5,FD5,FWS5,FWG5,FGS5,FGG5,FAS5,FAG5

COMMON /BLK12/US6,UG6,FD6,FWS6,FWG6,FGS6,FGG6,FAS6,FAG6

COMMON /BLK13/EC1,EC2,EC3,EC4,EC5,EC6

COMMON /BLK14/IT,XX,CK,CM,CDDP,CDDF

COMMON /BLK15/DPCR,IJ

EXTERNAL FCN

OPEN(UNIT=16,FILE='OUT6.DAT',STATUS='NEW')

OPEN(UNIT=17,FILE='OUT7.DAT',STATUS='NEW')

OPEN(UNIT=18,FILE='OUT8.DAT',STATUS='NEW')

C THIS INPUT DATA IN UNITS (M,KG,S)

C GEOMETRIC PROPERTIS

L1=0.0

L2=0.204

L3=0.838

L4=0.08

L5=0.305

L6=3.023

DBDM(1)=0.0508

DBDM(2)=0.0762

```

C      DB1=0.0762
      DB2=0.2032
      BAG=18.24
C GAS PROPERTIS
      RG=1.0
      GVIS=2.0D-5
C SOLIDS PROPERTIS
      RP=2.5D3
      DPS=0.116D-3
      ECI=0.38
      BT=34.
C DYNAMIC PROPERTIS
      QM=0.079
      EMF=0.43
      UMF=0.12
C CALCULATION DATA
      N=2
      MAXIT=50
      DELTA(1)=1.0D-4
      DELTA(2)=1.0D-3
      XTOL=1.0D-7
      FTOL=10.
      QAMXM=25.
      DPCR=2.0
      BA1=56.
      BA=65.
      G=9.81
      PI=3.1416

C
      DO 77 IJ=1,2
      DB1=DBDM(IJ)
      WRITE(11,2017)IJ
      WRITE(12,2018)IJ
      AREA1=0.25*PI*DB1**2
      QA=QAMXM*7.87D-6/100.0
      IK=0
30    IK=IK+1
      IF(DP4/249 .LT. DPCR) THEN

```



```

X1=0.0
X2=-1E-4
CALL MDLNIN(FCN,X1,X2,X,F,XTOL,FTOL,MAXIT,I)
F(2)=0.0
ELSE
CALL NLSYST(FCN,N,MAXIT,X,F,DELTA,XTOL,FTOL,I)
END IF
MS1=X(2)
IF(MS1 .LT. 0.0) MS1=0.005
IKK=(IK/10)*10
IF(IK .EQ. IKK) THEN
WRITE((IJ+16),2016)IKK/10,QA/7.87D-6,X(2)/0.126
& , -DP3/(3.28*L3*249),DP4/(3.28*(L4+L5)*249),DP6/(3.28*L6*249)
C   WRITE(6,2019)IK,QA/7.87D-6,X(1)/7.87D-6,X(2)/0.126,F
C   WRITE(16,2004)IJ,IK/10,QA/7.87D-6,X(1)/7.87D-6,X(2)/0.126
C   & ,F(1)/249,F(2)/249,FAG2/249,FAS2/249,FAG4/249,FAS4/249,EC6
C   WRITE(16,2005)DP2/249,DF2/249,FD2/249,FWG2/249,-FGG2/249
C   & ,FWS2/249,-FGS2/249
C   WRITE(16,2005)DP3/249,DF3/249,FD3/249,FWG3/249,-FGG2/249
C   & ,FWS3/249,-FGS3/249
C   WRITE(16,2005)DP4/249,DF4/249,FD4/249,FWG4/249,-FGG4/249
C   & ,FWS4/249,-FGS4/249
C   WRITE(16,2005)DP6/249,DF6/249,FD6/249,FWG6/249,FGG6/249
C   & ,FWS6/249,FGS6/249
C   WRITE(8,2014)UG2,US2,UG3,US3,UG4,US4,UG6,US6
END IF
IF(IK .EQ. 100) GO TO 77
QA=QA+QAMXM*7.87D-6/100.0
GO TO 30
77  CONTINUE
C
2020  FORMAT(1X,I2,1X,5E11.3,2E12.4)
2019  FORMAT(1X,I4,6E12.4)
2022  FORMAT(1X,4E12.4)
2016  FORMAT(I3,1X,7E10.3)
2017  FORMAT(/I3,/3X,'QA',6X,'X(1)',6X,'X(2)',6X,5X,'F(1)'
& ,5X,'F(2)')
2018  FORMAT(/I3,/3X,'-DP3',6X,'DPL',6X,'DP45',6X,'DPST',5X,'DP6'

```

```

& ,5X,'DPR' )
2014 FORMAT(8E10.3)
2004 FORMAT(//' CASE: ' I3,3X,' POINT: ' I3,/' QA=' D11.4,1X,' QC=' ,D11.4
& ,1X,' MS=' D11.4,1X,' DP=' ,D11.4,1X,' DF=' D11.4
& ,/' FAG2=' ,D11.4,1X,' FAS2=' ,D11.4,1X,' FAG4=' ,D11.4,1X
& , ' FAS4=' ,D11.4,1X,' EC6=' F7.4
& ,/4X,' DP2-6' ,6X,' DF2-6' ,6X,' FD2-6' ,6X,' FWG2-6' ,5X,' FGG2-6'
& ,5X,' FWS2-6' ,4X,' FGS2-6' )
2005 FORMAT(/7E11.4)
2031 FORMAT(1X,I3,1X,7E12.4)
STOP
END

```

C

```

SUBROUTINE FCN(X,F)
REAL X(2),F(2),MS,MG,L1,L2,L22,L3,L4,L5,L6
COMMON /BLK1/L1,L2,L3,L4,L5,L6,DB1,DB2
COMMON /BLK2/RG,GVIS
COMMON /BLK3/RP,DP,DPS,EC1,BT,BA,BA1,BAS,BAG,EMF,UMF
COMMON /BLK4/QA,QM,UUG5,FST
COMMON /BLK5/P1,P2,P3,P4,P5,P6,P11,F1,F2,F3,F4,F5,F6,F11
COMMON /BLK6/DF1,DF2,DF3,DF4,DF5,DF6,DP1,DP2,DP3,DP4,DP5,DP6
COMMON /BLK7/US1,UG1,FD1,FWS1,FWG1,FGS1,FGG1,FAS1,FAG1
COMMON /BLK8/US2,UG2,FD2,FWS2,FWG2,FGS2,FGG2,FAS2,FAG2
COMMON /BLK9/US3,UG3,FD3,FWS3,FWG3,FGS3,FGG3,FAS3,FAG3
COMMON /BLK10/US4,UG4,FD4,FWS4,FWG4,FGS4,FGG4,FAS4,FAG4
COMMON /BLK11/US5,UG5,FD5,FWS5,FWG5,FGS5,FGG5,FAS5,FAG5
COMMON /BLK12/US6,UG6,FD6,FWS6,FWG6,FGS6,FGG6,FAS6,FAG6
COMMON /BLK13/EC1,EC2,EC3,EC4,EC5,EC6
COMMON /BLK14/IT,XX,CK,CM,CDDP,CDDF
COMMON /BLK15/DPCR,IJ

```

C-----

```

X(2)=ABS(X(2))
PI=3.1416
BT1=PI*BT/180.
BA11=PI*BA/180.
BA12=PI*BA1/180.
BAG1=PI*BAG/180
BD=TAN(BA11)*(1.-SIN(BT1))/(1.+SIN(BT1))

```

```

BD1=TAN(BA12)*(1.-SIN(BT1))/(1.+SIN(BT1))
DB21=2.*L2*TAN(BAG1)+DB1
DB22=(DB1+DB21)*0.5
L22=L2/COS(BAG1)

```

C

```

G=9.81
Q4=X(1)+QA
Q6=Q4+QM
AREA1=PI*DB1**2.0/4.0
AREA2=PI*DB2**2.0/4.0
AREA21=PI*DB21*2.0/4.0
AREA22=PI*DB22*2.0/4.0
EC2=ECI
EC3=ECI
EC4=ECI
USS6=X(2)/(AREA2*RP)
EC6=0.999-5.8*SQRT(DB2/G)*USS6/(AREA2*L6)
US2=X(2)/(AREA22*(1-EC2)*RP)
UG2=X(1)/(AREA22*EC2)
US3=X(2)/(AREA1*(1-EC3)*RP)
UG3=X(1)/(AREA1*EC3)
US4=X(2)/(AREA1*(1-EC4)*RP)
UG4=Q4/(AREA1*EC4)
US6=USS6/(1-EC6)
UG6=Q6/(AREA2*EC6)
F2=0.0
P2=0.0

```

C-----

C L2

```

FD2=FUNFD(US2,UG2,EC2,DB22)*L2
FWG2=FUNFWG(UG2,DB22,EC2)*L22
FGS2=(1-EC2)*RP*G*L2
FGG2=EC2*RG*G*L2
FAG2=X(1)*RG*(UG3-UG1)
FAS2=X(2)*(US3-US1)
DP2=FD2-FGG2+FWG2+FAG2
BDD2=BD1*L22/DB22
DF2=(-FD2-FGS2+FAS2-4.*BDD2*F2)/(1.+2.*BDD2)

```

$P3 = P2 - DP2$
 $F3 = F2 - DF2$
 $FWS2 = -4 \cdot BDD2 \cdot (F2 + 0.5 \cdot DF2)$

C L3

$FD3 = FUNFD(US3, UG3, EC3, DB1) \cdot L3$
 $FWG3 = FUNFWG(UG3, DB1, EC3) \cdot L3$
 $FGS3 = (1 - EC3) \cdot RP \cdot G \cdot L3$
 $FGG3 = EC3 \cdot RG \cdot G \cdot L3$
 $DP3 = FD3 + FWG3 - FGG3$
 $BDD3 = BD \cdot L3 / DB1$
 $DF3 = (-FD3 - FGS3 - 4 \cdot BDD3 \cdot F3) / (1 + 2 \cdot BDD3)$
 $P4 = P3 - DP3$
 $F4 = F3 - DF3$
 $FWS3 = -4 \cdot BDD3 \cdot (F3 + 0.5 \cdot DF3)$

C L4 (L4+L5)

$FD4 = FDLR(US4, DB1, EC4, 4) \cdot (L4 + L5)$
 $FWG4 = FUNFWG(UG4, DB1, EC4) \cdot (L4 + L5)$
 $FWS4 = FUNFWS(UG4, US4, DB1, EC4, 4) \cdot (L4 + L5)$
 $FGS4 = (1 - EC4) \cdot RP \cdot G \cdot L4$
 $FGG4 = EC4 \cdot RG \cdot G \cdot L4$
 $FAG4 = Q4 \cdot RG \cdot (US6 - US3)$
 $FAS4 = X(2) \cdot (US6 - US3)$
 $DP4 = FD4 - FGG4 + FWG4 + FAG4$
 $DF4 = -FD4 - FGS4 + FWS4 + FAS4$
 $P5 = P4 - DP4$
 $F5 = F4 - DF4$

C L6

$FD6 = FDLR(US6, DB2, EC6, 6) \cdot L6$
 $FWG6 = FUNFWG(UG6, DB2, EC6) \cdot L6$
 $FWS6 = FUNFWS(UG6, US6, DB2, EC6, 6) \cdot L6$
 $FGS6 = (1 - EC6) \cdot RP \cdot G \cdot L6$
 $FGG6 = EC6 \cdot RG \cdot G \cdot L6$
 $DP6 = FD6 + FWG6 - FGG6$
 $DF6 = -FD6 + FWS6 + FGS6$
 $P11 = P5 - DP6$
 $F11 = F5 - DF6$

C

$F(1) = P2 - P11 - 300.$

```

      F(2)=F2-F11
      RETURN
      END

C
C THE MOUNTZIARIS AND JACKSON EQUATION
C
      FUNCTION FUNFD(S,G,EC,D)
      DOUBLE PRECISION W
      COMMON /BLK1/L1,L2,L3,L4,L5,L6,DB1,DB2
      COMMON /BLK2/RG,GVIS
      COMMON /BLK3/RP,DP,DPS,ECI,BT,BA,BA1,BAS,BAG,EMF,UMF
      COMMON /BLK14/IT,XX
      UR=G-S
      AFD=150*GVIS*(1-EC)**2.*UR/(DPS*EC)**2.
      BFD=1.75*RG*(1-EC)*UR*ABS(UR)/(DPS*EC)
      ENGUN=AFD+BFD
      CW=1.0
      IF((EC-ECI) .LT. 1.E-2) THEN
      W=0.0
      GO TO 10
      ENDIF
      UT=DPS**2.*(RP-RG)*9.81/(18.*GVIS)
      RET=UT*DPS*RG/GVIS
      CN=4.65+19.5*DPS/D
      IF (RET.GE.0.2) CN=(4.35+17.5*DPS/D)/RET**0.03
      IF (RET.GE.1.0) CN=(4.45+18.*DPS/D)/RET**0.1
      IF (RET.GE.200.) CN=4.45/RET**0.1
      IF (RET.GE.500.) CN=2.39
      RICH=RG*(1-EC)*9.81*UR/(UT*EC**(CN-1))
      W=1./EXP(CW*(1-EC)/(EC-ECI))
10    FD=(1.-W)*ENGUN+W*RICH
      FUNFD=FD
      RETURN
      END

C
C KNOWLTON MODIFIED THE KONNO AND SAITO EQUATION
C
      FUNCTION FDLR(S,D,E,J)

```

```

COMMON /BLK2/RG,GVIS
COMMON /BLK3/RP,DP,DPS,ECI,BT,BA,BA1,BAS,BAG,EMF,UMF
G=9.81
IF(S .EQ. 0.0) S=S+0.0001
IF(J .EQ. 6) THEN
SU=S*(1.-E)
CF=5.8*SQRT(G*D)/SU
FDLR=2.*CF*RP*SU*ABS(SU)
ELSE
FDLR=3.21*RP*(1-E)*G*(DPS/D)**0.284*ABS(S)**0.16145
END IF
100 RETURN
END

C
C THE FANNIG EQUATION
C
FUNCTION FUNFWG(G,D,E)
COMMON /BLK2/RG,GVIS
COMMON /BLK3/RP,DP,DPS,ECI,BT,BA,BA1,BAS,BAG,EMF,UMF
IF(G .EQ. 0.0) THEN
FUNFWG=0.0
RETURN
ENDIF
UUG=G*E
REWG=RG*D*UUG/GVIS
CFWG=0.079/ABS(REWG)**0.25
FUNFWG=2.*CFWG*RG*UUG*ABS(UUG)
RETURN
END

C
C GELDART AND LING EQUATION
C
FUNCTION FUNFWS(G,S,D,E,I)
COMMON /BLK2/RG,GVIS,USRS
COMMON /BLK3/RP,DP,DPS,ECI,BT,BA,BA1,BAS,BAG,EMF,UMF
COMMON /BLK14/IT,XX,CK,CM
GS=S*RP*(1-E)
USRS=GS/D

```

```

DO 20 IT=1,NLIM
X(1)=X2-F2*(X2-X1)/(F2-F1)
CALL FCN(X,F)
XERR=ABS(X1-X2)/2.0
IF (OUTPUT) THEN
C   WRITE(9,199) IT,X(1),F(1)
END IF

C
IF (XERR .LE. XTOL) THEN
I=1
C   WRITE(9,202) IT,X(1),F(1)
RETURN
END IF

C
IF (ABS(F(1)) .LE. FTOL) THEN
I=2
C   WRITE(9,203) IT,X(1),F(1)
RETURN
END IF

C
IF(F(1)*F1 .GT. 0.0) THEN
X1=X(1)
F1=F(1)
IF(F(1)*FSAVE .GT. 0.0) F2=F2/2.
FSAVE=F(1)
ELSE
X2=X(1)
F2=F(1)
IF(F(1)*FSAVE .GT. 0.0) F1=F1/2.
FSAVE=F(1)
END IF
20 CONTINUE

C
I=-1
C   WRITE(9,200) NLIM,X(1),F(1)
RETURN

C234567
199 FORMAT(' AT ITERATION ',I3,3X,'X(1)=' ,D12.4,3X,'F(1)=' ,D12.4)

```

```

      IF (ITEST .EQ. 0) THEN
        I=1
        IF (I .EQ. 1) WRITE(9,1002) IT, X
        RETURN
      END IF
100 CONTINUE
C
      I=-1
      RETURN
C
1000 FORMAT(/' AFTER ITERATION NUMBER',I3,' X AND F VALUES ARE'
+ //10E13.5)
1001 FORMAT(/ 10E13.5)
1002 FORMAT(/' AFTER ITERATION NUMBER',I3,' X VALUES (MEETING',
+ ' XTOL) ARE '//10E13.5)
1003 FORMAT(/' CANNOT SOLVE SYSTEM. MATRIX NEARLY SINGULAR.')
1004 FORMAT(/' NUMBER OF EQUATIONS PASSED TO NLSYST IS INVALID.',
+ ' MUST BE 1<N<11. VALUE WAS ',I3)
C
      END
C
C GAUSSIAN ELIMINATION METHOD
C
      SUBROUTINE ELIM(AB,N,NP,NDIM)
      REAL AB(NDIM,NP)
      INTEGER N,NP,NDIM
      REAL SAVE,RATIO,VALUE
      INTEGER NM1,IPVT,IP1,J,NVBL,L,KCOL,JCOL,JROW
C
      NM1=N-1
      DO 35 I=1,NM1
        IPVT=I
        IP1=I+1
        DO 10 J=IP1,N
          IF (ABS(AB(IPVT,I)) .LT. ABS(AB(J,I))) IPVT=J
10      CONTINUE
          IF (ABS(AB(IPVT,I)) .LT. 1.0E-6) THEN
C            WRITE(9,100)

```



```

        RETURN
    END IF
    IF (IPVT .NE. I) THEN
        DO 20 JCOL=1,NP
            SAVE=AB(I,JCOL)
            AB(I,JCOL)=AB(IPVT,JCOL)
            AB(IPVT,JCOL)=SAVE
20        CONTINUE
    END IF

C
    DO 32 JROW=IP1,N
        IF (AB(JROW,I) .EQ. 0) GO TO 32
        RATIO=AB(JROW,I)/AB(I,I)
        AB(JROW,I)=RATIO
        DO 30 KCOL=IP1,NP
            AB(JROW,KCOL)=AB(JROW,KCOL)-RATIO*AB(I,KCOL)
30        CONTINUE
32    CONTINUE
35    CONTINUE
C
    IF(ABS(AB(N,N)) .LT. 1.0E-6) THEN
C        WRITE(9,100)
        RETURN
    END IF
C
    NP1=N+1
    DO 50 KCOL=NP1,NP
        AB(N,KCOL)=AB(N,KCOL)/AB(N,N)
        DO 45 J=2,N
            NVBL=NP1-J
            L=NVBL+1
            VALUE=AB(NVBL,KCOL)
            DO 40 K=L,N
                VALUE=VALUE-AB(NVBL,K)*AB(K,KCOL)
40            CONTINUE
            AB(NVBL,KCOL)=VALUE/AB(NVBL,NVBL)
45        CONTINUE
50    CONTINUE

```

```
        RETURN
C
100  FORMAT(/'SOLUTION NOT FEASIBLE. A NEAR ZERO PIVOT',
+        'WAS ENCOUNTERED.')
```

END

C

ABSTRACT

It is reported on development, testing and verification of engineering models for predicting the pressure drop, the solids flow rate, and the downcoming gas flow rate through an L-valve for a given aeration flow rate. The models are, in particular, applicable for studying the one-dimensional gas-solids motion through the return loop of a circulating fluidized bed. A literature review is presented in a comparative manner. One-dimensional transient equations governing the dense two-phase flows are derived. Those equations are then used to deduce relevant characteristic dimensionless parameters. Experimental data from literature have been analyzed and empirical correlations are suggested. A calculation procedure is proposed for predicting relevant gas and solid flow parameters. The model is based on integrated conservation equations for mass and momentum for both phases. Some experiments of our own have been performed and the data have been analyzed. The model is calibrated against experimental data.

END

**DATE
FILMED**

12/02/93