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**Development of Liquid-Film Tracking Models for Analysis
of AP-600 Passive Containment Cooling System***

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The AP-600, an advanced pressurized water reactor, utilizes a passive containment cooling system (PCCS) to remove heat released inside the containment vessel following postulated design-base accidents such as a main-steam-line break or loss-of-coolant accident. To ensure the performance of the PCCS, two liquid-film tracking models for time-dependent flows (a simplified model and a comprehensive model) have been developed and implemented in the COMMIX code. The models compute the transient liquid-film thickness, its mean velocity, and its temperature on both sides of the steel containment vessel. The governing equations for the models are boundary-layer-type equations based on first principles. The difference between the simplified and comprehensive models is that the inertia terms in the momentum equation are neglected in the simplified model, whereas they are retained in the comprehensive model. Both models have been used to analyze AP-600 PCCS small-scale integral test data, and agreement was good. The two models gave essentially the same result. Computational results for a postulated containment system are presented and discussed.

1. Introduction

The AP-600, an advanced pressurized water reactor, utilizes a passive containment cooling system (PCCS) to remove heat released inside the containment vessel following postulated design-base accidents (DBAs) such as a main-steam-line break or loss-of-coolant accident. A schematic sketch of AP-600 PCCS is shown in Fig.1. The containment vessel consists of a vertical cylindrical shell and is capped at both top and bottom by a dome that is also a body of revolution with a meridian cross section in the shape of a semiellipse with a horizontal major axis. During a DBA, heat released to the interior of the steel containment vessel is removed by evaporation of a continuously flowing thin liquid film on the outside surface of the vessel, thus lowering the temperature of the steel vessel wall so that steam condenses on its inside surface. The external liquid film is formed by flooding water at top of the ellipsoidal dome. Evaporation of the falling liquid film is enhanced by buoyancy-driven flows of moist air in an annular space outside the steel containment vessel.

To ensure PCCS performance, it is necessary to predict both the evaporating film on the outside surface of the vessel and the condensate film on its inside. To this end, two liquid-film tracking models for time-dependent flows (a simplified model and a comprehensive model) have been developed and implemented in the COMMIX code.¹⁻⁵ COMMIX is a general-purpose, time-dependent, multidimensional computer code for thermal hydraulic analysis of single- or multicomponent engineering systems. It solves a system of conservation equations of continuities for up to six species, mixture momentum, and mixture energy, and a k - ϵ two-equation turbulent model. A unique feature of the COMMIX code is its porous-media formulation, which represents the first unified approach to thermal-hydraulic analysis. The tracking models compute the liquid-film thickness, its mean velocity, and its temperature on both sides of the steel containment vessel.

In Sec. 2 below, a complete description of the two liquid-film tracking models is presented. The calculational results and comparisons of the two models are presented in Sec. 3. A summary of the conclusions is given in Sec. 4.

2. Liquid-Film Tracking Models for Time-Dependent Flows with Unsteady Film Condensation or Evaporation

Because the performance of a reactor containment system following an accident (such as a main-steam-line break or sudden loss-of-coolant) is highly transient, it was decided to develop liquid-film tracking models suitable for time-dependent flows. The models are particularly relevant for the condensate inside the containment vessel. The final formulations of the models were expressed in finite-difference forms so they can be readily implemented in the COMMIX code. In both models, a differential equation describing the time rate of change of the film thickness is

rigorously derived from first principles. This equation relates the time rate of thickness change to the streamwise gradient of the mass flow rate and the condensate mass flux at the film's free surface. In the simplified model, the inertia terms in the momentum equation were ignored to obtain an explicit expression of the velocity field in the film. Arguments were given to show that the simplification could be justified except for brief times following a disturbance. In the comprehensive model, the inertia terms were retained in the momentum equation, and a parabolic velocity distribution was assumed across the film thickness. In both models, linear temperature distributions across the film thickness were assumed in the energy equation.

2.1 Governing Equations, Boundary Conditions, and Liquid/Vapor Interfacial Conditions

Experimental data from AP-600 PCCS small-scale tests indicate that the condensate flows at Reynolds numbers of 20 to 700, suggesting that the condensate film flow is laminar. This has indeed been experimentally observed. The condensate film thickness is ≈ 0.1 mm. The external evaporating liquid film flows at a Reynolds number of 570 to 900. At 300 °K, the Prandtl number of water is ≈ 6.5 and hence the flow in the evaporating liquid film is still laminar. Its thickness is greater than that of the condensate film, but it still only a few tenths of a millimeter. Thus, in both cases, the laminar-boundary-layer type of analysis is valid.

The dynamic equations for steady, axisymmetrical, boundary-layer flow over a body of revolution were first given by E. Boltze⁶ and are most conveniently expressed in orthogonal curvilinear coordinates (see Schlichting⁷). As seen in Fig. 2, x is the distance measured along a meridian from the stagnation point S . The coordinate y is normal to x and points inward for analysis of condensate film flow on the inside surface and outward for analysis of evaporating water film on the exterior. The body contour is specified by the radius $r(x)$ of the section perpendicular to the axis of symmetry. The contour has no sharp corners so that d^2r/dx^2 is small everywhere. The velocity components along the x - and y -directions are denoted by u and v , respectively. The gravitational acceleration vector $\mathbf{g}$ is colinear with the axis of rotational symmetry as illustrated, and the angle between $\mathbf{g}$ and x is β . The condensate flows along the undersurface of the dome and then down the vertical wall of the cylinder shell, mainly due to gravity, and is resisted by fluid viscosity. The pressure gradient in the streamwise direction is assumed to be small relative to the gravitational and viscous forces. All properties are considered constant in the derivation. The same can be said for the flow of the evaporating water film on the outside. The condensate flow rate increases with x from the stagnation point S , with a continuous increase in resistance to heat transfer along the vertical cylindrical wall. The water film on the outside evaporates such that its flow rate decreases with x with a corresponding change in heat transfer coefficient.

The time-dependent conservation equations are

Mass:

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial y} = 0. \quad (1)$$

To be consistent with boundary-layer approximation, r is regarded as a function of x only, and hence the second term can be written as $r \frac{\partial v}{\partial y}$.

Momentum:

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \mu \frac{\partial^2 u}{\partial y^2} + \rho g \cos \beta. \quad (2)$$

Energy:

$$\rho C_p \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2}. \quad (3)$$

In Eq. 2, μ and ρ are the viscosity and density of the liquid film, respectively. If desired, the second term on the right-hand side can be replaced by $(\rho - \rho_v)g \cos \beta$, where ρ_v is the vapor density. In Eq. 3, T is the temperature and k and C_p are the thermal conductivity and specific heat of the liquid, respectively.

The boundary conditions are

At the wall, $y = 0$:

$$u = v = 0, \quad T = T^w, \quad (4a,b)$$

At the liquid/vapor interface, $y = \delta(t,x)$:

$$\frac{\partial u}{\partial y} = 0, \quad \rho \frac{\partial \delta}{\partial t} - \rho v + \rho u \frac{\partial \delta}{\partial x} = m_c, \quad (5a,b)$$

$$k \frac{\partial T}{\partial y} = m_c h_g + h_c (T^e - T^i). \quad (5c)$$

In Eq. 4b, T^w is the local temperature on the wall surface. The film thickness δ is a function of time and meridian distance. The condition $\partial u / \partial y = 0$ implies that interfacial shear is negligible. Equation 5b is the interfacial kinetic relation, with m_c being the condensate mass flux. Equation 5c is the interfacial heat balance relation in which h_g is the enthalpy of the condensate steam. The

second term on the right-hand side of Eq. 5c accounts for the convective heat flux resulting from the bulk motion of the vapor/air mixture in the containment vessel. It is expressed in terms of mixed convection heat transfer coefficient h_c and a temperature difference $(T^e - T^i)$; here, T^e is the bulk temperature of the local vapor/air mixture and T^i is the local interfacial temperature, which is taken to be the saturation temperature corresponding to the partial pressure of the vapor at the interface. The initial condition is simply

$$\delta(t = 0, x) = 0. \quad (6)$$

In passing, we note that the well-known Nusselt's film condensation analysis for a flat plate is for steady flow and neglects convective transport in both the momentum equation and the energy equation. Consequently, Eq. 2 is replaced by

$$\mu \frac{\partial^2 u}{\partial y^2} + \rho g \cos \beta = 0, \quad (7)$$

and Eq. 3 is replaced by

$$k \frac{\partial^2 T}{\partial y^2} = 0. \quad (8)$$

Integration of Eq. 7 yields a parabolic profile with maximum occurring at $y = \delta$. Equation 8 leads to a linear temperature distribution across the film.

2.2 Simplified Model for Film Thickness and Mean Flow Velocity

If we integrate Eq. 1 across the film from $y = 0$ to $y = \delta$, there is obtained

$$\frac{1}{r} \frac{\partial r \bar{u} \delta}{\partial x} = u_\delta \frac{\partial \delta}{\partial x} - v_\delta, \quad (9)$$

in which $\bar{u}$ is the mean film flow velocity defined by

$$\bar{u} = \frac{1}{\delta} \int_0^\delta u \, dy, \quad (10)$$

and u_δ and v_δ are the velocity components at the film's free surface, $y = \delta$. Substituting the interfacial kinetic relation (Eq. 5b) into Eq. 9, we have

$$\rho \frac{\partial \delta}{\partial t} + \frac{\rho}{r} \frac{\partial r \bar{u} \delta}{\partial x} = m_c, \quad (11)$$

which is the rigorous result for constant ρ . Equation 11 can also be derived by consideration of mass conservation for an annular control volume bounded by the solid wall, the free surface of the condensate film, and two cross sections normal to the wall, namely cross section at x and at $x + dx$.

The velocity distribution in the film flow is determined from Eq. 2 by neglecting the inertia effect, denoted by $\rho \partial u / \partial t$, and the convective effect, denoted by $\rho [(u \partial u / \partial x) + (v \partial u / \partial y)]$. The conditions under which such simplification can be justified are discussed below.

The inertia effect is characterized by a time scale $\rho \delta^2 / \mu$ and is the time needed to approach a steady-state velocity profile following a disturbance. For a water film whose thickness is 0.1–1.0 mm, the time scale at room temperature is 0.01–1.0 s. Thus, the inertia effect is not significant if the time scale of the imposed disturbance is much larger than 0.01 to 1.0 s.

The convective effect can be estimated by using known results for parallel flow past a flat plate. For a vertical film of thickness δ , a disturbance at its free surface will penetrate a layer of certain thickness at a distance x downstream. From the boundary-layer analysis, this thickness is very roughly $5(\mu x / \rho \bar{u})^{1/2}$, where $\bar{u}$ is the mean flow velocity. If one assumes a parabolic velocity distribution for the film flow, the distance x for the disturbance to reach the entire thickness δ is $0.013 \rho^2 g \delta^4 / \mu^2$. At room temperature for a water film whose thickness is 0.1–1.0 mm, x ranges from 0.13 to 130 mm. The distance will be shorter for inclined layers. Thus, in view of the large dimensions of the containment geometry, neglect of the convective effect would not seriously influence the fluid dynamics of the condensate film flow.

With the foregoing simplifications, Eq. 2 reduces to Eq. 7, which upon integration with the boundary conditions (Eqs. 4a and 5a), yields the parabolic velocity distribution

$$u = \frac{\rho g \cos \beta}{\mu} \left(\delta y - \frac{y^2}{2} \right). \quad (12)$$

Hence, from Eq. 10, mean film flow velocity is

$$\bar{u} = \frac{\rho g \cos \beta}{3\mu} \delta^2, \quad (13)$$

which is time-dependent because δ is time-dependent.

Equation 13 gives an explicit relationship between instantaneous local film thickness and local mean velocity. One may eliminate $\bar{u}$ in Eq. 11 by introducing Eq. 13 and solve the resulting partial differential equation in δ . However, this was not pursued because the simplified liquid-film tracking model is linked with the COMMIX code, which is based on finite-difference formulations. For a finite surface area A_i of the steel shell defined by

$$A_i = \int_{x_{i-1/2}}^{x_{i+1/2}} \Delta\Phi \, r \, dx, \quad (14)$$

where $\Delta\Phi$ is the incremental azimuthal angle, the finite-difference formulation of Eq. 11 for A_i is

$$\rho A_i \frac{\Delta\delta_i}{\Delta t} + F_{i+1/2} - F_{i-1/2} = m_{ci} A_i, \quad (15)$$

in which

$$\delta_i = \frac{1}{2} (\delta_{i-1/2} + \delta_{i+1/2}), \quad (16)$$

and

$$F_{i+1/2} = \rho \bar{u}_{i+1/2} r_{i+1/2} \Delta\Phi \delta_{i+1/2}, \quad (17a)$$

$$F_{i-1/2} = \rho \bar{u}_{i-1/2} r_{i-1/2} \Delta\Phi \delta_{i-1/2}. \quad (17b)$$

Substituting Eq. 13 into Eq. 17, we have

$$F_{i+1/2} = \frac{1}{3\mu} \rho^2 g \Delta\Phi r_{i+1/2} \cos\beta_{i+1/2} \delta_{i+1/2}^3, \quad (18a)$$

$$F_{i-1/2} = \frac{1}{3\mu} \rho^2 g \Delta\Phi r_{i-1/2} \cos\beta_{i-1/2} \delta_{i-1/2}^3. \quad (18b)$$

Upon introducing Eq. 18 into Eqs. 15 and rearranging, we obtain the following algebraic equation for $\delta_{i+1/2}$:

$$a_s \delta_{i+1/2}^3 + b_s \delta_{i+1/2} + c_s = 0, \quad (19)$$

where

$$a_s = \frac{\rho^2 g \Delta \Phi r_{i+1/2} \cos \beta_{i+1/2}}{3\mu}, \quad (20a)$$

$$b_s = \frac{\rho A_i}{2\Delta t}, \quad (20b)$$

and

$$c_s = \frac{\rho A_i}{2\Delta t} \left(\delta_{i-1/2} - \delta_{i-1/2}^n - \delta_{i+1/2}^n \right) - \frac{\rho^2 g \Delta \Phi r_{i-1/2} \cos \beta_{i-1/2}}{3\mu} \delta_{i-1/2}^3 - m_{ci} A_i. \quad (20c)$$

The superscript n on δ denotes thickness at the previous time-step and is therefore known. The coefficients a_s and b_s are always positive. Hence, Eq. 19 has one real root and two complex conjugate roots. Only the real root is relevant. It is

$$\delta_{i+1/2} = \left\{ -\frac{c_s}{2a_s} + \left[\left(\frac{c_s}{2a_s} \right)^2 + \left(\frac{b_s}{3a_s} \right)^3 \right]^{1/2} \right\}^{1/3} + \left\{ -\frac{c_s}{2a_s} - \left[\left(\frac{c_s}{2a_s} \right)^2 + \left(\frac{b_s}{3a_s} \right)^3 \right]^{1/2} \right\}^{1/3}. \quad (21)$$

The corresponding mean film velocity at $x_{i+1/2}$ is

$$\bar{u}_{i+1/2} = \frac{\rho g \cos \beta_{i+1/2}}{3\mu} \delta_{i+1/2}^2. \quad (22)$$

Equations 21 and 22 are the desired results of the simplified film tracking model. They are used for the condensate film flow at the inside and for the evaporating water layer at the outside of the containment vessel, including the elliptical domes at top and bottom and the vertical cylindrical shell between. For the latter, $r = R$, the constant radius of the cylindrical shell, and $\cos \beta = 1$. For the evaporating film, $m_c < 0$ and becomes the evaporation mass flux.

2.3 Comprehensive Model for Film Thickness and Mean Flow Velocity

In the comprehensive liquid-film tracking model, the rigorous integral mass conservation equation (Eq. 11) is used. Its finite-difference formulation for the film thickness $\delta_{i+1/2}$ at $x_{i+1/2}$ is

$$a_\delta \delta_{i+1/2} = b_\delta, \quad (23)$$

where

$$a_{\delta} = \frac{\rho A_i}{2\Delta t}, \quad (24a)$$

$$b_{\delta} = \frac{\rho A_i}{2\Delta t} \left(\delta_{i+1/2}^n + \delta_{i-1/2}^n - \delta_{i-1/2} \right) - F_{i+1/2} + F_{i-1/2} + m_{ci} A_i, \quad (24b)$$

and $F_{i-1/2}$ and $F_{i+1/2}$ are expressed in Eq. 17.

The complete momentum equation (Eq. 2) is used in the comprehensive model. Formally, a rigorous integral momentum equation can be derived by using the approach similar to the boundary-layer analysis (see Schlichting⁷). However, because only one computation mesh of the film thickness is modeled, we are unable to resolve the velocity distribution across the film thickness. Consequently, we assume that the velocity distribution is parabolic across the film thickness. The assumption can be justified as shown in the previous section, i.e., when the inertia and convective terms are neglected, which have small effects for the film, the velocity distribution is indeed parabolic (Eq. 12).

The parabolic velocity distribution gives

$$u = 3\bar{u} \left(\frac{y}{\delta} - \frac{y^2}{2\delta^2} \right), \quad (25)$$

where $\bar{u}$ is the mean velocity defined in Eq. 10. In Eq. 25, both $\bar{u}$ and δ are treated as unknowns. Thus, the interface velocity u_{δ} becomes

$$u_{\delta} = \frac{3}{2} \bar{u}. \quad (26)$$

Multiplying the radius r into both sides of the momentum equation (Eq. 2) and integrating the resulting equation across the film thickness, from $y = 0$ to $y = \delta$, we obtain

$$\rho r \frac{\partial \delta \bar{u}}{\partial t} + \frac{6}{5} \rho \frac{\partial r \delta \bar{u}^2}{\partial x} + \rho r u_{\delta} \left(v_{\delta} - \frac{\partial \delta}{\partial t} - u_{\delta} \frac{\partial \delta}{\partial x} \right) = -3\mu r \frac{\bar{u}}{\delta} + \rho g r \cos \beta \delta. \quad (27)$$

Using the interfacial kinetic relation (Eq. 5b), we can eliminate v_{δ} , u_{δ} , and $\partial \delta / \partial t$ from the above equation. Equation 27 then becomes

$$\rho r \frac{\partial \delta \bar{u}}{\partial t} + \frac{6}{5} \rho \frac{\partial r \delta \bar{u}^2}{\partial x} = \frac{3}{2} r m_c \bar{u} - 3\mu r \frac{\bar{u}}{\delta} + \rho g r \cos \beta \delta. \quad (28)$$

For the momentum equation, the finite-difference formulation is based on a staggered mesh configuration by assuming the center of the mesh is at $x_{i+1/2}$. The finite surface area $A_{i+1/2}$ of the steel shell is defined by

$$A_{i+1/2} = \int_{x_i}^{x_{i+1}} \rho \Delta \Phi dx = A_{i(2)} + A_{i+1(1)}, \quad (29)$$

where

$$A_{i(2)} = \int_{x_i}^{x_{i+1/2}} \rho \Delta \Phi dx, \quad A_{i+1(1)} = \int_{x_{i+1/2}}^{x_{i+1}} \rho \Delta \Phi dx. \quad (30a,b)$$

The finite-difference formulation of Eq. 28 for $A_{i+1/2}$ is

$$\begin{aligned} & \rho A_{i+1/2} \frac{\Delta \delta_{i+1/2} \bar{u}_{i+1/2}}{\Delta t} + \frac{6}{5} \left(F_{i+1} \bar{u}_{i+1} - F_i \bar{u}_i \right) \\ &= \frac{3}{2} \left(m_{ci} A_{i(2)} + m_{ci+1} A_{i+1(1)} \right) \bar{u}_{i+1/2} - 3\mu \frac{A_{i+1/2}}{\delta_{i+1/2}} \bar{u}_{i+1/2} + \rho g \delta_{i+1/2} B_{i+1/2}, \end{aligned} \quad (31)$$

in which

$$\bar{u}_i = \bar{u}_{i-1/2}, \quad \bar{u}_{i+1} = \bar{u}_{i+1/2}, \quad (32a,b)$$

$$F_i = \frac{1}{2} \left(F_{i+1/2} + F_{i-1/2} \right), \quad F_{i+1} = \frac{1}{2} \left(F_{i+3/2} + F_{i+1/2} \right), \quad (33a,b)$$

and

$$B_{i+1/2} = \int_{x_i}^{x_{i+1}} r \cos \beta \Delta \Phi dx. \quad (34)$$

We used the upwind difference scheme for the velocity $\bar{u}$ and the central difference scheme for the film mass flux F . The central difference scheme gives a better approximation for change of the film mass flux due to condensation or evaporation.

Substituting Eqs. 32 and 33 into Eq. 31 and rearranging, we obtain the final form of the finite difference formulation for the mean film velocity $\bar{u}_{i+1/2}$ at $x_{i+1/2}$,

$$a_u \bar{u}_{i+1/2} = b_u, \quad (35)$$

where

$$a_u = \frac{\rho A_{i+1/2}}{\Delta t} \delta_{i+1/2} + \frac{3}{5} \left(F_{i+3/2} + F_{i+1/2} \right) - \frac{3}{2} \left(m_{ci} A_{i(2)} + m_{ci+1} A_{i+1(1)} \right) + 3\mu \frac{A_{i+1/2}}{\delta_{i+1/2}}, \quad (36a)$$

$$b_u = \frac{\rho A_{i+1/2}}{\Delta t} \delta_{i+1/2}^n \bar{u}_{i+1/2}^n + \frac{3}{5} \left(F_{i+1/2} + F_{i-1/2} \right) \bar{u}_{i-1/2} + \rho g \delta_{i+1/2} B_{i+1/2}. \quad (36b)$$

Equations 23 and 35 are the results of the comprehensive liquid–film tracking model for solution of film thickness and film mean velocity. These two equations are coupled and therefore require iteration for their solution. As in the simplified liquid–film tracking model, they are used for the condensate film on the inside wall of the containment vessel and for the evaporating film on the outside wall.

2.4 Film Temperature

In an approach similar to that used for the integral momentum equation, the integral energy equation is derived by assuming a known temperature distribution across the film thickness. If inertia and convective effects are neglected, the energy equation (Eq. 3) will result in a linear temperature distribution. However, this solution does not account for the heat capacity of the liquid film and, at the same time, imposes a constant heat flux across the entire film thickness, including the interface and the wall. While this is approximately true for steady–state films, it is unrealistic for transient liquid film flow, especially when the temperatures of the evaporating film and the condensate film are much different than the local equilibrium temperature.

Because the integral energy equation is used in the model, we need to know only the heat fluxes at the interface and at the wall. Hence, denoting the temperature at film center as $\bar{T}$, i.e., $\bar{T} = T(y=\delta/2)$, we assume the temperature distribution across the film as

$$T = T^w + 2(\bar{T} - T^w) \frac{y}{\delta}, \quad 0 \leq y < \delta/2, \quad (37a)$$

$$T = T^i + 2(\bar{T} - T^i) \left(1 - \frac{y}{\delta}\right), \quad \delta/2 < y \leq \delta. \quad (37b)$$

Multiplying the radius r into both sides of the energy equation (Eq. 3) and integrating the resulting equation across the film thickness, from $y = 0$ to $y = \delta$, we obtain

$$\rho C_p \left[r \frac{\partial \delta \bar{T}}{\partial t} + \frac{\partial r \delta \bar{u} \bar{T}}{\partial x} + r T^i \left(v_\delta - \frac{\partial \delta}{\partial t} - u_\delta \frac{\partial \delta}{\partial x} \right) \right] = r [m_c h_g + h_c (T^e - T^i) - q_w]. \quad (38)$$

where q_w is the local heat flux at the wall. In the above equation, we have made two approximations,

$$\int_0^\delta T \, dy = \delta \bar{T}, \quad (39a)$$

$$\int_0^\delta u T \, dy = \delta \bar{u} \bar{T}. \quad (39b)$$

These two approximations are consistent with the fact that the contributions from the inertia and the convective terms are small and their high-order terms can be neglected. The approximations greatly simplify the integral equation and make clear the physical meanings of the corresponding terms in the equation. However, if desired, the integrations in Eq. 39 can be evaluated rigorously by substituting u and T from Eqs. 25 and 37.

Using the interfacial kinetic relation (Eq. 5b), we can eliminate v_δ , u_δ , and $\partial \delta / \partial t$ from Eq. 38,

$$\rho C_p \left(r \frac{\partial \delta \bar{T}}{\partial t} + \frac{\partial r \delta \bar{u} \bar{T}}{\partial x} \right) = r [m_c h_g + h_c (T^e - T^i) - q_w] + r C_p m_c T^i. \quad (40)$$

The local interface temperature T^i can be obtained from the interface condition (Eq. 5c) and the temperature distribution (Eq. 37b),

$$T^i = \frac{m_c h_g + h_c T^e + \frac{2k}{\delta} \bar{T}}{\frac{2k}{\delta} + h_c}. \quad (41)$$

The wall heat flux q_w is evaluated in conjunction with the thermal structures, which are cylindrical shells in the COMMIX modeling of the containment vessel wall. Because the film is attached to the thermal structures, the local wall heat flux can be obtained from the local temperature of the film and the local temperature of the corresponding thermal structure. This leads to the wall heat flux as

$$q_w = \frac{\bar{T} - T^s}{\frac{\delta}{2k} + \frac{\Delta r_s}{2k_s}} \quad (42)$$

where k_s is the thermal conductivity of the solid material, Δr_s is the thickness of the thermal structure, and T^s is the temperature at the center of the thermal structure. Consequently, the local wall temperature is

$$T^w = \frac{\frac{k}{\delta} \bar{T} + \frac{k_s}{\Delta r_s} T^s}{\frac{k}{\delta} + \frac{k_s}{\Delta r_s}} \quad (43)$$

The finite-difference formulation of the energy equation (Eq. 40) for a finite surface area A_i is

$$\begin{aligned} & \rho C_p A_i \frac{\Delta \delta_i \bar{T}_i}{\Delta t} + C_p \left(F_{i+1/2} \bar{T}_{i+1/2} - F_{i-1/2} \bar{T}_{i-1/2} \right) \\ & = A_i \left[m_{ci} h_g + h_c \left(T_i^e - T_i^i \right) - q_{wi} + C_p m_{ci} T_i^i \right], \end{aligned} \quad (44)$$

where

$$\bar{T}_{i-1/2} = \bar{T}_{i-1}, \quad \bar{T}_{i+1/2} = \bar{T}_i. \quad (45)$$

It is seen that we used the upwind difference scheme for the temperature $\bar{T}$.

Substituting Eqs. 41, 42, and 45 into Eq. 44 and rearranging, we obtain the final form of the finite-difference formulation for the film temperature $\bar{T}_i$ at x_i ,

$$a_T \bar{T}_i = b_T, \quad (46)$$

where

$$a_T = \frac{\rho C_p A_i}{\Delta t} \delta_i + C_p F_{i+1/2} + A_i \left(\frac{1}{\frac{\delta_i}{2k} + \frac{\Delta r_s}{2k_s}} - \frac{m_{ci} C_p - h_{ci}}{1 + \frac{h_{ci} \delta_i}{2k}} \right), \quad (47a)$$

$$b_T = \frac{\rho C_p A_i}{\Delta t} \delta_i^n \bar{T}_i^n + C_p F_{i-1/2} \bar{T}_{i-1} + A_i \left(1 + \frac{m_{ci} C_p - h_{ci}}{\frac{2k}{\delta_i} + h_{ci}} \right) (m_{ci} h_g + h_{ci} T_i^e) + \frac{A_i}{\frac{\delta_i}{2k} + \frac{\Delta r_s}{2k_s}} T_i^s. \quad (47b)$$

It is seen that the solution for the film temperature $\bar{T}_i$ in Eq. 46 is explicit.

Equation 46 is applied for the condensate film on the inside wall and for the evaporating film on the outside wall of the containment vessel. It can be used with the simplified or the comprehensive liquid-film tracking model.

3. Calculational Results of the Liquid-Film Tracking Models

The liquid-film tracking models developed in the previous section have been implemented in the COMMIX computer code. The COMMIX calculational results were compared with the Westinghouse AP-600 PCCS small-scale integral test data for average wall heat flux, temperature rise of air flowing through the outside annulus, evaporation and condensation rates, and wall temperature distributions. Reasonably good agreement between the calculational results and the experimental data has been obtained.⁸ In the following, COMMIX calculational results of film thickness, velocity, and temperature for a postulated containment system are presented and discussed.

The containment system, illustrated in Fig. 3, consists of ellipsoidal upper and lower domes and a cylindrical shell between the domes. The domes have a horizontal major axis of 1.5 m and a vertical minor axis of 0.8 m. The cylinder has a height of 3 m. A cylindrical air baffle is placed outside of the cylindrical containment vessel with an air gap of 0.2 m. The whole system is initially stagnant at 25°C and 1 atm. To simulate a DBA, steam is introduced into the containment vessel by a jet at the bottom of the lower dome. The inlet steam has a temperature of 140°C and a flow rate of 0.1 kg/s. At the same time, water of 25°C starts to spray on the outside surface of the upper dome at a constant rate of 0.5 kg/s. Air enters the bottom of the annulus at a velocity 2 m/s, carrying heat and moisture as it flows out at the top chimney.

The system is discretized in a cylindrical coordinate system. There are nine horizontal and vertical partitions for the domes, and fifteen vertical partitions for the cylinder. Because the system is axisymmetric, only one grid is used in the azimuthal direction. The total number of computational cells in this model is 298.

Transient calculations were performed with the simplified and comprehensive liquid–film tracking models. Typically, the comprehensive model requires about 2.5 times more computer time than the simplified model. This was because the solution in the comprehensive model requires iterations and because a smaller time–step must be used due to the nonlinear coupling between film thickness and mean velocity. For calculations with the simplified model, the time–step size was typically 0.1 s. The solutions at each time–step are considered to be converged when the relative changes of all the dependent variables (pressure, velocities, temperature, and steam concentrations) are smaller than a convergent criterion, say 1×10^{-5} . The whole system is assumed to reach steady state when the steam condensation rate inside the containment vessel is 99% of the steam inlet flow rate, which is 0.1 kg/s. The transient calculation with the simplified model required about 10 h on the HP 730 workstation. Calculational results with the simplified model are presented below, and comparisons of film thickness and mean velocity are made between the simplified and the comprehensive models.

Figure 4 shows the steady–state velocity distribution and temperature contour in the system. It is seen from Fig. 4a that inlet steam flows upward along the center–line in the containment vessel with little spreading. It then flows down along the wall at a velocity of ≈ 0.1 m/s due to buoyancy. There is very little gas motion and mixing in the core region. In the annulus, the air goes upward with increasing velocity, mainly due to addition of steam from the evaporating film. The temperature in the containment vessel is stratified, as shown in Fig. 4b. At the bottom of the vessel, the temperature is only about 40°C.

The time–progressive film thickness distribution on the inside wall of the containment vessel is plotted in Fig. 5 for $t = 1, 10, 30, 50, 70, 100$, and 1200 s into the transient. Steam condensation starts near the top of the upper dome where the steam reaches first. The film thickness builds up before it gains enough momentum to flow downward such that a sharp film front can be observed at $t = 50$ s and 70 s into the transient. As the film reaches the lower dome, its thickness increases purely due to the decreasing surface areas in the flow direction. At steady state (denoted by $t = 1200$ s into the transient), the film thickness is smaller because the film has a higher temperature and consequently a lower viscosity (and hence a higher film flow velocity). For the evaporating film on the outside surface of the containment vessel, its thickness distribution is plotted in Fig. 6 for $t = 1, 5, 10, 14$, and 1200 s into the transient. The spray water takes about 14 s to reach the bottom of the cylindrical shell. The steady–state film thickness distribution (denoted by $t = 1200$ s into the transient) is thinner partially because of the higher film temperature and partially due to water evaporation.

Figures 7 and 8 show, respectively, the steady–state film thickness distributions for the condensate film and the evaporating film calculated with the simplified and comprehensive models. The two models give essentially the same results.

Steady-state mean flow velocities for the condensate film and the evaporating film are plotted in Figs. 9 and 10, respectively. For the inside film (Fig. 9), mean velocity increases with x on the upper dome and the upper half of the cylinder shell, and becomes approximately constant on the lower half of the cylinder shell and the lower dome. The increase of v with x is due to the increase of film temperature and the gravitational component on the film, and the constant v with x is due to the decrease of film temperature on the cylinder and the decrease of the gravitational component on the film on the lower dome. For the outside film (Fig. 10), mean velocity decreases on the upper half of the upper dome, due to the decrease of the film thickness as shown in Fig. 7, and increases on the lower half of the upper dome, due to the increase of the gravitational component in that region. The decrease in mean velocity on the cylinder is due to the decrease of film temperature. Again, it is seen that the simplified model and the comprehensive model give essentially the same results.

Steady-state temperature distributions for the condensate film and the evaporating film are plotted in Fig. 11. On the upper dome, the spray water on the outside of the dome effectively reduced the temperature of the inside film and, consequently, the temperature of the steel wall. As the outside film is heated by conduction from the condensate film through the wall, its temperature increases and reaches an maximum near the intersection of the upper dome and the cylinder. The temperatures of the inside and outside films decrease on the cylinder as the heat is convected by the air stream in the annulus between the containment vessel and the air baffle. The temperature of the inside film on the lower dome is higher because there is no film cooling on the outside wall.

4. Conclusions

Two liquid-film tracking models for time-dependent flows, a simplified model and a comprehensive model, have been developed and implemented in the COMMIX code. These models compute the transient liquid-film thickness, its mean velocity, and its temperature on both sides of the steel containment vessel. The difference between the simplified and comprehensive models is that the inertia terms in the momentum equation are neglected in the simplified model, whereas they are retained in the comprehensive model. The governing equations for the models are boundary-layer-type equations based on first principles. In both models, a differential equation describing the time rate of change of the film thickness is rigorously derived. It relates the time rate of thickness change to the streamwise gradient of the mass flow rate and the condensate mass flux at the film's free surface. In the simplified model, because the inertia terms are neglected in the momentum equation, a third-order algebraic equation for the film thickness was obtained, and the film velocity can then be explicitly calculated. Arguments were given to show that the

simplification could be justified except for small times following a disturbance. In the comprehensive model, the inertia terms are retained in the momentum equation and a parabolic velocity distribution was assumed across the film thickness. In both models, linear temperature distributions across the film thickness were assumed in the energy equation. The assumptions are necessary because only one computation mesh is used over the film thickness to reduce computation cost; they are also consistent with our estimation that contributions of the inertia effects are small for the problems under consideration. The final results of the models are expressed as finite-difference formulations so that they can be readily implemented in the COMMIX code.

The computer models have been used to analyze AP-600 PCCS small-scale integral test data and the agreement was good. The two models gave essentially the same results, which further supports our arguments for the assumptions. Computational results of liquid films for a postulated containment system have been presented and discussed. It was shown that the evaporating film on the outside of the containment vessel can effectively reduce the temperature of the steel shell of the containment vessel.

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Figure 1 The AP-600 Passive Containment Cooling System

Figure 2 Curvilinear Coordinates (x, y) Associated with Meridional Section of Elliptical Dome

Figure 3 A Postulated AP-600 Passive Containment Cooling System

(a)

(b)

Figure 4 Steady-State Velocity Distribution and Temperature Contour

Figure 5 Thickness of Film on Inside Containment Vessel Wall at Various Times

Figure 6 Thickness of Film on Outside Containment Vessel Wall at Various Times

Figure 7 Comparison of Steady-State Film Thicknesses Calculated with Simplified and Comprehensive Models

Figure 8 Comparison of Steady-State Film Thicknesses Calculated with Simplified and Comprehensive Models

Figure 9 Mean Film Flow Velocities Calculated with Simplified and Comprehensive Models

Figure 10 Mean Film Flow Velocities Calculated with Simplified and Comprehensive Models

Figure 11 Steady-State Temperature of Films on Inside and Outside Containment Vessel Walls

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