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RADIATIVE HEAT TRANSFER IN POROUS URANIUM DIOXIDE

A Dissertation

by

STEVEN LOWE HAYES

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MASTER

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STEVEN LOWE HAYES

Approved as to style and content by:

K.L. Peddicord
(Chair of Committee)

Y.A. Hassan
(Member)

C.P. Burger
(Member)

J.A. Reuscher
(Member)

J.W. Poston
(Head of Department)

December 1992

ABSTRACT

Radiative Heat Transfer in Porous Uranium Dioxide (December 1992)

Steven Lowe Hayes, B.S., Texas A&M University;

M.S., Texas A&M University

Chair of Advisory Committee: Dr. K.L. Peddicord

Due to the low thermal conductivity and high emissivity of UO_2 , it has been suggested that radiative heat transfer may play a significant role in the transport of heat through the pores of UO_2 used as a nuclear fuel. This possibility was computationally investigated and the contribution of radiative heat transfer within pores to the overall heat transport in porous UO_2 was quantified. A repeating unit cell was developed to model approximately a porous UO_2 fuel system, and the heat transfer through unit cells representing a wide variety of fuel conditions was calculated using an original finite element computer program. Conduction through the solid fuel matrix as well as the pore gas, and radiative exchange at the pore surface was incorporated in the heat transfer model. A variety of pore compositions were investigated; porosity, pore size, shape and orientation, temperature, and temperature gradient were parametrically varied to determine their particular influence on the radiative transfer mechanism. Calculations were made in which the pore surface radiation was both modeled and neglected. The difference between these two calculations yielded the integral contribution of the radiative heat transfer mechanism to the overall heat transport.

The results of these calculations indicate that the radiative component of heat transfer within pores is small for conditions representative of light water reactor fuel, typically less than 1% of the total heat transport. The radiative component is much larger, however, for conditions present in liquid metal fast breeder reactor fuel. During the restructuring of this fuel type early in life, the radiative heat transfer mode was shown to contribute as much as 10-20% of the total heat transport in the hottest regions of the fuel.

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Finally, I thank my wife, Mimi. Without her love and support I could not have completed this work.

DEDICATION

I dedicate this dissertation to my best friend, Mimi.

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NOMENCLATURE

Symbols

A	=	area, m^2
c_p	=	specific heat at constant pressure, $J/(kg \cdot K)$
d	=	major pore diameter, μm
e	=	electronic charge, C
E	=	radiation emissive power, W/m^2
f	=	porosity correction factor
F	=	geometrical view factor
g	=	gravitational acceleration, m/s^2 ; ratio of major to minor diameters for an ellipse
G	=	temperature gradient, K/m ; irradiation, W/m^2
h	=	fractional concentration of electron holes
J	=	radiosity, W/m^2
k	=	thermal conductivity, $W/(m \cdot K)$
k_B	=	Boltzmann's constant, J/K
L	=	altitude of a cylinder, m
n	=	fractional concentration of electrons
N	=	atomic number density, m^{-3}
P	=	volume fraction porosity
q	=	heat flux, W/m^2
Q	=	heat transfer rate, W
r	=	distance separating two differential surface areas, m
R	=	pore to matrix conductivity ratio, k_p / k_{100} ; pore radius, m
Ra	=	Rayleigh number
s	=	arclength, m
S	=	volumetric heat generation rate, W/m^3
T	=	absolute temperature, K
ΔT	=	temperature difference, K
U	=	activation energy, J
v	=	arbitrary weight function
x	=	spatial dimension in a rectangular coordinate system, m

y = spatial dimension in a rectangular coordinate system, m

Greek Letters

α = thermal diffusivity, m^2/s ; modified Loeb coefficient; hemispherical total absorptivity

β = volumetric thermal expansion coefficient, K^{-1} ; Maxwell-Eucken coefficient

Γ = boundary of a two-dimensional domain, m

ε = hemispherical total emissivity

γ = Loeb's geometrical factor

μ = mobility

η = unit normal vector for a differential surface area

ν = kinematic viscosity, m^2/s

Ω = two-dimensional domain, m^2

ρ = density, kg/m^3 ; hemispherical total reflectivity

ψ = angle measured from z-axis in x-z plane, radians

σ = Stefan-Boltzmann radiation constant; electrical conductivity, $(\Omega\text{-m})^{-1}$; standard deviation, %

θ = angle between a differential surface area and its surface normal vector, radians; elliptical pore orientation angle, degrees

Subscripts

b = black body

c = cross-sectional area

e = electron

gas = gas phase

h = electron hole

He = helium

Kr = krypton

L = longitudinal

mix = mixture

n = normal

net = a net quantity

o = value at pore center

p	=	effective property of the porous unit cell
phon	=	phonon
pore	=	property of the pore
rad	=	radiation
sp	=	small polaron
u	=	uranium
Xe	=	xenon
1	=	surface area 1
2	=	surface area 2
100	=	fully dense condition; a property of the matrix material

Superscripts

e	=	element
*	=	augmentation due to radiative heat transfer

CHAPTER I

INTRODUCTION

1.1 POROSITY CORRECTION FACTORS

Heat transfer in ceramic nuclear fuels as well as other materials is influenced by the presence of porosity entrained within the solid fuel matrix; generally, the heat transfer through a porous material is less than that through the same material in a fully dense condition. To account for the influence of porosity in thermal calculations, a porosity correction factor is typically employed which modifies the thermal conductivity of the fully dense solid to yield the correct macroscopic heat transfer. This statement can be expressed symbolically as:

$$k_p = f k_{100} , \quad (1.1)$$

where the porosity correction factor f usually has a numerical value between 0.0 and 1.0, but could theoretically take on values greater than unity. A simple rearrangement of equation (1.1) serves as the basis for finding or calculating such a porosity correction factor:

$$f = k_p / k_{100} . \quad (1.2)$$

Equation (1.2) can be used directly to determine f if thermal conductivity measurements can be made experimentally. The ratio of thermal conductivities of porous and fully dense specimens of the same material directly yields f . Any experimental method available can be employed in the thermal conductivity determinations.

The porosity correction factor can also be estimated by numerical computation. Fourier's heat conduction law gives the heat transfer by conduction as

$$Q_{100} = k_{100} A \nabla T , \quad (1.3)$$

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for a fully dense material. From equation (1.1) above, the heat transfer by conduction through a porous material of the same dimensions and same overall temperature gradient can be expressed as:

$$Q_p = k_p A \nabla T = f k_{100} A \nabla T. \quad (1.4)$$

Combining equations (1.3) and (1.4) yields a new formula for finding the porosity correction factor that provides a more fundamental approach to computing porosity correction factors:

$$f = Q_p / Q_{100}. \quad (1.5)$$

Many approaches for obtaining porosity correction factors have been used and range from simple empirical curve fits of porosity-dependent experimental data to analytical and numerical calculations. Previously derived analytical expressions for f will be reviewed in the next chapter, but all such expressions were obtained by calculating or estimating the heat transfer through a modeled porous system and dividing that by the heat transfer that would have occurred had the same modeled system been fully-dense, as specified by equation (1.5).

Though equation (1.5) was obtained by considering only the conduction heat transfer mode within the material, it can be regarded as a general rule that can be applied without reference to the actual heat transfer modes. If Q_p is computed by considering any and all operative heat transfer modes including conduction, convection and radiation, and Q_{100} is computed from conduction only through the fully-dense material, then application of equation (1.5) to obtain f and its use in equation (1.4) will yield the actual macroscopic heat transfer through the material from all heat transfer modes using a simple conduction-like heat transfer calculation. This is the attraction of porosity correction factors; if an applicable porosity correction factor is known, then the heat transfer through a porous medium can be calculated very simply regardless of the complex interplay of the actual heat transfer mechanisms.

Porosity correction factors are powerful tools for heat transfer calculations in porous media. In calculating porosity correction factors using computational models, all operative heat transfer modes can be included or only those modes believed to be the most

important. It is within this context that the present study of radiative heat transfer in porous uranium dioxide (UO_2) is conducted. By calculating porosity correction factors for porous UO_2 in which radiative heat transfer within pores is modeled and comparing that to the porosity correction factors obtained when the radiative heat transfer mode is not incorporated a quantitative understanding of the radiative contribution to overall heat transport through porous UO_2 will be obtained.

1.2 POROSITY IN UO_2 FUEL

In nuclear fuel performance modeling the most fundamental and important prediction that must be made with accuracy is the temperature profile within the fuel. Both high temperatures and steep temperature gradients within the fuel can dramatically influence fuel performance. High temperatures within the fuel drive phenomena such as grain growth, densification and fission product diffusion, which in turn govern processes such as fuel swelling and fission gas release. Many solid-state chemical reaction rates also increase with temperature. Steep temperature gradients within the fuel give rise to other important phenomena. Closed pores migrate from low temperature regions toward the center of the fuel pellet via a vapor transport mechanism in which solid material evaporates from the hotter side of the pore and condenses on the cooler side; this also leads to a redistribution of fuel constituents such as oxygen, plutonium and fission products from their initially flat concentration profiles since these species and their compounds have different vapor pressures. Additionally, thermal stresses resulting from the steep temperature gradient cause plastic deformation and cracking in regions of high and low temperature, respectively (Olander, 1976). Given that all of these complex phenomena are consequences of the fuel temperature profile, the ability to accurately predict that profile profoundly influences the accuracy achievable in any subsequent prediction based on the predicted temperatures. Thus, an accurate prediction of the fuel temperature profile is the first crucial step in any fuel performance calculation, and fuel porosity effects must be adequately incorporated.

Since porosity in a ceramic nuclear fuel effectively reduces its thermal conductivity, it would be desirable from a heat transfer perspective to minimize or eliminate porosity from the fuel. However, nuclear fuels such as UO_2 are intentionally fabricated to retain a specified amount of internal porosity. Solid and gaseous fission products accumulate during fuel irradiation and cause the fuel to swell; internal porosity is desirable since it can accommodate internally some of this swelling and reduce

macroscopic dimensional changes of the fuel pellet. As-fabricated UO_2 fuel designed for use in thermal reactors typically contains 5-8% porosity. Fast reactor designs fueled with UO_2 usually call for higher burnups, and higher burnup limits demand more internal porosity since more fission products will be produced during the lifetime of the fuel increasing fuel swelling. For this reason, many fast reactors employ UO_2 fuel with 10-20% porosity (Olander, 1976; Tang, 1978; Marbach et al., 1986; Vaidyanathan and Murata, 1986; Mühling et al., 1986; Shimada et al., 1986).

In fresh fuel the porosity is uniformly distributed with a range of pore sizes due primarily to fabrication effects. Small pores generally have diameters greater than 1 μm , and a few large pores can have diameters of 75-100 μm (Radford and Pope, 1977). This as-fabricated porosity initially contains 1-3 atm of helium which is used to provide an inert atmosphere during fuel preparation and fuel element fabrication. During irradiation, fission gases comprised primarily of xenon and krypton are formed which coalesce into bubbles and can effectively increase the fuel porosity (pellet cracking and relocation followed by sintering can also introduce additional porosity). Fission gas bubbles are usually small, having diameters less than 2 μm . Additionally, this Xe-Kr fission gas mixture increasingly pollutes the He-filled pores already present following the fabrication process (Olander, 1976).

Finally, a sequence of interesting phenomena occurs in UO_2 fuel when operated at the high linear heat generation rates typical of fast breeder reactors. A steep temperature gradient within a UO_2 fuel pellet gives rise to a migration of the initial porosity up the temperature gradient to the central region of the fuel (Nichols, 1967). This migration is believed to be due to a mass transfer via a net evaporation of volatile species from the hotter side of the pore and condensation on the pore's cooler side, giving the appearance of pore movement (Olander, 1973). As the migrating pores collect at the center of the fuel pellet a central hole or void is formed and grows in size until the hottest inner ring of the fuel pellet becomes a dense region having little porosity. During this evaporation-condensation driven pore migration, the pore shape is altered from its initially near-spherical shape to a very thin, oblate disk that appears to be a very short pancake shaped cylinder with its circular base oriented perpendicular to the heat flow direction, which is radially outward in a cylindrical fuel pellet (De Halas, 1967). Pores shaped and oriented in such a way would presumably represent a major impediment to heat flow within the fuel pellet.

1.3 HEAT TRANSPORT IN POROUS UO_2

The three modes of heat transfer are conduction, convection and radiation. Convection heat transfer can further come in either the free or forced variety. When contemplating the important heat transfer modes operative within porous UO_2 fuel, these are the modes that must be considered. Conduction within the solid fuel matrix as well as gas-filled pores or fission gas bubbles is the dominant mode of heat transfer in all cases within a nuclear fuel pellet. For convection within the gas-filled pores or bubbles to participate in the heat transfer process, it must take the form of free or natural convection.

For free convective flow to arise and persist, two necessary but *not* sufficient conditions must exist (Incropera and DeWitt, 1981). First, there must exist within the fluid a density gradient, and, second, a body force proportional to density must be present (usually gravity). For any gas existing within a pore its density will be inversely proportional to temperature, with the hotter regions of gas having a lower density relative to the cooler portions of the gas. This ensures that the first necessary condition for free convection within the gas-filled pore exists. Heat is being generated within the fuel pellet and transported from it, so a temperature gradient will exist within the fuel (both solid matrix and gas-filled pore) to satisfy the second condition for free convection heat transfer. In principle, these two conditions are not quite sufficient to allow for a free convection flow. The final consideration is the relative orientations of the density, temperature and gravity gradients. However, within a pore the temperature gradient is perpendicular to the direction of gravity which is conducive to free convection flow.

The fact that the necessary conditions for free convection flow are present within a gas-filled pore, however, does not mandate that such a flow exists. It has been shown that for free convection heat transfer to occur in an internal cavity the Rayleigh number must satisfy the following inequality (Incropera and DeWitt, 1981):

$$Ra = \frac{g \beta \Delta T d^3}{\alpha \nu} > 1700, \quad (1.6)$$

where the temperature difference is across the pore in the radial direction of the fuel pellet and all material properties are those of the gas within the pore. Substitution of He and Xe properties, a large ΔT of 300 K, and a large d of 100 μm into equation (1.6) yield Rayleigh numbers on the order of 10^{-6} for a He-filled pore and 10^{-4} for a Xe-filled pore. The pore dimension is simply too small to allow a free convective flow to develop. Thus,

heat transfer by convection within pores cannot significantly contribute to the overall heat transport within UO_2 fuel. The only remaining mode of heat transfer within the fuel is that of thermal radiative exchange from the hotter side of a pore to the cooler side. Such an exchange must occur, but its magnitude is determined by the temperature distribution on the pore surface, the pore geometry, and the material properties of the UO_2 rather than the gases, namely thermal conductivity and emissivity.

The importance of the temperature prediction in a UO_2 fuel element has led to many experimental studies of the thermal conductivity of UO_2 which will be reviewed in the next chapter. As such, the behavior of the thermal conductivity of theoretically dense UO_2 is well known; overall, the thermal conductivity is relatively low in value and is a strong function of temperature, decreasing from 7 W/(m-K) at 500 K to a minimum of less than 2.5 W/(m-K) near 2000 K, after which it increases to 3.5 W/(m-K) near its melting temperature of approximately 3120 K (Harding and Martin, 1989). Over this same temperature range the hemispherical total emissivity of UO_2 varies linearly but remains above 0.8 (Hagman et al., 1981). The relatively low thermal conductivity of UO_2 coupled with its high emissivity suggests that radiative heat transfer through internal pores in UO_2 should be considered when computing the total heat transport within the fuel.

1.4 OBJECTIVES AND SCOPE OF WORK

Based upon the preliminary considerations outlined in the previous two sections of this chapter, the objective of this study is the quantification of the relative contribution of radiative heat transfer within internal voids (fission gas bubbles and pores) to the overall heat transport in UO_2 under conditions representative of a nuclear fuel environment. Such conditions include the following: volume fraction porosities of 0-50%, circular and elliptical pore shapes with variable orientations, pore diameters of 1-200 μm for circular pores and major diameters of 1-50 μm for elliptical pores, temperatures of 1000-3000 K, and temperature gradients of 2000-10,000 K/cm. These parameter values include the ranges expected for normal operation of UO_2 fuel in both thermal and fast reactors and indeed go beyond normal operational limits to include accident conditions.

A computational approach was adopted where the heat transfer ratio in equation (1.5), and thus the porosity correction factor f , was calculated for a finite element model of a porous UO_2 fuel system. The model assumed the fuel system was comprised of a solid UO_2 matrix containing uniformly dispersed pores. The fuel matrix was assumed to

have properties of fully-dense UO_2 , and the interior of the pore was in separate calculations assigned the properties of helium, a fission gas mixture of 85% xenon and 15% krypton, and a vacuum (zero thermal conductivity). Conduction in both the solid UO_2 matrix and the gas-filled pores was modeled, but convective heat transfer within the pores was neglected based upon the considerations of the previous section. Calculations of the porosity correction factors were made which both modeled as well as neglected radiative exchange at the pore surfaces; comparison of the difference between calculated porosity correction factors when radiative heat transfer was explicitly modeled versus neglected allowed the radiative contribution to the overall heat transport to be quantified in an integral way.

The number of calculations required to span all of the parameter ranges given above was quite large. The results of these calculations are tabulated and included in an appendix for reference. These results were correlated to obtain a set of equations capable of reproducing with good precision the calculated porosity correction factors.

The effect of porosity on the overall thermal conductivity of UO_2 and other porous materials has been studied in many ways by many researchers. Previous efforts assumed the pore medium was either non-conducting or took on the conductivity of a representative gas. A few workers recognized that the radiative heat transfer mechanism could become important at high temperatures and suggested the gas conductivity be augmented to account for the increased heat transfer due to radiation. None, however, have directly treated the radiative heat transfer component within the pore. This investigation studied the radiative heat transfer component in an integral way: temperature-dependent thermal conductivity expressions were used for all materials involved, and the conduction and radiation heat transfer modes operating within the pore were recognized as being parallel and competing mechanisms that cannot be decoupled. The results of this work are the first to quantify the contribution that radiative heat transfer within pores makes to the overall heat transport in UO_2 fuel.

CHAPTER II

LITERATURE REVIEW

2.1 THERMAL CONDUCTIVITY OF UO_2

Due to the importance of predicting temperature profiles in nuclear fuel elements, the thermal conductivity of UO_2 is a crucial property to understand. This has led to its many experimental determinations, theoretical treatments and overall reviews and correlations. In this section the experimental data for the thermal conductivity of UO_2 are presented, the current understanding of the underlying physics of the thermal conductivity behavior of UO_2 is reviewed, and a correlation for best predicting the thermal conductivity of fully-dense UO_2 is selected.

Experimental measurements of the thermal conductivity of UO_2 appear in the literature as early as 1954. Both the radial heat flow and the transient heat pulse methods have been used for measurements. In the radial heat flow method a specimen is internally heated and its thermal conductivity is inferred from the known amount of input energy and the observed temperature difference produced across the specimen. In the transient heat pulse method, the thermal diffusivity α is determined and the thermal conductivity is calculated as $k = \alpha \rho c_p$. Hagerman et al. (1981) have given an excellent review of the experimental data, and their conclusions are reproduced here.

The available data for the thermal conductivity of UO_2 are contained in the following references: VanCraeynest and Stora, 1970; Goldsmith and Douglas, 1973; Hobson et al., 1974; Asamoto et al., 1969; Kingery et al., 1954; Hetzler, 1967; Godfrey, 1964; Bates, 1970; Christensen, 1964; Reiswig, 1961; Ferro et al., 1972; Weilbacher, 1972; and Gibby, 1971. Hagerman et al. (1981) have analyzed the data and consider some of it suspect. Asamoto et al. (1969), Reiswig (1961) and Hetzler (1967) employed radial heat flow methods in which the electrically heated center conductor probably contacted the oxide sample; Joule heating of the oxide would result and yield an anomalously high thermal conductivity. The data of Asamoto et al. (1969) and Hetzler (1967) also exhibit an unusually large degree of scatter, probably due to specimen cracking during the experiments. The data of Ferro et al. (1972) also show extreme scatter that cannot be accounted for from their specimen characterization or experimental procedures. The data of VanCraeynest and Stora (1970) show anomalously low conductivity compared to data

from specimens of similar density, and cannot be reconciled based on their specimen characterization; Hagerman et al. (1981) believe the cause to be specimen cracking prior to the conductivity measurements. These five data sets are not further considered.

Three of the remaining experimenters employed the radial heat flow method. Kingery et al. (1954) performed measurements on specimens of 26.7% porosity. Christensen (1964) performed measurements on specimens of 6% porosity. Godfrey (1964) performed measurements on specimens of 6.6% porosity.

The remaining five sets of thermal conductivity data were obtained using the transient heat pulse technique. Bates (1970) performed measurements on specimens of 1.6% porosity. Gibby (1971) performed measurements on specimens of 4.2% porosity. Wellbacher (1972) performed measurements on specimens of 2% porosity. Goldsmith and Douglas (1973) performed measurements on specimens for the porosity range of 1.4 to 9.6%, and Hobson et al. (1974) performed measurements on specimens of 5.1% porosity.

Since the radiation heat transport mode is of primary interest in this study, only temperatures above 1000 K are considered. A plot of the experimental thermal conductivity data as a function of temperature is shown in Fig. 2.1 along with the curve for fully-dense UO_2 (Harding and Martin, 1989). The data sets shown in this plot have been grouped by porosity, and obvious disagreement among the data sets of different experimenters exists since many data points of high porosity can be seen that have higher conductivities than data at the same temperature with a lower amount of total porosity. This must be due to differences between specimens. Impurity content, stoichiometry, pore sizes and shapes, and gas composition within the pores are all known to have a significant impact on the overall measured thermal conductivity. Unfortunately, most of this data cannot be analyzed with respect to these parameters since specimen characterization to this extent was not performed by the experimenters. More physically consistent trends can be seen when data from a single experimenter are analyzed, such as the data of Goldsmith and Douglas (1973) shown in Fig. 2.2.

Based upon consideration of the experimental data just reviewed, it is generally accepted that heat conduction in UO_2 is primarily due to the sum of a phonon component that dominates low temperature transport (< 1500 K) and a small polaron component (electron-hole pair creation) that is enhanced with temperature and becomes significant above 2000 K (Harding and Martin, 1989). Based on theoretical results the phonon component of the thermal conductivity behaves as depicted by (Klemens, 1985):

Fig. 2.1 Experimental thermal conductivity data for UO_2

Fig. 2.2 Experimental UO_2 thermal conductivity data of Goldsmith and Douglas (1973)

$$k_{\text{phon}} = (a + bT)^{-1}, \quad (2.1)$$

where a and b are constants that relate to phonon-impurity and phonon-phonon (Umklapp) scattering processes, respectively; bT can be interpreted as the ideal thermal resistivity, and a is a temperature-independent thermal resistivity due to point defects. Point defects serve to scatter phonons and reduce the thermal conductivity of a dielectric solid. Equation (2.1) is generally valid only if $a < bT/3$.

The thermal conductivity component due to small polarons is theoretically represented by the following expression (Hyland, 1983):

$$k_{\text{sp}} = (U/e)^2 \sigma / 4T, \quad (2.2)$$

where the required activation energy is that for the creation of electron-hole pairs. The electrical conductivity may be written as:

$$\sigma = eN_u (n\mu_n + h\mu_h). \quad (2.3)$$

For stoichiometric UO_2 , $n = h$. Furthermore, the Seebeck coefficient for UO_2 has been experimentally observed to tend toward a temperature-independent value at high temperatures that Hyland (1982) has equated with the condition $\mu_n = \mu_h$; finally, the hole mobility may be written in terms of the Einstein relationship:

$$\mu_h = (b'/T) \exp(-V/k_B T) \quad (2.4)$$

where b' and V are material-dependent constants. Using equations (2.3) and (2.4) and the conclusions of Hyland, the small polaron contribution to the thermal conductivity of UO_2 can be cast in the following form:

$$k_{\text{sp}} = (c/T^2) \exp(-W/k_B T), \quad (2.5)$$

where c and W can be considered constants. This small polaron conduction mechanism is now believed to be responsible for the increase in overall thermal conductivity of UO_2 above 2000 K. Early researchers believed this increase might be due to radiative heat

transport through the crystal lattice. However, more recent experiments on single crystals of UO_2 have shown this contribution to be extremely small. This process is now considered to be of minor importance in polycrystalline UO_2 even up to the melting point (Browning, 1980). Thus, the thermal conductivity of stoichiometric, fully-dense UO_2 should be adequately described by the sum of equations (2.1) and (2.5) as:

$$k = k_{\text{phon}} + k_{\text{sp}}, \quad (2.6)$$

where the four constants (a , b , c and W) must be determined based on the experimental data. Harding and Martin (1989) have recommended this form and determined the required constants based on the analysis given in the following paragraphs.

The low temperature experimental data for UO_2 can be fit quite well using the phonon conductivity expression given as equation (2.1), and several reviewers have reported values for the required constants a and b derived from this data (Washington, 1973; Brandt et al., 1977; Martin, 1982). These reported values are in general agreement with each other; some difference is present due to the spread in the experimental data and differences between reviewer's techniques in correcting the data to represent a fully-dense value. Harding and Martin selected the constants of Martin (1982) for use with equation (2.1): $a = 0.0375 \text{ W}^{-1} \text{ m-K}$ and $b = 2.165 \times 10^{-4} \text{ W}^{-1} \text{ m}$.

In order to obtain the constants for the small polaron component of thermal conductivity needed in equation (2.5), Harding and Martin analyzed the comprehensive set of raw data for the electrical conductivity of fully-dense, stoichiometric, single crystal UO_2 between 2000-3000 K (Bates et al., 1967) in terms of equation (2.5). This analysis provided the needed constants as: $c = 4.715 \times 10^9 \text{ W-K/m}$ and $W = 1.41 \text{ eV}$. These values are in good agreement with subsequent experimental results (Hampton et al., 1988). Thus, the overall recommendation of Harding and Martin (1989) for the thermal conductivity of fully-dense, solid, stoichiometric UO_2 over $773 \leq T \leq 3120 \text{ K}$ is:

$$k = \frac{1}{0.0375 + 2.165 \times 10^{-4} T} + \frac{4.715 \times 10^9}{T^2} \exp\{-16361/T\}. \quad (2.7)$$

Comparison of the predictions of equation (2.7) with the experimental data indicate the uncertainties in this recommendation are about 7% up to $\sim 1800 \text{ K}$ and increase to $\sim 15\%$ at 3120 K . This equation is employed throughout this work to provide the values needed

for the thermal conductivity of fully-dense UO_2 .

2.2 EXISTING POROSITY CORRECTION FACTOR EXPRESSIONS

Many investigators have provided expressions for estimating porosity correction factors. These include both empirical correlations obtained from experimentation as well as analytically derived equations. A number of these expressions will be discussed in this section.

Two early studies were performed on the equivalent electrical conductivity of heterogeneous media. Maxwell (1892) considered current in a system comprised of dispersed spheres within a solid spherical shell. Interpreting the dispersed spheres as pores, an expression completely analogous to the porosity correction factor for thermal conductivity can be extracted. Doing this, Maxwell's correction factor appears as:

$$f = \frac{2 + R - 2P(1 - R)}{2 + R + P(1 - R)}, \quad (2.8)$$

where R is the ratio of the thermal conductivity of the gas inside the pore to that of the solid matrix, namely $R = k_{\text{gas}} / k_{100}$. Maxwell obtained his solution with simplifications brought about by assuming the heterogeneous mixture was dilute. Thus, his results are expected to be valid only at low porosities. For the case of non-conducting pores, Maxwell's expression reduces to the familiar formula:

$$f = \frac{1 - P}{1 + 0.5P}. \quad (2.9)$$

Fricke (1924) considered the effect on electrical conductivity of spherical as well as ellipsoidal shaped phases dispersed in a rectangular solid and obtained the following expression:

$$f = \frac{1 + xP \left(\frac{R - 1}{R + x} \right)}{1 - P \left(\frac{R - 1}{R + x} \right)}, \quad (2.10)$$

where the parameter x is exactly 2 for spherical pores and is a function of R as well as the ratio of major to minor diameters for ellipsoidal shaped pores. For the case of spherical,

non-conducting pores, equation (2.10) reduces to equation (2.9).

Lord Rayleigh (1892) treated the effect of obstacles arranged in rectangular order on the properties of a medium by considering the current through a two-dimensional rectangular region containing a circular obstruction. For the case of non-conducting pores, Lord Rayleigh obtained a rapidly convergent series solution for the porosity correction factor in two dimensions. Lord Rayleigh gave the appropriately truncated expression as:

$$f = 1 - \frac{2P}{1 + P - 0.3058P^4} . \quad (2.11)$$

Lord Rayleigh also considered the three-dimensional analog of a conducting sphere within a rectangular solid and independently derived equation (2.8) as Maxwell had.

One of the first treatments of heat flow in porous ceramic insulators was given by Russell (1935). Russell modeled the porous system as a three dimensional solid filled uniformly with cubical pores. Assuming the isothermal surfaces are planes and summing thermal resistances in series and parallel, he obtained to following equation:

$$f = \frac{P^{2/3} + R^{-1} (1 - P^{2/3})}{P^{2/3} - P + R^{-1} (1 - P^{2/3} + P)} . \quad (2.12)$$

Loeb (1954) suggested the following expression based on a one-dimensional treatment:

$$f = 1 - P_c \frac{1 - R}{1 + R \left(\frac{1 - P_L}{P_L} \right)} . \quad (2.13)$$

This expression is often simplified to the form known as the classical Loeb formula that appears as: $f = 1 - P$. This expression proved unsatisfactory when compared to experimental data, however, and led to the modified Loeb formula:

$$f = 1 - \alpha P , \quad (2.14)$$

where an α between 1 and 4 is generally employed (Martin, 1982). Theoretical

considerations suggest that the coefficient α should decrease with temperature since pores should become better conductors of heat as temperature increases (increase in gaseous conduction and radiation). Van Craeynest and Stora (1970) found the expression $\alpha = 2.74 - 0.58 \times 10^{-3}T$ described their experimental data well to 1250 K. Hobson et al. (1974) suggest an α computed by $\alpha = 2.46 - 0.70 \times 10^{-3}T$ based on their own experimental data to a temperature of 1500 K. Brandt et al. (1977) modified this expression slightly to $\alpha = 2.6 - 0.50 \times 10^{-3}T$ to yield better agreement with the whole of the experimental data up to 3073 K.

Another popular empirical form is the Maxwell-Eucken expression, in which the form of Maxwell's classical expression is used but an empirically adjustable parameter is introduced to yield better agreement with experimental results:

$$f = \frac{1 - P}{1 + \beta P} \quad (2.15)$$

Based on the geometric considerations of Fricke (1924), Biancheria (1966) interpreted the coefficient β as being a function of the shape factor related to ellipsoidal relative dimensions. From this assumption he calculated β to be 0.5 for spherical pores, increasing to 0.667 for prolate ellipsoids having a major to minor diameter ratio of 10, and increasing to 2.107 for oblate ellipsoids of the same diameter ratio. Marino (1971) also reconciled the work of Fricke (1924) with the Maxwell-Eucken form. Marino obtained an expression for β from the analysis Fricke performed for the porosity correction for prolate and oblate ellipsoidal pores within rectangular solids. The form of β so obtained is a complicated function of the conductivity ratio R , the ratio of major to minor ellipsoid diameters, and the porosity. Marino showed that his expression reduces to the constant value 0.5 for spherical, non-conducting pores. From purely empirical considerations, Hagrman et al. (1981) have given the temperature dependent formula $\beta = 6.5 - 0.00469T$ for UO_2 below 1364 K.

Using a similar form to fit experimental thermal conductivity data from stainless steel and copper sintered spherical powders Koh and Fortini (1973) found good agreement with the data using the expression:

$$f = \frac{1 - P}{1 + 11P^2} \quad (2.16)$$

This formula, however, displays a behavior very different from other porosity correction factor expressions (namely, its second derivative is positive) and is not believed to be generally applicable for generic material systems and/or higher porosities (>10%).

Kämpf and Karsten (1970) obtained an analytical porosity correction factor for a cubical, non-conducting pore located within a cubical solid. Heat flow was assumed to be in one dimension. By summing the resistance to heat flow and averaging over the cross-sectional area of the cubical solid the porosity correction factor was estimated to be:

$$f = 1 - P^{2/3} . \quad (2.17)$$

An analogous approach was given by Peddicord (1976) using a spherical pore in the center of a cubical solid. With this modification, Peddicord found the porosity correction factor to be:

$$f = 1 - \left(\frac{3\pi^{1/2}}{4} \right)^{2/3} P^{2/3} . \quad (2.18)$$

where the coefficient preceding the porosity term in equation (2.18) is 1.209 as compared to 1.0 in equation (2.17). Peddicord has suggested that this expression constitutes a lower bound on the effect of spherical pores on the thermal conductivity of a porous material.

Equation (2.18) was derived assuming one-dimensional heat flow. Cunningham and Peddicord (1981) considered multi-dimensional heat flow in the cubical system containing a spherical pore and analytically calculated temperatures and heat fluxes throughout the system. Comparing the net heat flow through the system with those found in the fully-dense system, porosity correction factors were obtained. Their analytical results are adequately described by the expressions:

$$f = e^{-2.14P} , \text{ for } 0 \leq P < 0.3, \text{ and} \quad (2.19a)$$

$$f = 0.92 - 1.34P, \text{ for } 0.3 \leq P \leq 0.5. \quad (2.19b)$$

Cunningham and Peddicord were the first to explicitly model three-dimensional heat flow around a pore.

Finally, the most recent and innovative approach to calculating the porosity correction factor has been provided by Tzou (1991). Tzou employed an analytical model based on a self-consistent approach; the model is developed on a continuum basis which absorbs the effect of pores on the overall degradation of thermal conductivity in an added thermal resistance tensor. The added tensor thus formulated depends on the temperature distribution along the surface of the pore which can be computed analytically for pores of simple geometry or numerically if the geometry is more complicated. The approach, in theory, allows for the incorporation of energy exchange between the solid matrix and fluid within the pore by both conduction and convection, as well as anisotropic thermal conductivity behavior via the tensor mathematics. In principle, thermal radiative exchange between the pore surfaces could be handled using this approach. This complex approach offers immense modeling flexibility, but has yet to be fully explored. Tzou gives the simple result obtained for randomly dispersed, spherical, non-conducting pores embedded in a matrix having an isotropic thermal conductivity as:

$$f = 1 - \frac{2\pi}{3} P, \quad (2.20)$$

which is in general agreement with the empirical Loeb formula.

Equations (2.8) through (2.20) comprise the wide variety of generic or semi-generic porosity correction factor expressions that are available in the literature. The predictions of these formulae have been tabulated for comparison in Table 2.1 for the simplest case of spherical, non-conducting pores. The empirical expressions derived from experimental data have proven applicable only to porosities of ~10%, although this is usually suitable for application to the commercial light water reactor fuel environment; UO₂ fuel pellets in a fast reactor, in contrast, can experience local porosity conditions of up to 50% during restructuring. Those analytical expressions derived based on one-dimensional heat flow assumptions are likewise accurate only at low porosity, and generally overestimate the effect of porosity by ignoring multi-dimensional heat flow. Finally, while some of these expressions provide for the conduction of heat by the fluid medium within the pore, none address the contribution of thermal radiative exchange at the pore surface.

Table 2.1 Porosity correction factor predictions for spherical, non-conducting pores

Porosity	Equation:												
	2.1-2.3	2.4	2.5	2.6	2.7†	2.8††	2.9	2.10	2.11	2.12	2.13		
0.000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000		
0.025	0.9630	0.9512	0.9734	0.8966	0.9500	0.9262	0.9683	0.9145	0.8966	0.9479	0.9476		
0.050	0.9268	0.9048	0.9453	0.8359	0.9000	0.8595	0.9246	0.8643	0.8359	0.8985	0.8953		
0.075	0.8916	0.8605	0.9164	0.7850	0.8500	0.7988	0.8711	0.8222	0.7850	0.8517	0.8429		
0.100	0.8571	0.8182	0.8869	0.7395	0.8000	0.7434	0.8108	0.7846	0.7395	0.8073	0.7906		
0.125	0.8235	0.7778	0.8571	0.6978	0.7500	0.6926	0.7467	0.7500	0.6978	0.7653	0.7382		
0.150	0.7907	0.7391	0.8271	0.6587	0.7000	0.6459	0.6814	0.7177	0.6587	0.7254	0.6858		
0.175	0.7586	0.7021	0.7970	0.6217	0.6500	0.6028	0.6171	0.6871	0.6217	0.6876	0.6335		
0.200	0.7273	0.6665	0.7669	0.5865	0.6000	0.5628	0.5556	0.6580	0.5865	0.6518	0.5811		
0.225	0.6966	0.6324	0.7369	0.5528	0.5500	0.5258	0.4978	0.6301	0.5528	0.6179	0.5288		
0.250	0.6667	0.5996	0.7070	0.5202	0.5000	0.4912	0.4444	0.6031	0.5202	0.5857	0.4764		
0.275	0.6374	0.5680	0.6773	0.4887	0.4500	0.4590	0.3958	0.5771	0.4887	0.5552	0.4240		
0.300	0.6087	0.5376	0.6478	0.4582	0.4000	0.4289	0.3518	0.5519	0.4582	0.5262	0.3717		

† Alpha coefficient of 2.0 used.

†† Beta coefficient of 2.107 used.

2.3 RADIATION CONDUCTIVITY

While none of the previously developed expressions directly treat the radiative component of heat transfer within the pore, several of those expressions allow for the conduction of heat by the fluid within the pore. Equations (2.8), (2.10), (2.12) and (2.13) contain the thermal conductivity ratio R of the fluid within the pore to that of the solid matrix. For these expressions, any temperature dependent behavior of the thermal conductivities of either the gas or solid materials can be accommodated via this factor. Evidence was given in Chapter I that natural convection is not an active heat transfer mode in pores with diameters of $\sim 100 \mu\text{m}$ or less filled with a mono-atomic gas such as He, Kr or Xe. Therefore, the previous equations given for calculating porosity correction factors can presumably suffer only from inadequate geometry modeling or the neglect of radiant heat transfer.

It was realized fairly early that radiation across pores can contribute significantly to the total heat transport in some porous materials at high temperatures (Kingery, 1960). To appropriately address the radiative heat transfer component in a porous material, the conduction and radiation processes must be modeled in an integral way. Heat transfer by conduction through the solid and gas phases and by radiation through the pores cannot be treated as independent mechanisms. The relative magnitude of the radiative component will be dependent, for example, on the thermal conductivities of both the solid matrix and gas filling the pore since the conduction and radiation processes are parallel, competing heat transfer modes. The interaction between these mechanisms should be incorporated in any calculation of heat transfer through porous materials at high temperatures (Ganguly and Hasselman, 1976).

The interaction between conduction and radiation could not be modeled by early investigators due to the mathematical complexity of analytical treatments and/or computational limitations. For this reason, previous investigators sought to augment the thermal conductivity of the pore gas with a "radiation conductivity" component:

$$k_{\text{pore}} = k_{\text{gas}} + k_{\text{rad}} . \quad (2.21)$$

Loeb (1954) assumed the radiation heat transfer expression could be linearized since pores are small and the temperature drop across them is correspondingly small. With that assumption Loeb gives the following expression for the radiation conductivity of a pore:

$$k_{\text{rad}} = 4 \gamma \epsilon \sigma T^3 d, \quad (2.22)$$

where the temperature is the mean temperature of the pore, d is taken as the diameter in the direction of heat flow, and γ is a geometrical factor ($\gamma = 1$ for laminar pores and cylindrical pores with axes parallel to the heat flow direction, $2/3$ for spherical pores and $\pi/4$ for cylindrical pores with axes perpendicular to the heat flow direction). This expression for the radiation conductivity allows for non-ideal thermal emission through the emissivity ϵ . However, reflection within the pore was neglected by Loeb.

Whereas the radiation conductivity computed from equation (2.22) is only approximate due to linearization, Marino (1969) has given an exact expression for the radiation conductivity of spherical pores in a linear temperature gradient. The following expression was obtained by integration of the general expression for the net heat transfer between two incremental areas on the surface of a spherical pore:

$$k_{\text{rad}} = 2 \epsilon \sigma T^3 d + 0.25 \epsilon \sigma T G^2 d^3. \quad (2.23)$$

For conditions representative of a UO_2 fuel pellet ($T \approx 2000$ K, $G = 6 \times 10^5$ K/m and $d = 100 \mu\text{m}$) the first term in equation (2.23) is roughly 10^4 times larger than the second term. Thus the second term can be neglected with little error. The first term gives a value for the radiation conductivity for a spherical pore that is $\sim 75\%$ of the conductivity predicted by equation (2.22). The radiation conductivity expression of Marino also neglects reflection within the pore. Using the procedure of Marino (1969), the radiation conductivity of a circular pore in a two-dimensional domain (or equivalently an infinitely long cylindrical pore in a three-dimensional domain) has been calculated in Appendix A. The result is $\sim 64\%$ of the conductivity predicted by equation (2.22).

Finally, Wolf et al. (1990) have recently reconsidered the radiation conductivity for spherical pores. A variational principle was formulated for radiant heat transfer through the pores and conduction within the solid matrix of an arbitrary, randomly generated and overlapping pore-solid system with diffusely reflecting surfaces. For thermal radiative equilibrium the solid conduction component can be neglected, and the variational result gives a rigorous upper bound on the radiation conductivity as:

$$k_{\text{rad}} \leq \frac{48 \sigma T^3 d}{9 + 4(1 - \epsilon)}. \quad (2.24)$$

All three of the above suggested expressions for the radiation conductivity of a pore were obtained from analyses that omitted the interaction between the competing conduction and radiation mechanisms that would actually be operative in a material. Consequently, when any of these expressions are used to augment the value of the pore thermal conductivity for use in the previous porosity correction factor formulae the effect of radiative heat transfer is usually overestimated. Furthermore, none of these radiation conductivity expressions indicate any dependence on pore shape; experimental data as well as physical reasoning indicate a pore shape dependency (Chen and Churchill, 1963).

CHAPTER III

METHODOLOGY

3.1 DEFINITION OF THE UNIT CELL

The unit cell approach is widely used in the calculation of porosity correction factors. A unit cell approach is a simplification of the geometrical model of the overall porous system. A real UO_2 fuel pellet would contain many pores having a distribution of sizes and shapes randomly scattered throughout the pellet in no repeating pattern. Since a complete modeling of such a system would require enormous computational resources, a simpler geometrical description that is assumed to be repeated uniformly is sought to approximate the overall solid-pore system. This simpler, repeating geometrical model is termed a "unit cell." Symmetry is the fundamental advantage to the unit cell approach.

The unit cell most often employed for calculating porosity correction factors is one containing a single pore centered in a matrix of solid, fully-dense material. The geometry of the unit cell can be either two- or three-dimensional. A two-dimensional unit cell would be a square domain with a central pore of circular or elliptical shape. Two parallel faces can be assigned insulation boundary conditions from symmetry arguments, while the remaining parallel faces are assigned constant temperature conditions. This forces a net heat flow through the unit cell. The challenge is to model all of the operative heat transport mechanisms within the unit cell and to find the net heat flow through the unit cell by analytical or numerical computation. The ratio of the heat flow value for this pore-containing unit cell to the heat flow through the fully-dense unit cell (which is an easy analytical calculation) yields the porosity correction factor for the specific unit cell conditions of porosity, temperature, etc., as given in equation (1.5). The analogous unit cell in three dimensions would be a spherical or ellipsoidal pore centered within a cube of solid matrix material. Two parallel faces of the cubical domain would be assigned constant temperature conditions, and the four remaining faces would be insulated.

In this work the unit cell is discretized into subdomains for which the finite element method is used to calculate nodal temperatures throughout the unit cell; the finite element method is the obvious choice due to its ability to easily and accurately model curved surfaces. Following the solution for the nodal temperatures, the nodal heat fluxes along one of the constant temperature boundaries of the unit cell can be extracted from the

problem source vector. The sum of these nodal heat fluxes along such a domain boundary yields the heat flow through the unit cell, Q_p . Conduction is the only heat transport mode within the solid matrix material. Within the pore, conduction is modeled through the gas (unless a vacuum condition is assumed to exist within the pore) as well as radiative exchange between all elements having sides that make up the pore surface.

A two-dimensional unit cell is employed for all calculations. This decision was necessitated primarily by two considerations. First, a simple method for calculating the required view factors for use in the radiative exchange equations exists in two dimensions, namely Hottel's crossed-string method. In three dimensions, calculation of view factors between arbitrarily oriented planes is computationally difficult and intensive and would severely limit the number of different unit cells that could be studied. Second, a three-dimensional finite element solution, even disregarding the view factor calculation, is very computationally intensive and would further drastically reduce the number of different unit cells that could be studied. Given that this work is a first study of the radiative component of the heat transport in porous fuels, it seems more informative to perform a comprehensive study of a two-dimensional model system than a mere survey of a few three-dimensional systems. Hopefully, the results of this work will provide the information necessary to conduct a directed and productive analysis of the more interesting three dimensional systems.

In this investigation, the unit cell descriptions are determined in the following way. First, a pore size and shape as well as a porosity are selected, say a 50 μm circular pore and a porosity of 10% for example. The porosity is defined as the area of the pore divided by the area of the square unit cell, so these selections determine the unit cell edge dimension to be 140.125 μm . Next, a temperature and an average temperature gradient are chosen; the pore center is assumed to be at the temperature of interest, and the two boundary temperatures are then calculated to yield the desired temperature gradient. If a pore temperature of 2000 K is desired with an average temperature gradient of 6000 K/cm, the boundary conditions of two opposite unit cell sides must be 1957.962 K and 2042.038 K. The description of these parameters completely describes a specific problem. Those elements within the solid region of the problem are assigned the temperature-dependent properties of fully-dense UO_2 . The elements within the pore are assigned either vacuum properties ($k = 0$) or the temperature-dependent properties of helium or fission gas. The sides of the elements that lie within the UO_2 region but make up the pore surface are assign the temperature-dependent emissivity of UO_2 . The next

section describes the finite element formulation of the problem.

3.2 THE FINITE ELEMENT METHOD

In this section the problem defined in section 3.1 is formulated in a way that can be solved using the Galerkin finite element method. The method and solution procedure that are developed in this section have been incorporated into an original FORTRAN program called CARESS: Conduction And Radiation Enclosure Simulation Software. The program was written to be a very general software package for the solution of two-dimensional conduction problems. The program can handle a variety of external boundary conditions as well as radiating enclosures within the calculational domain. The program is fully nonlinear using an iterative solution technique, and for that reason can employ user-supplied temperature dependent material property correlations. For brevity, however, only those aspects of the program specific to the solutions required in this work are developed and described in this section.

In the following paragraphs the equations governing heat conduction in a solid material are presented and manipulated to give a form suitable for solution by the finite element method, the finite element formulation of the problem is developed, the required interpolation functions are obtained from heuristic arguments, the numerical integration method to be employed is presented, and assembly of the element equations, imposition of the boundary conditions, and the solution of the resulting system of equations is described. The complete FORTRAN listing of the program can be found in Appendix B.

3.2.1 Variational Formulation

The equation that must be solved to obtain the steady-state temperatures within an arbitrary two dimensional solid region is the Poisson or heat conduction equation (Carslaw and Jaeger, 1947):

$$\frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + S = 0. \quad (3.1)$$

In order to implement a finite element solution of equation (3.1), it is necessary to recast the equation in an equivalent weighted residual form. It is further desirable to reduce the weighted residual form to a weak form using integration by parts. Casting equation (3.1) in weighted residual form yields:

$$\int_{\Omega^e} v \left\{ \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + S \right\} dx dy = 0, \quad (3.2)$$

where v is an arbitrary test or weight function that can be thought of as a variation of T and Ω^e denotes the two dimensional domain of the problem. Moving the weight function within the spatial derivatives and subtracting off the additional terms generated by this procedure gives rise to the following equation:

$$\int_{\Omega^e} \left\{ \frac{\partial}{\partial x} \left(v k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(v k \frac{\partial T}{\partial y} \right) + v S - k \frac{\partial v}{\partial x} \frac{\partial T}{\partial x} - k \frac{\partial v}{\partial y} \frac{\partial T}{\partial y} \right\} dx dy = 0. \quad (3.3)$$

The first two terms of equation (3.3) can now be transformed from two dimensional integrals into a line integral along the boundary of the problem domain by using the gradient theorem. Performing this transformation yields the weak form of the governing equation for heat conduction within the two dimensional domain:

$$\int_{\Gamma^e} (v q_n) ds - \int_{\Omega^e} \left\{ k \frac{\partial v}{\partial x} \frac{\partial T}{\partial x} + k \frac{\partial v}{\partial y} \frac{\partial T}{\partial y} - v S \right\} dx dy = 0, \quad (3.4)$$

where q_n is the normal heat flux along the boundary of the computational domain Γ^e . Equation (3.4) is the weak variational form of the two dimensional heat conduction equation and represents the governing equation for heat conduction in integral form that must be solved; it is ideally suited to solution by the finite element method (Reddy, 1984).

3.2.2 Finite Element Formulation

To formulate the problem of solving equation (3.4) using the finite element method, it is necessary to introduce an approximating function for the solution variable, temperature. Since first derivatives are the highest order of differentiation appearing in the weak form, a bilinear approximating function is the minimal approximation that can be made while assuring that the heat flux q_n and the remaining derivatives in equation (3.4) are not zero by default. Higher order approximations are often used, however, to achieve

the same or better accuracy with fewer elements. In this work, 8-node biquadratic serendipity elements are used. Let the temperature anywhere within the element be approximated as the sum of the product of the nodal temperatures and interpolation functions:

$$T = \sum_{j=1}^8 T_j \Psi_j, \quad (3.5)$$

where the T_j are the values of temperature at the nodes located at (x_j, y_j) , and Ψ_j are biquadratic interpolation functions (the explicit form of the interpolation functions will be derived in a subsequent section). Furthermore, the arbitrary weight function v in the weak variational form represents a variation in temperature. Thus, from equation (3.5) the appropriate approximation to make for v is the following:

$$v = \delta T = \sum_{j=1}^8 \delta T_j \Psi_j, \quad (3.6)$$

where δ is the variational operator. By substituting equations (3.5) and (3.6) into equation (3.4), performing the required differentiations, and grouping terms appropriately one obtains the following finite element equations:

$$\sum_{j=1}^8 \left\{ \int_{\Omega^e} \left[k \frac{\partial \Psi_i}{\partial x} \frac{\partial \Psi_j}{\partial x} + k \frac{\partial \Psi_i}{\partial y} \frac{\partial \Psi_j}{\partial y} \right] dx dy \right\} T_j = \int_{\Omega^e} \Psi_i Q dx dy + \int_{\Gamma^e} \Psi_i q_n ds. \quad (3.7)$$

$i = 1, 2, \dots, 8$

This equation can be written in a more compact matrix form as:

$$\sum_{j=1}^8 K_{ij}^{(e)} T_j^{(e)} = F_i^{(e)}, \quad i = 1, 2, \dots, 8 \quad (3.8)$$

where K_{ij} is termed the conductivity matrix, F_i is the nodal heat flux vector, and T_j is the unknown temperature vector. Comparison of equations (3.7) and (3.8) give rise to the

following definitions:

$$K_{ij}^{(e)} = \int_{\Omega^e} \left[k \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial x} + k \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial y} \right] dx dy, \text{ and} \quad (3.9)$$

$$F_i^{(e)} = \int_{\Omega^e} \psi_i Q dx dy + \int_{\Gamma^e} \psi_i q_n ds. \quad (3.10)$$

Equations (3.8)-(3.10) represent the finite element formulation of equation (3.4).

3.2.3 Interpolation Functions

The element type employed by CARESS in the finite element solution of equation (3.8) is an 8-node quadrilateral element. The 8-node quadrilateral element gives rise to the biquadratic interpolation functions introduced in equation (3.5), and the form of these interpolation functions are derived in this section. To make the derivation easier and to aid in performing subsequent integrations, the interpolation functions are derived for the quadrilateral element shown in Fig. 3.1; this normalized element defined in a natural coordinate system is called the "master element."

Fig. 3.1 The master element

The element shown in Fig. 3.1 has its eight nodes labeled and located as shown in (ξ, η) space. The biquadratic serendipity approximation for T in this coordinate system is of the form:

$$T(\xi, \eta) = c_1 + c_2 \xi + c_3 \eta + c_4 \xi \eta + c_5 \xi^2 + c_6 \eta^2 + c_7 \xi \eta^2 + c_8 \xi^2 \eta. \quad (3.11)$$

Unlike the Lagrangian class of interpolation functions, however, the interpolation functions for a serendipity element cannot be obtained directly from tensor products of one-dimensional interpolation functions that are easily obtained. The heuristic approach of Reddy (1984) will be used here to construct the 8-node serendipity interpolation functions in (ξ, η) space. Consider the functions shown in Fig. 3.2.

Fig. 3.2 Consideration of the interpolation functions

The interpolation function for node 1, ψ_1 , should have a value of one at node 1 and a value of zero at the remaining seven nodes; ψ_1 should vanish on the sides of the master element defined by the equations $1 - \xi = 0$ and $1 - \eta = 0$, as well as on the line defined by $1 + \xi + \eta = 0$. To satisfy these requirements, ψ_1 must have the form:

$$\psi_1(\xi, \eta) = c (1 - \xi) (1 - \eta) (1 + \xi + \eta) . \quad (3.12)$$

The value of the constant c must be -0.25 to yield a value of one at node 1; thus the interpolation function ψ_1 is determined. The remaining seven interpolation functions are found in exactly the same way. The full set of interpolation functions for this 8-node, biquadratic serendipity element are:

$$\psi_1 = 0.25 (1 - \xi) (1 - \eta) (-1 - \xi - \eta) , \quad (3.13a)$$

$$\psi_2 = 0.50 (1 - \xi^2) (1 - \eta) , \quad (3.13b)$$

$$\psi_3 = 0.25 (1 - \xi) (1 - \eta) (-1 + \xi - \eta) , \quad (3.13c)$$

$$\psi_4 = 0.50 (1 + \xi) (1 - \eta^2) , \quad (3.13d)$$

$$\psi_5 = 0.25 (1 + \xi) (1 + \eta) (-1 + \xi + \eta) , \quad (3.13e)$$

$$\psi_6 = 0.50 (1 - \xi^2) (1 + \eta) , \quad (3.13f)$$

$$\psi_7 = 0.25 (1 - \xi) (1 + \eta) (-1 - \xi + \eta) , \text{ and} \quad (3.13g)$$

$$\psi_8 = 0.50 (1 - \xi) (1 - \eta^2) . \quad (3.13h)$$

These ψ_j are the interpolation functions required in equation (3.5).

3.2.4 Numerical Integration and the Use of the Interpolation Functions

The interpolation functions were derived for the master element as defined in Fig. 3.1 for an important reason, namely so that Gauss-Legendre quadrature could be easily implemented to evaluate the integrals appearing in equations (3.9) and (3.10). Gauss-Legendre quadrature approximates an integral in the following way (Press et al., 1986):

$$\int_{-1}^1 F(x) dx \approx \sum_{i=1}^n w_i F(x_i) , \quad (3.14)$$

where $F(x)$ is the integrand, $F(x_i)$ is that integrand evaluated at quadrature points x_i , w_i are weights, and n is the number of quadrature points. The evaluation points x_i and weights w_i are tabulated parameters that depend on n , the number of quadrature points. This method can easily be extended to integrations carried out in two dimensions. Written for an integration over the "master element," Gauss-Legendre quadrature takes the form:

$$\int_{-1}^1 \int_{-1}^1 F(\xi, \eta) d\xi d\eta \approx \sum_{i=1}^n \sum_{j=1}^n w_i w_j F(\xi_i, \eta_j) , \quad (3.15)$$

where $F(\xi_i, \eta_j)$ is the integrand evaluated at the quadrature points on the master element. Three point Gauss-Legendre quadrature is used in conjunction with quadratic elements; this quadrature rule dictates that $n=3$, $\xi_i, \eta_j = \pm 0.774596669241483...$, and 0.0 with corresponding weights of $w_i, w_j = 0.555...$, and 0.888... This quadrature rule integrates exactly a fourth order polynomial. Implementation of this integration procedure within CARESS is straightforward; the integrand is simply evaluated at all of the quadrature points (3 for a single integral, 9 for a double integral), multiplied by the appropriate weights, and the results summed.

The integration carried out over the master element in (ξ, η) space must be transformed to (x, y) space to obtain the value of that integral over the real element. This transformation is accomplished using the following procedure (Reddy, 1984):

$$\iint dx dy = \iint \det[J] d\xi d\eta , \quad (3.16)$$

where J is termed the Jacobian matrix. An integral including the determinant of the Jacobian matrix carried out in (ξ, η) space is equivalent to the integral in (x, y) space. The Jacobian matrix and its inverse have the following definitions:

$$[J] = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix} , \text{ and} \quad (3.17)$$

$$[J^{-1}] = \begin{bmatrix} \frac{\partial \xi}{\partial x} & \frac{\partial \eta}{\partial x} \\ \frac{\partial \xi}{\partial y} & \frac{\partial \eta}{\partial y} \end{bmatrix} . \quad (3.18)$$

Expressions for the derivatives appearing in equation (3.17) are known if isoparametric elements are used. Isoparametric elements are elements in which the same interpolation functions are used to describe both the geometry of the element and the variation of the temperature over the element. The following equations allow for a 1-to-1 mapping

between the (x,y) and (ξ,η) spaces and provide a means of obtaining explicit expressions for the derivatives in equation (3.17):

$$x = \sum_{j=1}^8 x_j \psi_j, \text{ and} \quad (3.19a)$$

$$y = \sum_{j=1}^8 y_j \psi_j. \quad (3.19b)$$

where (x_j, y_j) are the nodal coordinates of an element and the ψ_j are the interpolation functions defined in equations (3.13). Differentiation of equations (3.19) with respect to ξ and η yield the needed expressions for the derivatives appearing in the Jacobian matrix:

$$\frac{\partial x}{\partial \xi} = \sum_{j=1}^8 x_j \frac{\partial \psi_j}{\partial \xi}, \quad (3.20a)$$

$$\frac{\partial y}{\partial \xi} = \sum_{j=1}^8 y_j \frac{\partial \psi_j}{\partial \xi}, \quad (3.20b)$$

$$\frac{\partial x}{\partial \eta} = \sum_{j=1}^8 x_j \frac{\partial \psi_j}{\partial \eta}, \text{ and} \quad (3.20c)$$

$$\frac{\partial y}{\partial \eta} = \sum_{j=1}^8 y_j \frac{\partial \psi_j}{\partial \eta}. \quad (3.20d)$$

The derivatives of the interpolation functions with respect to ξ and η can be computed explicitly from equations (3.13). Thus, the Jacobian matrix can be filled at a given quadrature point by knowing the coordinates (x_j, y_j) of the nodes of an element; the determinant of the Jacobian matrix is computed directly as:

$$\det [J] = \frac{\partial x}{\partial \xi} \frac{\partial y}{\partial \eta} - \frac{\partial y}{\partial \xi} \frac{\partial x}{\partial \eta}, \quad (3.21)$$

and the inverse Jacobian matrix is:

$$[J^{-1}] = \begin{bmatrix} \frac{\partial \xi}{\partial x} & \frac{\partial \eta}{\partial x} \\ \frac{\partial \xi}{\partial y} & \frac{\partial \eta}{\partial y} \end{bmatrix} = \frac{1}{\det[J]} \begin{bmatrix} \frac{\partial y}{\partial \eta} & -\frac{\partial y}{\partial \xi} \\ -\frac{\partial x}{\partial \eta} & \frac{\partial x}{\partial \xi} \end{bmatrix}. \quad (3.22)$$

Calculation of the inverse Jacobian matrix is necessary since the derivatives obtained from that matrix are required to express the derivatives of the interpolation functions with respect to global coordinates in terms of known derivatives with respect to the natural coordinates. For example, the derivatives appearing in equation (3.9) are defined by the chain rule as:

$$\frac{\partial \psi_I}{\partial x} = \frac{\partial \psi_I}{\partial \xi} \frac{\partial \xi}{\partial x} + \frac{\partial \psi_I}{\partial \eta} \frac{\partial \eta}{\partial x}, \text{ and} \quad (3.23)$$

$$\frac{\partial \psi_I}{\partial y} = \frac{\partial \psi_I}{\partial \xi} \frac{\partial \xi}{\partial y} + \frac{\partial \psi_I}{\partial \eta} \frac{\partial \eta}{\partial y}. \quad (3.24)$$

Here, the derivatives of the interpolation functions with respect to the natural coordinates are known explicitly from differentiating equations (3.13), and the derivatives of the natural coordinates with respect to the global coordinates are simply the entries in the inverse Jacobian matrix, equation (3.22).

Equations (3.15) and (3.16) along with the procedure described above for obtaining the necessary derivatives provide all the tools needed for evaluating the double integrals that appear in the finite element equations (3.9) and (3.10). The single integral for the heat flux along a boundary appearing in equation (3.10) requires special treatment. As an example, consider the line integral along the boundary from nodes 1 to 3 in the master element of Fig. 3.1. The integral must be performed along the arc s , whose trajectory in (x,y) space is given by:

$$ds = [(dx)^2 + (dy)^2]^{1/2}, \quad (3.25)$$

or in terms of the local coordinates:

$$ds = [\left(\frac{\partial x}{\partial \xi} \right)^2 + \left(\frac{\partial y}{\partial \xi} \right)^2]^{1/2} d\xi, \quad (3.26)$$

where the square-root term preceding the differential on the right hand side of equation (3.26) is in effect a one-dimensional Jacobian. In this example, one dimensional Gauss-Legendre quadrature is employed to evaluate the boundary integral. Along the boundary between nodes 1 and 3 of the master element $\eta = -1$, so the function is evaluated at the quadrature points for ξ only $((\xi = -0.77459..., \eta = -1), (\xi = 0.0, \eta = -1), \text{ and } (\xi = +0.77459..., \eta = -1))$; the contributions of each of these evaluations times the corresponding weights are summed to approximate the boundary integral.

3.2.5 Assembly of the Element Equations

Assembly of the element equations is straightforward. Equation (3.8) is written and all of the matrix entries computed for each element within the calculational domain. The element conductivity matrix $K^{(e)}$ is an 8x8 matrix that is symmetric with respect to the diagonal. Equation (3.9) defines each of the sixty-four entries that appear in $K^{(e)}$. The element heat flux vector $F^{(e)}$ is a column vector with 8 entries defined by equation (3.10). If heat generation were present within an element, the first integral of equation (3.10) would generate equivalent nodal heat loads; this term is zero for all computations required in this work since no internal heat sources are present within elements. If a heat flux is specified along an element boundary, the second integral of equation (3.10) generates equivalent nodal heat fluxes. These equivalent nodal heat fluxes are non-zero only when a heat flux along an element boundary has been "specified" (e.g. most often a boundary condition, but also a thermal radiation heat flux on an element side that borders the pore surface). Otherwise these heat fluxes are cancelled by opposing normal vectors of adjacent elements when the global heat flux vector is assembled since continuity of the heat flux between two elements sharing a common boundary is required. In this work, this heat flux component to the element heat flux vector is only computed for elements having a side that defines a portion of the pore surface where thermal radiative exchange occurs.

Once the element conductivity matrix and heat flux vector is constructed, it is entered into its appropriate location in a global conductivity matrix and heat flux vector that represents the collection of all the elements that make up the entire calculational domain. The location of the element level entries in the global conductivity matrix and heat flux vector is obtained via the Boolean connectivity matrix. This matrix is simply a collection of the global node numbers for each of the nodes of an element. In CARESS, this

connectivity matrix is represented by the array NODCON(I = element number, J = local node number as in Fig. 3.1), where the node numbers run from 1 to 8, and the I,J entry of NODCON(I,J) holds the global node number for the node. The element conductivity matrix and heat flux vector entries are summed with any values already occupying their various proper locations in the global conductivity matrix and heat flux vector since more than one element level entry can contribute to a global level entry.

3.2.6 Imposition of the Boundary Conditions

Once the global conductivity matrix and heat flux vector are assembled, the imposition of the essential boundary conditions (i.e. specified temperature conditions) remains before the system is solved; specified heat fluxes have already been incorporated into the heat flux vector at the element level. There are several techniques available for imposing the specified temperature conditions. In CARESS, an "engineering" solution to the problem is used. If the temperature at node i is specified, then the diagonal entry on row i of the conductivity matrix is changed to $1.0D+30$, and the i -th row of the heat flux vector is changed to $1.0D+30$ times the value of the specified temperature. This is the simplest way of imposing the specified temperature condition.

3.2.7 Solution of the Equations

After imposition of the essential boundary conditions, the resulting global conductivity matrix and heat flux vector represent a system of N linear equations in N unknowns, where N is the total number of nodes in the finite element model. This system can be solved by any of the standard techniques for solving such linear systems including matrix inversion, Gauss-Seidel iteration and Gaussian elimination. In CARESS, the system is solved by Gaussian elimination. As stated previously, the global conductivity matrix is diagonally symmetric and sparsely populated. This allows an optimized banded procedure to be used in computing and storing the upper triangular portion of the conductivity matrix only. Since the structure of the conductivity matrix is known in advance, an optimized form of Gaussian elimination is employed to solve the system (Becker et al., 1981). This results in vastly less memory use as well as a major reduction in the time required to solve the system of equations. This time savings is important since the overall problem must be solved iteratively. Temperature dependent material properties make the conductivity matrix nonlinear; temperature dependent radiation heat fluxes within the enclosure make the heat flux vector nonlinear. Thus, temperatures must initially be

guessed, the material properties and radiation heat fluxes computed based on these guessed temperatures, and these estimates used to construct the initial system of equations. The new temperatures obtained by solution of the system are saved and used to compute updated property and radiation heat flux estimates to resolve the system. Convergence is declared when the temperatures cease to change from iteration to iteration.

A copy of the global conductivity matrix and heat flux vector is retained in which no boundary conditions have been incorporated. Additionally, the heat flux vector is saved in its two components, the nodal heat loads due to internal heat generation (which is always zero in the present calculations) and the nodal heat fluxes due to Neumann boundary conditions. These copies are retained so that the nodal heat fluxes along a constant temperature problem boundary can be extracted from the heat flux vector once all the nodal temperatures are known. The sum of these particular boundary heat fluxes gives the heat flow through the unit cell, Q_p .

3.3 ENCLOSURE RADIATION PROBLEM

Enclosure radiation occurring along the pore surface is readily handled within the finite element formulation previously presented. Those matrix elements having sides that form a part of the pore surface have a non-zero q_n that is due to a net heat flux either entering or leaving the element due to the radiative exchange with all the other elements that constitute the pore surface. Determination of the net heat fluxes for each element side adjacent to the enclosure is performed following each iteration using the current estimates for nodal temperatures. Each element side is assumed to be at a constant temperature equal to the average of the three nodal temperatures that define the element's side. Thus, the enclosure problem is one of determining the net heat fluxes within an enclosure of known temperature. CARESS employs the diffuse-gray approximation and radiosity formulation of Siegel and Howell (1981) to find the net heat fluxes on the enclosure boundary that is described in this section.

3.3.1 Assumptions

The diffuse-gray approximation is employed by CARESS to compute the radiosities and subsequently the net heat transferred at each surface within the enclosure. The medium bounded by the enclosure is assumed to be non-participating, but can conduct heat. The following section outlines the assumptions that are used to develop and solve the radiative transfer equations within the enclosure. It is assumed that the enclosure

boundary is divided into areas (element sides) such that over each of these areas the following assumptions are reasonable: the temperature is uniform, the surface radiation properties are independent of radiation wavelength or direction so that $\epsilon(T) = \alpha(T) = 1 - \rho(T)$, all energy is emitted and reflected diffusely from each surface, the incident and hence reflected energy flux is uniform over each individual area, and the temperature of each area is known. With these assumptions, the equations that describe the radiative transfer within the enclosure can be derived in a very straightforward manner.

3.3.2 Derivation of Equations of Transfer

Consider the general enclosure shown below having N sides.

Fig. 3.3 Radiosity notation

All of the mechanisms of energy incidence and emission are shown above for an arbitrary enclosure surface i . By definition, the radiosity of surface i is the sum of the radiant energy leaving the surface, both due to thermal emission and reflection of incident energy:

$$J_i = \epsilon_i E_{bi} + \rho_i G_i, \quad (3.27)$$

and the irradiation G is computed from the energy incident on surface i from all the other surfaces within the enclosure:

$$A_i G_i = \sum_{j=1}^N A_j F_{ji} J_j . \quad (3.28)$$

Equation (3.28) can be transformed by the view factor reciprocity relation to appear as:

$$G_i = \sum_{j=1}^N F_{ij} J_j . \quad (3.29)$$

From the second assumption of the previous section, the reflectivity of surface i is related to the emissivity of the surface as:

$$\rho_i = 1 - \epsilon_i . \quad (3.30)$$

Substituting equation (3.30) into equation (3.27) yields:

$$J_i = \epsilon_i E_{bi} + (1 - \epsilon_i) G_i . \quad (3.31)$$

Rearranging equation (3.31) gives rise to the following expression for the irradiation incident upon surface i :

$$G_i = \frac{J_i - \epsilon_i E_{bi}}{1 - \epsilon_i} . \quad (3.32)$$

Finally, the net heat transferred to the surface i is found directly from the simple heat balance:

$$Q_i = (J_i - G_i) A_i . \quad (3.33)$$

Substituting equation (3.32) into equation (3.33) provides an expression for the net heat transferred to surface i that is a function of only one unknown, the radiosity of surface i

(the emissive power of a black body can be found from the Stefan-Boltzmann law since the temperature of the surface is known):

$$Q_i = \frac{A_i \epsilon_i}{1 - \epsilon_i} [E_{bi} - J_i] . \quad (3.34)$$

A second expression for the net heat transferred to the surface i is obtainable by substituting equation (3.29) into equation (3.33) to give:

$$Q_i = A_i \left[J_i - \sum_{j=1}^N F_{ij} J_j \right] . \quad (3.35)$$

Equating equations (3.34) and (3.35) and rearranging the resulting expression yields the following equation for surface i :

$$\left(\frac{1}{1 - \epsilon_i} \right) J_i - \sum_{j=1}^N F_{ij} J_j = \left(\frac{\epsilon_i}{1 - \epsilon_i} \right) \sigma T_i^4 , \quad \text{for } j=1, \dots, N. \quad (3.36)$$

Note that this equation can be applied to each surface of the enclosure and results in a system of N equations in N unknowns, namely the radiosities J_i . The Stefan-Boltzmann law for computing the black body emissive power has been inserted into equation (3.36); the temperature of each surface of the enclosure is taken as known.

This system of linear simultaneous equations is solved using Gaussian elimination following each iteration cycle to obtain an improved estimate of the radiosity of each enclosure surface based on updated temperatures. The net heat transfer out (+) of each surface is computed using equation (3.34) and incorporated into the heat flux vector during the next iteration cycle.

3.4 PROGRAM EXECUTION

The previous section described how the radiation heat fluxes at the pore surface are incorporated into the overall finite element solution to yield the temperatures within the unit cell and subsequently the net heat flow through the unit cell. This section describes the way the program CARESS integrates all the above procedures to obtain the desired temperatures and heat transfer rate.

Since the overall problem is nonlinear due to temperature dependent material

properties as well as the enclosure radiation problem, the solution is accomplished iteratively. Initially, the unit cell is assumed to be at a uniform, arbitrary temperature (1000 K). The material properties for each element are evaluated at this guessed temperature, and the radiation heat fluxes are computed from a radiosity solution within the pore. With the material properties and enclosure heat fluxes assumed known from this initial estimate, the global conductivity matrix and heat flux vector are assembled and solved by Gaussian elimination. The resulting temperatures are saved. These new temperatures are used to begin a new iteration by re-evaluating the material properties of each element and recomputing the radiation heat fluxes within the enclosure. A new global conductivity matrix and heat flux vector are assembled and solved, and the new temperatures are compared with those obtained from the previous iteration. This process is repeated until the maximum temperature change between iterations is no more than 10^{-10} . When the nodal temperatures have converged, these temperatures are used to extract the nodal heat fluxes from the heat flux vector for nodes along one of the constant temperature edges of the unit cell. The sum of these heat flux values yields the net heat flow through the unit cell. This iterative procedure employed by CARESS for the unit cell calculations of this work are shown in a flow chart given in Fig. 3.4.

Fig. 3.4 Calculation flow chart

CHAPTER IV

PRELIMINARY CONSIDERATIONS

4.1 INTRODUCTION

The previous chapter presented the numerical methods used in the required calculations and their incorporation into an original FORTRAN program. In this chapter a few preliminary considerations are addressed. These considerations include benchmarking studies of the program, generation of the finite element meshes, and convergence studies to qualify candidate mesh designs. This chapter concludes by determining and presenting a list of required calculations to fully investigate the radiative heat transfer component in porous UO_2 fuel.

4.2 BENCHMARKING STUDIES

Extensive benchmarking studies were conducted to verify the ability of CARESS to obtain accurate solutions. Three simple benchmark calculations are presented in this section with two goals in mind. First, the benchmark calculations are necessary to verify that the program functions physically and as expected. Second, comparison of the finite element solutions with the exact solutions for these problems will provide insight into the accuracy of the finite element method. This section presents the results of three representative benchmark calculations.

4.2.1 Benchmark Problem 1

The first benchmark problem was selected to verify that CARESS handles inhomogeneous essential and natural boundary conditions properly. The calculations consisted of computing the temperature field within a square domain with sides of length 1 m. On the top boundary a uniform heat flux of 5 W/m^2 was imposed, while the bottom boundary was held at a constant temperature of 10 K. Insulation boundary conditions (heat flux specified to be zero) were imposed on the left and right boundaries. No internal heat generation within the square domain was modeled, and the thermal conductivity was taken everywhere within the region to be 1 W/(m-K) . This is a simple problem that reduces to a one-dimensional heat conduction problem; no heat flow occurs in the left or right directions. The temperature at the top boundary should be 15 K, and the temperature

varies linearly between the top and bottom boundaries.

To make the calculation of the temperature field for this problem using CARESS, the square domain of the problem was divided uniformly into 16 square elements (4x4). Since the exact solution is known to be a linear function (the interpolation functions are biquadratic) and the elements used were square (the three-point Gauss-Legendre quadrature rule performs the necessary integrations exactly since the Jacobian matrix is constant for square elements), the finite element solution returned by CARESS should be the exact solution. Indeed, inspection of the temperatures throughout the domain and the heat fluxes along the top and bottom boundaries calculated using CARESS show exact agreement with the analytical solution as expected. Experimentation was performed by deforming the 16 elements such that they were no longer squares but general quadrilaterals. In all cases the nodal temperatures calculated by CARESS were exact to the number of significant digits provided in the output file. This result provided confidence in the ability of CARESS to provide accurate solutions when the Jacobian matrix for an element is not constant within elements.

4.2.2 Benchmark Problem 2

The second benchmark problem was chosen to be a fully two-dimensional problem including internal heat generation. The domain of the problem was identical to that in problem 1. The top and right boundaries were held at a constant temperature of 0.0 K, and insulation boundary conditions were imposed on the bottom and left boundaries. The thermal conductivity of the region was again taken as 1 W/(m-K), and a constant heat generation rate of 1 W/m³ was assigned within the region. Physically, the hottest temperature must occur in the bottom-left corner, the coolest temperatures will be zero along the top and right boundaries (as imposed), and symmetry will exist about the diagonal extending from the bottom-left to the top-right corners. Within the domain, however, the variation of temperature will be a complex function that biquadratic interpolation functions will not be able to match exactly. The exact solution for this problem can be obtained as a convergent series using the method of separation of variables and has been supplied by Reddy (1984).

Since the finite element method as employed by CARESS is not expected to yield the exact solution to this problem, three different meshes were used for this calculation in an attempt observe the rate of convergence toward the exact solution with mesh refinement. The first mesh employed only 1 square element, the second mesh employed 4

square elements (2x2), and the third mesh employed 16 square elements (4x4) to model the calculational domain. The results of these calculations are summarized and compared with the series solution in Table 4.1.

Table 4.1 Convergence of the finite element solution with mesh refinement for benchmark problem 2

Location (m)		Finite Element Solution			Series Solution
x	y	1x1	2x2	4x4	
0.00	0.00	0.26966	0.29410	0.29465	0.29469
0.25	0.00	-	0.27904	0.27885	0.27888
0.50	0.00	0.24017	0.22917	0.22931	0.22934
0.75	0.00	-	0.13962	0.13971	0.13973
0.25	0.25	-	-	0.26411	0.26415
0.50	0.50	-	0.17924	0.18109	0.18114
0.75	0.75	-	-	0.07239	0.07332

As seen from Table 4.1, the very crude 1x1 mesh yields temperatures that differ from the series solution by roughly 5-8%. Refinement of the mesh to a still coarse mesh of 2x2 elements brings the temperatures to within 0.1% agreement; the mesh of 4x4 elements yields temperatures within ~0.01% of the series solution. Further refinement could bring the agreement even closer if greater accuracy is required.

4.2.3 Benchmark Problem 3

A final benchmark problem is presented to illustrate the ability of CARESS to accurately solve problems involving radiation heat transfer. CARESS is a two-dimensional finite element program, but two-dimensional analytical solutions to problems involving radiation heat transfer are not abundant. Consider, however, the radiant heat transfer between two infinitely long, parallel, gray plates. It is easily shown that the net radiant heat flux between two such plates, labelled 1 and 2, is (Bird et al., 1960):

$$q_{12} = \frac{\sigma (T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}. \quad (4.1)$$

If the two plates are at temperatures of 2000 K and 1000 K and both have emissivities of 0.1, the net radiant heat flux between the two plates is 32,979.44 W/m². This exact

problem cannot be solved directly using the capabilities of CARESS, but it is possible to obtain an approximate solution by studying the model shown below in Fig. 4.1.

Fig. 4.1 Benchmark problem 3

The eight-element finite element model shown in Fig. 4.1 was analyzed using CARESS for increasingly large ratios of w/t . As the w/t ratio becomes large, the modeled system appears increasingly similar to two infinite plates and the net heat transfer solution must asymptotically approach the value expected for two infinite plates. This convergence is indeed observed in the CARESS analysis. For the w/t ratios of $10^2:1$, $10^4:1$ and $10^8:1$, the following net heat transfer rates were obtained: $32,970.76 \text{ W/m}^2$; $32,979.35 \text{ W/m}^2$; and $32,979.44 \text{ W/m}^2$, respectively.

These three benchmark problems were not the only benchmarking studies performed. Many other academic problems were successfully analyzed using CARESS. The three problems discussed in this section were selected for presentation to illustrate the capabilities of CARESS that are specifically needed to perform the unit cell heat transfer calculations required in this work.

4.3 MESH GENERATION

A multitude of different meshes are required for all the various unit cells needed to adequately explore the dependence of the radiative heat transfer component on porosity, pore size, pore shape and orientation. Mesh generation is the process of subdividing the entire calculational domain into interlocking subdomains called elements and determining the coordinates of the nodes. It is simply a geometrical problem. The meshes used in the previous benchmarking problems were simple and could be designed by hand. More complex meshes such as those required in the unit cell calculations are rarely designed without computer aid. For generation of the meshes used in this work the commercial software package PATRAN® was employed.

Two different types of meshes are required in the present investigation. The simplest mesh needed is a square domain containing a central circle. A mesh containing a circular pore requires the specification of the total porosity and the pore diameter for complete definition. Also required are square domains containing a central ellipse. For ellipses, two additional parameters are required for an unambiguous description of the mesh. First is a geometrical factor g that is the ratio of the ellipse's major to minor diameters; this factor is 1 for a circle, and greater than 1 for ellipses. Second is the pore orientation angle θ . This angle is 0° for an oblate ellipse (one whose major diameter is perpendicular to the direction of heat flow) and 90° for a prolate ellipse (one whose major diameter is parallel to the direction of heat flow). Five example meshes are shown in Figs. 4.2 through 4.6. In each mesh, the direction of heat flow is assumed to be from top to bottom.

4.4 CONVERGENCE STUDY

The five example meshes shown in Figs. 4.2 through 4.6 are meshes that are used in actual calculations, the results of which will be given in the next chapter. Prior to the selection of those meshes acceptable for use in calculations, however, a sequence of convergence studies was necessary. As seen in the results of benchmark problem 2 given in Table 4.1, accurate solutions are obtained from finite element analyses only if the mesh employed is adequately fine. Convergence studies were performed to determine the level of refinement necessary to yield accurate solutions using meshes of the type shown in Figs. 4.2 through 4.6.

One such convergence study was performed for a mesh that was qualitatively equivalent to the one shown in Fig. 4.2. The mesh represented a unit cell containing a

Fig. 4.2 Circular pore mesh; $P=10\%$

Fig. 4.3 Circular pore mesh; $P=30\%$

Fig. 4.4 Elliptical pore mesh; $P=10\%$, $g=2$, $\theta=0^\circ$

Fig. 4.5 Elliptical pore mesh; $P=10\%$, $g=5$, $\theta=0^\circ$

Fig. 4.6 Elliptical pore mesh; $P=10\%$, $g=2$, $\theta=45^\circ$

200 μm circular pore. The porosity of the unit cell was 20%. The matrix elements were assigned UO_2 properties, the pore was assumed to be at vacuum, and the surface of the pore was allowed to emit and reflect radiant energy with the emissivity of UO_2 . This system was studied for various numbers of elements and nodes used to model the unit cell. The change in the calculated heat flow through the unit cell was monitored as the mesh was refined, and the results of this study are summarized in Table 4.2.

Table 4.2 Results of a typical convergence study

No. Elements	No. Nodes	Q	% Change
16	64	697.5492	-
64	224	699.6032	0.29359
144	480	699.6405	0.00533
256	832	699.6419	0.00020

The heat flow through the described unit cell is given in W in Table 4.2. Also shown is the change (in percent) in the calculated heat flow through the unit cell with each mesh refinement. This study and other similar studies using elliptical pore shapes and various porosities showed that meshes made up of ~ 100 elements and ~ 350 nodes yielded solutions that were consistently within 0.01% of the fine mesh limit solution. Thus, all of the meshes used in the calculations required for this work were designed in this range. The meshes shown in Figs. 4.2 and 4.3, for example, are made up of 96 elements having 305 nodes. The meshes of Figs. 4.4 and 4.5 are comprised of 112 elements having 353 nodes, and the mesh in Fig. 4.6 contains 112 elements having 321 nodes.

4.5 CALCULATIONS PERFORMED

Using CARESS and constructing the meshes as described in the previous section, all of the calculations shown in Table 4.3 were performed. For each pore shape described by the geometrical factor g , Table 4.3 shows the range of the parameters encompassed within the calculations. The porosity was varied in increments of 0.05; the pore major diameter was varied as 1, 10, 25, 50, 100, and 200 μm ; the pore orientation angle was varied as 0° , 5° , 10° , 15° , 22.5° , 30° , 45° , 60° , 67.5° , 75° , 80° , 85° , and 90° ; the temperature was varied in increments of 500 K; and the temperature gradient was varied in increments of 2000 K/cm. For circular pores, calculations were performed for pore compositions of vacuum, FP (fission gas, 85% Xe - 15% Kr), and helium. Since the

circular pore results showed only a slight difference between vacuum-filled and FP-filled pore compositions, the elliptical pore studies did not consider FP-filled pores. Finally, each calculation was performed both incorporating radiant energy emission and reflection within the pore and excluding the radiant component; comparison of the difference between these two cases yields the integral effect of the radiative heat transfer component on the overall heat transport through the unit cell.

Table 4.3 Calculations performed

g	P	d	θ	T	G	Pore Composition
1	.05-.50	1-200 μm	-	1000-3000 K	2000-10,000 K/cm	vacuum, FP, He
2	.05-.30	1-50 μm	0-90°	1000-3000 K	6000 K/cm	vacuum, He
3	.05-.20	1-50 μm	0,90°	1000-3000 K	6000 K/cm	vacuum, He
4	.05-.15	1-50 μm	0°	1000-3000 K	6000 K/cm	vacuum, He
5	.05-.10	1-50 μm	0°	1000-3000 K	6000 K/cm	vacuum, He
6	.05-.10	1-50 μm	0°	1000-3000 K	6000 K/cm	vacuum, He
7	0.05	1-50 μm	0°	1000-3000 K	6000 K/cm	vacuum, He
8	0.05	1-50 μm	0°	1000-3000 K	6000 K/cm	vacuum, He
9	0.05	1-50 μm	0°	1000-3000 K	6000 K/cm	vacuum, He
10	0.05	1-50 μm	0°	1000-3000 K	6000 K/cm	vacuum, He

The porosity correction factors obtained as a result of the calculations shown in Table 4.3 are tabulated in Appendix C. The next chapter describes these results and presents them in graphical form as a sequence of parametric plots. The contribution of the radiative heat transfer component is quantified by comparing calculations for identical unit cells in which the radiation mode was modeled versus neglected.

CHAPTER V

RESULTS AND DISCUSSION

5.1 CALCULATIONAL PROCEDURE

In this chapter the results of the porosity correction factor calculations specified in Table 4.3 are presented and discussed. As indicated by equation (1.5), each porosity correction factor was obtained as the ratio of two heat transfer calculations made using the program CARESS. The unit cells were defined and the required boundary conditions determined using the methodology outlined in section 3.1; these unit cells were then discretized into acceptable finite element meshes using the procedure described in section 4.3.

For each mesh with a specific set of boundary conditions, seven heat transfer calculations (CARESS runs) were made. The first heat transfer calculation was made with all elements within the unit cell, including those elements within the pore, assigned the thermal conductivity of UO_2 . This calculation provides Q_{100} . The next six calculations yield the Q_p values for the following pore conditions: pore at vacuum without radiative heat transfer within the pore, pore at vacuum with radiative heat transfer, fission gas-filled pore without radiative heat transfer within the pore, fission gas-filled pore with radiative heat transfer, helium-filled pore without radiative heat transfer within the pore, and helium-filled pore with radiative heat transfer. The vacuum pore condition is obtained by assigning the elements within the pore zero thermal conductivity; fission gas or helium properties are assigned to the elements within the pore for the other calculations. Finally, to obtain the results without radiative heat transfer occurring within the pore, the surface of the pore is assigned a zero emissivity instead of that of UO_2 . The relative difference between identical calculations for which radiation is and is not included gives the integral contribution radiative heat transfer provides to the overall heat transport in porous UO_2 .

The number of calculations that must be made to encompass the conditions outlined in Table 4.3 is quite large, so any reduction in parameters that can be made a priori is desirable. Analytical analyses of the radiation conductivity of a pore, such as that given in Appendix A, suggest a very weak dependence on temperature gradient. This weak dependence was investigated and confirmed numerically prior to the start of "production" runs. The temperature gradient was shown to influence the calculated

porosity correction factors only slightly. In Table 5.1 the porosity correction factors obtained for a unit cell of 5% porosity containing a circular, radiating, 200 μm pore at vacuum are shown for the extreme conditions of temperature gradient, G . Identical results for a unit cell of 30% porosity with a 100 μm pore are given in Table 5.2.

Table 5.1 Dependence of f on G ; $P=5\%$, $d=200 \mu\text{m}$

T (K)	f		Relative Change (%)
	$G=2,000 \text{ K/cm}$	$G=10,000 \text{ K/cm}$	
1000	0.9059013	0.9053309	0.0630070
1500	0.9097211	0.9089987	0.0794626
2000	0.9176499	0.9168447	0.0878178
2500	0.9259376	0.9263787	-0.0476194
3000	0.9319310	0.9322808	-0.0375216

Table 5.2 Dependence of f on G ; $P=30\%$, $d=100 \mu\text{m}$

T (K)	f		Relative Change (%)
	$G=2,000 \text{ K/cm}$	$G=10,000 \text{ K/cm}$	
1000	0.5396035	0.5395987	0.0009057
1500	0.5473161	0.5473197	0.0006404
2000	0.5638772	0.5639053	0.0049905
2500	0.5823023	0.5823267	0.0041858
3000	0.5963920	0.5964057	0.0023033

In the low porosity case the change in the calculated porosity correction factors between the two extremes of temperature gradient is on the order of $\sim 0.01\%$; the change for the high porosity case is even lower, $\sim 0.001\%$. The pore sizes employed (100 and 200 μm) are large and not often observed in nuclear fuel. The relative changes decrease with pore size, so the changes shown in Tables 5.1 and 5.2 can be regarded as upper limits; for more typical pore diameters of 10-30 μm the relative changes observed in f between 2,000 and 10,000 K/cm are $\sim 10^{-4}\%$. The very weak dependence on temperature gradient suggested by analytical studies and confirmed by numerical results allows a mean value for the temperature gradient (6000 K/cm) to be employed in all the subsequent porosity correction factor calculations.

5.2 POROSITY CORRECTION FACTOR RESULTS

The results of the porosity correction factor calculations for all of the different

geometries and conditions of porosity and temperature have been tabulated and appear in the 45 tables included in Appendix C. Tables C.1 through C.10 contain the results for circular pores of increasing porosity. Tables C.11 through C.15 contain the results for $g=2$ ellipses of 5% porosity and increasing in orientation angle from 0 to 90°. Tables C.16 through C.18 contain the results for $g=2$ ellipses of 10% porosity and orientation angles of 0, 45, and 90°. The remaining results for $g=2$ ellipses are ordered similarly in Tables C.19 through C.31. For ellipses with $g \geq 3$, only orientations of 0° were considered; Tables C.32 through C.45 contain the results for these ellipses. Within each table the results are broken down by pore diameters and mean pore temperatures and are collimated according to pore composition and whether or not the radiation mechanism is included.

If the porosity correction factors are plotted versus temperature with the pore gas and radiation condition as parameters, the influence due to pore composition can be seen. All such plots appear qualitatively similar to those shown in Figs. 5.1 and 5.2. Figure 5.1 shows the porosity correction factors for a unit cell of 20% porosity containing a 200 μm circular pore. The bottom, flat curve with solid square data points is the result obtained when the pore is assumed to be at vacuum and non-radiating (vac in legend). The next lowest curve, solid diamonds, results if the pore is non-radiating, but filled with fission gas (FP in legend). The conductivity of the fission gas mixture (85% Xe and 15% Kr) is so low that only a slight increase in heat transfer results when fission gas is added to the pore; for this reason, calculations with FP-filled pores were not made for most of the unit cells containing elliptical pores. The two intermediate curves show the role the radiation mechanism plays for vacuum-filled (vac+rad in legend) and fission gas-filled (FP+rad in legend) pores. The increase observed in f with temperature is due solely to the increase in radiation heat transfer within the pore. Finally, the two uppermost curves result when pure helium gas is present in the pore. In the case of a He-filled pore, significant temperature dependence is observed in f even when the radiation mechanism is not active (He in legend). This results from the fact that the thermal conductivity of helium gas is fairly high in general (about an order of magnitude less than the conductivity of UO_2) and increases logarithmically by a factor of two between 1000 and 3000 K. As seen from Fig. 2.1, the thermal conductivity of UO_2 decreases from 1000 to 2000 K and increases from 2000 to 3000 K. The composite result of these two temperature dependencies results in the temperature dependence shown in Fig. 5.1. When the radiation mechanism is active (He+rad in legend), however, the porosity correction factor increases with temperature

Fig. 5.1 f for $P=20\%$, $d=200\ \mu\text{m}$, and $g=1.0$

Fig. 5.2 f for $P=20\%$, $d=100\ \mu\text{m}$, and $g=1.0$

over the entire temperature range.

Note that the magnitude of the increase due to radiative heat transfer is least for the He-filled pore. This is due to the better conductivity of the helium which allows for more gas conduction. The gas conduction and radiation components are parallel, competing modes of heat transfer. Their influences on the overall heat transport through the unit cell are integrally inter-connected and cannot be correctly studied as separate phenomena. Figure 5.2 shows the same results as Fig. 5.1 except the pore diameter has been reduced from 200 to 100 μm . The radiative contribution in this situation is reduced. The parameters for these two figures were chosen to magnify the radiative component (e.g. high porosity and large pore diameter). The radiation contribution of more prototypical fuel is somewhat less and will be quantified in section 5.4.

5.3 PARAMETRIC VARIATION OF f

In the previous section the tabulated values for the calculated porosity correction factors were described, and the dependence of f on the pore composition and radiation condition was observed from Figs. 5.1 and 5.2. Plots similar to these can be constructed for all the various conditions of porosity, pore size, etc., but each appears qualitatively the same. In this section, the variation of f with the parameters porosity, temperature, pore size, pore shape, and pore orientation is observed. This is accomplished by a sequence of parametric plots given as Figs. 5.3 through 5.11.

Figures 5.3 and 5.4 show the dependence of f on the total porosity for a pore at vacuum and filled with helium, respectively, with the pore shape as a parameter. The abscissas for both of these figures have the same scale for ease of comparison. The porosity correction factor decreases with porosity in all cases with a "concave-up" behavior. Clearly, helium gas within the pore increases the overall heat transport through the unit cell via gas conduction and yields higher porosity correction factors. Also apparent is a decrease in f with increasing g for oblate elliptical pores, as expected. This decrease is less for helium-filled pores than for the vacuum pore condition due to helium conduction. As the g of an oblate ellipse is increased while the total porosity of the unit cell remains constant, the major diameter increases and the minor diameter decreases. An increase of the major pore diameter reduces conduction through the solid matrix by decreasing the cross-sectional area of the solid matrix perpendicular to the heat flow direction. If a vacuum exists within the pore, only an increase in radiation across the pore can counter the reduced heat transfer through the solid matrix; the radiation component

Fig. 5.3 Dependence of f on P ; vac+rad, $d=50 \mu\text{m}$, $\theta=0^\circ$, $T=3000 \text{ K}$

Fig. 5.4 Dependence of f on P ; He+rad, $d=50 \mu\text{m}$, $\theta=0^\circ$, $T=3000 \text{ K}$

Fig. 5.5 Dependence of f on T ; vac+rad, $d=50 \mu\text{m}$, $\theta=0^\circ$, $P=10\%$

Fig. 5.6 Dependence of f on T ; He+rad, $d=50 \mu\text{m}$, $\theta=0^\circ$, $P=10\%$

Fig. 5.7 Dependence of f on d ; vac+rad, $g=1.0$, $P=20\%$

Fig. 5.8 Dependence of f on d ; He+rad, $g=1.0$, $P=20\%$

Fig. 5.9 Dependence of f on g ; vac+rad, $d=50 \mu\text{m}$, $\theta=0^\circ$, $T=3000 \text{ K}$

Fig. 5.10 Dependence of f on g ; He+rad, $d=50 \mu\text{m}$, $\theta=0^\circ$, $T=3000 \text{ K}$

Fig. 5.11 Dependence of f on pore orientation; vac+rad, $d=50\text{ }\mu\text{m}$, $P=5\%$, $g=2.0$

does increase in this case (pore at vacuum) as g increases, but at a rate significantly less than the rate of decrease in solid conduction. When helium is present within the pore, however, gas conduction through the pore increases as g increases since the minor pore diameter (the conduction length through the pore) decreases. An increase in conduction through the pore as g increases serves to moderate the reduction in f compared to the vacuum condition.

Figures 5.5 and 5.6 show the dependence of f on the pore mean temperature for vacuum and helium-filled pores, respectively, again with the pore shape factor g for an oblate elliptical pore as a parameter. In Fig. 5.5 the slight increase in f with temperature is due entirely to radiative heat transfer within the pore since no pore conduction occurs; the dependence of f on the pore shape factor g for an oblate elliptical pore appears near-linear for conditions of constant porosity, pore size, and orientation. This is justifiable since the increase in g is linearly related to the decrease in the cross-sectional area of the solid matrix, and the heat conduction through the solid is directly proportional to the cross-sectional area from Fourier's law. The dependence of f on g is clearly not linear when the pore is filled with helium, as seen in Fig. 5.6. Again, an increase in helium gas conduction with increasing g explains this behavior. The overall temperature dependence observed in Fig. 5.6 is due primarily to the contrasting temperature dependencies of the UO_2 and helium thermal conductivities.

Figures 5.7 and 5.8 show the dependence of f on the pore diameter for vacuum and helium-filled pores (circular pores, $g=1$), respectively, with mean pore temperature as a parameter. As seen in these figures, the increase in radiative heat transfer is a near-linear function of pore size for a circular pore (actual exponent ~ 0.95). In Fig. 5.7 the increase in f with pore diameter is solely due to radiation heat transfer and can be quite large for pores $\sim 100 \mu\text{m}$ and larger. The curves for all three pore temperatures appear to meet for $1 \mu\text{m}$ diameter pores indicating that relatively no radiation heat transfer occurs through such small pores even at high temperatures and porosities. In Fig. 5.8 the increase in f with pore diameter for each constant temperature curve is due to radiation heat transfer.

Figures 5.9 and 5.10 show the dependence of f on the pore major-to-minor diameter ratio g for vacuum and helium-filled pores, respectively, with the total porosity of the unit cell as a parameter. All of these elliptical pores were oblate. For the pore at vacuum, a near-linear dependence of f on g is observed; again, this dependence is consistent with the linear relationship between increasing g , decreasing cross-sectional area of the solid matrix, and decreasing heat transfer through the solid by conduction.

When helium is present within the pore, the decrease in f with increasing g is moderated by improved gas conduction.

Finally, Fig. 5.11 shows the dependence of f on the pore orientation for a $\kappa=2$ elliptical pore at vacuum and a constant porosity of 5%. In this figure the change in f can be seen as the pore orientation changes from an oblate orientation (0°) to a prolate orientation (90°). Two very interesting phenomena are made evident by this figure. First, the minimum in the porosity correction factor occurs not at an orientation of 0° (an oblate pore), but rather at some orientation angle between 0 - 5° (the exact minimum orientation has not been resolved); the porosity correction factors for orientations of $0 < \theta < 15^\circ$ are lower than those for $\theta=0^\circ$. This phenomenon is also observed with helium in the pore and in the absence of radiation within the pore; it is a solid matrix conduction effect. Second, at pore orientations of 0° (oblate pore) and 90° (prolate pore) the radiative component of heat transfer within the pore serves to increase the overall heat transport through the unit cell. However, for all orientations between 0 and 90° the radiative component acts to reduce the overall heat transport through the unit cell, with the reduction increasing in magnitude as temperature increases. This phenomenon is caused by a skewing of the net radiative transfer within a pore that is asymmetric relative to the direction of overall heat transfer. Consider the elliptical pore having $\theta=45^\circ$ orientation shown in Fig. 4.6. If the top boundary of the unit cell is assumed to be at a uniform temperature condition greater than the bottom boundary and the left and right boundaries are insulated, then heat conduction through the solid matrix occurs more readily around the left side of the pore than the right side since the reduction in cross-sectional area occurs more gradually along this path. As a result, the element labelled "1" is at a higher temperature than the element labelled "2" even though it is at a lower "elevation" (farther from the high temperature boundary). The pore surface at "1" is a net emitter of radiation, and the pore surface at "2" is a net absorber of radiation. This asymmetric condition causes a portion of the net radiant heat transfer to occur from a lower to a higher elevation within the unit cell, opposite in direction to that of the overall net heat transfer from the top (hot) boundary to the bottom (cool) boundary and acting as a thermal resistance. The radiant thermal resistance increases with temperature since the overall radiant emission increases with temperature. This radiant thermal resistance was observed for all asymmetric pore orientations investigated. When the pore orientation is symmetric (oblate or prolate pore), the conduction paths to the left and right of the pore are equivalent and a symmetry of temperature exists about a vertical line through the pore center, thus preventing a net

radiative heat transfer from a lower to higher elevation.

5.4 INTEGRAL RADIATION CONTRIBUTION

The previous section presented the raw results obtained for the porosity correction factors calculated for the conditions outlined in Table 4.3 and analyzed the observed parametric dependencies. Overall trends and interesting phenomena attributable to radiative heat transfer within the pores were discussed qualitatively. In this section, the contribution of radiative heat transfer to the overall heat transport through the unit cell is quantified.

For each unit cell with specified parameters and a particular pore composition two calculations were made: one in which radiative heat transfer within the pore was included and one in which it was excluded. The relative difference in the overall heat transfer through the unit cell Q_p (or equivalently the difference in f) between these two calculations represents the integral contribution made by the radiative component to the total heat transport. The integral radiative contributions were computed as shown:

$$\text{Integral Radiative Contribution} = \frac{f_{\text{with radiation}} - f_{\text{without radiation}}}{f_{\text{with radiation}}} \cdot 100\%.$$

This quantity, expressed as a percent of the total heat transport through the unit cell, is shown for circular porosity having diameters of 25 and 50 μm in Table 5.3 for a variety of temperatures. Table 5.4 shows the radiative contribution for oblate, elliptical porosity having 50 μm major diameters at 3000 K.

It is evident from Table 5.3 that the role that radiative heat transfer within pores play in the overall heat transport in porous UO_2 is small for the conditions common in a reactor environment. Typically, porosities up to $\sim 10\%$, pore diameters up to $\sim 50 \mu\text{m}$, and temperatures up to $\sim 2000 \text{ K}$ encompass normal operation conditions for light water reactor (LWR) fuel. Under these conditions the radiative component of heat transfer is less than 1% of the total. In a fast reactor environment with a high linear heat generation rate, in contrast, central fuel temperatures can exceed 2500 K under normal operating conditions, and local porosity near the center of the fuel pellet can reach $\sim 50\%$ during restructuring. In these regions the radiative heat transfer component could contribute as much as 10% or more to the total heat transport through the fuel. Restructuring is a transient phenomenon, however, and is important only early in the life of a fuel element.

As previously indicated and apparent from Table 5.3, the difference in overall heat transport (as well as radiative component) between pore compositions of vacuum and fission gas are slight. The radiative component is less in magnitude when helium is present due to significant gas conduction within the pore. Pore conduction and radiation are parallel and competing modes of heat transfer; when the conduction through the pore increases the radiative transfer through the pore must decrease.

Table 5.3 Integral radiative contribution (in %) to total heat transport for circular porosity

	25 μm Pore Diameter			50 μm Pore Diameter		
	1000 K	2000 K	3000 K	1000 K	2000 K	3000 K
P=5%						
Vacuum:	0.014	0.186	0.422	0.028	0.368	0.824
FP:	0.014	0.181	0.411	0.028	0.359	0.802
He:	0.012	0.122	0.290	0.024	0.243	0.570
P=10%						
Vacuum:	0.029	0.376	0.849	0.058	0.743	1.656
FP:	0.029	0.366	0.828	0.057	0.723	1.616
He:	0.024	0.246	0.582	0.048	0.487	1.144
P=20%						
Vacuum:	0.060	0.774	1.744	0.119	1.527	3.382
FP:	0.059	0.752	1.693	0.118	1.484	3.296
He:	0.050	0.497	1.176	0.099	0.983	2.298
P=30%						
Vacuum:	0.095	1.226	2.752	0.190	2.411	5.298
FP:	0.094	1.189	2.673	0.187	2.338	5.151
He:	0.077	0.760	1.800	0.155	1.502	3.502
P=40%						
Vacuum:	0.139	1.790	3.991	0.281	3.501	7.609
FP:	0.137	1.725	3.858	0.274	3.377	7.366
He:	0.110	1.045	2.479	0.220	2.061	4.800
P=50%						
Vacuum:	0.202	2.577	5.698	0.402	5.009	10.708
FP:	0.198	2.461	5.464	0.389	4.789	10.291
He:	0.151	1.368	3.253	0.302	2.691	6.261

Table 5.4 shows the integral radiative contribution for some oblate, elliptical pores

having major pore diameters of 50 μm at 3000 K. Oblate elliptical pores are of interest due to their similarity in appearance to the lenticular pores observed in fuel elements having high linear heat generation rates (usually associated with fast reactors). The migration of these pores to the central region of the fuel pellet are responsible for fuel restructuring and the formation of a central void. For pores filled with vacuum or fission gas the overall heat transport through the unit cell is reduced, but the radiative contribution increases, as the geometrical factor g increases (with porosity remaining constant). With helium in the pore, however, the total heat transport through the unit cell is reduced as g increases, but in this case the radiative contribution also decreases. To reiterate, this is due to improved gas conduction (since the minor pore diameter, the conduction length through the pore, is reduced) as g increases.

Table 5.4 Integral radiative contribution (in %) to total heat transport for oblate, elliptical porosity at 3000 K

	Pore Shape Factor			
	$g=2$	$g=3$	$g=5$	$g=10$
P=5%				
Vacuum:	0.924	1.130	1.608	3.247
He:	0.477	0.446	0.397	0.309
P=10%				
Vacuum:	1.962	2.481	3.983	---
He:	0.983	0.926	0.846	---
P=20%				
Vacuum:	4.528	7.212	---	---
He:	2.090	2.106	---	---
P=30%				
Vacuum:	9.271	---	---	---
He:	3.513	---	---	---

An oblate elliptical pore with $g=10$ is a fairly good approximation of a lenticular pore. Even at the low porosity of 5% the radiative contribution to the overall heat transport in regions where this type of porosity is prevalent is over 3% at high temperatures. Unit cells containing this type of ellipse could not be constructed for porosities over 5%; this simply means that the type of repeating unit cell as envisioned and

employed in this work cannot adequately represent this type of pore at high porosities. Since porosities higher than 5% made up of lenticular pores are observed in fast reactor fuel, it is probable that the radiative contribution in these regions of real fuel are higher than the values shown in Table 5.4.

5.5 ERROR ESTIMATION

The porosity correction factors as calculated in this work and tabulated in Appendix C are subject to uncertainty from a variety of sources. First, the solution of the finite element equations could not be obtained directly due to nonlinearities within the equations. The temperatures were obtained by an iterative procedure that is not exact; however, the temperatures were converged to a maximum change of 10^{-10} between iteration cycles and are assumed to be accurate to that level of precision. Second, the meshes employed for the discretization of the unit cells were not infinitely refined and introduce some uncertainty in the calculated values of Q_p . Convergence studies on representative meshes during their design phase, however, indicate that the meshes employed yield values of Q_p that are consistently within 0.01% of the total heat transport obtained in the fine mesh limit (which is taken as an accurate reference value); this source of uncertainty is relatively small. The final source of uncertainty arises from the use of material property correlations in CARESS.

To estimate the material properties required in this work, correlations are employed by CARESS for the thermal conductivity of UO_2 , He, Xe and Kr and for the emissivity of UO_2 . The correlation used for the thermal conductivity of UO_2 was previously given as equation (2.7). Harding and Martin (1989) report the standard error of this correlation with respect to the experimental data as 7% up to 1800 K, increasing to 15% at 3120 K. The gas thermal conductivity correlations and their associated standard errors were obtained from Hagrman et al. (1981):

$$k_{He} = 2.639 \times 10^{-3} T^{0.7085} \pm 8.00 \times 10^{-7} T^{1.5}, \quad (5.1)$$

$$k_{Xe} = 4.351 \times 10^{-5} T^{0.8616} \pm 2.77 \times 10^{-8} T^{1.5}, \text{ and} \quad (5.2)$$

$$k_{Kr} = 8.247 \times 10^{-5} T^{0.8363} \pm 1.45 \times 10^{-9} T^{2.0}. \quad (5.3)$$

The thermal conductivity of an 85% Xe - 15% Kr fission gas mixture is computed as

(Olander, 1976):

$$k_{\text{mix}} = (k_{\text{Xe}})^{0.85} (k_{\text{Kr}})^{0.15} . \quad (5.4)$$

Finally, the hemispherical total emissivity of UO_2 was estimated using the correlation of Hagerman et al. (1981):

$$\epsilon = 0.7856 + 1.5263 \times 10^{-5} T , \quad (5.5)$$

which was reported to have a standard error of 6.8% with respect to the experimental data.

These known standard errors for the material property correlations used in CARESS were used to perturb the nominal material property predictions alternatively to reflect the maxima and minima for each of the properties. All permutations (for the set of correlations as a collection) of the maximum and minimum material property predictions were used in representative porosity correction factor calculations on a variety of unit cells to observe the sensitivity of f on the perturbations of the material properties. The uncertainty estimates obtained for 25 μm diameter, circular porosity are summarized in Table 5.5.

Table 5.5 gives the maximum positive and negative variations from the nominal f values observed when the material properties were varied between the extremes of their respective uncertainties. The uncertainty of all the material property correlations increase with temperature, and so do the associated uncertainties in f . The uncertainties also increase from pore compositions of vacuum, to fission gas, to helium as increased pore conduction increasingly weighs the gas conductivity uncertainties. For this same reason (increasing conduction through the pore), the uncertainties in f increase with porosity.

Table 5.5 Uncertainty estimates on the porosity correction factor calculations for 25 μm diameter, circular pores

	Pore Composition		
	Vacuum	Fission Gas	Helium
P=5%			
1000 K:	+0.002%,-0.002%	+0.014%,-0.012%	+0.229%,-0.203%
2000 K:	+0.034%,-0.028%	+0.080%,-0.067%	+0.676%,-0.617%
3000 K:	+0.110%,-0.083%	+0.178%,-0.134%	+1.020%,-0.850%
P=10%			
1000 K:	+0.005%,-0.004%	+0.029%,-0.025%	+0.461%,-0.409%
3000 K:	+0.223%,-0.167%	+0.359%,-0.271%	+2.058%,-1.701%
P=20%			
1000 K:	+0.010%,-0.009%	+0.059%,-0.052%	+0.945%,-0.835%
3000 K:	+0.460%,-0.343%	+0.740%,-0.557%	+4.205%,-3.422%
P=30%			
1000 K:	+0.016%,-0.014%	+0.095%,-0.082%	+1.479%,-1.304%
3000 K:	+0.729%,-0.542%	+1.169%,-0.879%	+6.501%,-5.220%

CHAPTER VI

ANALYSIS AND APPLICATIONS

6.1 INTRODUCTION

The previous chapter presented and discussed the results of the porosity correction factor calculations incorporating radiative heat transfer within pores. The trends in the calculated porosity correction factors were identified and explained. In this chapter, further analyses of these results are performed. The porosity correction factors obtained in this work are compared and contrasted to those expressions for f suggested by previous investigators and reviewed in chapter II.

The porosity correction factors generated in this work were obtained assuming a uniform distribution of pores of identical size and shape throughout the fuel. The influence of having a distribution of pore sizes within the fuel is addressed in this chapter. A procedure for transforming the available, single pore size porosity correction factors into values appropriate for use in fuel with a distribution of pore sizes is presented in this chapter. This procedure is used in conjunction with a simple temperature program to calculate temperature profiles within three example fuel element designs: a light water reactor, a liquid metal fast breeder reactor, and a space-based thermionic reactor. The importance of the radiative heat transfer mechanism is analyzed with respect to its influence on fuel temperatures. Finally, since the tabulated porosity correction factors given in Appendix C and the reduction procedure necessary to adapt this data to a more realistic distribution of pore sizes is not suited to easy implementation in a fuel performance code, a set of correlations are provided at the conclusion of this chapter.

6.2 COMPARISON WITH PREVIOUS WORK

In this section the porosity correction factors obtained in this study are compared to previous generic expressions and UO_2 specific formulae for estimating f . Table 6.1 shows the analytical results of Lord Rayleigh (1892) for the porosity correction factor for a two-dimensional domain containing uniformly distributed, circular, non-conducting pores along with those calculated in this work. The Rayleigh values were obtained from equation (2.11); the values employed from this work are those for vacuum-filled, circular pores of 25 μm diameter. The relative agreement between the results shown in Table 6.1

is excellent. The difference seen is due both to the approximate nature of the finite element solution employed in this work as well as Rayleigh's truncation of his series solution.

Table 6.1 Comparison with analytical solution for circular, non-conducting pores in a two-dimensional medium

Porosity	f		Relative Difference (%)
	Rayleigh (1892)	This work	
0.05	0.9047617	0.9048924	0.0144423
0.10	0.8181768	0.8182202	0.0053089
0.15	0.7390953	0.7391070	0.0015814
0.20	0.6665307	0.6665280	-0.0004051
0.25	0.5996174	0.5996086	-0.0014650
0.30	0.5375805	0.5375722	-0.0015362
0.35	0.4797129	0.4797080	-0.0010267
0.40	0.4253582	0.4253577	-0.0001088
0.45	0.3738957	0.3738903	-0.0014569
0.50	0.3247293	0.3246898	-0.0121512

Figure 6.1 compares the theoretically obtained f results from various investigators with the results of this work for vacuum-filled, non-radiating circular porosity. The results of this work for this particular case are very similar to equation (2.19) given by Cunningham and Peddicord (1981). Equation (2.13) given by Loeb (1954) predicts the most severe degradation in conductivity by porosity; however, it was developed from a one-dimensional treatment that did not account for multi-dimensional heat flow around the pore. Another one-dimensional treatment given by Peddicord (1976), equation (2.18), is equivalent to Loeb's equation (2.13) for these conditions and is not shown in this figure.

Figures 6.2 and 6.3 provide comparisons with those theoretically obtained expressions for f that modeled pore conduction. The results of this work for helium-filled, non-radiating pores are shown for 5% porosity in Fig. 6.2 and 10% porosity in Fig. 6.3 with the previous expressions. Both figures are qualitatively similar. In both figures, equations (2.17), (2.18) and (2.19) are shown for reference; these expressions do not allow for pore conduction, and as a result show no temperature dependence. The other four curves shown model pore conduction and show the identical temperature dependence resulting from the contrasting temperature dependencies of the helium and UO_2 thermal conductivities. Once again, the curve of Loeb (1954) shown as equation (2.13) predicts the lowest values of f . The two curves that lie above the curve of this work are those of Maxwell (1892) given as equation (2.8), and Russell (1935), equation

Fig. 6.1 Comparison of theory-based f results for non-conducting, non-radiating pores

Fig. 6.2 Comparison of theory-based f results for He-filled, non-radiating pores; $P=5\%$

Fig. 6.3 Comparison of theory-based f results for He-filled, non-radiating pores; $P=10\%$

(2.12). Both of these f expressions were developed from three-dimensional models, although Maxwell made a dilute pore concentration assumption and Russell did not treat the heat flow in multi-dimensions. As a result, the f values of this work agree well with Maxwell (1892) and Russell (1935) at low porosities, but differ increasingly as porosity increases and the approximations used by Maxwell and Russell break down.

Although none of the previous expressions for f incorporated radiative heat transfer through the pores directly in their model development, some can employ the radiation conductivity expressions reviewed in section 2.3 to augment the pore gas conduction in an attempt to account for the radiant contribution. Figures 6.4 and 6.5 show the effects of using the radiation conductivity expression of Loeb (1954) in the previous f formulae that modeled gas conduction. The parameter shown plotted as a function of temperature, f^* , is the difference between a particular expression's prediction for f using the radiation conductivity and omitting the radiation within the pore. Thus, f^* represents the amount that f is increased when the radiation mode is active. It is assumed that the pore is at vacuum, thus no heat transfer through the pore occurs by gas conduction. As seen in Fig. 6.4 for 10% porosity and in Fig. 6.5 for 30% porosity, the use of a radiation conductivity expression predicts a much larger radiation contribution to the overall heat transfer through the material than the result obtained in the present work. The model of this work incorporates the competition between the conduction and radiation modes in an integral way. The use of a radiation conductivity decouples the interaction present between these two competing heat transfer modes, and probably overpredicts the radiant contribution.

The previous comparisons were made with generic, material non-specific expressions for predicting porosity correction factors. Figures 6.6 and 6.7 show comparisons of the results of this work with an expression developed for porous UO_2 . The expression employed was the widely-used, modified Loeb formula with the equation provided by Brandt et al. (1977) used to compute the required coefficient α based on temperature. Similar α expressions have been given by Van Craeynest and Stora (1970) and Hobson et al. (1974). Each of these α expressions were developed based on the observed porosity dependence of UO_2 thermal conductivity data up to ~ 1500 K. As seen in Figs. 6.6 and 6.7, the agreement in slope between the helium-filled, radiating porosity result of this work and that of Brandt et al. is excellent up to ~ 2000 K (i.e. the dependence of f on T is the same for both curves); the magnitudes, however, differ. This difference in magnitude is probably due to two effects: the model employed in the present calculations

Fig. 6.4 Predicted contribution to f due to radiative heat transfer by incorporating a radiation conductivity expression; $P=10\%$

Fig. 6.5 Predicted contribution to f due to radiative heat transfer by incorporating a radiation conductivity expression; $P=30\%$

Fig. 6.6 Modified Loeb expression with temperature dependent α for UO_2 ; $d=25 \mu\text{m}$, $P=5\%$

Fig. 6.7 Modified Loeb expression with temperature dependent α for UO_2 ; $d=25 \mu\text{m}$, $P=10\%$

is two-dimensional (reality is three-dimensional), and the gas contained within the pores of the UO_2 was probably not pure helium (probably contaminations of air and water vapor present), resulting in a degradation in pore gas conduction. This assertion is supported by the fact that over the temperature range 1000-3000 K, most of the Brandt et al. curve is bounded between the vacuum- and helium-filled porosity results from this work. Porosity dependent thermal conductivity data for UO_2 are not available above ~ 1500 K, and the Brandt et al. curve at higher temperatures is merely an extrapolation. Thus, the dependence of f on temperature obtained from the calculations of this work are in general agreement with experimental results at low temperatures; experimental results at the higher temperatures where radiant heat transfer would be more important are not available.

6.3 PORE SIZE DISTRIBUTIONS

In UO_2 fuel, the porosity present does not consist of a single size or shape. Rather, a distribution of pore sizes and shapes exists. Due to surface tension forces acting on a pore's surface during fuel fabrication and sintering, most pores present in as-fabricated fuel are nearly spherical in shape. It is the distribution of pore sizes that is addressed in this section, since the radiative component of heat transfer in porous UO_2 has been shown to depend on pore size.

Early fuel fabrication methods produced UO_2 fuel of variable total porosity, but with essentially all the porosity present as small, intra-granular pores of less than $2\text{ }\mu\text{m}$ in diameter. Such small pores lead to irradiation instability with respect to densification which is not desirable. Consequently, pore forming compounds (pore formers) are generally used during fuel fabrication to produce a fuel made up of larger, inter-granular pores. Pore formers give rise to a very consistent, reproducible pore size distribution that has been well documented (Freshley et al., 1976; Radford and Pope, 1977; Vollath, 1979; Archer et al., 1979; Elbel and Vollath, 1982; Goebelbecker, 1988). Typical pore size distributions resulting from fuel fabricated both with and without the use of a pore former are shown in Fig. 6.8 (Radford and Pope, 1977). In Fig. 6.8, $p(d)$ is the fraction of the total porosity that can be found within the various pore diameter ranges shown.

Since the pore size distribution in UO_2 fuel is well characterized, it can be used to adjust the tabulated f data in Appendix C to reflect a distribution of pore sizes rather than a single pore size. Mathematically, the correct adjustment to the tabulated data is the following:

$$f(P, T, g=1) = \frac{\int_{d'=0}^{d'=d\text{-cutoff}} f(P, T, d', g=1) p(d') dd'}{\int_{d'=0}^{d'=d\text{-cutoff}} p(d') dd'} \quad (6.1)$$

All the porosity is assumed to be spherical. Essentially, the dependence of f on d can be integrated out since the distribution of pore sizes is assumed known. In principle, this is a great advantage; in practice, only the small portion of f due to radiative heat transfer is dependent upon d , and the integration shown in equation (6.1) adjusts the data by only a small amount. An original FORTRAN program was written to perform the necessary integrations numerically using Simpson's extended trapezoid quadrature rule and a user-input pore size distribution (e.g. the distributions shown in Fig. 6.8). A listing of this program, entitled PREFP, is provided in Appendix D.

Fig. 6.8 Typical pore size distributions in UO_2 fuel

PREFP reads tables of f data similar to those shown in Appendix C and returns a simplified table with the dependence on pore size integrated out based on an input pore size distribution. However, simply using the average pore size to determine f from the tables in Appendix C is much simpler, and yields approximately the same result; significant deviation occurs only at higher temperatures.

6.4 CALCULATED TEMPERATURE PROFILES

In this section the radiative heat transfer mode within pores is examined in the context of its effect on calculated temperature profiles in reactor fuel elements. Assuming the net radiative transfer within the pores serves to increase the total heat transport through the unit cell, or fuel material, the calculated fuel temperature profiles should be lower when radiation is incorporated into f than when radiation is neglected. The reduction in temperature achieved is dependent upon the temperatures, pore sizes and shapes, and porosity levels present in a particular fuel design. Representative temperature profile calculations for three UO_2 fuel designs are made in this section: a light water reactor (LWR), a liquid metal fast breeder reactor (LMFBR), and a space-based thermionic reactor.

An original FORTRAN program was written utilizing the finite-difference methodology in a radial geometry to perform the nonlinear temperature profile calculations. The nonlinearity arises from the temperature-dependent thermal conductivity of UO_2 . The porosity correction factor value is called for from an independent subroutine and the appropriate f value is obtained by quadratic interpolation within the tables of f given in Appendix C. A listing of this program, called TEMP, is provided in Appendix E.

The first UO_2 fuel environment considered was that of an LWR (specifically, a Westinghouse pressurized water reactor). For this type of LWR, the fuel pellet is 8.255 mm in diameter containing 6% porosity uniformly distributed within the fuel. The fuel surface temperature at the beginning of life is approximately 800 K, and the peak rod operates at a linear heat generation rate of 410 W/cm; an average rod operates at 175 W/cm (Rust, 1979). With these fuel parameters, temperature profile calculations were made for a variety of pore sizes (all circular in shape). The calculations were made with vacuum- and He-filled pores, and with and without inclusion of pore radiative heat transfer. The fuel central temperature was observed to decrease by the amounts shown in Table 6.2. For the relatively low temperature and low porosity environment of an LWR fuel pin, little decrease in fuel temperature is expected when radiative heat transfer is incorporated into the porosity correction factor, only 2 to 4 K for large pores. The calculations shown in Table 6.2 are for the peak fuel pin in the reactor; an average pin would be affected even less by radiative heat transfer.

Table 6.3 shows the results of similar temperature calculations for an LMFBR fuel environment. The fuel pellet is 5 mm in diameter and contains 15% porosity uniformly

distributed within the fuel. The fuel surface temperature at the beginning of life is approximately 1100 K, and the peak rod operates at a linear heat generation rate of 500

Table 6.2 Reduction of LWR fuel central temperature in peak rod due to radiative heat transfer within pores

Pore Size	Vacuum-filled pores		He-filled pores	
	Central Temp. without Rad.	Reduction due to Rad.	Central Temp. without Rad.	Reduction due to Rad.
10 μm	1953 K	-1	1900 K	0
20 μm	"	-1	"	-1
30 μm	"	-2	"	-1
40 μm	"	-3	"	-2
50 μm	"	-4	"	-2

W/cm (Rust, 1979). In LMFBR fuel, a restructuring phenomenon occurs due to the high temperatures, porosities and temperature gradients present. During restructuring, much of the initially uniform and spherical porosity transforms into lenticular pores that migrate toward the central region of the fuel pellet. When this transient phenomenon equilibrates, a central void exists within the fuel along with a three-ring structure in the remaining annulus of solid fuel. The inner-most ring is a dense zone comprised of columnar grains, the intermediate ring is a less dense zone comprised of equiaxed grains, and the outermost

Table 6.3 Reduction of LMFBR fuel central and void temperatures in peak rod due to radiative heat transfer within pores

Pore Size	Vacuum (He)		Vacuum (He)	
	Central Temp. without Rad.	Reduction due to Rad.	Void Temp. without Rad.	Reduction due to Rad.
10 μm	2989K (2816K)	-7 (-3)	2469K (2431K)	-2 (-1)
20 μm	"	-13 (-7)	"	-3 (-1)
30 μm	"	-19 (-10)	"	-4 (-2)
40 μm	"	-24 (-13)	"	-4 (-2)
50 μm	"	-30 (-16)	"	-5 (-3)

ring extending to the surface of the pellet remains in its as-fabricated condition. TEMP

employs the restructuring model suggested from experimental work by Lackey et al. (1972). For an initial porosity level of 15%, their model assumes a columnar grain region of 3% porosity occurs for $T > 2023$ K, an equiaxed grain region of 8% porosity occurs for $1773 \leq T \leq 2023$ K, and the fuel remains in its as fabricated condition for $T < 1773$ K; the radius of the central void is computed from the change in density that occurs following restructuring. It is further assumed that the as-fabricated porosity is spherical, and the porosity within the equiaxed and columnar grained regions consists of oblate elliptical pores of $g=2$ and $g=10$, respectively. In Table 6.3 the values shown in parenthesis are for He-filled pores (those values not in parentheses are for pores at vacuum). Results are shown for the reduction in central temperature prior to restructuring, and the reduction in the void temperature following restructuring. The temperature reduction due to radiative heat transfer is greater for the LMFBR than for the LWR since the fuel operates at a higher temperature and greater porosity level. Note that following restructuring, the porosity and temperature in the LMFBR are similar to that of the LWR, and the reduction in temperatures due to radiation decreases accordingly.

Finally, Table 6.4 shows the temperature reductions due to radiation obtained for space-based thermionic reactor fuel. This fuel consists of pellets 27.9 mm in diameter and having 8% porosity. The emitter surface, which is essentially the fuel surface, is designed to operate at 1800 K, and the peak element operates at 330 W/cm (Boltax and Sundquist, 1986). The results shown in Table 6.4 assume spherical pores. The reduction in temperatures obtained in this fuel environment lie between those for the LWR and LMFBR.

Table 6.4 Reduction of thermionic fuel central temperature in peak rod due to radiative heat transfer within pores

Pore Size	Vacuum-filled pores		He-filled pores	
	Central Temp. without Rad.	Reduction due to Rad.	Central Temp. without Rad.	Reduction due to Rad.
10 μm	2912 K	-2	2861 K	-1
20 μm	"	-4	"	-2
30 μm	"	-6	"	-4
40 μm	"	-8	"	-5
50 μm	"	-10	"	-6

6.5 CORRELATION CONSTRUCTION

The temperature calculations made using TEMP employed quadratic interpolations within the tables in Appendix C to obtain porosity correction factors. This procedure produces excellent results; the tabulated values are reproduced exactly, and interpolated values are guaranteed to be bounded by adjacent values that appear within the tables. However, repeated interpolation is a time consuming procedure. For incorporation into a fuel performance code, it is more useful to have algebraic equations capable of reproducing the tabulated f data to within an acceptable level of precision. A set of such equations, or correlations, was developed to reproduce approximately the f data for vacuum- and helium-filled radiating pores. All correlations are of the following form:

$$f = a + bT + cT^2 + dT^3, \quad (6.2)$$

with the coefficients in equation (6.2) determined from the set of equations:

$$a = c_{11} + c_{12}P + c_{13}P^2 + c_{14}P^3, \quad (6.3a)$$

$$b = c_{21} + c_{22}P + c_{23}P^2 + c_{24}P^3, \quad (6.3b)$$

$$c = c_{31} + c_{32}P + c_{33}P^2 + c_{34}P^3, \text{ and} \quad (6.3c)$$

$$d = c_{41} + c_{42}P + c_{43}P^2 + c_{44}P^3. \quad (6.3d)$$

The set of coefficients c_{ij} are tabulated for vacuum-filled pores in Table 6.5 and for helium-filled pores in Table 6.6. Note that all c_{11} coefficients are 1.0 and $c_{21}=c_{31}=c_{41}=0.0$. This assures that all correlations converge to yield the physical result $f=1.0$ as $P \rightarrow 0.0$. Shown in the tables for each set of coefficients are the ranges of validity for the expressions and the standard deviation of the correlation predictions with respect to the tabulated data in Appendix C; most of the standard deviations are on the order of 10^{-2} to $10^{-3}\%$.

Table 6.5 Correlation coefficients for vacuum-filled pores

g : 1 P : 0-0.30 T : 1000-3000 K σ : 3.10E-03 %				
$c(i,j)$	1	2	3	4
1	1.00E+00	-1.9384E+00	1.7730E+00	-1.1349E+00
2	0.00E+00	-1.1713E-04	2.0509E-04	-1.7537E-04
3	0.00E+00	7.8210E-08	-1.3801E-07	1.1865E-07
4	0.00E+00	-1.2051E-11	2.1482E-11	-1.8639E-11
g : 2 θ : 0° P : 0-0.30 T : 1000-3000 K σ : 1.11E-01 %				
$c(i,j)$	1	2	3	4
1	1.00E+00	-2.8081E+00	3.1907E+00	-3.9355E+00
2	0.00E+00	-1.2915E-04	2.7100E-04	-3.4882E-04
3	0.00E+00	8.6074E-08	-1.8168E-07	2.3434E-07
4	0.00E+00	-1.3218E-11	2.8072E-11	-3.6277E-11
g : 2 θ : 45° P : 0-0.30 T : 1000-3000 K σ : 4.46E-03 %				
$c(i,j)$	1	2	3	4
1	1.00E+00	-2.2536E+00	2.3439E+00	-1.8111E+00
2	0.00E+00	8.2205E-05	-1.6168E-04	1.5843E-04
3	0.00E+00	-5.3561E-08	1.0495E-07	-1.0272E-07
4	0.00E+00	7.3536E-12	-1.4335E-11	1.3995E-11
g : 2 θ : 90° P : 0-0.30 T : 1000-3000 K σ : 1.27E-02 %				
$c(i,j)$	1	2	3	4
1	1.00E+00	-1.4505E+00	1.0036E+00	-2.5470E-02
2	0.00E+00	-4.6189E-05	6.4536E-05	-2.7347E-05
3	0.00E+00	3.0490E-08	-4.2748E-08	1.8125E-08
4	0.00E+00	-4.2626E-12	6.0134E-12	-2.5749E-12
g : 3 θ : 0° P : 0-0.20 T : 1000-3000 K σ : 8.99E-02 %				
$c(i,j)$	1	2	3	4
1	1.00E+00	-3.6899E+00	5.3688E+00	-1.1089E+01
2	0.00E+00	-1.7280E-04	4.7276E-04	-9.6929E-04
3	0.00E+00	1.1488E-07	-3.1572E-07	6.4769E-07
4	0.00E+00	-1.7597E-11	4.8600E-11	-9.9793E-11

Table 6.5 (Continued)

g: 4 θ : 0° P: 0-0.15 T: 1000-3000 K σ : 1.48E-03 %				
c(i,j)	1	2	3	4
1	1.00E+00	-4.5677E+00	8.1900E+00	-2.3827E+01
2	0.00E+00	-2.3311E-04	7.9450E-04	-2.2609E-03
3	0.00E+00	1.5522E-07	-5.3210E-07	1.5159E-06
4	0.00E+00	-2.3800E-11	8.1960E-11	-2.3347E-10
g: 5 θ : 0° P: 0-0.10 T: 1000-3000 K σ : 1.21E-03 %				
c(i,j)	1	2	3	4
1	1.00E+00	-5.2499E+00	5.4500E+00	0.0000E+00
2	0.00E+00	-2.7872E-04	5.7880E-04	0.0000E+00
3	0.00E+00	1.8573E-07	-3.8852E-07	0.0000E+00
4	0.00E+00	-2.8503E-11	6.0020E-11	0.0000E+00
g: 6 θ : 0° P: 0-0.10 T: 1000-3000 K σ : 1.45E-03 %				
c(i,j)	1	2	3	4
1	1.00E+00	-5.9786E+00	5.1400E+00	0.0000E+00
2	0.00E+00	-3.3324E-04	6.1120E-04	0.0000E+00
3	0.00E+00	2.2212E-07	-4.1040E-07	0.0000E+00
4	0.00E+00	-3.4106E-11	6.3520E-11	0.0000E+00
g: 7 θ : 0° P: 0-0.05 T: 1000-3000 K σ : 1.17E-03 %				
c(i,j)	1	2	3	4
1	1.00E+00	-6.4528E+00	0.0000E+00	0.0000E+00
2	0.00E+00	-3.5764E-04	0.0000E+00	0.0000E+00
3	0.00E+00	2.3826E-07	0.0000E+00	0.0000E+00
4	0.00E+00	-3.6560E-11	0.0000E+00	0.0000E+00
g: 8 θ : 0° P: 0-0.05 T: 1000-3000 K σ : 1.40E-03 %				
c(i,j)	1	2	3	4
1	1.00E+00	-7.1762E+00	0.0000E+00	0.0000E+00
2	0.00E+00	-4.1478E-04	0.0000E+00	0.0000E+00
3	0.00E+00	2.7636E-07	0.0000E+00	0.0000E+00
4	0.00E+00	-4.2418E-11	0.0000E+00	0.0000E+00

Table 6.5 (Continued)

$g:$ 9 $\theta:$ 0° $P:$ 0-0.05 $T:$ 1000-3000 K $\sigma:$ 1.97E-03 %				
c(l,i)	1	2	3	4
1	1.00E+00	-7.8988E+00	0.0000E+00	0.0000E+00
2	0.00E+00	-4.7456E-04	0.0000E+00	0.0000E+00
3	0.00E+00	3.1626E-07	0.0000E+00	0.0000E+00
4	0.00E+00	-4.8556E-11	0.0000E+00	0.0000E+00
$g:$ 10 $\theta:$ 0° $P:$ 0-0.05 $T:$ 1000-3000 K $\sigma:$ 5.98E-03 %				
c(l,i)	1	2	3	4
1	1.00E+00	-8.6220E+00	0.0000E+00	0.0000E+00
2	0.00E+00	-5.4414E-04	0.0000E+00	0.0000E+00
3	0.00E+00	3.6238E-07	0.0000E+00	0.0000E+00
4	0.00E+00	-5.5814E-11	0.0000E+00	0.0000E+00

Table 6.6 Correlation coefficients for helium-filled pores

$g: 1$ $P: 0-0.30$ $T: 1000-3000 \text{ K}$ $\sigma: 2.46\text{E-}01 \quad \%$				
$c(i,j)$	1	2	3	4
1	1.00E+00	-2.5783E+00	2.5486E+00	-1.6450E+00
2	0.00E+00	1.1379E-03	-1.5350E-03	1.1052E-03
3	0.00E+00	-2.3200E-07	3.1377E-07	-2.2642E-07
4	0.00E+00	0.0000E+00	0.0000E+00	0.0000E+00
$g: 2$ $\theta: 0^\circ$ $P: 0-0.30$ $T: 1000-3000 \text{ K}$ $\sigma: 3.41\text{E-}01 \quad \%$				
$c(i,j)$	1	2	3	4
1	1.00E+00	-3.7300E+00	4.3119E+00	-5.2240E+00
2	0.00E+00	1.8800E-03	-2.8350E-03	3.5512E-03
3	0.00E+00	-3.8857E-07	5.8687E-07	-7.3641E-07
4	0.00E+00	0.0000E+00	0.0000E+00	0.0000E+00
$g: 2$ $\theta: 45^\circ$ $P: 0-0.30$ $T: 1000-3000 \text{ K}$ $\sigma: 2.06\text{E-}01 \quad \%$				
$c(i,j)$	1	2	3	4
1	1.00E+00	-2.8238E+00	2.9884E+00	-2.2344E+00
2	0.00E+00	1.3037E-03	-1.8752E-03	1.5595E-03
3	0.00E+00	-2.7372E-07	3.9668E-07	-3.3136E-07
4	0.00E+00	0.0000E+00	0.0000E+00	0.0000E+00
$g: 2$ $\theta: 90^\circ$ $P: 0-0.30$ $T: 1000-3000 \text{ K}$ $\sigma: 1.60\text{E-}01 \quad \%$				
$c(i,j)$	1	2	3	4
1	1.00E+00	-1.8509E+00	1.5030E+00	-3.7641E-01
2	0.00E+00	7.0257E-04	-9.0270E-04	5.5709E-04
3	0.00E+00	-1.3972E-07	1.8757E-07	-1.3228E-07
4	0.00E+00	0.0000E+00	0.0000E+00	0.0000E+00
$g: 3$ $\theta: 0^\circ$ $P: 0-0.20$ $T: 1000-3000 \text{ K}$ $\sigma: 2.72\text{E-}01 \quad \%$				
$c(i,j)$	1	2	3	4
1	1.00E+00	-4.8970E+00	7.8965E+00	-1.6304E+01
2	0.00E+00	2.7457E-03	-6.7134E-03	1.4269E-02
3	0.00E+00	-6.0455E-07	2.0621E-06	-4.4906E-06
4	0.00E+00	9.5013E-12	-1.8129E-10	4.1407E-10

Table 6.6 (Continued)

$g: 4$ $\theta: 0^\circ$ $P: 0-0.15$ $T: 1000-3000 \text{ K}$ $\sigma: 2.42\text{E-}01 \%$				
$c(i,j)$	1	2	3	4
1	1.00E+00	-6.1503E+00	1.1800E+01	-3.4173E+01
2	0.00E+00	3.8866E-03	-1.0874E-02	3.1956E-02
3	0.00E+00	-9.7797E-07	3.5856E-06	-1.0667E-05
4	0.00E+00	4.6320E-11	-3.6002E-10	1.0931E-09
$g: 5$ $\theta: 0^\circ$ $P: 0-0.10$ $T: 1000-3000 \text{ K}$ $\sigma: 1.87\text{E-}01 \%$				
$c(i,j)$	1	2	3	4
1	1.00E+00	-7.0860E+00	7.6560E+00	0.0000E+00
2	0.00E+00	4.7854E-03	-7.3320E-03	0.0000E+00
3	0.00E+00	-1.2856E-06	2.4980E-06	0.0000E+00
4	0.00E+00	7.8901E-11	-2.6414E-10	0.0000E+00
$g: 6$ $\theta: 0^\circ$ $P: 0-0.10$ $T: 1000-3000 \text{ K}$ $\sigma: 1.96\text{E-}01 \%$				
$c(i,j)$	1	2	3	4
1	1.00E+00	-8.0487E+00	6.9860E+00	0.0000E+00
2	0.00E+00	5.7463E-03	-6.9540E-03	0.0000E+00
3	0.00E+00	-1.6247E-06	2.4428E-06	0.0000E+00
4	0.00E+00	1.1654E-10	-2.7062E-10	0.0000E+00
$g: 7$ $\theta: 0^\circ$ $P: 0-0.05$ $T: 1000-3000 \text{ K}$ $\sigma: 1.24\text{E-}01 \%$				
$c(i,j)$	1	2	3	4
1	1.00E+00	-8.6336E+00	0.0000E+00	0.0000E+00
2	0.00E+00	6.3478E-03	0.0000E+00	0.0000E+00
3	0.00E+00	-1.8423E-06	0.0000E+00	0.0000E+00
4	0.00E+00	1.4147E-10	0.0000E+00	0.0000E+00
$g: 8$ $\theta: 0^\circ$ $P: 0-0.05$ $T: 1000-3000 \text{ K}$ $\sigma: 1.23\text{E-}01 \%$				
$c(i,j)$	1	2	3	4
1	1.00E+00	-9.5142E+00	0.0000E+00	0.0000E+00
2	0.00E+00	7.2648E-03	0.0000E+00	0.0000E+00
3	0.00E+00	-2.1768E-06	0.0000E+00	0.0000E+00
4	0.00E+00	1.8033E-10	0.0000E+00	0.0000E+00

Table 6.6 (Continued)

$g:$ 9 $\theta:$ 0° $P:$ 0-0.05 $T:$ 1000-3000 K $\sigma:$ 1.21E-01 %				
$c(l,i)$	1	2	3	4
1	1.00E+00	-1.0350E+01	0.0000E+00	0.0000E+00
2	0.00E+00	8.1520E-03	0.0000E+00	0.0000E+00
3	0.00E+00	-2.5052E-06	0.0000E+00	0.0000E+00
4	0.00E+00	2.1924E-10	0.0000E+00	0.0000E+00
$g:$ 10 $\theta:$ 0° $P:$ 0-0.05 $T:$ 1000-3000 K $\sigma:$ 1.17E-01 %				
$c(l,i)$	1	2	3	4
1	1.00E+00	-1.1147E+01	0.0000E+00	0.0000E+00
2	0.00E+00	9.0124E-03	0.0000E+00	0.0000E+00
3	0.00E+00	-2.8278E-06	0.0000E+00	0.0000E+00
4	0.00E+00	2.5804E-10	0.0000E+00	0.0000E+00

CHAPTER VII

SUMMARY AND CONCLUSION

7.1 SUMMARY OF WORK

Since the thermal conductivity of UO_2 is low and its emissivity is high, it has been suggested that radiative heat transfer may play a significant role in the heat transport through the pores of UO_2 fuel. The objective of this work was the study and quantification of the relative contribution that radiative heat transfer within the pores of UO_2 makes to the overall heat transport in this porous ceramic fuel. This objective was accomplished by performing a numerical study on repeating unit cells representing a two-dimensional model of a porous UO_2 fuel system.

The heat transfer through the unit cells used to model various fuel conditions was obtained by discretizing the unit cell into nodes and elements and employing the finite element method to compute nodal temperatures and heat fluxes throughout the unit cell. The ratio of the heat transfer through the pore-bearing unit cell to an equivalent fully-dense unit cell supplied the porosity correction factor, f , for the specific conditions under consideration. By first neglecting and subsequently modeling radiative heat transfer at the pore surface, the integral contribution of the radiant heat transfer component to the overall heat transport through the unit cell was determined. Many such calculations were made to provide porosity correction factors for far-reaching conditions that encompass and go beyond the typical nuclear fuel thermal environment. By parametrically varying the pore size, shape, orientation, temperature, temperature gradient and total porosity the influence of these parameters on the radiative heat transfer mechanism in particular and the overall heat transport through the porous system in general was obtained. Additionally, calculations were made for pore compositions of vacuum, fission gas and helium. For the fission gas and helium compositions, gas conduction within the pore exists and competes with the radiation mode.

The radiative heat transfer component was shown by both analytical and numerical results to be very weakly dependent on temperature gradient over the range 2,000-10,000 K/cm, which includes the normal regime for nuclear fuels. For a constant pore shape, the radiative component of heat transport increases with temperature to the 2nd-3rd power and increases roughly linearly with porosity. The radiative component was shown to be

almost exactly proportional to pore diameter. Oblate elliptical pores were studied due to their similarity in appearance to the lenticular pores often present in fast reactor fuel. For oblate elliptical pores of constant porosity, the porosity correction factor decreases as the major-to-minor diameter ratio of the ellipse increases; however, the radiative component of the heat transport increases with the major-to-minor diameter ratio for pores at vacuum or filled with fission gas; it decreases with this diameter ratio when helium exists within the pore due to increasing gas conduction as the diameter ratio increases. The radiative component of heat transfer within the pore is essentially the same when either a vacuum or fission gas exists in the pore (the thermal conductivity of the fission gas mixture is so low it is virtually non-conducting), but always decreases for the same conditions when helium is present due to gas conduction within the pore that competes with the radiation mechanism in the transfer of heat through the pore.

The most interesting result obtained from this work was the dependence of both f as well as the radiative heat transfer component on pore orientation. Figure 5.11 showed the variation of f with the pore orientation angle between 0-90° for an elliptical pore of 5% porosity and $g=2$. At a given temperature, the minimum in f occurs not for an oblate (0°) ellipse, but for an ellipse oriented at ~5°. Furthermore, the radiative heat transfer mechanism within the pore serves to increase the overall heat transport through the unit cell, or equivalently through the porous fuel system, only for the symmetric orientations of 0° (oblate) and 90° (prolate). For all asymmetric orientations $0 < \theta < 90^\circ$ the net radiative heat transfer that occurs within the pore acts as a thermal resistance and actually decreases the overall heat transport through the unit cell with increasing temperature.

The porosity correction factors obtained in this work were compared to expressions obtained by other researchers. The f values obtained in this work suffered somewhat from the two-dimensional nature of the model employed in their calculation. However, the general trends and overall agreement at lower porosities between this and previous work are reasonably good. The results of this work were compared with a widely-used empirical expression for f developed by curve-fitting porosity-dependent thermal conductivity data of UO_2 . While the magnitudes of the f values differed somewhat between the two, the temperature dependency of the empirical expression and that for helium-filled porosity generated in this work were essentially identical over the range of validity of the empirical expression. This agreement in temperature dependencies between the experimental data and the calculated porosity correction factors of this work suggest that the model of pore gas conduction and radiation employed in the calculations

of this study is accurate. The two-dimensional nature of the calculational model and uncertainties in pore gas contamination in the experimental specimens is probably responsible for the differences in magnitude. Reliable, porosity-dependent experimental data for the thermal conductivity of UO_2 , however, only exist to ~ 1500 K; model validation would be aided by similar data at the higher temperatures (above 2000 K) where the radiative heat transfer contribution becomes more pronounced.

The porosity correction factors generated in this work by both including and neglecting pore radiant heat transfer were used to ascertain the effect the radiative component has on calculated temperature profiles in real nuclear fuel elements. The reduction in temperature observed when radiation is explicitly modeled was slight for light water reactor fuel, probably only a few degrees Celsius at most; the reduction in temperatures seen for thermionic reactor fuel was 6-10 K. The reduction observed for liquid metal fast breeder reactor fuel was the most pronounced of the three fuel designs, 15-30 K.

7.2 CONCLUSIONS

The results of this study indicate that radiative heat transfer within the pores of UO_2 fuel is a minor component of the overall heat transport in most nuclear fuel operating regimes (pore gas composition, however, has a significant impact on heat transfer via gas conduction). For temperatures below ~ 2000 K, porosities below $\sim 10\%$, and pore sizes below ~ 2 μm radiative heat transfer contributes less than 1% to the total heat transport. LMFBR fuel during restructuring is probably the only environment in which radiation plays an important role; the results of this work suggest that the radiative heat transfer mechanism near the central region of LMFBR fuel undergoing restructuring could be responsible for 10-20% of the total heat transport. However, restructuring is a transient phenomenon that occurs only early in the life of a fuel element.

The radiative heat transfer component is extremely sensitive to pore orientation. The radiative mechanism acts to improve the overall heat transport only for circular pores, or elliptical pores oriented exactly oblately or prolately. As-fabricated pores are nearly but not always exactly spherical in shape. The sum of the deviations from sphericity throughout the fuel probably serve to make the radiative heat transfer within as-fabricated porosity act as a thermal resistance that increases with temperature. The creation of lenticular pores in fast reactor fuel, in contrast, is driven by the temperature gradient. For this reason, lenticular pores are oriented oblately with respect to the local temperature

gradient assuring that the radiation mechanism within these pores serves to increase the overall heat transfer.

The results obtained from this two dimensional study are valuable in establishing overall trends in the porosity correction factor behavior and the interdependencies between the many parameters that influence the radiative heat transfer mode. It is clear from this study, for example, that pore gas conduction and radiation are coupled, competing heat transport modes within the pore and cannot be appropriately addressed independently as has been suggested previously. It is limited, however, by its two-dimensional modeling of the geometry of the fuel-pore system. This was necessary to allow a large number of unit cells representing a wide variety of fuel conditions to be studied. Future efforts could improve the analysis by employing a three-dimensional unit cell model. A 3-D model would require dramatically more computational effort and reduce the number of unit cells that could be studied, but the results of this work could be used to direct attention to the most interesting unit cells.

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APPENDIX A

RADIATION CONDUCTIVITY FOR A CIRCULAR PORE

In this appendix, the radiation conductivity for a circular pore in a two-dimensional domain (or equivalently an infinitely long oblate cylinder in a three-dimensional domain) under a linear temperature gradient is calculated using the method of Marino (1969). The geometry of the problem is defined in Fig. A.1. The total emissive power for a gray body is given by:

$$E = \epsilon \sigma T^4. \quad (\text{A.1})$$

The pore depicted in Fig. A.1 represents an enclosure problem in which the net exchange of thermal radiation between all the differential surface elements comprising the pore surface is desired. In the following development three assumptions are made: the hemispherical total emissivity of the pore surface is constant, the medium within the pore does not participate in the radiative exchange, and no reflection of radiation at the pore surface occurs.

Consider two differential surface area elements dA_1 and dA_2 having an arbitrary relative orientation, with the geometry defined as shown in Fig. A.2. Let dA_1 be at temperature T_1 and dA_2 at T_2 . Using equation (A.1) and Lambert's cosine law for diffuse radiation, the radiation emitted from each differential area that strikes the other differential area is given as:

$$dQ_{dA_1 \rightarrow dA_2} = \epsilon \sigma T_1^4 \frac{\cos \theta_1 \cos \theta_2}{\pi^2} dA_1 dA_2, \text{ and} \quad (\text{A.2a})$$

$$dQ_{dA_2 \rightarrow dA_1} = \epsilon \sigma T_2^4 \frac{\cos \theta_1 \cos \theta_2}{\pi^2} dA_1 dA_2. \quad (\text{A.2b})$$

Equations (A.2) are general expressions for the case of diffuse radiation between two differential surfaces. If these two differential areas are assumed to be on the surface of the circular pore shown in Fig. A.1, then $\theta_1 = \theta_2$ and $r = 2R \cos \theta_1 = 2R \cos \theta_2$. With these observations Equations (A.2) become:

Fig. A.1 Circular pore in a linear temperature gradient

Fig. A.2 Geometry of two surfaces of arbitrary orientation

$$dQ_{dA_1 \rightarrow dA_2} = \frac{\epsilon \sigma T_1^4}{4\pi R^2} dA_1 dA_2, \text{ and} \quad (\text{A.3a})$$

$$dQ_{dA_2 \rightarrow dA_1} = \frac{\epsilon \sigma T_2^4}{4\pi R^2} dA_1 dA_2. \quad (\text{A.3b})$$

Now consider the radiation from an area A_1 defined by a circular arc $-\psi_0 \leq \psi \leq \psi_0$ to the remainder of the circular pore denoted as A_2 . If dA_1 is a differential area on A_1 , then the radiation emitted by dA_1 that falls on A_2 is found as:

$$dQ_{dA_1 \rightarrow A_2} = \frac{\epsilon \sigma T_1^4}{4\pi R^2} dA_1 \int_{A_2} dA_2, \quad (\text{A.4})$$

where,

$$\int_{A_2} dA_2 = \int_{\psi_0}^{\pi} 2LR d\psi = 2LR (\pi - \psi_0). \quad (\text{A.5})$$

Substitution of equation (A.5) into (A.4) yields:

$$dQ_{dA_1 \rightarrow A_2} = \frac{\epsilon \sigma T_1^4}{2\pi} \left(\frac{L}{R}\right) (\pi - \psi_0) dA_1. \quad (\text{A.6})$$

Finally, to get the radiation from the whole of A_1 falling on A_2 , an integration over A_1 must be performed; the temperature T_1 must be incorporated into the integration, however, since it varies over the area A_1 :

$$Q_{A_1 \rightarrow A_2} = \frac{\epsilon \sigma}{2\pi} \left(\frac{L}{R}\right) (\pi - \psi_0) \int_{A_1} T^4 dA_1. \quad (\text{A.7})$$

The steps taken to obtain the expression for the radiative energy leaving area A_1 and striking A_2 , equations (A.3a) to (A.7), can be performed in an analogous manner starting with equation (A.3b) to find the radiative energy that leaves area A_2 and strikes A_1 . This process results in the following expression:

$$Q_{A_2 \rightarrow A_1} = \frac{\epsilon \sigma}{2\pi} \left(\frac{L}{R} \right) \psi_0 \int_{A_2} T^4 dA_2, \quad (\text{A.8})$$

where the temperatures involved in the integrals appearing in equations (A.7) and (A.8) are functions of their position on the surface of the circle and are found as:

$$T = T_0 + GR \cos \psi. \quad (\text{A.9})$$

The parameter T_0 is the mean temperature of the pore, or simply the temperature at the midplane of the pore $T_0 = T(\psi = \pi/2)$.

An enclosure radiation problem such as this is a problem of net radiative *exchange*. Area A_1 emits energy that is absorbed by A_2 , but A_2 also emits energy that is absorbed by A_1 . To find the overall energy transfer through the pore, the net energy transported from area A_1 to A_2 is calculated by subtracting equation (A.8) from (A.7):

$$Q_{\text{net}} = \frac{\epsilon \sigma}{2\pi} \left(\frac{L}{R} \right) \left[(\pi - \psi_0) \int_{A_1} T^4 dA_1 - \psi_0 \int_{A_2} T^4 dA_2 \right], \quad (\text{A.10})$$

where the differential areas can be expressed as $dA = 2L R d\psi$, with $0 \leq A_1 \leq \psi_0$ and $\psi_0 \leq A_2 \leq \pi$. To evaluate Q_{net} , equation (A.9) must be substituted into (A.10), the binomial expressions must be expanded, and the lengthy integrations performed. If it is further specified that the area A_1 is the "top" semicircle and A_2 is the "bottom" semicircle, then the resulting expression for Q_{net} can be evaluated for $\psi_0 = \pi/2$. Performing all of these steps yields the following expression for the net energy transfer through the pore in the $-z$ direction:

$$Q_{\text{net}} = 4\epsilon \sigma T_0^3 G R L^2 + \frac{8}{3} \epsilon \sigma T_0 G^3 R^3 L^2. \quad (\text{A.11})$$

From equation (A.11) the net emissive power in the $-z$ direction can be obtained simply as the net heat transfer through the pore divided by a representative cross-sectional area:

$$E_{\text{net}} = - \frac{Q_{\text{net}}}{A_c}. \quad (\text{A.12})$$

To find the radiation conductivity of the pore, equation (A.12) must be equated to the Fourier law of heat conduction:

$$-\frac{Q_{\text{net}}}{A_c} = -k_{\text{rad}} G. \quad (\text{A.13})$$

Solving equation (A.13) for k_{rad} and substituting in the expression in equation (A.11) for the net heat transfer, the radiation conductivity of the circular pore is given by:

$$k_{\text{rad}} = \frac{4\epsilon\sigma T_0^3 RL^2 + \frac{8}{3}\epsilon\sigma T_0 G^2 R^3 L^2}{A_c}. \quad (\text{A.14})$$

The cross-sectional area needed is that at the midplane of the circle, $A_c = 2RL$. Substitution of this expression into equation (A.14) gives:

$$k_{\text{rad}} = 2\epsilon\sigma T_0^3 L + \frac{4}{3}\epsilon\sigma T_0 G^2 R^2 L. \quad (\text{A.15})$$

Equation (A.15) gives the "radiation conductivity" of a circular pore in a linear temperature gradient. Consider, however, the relative sizes of the two terms appearing in equation (A.15). For conditions representative of a UO_2 fuel pellet ($T_0 = 2000$ K, $G = 6 \times 10^5$ K/m and $R = 50$ μm), the first term in equation (A.15) is almost 7,000 times larger than the second term; thus, the second term can be neglected relative to the first term. The radiation conductivity of a circular pore in a linear temperature gradient is adequately given by:

$$k_{\text{rad}} = 2\epsilon\sigma T_0^3 L. \quad (\text{A.16})$$

APPENDIX B
CARESS FORTRAN LISTING

PROGRAM CARESS

 <C>onduction <A>nd <R>adiation <E>nclosure <S>imulation <S>oftware

PROGRAM AUTHOR :

Steven L. Hayes
 Texas A&M University
 Department of Nuclear Engineering
 College Station, Texas 77843-3133
 (409) 845-8101

PROGRAM EVOLUTION :

Jan - Apr 1991: Development of 2-D conduction, finite-element code for linear, non-iterative problems involving only Dirichlet and Neumann boundary conditions; extensive benchmarking studies performed.

May 1991: Incorporation of Robin boundary condition capability; iterative solution procedure implemented; temperature dependent material properties introduced via user-supplied, externally linked correlation subroutines.

Jun - Aug 1991: Internal enclosure(s) radiation capability initiated. Construction of view factor(s) computation subroutine, radiosity solution subroutine, and interfacing with solution procedure conducted.

Oct 1991: Incorporation of banded storage of the global stiffness or conductivity matrix and development of the optimized Gaussian elimination for a banded system routine.

PROGRAM DESCRIPTION :

This program solves the steady-state, two-dimensional heat conduction equation in x-y geometry:

$$d/dx(k dT/dx) + d/dy(k dT/dy) + Q = 0,$$

using the finite element method. Dirichlet, Neumann, and Robin boundary conditions can be imposed on the external boundaries of the modelled system. Additionally, the computational domain can contain a multiple enclosure(s) in which radiative heat transfer is modelled. The enclosures are assumed to bound non-participating media, and the diffuse-gray approximation is employed in the transfer calculations.

This program requires the following external functions to be

C linked with CARESS to provide problem specific material property
C correlations:

- C 1) COND => Function containing thermal conductivity
C correlations.
C 2) EMISS => Function containing emissivity correlations.
C

C -----
C
C Variable Definitions:

C AJ(2,2).....Jacobian matrix.
C AREA(NE).....Area of an element.
C AVGCOND.....Calculated average thermal conductivity within
C the problem domain.
C AVGEEM(MAXENCL)...Average enclosure emissivity.
C AVGETEMP(MAXENCL).Average enclosure temperature.
C DIFFMAX.....Maximum relative difference in nodal
C temperatures between two consecutive iteration
C cycles.
C DPDE(NNPE).....Value of the derivative of a shape function
C w.r.t. SI.
C DPDN(NNPE).....Value of the derivative of a shape function
C w.r.t. ETA.
C DPDX(NNPE).....Value of the derivative of a shape function
C w.r.t. X.
C DPDY(NNPE).....Value of the derivative of a shape function
C w.r.t. Y.
C DJACOB.....Determinant of the Jacobian matrix.
C DXDE.....Value of the derivative of X w.r.t. SI.
C DXDN.....Value of the derivative of X w.r.t. ETA.
C DYDE.....Value of the derivative of Y w.r.t. SI.
C DYDN.....Value of the derivative of Y w.r.t. ETA.
C ECOND(NE)Converged values of element thermal
C conductivities.
C EF(NNPE).....Element heat flux vector.
C EK(NNPE,NNPE)....Element conductivity matrix.
C EKU(NNPE,NNPE)....Element conductivity matrix with no boundary
C conditions
C incorporated (unmodified).
C ERROR.....User specified error criterion for declaration
C of a converged solution.
C ETA.....Natural 'Y' coordinate.
C FLUX(NE,NSPE)....Element boundary heat flux (specified).
C GF(NN).....Global heat flux vector.
C GK(NN,IHBW).....Global conductivity matrix stored in banded,
C upper-triangular format.
C GKU(NN,NN).....Unmodified global conductivity matrix (before
C constraints have been imposed).
C GQ(NN).....Vector holding the equivalent nodal heat loads
C due to internal heat generation with an
C element.
C H(NE,NSPE).....Element side convection heat transfer
C coefficient.
C IE.....Element currently working on.
C IEM(I,J).....Emissivity specifier used in the function EMISS

```

C          to determine the emissivity surface J in
C          enclosure I.
C  IENCL.....Enclosure specification flag; specifies whether
C          or not the computational domain contains a
C          radiation enclosure(s). (IENCL=0,==> NO;
C          IENCL=1,==> YES)
C  IESURF(I,J).....Identification array holding the element number
C          that corresponds to enclosure surface J in
C          enclosure I.
C  IFLUXES.....Output control parameter; specifies how many
C          nodes for which to compute nodal heat fluxes.
C  IHBW.....Half bandwidth of the global stiffness matrix.
C  ISIDE.....Side of an element having a specified boundary
C          condition.
C  ISFTYPE.....Shape function type specifier;
C          ISFTYPE=1,==> 4-node Lagrangian quadrilateral.
C          ISFTYPE=2,==> 8-node serendipity quadrilateral.
C          ISFTYPE=3,==> 9-node Lagrangian quadrilateral.
C  ISSURF(I,J).....Identification array holding the side number
C          that corresponds to enclosure surface J in
C          enclosure I.
C  ITER.....Number of current iteration cycle.
C  ITOLD.....Specification flag; if ITOLD=1, initial
C          "guessed" temperature field is read in from an
C          external file.
C  ITREQ.....Control specifier indicating whether or not
C          iteration is required to achieve the solution;
C          ITREQ=0, ==> Iteration is NOT required.
C  KTH(NE).....Thermal conductivity specifier used in the
C          function COND to determine the conductivity for
C          an element.
C  MAXBW.....Maximum half-bandwidth of the global stiffness
C          matrix allowable.
C  MAXENCL.....Maximum number of independent enclosures
C          allowed in the computational model.
C  MAXITER.....User-specified maximum number of iterations
C          allowed.
C  MNPE.....Maximum number of nodes per element of any
C          incorporated
C  MQUAD.....Maximum incorporated Gauss-Legendre quadrature
C          order.
C          element (used for dimensioning purposes).
C  MSURF.....Maximum number of surfaces allowed for each
C          independent enclosure.
C  NDIM.....Maximum number of nodes allowed in the
C          computational model.
C  NE.....Number of elements.
C  NENCL.....Number of enclosure(s) within the computational
C          domain.
C  NEWC.....Number of element sides having convection
C          boundary conditions.
C  NEWF.....Number of element sides having non-zero
C          specified heat flux boundary conditions.
C  NN.....Number of nodes.
C  NNFLUX(NN).....List of nodes for which to compute nodal heat

```



```

C          fluxes as output.
C  NNPE.....Number of nodes per element.
C  NODCON(NE,NNPE)...Element Boolean connectivity matrix.
C  NQUAD.....Order of Gauss-Legendre quadrature employed in
C              the integrations.
C  NRL(NN).....Nodal restraint list (0=no, 1=yes).
C  NSPE.....Number of sides per element.
C  NSURF(MAXENCL)...Array holding the number of surfaces defining
C              each of the enclosures.
C  POINTS(NQUAD)....Gauss-Legendre quadrature points.
C  PSI(NNPE).....Value of a shape function.
C  Q(NE).....Internal heat generation rate for an element.
C  QOUT(NN).....Computed nodal heat flux based on nodal
C              temperature solution.
C  QSUM.....Sum of all the requested output nodal heat
C              fluxes.
C  RELAX.....Relaxation factor used in obtaining "improved"
C              temperature estimates during the iteration
C              process.
C  SI.....Natural 'X' coordinate.
C  SJ(2,2).....Inverse of the Jacobian matrix.
C  SURFEM(I,J).....Converged average emissivity of the Jth surface
C              in enclosure I.
C  SURFTEMP(I,J)....Converged average temperature of the Jth
C              surface in enclosure I.
C  T(NN).....Solution variable.
C  THERM.....Current best estimate of the thermal
C              conductivity of an element.
C  TINF(NE,NSPE)....Element side fluid temperature for a convection
C              boundary condition.
C  TOLD(NN).....Value of the solution variable at previous
C              iteration.
C  TOTAREA.....Total area of all elements.
C  WE.....Current quadrature weight for the SI
C              coordinate.
C  WN.....Current quadrature weight for the ETA
C              coordinate.
C  WTS(NQUAD).....Gauss-Legendre quadrature weights.
C  X(NNPE).....X coordinates of master element.
C  XN(NN).....X coordinate of a node.
C  Y(NNPE).....Y coordinates of master element.
C  YN(NN).....Y coordinate of a node.

```

```

-----
C  IMPLICIT REAL*8 (A-H,O-Z)
C  EXTERNAL COND, EMISS
C  PARAMETER (NDIM=1300, MAXBW=105, MAXENCL=50, MSURF=99, MNPE=9,
+           MQUAD=5, NSPE=4)
C  COMMON/TITLTK/ TITLE1, TITLE2, TITLE3, TITLE4, TITLE5
C  COMMON/CONTRL/ ITOLD, ISFTYPE, NN, NE, NNPE, NEWF, NEWC, NQUAD, IHBW
C  COMMON/ENCLOS/ IENCL, NENCL, NSURF (MAXENCL), IESURF (MAXENCL, MSURF),
+           ISSURF (MAXENCL, MSURF), IEM (MAXENCL, MSURF),
+           ASURF (MAXENCL, MSURF)
C  COMMON/NUMINT/ POINTS (MQUAD), WTS (MQUAD)
C  COMMON/OUTPUT/ IFLUXES, NNFLUX (NDIM)

```

```

REAL*8 XN(NDIM),YN(NDIM),AREA(NDIM)
REAL*8 H(NDIM,NSPE),TINF(NDIM,NSPE),FLUX(NDIM,NSPE),Q(NDIM)
REAL*8 T(NDIM),TOLD(NDIM),QOUT(NDIM),ECOND(NDIM)
REAL*8 GF(NDIM),GQ(NDIM),GK(NDIM,MAXBW),GKU(NDIM,NDIM)
REAL*8 SURFEM(MAXENCL,MSURF),SURFTEMP(MAXENCL,MSURF)
REAL*8 AVGEEM(MAXENCL),AVGETEMP(MAXENCL)
INTEGER NRL(NDIM),NODCON(NDIM,MNPE),KTH(NDIM)
CHARACTER*72 TITLE1,TITLE2,TITLE3,TITLE4,TITLE5
CHARACTER CDATE*9,CTIME*8
OPEN (UNIT=10,STATUS='OLD')
OPEN (UNIT=11,FILE='RUN.DAT',STATUS='NEW')
OPEN (UNIT=12,FILE='CARE.OUT',STATUS='NEW')
CALL DATE(CDATE)
CALL TIME(CTIME)
WRITE (11,999) CDATE,CTIME
WRITE (12,999) CDATE,CTIME
WRITE (11,2999)
WRITE (12,1000)

C
C * READ IN THE PROBLEM DATA *
C
CALL INPUT(ERROR,RELAX,MAXITER,IOUT,XN,YN,NRL,NODCON,KTH,
+      T,Q,FLUX,H,TINF)

C
C Write input data to output file.
WRITE (12,998) TITLE1,TITLE2,TITLE3,TITLE4,TITLE5
WRITE (12,498)
WRITE (12,500) NN,NE,NNPE,ISFTYPE,IENCL,ITOLD,NQUAD,
+      MAXITER,RELAX,ERROR
WRITE (12,502)
DO I=1,NE
  WRITE (12,504) I,(NODCON(I,J),J=1,NNPE)
END DO
WRITE (12,505) IHBW
WRITE (12,510)
DO I=1,NN
  WRITE (12,512) I,XN(I),YN(I)
END DO
WRITE (12,520)
ITREQ=0
DO I=1,NE
  WRITE (12,522) I,KTH(I),Q(I)
  IF (KTH(I).GT.3) ITREQ=1
END DO
WRITE (12,524)
DO I=1,NN
  IF (NRL(I).EQ.1) THEN
    WRITE (12,526) I,T(I)
  ENDIF
END DO
WRITE (12,540)
K=0
DO I=1,NE
  DO J=1,4
    IF (FLUX(I,J).NE.0.0) THEN

```

```

        WRITE (12,544) I,J,FLUX(I,J)
        K=K+1
    ENDIF
END DO
END DO
IF (K.EQ.0) WRITE (12,546)
WRITE (12,547)
K=0
DO I=1,NE
    DO J=1,4
        IF (H(I,J).NE.0.0) THEN
            WRITE (12,548) I,J,H(I,J),TINF(I,J)
            K=K+1
        ENDIF
    END DO
END DO
IF (K.EQ.0) WRITE (12,550)
IF (IENCL.EQ.1) THEN
    ITREQ=1
    WRITE (12,560)
    DO N=1,NENCL
        WRITE (12,562) N
        DO I=1,NSURF(N)
            WRITE (12,565) I,IESURF(N,I),ISSURF(N,I),IEM(N,I)
        END DO
    END DO
ENDIF
C
C   If requested, retrieve initial "guessed" temperature field from
C   an external file:
IF (ITOLD.EQ.1) THEN
    OPEN (UNIT=13,FILE='TOLD.DAT',STATUS='OLD')
    DO I=1,NDIM
        READ (13,*,END=40) N,TOLD(N)
        IF (NRL(N).EQ.0) T(N)=TOLD(N)
    END DO
40    CLOSE (13)
C
C   If enclosures are present, call for calculation of radiation heat
C   fluxes based on initial "guessed" temperature field:
IF (IENCL.EQ.1) THEN
    CALL ENCLOSURES(XN,YN,NODCON,T,FLUX)
ENDIF
ENDIF
C
C * A S S E M B L E   S T I F F N E S S   M A T R I X   A N D   L O A D
C   V E C T O R *
C
C   Begin the iteration loop:
DO 600 ITER=1,MAXITER
C
C   If enclosures exist within the computational domain, call for the
C   sub routines to set up and solve for the radiosities/heat fluxes
C   on the boundaries of the enclosures:
IF (IENCL.EQ.1.AND.ITER.GT.1) THEN

```

```

      CALL ENCLOSURES(XN,YN,NODCON,T,FLUX)
    ENDIF
C
C   Assemble the set of simultaneous equations; fill the stiffness
C   (conductivity) matrix and load (heat flux) vector entries:
      CALL ASSEMBLE(XN,YN,KTH,Q,FLUX,H,TINF,NODCON,GF,GQ,GK,
+           GKU,NRL,T)
C
      IF (IOUT.EQ.0.OR.ITER.GE.2) GOTO 55
C   Write assembled conductivity matrix and heat flux vector to
C   output file if requested:
      WRITE (12,1001)
      DO I=1,NN
        MAX=IHBW
        IF (IHBW.GT.8) MAX=8
        WRITE (12,1002) (GK(I,J),J=1,MAX)
      END DO
      WRITE (12,1003)
      DO I=1,NN
        WRITE (12,1004) GF(I)
      END DO
55    CONTINUE
C
C   *   S O L V E   F O R   U N K N O W N S   *
C
      CALL BANDED_SOLVER(GK,GF)
      DO I=1,NN
        T(I)=GF(I)
      END DO
C
C   If enclosures are present within the domain, heat transfer within
C   the enclosure was neglected during the first calculational pass.
C   Now go back and allow heat transfer within the enclosures to
C   "kick in."
      IF (IENCL.EQ.1.AND.ITER.EQ.1) THEN
        DO I=1,NN
          TOLD(I)=T(I)
        END DO
        GOTO 599
      ENDIF
C
C   Check for a converged temperature field; write convergence data
C   to the RUN.DAT file and to the screen:
      IF (ITREQ.EQ.0) GOTO 601
      DIFFTOP=0.0
      DIFFBOT=0.0
      DIFFMAX=0.0
      DO I=1,NN
        IF (NRL(I).EQ.0) THEN
          DIFFTOP=DABS(T(I)-TOLD(I))
          DIFFBOT=DABS(T(I))
          DIFF=DIFFTOP/DIFFBOT
          IF (DIFF.GT.DIFFMAX) DIFFMAX=DIFF
        ENDIF
      END DO

```

```

      IF (ITER.GE.2) THEN
        WRITE (11,3000) ITER,DIFFMAX
        WRITE (*,*) ITER,DIFFMAX
      ENDIF
      IF (DIFFMAX.LT.ERROR) GOTO 601
      DO 598 I=1,NN
        IF (NRL(I).EQ.0) THEN
          T(I)=TOLD(I)+RELAX*(T(I)-TOLD(I))
          TOLD(I)=T(I)
        ENDIF
598      CONTINUE
C
599  CONTINUE
600  CONTINUE
601  CONTINUE
      IF (ITREQ.EQ.0) ITER=0
C
C * COMPUTE ADDITIONAL QUANTITIES *
C
      CALL RESULTS(XN,YN,T,GKU,GQ,NODCON,KTH,ECOND,AVGCOND,AREA,
+        TOTAREA,QOUT,QSUM,SURFEM,SURFTEMP,AVGEEM,AVGETEMP)
C
C Write results of calculation to output file(s):
      WRITE (12,1900) ITER
      OPEN (UNIT=13,FILE='TEMPERATURES.OUT',STATUS='NEW')
      DO I=1,NN
        WRITE(12,526) I,T(I)
        WRITE(13,*) I,T(I)
      END DO
      CLOSE (13)
      WRITE (12,2003)
      DO I=1,NE
        WRITE (12,2004) I,ECOND(I),AREA(I)
      END DO
      WRITE (12,2006) AVGCOND,TOTAREA
      IF (IENCL.EQ.1) THEN
        WRITE (12,1980)
        DO N=1,NENCL
          IF (N.EQ.1) THEN
            WRITE (12,1981) N
          ELSE
            WRITE (12,1982) N
          ENDIF
          DO I=1,NSURF(N)
            WRITE (12,1985) I,ASURF(N,I),
+              FLUX(IESURF(N,I),ISSURF(N,I)),
+              SURFTEMP(N,I),SURFEM(N,I)
          END DO
          WRITE (12,1987) AVGETEMP(N),AVGEEM(N)
        END DO
      ENDIF
      IF (IFLUXES.NE.0) THEN
        WRITE (12,2002)
        DO I=1,IFLUXES
          WRITE (12,526) NNFLUX(I),QOUT(I)
        END DO
      ENDIF

```

```

      END DO
      WRITE (12,527) QSUM
      WRITE (*,528) QSUM
    ENDIF

C
      CALL TERMINATE

C
C --- Format Statements -----
498  FORMAT(X,'----- INPUT DATA -----
      +----',/)
500  FORMAT(X,'NN          = ',I4,/,
      +      X,'NE          = ',I4,/,
      +      X,'NNPE        = ',I4,/,
      +      X,'ISFTYPE     = ',I4,/,
      +      X,'IENCL       = ',I4,/,
      +      X,'ITOLD       = ',I4,/,
      +      X,'NQUAD       = ',I4,/,
      +      X,'MAXITER     = ',I4,/,
      +      X,'RELAX       = ',D14.7,/,
      +      X,'ERROR       = ',D14.7,/)
502  FORMAT(X,'BOOLEAN CONNECTIVITY MATRIX:',/,
      +      X,'Element',3X,'Nodes')
504  FORMAT(X,I4,4X,<NNPE-1>(I4,' '),I4)
505  FORMAT(X,'Half-Bandwidth = ',I4)
510  FORMAT(/,X,'NODAL DATA:',/,
      +      X,'Node',10X,'XN',16X,'YN')
512  FORMAT(X,I4,4X,D14.7,4X,D14.7)
520  FORMAT(/,X,'ELEMENT DATA:',/,
      +      X,'Element',6X,'KTH',14X,' Q')
522  FORMAT(X,I4,7X,I4,10X,D14.7)
524  FORMAT(/,X,'DIRICHLET BOUNDARY CONDITIONS:',/,
      +      X,'Node',10X,'T')
526  FORMAT(X,I4,4X,D14.7)
527  FORMAT(/,X,'Sum of Requested Fluxes: ',D14.7)
528  FORMAT(X,'QSUM = ',D14.7)
540  FORMAT(/,X,'NEUMANN BOUNDARY CONDITIONS (NON-ZERO):',/,
      +      X,'Element',3X,'Side',7X,'Heat Flux')
544  FORMAT(X,I4,8X,I1,5X,D14.7)
546  FORMAT(X,'(No non-zero fluxes specified)')
547  FORMAT(/,X,'ROBIN BOUNDARY CONDITIONS:',/,
      +      X,'Element',3X,'Side',6X,'H.T. Coeff.',8X,'Fluid Temp.')
548  FORMAT(X,I4,8X,I1,5X,D14.7,5X,D14.7)
550  FORMAT(X,'(No Robin boundary conditions)')
560  FORMAT(/,X,'ENCLOSURE(S) DEFINITIONS:')
562  FORMAT(X,'Enclosure #',I2,':',/,
      +      X,'Surface',3X,'Element',2X,'Side',4X,'IEM')
565  FORMAT(X,3X,I2,6X,I4,6X,I1,5X,I3)
998  FORMAT(X,'CALCULATION DESCRIPTION:',/,
      +      X,76('*'),/,
      +      X,'* ',A72,'*',/,
      +      X,'* ',A72,'*',/,
      +      X,'* ',A72,'*',/,
      +      X,'* ',A72,'*',/,
      +      X,'* ',A72,'*',/,
      +      X,76('*'),/,/)

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999  FORMAT(X,'<This calculation was begun on ',A9,' at ',A8,'>')
1000 FORMAT(X,76('*'),
+      /,X,19('*'),' CARESS  OUTPUT  FILE ',20('*'),
+      /,X,76('*'),
+      /,X,'<C>onduction',
+      /,X,'<A>nd',
+      /,X,'<R>adiation',
+      /,X,'<E>nclosure',
+      /,X,'<S>imulation',
+      /,X,'<S>oftware',/,/)
1001 FORMAT(/,/,X,'----- INTERMEDIATE RESULTS -----'
+-----'
+      /,/,X,'Conductivity Matrix (may be incomplete):')
1002 FORMAT(X,<NN>(D14.7,X))
1003 FORMAT(/,X,'Heat Flux Vector:')
1004 FORMAT(X,D14.7)
1900 FORMAT(/,/,X,'----- RESULTS -----'
+-----',/,/,X,'NUMBER OF ITERATIONS REQUIRED: ',I8,
+      /,/,X,'TEMPERATURES:',/,
+      X,'Node',9X,'T')
1980 FORMAT(/,X,'ENCLOSURE RESULTS:')
1981 FORMAT(X,'Enclosure #',I2,':',/,
+      X,'Surface',7X,'Area',11X,'Heat Flux',7X,'Temperature',6X,
+      'Emissivity')
1982 FORMAT(/,X,'Enclosure #',I2,':',/,
+      X,'Surface',7X,'Area',11X,'Heat Flux',7X,'Temperature'
+      ,6X,'Emissivity')
1985 FORMAT(X,3X,I2,4X,D14.7,3X,D14.7,3X,D14.7,3X,D14.7)
1987 FORMAT(/,X,'Average Temperature: ',D14.7,/,
+      X,'Average Emissivity: ',D14.7)
2002 FORMAT(/,X,'REQUESTED NODAL HEAT LOADS:',/,
+      X,'Node',8X,'QOUT')
2003 FORMAT(/,X,'THERMAL CONDUCTIVITIES AND AREAS:',/,
+      X,'Element',7X,'ECOND',12X,'AREA')
2004 FORMAT(X,I4,5X,D14.7,3X,D14.7)
2006 FORMAT(/,X,'Average Thermal Conductivity: ',D14.7,/,
+      X,'Total Area: ',D14.7)
2999 FORMAT(X,'***** CARESS CONVERGENCE DATA *****'
+-----',/,/,
+      X,'Iteration',10X,'DIFF')
3000 FORMAT(X,I6,8X,D14.7)
C  -----
C      END
C  -----
C      SUBROUTINE TERMINATE
C  -----
C      This subroutine simply terminates the execution of the
C      calculation.
C
C      CLOSE (11)
C      CLOSE (12)
C
C      STOP
C      END
C  -----

```

```

      SUBROUTINE INPUT (ERROR, RELAX, MAXITER, IOUT, XN, YN, NRL, NODCON, KTH,
+      T, Q, FLUX, H, TINF)
C -----
C   This subroutine reads in all the input data required to construct
C   the conductivity matrix and heat flux vector.
C
      IMPLICIT REAL*8 (A-H, O-Z)
      PARAMETER (NDIM=1300, MAXBW=105, MAXENCL=50, MSURF=99, MNPE=9,
+      MQUAD=5, NSPE=4)
      COMMON/TITLBN/ TITLE1, TITLE2, TITLE3, TITLE4, TITLE5
      COMMON/CONTRL/ ITOLD, ISFTYPE, NN, NE, NNPE, NEWF, NEWC, NQUAD, IHBW
      COMMON/ENCLOS/ IENCL, NENCL, NSURF (MAXENCL), IESURF (MAXENCL, MSURF),
+      ISSURF (MAXENCL, MSURF), IEM (MAXENCL, MSURF),
+      ASURF (MAXENCL, MSURF)
      COMMON/NUMINT/ POINTS (MQUAD), WTS (MQUAD)
      COMMON/OUTPUT/ IFLUXES, NNFLUX (NDIM)
      REAL*8 XN (NDIM), YN (NDIM)
      REAL*8 Q (NDIM), T (NDIM), FLUX (NDIM, NSPE)
      REAL*8 H (NDIM, NSPE), TINF (NDIM, NSPE)
      INTEGER NRL (NDIM), NODCON (NDIM, MNPE), KTH (NDIM)
      CHARACTER*72 TITLE1, TITLE2, TITLE3, TITLE4, TITLE5
C
C   Read in the calculation control parameters:
      READ (10, 99) TITLE1
      READ (10, 99) TITLE2
      READ (10, 99) TITLE3
      READ (10, 99) TITLE4
      READ (10, 99) TITLE5
      READ (10, *) NN, NE, ISFTYPE, NQUAD, IENCL, ITOLD
      READ (10, *) ERROR, RELAX, MAXITER, IOUT
C
C   Check for errors in the input data:
      IF (NN.GT.NDIM) THEN
        WRITE (*, *) 'NUMBER OF NODES EXCEEDS LIMIT!'
        CALL TERMINATE
      ENDIF
      IF (ISFTYPE.NE.1.AND.ISFTYPE.NE.2.AND.ISFTYPE.NE.3) THEN
        WRITE (*, *) 'INTERPOLATION FUNCTION TYPE REQUESTED NOT',
+      ' AVAILABLE!'
        CALL TERMINATE
      ENDIF
C
C   Assign number of nodes per element based on type of interpolation
C   function requested:
      IF (ISFTYPE.EQ.1) NNPE=4
      IF (ISFTYPE.EQ.2) NNPE=8
      IF (ISFTYPE.EQ.3) NNPE=9
C
C   Assign quadrature points and weights based on the order
C   requested:
      IF (NQUAD.EQ.2) THEN
        POINTS(1)=0.577350269189626
        POINTS(2)=-.577350269189626
        WTS(1)=1.0
        WTS(2)=1.0

```



```

ELSEIF (NQUAD.EQ.3) THEN
  POINTS(1)=0.774596669241483
  POINTS(2)=0.0
  POINTS(3)=-.774596669241483
  WTS(1)=0.5555555555555556
  WTS(2)=0.8888888888888889
  WTS(3)=0.5555555555555556
ELSEIF (NQUAD.EQ.4) THEN
  POINTS(1)=0.861136311594053
  POINTS(2)=0.339981043584856
  POINTS(3)=-.339981043584856
  POINTS(4)=-.861136311594053
  WTS(1)=0.347854845137454
  WTS(2)=0.652145154862546
  WTS(3)=0.652145154862546
  WTS(4)=0.347854845137454
ELSEIF (NQUAD.EQ.5) THEN
  POINTS(1)=0.906179845938664
  POINTS(2)=0.538469310105683
  POINTS(3)=0.0
  POINTS(4)=-.538469310105683
  POINTS(5)=-.906179845938664
  WTS(1)=0.236926885056189
  WTS(2)=0.478628670499366
  WTS(3)=0.5688888888888889
  WTS(4)=0.478628670499366
  WTS(5)=0.236926885056189
ELSE
  WRITE (*,*) 'REQUESTED QUADRATURE ORDER UNAVAILABLE!'
  CALL TERMINATE
ENDIF

```

```

C
C      Read in the nodal coordinates and restraint list:
      DO I=1,NN
        READ (10,*) XN(I),YN(I),NRL(I)
      END DO
C
C      Read in the Boolean connectivity matrix and compute the maximum
C      half-bandwidth:
      DO I=1,NE
        READ (10,*) (NODCON(I,J),J=1,NNPE)
      END DO
      CALL BANDWIDTH(NODCON)
      IF (IHBW.GT.MAXBW) THEN
        WRITE (*,*) 'HALF-BANDWIDTH EXCEEDS MAXIMUM!'
        CALL TERMINATE
      ENDIF
C
C      Read in the element thermal conductivity specifier and internal
C      heat generation rate:
      DO I=1,NE
        READ (10,*) N,KTH(N),Q(N)
      END DO
C
C      Read in any specified nodal temperatures:

```

```

DO 10 I=1,NN
  IF (NRL(I).EQ.1) THEN
    READ (10,*) N,T(N)
  ENDIF
10 CONTINUE
C
C Read in any specified heat fluxes:
READ (10,*) NEWF
IF (NEWF.GT.0) THEN
  DO 40 I=1,NEWF
    READ (10,*) N,ISIDE,FLUX(N,ISIDE)
40 CONTINUE
ENDIF
C
C Read in any convective boundary conditions:
READ (10,*) NEWC
IF (NEWC.GT.0) THEN
  DO I=1,NEWC
    READ (10,*) N,ISIDE,H(N,ISIDE),TINF(N,ISIDE)
  END DO
ENDIF
C
C Read in the enclosure information:
IF (IENCL.EQ.1) THEN
  READ (10,*) NENCL
  IF (NENCL.GT.MAXENCL) THEN
    WRITE (*,*) 'NUMBER OF ENCLOSURES EXCEEDS LIMIT!'
    CALL TERMINATE
  ENDIF
  DO I=1,NENCL
    READ (10,*) NSURF(I)
    DO J=1,NSURF(I)
      READ (10,*) IESURF(I,J),ISSURF(I,J),IEM(I,J)
    END DO
  END DO
ENDIF
C
C Read in the nodes for any requested output heat fluxes:
READ (10,*) IFLUXES
IF (IFLUXES.GT.0) THEN
  DO I=1,IFLUXES
    READ (10,*) NNFLUX(I)
  END DO
ENDIF
CLOSE (10)
C
99 FORMAT(A72)
C
RETURN
END
C -----
SUBROUTINE ASSEMBLE (XN,YN,KTH,Q,FLUX,H,TINF,NODCON,GF,GQ,GK,
+                   GKU,NRL,T)
C -----
C This subroutine constructs the conductivity matrix and heat flux

```

```

C      vector for this problem.  Essential boundary conditions are also
C      imposed on the system in this subroutine.
C
      IMPLICIT REAL*8 (A-H,O-Z)
      PARAMETER (NDIM=1300,MAXBW=105,MAXENCL=50,MSURF=99,MNPE=9,
+               MQUAD=5,NSPE=4)
      COMMON/CONTRL/ ITOLD,ISFTYPE,NN,NE,NNPE,NEWF,NEWC,NQUAD,IHBW
      REAL*8 EK(MNPE,MNPE),Q(NDIM),FLUX(NDIM,NSPE),EQ(MNPE),
+      GQ(NDIM),EF(MNPE),GF(NDIM),GK(NDIM,MAXBW),GKU(NDIM,NDIM),
+      T(NDIM),XN(NDIM),YN(NDIM),H(NDIM,NSPE),TINF(NDIM,NSPE)
      REAL*8 EKU(MNPE,MNPE)
      INTEGER NODCON(NDIM,MNPE),NRL(NDIM),KTH(NDIM)

C
C      Initialize arrays to be used:
      DO I=1,NN
        GF(I)=0.0
        GQ(I)=0.0
        DO J=1,IHBW
          GK(I,J)=0.0
          GKU(I,J)=0.0
        END DO
        DO J=IHBW+1,NN
          GKU(I,J)=0.0
        END DO
      END DO

C
C      Construct the global conductivity matrix by computing each
C      element conductivity matrix and adding it to the global
C      conductivity matrix.  Construct the global heat flux vector by
C      computing each element heat flux vector and adding it to the
C      global heat flux vector.
      DO I=1,NE
        CALL ESTIFF(I,NODCON,KTH,XN,YN,EK,EKU,T,H)
        CALL ELOAD(I,XN,YN,NODCON,Q,FLUX,EF,EQ,H,TINF)
        DO IROWE=1,NNPE
          IROWG=NODCON(I,IROWE)
          GF(IROWG)=GF(IROWG)+EF(IROWE)
          GQ(IROWG)=GQ(IROWG)+EQ(IROWE)
          DO 10 ICOLE=1,NNPE
            ICOLG=NODCON(I,ICOLE)
            ICOLGB=NODCON(I,ICOLE)-IROWG+1
            IF (ICOLGB.LE.0) GOTO 9
            GK(IROWG,ICOLGB)=GK(IROWG,ICOLGB)+EK(IROWE,ICOLE)
            GKU(IROWG,ICOLG)=GKU(IROWG,ICOLG)+EKU(IROWE,ICOLE)
          9      CONTINUE
        10      CONTINUE
          END DO
        END DO

C
C      Impose constraints, i.e. incorporate the specified nodal
C      temperatures.
      DO J=1,NN
        IF (NRL(J).EQ.1) THEN
          GK(J,1)=1.0D+30
          GF(J)=T(J)*1.0D+30
        ENDIF
      END DO

```

```

      END DO
C
      RETURN
      END
C -----
      SUBROUTINE ESTIFF(IE,NODCON,KTH,XN,YN,EK,EKU,T,H)
C -----
C      This subroutine constructs the element conductivity or stiffness
C      matrix.
C
      IMPLICIT REAL*8(A-H,O-Z)
      PARAMETER (NDIM=1300,MAXBW=105,MAXENCL=50,MSURF=99,MNPE=9,
+              MQUAD=5,NSPE=4)
      COMMON/CONTRL/ ITOLD,ISFTYPE,NN,NE,NNPE,NEWF,NEWC,NQUAD,IHBW
      COMMON/NUMINT/ POINTS(MQUAD),WTS(MQUAD)
      REAL*8 XN(NDIM),YN(NDIM),EK(MNPE,MNPE),EKU(MNPE,MNPE)
      REAL*8 X(MNPE),Y(MNPE),T(NDIM)
      REAL*8 H(NDIM,NSPE)
      REAL*8 PSI(MNPE),DPDE(MNPE),DPDN(MNPE)
      REAL*8 DPDX(MNPE),DPDY(MNPE)
      INTEGER NODCON(NDIM,MNPE),KTH(NDIM)
C
C      Initialize the element stiffness matrix:
      DO 20 I=1,NNPE
          DO 25 J=1,NNPE
              EK(I,J)=0.0
              EKU(I,J)=0.0
          25 CONTINUE
      20 CONTINUE
C
C      Strip out the coordinates for this element.
      DO 10 I=1,NNPE
          X(I)=XN(NODCON(IE,I))
          Y(I)=YN(NODCON(IE,I))
      10 CONTINUE
C
C      Compute the average temperature of the element for the thermal
C      conductivity calculation; if the resulting temperature is zero,
C      set TEMP equal to an arbitrary value to avoid problems with
C      thermal conductivity correlations; call for the element thermal
C      conductivity:
      TEMP=0.0
      DO 50 M=1,NNPE
          TEMP=TEMP+(1.0/(REAL(NNPE)))*T(NODCON(IE,M))
      50 CONTINUE
      IF (TEMP.LE.0.0) TEMP=1000.0
      THERM=COND(KTH(IE),TEMP)
C
C      Loop through all the entries in the element stiffness matrix:
      DO 60 IN=1,NNPE
          DO 70 JN=1,NNPE
C
C      Compute the functions to be integrated evaluated at SI(I),ETA(J).
          SUM=0.0
          DO 100 I=1,NQUAD

```

```

      DO 110 J=1,NQUAD
C
C      Assign values of SI and ETA for the current quadrature point:
      SI=POINTS(I)
      ETA=POINTS(J)
C
C      Obtain weights for the current quadrature point:
      WE=WTS(I)
      WN=WTS(J)
C
C      Call for the evaluation of the derivatives of the functions
C      w.r.t. X and Y and the determinant of the Jacobian:
      CALL DERIVATIVES(X,Y,SI,ETA,PSI,DXDE,DXDN,DYDE,DYDN
+      ,DPDX,DPDY,DJACOB)
C
C      Sum up function evaluations at the current quadrature points:
      SUM=SUM+WE*WN*THERM*(DPDX(IN)*DPDX(JN)+
+      DPDY(IN)*DPDY(JN))*DJACOB
110      CONTINUE
100      CONTINUE
C
C      Assign the integrated function to its place in the stiffness
C      matrix:
      EK(IN,JN)=SUM
      EKV(IN,JN)=SUM
C
70      CONTINUE
60      CONTINUE
C
C      Impose convective boundary conditions; consider the 1-D boundary
C      integral for convection boundary conditions along an element
C      edge:
      IF (NEWC.GT.0) THEN
      DO IN=1,NNPE
      DO ISIDE=1,4
      IF (H(IE,ISIDE).NE.0.0) THEN
      SUM=0.0
      DO 200 INN=1,NQUAD
C
C      Set up the quadrature and evaluation points and weights:
      SI=POINTS(INN)
      ETA=POINTS(INN)
      WE=WTS(INN)
      IF (ISIDE.EQ.1) THEN
      ETA=-1.0
      ELSEIF (ISIDE.EQ.2) THEN
      SI=1.0
      ELSEIF (ISIDE.EQ.3) THEN
      ETA=1.0
      ELSE
      SI=-1.0
      ENDIF
C
C      Call for the evaluation of the derivatives of the functions
C      w.r.t. X and Y and the determinant of the Jacobian:

```

```

      CALL DERIVATIVES (X,Y,SI,ETA,PSI,DXDE,DXDN,DYDE,DYDN
+      ,DPDX,DPDY,DJACOB)
C
C   Evaluate the function at the current quadrature points:
      IF (ISIDE.EQ.1.OR.ISIDE.EQ.3) THEN
        SUM=SUM+WE*H(IE,ISIDE)*PSI(IN)*
+      (DSQRT(DXDE*DXDE+DYDE*DYDE))
      ELSE
        SUM=SUM+WE*H(IE,ISIDE)*PSI(IN)*
+      (DSQRT(DXDN*DXDN+DYDN*DYDN))
      ENDIF
200    CONTINUE
C
C   Assign this entry to its appropriate location in the stiffness
C   matrix:
      EK(IN,IN)=EK(IN,IN)+SUM
      ENDIF
    END DO
  END DO
ENDIF
C
  RETURN
  END
C -----
  SUBROUTINE ELOAD(IE,XN,YN,NODCON,Q,FLUX,EF,EQ,H,TINF)
C -----
C   This subroutine constructs the element heat flux or load vector.
C
  IMPLICIT REAL*8(A-H,O-Z)
  PARAMETER (NDIM=1300,MAXBW=105,MAXENCL=50,MSURF=99,MNPE=9,
+    MQUAD=5,NSPE=4)
  COMMON/CONTRL/ ITOLD,ISFTYPE,NN,NE,NNPE,NEWF,NEWC,NQUAD,IHBW
  COMMON/NUMINT/ POINTS(MQUAD),WTS(MQUAD)
  REAL*8 Q(NDIM),FLUX(NDIM,NSPE),XN(NDIM),YN(NDIM),EF(MNPE),
+    EQ(MNPE)
  REAL*8 H(NDIM,NSPE),TINF(NDIM,NSPE)
  REAL*8 X(MNPE),Y(MNPE)
  REAL*8 PSI(MNPE),DPDE(MNPE),DPDN(MNPE)
  REAL*8 DPDX(MNPE),DPDY(MNPE)
  INTEGER NODCON(NDIM,MNPE)
C
C   Initialize the element load vector:
  DO 20 I=1,MNPE
    EF(I)=0.0
    EQ(I)=0.0
20  CONTINUE
C
C   Strip out the coordinates for this element.
  DO 11 I=1,MNPE
    X(I)=XN(NODCON(IE,I))
    Y(I)=YN(NODCON(IE,I))
11  CONTINUE
C
C   Loop through all the entries in the element load vector:
  DO 10 IN=1,MNPE

```

```

C
C   Compute the functions to be integrated evaluated at SI(1),ETA(J).
C   First consider the 2-D integral involving the internal heat
C   generation:
      IF (Q(IE).NE.0.0) THEN
        SUM=0.0
        DO 100 I=1,NQUAD
          DO 110 J=1,NQUAD
C
C     Assign values of SI and ETA for the current quadrature point:
          SI=POINTS(I)
          ETA=POINTS(J)
C
C     Obtain weights for the current quadrature point:
          WE=WTS(I)
          WN=WTS(J)
C
C     Call for the evaluation of the derivatives of the functions
C     w.r.t. X and Y and the determinant of the Jacobian:
          CALL DERIVATIVES(X,Y,SI,ETA,PSI,DXDE,DXDN,DYDE,
+             DYDN,DPDX,DPDY,DJACOB)
C
C     Sum up function evaluations at the current quadrature points:
          SUM=SUM+WE*WN*Q(IE)*PSI(IN)*DJACOB
110      CONTINUE
100      CONTINUE
C
C   Assign this entry to its appropriate location in the load vector:
      EF(IN)=EF(IN)+SUM
      EQ(IN)=EQ(IN)+SUM
      ENDIF
C
C   Now consider the 1-D boundary integral for specified heat fluxes
C   or convection boundary conditions along an element edge:
      DO 199 ISIDE=1,4
C
C     If there is no specified heat flux on this element side and no
C     convection is occurring at this element side either, then the
C     following calculation is going to result in zero being added to
C     this entry of the element load vector; therefore, simply skip the
C     calculation:
          IF (FLUX(IE,ISIDE).EQ.0.0.AND.H(IE,ISIDE).EQ.0.0) THEN
            GOTO 198
          ENDIF
C
C     Non-zero terms are present, so proceed with the calculation:
          SUM1=0.0
          SUM2=0.0
          DO 200 INN=1,NQUAD
C
C     Set up the quadrature and evaluation points and weights:
          SI=POINTS(INN)
          ETA=POINTS(INN)
          WE=WTS(INN)
          IF (ISIDE.EQ.1) THEN

```

```

      ETA=-1.0
      ELSEIF (ISIDE.EQ.2) THEN
        SI=1.0
      ELSEIF (ISIDE.EQ.3) THEN
        ETA=1.0
      ELSE
        SI=-1.0
      ENDIF
C
C      Call for the evaluation of the derivatives of the functions
C      w.r.t. X and Y and the determinant of the Jacobian:
      CALL DERIVATIVES(X,Y,SI,ETA,PSI,DXDE,DXDN,DYDE,DYDN
+      ,DPDX,DPDY,DJACOB)
C
C      Evaluate the function at the current quadrature points:
      IF (ISIDE.EQ.1.OR.ISIDE.EQ.3) THEN
        SUM1=SUM1+WE*FLUX(IE,ISIDE)*PSI(IN)
+      *(DSQRT(DXDE*DXDE+DYDE*DYDE))
        SUM2=SUM2+WE*H(IE,ISIDE)*TINF(IE,ISIDE)*PSI(IN)
+      *(DSQRT(DXDE*DXDE+DYDE*DYDE))
      ELSE
        SUM1=SUM1+WE*FLUX(IE,ISIDE)*PSI(IN)
+      *(DSQRT(DXDN*DXDN+DYDN*DYDN))
        SUM2=SUM2+WE*H(IE,ISIDE)*TINF(IE,ISIDE)*PSI(IN)
+      *(DSQRT(DXDN*DXDN+DYDN*DYDN))
      ENDIF
200      CONTINUE
C
C      Assign this entry to its appropriate location in the load vector:
      EF(IN)=EF(IN)+SUM1+SUM2
198      CONTINUE
199      CONTINUE
C
10      CONTINUE
C
      RETURN
      END
C -----
      SUBROUTINE DERIVATIVES(X,Y,SI,ETA,PSI,DXDE,DXDN,DYDE,DYDN,DPDX
+      ,DPDY,DJACOB)
C -----
C      This subroutine computes, for passed values of SI and ETA, the
C      values of the shape function, its derivatives w.r.t. local and
C      global coordinates, the Jacobian matrix, its determinant, and
C      its inverse for an element.
C
      IMPLICIT REAL*8(A-H,O-Z)
      PARAMETER (NDIM=1300,MAXBW=105,MAXENCL=50,MSURF=99,MNPE=9,
+      MQUAD=5,NSPE=4)
      COMMON/CONTRL/ ITOLD,ISFTYPE,NN,NE,NNPE,NEWF,NEWC,NQUAD,IHBW
      REAL*8 AJ(2,2),SJ(2,2),X(MNPE),Y(MNPE)
      REAL*8 PSI(MNPE),DPDE(MNPE),DPDN(MNPE)
      REAL*8 DPDX(MNPE),DPDY(MNPE)
C
C      Evaluate the value of the shape functions:

```



```

IF (ISFTYPE.EQ.1) THEN
  PSI(1)=0.25*(1.-SI)*(1.-ETA)
  PSI(2)=0.25*(1.+SI)*(1.-ETA)
  PSI(3)=0.25*(1.+SI)*(1.+ETA)
  PSI(4)=0.25*(1.-SI)*(1.+ETA)
ELSEIF (ISFTYPE.EQ.2) THEN
  PSI(1)=0.25*(1.-SI)*(1.-ETA)*(-1.-SI-ETA)
  PSI(2)=0.50*(1.-SI*SI)*(1.-ETA)
  PSI(3)=0.25*(1.+SI)*(1.-ETA)*(-1.+SI-ETA)
  PSI(4)=0.50*(1.+SI)*(1.-ETA*ETA)
  PSI(5)=0.25*(1.+SI)*(1.+ETA)*(-1.+SI+ETA)
  PSI(6)=0.50*(1.-SI*SI)*(1.+ETA)
  PSI(7)=0.25*(1.-SI)*(1.+ETA)*(-1.-SI+ETA)
  PSI(8)=0.50*(1.-SI)*(1.-ETA*ETA)
ELSEIF (ISFTYPE.EQ.3) THEN
  PSI(1)=0.25*(SI*SI-SI)*(ETA*ETA-ETA)
  PSI(2)=0.50*(1.-SI*SI)*(ETA*ETA-ETA)
  PSI(3)=0.25*(SI*SI+SI)*(ETA*ETA-ETA)
  PSI(4)=0.50*(SI*SI+SI)*(1.-ETA*ETA)
  PSI(5)=0.25*(SI*SI+SI)*(ETA*ETA+ETA)
  PSI(6)=0.50*(1.-SI*SI)*(ETA*ETA+ETA)
  PSI(7)=0.25*(SI*SI-SI)*(ETA*ETA+ETA)
  PSI(8)=0.50*(SI*SI-SI)*(1.-ETA*ETA)
  PSI(9)=(1.-SI*SI)*(1.-ETA*ETA)
ENDIF

```

C
C
C

Evaluate the value of the derivatives of the shape functions
w.r.t. SI:

```

IF (ISFTYPE.EQ.1) THEN
  DPDE(1)=0.25*(ETA-1.)
  DPDE(2)=0.25*(1.-ETA)
  DPDE(3)=0.25*(1.+ETA)
  DPDE(4)=0.25*(-1.-ETA)
ELSEIF (ISFTYPE.EQ.2) THEN
  DPDE(1)=0.25*(2.*SI+ETA-2.*SI*ETA-ETA*ETA)
  DPDE(2)=-SI+ETA*SI
  DPDE(3)=0.25*(-ETA+2.*SI-2.*SI*ETA+ETA*ETA)
  DPDE(4)=0.50*(1.-ETA*ETA)
  DPDE(5)=0.25*(ETA+2.*ETA*SI+2.*SI+ETA*ETA)
  DPDE(6)=-SI-SI*ETA
  DPDE(7)=0.25*(-ETA+2.*SI+2.*SI*ETA-ETA*ETA)
  DPDE(8)=0.5*(-1.+ETA*ETA)
ELSEIF (ISFTYPE.EQ.3) THEN
  DPDE(1)=0.25*(2.*SI*ETA*ETA-2.*ETA*SI-ETA*ETA+ETA)
  DPDE(2)=0.5*(-2.*SI*ETA*ETA+2.*ETA*SI)
  DPDE(3)=0.25*(2.*SI*ETA*ETA-2.*ETA*SI+ETA*ETA-ETA)
  DPDE(4)=0.5*(2.*SI-2.*SI*ETA*ETA+1.-ETA*ETA)
  DPDE(5)=0.25*(2.*SI*ETA*ETA+2.*ETA*SI+ETA*ETA+ETA)
  DPDE(6)=0.5*(-2.*SI*ETA*ETA-2.*ETA*SI)
  DPDE(7)=0.25*(2.*SI*ETA*ETA+2.*SI*ETA-ETA*ETA-ETA)
  DPDE(8)=0.5*(2.*SI-2.*SI*ETA*ETA-1.+ETA*ETA)
  DPDE(9)=-2.*SI+2.*SI*ETA*ETA
ENDIF

```

C
C

Evaluate the value of the derivatives of the shape functions

```

C      w.r.t. ETA:
      IF (ISFTYPE.EQ.1) THEN
        DPDN(1)=0.25*(SI-1.)
        DPDN(2)=0.25*(-1.-SI)
        DPDN(3)=0.25*(1.+SI)
        DPDN(4)=0.25*(1.-SI)
      ELSEIF (ISFTYPE.EQ.2) THEN
        DPDN(1)=0.25*(2.*ETA+SI-SI*SI-2.*SI*ETA)
        DPDN(2)=0.50*(-1.+SI*SI)
        DPDN(3)=0.25*(-SI+2.*ETA-SI*SI+2.*ETA*SI)
        DPDN(4)=-ETA-SI*ETA
        DPDN(5)=0.25*(SI+SI*SI+2.*ETA+2.*ETA*SI)
        DPDN(6)=0.50*(1.-SI*SI)
        DPDN(7)=0.25*(-SI+2.*ETA+SI*SI-2.*ETA*SI)
        DPDN(8)=-ETA+ETA*SI
      ELSEIF (ISFTYPE.EQ.3) THEN
        DPDN(1)=0.25*(2.*ETA*SI*SI-SI*SI-2.*SI*ETA+SI)
        DPDN(2)=0.5*(2.*ETA-1.-2.*ETA*SI*SI+SI*SI)
        DPDN(3)=0.25*(2.*ETA*SI*SI-SI*SI+2.*ETA*SI-SI)
        DPDN(4)=0.5*(-2.*ETA*SI*SI-2.*ETA*SI)
        DPDN(5)=0.25*(2.*ETA*SI*SI+SI*SI+2.*ETA*SI+SI)
        DPDN(6)=0.5*(2.*ETA+1.-2.*ETA*SI*SI-SI*SI)
        DPDN(7)=0.25*(2.*ETA*SI*SI+SI*SI-2.*ETA*SI-SI)
        DPDN(8)=0.5*(-2.*ETA*SI*SI+2.*ETA*SI)
        DPDN(9)=-2.*ETA+2.*ETA*SI*SI
      ENDIF

C
C      Initialize matrices to be used:
      DO 2 I=1,2
        DO 3 J=1,2
          AJ(I,J)=0.0
          SJ(I,J)=0.0
        3    CONTINUE
      2    CONTINUE

C
C      Compute the Jacobian matrix:
      DO 10 I=1,NNPE
        AJ(1,1)=AJ(1,1)+X(I)*DPDE(I)
        AJ(1,2)=AJ(1,2)+Y(I)*DPDE(I)
        AJ(2,1)=AJ(2,1)+X(I)*DPDN(I)
        AJ(2,2)=AJ(2,2)+Y(I)*DPDN(I)
      10    CONTINUE
      DXDE=AJ(1,1)
      DYDE=AJ(1,2)
      DXDN=AJ(2,1)
      DYDN=AJ(2,2)

C
C      Compute the determinant of the Jacobian:
      DJACOB=AJ(1,1)*AJ(2,2)-AJ(1,2)*AJ(2,1)

C
C      Compute the inverse of the Jacobian matrix:
      SJ(1,1)=AJ(2,2)/DJACOB
      SJ(1,2)=-AJ(1,2)/DJACOB
      SJ(2,1)=-AJ(2,1)/DJACOB
      SJ(2,2)=AJ(1,1)/DJACOB

```

```

C
C   Compute the values of the derivatives of the shape functions
C   w.r.t.
C   X and Y:
C   DO 20 I=1,NNPE
C       DPDX(I)=DPDE(I)*SJ(1,1)+DPDN(I)*SJ(1,2)
C       DPDY(I)=DPDE(I)*SJ(2,1)+DPDN(I)*SJ(2,2)
20  CONTINUE
C
C   RETURN
C   END
C -----
C   SUBROUTINE ENCLOSURES (XN,YN,NODCON,T,FLUX)
C -----
C   This subroutine organizes the geometries of the enclosure(s) and
C   calls the routines necessary to calculate first the radiosities
C   and then the net heat fluxes at each surface of the enclosure(s).
C   This information is returned to the main program so the enclosure
C   surface heat fluxes can be incorporated into the global heat flux
C   vector.
C
C   IMPLICIT REAL*8 (A-H,O-Z)
C   PARAMETER (NDIM=1300,MAXBW=105,MAXENCL=50,MSURF=99,MNPE=9,
C +           MQUAD=5,NSPE=4)
C   COMMON/CONTRL/ ITOLD,ISFTYPE,NN,NE,NNPE,NEWF,NEWC,NQUAD,IHBW
C   COMMON/ENCLOS/ IENCL,NENCL,NSURF(MAXENCL),IESURF(MAXENCL,MSURF),
C +           ISSURF(MAXENCL,MSURF),IEM(MAXENCL,MSURF),
C +           ASURF(MAXENCL,MSURF)
C   REAL*8 XN(NDIM),YN(NDIM),X1(MSURF),X2(MSURF),Y1(MSURF),Y2(MSURF)
C   REAL*8 EM(MSURF),TSURF(MSURF),RADHEAT(MSURF)
C   REAL*8 T(NDIM),FLUX(NDIM,NSPE)
C   INTEGER NODCON(NDIM,MNPE)
C
C   Visit each enclosure:
C   DO 10 N=1,NENCL
C
C   Identify the coordinates of the sides defining the enclosure and
C   load these coordinates into the vectors needed the the subroutine
C   RADIOSITY:
C       DO I=1,NSURF(N)
C           IF (ISFTYPE.EQ.1) THEN
C               IF (ISSURF(N,I).LT.4) THEN
C                   NODE=ISSURF(N,I)
C                   NODEP1=NODE+1
C               ELSE
C                   NODE=4
C                   NODEP1=1
C               ENDIF
C           ELSEIF (ISFTYPE.EQ.2.OR.ISFTYPE.EQ.3) THEN
C               IF (ISSURF(N,I).LT.4) THEN
C                   NODE=ISSURF(N,I)*2-1
C                   NODEPH=NODE+1
C                   NODEP1=NODE+2
C               ELSE
C                   NODE=7

```

```

      NODEPH=8
      NODEP1=1
      ENDIF
      ENDIF
C
C      Compute the global node numbers:
      MNOD=NODCON(IESURF(N,I),NODE)
      MNODPH=NODCON(IESURF(N,I),NODEPH)
      MNODP1=NODCON(IESURF(N,I),NODEP1)
C
C      Extract the coordinates:
      X1(I)=XN(MNOD)
      Y1(I)=YN(MNOD)
      X2(I)=XN(MNODP1)
      Y2(I)=YN(MNODP1)
C
C      Compute the area of this surface:
      ASURF(N,I)=DSQRT((X1(I)-X2(I))**2+(Y1(I)-Y2(I))**2)
C
C      Compute average temperature of this enclosure surface:
      IF (ISFTYPE.EQ.1) THEN
        TSURF(I)=(T(MNOD)+T(MNODP1))/2.0
      ELSEIF (ISFTYPE.EQ.2.OR.ISFTYPE.EQ.3) THEN
        TSURF(I)=(T(MNOD)+T(MNODPH)+T(MNODP1))/3.0
      ENDIF
C
C      Call for the emissivity of this enclosure surface:
      EM(I)=EMISS(IEM(N,I),TSURF(I))
C
      END DO
C
C      Call for the subroutine to compute the radiosities of each of the
C      enclosure's surfaces, and returns the value of the net heat flux
C      at each surface:
      CALL RADIOSITY(NSURF(N),X1,X2,Y1,Y2,EM,TSURF,RADHEAT)
C
C      Assign the computed net heat fluxes to their appropriate
C      locations in the heat flux vector FLUX:
      DO 20 J=1,NSURF(N)
        FLUX(IESURF(N,J),ISSURF(N,J))=RADHEAT(J)
20    CONTINUE
C
10  CONTINUE
C
      RETURN
      END
C -----
      SUBROUTINE RADIOSITY(NUMSURF,X1,X2,Y1,Y2,EM,TSURF,RADHEAT)
C -----
C      This subroutine computes the radiosity of each surface within an
C      enclosure having sides of known temperatures. Each side of the
C      enclosure is assumed to be diffuse-gray and have a uniform
C      temperature. Knowing the radiosity of each side of the enclosure
C      then enables the net heat transfer to each enclosure side to be
C      computed.

```

```

C
C   Local Variable Definitions:
C   ASURF'.....Area (length * 1 unit) of a surface.
C   EM().....Array holding the surface emissivities.
C   FIJ'..... View factor for surface I "looking at" surface J.
C   NUMSURF'....Number of surfaces defining the enclosure.
C   RADHEAT()...The net heat flux out of (+) each enclosure surface.
C   SIGMA'.....Stefan-Boltzmann constant in units of W/(m**2-K**4).
C   TSURF()....Array holding the surface temperatures.
C   U'.....Array holding, upon solution, the radiosities of each
C           surface.
C   X1,X2()....Arrays holding x-coordinates of the surface endpoints
C           as depicted in the function VIEW.
C   Y1,Y2()....Arrays holding y-coordinates of the surface endpoints
C           as depicted in the function VIEW.
C   -----
C
C   IMPLICIT REAL*8(A-H,O-Z)
C   PARAMETER (NDIM=1300,MAXBW=105,MAXENCL=50,MSURF=99,MNPE=9,
C   +          MQUAD=5,NSPE=4)
C   COMMON/CONTRL/ ITOLD,ISFTYPE,NN,NE,NNPE,NEWF,NEWC,NQUAD,IHBW
C   COMMON/ENCLOS/ IENCL,NENCL,NSURF(MAXENCL),IESURF(MAXENCL,MSURF),
C   +             ISSURF(MAXENCL,MSURF),IEM(MAXENCL,MSURF),
C   +             ASURF(MAXENCL,MSURF)
C   REAL*8 X(4),Y(4)
C   REAL*8 X1(MSURF),X2(MSURF),Y1(MSURF),Y2(MSURF)
C   REAL*8 EM(MSURF),TSURF(MSURF),RADHEAT(MSURF)
C   REAL*8 A(MSURF,MSURF),B(MSURF),U(MSURF)
C
C   Assign constant values:
C   SIGMA=5.729D-08
C
C   Initialize arrays to be used:
C   DO I=1,NUMSURF
C       B(I)=0.0
C       U(I)=0.0
C       RADHEAT(I)=0.0
C       DO J=1,NUMSURF
C           A(I,J)=0.0
C       END DO
C   END DO
C
C   Begin constructing the radiosity equations and storing the
C   radiosity coefficients in the matrix A and the force term in the
C   vector B, such that the radiosity system is defined by the matrix
C   equation  $A*U=B$ , where U is the unknown radiosity vector:
C   DO I=1,NUMSURF
C       X(1)=X1(I)
C       X(2)=X2(I)
C       Y(1)=Y1(I)
C       Y(2)=Y2(I)
C       B(I)=(EM(I)/(1.0-EM(I)))*SIGMA*(TSURF(I)**4)
C
C       DO J=1,I-1
C           X(3)=X1(J)

```

```

X(4)=X2(J)
Y(3)=Y1(J)
Y(4)=Y2(J)
FIJ=VIEW(X,Y)
A(I,J)=-FIJ
END DO

C
J=1
A(I,J)=1.0/(1.0-EM(I))

C
DO J=I+1,NUMSURF
X(3)=X1(J)
X(4)=X2(J)
Y(3)=Y1(J)
Y(4)=Y2(J)
FIJ=VIEW(X,Y)
A(I,J)=-FIJ
END DO
END DO

C
C The coefficient matrix A and force vector B are now filled; call
C for the solution vector U of the system of equations:
CALL SOLVER(A,B,U,NUMSURF)

C
C Knowing the surface radiosities, compute the net heat flux out
C (+) of each surface of the enclosure:
DO I=1,NUMSURF
RADHEAT(I)=EM(I)*(SIGMA*(TSURF(I)**4)-U(I))
+ / (1.0-EM(I))
END DO

C
RETURN
END

C -----
FUNCTION VIEW(X,Y)
C -----
C This subroutine computes the view factor between two element
C sides using Hottel's crossed-string method; the "canonical"
C geometry orientation assumed is shown below:
C CANONICAL GEOMETRY:
C
C          STR1
C      X1 ( ) \-----/ ( ) X3
C          ( ) \       / ( )
C          ( ) \     / <-( ) ----- STR3
C          ( )  \   /   ( )
C      A  ( )   /\    ( ) B
C          ( )  / \   <-( ) ----- STR2
C          ( ) /   \   ( )
C      X2 ( ) /-----\ ( ) X4
C          STR4
C
C Local Variable Definitions:
C DIST1(4)...Distance from 4 surface endpoints to the point
C (XINT,YINT).

```

```

C      IPARALLEL..Specification flag indicating whether or not the two
C                  radiating surfaces are parallel or not (0=NO, 1=YES).
C      SIDEA.....Length of radiation surface A.
C      SLOPEA.....Slope of the line defined by surface A.
C      SLOPEB.....Slope of the line defined by surface B.
C      STR1(4)....The lengths of the four "strings".
C      VIEW.....The view factor for transfer from surface A to B.
C      XINT.....X coordinate of the point of intersection of the two
C                  lines defined by the two radiating surfaces.
C      X(4).....Array holding the X coordinates of the four endpoints
C                  of the radiating surfaces.
C      X1..X4....Values of the four X coordinates of the surface
C                  endpoints in "canonical" (known) geometry orientation.
C      YINT.....Y coordinate of the point of intersection of the two
C                  lines defined by the two radiating surfaces.
C      Y(4).....Array holding the Y coordinates of the four endpoints
C                  of the radiating surfaces.
C      Y1..Y4....Values of the four Y coordinates of the surface
C                  endpoints in "canonical" (known) geometry orientation.
C      -----

```

```

C      IMPLICIT REAL*8 (A-H,O-Z)
C      PARAMETER (NDIM=1300,MAXBW=105,MAXENCL=50,MSURF=99,MNPE=9,
+                MQUAD=5,NSPE=4)
C      REAL*8 X(4),Y(4)
C
C      Check to see if side A is viewing itself; if it is, the view
C      factor is zero:
C      IF (X(1).EQ.X(3).AND.X(2).EQ.X(4).AND.Y(1).EQ.Y(3).AND
+        .Y(2).EQ.Y(4)) THEN
C          VIEW=0.0
C          RETURN
C      ENDIF
C      IF (X(1).EQ.X(4).AND.X(2).EQ.X(3).AND.Y(1).EQ.Y(4).AND
+        .Y(2).EQ.Y(3)) THEN
C          VIEW=0.0
C          RETURN
C      ENDIF
C
C      Compute the slopes of the two lines defined by the two surfaces
C      for which the view factor is being computed:
C      DENA=X(1)-X(2)
C      DENB=X(3)-X(4)
C      IF (DENA.EQ.0.0) THEN
C          SLOPEA=1.0D+20
C      ELSE
C          SLOPEA=(Y(1)-Y(2))/DENA
C      ENDIF
C      IF (DENB.EQ.0.0) THEN
C          SLOPEB=1.0D+20
C      ELSE
C          SLOPEB=(Y(3)-Y(4))/DENB
C      ENDIF
C
C      As a special case, check to see if the two surfaces are parallel:

```

```

IPARALLEL=0
IF (SLOPEA.EQ.SLOPEB) IPARALLEL=1
C
C      Thus, two different procedures must be used to determine which
C      strings are crossed and which are uncrossed dependent upon
C      whether the two surfaces are parallel or not; the case where the
C      two surfaces are parallel is the easiest situation, so consider
C      it first:
IF (IPARALLEL.EQ.1) THEN
  IF (X(1)-X(2).EQ.0.0) GOTO 100
  IF (X(1).LT.X(2)) THEN
    X1=X(1)
    X2=X(2)
    Y1=Y(1)
    Y2=Y(2)
  ELSE
    X1=X(2)
    X2=X(1)
    Y1=Y(2)
    Y2=Y(1)
  ENDIF
C
  IF (X(3).LT.X(4)) THEN
    X3=X(3)
    X4=X(4)
    Y3=Y(3)
    Y4=Y(4)
  ELSE
    X3=X(4)
    X4=X(3)
    Y3=Y(4)
    Y4=Y(3)
  ENDIF
  GOTO 101
100 CONTINUE
  IF (Y(1).LT.Y(2)) THEN
    X1=X(1)
    X2=X(2)
    Y1=Y(1)
    Y2=Y(2)
  ELSE
    X1=X(2)
    X2=X(1)
    Y1=Y(2)
    Y2=Y(1)
  ENDIF
C
  IF (Y(3).LT.Y(4)) THEN
    X3=X(3)
    X4=X(4)
    Y3=Y(3)
    Y4=Y(4)
  ELSE
    X3=X(4)
    X4=X(3)

```



```

        Y3=Y(4)
        Y4=Y(3)
    ENDIF
101    CONTINUE
ELSE
C
C    The two surfaces are not parallel; compute the intersection of
C    the lines defined by the two surfaces:
        XINT=(SLOPEA*X(1)-SLOPEB*X(3)+Y(3)-Y(1))/(SLOPEA-SLOPEB)
        YINT=SLOPEA*(XINT-X(1))+Y(1)
C
C    Determine which endpoints on the surfaces are nearest (and
C    farthest) from the intersection point. This will allow for the
C    orientation of the surfaces to be known:
        DIST1=DSQRT((X(1)-XINT)**2+(Y(1)-YINT)**2)
        DIST2=DSQRT((X(2)-XINT)**2+(Y(2)-YINT)**2)
        DIST3=DSQRT((X(3)-XINT)**2+(Y(3)-YINT)**2)
        DIST4=DSQRT((X(4)-XINT)**2+(Y(4)-YINT)**2)
C
        IF (DIST1.LT.DIST2) THEN
            X1=X(1)
            X2=X(2)
            Y1=Y(1)
            Y2=Y(2)
        ELSE
            X1=X(2)
            X2=X(1)
            Y1=Y(2)
            Y2=Y(1)
        ENDIF
C
        IF (DIST3.LT.DIST4) THEN
            X3=X(3)
            X4=X(4)
            Y3=Y(3)
            Y4=Y(4)
        ELSE
            X3=X(4)
            X4=X(3)
            Y3=Y(4)
            Y4=Y(3)
        ENDIF
    ENDIF
C
C    Now that the crossed and uncrossed strings have been determined,
C    compute the view factor using Hottel's crossed string method:
        STR1=DSQRT((X1-X3)**2+(Y1-Y3)**2)
        STR2=DSQRT((X1-X4)**2+(Y1-Y4)**2)
        STR3=DSQRT((X2-X3)**2+(Y2-Y3)**2)
        STR4=DSQRT((X2-X4)**2+(Y2-Y4)**2)
        SIDEA=DSQRT((X1-X2)**2+(Y1-Y2)**2)
        VIEW=0.5*DABS(STR2+STR3-STR1-STR4)/SIDEA
C
    RETURN
END

```

```

C -----
C      SUBROUTINE RESULTS (XN,YN,T,GKU,GQ,NODCON,KTH,ECOND,AVGCOND,AREA,
+      TOTAREA,QOUT,QSUM,SURFEM,SURFTEMP,AVGEEM,
+      AVGETEMP)
C -----
C      This subroutine computes additional quantities based on the now
C      known solution variable T. These quantities include:
C
C      - Element thermal conductivities.
C      - Requested nodal heat fluxes.
C      - Element areas.
C      - Area of entire computational domain.
C      - Average thermal conductivity of the computational domain.
C
C      IMPLICIT REAL*8 (A-H,O-Z)
C      PARAMETER (NDIM=1300,MAXBW=105,MAXENCL=50,MSURF=99,MNPE=9,
+      MQUAD=5,NSPE=4)
C      COMMON/CONTRL/ ITOLD,ISFTYPE,NN,NE,NNPE,NEWF,NEWC,NQUAD,IHBM
C      COMMON/ENCLOS/ IENCL,NENCL,NSURF(MAXENCL),IESURF(MAXENCL,MSURF),
+      ISSURF(MAXENCL,MSURF),IEM(MAXENCL,MSURF),
+      ASURF(MAXENCL,MSURF)
C      COMMON/NUMINT/ POINTS(MQUAD),WTS(MQUAD)
C      COMMON/OUTPUT/ IFLUXES,NNFLUX(NDIM)
C      REAL*8 T(NDIM),ECOND(NDIM),QOUT(NDIM),GQ(NDIM),GKU(NDIM,NDIM)
C      REAL*8 AREA(NDIM),XN(NDIM),YN(NDIM),X(MNPE),Y(MNPE)
C      REAL*8 SURFEM(MAXENCL,MSURF),SURFTEMP(MAXENCL,MSURF)
C      REAL*8 AVGEEM(MAXENCL),AVGETEMP(MAXENCL)
C      REAL*8 PSI(MNPE),DPDE(MNPE),DPDN(MNPE)
C      REAL*8 DPDX(MNPE),DPDY(MNPE)
C      INTEGER NODCON(NDIM,MNPE),KTH(NDIM)
C
C      Compute the element thermal conductivity values from the
C      converged temperature field:
C      DO I=1,NE
C          TEMP=0.0
C          DO M=1,NNPE
C              TEMP=TEMP+(1.0/(REAL(NNPE)))*T(NODCON(I,M))
C          END DO
C          ECOND(I)=COND(KTH(I),TEMP)
C      END DO
C
C      Extract the nodal heat fluxes from the heat flux vector now that
C      the temperatures are known; loop through all the nodes:
C      QSUM=0.0
C      DO I=1,IFLUXES
C          IN=NNFLUX(I)
C          SUM=0.0
C          DO J=1,NN
C              SUM=SUM+T(J)*GKU(IN,J)
C          END DO
C          QOUT(I)=SUM-GQ(IN)
C          QSUM=QSUM+QOUT(I)
C      END DO
C
C      Compute the area of each element and the total area of all

```

```

C      elements:
      TOTAREA=0.0
      DO IE=1,NE
C
C      Strip out the coordinates for this element.
      DO 10 I=1,NNPE
        X(I)=XN(NODCON(IE,I))
        Y(I)=YN(NODCON(IE,I))
10      CONTINUE
C
C      Perform the area calculation using G-L quadrature:
      AREA(IE)=0.0
      DO 100 I=1,NQUAD
        DO 110 J=1,NQUAD
C
C      Assign values of SI and ETA for the current quadrature point:
          SI=POINTS(I)
          ETA=POINTS(J)
C
C      Obtain weights for the current quadrature point:
          WE=WTS(I)
          WN=WTS(J)
C
C      Call for the evaluation of the derivatives of the functions
C      w.r.t. X and Y and the determinant of the Jacobian:
          CALL DERIVATIVES(X,Y,SI,ETA,PSI,DXDE,DXDN,DYDE,DYDN
+          ,DPDX,DPDY,DJACOB)
C
C      Sum up contribution from the current quadrature point:
          ADD=WE*WN*DJACOB
          ADD=DABS(ADD)
          AREA(IE)=AREA(IE)+ADD
110      CONTINUE
100      CONTINUE
      TOTAREA=TOTAREA+AREA(IE)
    END DO
C
C      Compute the average thermal conductivity for the problem:
      AVGCOND=0.0
      DO I=1,NE
        AVGCOND=AVGCOND+ECOND(I)*AREA(I)
      END DO
      AVGCOND=AVGCOND/TOTAREA
C
C      Compute the averaged temperature and emissivity for each surface
C      of each enclosure:
      IF (IENCL.EQ.1) THEN
        DO 20 N=1,NENCL
C
C      Identify the nodes on the sides of the elements that make up the
C      surfaces of the enclosure:
          DO I=1,NSURF(N)
            IF (ISFTYPE.EQ.1) THEN
              IF (ISSURF(N,I).LT.4) THEN
                NODE=ISSURF(N,I)

```

```

        NODEP1=NODE+1
      ELSE
        NODE=4
        NODEP1=1
      ENDIF
    ELSEIF (ISFTYPE.EQ.2.OR.ISFTYPE.EQ.3) THEN
      IF (ISSURF(N,I).LT.4) THEN
        NODE=ISSURF(N,I)*2-1
        NODEPH=NODE+1
        NODEP1=NODE+2
      ELSE
        NODE=7
        NODEPH=8
        NODEP1=1
      ENDIF
    ENDIF
  ENDIF

C
C   Compute the global node numbers:
      MNOD=NODCON(IESURF(N,I),NODE)
      MNODPH=NODCON(IESURF(N,I),NODEPH)
      MNODP1=NODCON(IESURF(N,I),NODEP1)
C
C   Compute average temperature of this enclosure surface:
      IF (ISFTYPE.EQ.1) THEN
        SURFTEMP(N,I)=(T(MNOD)+T(MNODP1))/2.0
      ELSEIF (ISFTYPE.EQ.2.OR.ISFTYPE.EQ.3) THEN
        SURFTEMP(N,I)=(T(MNOD)+T(MNODPH)+T(MNODP1))/3.0
      ENDIF
C
C   Call for the emissivity of this enclosure surface:
      SURFEM(N,I)=EMISS(IEM(N,I),SURFTEMP(N,I))
C
      END DO
20  CONTINUE
C
C   Compute the average temperature and emissivity of each enclosure:
      DO N=1,NENCL
        AVGETEMP(N)=0.0
        DO I=1,NSURF(N)
          AVGETEMP(N)=AVGETEMP(N)+SURFTEMP(N,I)
          AVGEEM(N)=AVGEEM(N)+SURFEM(N,I)
        END DO
        AVGETEMP(N)=AVGETEMP(N)/NSURF(N)
        AVGEEM(N)=AVGEEM(N)/NSURF(N)
      END DO
    ENDIF
C
    RETURN
  END
C -----
  SUBROUTINE SOLVER(A,B,X,N)
C -----
C   This subroutine executes the solution procedure to solve the
C   matrix equation  $A \cdot X = B$ , where the matrix and vectors are defined
C   below. This represents the solution of N simultaneous linear

```

```

C      equations.
C
C      Internal variables:
C      A...Coefficient matrix.
C      B...Force vector.
C      X...Unknown vector.
C      N...Number of equations.
C
C      IMPLICIT REAL*8 (A-H,O-Z)
C      PARAMETER (NDIM=1300,MAXBW=105,MAXENCL=50,MSURF=99,MNPE=9,
+      MQUAD=5,NSPE=4)
C      REAL*8 A (MSURF,MSURF), B (MSURF), X (MSURF)
C
C      CALL GAUSSY (A,B,X,N)
C
C      RETURN
C      END
C -----
C      SUBROUTINE BANDWIDTH (NODCON)
C -----
C      This subroutine computes the maximum half-bandwidth of the global
C      stiffness or conductivity matrix.
C
C      IMPLICIT REAL*8 (A-H,O-Z)
C      PARAMETER (NDIM=1300,MAXBW=105,MAXENCL=50,MSURF=99,MNPE=9,
+      MQUAD=5,NSPE=4)
C      COMMON/CONTRL/ ITOLD,ISFTYPE,NN,NE,NNPE,NEWF,NEWC,NQUAD,IHBW
C      INTEGER NODCON (NDIM,MNPE)
C
C      IHBW=0
C      DO 3 I=1,NE
C          IMIN=10000
C          IMAX=-10000
C          DO 4 J=1,NNPE
C              IF (NODCON (I,J).LT.IMIN) IMIN=NODCON (I,J)
C              IF (NODCON (I,J).GT.IMAX) IMAX=NODCON (I,J)
C          4      CONTINUE
C          ICMAX=IMAX-IMIN+1
C          IF (ICMAX.GT.IHBW) IHBW=ICMAX
C      3      CONTINUE
C      RETURN
C      END
C -----
C      SUBROUTINE BANDED__SOLVER (A,B)
C -----
C      This subroutine takes the linear system  $A \cdot X = B$  that has been
C      stored in banded format and solves for the unknowns X. If
C      IRES.GT.0, elimination process on A is skipped (assumed already
C      complete) and back substitution is begun immediately.
C
C      IMPLICIT REAL*8 (A-H,O-Z)
C      PARAMETER (NDIM=1300,MAXBW=105,MAXENCL=50,MSURF=99,MNPE=9,
+      MQUAD=5,NSPE=4)
C      COMMON/CONTRL/ ITOLD,ISFTYPE,NN,NE,NNPE,NEWF,NEWC,NQUAD,IHBW
C      REAL*8 A (NDIM,MAXBW), B (NDIM), X (NDIM)

```

```

C
  IRES=0
  MEQNS=NN-1
  IF (IRES.GT.0) GOTO 90
  DO 500 NPIV=1,MEQNS
    NPIVOT=NPIV+1
    LSTSUB=NPIV+IHBW-1
    IF (LSTSUB.GT.NN) LSTSUB=NN
    DO 400 NROW=NPIVOT,LSTSUB
      NCOL=NROW-NPIV+1
      FACTOR=A(NPIV,NCOL)/A(NPIV,1)
      DO 200 NCOL=NROW,LSTSUB
        ICOL=NCOL-NROW+1
        JCOL=NCOL-NPIV+1
200      A(NROW,ICOL)=A(NROW,ICOL)-FACTOR*A(NPIV,JCOL)
400      B(NROW)=B(NROW)-FACTOR*B(NPIV)
500    CONTINUE
      GOTO 101

```

```

C
90  DO 100 NPIV=1,MEQNS
    NPIVOT=NPIV+1
    LSTSUB=NPIV+IHBW-1
    IF (LSTSUB.GT.NN) LSTSUB=NN
    DO 110 NROW=NPIVOT,LSTSUB
      NCOL=NROW-NPIV+1
      FACTOR=A(NPIV,NCOL)/A(NPIV,1)
110    B(NROW)=B(NROW)-FACTOR*B(NPIV)
100  CONTINUE

```

```

C
C      Begin back substitution:
101  DO 800 IJK=2,NN
    NPIV=NN-IJK+2
    B(NPIV)=B(NPIV)/A(NPIV,1)
    LSTSUB=NPIV-IHBW+1
    IF (LSTSUB.LT.1) LSTSUB=1
    NPIVOT=NPIV-1
    DO 700 JKI=LSTSUB,NPIVOT
      NROW=NPIVOT-JKI+LSTSUB
      NCOL=NPIV-NROW+1
      FACTOR=A(NROW,NCOL)
700    B(NROW)=B(NROW)-FACTOR*B(NPIV)
800  CONTINUE
    B(1)=B(1)/A(1,1)
    DO I=1,NN
      X(I)=B(I)
    END DO

```

```

C
  RETURN
  END

```

```

C -----
  SUBROUTINE GAUSSY(A,B,X,N)

```

```

C -----

```

```

C      Solution of simultaneous equations by Gaussian Elimination.

```

```

C
  IMPLICIT REAL*8(A-H,O-Z)

```

```

PARAMETER (NDIM=1300,MAXBW=105,MAXENCL=50,MSURF=99,MNPE=9,
+          MQUAD=5,NSPE=4)
REAL*8 A(MSURF,MSURF),X(MSURF),B(MSURF)
C
C   Beginning elimination process:
DO 28 K=1,N
C
C   Moving largest coefficient into diagonal position:
AMAX=0.
DO 4 I=K,N
    IF(DABS(A(I,K))-DABS(AMAX)) 4,4,2
2    AMAX=A(I,K)
    IMAX=I
4    CONTINUE
C
C   Testing for independence of equations:
IF(DABS(AMAX)-0.1E-15) 10,10,14
10   WRITE(6,12)
12   FORMAT ('0EQUATIONS ARE NOT INDEPENDENT')
RETURN
C
C   Exchanging row IMAX and row K:
14   BTEMP=B(K)
    B(K)=B(IMAX)
    B(IMAX)=BTEMP
DO 18 J=K,N
    ATEMP=A(K,J)
    A(K,J)=A(IMAX,J)
    A(IMAX,J)=ATEMP
18   CONTINUE
C
C   Subtracting A(I,K)/A(K,K) times term in first EQ. from others:
KPLUS=K+1
IF(K-N) 22, 28, 28
22   DO 24 I=KPLUS,N
        B(I)=B(I)-B(K)*A(I,K)/A(K,K)
        ACON=A(I,K)
        DO 24 J=K,N
            A(I,J)=A(I,J)-A(K,J)*ACON/A(K,K)
24   CONTINUE
28   CONTINUE
C
C   Back substitution:
L=N
32   SUM=0.0
    IF(L-N) 34, 38, 38
34   LPLUS=L+1
    DO 36 J=LPLUS,N
        SUM=SUM+A(L,J)*X(J)
36   CONTINUE
38   CONTINUE
    X(L)=(B(L)-SUM)/A(L,L)
    IF(L-1) 42, 42, 40
40   L=L-1
    GO TO 32

```

```

42  RETURN
    END
C  -----
C  FUNCTION COND(N,TEMP)
C  -----
C  This function written by:
C  STEVEN L. HAYES
C  TEXAS A&M UNIVERSITY
C  DEPARTMENT OF NUCLEAR ENGINEERING
C  COLLEGE STATION, TX 77843-3133
C  (409) 845-8101
C  -----
C  This external, user-supplied function takes the temperature of an
C  element and a specification flag and computes the thermal
C  conductivity of the element.
C
C  Variable definitions:
C  COND.....Element thermal conductivity in W/m-K.
C  N.....Element thermal conductivity specification flag.
C  TEMP.....Element temperature in Kelvin.
C  -----
C  IMPLICIT REAL*8 (A-H,O-Z)
C
C  Arbitrary constant conductivity:
C  IF (N.EQ.0) COND=1.0
C
C  Insulating material (zero conductivity):
C  IF (N.EQ.1) COND=1.0D-10
C
C  Perfectly conducting material (infinite conductivity):
C  IF (N.EQ.2) COND=1.0D+10
C
C  Approximate UO2 conductivity:
C  (Average temperature-weighted conductivity for 1000 < T < 3000K).
C  IF (N.EQ.3) COND=2.9147
C
C  ==> URANIUM DIOXIDE
C  Valid for 773 < T < 3120K;
C  Taken from:
C  J.H. Harding and D.G. Martin, "A Recommendation for the Thermal
C  Conductivity of UO2," J. Nucl. Mater., 166 (1989) 223.
C
C  IF (N.EQ.4) THEN
C      COND=1.0/(0.0375+2.165D-04*TEMP)+(4.715D+09/(TEMP*TEMP))*
C      +      DEXP(-16361.0/TEMP)
C  ENDIF
C
C  ==> URANIUM MONONITRIDE
C  Valid for 298 < T < 1923K;
C  Taken from:
C  S.L. Hayes, J.K. Thomas and K.L. Peddicord, "Material Property
C  Correlations for Uranium Mononitride - III. Transport
C  Properties," J. Nucl. Mater.,
C  171 (1990) 289.

```



```

C
  IF (N.EQ.5) THEN
    COND=1.864*(TEMP**0.361)
  ENDIF

C
C ==> URANIUM MONOCARBIDE
C   Valid for 298 < T < 2273K (or higher???)
C   Taken from:
C   T. Preusser, Nucl. Tech., 57 (1982) 343.
C
  IF (N.EQ.6) THEN
    IF (TEMP.LE.773.0) THEN
      COND=20.0
    ELSE
      COND=20.0+1.3D-03*(TEMP-773.0)
    ENDIF
  ENDIF

C
C ==> NOBLE GASES
C   Taken from:
C   D.L. Hagrman, G.A. Reymann and R.E. Mason (1981) _MATPRO-VERSION
C   11 (Revision 2) A Handbook of Materials Properties for Use in the
C   Analysis of Light Water Reactor Fuel Rod Behavior_, TREE-1280,
C   EG&G Idaho, p. 485.
C   Pure Helium:
C   IF (N.EQ.7) THEN
C     COND=2.639D-03*(TEMP**0.7085)
C   ENDIF
C   Pure Xenon:
C   IF (N.EQ.8) THEN
C     COND=4.351D-05*(TEMP**0.8616)
C   ENDIF
C   Pure Krypton:
C   IF (N.EQ.9) THEN
C     COND=8.247D-05*(TEMP**0.8363)
C   ENDIF
C   Fission Product Gas (85% Xenon, 15% Krypton):
C   IF (N.EQ.10) THEN
C     CXE=4.351D-05*(TEMP**0.8616)
C     CKR=8.247D-05*(TEMP**0.8363)
C     COND=(CXE**0.85)*(CKR**0.15)
C   ENDIF

C
  RETURN
END

C -----
C   FUNCTION EMISS(N,TEMP)
C -----
C   This function written by:
C   STEVEN L. HAYES
C   TEXAS A&M UNIVERSITY
C   DEPARTMENT OF NUCLEAR ENGINEERING
C   COLLEGE STATION, TX 77843-3133
C   (409) 845-8101
C

```

```

C -----
C   This external, user-supplied function takes the temperature of a
C   surface and a specification flag and computes the hemispherical
C   total emittance of the surface.
C
C   Variable definitions:
C   EMISS.....Element hemispherical total emittance.
C   N.....Element emissivity specification flag.
C   TEMP.....Surface temperature in Kelvin.
C -----
C   IMPLICIT REAL*8 (A-H,O-Z)
C
C   Non-Radiating surface:
C   IF (N.EQ.0) EMISS=1.0D-10
C
C   Black surface:
C   IF (N.EQ.1) EMISS=0.9999999999
C
C   Arbitrary constant emissivity surface:
C   IF (N.EQ.2) EMISS=0.1
C
C   Approximate UO2 surface:
C   (Average temperature-weighted emissivity for 1000 < T < 3000K).
C   IF (N.EQ.3) EMISS=0.816126
C
C   ==> URANIUM DIOXIDE
C   Valid for 300 < T < 3000K;
C   Taken from:
C   D.L. Hagerman, G.A. Reymann and R.E. Mason (1981) _MATPRO-VERSION
C   11 (Revision 2) A Handbook of Materials Properties for Use in the
C   Analysis of Light Water Reactor Fuel Rod Behavior_, TREE-1280,
C   EG&G Idaho, p. 48.
C
C   IF (N.EQ.4) THEN
C       EMISS=0.7856+1.5263D-05*TEMP
C   ENDIF
C
C   RETURN
C   END

```

APPENDIX C

CALCULATED POROSITY CORRECTION FACTORS

Table C.1 Porosity correction factors for 5 vol-% circular porosity

T (K)	Vacuum		Fission Gas		Helium	
	No Rad.	Rad.	No Rad.	Rad.	No Rad.	Rad.
Pore Diameter: 200 μm						
1000	0.9047558	0.9057179	0.9055473	0.9065015	0.9192673	0.9200919
1500	0.9047531	0.9094819	0.9063573	0.9110103	0.9307912	0.9343732
2000	0.9049664	0.9177161	0.9073262	0.9197861	0.9399093	0.9487003
2500	0.9049052	0.9260893	0.9073987	0.9280862	0.9404683	0.9550864
3000	0.9048584	0.9320492	0.9070767	0.9337089	0.9363542	0.9560991
Pore Diameter: 100 μm						
1000	0.9048872	0.9053954	0.9057007	0.9062048	0.9197484	0.9201836
1500	0.9048990	0.9073417	0.9065210	0.9089244	0.9311776	0.9330236
2000	0.9049113	0.9114949	0.9072756	0.9137070	0.9399015	0.9444143
2500	0.9048960	0.9160120	0.9073940	0.9182413	0.9405275	0.9481203
3000	0.9048836	0.9193952	0.9071078	0.9213096	0.9364679	0.9468875
Pore Diameter: 50 μm						
1000	0.9048913	0.9051491	0.9057102	0.9059659	0.9198391	0.9200595
1500	0.9048942	0.9061293	0.9065203	0.9077357	0.9312315	0.9321639
2000	0.9048974	0.9082439	0.9072626	0.9105310	0.9398993	0.9421865
2500	0.9048935	0.9106011	0.9073927	0.9129601	0.9405421	0.9444206
3000	0.9048901	0.9124090	0.9071157	0.9144702	0.9364968	0.9418635
Pore Diameter: 25 μm						
1000	0.9048924	0.9050218	0.9057126	0.9058407	0.9198616	0.9199722
1500	0.9048929	0.9055131	0.9065203	0.9071303	0.9312449	0.9317130
2000	0.9048934	0.9065813	0.9072595	0.9089069	0.9398988	0.9410503
2500	0.9048932	0.9077872	0.9073925	0.9102146	0.9405458	0.9425074
3000	0.9048917	0.9087220	0.9071176	0.9108628	0.9365038	0.9392291
Pore Diameter: 10 μm						
1000	0.9048927	0.9049446	0.9057132	0.9057646	0.9198678	0.9199123
1500	0.9048926	0.9051414	0.9065203	0.9067649	0.9312488	0.9314364
2000	0.9048929	0.9055712	0.9072586	0.9079210	0.9398988	0.9403612
2500	0.9048927	0.9060605	0.9073922	0.9085307	0.9405467	0.9413366
3000	0.9048925	0.9064421	0.9071183	0.9086335	0.9365060	0.9376062
Pore Diameter: 1 μm						
1000	0.9048928	0.9048979	0.9057132	0.9057183	0.9198688	0.9198733
1500	0.9048927	0.9049176	0.9065202	0.9065446	0.9312495	0.9312684
2000	0.9048926	0.9049607	0.9072584	0.9073248	0.9398988	0.9399452
2500	0.9048928	0.9050102	0.9073920	0.9075064	0.9405467	0.9406260
3000	0.9048927	0.9050488	0.9071185	0.9072711	0.9365062	0.9366169

Table C.2 Porosity correction factors for 10 vol-% circular porosity

T (K)	Vacuum		Fission Gas		Helium	
	No Rad.	Rad.	No Rad.	Rad.	No Rad.	Rad.
Pore Diameter: 200 μm						
1000	0.8175773	0.8194135	0.8190515	0.8208742	0.8447120	0.8463046
1500	0.8182342	0.8270402	0.8211896	0.8298672	0.8666930	0.8735177
2000	0.8182734	0.8417070	0.8225915	0.8455418	0.8831950	0.8998632
2500	0.8182255	0.8571450	0.8227889	0.8608770	0.8843561	0.9120292
3000	0.8181897	0.8683289	0.8222528	0.8714485	0.8766912	0.9140396
Pore Diameter: 100 μm						
1000	0.8182133	0.8191518	0.8197068	0.8206387	0.8456630	0.8464776
1500	0.8182241	0.8227100	0.8211945	0.8256146	0.8668949	0.8703680
2000	0.8182340	0.8302978	0.8225579	0.8343682	0.8832158	0.8917533
2500	0.8182222	0.8386098	0.8227910	0.8427305	0.8844237	0.8987993
3000	0.8182127	0.8448764	0.8222812	0.8484259	0.8767946	0.8964766
Pore Diameter: 50 μm						
1000	0.8182190	0.8186909	0.8197170	0.8201856	0.8457421	0.8461522
1500	0.8182219	0.8204816	0.8211961	0.8234226	0.8669457	0.8686943
2000	0.8182244	0.8243468	0.8225499	0.8285426	0.8832209	0.8875430
2500	0.8182215	0.8286721	0.8227915	0.8330084	0.8844406	0.8917780
3000	0.8182185	0.8319975	0.8222883	0.8357936	0.8768205	0.8869420
Pore Diameter: 25 μm						
1000	0.8182202	0.8184570	0.8197196	0.8199540	0.8457615	0.8459669
1500	0.8182210	0.8193547	0.8211961	0.8223130	0.8669577	0.8678347
2000	0.8182216	0.8213064	0.8225474	0.8255663	0.8832219	0.8853965
2500	0.8182212	0.8235144	0.8227915	0.8279648	0.8844449	0.8881522
3000	0.8182203	0.8252289	0.8222900	0.8291571	0.8768266	0.8819615
Pore Diameter: 10 μm						
1000	0.8182208	0.8183156	0.8197202	0.8198141	0.8457673	0.8458495
1500	0.8182209	0.8186752	0.8211961	0.8216438	0.8669615	0.8673128
2000	0.8182211	0.8194605	0.8225471	0.8237601	0.8832225	0.8840956
2500	0.8182209	0.8203549	0.8227914	0.8248771	0.8844457	0.8859382
3000	0.8182208	0.8210533	0.8222907	0.8250661	0.8768284	0.8789000
Pore Diameter: 1 μm						
1000	0.8182210	0.8182304	0.8197205	0.8197299	0.8457682	0.8457763
1500	0.8182211	0.8182664	0.8211965	0.8212414	0.8669622	0.8669973
2000	0.8182209	0.8183453	0.8225477	0.8226691	0.8832224	0.8833099
2500	0.8182211	0.8184355	0.8227912	0.8230009	0.8844456	0.8845956
3000	0.8182210	0.8185059	0.8222906	0.8225699	0.8768286	0.8770371

Table C.3 Porosity correction factors for 15 vol-% circular porosity

T (K)	Vacuum		Fission Gas		Helium	
	No Rad.	Rad.	No Rad.	Rad.	No Rad.	Rad.
Pore Diameter: 200 μm						
1000	0.7390784	0.7416373	0.7411187	0.7436604	0.7768231	0.7790710
1500	0.7391177	0.7512994	0.7431891	0.7552101	0.8065828	0.8162389
2000	0.7391517	0.7715154	0.7450898	0.7768462	0.8297242	0.8534307
2500	0.7391103	0.7929847	0.7453867	0.7982122	0.8314131	0.8708388
3000	0.7390793	0.8086895	0.7446686	0.8130850	0.8206071	0.8737581
Pore Diameter: 100 μm						
1000	0.7391005	0.7403939	0.7411553	0.7424402	0.7770977	0.7782353
1500	0.7391100	0.7452848	0.7431947	0.7492879	0.8067676	0.8116596
2000	0.7391187	0.7557335	0.7450637	0.7613617	0.8297618	0.8418810
2500	0.7391084	0.7672330	0.7453903	0.7729540	0.8314845	0.8519240
3000	0.7391001	0.7759401	0.7446939	0.7808827	0.8206998	0.8486189
Pore Diameter: 50 μm						
1000	0.7391061	0.7397545	0.7411646	0.7418086	0.7771662	0.7777369
1500	0.7391079	0.7422142	0.7431962	0.7462604	0.8068139	0.8092733
2000	0.7391102	0.7475309	0.7450568	0.7533156	0.8297711	0.8359004
2500	0.7391075	0.7534939	0.7453908	0.7594857	0.8315017	0.8419192
3000	0.7391051	0.7580880	0.7447000	0.7633402	0.8207224	0.8350503
Pore Diameter: 25 μm						
1000	0.7391070	0.7394318	0.7411662	0.7414889	0.7771828	0.7774685
1500	0.7391075	0.7406650	0.7431966	0.7447331	0.8068253	0.8080582
2000	0.7391084	0.7433477	0.7450554	0.7492128	0.8297734	0.8328558
2500	0.7391080	0.7463859	0.7453917	0.7525203	0.8315066	0.8367666
3000	0.7391068	0.7487470	0.7447017	0.7541657	0.8207283	0.8279878
Pore Diameter: 10 μm						
1000	0.7391075	0.7392375	0.7411672	0.7412961	0.7771878	0.7773023
1500	0.7391076	0.7397313	0.7431966	0.7438120	0.8068282	0.8073221
2000	0.7391078	0.7408103	0.7450547	0.7467246	0.8297741	0.8310109
2500	0.7391077	0.7420393	0.7453912	0.7482633	0.8315073	0.8336239
3000	0.7391075	0.7429998	0.7447021	0.7485237	0.8207302	0.8236574
Pore Diameter: 1 μm						
1000	0.7391077	0.7391206	0.7411671	0.7411800	0.7771888	0.7772002
1500	0.7391078	0.7391702	0.7431968	0.7432585	0.8068292	0.8068786
2000	0.7391078	0.7392784	0.7450543	0.7452215	0.8297744	0.8298983
2500	0.7391078	0.7394022	0.7453913	0.7456798	0.8315077	0.8317200
3000	0.7391076	0.7394991	0.7447024	0.7450866	0.8207304	0.8210246

Table C.4 Porosity correction factors for 20 vol-% circular porosity

T (K)	Vacuum		Fission Gas		Helium	
	No Rad.	Rad.	No Rad.	Rad.	No Rad.	Rad.
Pore Diameter: 200 μm						
1000	0.6665022	0.6696581	0.6690115	0.6721480	0.7131679	0.7159740
1500	0.6665373	0.6815300	0.6715375	0.6863507	0.7502014	0.7623383
2000	0.6665681	0.7064325	0.6738550	0.7130406	0.7791962	0.8092076
2500	0.6665310	0.7330737	0.6742333	0.7395986	0.7813513	0.8313792
3000	0.6665030	0.7527090	0.6733619	0.7582233	0.7677591	0.8351291
Pore Diameter: 100 μm						
1000	0.6665221	0.6681098	0.6690437	0.6706218	0.7134065	0.7148198
1500	0.6665305	0.6741096	0.6715420	0.6790304	0.7503690	0.7565027
2000	0.6665385	0.6869528	0.6738326	0.6938940	0.7792449	0.7945594
2500	0.6665291	0.7011427	0.6742371	0.7082234	0.7814242	0.8072969
3000	0.6665218	0.7119263	0.6733847	0.7180600	0.7678431	0.8031051
Pore Diameter: 50 μm						
1000	0.6665269	0.6673226	0.6690516	0.6698422	0.7134660	0.7141741
1500	0.6665289	0.6703379	0.6715438	0.6753066	0.7504118	0.7534925
2000	0.6665312	0.6768641	0.6738272	0.6839796	0.7792573	0.7869947
2500	0.6665289	0.6841983	0.6742383	0.6915829	0.7814423	0.7946096
3000	0.6665267	0.6898597	0.6733905	0.6963406	0.7678637	0.7859226
Pore Diameter: 25 μm						
1000	0.6665280	0.6669263	0.6690535	0.6694491	0.7134805	0.7138352
1500	0.6665287	0.6684377	0.6715439	0.6734299	0.7504220	0.7519660
2000	0.6665296	0.6717280	0.6738258	0.6789334	0.7792602	0.7831499
2500	0.6665291	0.6754577	0.6742386	0.6830018	0.7814469	0.7880902
3000	0.6665279	0.6783591	0.6734301	0.6850262	0.7678685	0.7770092
Pore Diameter: 10 μm						
1000	0.6665285	0.6666879	0.6690540	0.6692122	0.7134847	0.7136265
1500	0.6665287	0.6672932	0.6715439	0.6722993	0.7504251	0.7510435
2000	0.6665287	0.6686160	0.6738252	0.6758762	0.7792610	0.7808215
2500	0.6665288	0.6701232	0.6742384	0.6777663	0.7814479	0.7841189
3000	0.6665285	0.6713012	0.6733924	0.6780864	0.7678701	0.7715523
Pore Diameter: 1 μm						
1000	0.6665289	0.6665447	0.6690541	0.6690699	0.7134856	0.7134997
1500	0.6665285	0.6666052	0.6715434	0.6716191	0.7504254	0.7504873
2000	0.6665286	0.6667379	0.6738245	0.6740300	0.7792613	0.7794175
2500	0.6665286	0.6668895	0.6742385	0.6745925	0.7814481	0.7817160
3000	0.6665287	0.6670083	0.6733923	0.6738642	0.7678704	0.7682404

Table C.5 Porosity correction factors for 25 vol-% circular porosity

T (K)	Vacuum		Fission Gas		Helium	
	No Rad.	Rad.	No Rad.	Rad.	No Rad.	Rad.
Pore Diameter: 200 μm						
1000	0.5995873	0.6032403	0.6024890	0.6061219	0.6538184	0.6571027
1500	0.5996188	0.6169651	0.6053968	0.6225551	0.6971713	0.7114885
2000	0.5996470	0.6458468	0.6080652	0.6535513	0.7313460	0.7670188
2500	0.5996128	0.6769443	0.6085111	0.6846002	0.7289075	0.7935291
3000	0.5995867	0.7000123	0.6075101	0.7065105	0.7178428	0.7980379
Pore Diameter: 100 μm						
1000	0.5996038	0.6014369	0.6025161	0.6043394	0.6540260	0.6556758
1500	0.5996116	0.6083647	0.6054000	0.6140577	0.6973237	0.7045465
2000	0.5996189	0.6232234	0.6080445	0.6312789	0.7314011	0.7495734
2500	0.5996101	0.6396957	0.6085144	0.6479413	0.7339874	0.7647390
3000	0.5996034	0.6522486	0.6075303	0.6594123	0.7179185	0.7597434
Pore Diameter: 50 μm						
1000	0.5996081	0.6005260	0.6025231	0.6034357	0.6540781	0.6549043
1500	0.5996098	0.6040054	0.6054007	0.6097483	0.6973617	0.7009883
2000	0.5996118	0.6115456	0.6080392	0.6197841	0.7314151	0.7405886
2500	0.5996099	0.6200330	0.6085153	0.6285977	0.7340058	0.7496341
3000	0.5996078	0.6265946	0.6075355	0.6341221	0.7179376	0.7393154
Pore Diameter: 25 μm						
1000	0.5996086	0.6000680	0.6025241	0.6029807	0.6540903	0.6545038
1500	0.5996093	0.6018118	0.6054008	0.6075792	0.6973714	0.6991881
2000	0.5996099	0.6056101	0.6080378	0.6139431	0.7314184	0.7360273
2500	0.5996096	0.6099192	0.6085152	0.6186520	0.7340098	0.7418889
3000	0.5996088	0.6132745	0.6075365	0.6209970	0.7179420	0.7287513
Pore Diameter: 10 μm						
1000	0.5996091	0.5997930	0.6025248	0.6027075	0.6540942	0.6542595
1500	0.5996093	0.6004916	0.6054009	0.6062734	0.6973741	0.6981019
2000	0.5996094	0.6020175	0.6080376	0.6104074	0.7314197	0.7332682
2500	0.5996093	0.6037569	0.6085149	0.6125933	0.7340109	0.7371776
3000	0.5996090	0.6051173	0.6075368	0.6129632	0.7179433	0.7222951
Pore Diameter: 1 μm						
1000	0.5996088	0.5996272	0.6025243	0.6025427	0.6540948	0.6541112
1500	0.5996084	0.5996968	0.6054310	0.6054878	0.6973747	0.6974473
2000	0.5996081	0.5998494	0.6080365	0.6082743	0.7314195	0.7316048
2500	0.5996080	0.6000242	0.6085140	0.6089235	0.7340110	0.7343289
3000	0.5996075	0.6001608	0.6075361	0.6080810	0.7179435	0.7183806

Table C.6 Porosity correction factors for 30 vol-% circular porosity

T (K)	Vacuum		Fission Gas		Helium	
	No Rad.	Rad.	No Rad.	Rad.	No Rad.	Rad.
Pore Diameter: 200 μm						
1000	0.5375544	0.5416247	0.5407882	0.5448382	0.5982628	0.6019594
1500	0.5375825	0.5569150	0.5440198	0.5631625	0.6471437	0.6633852
2000	0.5376085	0.5891986	0.5469861	0.5978529	0.6859247	0.7266960
2500	0.5375767	0.6241620	0.5474892	0.6328091	0.6888626	0.7571688
3000	0.5375520	0.6501155	0.5463778	0.6576132	0.6705738	0.7623699
Pore Diameter: 100 μm						
1000	0.5375680	0.5396077	0.5408109	0.5428404	0.5984450	0.6002987
1500	0.5375770	0.5473191	0.5440233	0.5536697	0.6472848	0.6554674
2000	0.5375815	0.5638847	0.5469665	0.5728957	0.6859839	0.7067200
2500	0.5375737	0.5823079	0.5474914	0.5915597	0.6889334	0.7240813
3000	0.5375675	0.5963883	0.5463963	0.6044435	0.6706435	0.7183543
Pore Diameter: 50 μm						
1000	0.5375710	0.5385917	0.5408161	0.5418316	0.5984900	0.5994180
1500	0.5375729	0.5424625	0.5440216	0.5488630	0.6473166	0.6514229
2000	0.5375748	0.5508572	0.5469616	0.5600549	0.6859987	0.6964576
2500	0.5375728	0.5603237	0.5474922	0.5698995	0.6889517	0.7067887
3000	0.5375711	0.5676473	0.5464007	0.5760762	0.6706602	0.6950005
Pore Diameter: 25 μm						
1000	0.5375722	0.5390828	0.5408179	0.5413259	0.5985016	0.5989658
1500	0.5375726	0.5400220	0.5440219	0.5464472	0.6473255	0.6493821
2000	0.5375732	0.5442484	0.5469604	0.5535400	0.6860024	0.6912549
2500	0.5375729	0.5490462	0.5474923	0.5587926	0.6889560	0.6979418
3000	0.5375721	0.5527840	0.5464018	0.5614091	0.6706646	0.6829577
Pore Diameter: 10 μm						
1000	0.5375723	0.5377768	0.5408182	0.5410214	0.5985045	0.5986904
1500	0.5375727	0.5385535	0.5440217	0.5449930	0.6473281	0.6481516
2000	0.5375726	0.5402508	0.5469598	0.5495998	0.6860033	0.6881101
2500	0.5375724	0.5421861	0.5474921	0.5520361	0.6889573	0.6925675
3000	0.5375724	0.5437000	0.5464021	0.5524471	0.6706657	0.6756130
Pore Diameter: 1 μm						
1000	0.5375727	0.5375931	0.5408183	0.5408387	0.5985052	0.5985238
1500	0.5375726	0.5376705	0.5440215	0.5441187	0.6473281	0.6474105
2000	0.5375725	0.5378409	0.5469595	0.5472238	0.6860036	0.6862146
2500	0.5375725	0.5380354	0.5474917	0.5479473	0.6889572	0.6893194
3000	0.5375725	0.5381878	0.5464013	0.5470083	0.6706657	0.6711627

Table C.7 Porosity correction factors for 35 vol-% circular porosity

T (K)	Vacuum		Fission Gas		Helium	
	No Rad.	Rad.	No Rad.	Rad.	No Rad.	Rad.
Pore Diameter: 200 μm						
1000	0.4796860	0.4841104	0.4832047	0.4876093	0.5460052	0.5500598
1500	0.4797041	0.5007310	0.4867079	0.5075474	0.5997593	0.6177082
2000	0.4797168	0.5359382	0.4899194	0.5454246	0.6426487	0.6880299
2500	0.4797038	0.5743007	0.4904888	0.5838284	0.6459410	0.7221579
3000	0.4796952	0.6030614	0.4892972	0.6112142	0.6256795	0.7280201
Pore Diameter: 100 μm						
1000	0.4797023	0.4819174	0.4832296	0.4854346	0.5461781	0.5482093
1500	0.4797072	0.4902919	0.4867195	0.4972101	0.5999037	0.6089379
2000	0.4797102	0.5083212	0.4899210	0.5181628	0.6427413	0.6657866
2500	0.4797074	0.5284337	0.4904984	0.5385631	0.6460249	0.6851478
3000	0.4797047	0.5438491	0.4893100	0.5526816	0.6257412	0.6787539
Pore Diameter: 50 μm						
1000	0.4797068	0.4808150	0.4832361	0.4843393	0.5462215	0.5472380
1500	0.4797078	0.4850181	0.4867226	0.4919853	0.5999399	0.6044713
2000	0.4797089	0.4941440	0.4899215	0.5041684	0.6427645	0.6543789
2500	0.4797080	0.5044495	0.4905005	0.5149003	0.6460455	0.6658744
3000	0.4797071	0.5124388	0.4893132	0.5216423	0.6257565	0.6527524
Pore Diameter: 25 μm						
1000	0.4797080	0.4802623	0.4832377	0.4837895	0.5462324	0.5467409
1500	0.4797081	0.4823679	0.4867234	0.4893593	0.5999490	0.6022183
2000	0.4797083	0.4869587	0.4899215	0.4970776	0.6427703	0.6486003
2500	0.4797085	0.4921755	0.4905012	0.5027967	0.6460509	0.6560334
3000	0.4797080	0.4962429	0.4893141	0.5056456	0.6257605	0.6393836
Pore Diameter: 10 μm						
1000	0.4797083	0.4799301	0.4832382	0.4834589	0.5462354	0.5464389
1500	0.4797081	0.4807730	0.4867237	0.4877790	0.5999515	0.6008602
2000	0.4797083	0.4826162	0.4899215	0.4927919	0.6427717	0.6451097
2500	0.4797085	0.4847185	0.4905014	0.4954425	0.6460523	0.6500615
3000	0.4797080	0.4863635	0.4893142	0.4958881	0.6257616	0.6312410
Pore Diameter: 1 μm						
1000	0.4797083	0.4797304	0.4832382	0.4832603	0.5462358	0.5462561
1500	0.4797082	0.4798147	0.4867234	0.4868292	0.5999521	0.6000431
2000	0.4797059	0.4799973	0.4899210	0.4902083	0.6427722	0.6430064
2500	0.4797084	0.4802108	0.4905005	0.4909962	0.6460526	0.6464545
3000	0.4797082	0.4803764	0.4893135	0.4899735	0.6257616	0.6263119

Table C.8 Porosity correction factors for 40 vol-% circular porosity

T (K)	Vacuum		Fission Gas		Helium	
	No Rad.	Rad.	No Rad.	Rad.	No Rad.	Rad.
Pore Diameter: 200 μm						
1000	0.4253380	0.4300683	0.4291085	0.4338193	0.4966334	0.5010028
1500	0.4253537	0.4478493	0.4328579	0.4551708	0.5547567	0.5742329
2000	0.4253647	0.4856283	0.4362963	0.4958592	0.6013749	0.6509483
2500	0.4253537	0.5269840	0.4369094	0.5373066	0.6049685	0.6884193
3000	0.4253463	0.5581267	0.4356340	0.5669901	0.5828997	0.6948644
Pore Diameter: 100 μm						
1000	0.4253530	0.4277195	0.4291311	0.4314879	0.4967896	0.4989770
1500	0.4253569	0.4366713	0.4328691	0.4440914	0.5548911	0.5646851
2000	0.4253597	0.4559756	0.4362991	0.4665541	0.6014701	0.6266084
2500	0.4253569	0.4775610	0.4369184	0.4884761	0.6050513	0.6477887
3000	0.4253549	0.4941430	0.4356454	0.5036781	0.5829566	0.6407707
Pore Diameter: 50 μm						
1000	0.4253566	0.4303156	0.4291366	0.4303156	0.4968286	0.4979229
1500	0.4253576	0.4310318	0.4328717	0.4384997	0.5549246	0.5598355
2000	0.4253583	0.4407910	0.4362996	0.4515490	0.6014935	0.6141534
2500	0.4253577	0.4518256	0.4369207	0.4630557	0.6050718	0.6267060
3000	0.4253570	0.4603898	0.4356482	0.4702888	0.5829706	0.6123635
Pore Diameter: 25 μm						
1000	0.4253577	0.4259499	0.4291381	0.4297278	0.4968385	0.4973858
1500	0.4253579	0.4281992	0.4328724	0.4356906	0.5549329	0.5573919
2000	0.4253583	0.4331060	0.4363001	0.4439562	0.6014996	0.6078523
2500	0.4253581	0.4386850	0.4369213	0.4500811	0.6050770	0.6159614
3000	0.4253578	0.4430375	0.4356491	0.4531306	0.5829743	0.5977944
Pore Diameter: 10 μm						
1000	0.4253578	0.4255949	0.4291384	0.4293744	0.4968412	0.4970602
1500	0.4253582	0.4264958	0.4328727	0.4340009	0.5549353	0.5559199
2000	0.4253582	0.4284650	0.4363001	0.4393701	0.6015013	0.6040480
2500	0.4253583	0.4307118	0.4369215	0.4422078	0.6050784	0.6094483
3000	0.4253580	0.4324696	0.4356493	0.4426819	0.5829751	0.5889331
Pore Diameter: 1 μm						
1000	0.4253581	0.4253820	0.4291391	0.4291626	0.4968418	0.4968635
1500	0.4253595	0.4254734	0.4328727	0.4329853	0.5549357	0.5550343
2000	0.4253603	0.4256714	0.4363003	0.4366077	0.6015019	0.6017567
2500	0.4253607	0.4258973	0.4369214	0.4374516	0.6050786	0.6055166
3000	0.4253586	0.4260722	0.4356493	0.4363549	0.5829755	0.5835730

Table C.9 Porosity correction factors for 45 vol-% circular porosity

T (K)	Vacuum		Fission Gas		Helium	
	No Rad.	Rad.	No Rad.	Rad.	No Rad.	Rad.
Pore Diameter: 200 μm						
1000	0.3738737	0.3788740	0.3778736	0.3828548	0.4496952	0.4543451
1500	0.3738875	0.3976835	0.3818471	0.4054660	0.5118011	0.5326571
2000	0.3738970	0.4377575	0.3854929	0.4486702	0.5618356	0.6152465
2500	0.3738874	0.4818069	0.3861453	0.4928615	0.5657045	0.6558120
3000	0.3738808	0.5151089	0.3847928	0.5246290	0.5419765	0.6627899
Pore Diameter: 100 μm						
1000	0.3738865	0.3763873	0.3778933	0.3803845	0.4498365	0.4521631
1500	0.3738898	0.3858501	0.3818571	0.3937279	0.5119257	0.5224061
2000	0.3738924	0.4062861	0.3854958	0.4175389	0.5619310	0.5889798
2500	0.3738899	0.4291858	0.3861534	0.4408206	0.5657853	0.6118305
3000	0.3738880	0.4468128	0.3848029	0.4569898	0.5420288	0.6042294
Pore Diameter: 50 μm						
1000	0.3738899	0.3751406	0.3778981	0.3791442	0.4498720	0.4510358
1500	0.3738904	0.3798865	0.3818596	0.3878107	0.5119569	0.5172102
2000	0.3738912	0.3902073	0.3854965	0.4016347	0.5647290	0.5755675
2500	0.3738907	0.4018892	0.3861555	0.4138309	0.5658056	0.5890882
3000	0.3738900	0.4109649	0.3848053	0.4214994	0.5420418	0.5736132
Pore Diameter: 25 μm						
1000	0.3738903	0.3745434	0.3778992	0.3785223	0.4498807	0.4504628
1500	0.3738904	0.3768926	0.3818599	0.3848396	0.5119645	0.5145946
2000	0.3738907	0.3820788	0.3854966	0.3935956	0.5619607	0.5687890
2500	0.3738905	0.3879788	0.3861559	0.4000818	0.5658103	0.5775174
3000	0.3738903	0.3925842	0.3848058	0.4033075	0.5420451	0.5579523
Pore Diameter: 10 μm						
1000	0.3738908	0.3741410	0.3778997	0.3781490	0.4498834	0.4501160
1500	0.3738906	0.3750924	0.3818603	0.3830531	0.5119670	0.5130195
2000	0.3738907	0.3771733	0.3854968	0.3887437	0.5619624	0.5646997
2500	0.3738906	0.3795474	0.3861559	0.3917478	0.5658119	0.5705105
3000	0.3738905	0.3814060	0.3848061	0.3922449	0.5420461	0.5484379
Pore Diameter: 1 μm						
1000	0.3738906	0.3739156	0.3778995	0.3779242	0.4498838	0.4499068
1500	0.3738893	0.3740095	0.3818601	0.3819795	0.5119675	0.5120725
2000	0.3738889	0.3742176	0.3854968	0.3858219	0.5619629	0.5622367
2500	0.3738884	0.3744553	0.3861561	0.3867166	0.5658118	0.5662828
3000	0.3738880	0.3746416	0.3848059	0.3855520	0.5420462	0.5426876

Table C.10 Porosity correction factors for 50 vol-% circular porosity

T (K)	Vacuum		Fission Gas		Helium	
	No Rad.	Rad.	No Rad.	Rad.	No Rad.	Rad.
Pore Diameter: 200 μm						
1000	0.3246746	0.3299205	0.3288923	0.3341196	0.4047618	0.4096668
1500	0.3246867	0.3496685	0.3330797	0.3578886	0.4705867	0.4927054
2000	0.3246953	0.3918434	0.3369222	0.4034030	0.5238046	0.5807606
2500	0.3246866	0.4383703	0.3376114	0.4501161	0.5279293	0.6242119
3000	0.3246807	0.4736686	0.3361860	0.4838080	0.5026548	0.6316751
Pore Diameter: 100 μm						
1000	0.3246862	0.3273090	0.3289102	0.3315238	0.4048914	0.4073446
1500	0.3246892	0.3372380	0.3330890	0.3455506	0.4707031	0.4818111
2000	0.3246914	0.3587075	0.3369256	0.3705994	0.5238997	0.5527085
2500	0.3246893	0.3828102	0.3376195	0.3951265	0.5280084	0.5771094
3000	0.3246875	0.4013955	0.3361952	0.4121827	0.5027037	0.5689492
Pore Diameter: 50 μm						
1000	0.3246890	0.3260008	0.3289146	0.3369759	0.4049238	0.4061508
1500	0.3246898	0.3309793	0.3330913	0.3393370	0.4707323	0.4762986
2000	0.3246904	0.3418121	0.3369263	0.3538745	0.5239235	0.5384129
2500	0.3246900	0.3540851	0.3376215	0.3667033	0.5280283	0.5528283
3000	0.3246893	0.3636283	0.3361975	0.3747627	0.5027160	0.5362927
Pore Diameter: 25 μm						
1000	0.3246898	0.3253458	0.3289158	0.3295693	0.4049319	0.4055456
1500	0.3246900	0.3278386	0.3330919	0.3362185	0.4707394	0.4735258
2000	0.3246902	0.3332798	0.3369265	0.3454292	0.5239295	0.5311956
2500	0.3246902	0.3394723	0.3376217	0.3522460	0.5280330	0.5404956
3000	0.3246898	0.3443086	0.3361982	0.3556294	0.5027193	0.5196249
Pore Diameter: 10 μm						
1000	0.3246900	0.3249524	0.3289160	0.3291775	0.4049342	0.4051808
1500	0.3246900	0.3259503	0.3330920	0.3343436	0.4707414	0.4718567
2000	0.3246900	0.3281327	0.3369267	0.3403346	0.5239311	0.5268429
2500	0.3246900	0.3306235	0.3376217	0.3434919	0.5280341	0.5330350
3000	0.3246899	0.3325737	0.3361982	0.3440070	0.5027201	0.5095100
Pore Diameter: 1 μm						
1000	0.3246900	0.3247163	0.3289162	0.3289423	0.4049346	0.4049592
1500	0.3246899	0.3248160	0.3330925	0.3332177	0.4707418	0.4708534
2000	0.3246896	0.3250343	0.3369276	0.3372688	0.5239315	0.5242230
2500	0.3246894	0.3252840	0.3376207	0.3382090	0.5280336	0.5285347
3000	0.3246899	0.3254805	0.3361978	0.3369809	0.5027201	0.5034011

Table C.11 Porosity correction factors for 5 vol-% elliptical porosity; $g=2.0$, 0°

T (K)	Vacuum		Helium	
	No Rad.	Rad.	No Rad.	Rad.
Major Pore Diameter: 50 μm				
1000	0.8655463	0.8658214	0.8939024	0.8941086
1500	0.8655488	0.8668683	0.9131713	0.9139648
2000	0.8655517	0.8691306	0.9266722	0.9284929
2500	0.8655481	0.8716626	0.9276410	0.9307230
3000	0.8655454	0.8736172	0.9214849	0.9259011
Major Pore Diameter: 25 μm				
1000	0.8655471	0.8656849	0.8939219	0.8940252
1500	0.8655484	0.8662105	0.9131823	0.9135800
2000	0.8655484	0.8673532	0.9266716	0.9275875
2500	0.8655476	0.8686487	0.9276435	0.9292004
3000	0.8655464	0.8696599	0.9214532	0.9237290
Major Pore Diameter: 10 μm				
1000	0.8655475	0.8656026	0.8939277	0.8939691
1500	0.8655478	0.8658132	0.9131854	0.9133450
2000	0.8655476	0.8662732	0.9266716	0.9270393
2500	0.8655477	0.8667991	0.9276448	0.9282713
3000	0.8655474	0.8672121	0.9214921	0.9223955
Major Pore Diameter: 1 μm				
1000	0.8655477	0.8655532	0.8939292	0.8939332
1500	0.8655478	0.8655744	0.9131864	0.9132025
2000	0.8655471	0.8656200	0.9266720	0.9267088
2500	0.8655480	0.8656738	0.9276462	0.9277090
3000	0.8655479	0.8657156	0.9214928	0.9215835

Table C.12 Porosity correction factors for 5 vol-% elliptical porosity; $d=50\mu\text{m}$, $g=2.0$, $0-30^\circ$

T (K)	Vacuum		Helium	
	No Rad.	Rad.	No Rad.	Rad.
Pore Orientation: 5°				
1000	0.8627849	0.8624953	0.8921896	0.8919740
1500	0.8627868	0.8613832	0.9120258	0.9111945
2000	0.8627887	0.8588936	0.9258551	0.9239254
2500	0.8627865	0.8559584	0.9268462	0.9235212
3000	0.8627848	0.8535871	0.9205496	0.9156980
Pore Orientation: 10°				
1000	0.8640634	0.8637765	0.8930921	0.8928781
1500	0.8640651	0.8626747	0.9126877	0.9118614
2000	0.8640671	0.8602092	0.9263601	0.9244392
2500	0.8640646	0.8573028	0.9273400	0.9240310
3000	0.8640629	0.8549552	0.9211130	0.9162894
Pore Orientation: 15°				
1000	0.8663175	0.8660357	0.8946677	0.8944570
1500	0.8663193	0.8649539	0.9138360	0.9130191
2000	0.8663211	0.8625330	0.9272312	0.9253282
2500	0.8663188	0.8596805	0.9281923	0.9249133
3000	0.8663169	0.8573772	0.9220881	0.9173159
Pore Orientation: 22.5°				
1000	0.8713680	0.8710980	0.8981792	0.8979757
1500	0.8713700	0.8700616	0.9163858	0.9155910
2000	0.8713716	0.8677438	0.9291628	0.9272985
2500	0.8713695	0.8650145	0.9300808	0.9268703
3000	0.8713676	0.8628131	0.9242509	0.9195954
Pore Orientation: 30°				
1000	0.8780910	0.8778368	0.9028356	0.9026415
1500	0.8780929	0.8768617	0.9197589	0.9189934
2000	0.8780946	0.8746837	0.9317132	0.9299022
2500	0.8780924	0.8721212	0.9325751	0.9294564
3000	0.8780906	0.8700574	0.9271086	0.9226101

Table C.13 Porosity correction factors for 5 vol-% elliptical porosity; $g=2.0$, 45°

T (K)	Vacuum		Helium	
	No Rad.	Rad.	No Rad.	Rad.
Major Pore Diameter: 50 μm				
1000	0.8946766	0.8944622	0.9142678	0.9140975
1500	0.8946752	0.8936382	0.9280138	0.9273201
2000	0.8946748	0.8918077	0.9379401	0.9362603
2500	0.8946754	0.8896682	0.9386668	0.9357746
3000	0.8946762	0.8879541	0.9340974	0.9299870
Major Pore Diameter: 25 μm				
1000	0.8946758	0.8945683	0.9142802	0.9141950
1500	0.8946761	0.8941585	0.9280223	0.9276764
2000	0.8946750	0.8932524	0.9379421	0.9371066
2500	0.8946751	0.8922057	0.9386694	0.9372360
3000	0.8946753	0.8913749	0.9340992	0.9320704
Major Pore Diameter: 10 μm				
1000	0.8946753	0.8946323	0.9142835	0.9142494
1500	0.8946753	0.8944688	0.9280247	0.9278863
2000	0.8946749	0.8941084	0.9379426	0.9376093
2500	0.8946751	0.8936951	0.9386699	0.9380997
3000	0.8946751	0.8933689	0.9341003	0.9332945
Major Pore Diameter: 1 μm				
1000	0.8946747	0.8946703	0.9142835	0.9142802
1500	0.8946741	0.8946535	0.9280241	0.9280101
2000	0.8946732	0.8946166	0.9379407	0.9379075
2500	0.8946735	0.8945759	0.9386682	0.9386113
3000	0.8946730	0.8945432	0.9340983	0.9340181

Table C.14 Porosity correction factors for 5 vol-% elliptical porosity; $d=50\mu\text{m}$, $g=2.0$, $60-85^\circ$

T (K)	Vacuum		Helium	
	No Rad.	Rad.	No Rad.	Rad.
Pore Orientation: 60°				
1000	0.9115872	0.9114139	0.9258597	0.9257139
1500	0.9115887	0.9107535	0.9363620	0.9357426
2000	0.9115903	0.9092888	0.9442304	0.9426843
2500	0.9115886	0.9075837	0.9448138	0.9421525
3000	0.9115867	0.9062285	0.9411505	0.9374360
Pore Orientation: 67.5°				
1000	0.9186762	0.9185207	0.9307046	0.9305690
1500	0.9186782	0.9179279	0.9398438	0.9392559
2000	0.9186793	0.9166175	0.9468506	0.9453601
2500	0.9186771	0.9150980	0.9473749	0.9448108
3000	0.9186760	0.9138964	0.9440918	0.9405436
Pore Orientation: 75°				
1000	0.9241609	0.9240190	0.9344497	0.9343220
1500	0.9241627	0.9234791	0.9425347	0.9419709
2000	0.9241642	0.9222888	0.9488751	0.9474282
2500	0.9241625	0.9209140	0.9493543	0.9468655
3000	0.9241611	0.9198315	0.9463649	0.9429460
Pore Orientation: 80°				
1000	0.9267360	0.9266004	0.9362095	0.9360857
1500	0.9267375	0.9260859	0.9438002	0.9432480
2000	0.9267388	0.9249520	0.9498276	0.9484025
2500	0.9267376	0.9236450	0.9502861	0.9478336
3000	0.9267362	0.9226194	0.9474342	0.9440770
Pore Orientation: 85°				
1000	0.9283542	0.9282225	0.9373192	0.9371975
1500	0.9283561	0.9277240	0.9446006	0.9440556
2000	0.9283571	0.9266255	0.9504314	0.9490199
2500	0.9283559	0.9253619	0.9508762	0.9484479
3000	0.9283544	0.9243718	0.9481115	0.9447948

Table C.15 Porosity correction factors for 5 vol-% elliptical porosity; $g=2.0$, 90°

T (K)	Vacuum		Helium	
	No Rad.	Rad.	No Rad.	Rad.
Major Pore Diameter: 50 μm				
1000	0.9291696	0.9292979	0.9379058	0.9380248
1500	0.9291698	0.9297836	0.9450395	0.9455727
2000	0.9291713	0.9308380	0.9507744	0.9521472
2500	0.9291689	0.9320196	0.9512115	0.9535511
3000	0.9291666	0.9329290	0.9484886	0.9516567
Major Pore Diameter: 25 μm				
1000	0.9291677	0.9292317	0.9379103	0.9379701
1500	0.9291679	0.9294755	0.9450430	0.9453099
2000	0.9291676	0.9300040	0.9507720	0.9514607
2500	0.9291666	0.9306009	0.9512116	0.9523879
3000	0.9291669	0.9310625	0.9484903	0.9500856
Major Pore Diameter: 10 μm				
1000	0.9291667	0.9291924	0.9379116	0.9379353
1500	0.9291667	0.9292899	0.9450430	0.9451499
2000	0.9291667	0.9295018	0.9507713	0.9510474
2500	0.9291667	0.9297424	0.9512117	0.9516839
3000	0.9291665	0.9299289	0.9484910	0.9491320
Major Pore Diameter: 1 μm				
1000	0.9291659	0.9291686	0.9379110	0.9379134
1500	0.9291653	0.9291778	0.9450418	0.9450524
2000	0.9291642	0.9291978	0.9507691	0.9507967
2500	0.9291645	0.9292222	0.9512097	0.9512571
3000	0.9291643	0.9292407	0.9484889	0.9485532

Table C.16 Porosity correction factors for 10 vol-% elliptical porosity; $g=2.0$, 0°

T (K)	Vacuum		Helium	
	No Rad.	Rad.	No Rad.	Rad.
Major Pore Diameter: 50 μm				
1000	0.7418231	0.7423284	0.7945155	0.7949012
1500	0.7418248	0.7442476	0.8308296	0.8323324
2000	0.7418270	0.7484007	0.8565291	0.8600050
2500	0.7418244	0.7530634	0.8583880	0.8642771
3000	0.7418228	0.7566692	0.8466334	0.8550423
Major Pore Diameter: 25 μm				
1000	0.7418241	0.7420770	0.7945331	0.7947262
1500	0.7418244	0.7430394	0.8308403	0.8315933
2000	0.7418249	0.7451377	0.8565299	0.8582776
2500	0.7418243	0.7475197	0.8583913	0.8613641
3000	0.7418237	0.7493802	0.8466380	0.8508984
Major Pore Diameter: 10 μm				
1000	0.7418242	0.7419252	0.7945376	0.7946149
1500	0.7418242	0.7423110	0.8308433	0.8311450
2000	0.7418241	0.7431558	0.8565300	0.8572314
2500	0.7418243	0.7441213	0.8583921	0.8595884
3000	0.7418241	0.7448799	0.8466393	0.8483577
Major Pore Diameter: 1 μm				
1000	0.7418245	0.7418348	0.7945388	0.7945465
1500	0.7418251	0.7418738	0.8308440	0.8308741
2000	0.7418263	0.7419595	0.8565302	0.8566004
2500	0.7418259	0.7420567	0.8583922	0.8585124
3000	0.7418256	0.7421333	0.8466399	0.8468128

Table C.17 Porosity correction factors for 10 vol-% elliptical porosity; $g=2.0$, 45°

T (K)	Vacuum		Helium	
	No Rad.	Rad.	No Rad.	Rad.
Major Pore Diameter: 50 μm				
1000	0.7993868	0.7989973	0.8352870	0.8349723
1500	0.7993871	0.7975057	0.8608311	0.8595356
2000	0.7993880	0.7941943	0.8794694	0.8763044
2500	0.7993875	0.7903268	0.8808408	0.8753917
3000	0.7993868	0.7872307	0.8722294	0.8645224
Major Pore Diameter: 25 μm				
1000	0.7993876	0.7991928	0.8352989	0.8351415
1500	0.7993871	0.7984489	0.8608394	0.8601927
2000	0.7993875	0.7968088	0.8794713	0.8778965
2500	0.7993875	0.7949132	0.8808435	0.8781420
3000	0.7993875	0.7934101	0.8722337	0.8684250
Major Pore Diameter: 10 μm				
1000	0.7993872	0.7993095	0.8353018	0.8352390
1500	0.7993875	0.7990128	0.8608418	0.8605833
2000	0.7993875	0.7983603	0.8794719	0.8788437
2500	0.7993878	0.7976108	0.8808446	0.8797691
3000	0.7993876	0.7970196	0.8722345	0.8707212
Major Pore Diameter: 1 μm				
1000	0.7993883	0.7993804	0.8353034	0.8352970
1500	0.7993892	0.7993516	0.8608439	0.8608181
2000	0.7993903	0.7992878	0.8794758	0.8794128
2500	0.7993904	0.7992134	0.8808482	0.8807409
3000	0.7993898	0.7991544	0.8722378	0.8720870

Table C.18 Porosity correction factors for 10 vol-% elliptical porosity; $g=2.0$, 90°

T (K)	Vacuum		Helium	
	No Rad.	Rad.	No Rad.	Rad.
Major Pore Diameter: 50 μm				
1000	0.8630344	0.8632745	0.8794409	0.8796652
1500	0.8630345	0.8641828	0.8929285	0.8939410
2000	0.8630364	0.8661520	0.9038329	0.9064526
2500	0.8630340	0.8683671	0.9046713	0.9091398
3000	0.8630315	0.8700726	0.8994826	0.9055204
Major Pore Diameter: 25 μm				
1000	0.8630335	0.8631531	0.8794452	0.8795580
1500	0.8630330	0.8636081	0.8929316	0.8934384
2000	0.8630337	0.8645970	0.9038324	0.9051462
2500	0.8630333	0.8657144	0.9046727	0.9069178
3000	0.8630330	0.8665789	0.8994860	0.9025246
Major Pore Diameter: 10 μm				
1000	0.8630330	0.8630809	0.8794463	0.8794911
1500	0.8630329	0.8632631	0.8929325	0.8931353
2000	0.8630332	0.8636596	0.9038321	0.9043586
2500	0.8630332	0.8641094	0.9046729	0.9055738
3000	0.8630330	0.8644578	0.8994863	0.9007068
Major Pore Diameter: 1 μm				
1000	0.8630337	0.8630386	0.8794473	0.8794518
1500	0.8630348	0.8630576	0.8929343	0.8929548
2000	0.8630359	0.8630989	0.9038356	0.9038883
2500	0.8630361	0.8631440	0.9046760	0.9047663
3000	0.8630353	0.8631781	0.8994890	0.8996113

Table C.19 Porosity correction factors for 15 vol-% elliptical porosity; $g=2.0$, 0°

T (K)	Vacuum		Helium	
	No Rad.	Rad.	No Rad.	Rad.
Major Pore Diameter: 50 μm				
1000	0.6300169	0.6307070	0.7028257	0.7033624
1500	0.6300188	0.6333274	0.7537778	0.7558952
2000	0.6300203	0.6390036	0.7902277	0.7951742
2500	0.6300184	0.6453903	0.7928804	0.8012704
3000	0.6300172	0.6503371	0.7761541	0.7880902
Major Pore Diameter: 25 μm				
1000	0.6300181	0.6303633	0.7028408	0.7031094
1500	0.6300187	0.6316774	0.7537875	0.7548487
2000	0.6300188	0.6345434	0.7902291	0.7927158
2500	0.6300184	0.6378003	0.7928833	0.7971168
3000	0.6300179	0.6403465	0.7761579	0.7822012
Major Pore Diameter: 10 μm				
1000	0.6300185	0.6301566	0.7028455	0.7029530
1500	0.6300186	0.6306832	0.7537899	0.7542150
2000	0.6300183	0.6318364	0.7902299	0.7912275
2500	0.6300187	0.6331553	0.7928840	0.7945873
3000	0.6300184	0.6341917	0.7761593	0.7785957
Major Pore Diameter: 1 μm				
1000	0.6300186	0.6300325	0.7028461	0.7028571
1500	0.6300185	0.6300851	0.7537902	0.7538325
2000	0.6300186	0.6302006	0.7902287	0.7903290
2500	0.6300210	0.6303363	0.7928830	0.7930540
3000	0.6300190	0.6304390	0.7761602	0.7764050

Table C.20 Porosity correction factors for 15 vol-% elliptical porosity; $g=2.0$, 45°

T (K)	Vacuum		Helium	
	No Rad.	Rad.	No Rad.	Rad.
Major Pore Diameter: 50 μm				
1000	0.7129090	0.7123764	0.7623917	0.7619549
1500	0.7129096	0.7103412	0.7980697	0.7962518
2000	0.7129104	0.7058255	0.8243567	0.8198808
2500	0.7129096	0.7005613	0.8263005	0.8185938
3000	0.7129097	0.6963538	0.8141206	0.8032679
Major Pore Diameter: 25 μm				
1000	0.7129095	0.7126435	0.7624022	0.7621839
1500	0.7129099	0.7116286	0.7980775	0.7971698
2000	0.7129099	0.7093901	0.8243589	0.8221306
2500	0.7129097	0.7068052	0.8263034	0.8224797
3000	0.7129097	0.7047570	0.8141232	0.8087548
Major Pore Diameter: 10 μm				
1000	0.7129101	0.7128039	0.7624056	0.7623182
1500	0.7129100	0.7123981	0.7980796	0.7977167
2000	0.7129106	0.7115078	0.8243603	0.8234714
2500	0.7129100	0.7104846	0.8263047	0.8247820
3000	0.7129100	0.7096778	0.8141244	0.8119902
Major Pore Diameter: 1 μm				
1000	0.7129104	0.7129000	0.7624065	0.7623979
1500	0.7129116	0.7128604	0.7980816	0.7980456
2000	0.7129141	0.7127740	0.8243649	0.8242760
2500	0.7129128	0.7126711	0.8263080	0.8261560
3000	0.7129117	0.7125903	0.8141265	0.8139140

Table C.21 Porosity correction factors for 15 vol-% elliptical porosity; $g=2.0$, 90°

T (K)	Vacuum		Helium	
	No Rad.	Rad.	No Rad.	Rad.
Major Pore Diameter: 50 μm				
1000	0.8022563	0.8025900	0.8251653	0.8254797
1500	0.8022567	0.8038537	0.8441485	0.8455793
2000	0.8022581	0.8065935	0.8595957	0.8633220
2500	0.8022569	0.8096802	0.8607892	0.8671506
3000	0.8022563	0.8120612	0.8534253	0.8619990
Major Pore Diameter: 25 μm				
1000	0.8022562	0.8024231	0.8251698	0.8253271
1500	0.8022563	0.8030558	0.8441518	0.8448678
2000	0.8022564	0.8044308	0.8595962	0.8614642
2500	0.8022561	0.8059857	0.8607908	0.8639853
3000	0.8022559	0.8071896	0.8534272	0.8577394
Major Pore Diameter: 10 μm				
1000	0.8022562	0.8023232	0.8251712	0.8252343
1500	0.8022563	0.8025761	0.8441527	0.8444394
2000	0.8022568	0.8031281	0.8595969	0.8603453
2500	0.8022564	0.8037528	0.8607915	0.8620725
3000	0.8022562	0.8042374	0.8534278	0.8551589
Major Pore Diameter: 1 μm				
1000	0.8022569	0.8022634	0.8251718	0.8251780
1500	0.8022580	0.8022900	0.8441546	0.8441833
2000	0.8022608	0.8023480	0.8596014	0.8596765
2500	0.8022592	0.8024093	0.8607946	0.8609227
3000	0.8022582	0.8024568	0.8534301	0.8536036

Table C.22 Porosity correction factors for 20 vol-% elliptical porosity; $g=2.0$, 0°

T (K)	Vacuum		Helium	
	No Rad.	Rad.	No Rad.	Rad.
Major Pore Diameter: 50 μm				
1000	0.5263582	0.5272040	0.6166036	0.6172724
1500	0.5263592	0.5304161	0.6805832	0.6832515
2000	0.5263604	0.5373821	0.7267822	0.7330674
2500	0.5263591	0.5452300	0.7301600	0.7408319
3000	0.5263579	0.5513206	0.7089017	0.7240326
Major Pore Diameter: 25 μm				
1000	0.5263589	0.5267821	0.6166170	0.6169516
1500	0.5263594	0.5283929	0.6805929	0.6819297
2000	0.5263594	0.5319078	0.7267846	0.7299435
2500	0.5263591	0.5359057	0.7301635	0.7355457
3000	0.5263588	0.5390334	0.7089060	0.7165617
Major Pore Diameter: 10 μm				
1000	0.5263593	0.5265284	0.6166211	0.6167549
1500	0.5263596	0.5271741	0.6805952	0.6811306
2000	0.5263595	0.5285880	0.7267859	0.7280530
2500	0.5263593	0.5302050	0.7301647	0.7323292
3000	0.5263590	0.5314767	0.7089073	0.7119921
Major Pore Diameter: 1 μm				
1000	0.5263590	0.5263759	0.6166219	0.6166352
1500	0.5263599	0.5264415	0.6805975	0.6806510
2000	0.5263602	0.5265835	0.7267886	0.7269155
2500	0.5263606	0.5267471	0.7301693	0.7303863
3000	0.5263586	0.5268735	0.7089086	0.7092183

Table C.23 Porosity correction factors for 20 vol-% elliptical porosity; $g=2.0$, 45°

T (K)	Vacuum		Helium	
	No Rad.	Rad.	No Rad.	Rad.
Major Pore Diameter: 50 μm				
1000	0.6337855	0.6331355	0.6947407	0.6941995
1500	0.6337858	0.6306508	0.7392048	0.7369311
2000	0.6337868	0.6251443	0.7722540	0.7666153
2500	0.6337862	0.6187345	0.7747091	0.7649994
3000	0.6337859	0.6136169	0.7593546	0.7457366
Major Pore Diameter: 25 μm				
1000	0.6337864	0.6334615	0.6947495	0.6944792
1500	0.6337867	0.6322224	0.7392120	0.7380769
2000	0.6337866	0.6294910	0.7722571	0.7694490
2500	0.6337868	0.6263390	0.7747124	0.7698917
3000	0.6337866	0.6238427	0.7593575	0.7526157
Major Pore Diameter: 10 μm				
1000	0.6337867	0.6336568	0.6947522	0.6946441
1500	0.6337869	0.6331618	0.7392141	0.7387602
2000	0.6337871	0.6320750	0.7722585	0.7711377
2500	0.6337870	0.6308259	0.7747135	0.7727930
3000	0.6337869	0.6298418	0.7593584	0.7566772
Major Pore Diameter: 1 μm				
1000	0.6337876	0.6337747	0.6947538	0.6947429
1500	0.6337888	0.6337263	0.7392166	0.7391712
2000	0.6337896	0.6336191	0.7722620	0.7721501
2500	0.6337904	0.6334957	0.7747180	0.7745265
3000	0.6337903	0.6333980	0.7593629	0.7590957

Table C.24 Porosity correction factors for 20 vol-% elliptical porosity; $g=2.0$, 90°

T (K)	Vacuum		Helium	
	No Rad.	Rad.	No Rad.	Rad.
Major Pore Diameter: 50 μm				
1000	0.7466833	0.7470947	0.7750260	0.7754169
1500	0.7466835	0.7486516	0.7986985	0.8004906
2000	0.7466844	0.7520305	0.8180907	0.8227881
2500	0.7466835	0.7558427	0.8195952	0.8276221
3000	0.7466828	0.7587860	0.8103317	0.8211239
Major Pore Diameter: 25 μm				
1000	0.7466832	0.7468890	0.7750298	0.7752256
1500	0.7466836	0.7476686	0.7987024	0.7995992
2000	0.7466836	0.7493635	0.8180924	0.8204465
2500	0.7466835	0.7512829	0.8195974	0.8236259
3000	0.7466833	0.7527692	0.8103338	0.8157573
Major Pore Diameter: 10 μm				
1000	0.7466833	0.7467658	0.7750314	0.7751096
1500	0.7466834	0.7470778	0.7987031	0.7990622
2000	0.7466836	0.7477574	0.8180933	0.8190363
2500	0.7466835	0.7485275	0.8195979	0.8212131
3000	0.7466836	0.7491256	0.8103343	0.8125103
Major Pore Diameter: 1 μm				
1000	0.7466846	0.7466928	0.7750325	0.7750404
1500	0.7466861	0.7467256	0.7987061	0.7987418
2000	0.7466870	0.7467945	0.8180967	0.8181912
2500	0.7466878	0.7468726	0.8196028	0.8197648
3000	0.7466878	0.7469325	0.8103389	0.8105569

Table C.25 Porosity correction factors for 25 vol-% elliptical porosity; $g=2.0$, 0°

T (K)	Vacuum		Helium	
	No Rad.	Rad.	No Rad.	Rad.
Major Pore Diameter: 50 μm				
1000	0.4270569	0.4280438	0.5335051	0.5342949
1500	0.4270582	0.4317932	0.6096944	0.6128745
2000	0.4270592	0.4399296	0.6650980	0.6726402
2500	0.4270582	0.4491078	0.6691631	0.6819797
3000	0.4270570	0.4562327	0.6436155	0.6617391
Major Pore Diameter: 25 μm				
1000	0.4270580	0.4275516	0.5335175	0.5339125
1500	0.4270584	0.4294314	0.6097035	0.6112966
2000	0.4270587	0.4335349	0.6651010	0.6688907
2500	0.4270583	0.4382050	0.6691666	0.6756279
3000	0.4270579	0.4418607	0.6436197	0.6527842
Major Pore Diameter: 10 μm				
1000	0.4270582	0.4272558	0.5335212	0.5336793
1500	0.4270583	0.4280090	0.6097057	0.6103438
2000	0.4270582	0.4296603	0.6651016	0.6666223
2500	0.4270582	0.4315466	0.6691677	0.6717651
3000	0.4270582	0.4330319	0.6436211	0.6473126
Major Pore Diameter: 1 μm				
1000	0.4270580	0.4270776	0.5335217	0.5335375
1500	0.4270584	0.4271535	0.6097064	0.6097700
2000	0.4270585	0.4273192	0.6651056	0.6652577
2500	0.4270585	0.4275093	0.6691703	0.6694312
3000	0.4270583	0.4276588	0.6436215	0.6439923

Table C.26 Porosity correction factors for 25 vol-% elliptical porosity; $g=2.0$, 45°

T (K)	Vacuum		Helium	
	No Rad.	Rad.	No Rad.	Rad.
Major Pore Diameter: 50 μm				
1000	0.5607571	0.5600084	0.6315812	0.6309509
1500	0.5607576	0.5571470	0.6837585	0.6810845
2000	0.5607584	0.5508109	0.7228385	0.7161633
2500	0.5607581	0.5434443	0.7257529	0.7142577
3000	0.5607575	0.5375685	0.7075536	0.6914909
Major Pore Diameter: 25 μm				
1000	0.5607579	0.5603837	0.6315894	0.6312744
1500	0.5607581	0.5589564	0.6837651	0.6824300
2000	0.5607582	0.5558117	0.7228418	0.7195165
2500	0.5607582	0.5521846	0.7257557	0.7200454
3000	0.5607581	0.5493134	0.7075565	0.6995978
Major Pore Diameter: 10 μm				
1000	0.5607584	0.5606088	0.6315922	0.6314662
1500	0.5607584	0.5600387	0.6837667	0.6832331
2000	0.5607582	0.5587864	0.7228426	0.7215158
2500	0.5607582	0.5573482	0.7257569	0.7234810
3000	0.5607585	0.5562151	0.7075575	0.7043907
Major Pore Diameter: 1 μm				
1000	0.5607586	0.5607437	0.6315926	0.6315798
1500	0.5607589	0.5606871	0.6837675	0.6837143
2000	0.5607596	0.5605624	0.7228441	0.7227114
2500	0.5607594	0.5604195	0.7257596	0.7255313
3000	0.5607591	0.5603064	0.7075584	0.7072427

Table C.27 Porosity correction factors for 25 vol-% elliptical porosity; $g=2.0$, 90°

T (K)	Vacuum		Helium	
	No Rad.	Rad.	No Rad.	Rad.
Major Pore Diameter: 50 μm				
1000	0.6961353	0.6966101	0.7289534	0.7294079
1500	0.6961359	0.6984067	0.7565734	0.7586733
2000	0.6961370	0.7023071	0.7793464	0.7848855
2500	0.6961365	0.7067134	0.7811196	0.7905943
3000	0.6961356	0.7101185	0.7702183	0.7829267
Major Pore Diameter: 25 μm				
1000	0.6961362	0.6963735	0.7289572	0.7291848
1500	0.6961365	0.6972727	0.7565771	0.7576279
2000	0.6961365	0.6992282	0.7793485	0.7821234
2500	0.6961363	0.7014435	0.7811215	0.7858738
3000	0.6961362	0.7031602	0.7702206	0.7766016
Major Pore Diameter: 10 μm				
1000	0.6961364	0.6962316	0.7289586	0.7290495
1500	0.6961365	0.6965912	0.7565781	0.7569985
2000	0.6961361	0.6973746	0.7793494	0.7804601
2500	0.6961364	0.6982634	0.7811220	0.7830268
3000	0.6961365	0.6989535	0.7702211	0.7727802
Major Pore Diameter: 1 μm				
1000	0.6961367	0.6961461	0.7289587	0.7289678
1500	0.6961369	0.6961826	0.7565787	0.7566210
2000	0.6961375	0.6962615	0.7793506	0.7794616
2500	0.6961377	0.6963509	0.7811236	0.7813146
3000	0.6961374	0.6964193	0.7702221	0.7704785

Table C.28 Porosity correction factors for 30 vol-% elliptical porosity; $g=2.0$, 0°

T (K)	Vacuum		Helium	
	No Rad.	Rad.	No Rad.	Rad.
Major Pore Diameter: 50 μm				
1000	0.3271834	0.3283129	0.4504676	0.4513778
1500	0.3271846	0.3326046	0.5390752	0.5427598
2000	0.3271854	0.3419231	0.6037359	0.6125137
2500	0.3271843	0.3524436	0.6084892	0.6234139
3000	0.3271833	0.3606170	0.5786398	0.5997081
Major Pore Diameter: 25 μm				
1000	0.3271841	0.3277490	0.4504786	0.4509338
1500	0.3271844	0.3299005	0.5390834	0.5409291
2000	0.3271846	0.3345986	0.6037393	0.6081489
2500	0.3271844	0.3399476	0.6084927	0.6160147
3000	0.3271841	0.3441363	0.5786435	0.5892926
Major Pore Diameter: 10 μm				
1000	0.3271846	0.3274106	0.4504820	0.4506640
1500	0.3271846	0.3282726	0.5390858	0.5398249
2000	0.3271847	0.3301629	0.6037403	0.6055090
2500	0.3271846	0.3323224	0.6084939	0.6115171
3000	0.3271844	0.3340228	0.5786448	0.5829333
Major Pore Diameter: 1 μm				
1000	0.3271836	0.3272062	0.4504818	0.4504999
1500	0.3271847	0.3272935	0.5390870	0.5391609
2000	0.3271840	0.3274823	0.6037397	0.6039171
2500	0.3271846	0.3277004	0.6084931	0.6087961
3000	0.3271817	0.3278691	0.5786436	0.5790742

Table C.29 Porosity correction factors for 30 vol-% elliptical porosity; $g=2.0$, 45°

T (K)	Vacuum		Helium	
	No Rad.	Rad.	No Rad.	Rad.
Major Pore Diameter: 50 μm				
1000	0.4926698	0.4918363	0.5722108	0.5715023
1500	0.4926706	0.4886513	0.6312740	0.6282453
2000	0.4926709	0.4816027	0.6757962	0.6681906
2500	0.4926706	0.4734164	0.6791269	0.6660290
3000	0.4926700	0.4668923	0.6583522	0.6401095
Major Pore Diameter: 25 μm				
1000	0.4926706	0.4922540	0.5722180	0.5718638
1500	0.4926707	0.4906649	0.6312798	0.6297676
2000	0.4926708	0.4871653	0.6757999	0.6720101
2500	0.4926708	0.4831301	0.6791299	0.6726203
3000	0.4926707	0.4799372	0.6583549	0.6493102
Major Pore Diameter: 10 μm				
1000	0.4926710	0.4925041	0.5722204	0.5720787
1500	0.4926708	0.4918693	0.6312815	0.6306767
2000	0.4926710	0.4904755	0.6758008	0.6742885
2500	0.4926709	0.4888749	0.6791312	0.6765363
3000	0.4926710	0.4876135	0.6583561	0.6547558
Major Pore Diameter: 1 μm				
1000	0.4926714	0.4926547	0.5722210	0.5722070
1500	0.4926719	0.4925918	0.6312831	0.6312228
2000	0.4926714	0.4924522	0.6758022	0.6756509
2500	0.4926725	0.4922941	0.6791338	0.6788749
3000	0.4926726	0.4921691	0.6583591	0.6579998

Table C.30 Porosity correction factors for 30 vol-% elliptical porosity; $g=2.0$, 90°

T (K)	Vacuum		Helium	
	No Rad.	Rad.	No Rad.	Rad.
Major Pore Diameter: 50 μm				
1000	0.6503796	0.6509050	0.6868480	0.6873555
1500	0.6503802	0.6528928	0.7177521	0.7201100
2000	0.6503806	0.6572117	0.7433859	0.7496442
2500	0.6503803	0.6620964	0.7453880	0.7561029
3000	0.6503795	0.6658751	0.7330944	0.7474322
Major Pore Diameter: 25 μm				
1000	0.6503802	0.6506429	0.6868516	0.6871054
1500	0.6503804	0.6516376	0.7177552	0.7189350
2000	0.6503804	0.6538020	0.7433884	0.7486580
2500	0.6503804	0.6562551	0.7453901	0.7507612
3000	0.6503802	0.6581569	0.7330961	0.7402895
Major Pore Diameter: 10 μm				
1000	0.6503804	0.6504855	0.6868527	0.6869545
1500	0.6503802	0.6508835	0.7177558	0.7182279
2000	0.6503806	0.6517506	0.7433895	0.7446438
2500	0.6503806	0.6527341	0.7453908	0.7475429
3000	0.6503806	0.6534979	0.7330969	0.7359802
Major Pore Diameter: 1 μm				
1000	0.6503813	0.6503917	0.6868538	0.6868638
1500	0.6503818	0.6504323	0.7177578	0.7178051
2000	0.6503814	0.6505184	0.7433900	0.7435157
2500	0.6503826	0.6506182	0.7453938	0.7456092
3000	0.6503832	0.6506952	0.7330998	0.7333886

Table C.31 Porosity correction factors for 5 vol-% elliptical porosity; $g=3.0$, 0°

T (K)	Vacuum		Helium	
	No Rad.	Rad.	No Rad.	Rad.
Major Pore Diameter: 50 μm				
1000	0.8245739	0.8248945	0.8699685	0.8701804
1500	0.8245757	0.8261149	0.8977855	0.8985390
2000	0.8245779	0.8287540	0.9159977	0.9176365
2500	0.8245750	0.8317121	0.9172697	0.9200369
3000	0.8245727	0.8339996	0.9091189	0.9131885
Major Pore Diameter: 25 μm				
1000	0.8245744	0.8247350	0.8699881	0.8700946
1500	0.8245747	0.8253469	0.8977955	0.8981733
2000	0.8245753	0.8266813	0.9159975	0.9168211
2500	0.8245745	0.8281957	0.9172722	0.9186681
3000	0.8245735	0.8293797	0.9091238	0.9111838
Major Pore Diameter: 10 μm				
1000	0.8245744	0.8246386	0.8699934	0.8700360
1500	0.8245744	0.8248838	0.8977983	0.8979498
2000	0.8245743	0.8254211	0.9159972	0.9163280
2500	0.8245744	0.8260358	0.9172728	0.9178345
3000	0.8245741	0.8265197	0.9091251	0.9099558
Major Pore Diameter: 1 μm				
1000	0.8245741	0.8245806	0.8699946	0.8699988
1500	0.8245737	0.8246049	0.8977985	0.8978136
2000	0.8245738	0.8246586	0.9159976	0.9160306
2500	0.8245735	0.8247205	0.9172730	0.9173295
3000	0.8245732	0.8247691	0.9091254	0.9092089

Table C.32 Porosity correction factors for 10 vol-% elliptical porosity; $g=3.0$, 0°

T (K)	Vacuum		Helium	
	No Rad.	Rad.	No Rad.	Rad.
Major Pore Diameter: 50 μm				
1000	0.6666091	0.6671846	0.7496142	0.7500060
1500	0.6666107	0.6693730	0.8015619	0.8029757
2000	0.6666125	0.6741112	0.8360344	0.8391434
2500	0.6666104	0.6794390	0.8384608	0.8437170
3000	0.6666095	0.6835659	0.8229729	0.8306676
Major Pore Diameter: 25 μm				
1000	0.6666098	0.6668979	0.7496318	0.7498277
1500	0.6666102	0.6679957	0.8015715	0.8022800
2000	0.6666107	0.6703902	0.8360348	0.8375973
2500	0.6666101	0.6731121	0.8384636	0.8411143
3000	0.6666098	0.6752420	0.8229767	0.8268707
Major Pore Diameter: 10 μm				
1000	0.6666102	0.6667258	0.7496367	0.7497152
1500	0.6666102	0.6671656	0.8015740	0.8018577
2000	0.6666107	0.6681297	0.8360356	0.8366619
2500	0.6666104	0.6692332	0.8384644	0.8395304
3000	0.6666099	0.6701017	0.8229780	0.8245476
Major Pore Diameter: 1 μm				
1000	0.6666105	0.6667259	0.7496373	0.7497158
1500	0.6666111	0.6671664	0.8015746	0.8018583
2000	0.6666092	0.6681303	0.8360341	0.8366612
2500	0.6666119	0.6692398	0.8384651	0.8395312
3000	0.6666112	0.6701114	0.8229777	0.8245469

Table C.33 Porosity correction factors for 15 vol-% elliptical porosity; $g=3.0$, 0°

T (K)	Vacuum		Helium	
	No Rad.	Rad.	No Rad.	Rad.
Major Pore Diameter: 50 μm				
1000	0.5218631	0.5226483	0.6369557	0.6375019
1500	0.5218646	0.5256326	0.7102677	0.7122694
2000	0.5218663	0.5321019	0.7594815	0.7639273
2500	0.5218646	0.5393892	0.7629654	0.7704897
3000	0.5218627	0.5450413	0.7407807	0.7517520
Major Pore Diameter: 25 μm				
1000	0.5218640	0.5222568	0.6369708	0.6372440
1500	0.5218641	0.5237534	0.7102766	0.7112793
2000	0.5218645	0.5270206	0.7594818	0.7617159
2500	0.5218642	0.5307382	0.7629676	0.7667609
3000	0.5218637	0.5336492	0.7407849	0.7463339
Major Pore Diameter: 10 μm				
1000	0.5218639	0.5220212	0.6369749	0.6370842
1500	0.5218638	0.5226206	0.7102786	0.7106802
2000	0.5218637	0.5239357	0.7594820	0.7603780
2500	0.5218636	0.5254409	0.7629676	0.7644925
3000	0.5218638	0.5266270	0.7407856	0.7430210
Major Pore Diameter: 1 μm				
1000	0.5218639	0.5218796	0.6369747	0.6369859
1500	0.5218614	0.5219372	0.7102761	0.7103167
2000	0.5218611	0.5220686	0.7594780	0.7595674
2500	0.5218601	0.5222194	0.7629626	0.7631156
3000	0.5218641	0.5223434	0.7407818	0.7410063

Table C.34 Porosity correction factors for 20 vol-% elliptical porosity; $g=3.0$, 0°

T (K)	Vacuum		Helium	
	No Rad.	Rad.	No Rad.	Rad.
Major Pore Diameter: 50 μm				
1000	0.3771556	0.3781473	0.5247039	0.5254016
1500	0.3771570	0.3819164	0.6194032	0.6219796
2000	0.3771585	0.3900924	0.6833070	0.6890613
2500	0.3771568	0.3993116	0.6878432	0.6975877
3000	0.3771555	0.4064697	0.6589926	0.6731700
Major Pore Diameter: 25 μm				
1000	0.3771566	0.3776528	0.5247177	0.5250666
1500	0.3771571	0.3795431	0.6194120	0.6207028
2000	0.3771576	0.3836708	0.6833091	0.6861998
2500	0.3771571	0.3883695	0.6878461	0.6927576
3000	0.3771565	0.3920510	0.6589967	0.6661646
Major Pore Diameter: 10 μm				
1000	0.3771573	0.3773556	0.5247217	0.5248616
1500	0.3771572	0.3781130	0.6194144	0.6199313
2000	0.3771578	0.3797743	0.6833097	0.6844696
2500	0.3771572	0.3816755	0.6878472	0.6898216
3000	0.3771572	0.3831742	0.6589982	0.6618851
Major Pore Diameter: 1 μm				
1000	0.3771568	0.3771765	0.5247222	0.5247359
1500	0.3771565	0.3772522	0.6194164	0.6194680
2000	0.3771602	0.3774225	0.6833111	0.6834274
2500	0.3771559	0.3776098	0.6878493	0.6880477
3000	0.3771555	0.3777609	0.6589977	0.6592876

Table C.35 Porosity correction factors for 5 vol-% elliptical porosity; $g=4.0$, 0°

T (K)	Vacuum		Helium	
	No Rad.	Rad.	No Rad.	Rad.
Major Pore Diameter: 50 μm				
1000	0.7845276	0.7848953	0.8492422	0.8494592
1500	0.7845295	0.7862943	0.8854204	0.8861402
2000	0.7845316	0.7893205	0.9078320	0.9093320
2500	0.7845285	0.7927156	0.9093628	0.9118921
3000	0.7845269	0.7953440	0.8994806	0.9032757
Major Pore Diameter: 25 μm				
1000	0.7845282	0.7847123	0.8492619	0.8493704
1500	0.7845286	0.7854139	0.8854301	0.8857907
2000	0.7845289	0.7869442	0.9078318	0.9085857
2500	0.7845283	0.7886825	0.9093654	0.9106402
3000	0.7845277	0.7900425	0.8994850	0.9014043
Major Pore Diameter: 10 μm				
1000	0.7845279	0.7846016	0.8492672	0.8493105
1500	0.7845281	0.7848830	0.8854327	0.8855772
2000	0.7845284	0.7854994	0.9078319	0.9081343
2500	0.7845281	0.7862050	0.9093661	0.9098786
3000	0.7845280	0.7867608	0.8994862	0.9002595
Major Pore Diameter: 1 μm				
1000	0.7845280	0.7846017	0.8492685	0.8493121
1500	0.7845284	0.7848836	0.8854335	0.8855777
2000	0.7845290	0.7855000	0.9078313	0.9081334
2500	0.7845288	0.7862101	0.9093659	0.9098784
3000	0.7845284	0.7867688	0.8994866	0.9002599

Table C.36 Porosity correction factors for 10 vol-% elliptical porosity; $g=4.0$, 0°

T (K)	Vacuum		Helium	
	No Rad.	Rad.	No Rad.	Rad.
Major Pore Diameter: 50 μm				
1000	0.5931823	0.5938326	0.7104520	0.7108495
1500	0.5931838	0.5963047	0.7777804	0.7791266
2000	0.5931849	0.6016595	0.8201509	0.8229937
2500	0.5931831	0.6076868	0.8230693	0.8278671
3000	0.5931816	0.6123594	0.8043034	0.8114645
Major Pore Diameter: 25 μm				
1000	0.5931829	0.5935083	0.7104691	0.7106700
1500	0.5931835	0.5947485	0.7777896	0.7784641
2000	0.5931838	0.5974551	0.8201519	0.8215799
2500	0.5931831	0.6005338	0.8230720	0.8254895
3000	0.5931826	0.6029451	0.8043077	0.8079275
Major Pore Diameter: 10 μm				
1000	0.5931835	0.5933134	0.7104741	0.7105536
1500	0.5931835	0.5938107	0.7777920	0.7780623
2000	0.5931834	0.5949000	0.8201520	0.8207250
2500	0.5931833	0.5961483	0.8230728	0.8240445
3000	0.5931835	0.5971320	0.8043093	0.8057669
Major Pore Diameter: 1 μm				
1000	0.5931840	0.5931970	0.7104756	0.7104835
1500	0.5931850	0.5932474	0.7777931	0.7778198
2000	0.5931820	0.5933544	0.8201525	0.8202096
2500	0.5931858	0.5934839	0.8230734	0.8231706
3000	0.5931851	0.5935830	0.8043104	0.8044567

Table C.37 Porosity correction factors for 15 vol-% elliptical porosity; $g=4.0$, 0°

T (K)	Vacuum		Helium	
	No Rad.	Rad.	No Rad.	Rad.
Major Pore Diameter: 50 μm				
1000	0.4073120	0.4082230	0.5747715	0.5753379
1500	0.4073136	0.4116863	0.6720934	0.6740338
2000	0.4073152	0.4191945	0.7338021	0.7379296
2500	0.4073134	0.4276555	0.7380678	0.7450369
3000	0.4073118	0.4342225	0.7106795	0.7210535
Major Pore Diameter: 25 μm				
1000	0.4073131	0.4077688	0.5747873	0.5750706
1500	0.4073136	0.4095060	0.6721024	0.6730744
2000	0.4073141	0.4132992	0.7338034	0.7358764
2500	0.4073136	0.4176164	0.7380705	0.7415822
3000	0.4073130	0.4209998	0.7106839	0.7159259
Major Pore Diameter: 10 μm				
1000	0.4073132	0.4074956	0.5747916	0.5749052
1500	0.4073134	0.4081918	0.6721048	0.6724941
2000	0.4073136	0.4097186	0.7338041	0.7346353
2500	0.4073135	0.4114674	0.7380712	0.7394826
3000	0.4073133	0.4128459	0.7106849	0.7127956
Major Pore Diameter: 1 μm				
1000	0.4073147	0.4073310	0.5747922	0.5748033
1500	0.4073144	0.4074023	0.6721066	0.6721456
2000	0.4073136	0.4075548	0.7338057	0.7338893
2500	0.4073150	0.4077325	0.7380736	0.7382150
3000	0.4073118	0.4078688	0.7106849	0.7108969

Table C.38 Porosity correction factors for 5 vol-% elliptical porosity; $g=5.0$, 0°

T (K)	Vacuum		Helium	
	No Rad.	Rad.	No Rad.	Rad.
Major Pore Diameter: 50 μm				
1000	0.7453931	0.7458071	0.8311097	0.8313289
1500	0.7453960	0.7473819	0.8752553	0.8759432
2000	0.7453991	0.7507896	0.9013753	0.9027614
2500	0.7453956	0.7546127	0.9031272	0.9054597
3000	0.7453916	0.7575725	0.8917449	0.8953015
Major Pore Diameter: 25 μm				
1000	0.7453935	0.7456007	0.8311295	0.8312393
1500	0.7453942	0.7463909	0.8752639	0.8756085
2000	0.7453954	0.7481139	0.9013734	0.9020694
2500	0.7453947	0.7500712	0.9031281	0.9043031
3000	0.7453937	0.7516029	0.8917497	0.8935467
Major Pore Diameter: 10 μm				
1000	0.7453941	0.7454770	0.8311354	0.8311794
1500	0.7453944	0.7457936	0.8752663	0.8754043
2000	0.7453944	0.7464877	0.9013727	0.9016519
2500	0.7453945	0.7472821	0.9031279	0.9036003
3000	0.7453940	0.7479081	0.8917508	0.8924743
Major Pore Diameter: 1 μm				
1000	0.7453938	0.7454022	0.8311357	0.8311400
1500	0.7453939	0.7454339	0.8752650	0.8752788
2000	0.7453935	0.7455034	0.9013696	0.9013975
2500	0.7453936	0.7455835	0.9031246	0.9031751
3000	0.7453934	0.7456467	0.8917471	0.8918196

Table C.39 Porosity correction factors for 10 vol-% elliptical porosity; $g=5.0$, 0°

T (K)	Vacuum		Helium	
	No Rad.	Rad.	No Rad.	Rad.
Major Pore Diameter: 50 μm				
1000	0.5193539	0.5200839	0.6750401	0.6754429
1500	0.5193557	0.5228596	0.7575477	0.7588378
2000	0.5193576	0.5288738	0.8071484	0.8097827
2500	0.5193557	0.5356456	0.8105024	0.8149403
3000	0.5193535	0.5408974	0.7887972	0.7955278
Major Pore Diameter: 25 μm				
1000	0.5193544	0.5197196	0.6750572	0.6752586
1500	0.5193546	0.5211118	0.7575560	0.7582021
2000	0.5193552	0.5241516	0.8071479	0.8084705
2500	0.5193548	0.5276104	0.8105036	0.8127388
3000	0.5193542	0.5303205	0.7888013	0.7922004
Major Pore Diameter: 10 μm				
1000	0.5193541	0.5195004	0.6750619	0.6751423
1500	0.5193541	0.5200582	0.7575579	0.7578168
2000	0.5193543	0.5212821	0.8071476	0.8076777
2500	0.5193542	0.5226843	0.8105037	0.8114015
3000	0.5193539	0.5237897	0.7888018	0.7901700
Major Pore Diameter: 1 μm				
1000	0.5193535	0.5193684	0.6750618	0.6750699
1500	0.5193530	0.5194233	0.7575563	0.7575820
2000	0.5193525	0.5195459	0.8071443	0.8071974
2500	0.5193513	0.5196860	0.8104985	0.8105884
3000	0.5193518	0.5197987	0.7887980	0.7889354

Table C.40 Porosity correction factors for 5 vol-% elliptical porosity; $g=6.0$, 0°

T (K)	Vacuum		Helium	
	No Rad.	Rad.	No Rad.	Rad.
Major Pore Diameter: 50 μm				
1000	0.7069607	0.7074195	0.8150584	0.8152787
1500	0.7069639	0.7091650	0.8667245	0.8673817
2000	0.7069672	0.7129425	0.8961242	0.8974119
2500	0.7069635	0.7171826	0.8980659	0.9002300
3000	0.7069600	0.7204671	0.8853829	0.8887253
Major Pore Diameter: 25 μm				
1000	0.7069618	0.7071914	0.8150789	0.8151890
1500	0.7069623	0.7080667	0.8667328	0.8670619
2000	0.7069633	0.7099766	0.8936818	0.8967682
2500	0.7069627	0.7121471	0.8980672	0.8991569
3000	0.7069616	0.7138461	0.8853872	0.8870746
Major Pore Diameter: 10 μm				
1000	0.7069619	0.7070538	0.8150844	0.8151284
1500	0.7069620	0.7074047	0.8667353	0.8668668
2000	0.7069621	0.7081738	0.8961209	0.8963804
2500	0.7069622	0.7090546	0.8980675	0.8985051
3000	0.7069618	0.7097490	0.8853884	0.8860675
Major Pore Diameter: 1 μm				
1000	0.7069613	0.7069707	0.8150856	0.8150900
1500	0.7069620	0.7070062	0.8667361	0.8667493
2000	0.7069619	0.7070836	0.8961222	0.8961482
2500	0.7069620	0.7071725	0.8980688	0.8981128
3000	0.7069613	0.7072420	0.8853889	0.8854571

Table C.41 Porosity correction factors for 10 vol-% elliptical porosity; $g=6.0$, 0°

T (K)	Vacuum		Helium	
	No Rad.	Rad.	No Rad.	Rad.
Major Pore Diameter: 50 μm				
1000	0.4410289	0.4418519	0.6411721	0.6415816
1500	0.4410308	0.4449811	0.7392183	0.7404646
2000	0.4410324	0.4517616	0.7957417	0.7982118
2500	0.4410303	0.4593980	0.7995018	0.8036572
3000	0.4410287	0.4653227	0.7750339	0.7814229
Major Pore Diameter: 25 μm				
1000	0.4410299	0.4414418	0.6411898	0.6413947
1500	0.4410306	0.4430116	0.7392270	0.7398511
2000	0.4410310	0.4464392	0.7957422	0.7969821
2500	0.4410305	0.4503396	0.7995037	0.8015953
3000	0.4410300	0.4533969	0.7750379	0.7782621
Major Pore Diameter: 10 μm				
1000	0.4410304	0.4411951	0.6411949	0.6412770
1500	0.4410304	0.4418242	0.7392290	0.7394794
2000	0.4410306	0.4432043	0.7957428	0.7962394
2500	0.4410305	0.4447855	0.7995046	0.8003442
3000	0.4410305	0.4460328	0.7750393	0.7763365
Major Pore Diameter: 1 μm				
1000	0.4410298	0.4410465	0.6411958	0.6412039
1500	0.4410314	0.4411108	0.7392311	0.7392561
2000	0.4410305	0.4412485	0.7957446	0.7957944
2500	0.4410320	0.4414095	0.7995073	0.7995912
3000	0.4410322	0.4415361	0.7750423	0.7751723

Table C.42 Porosity correction factors for 5 vol-% elliptical porosity; $g=7.0$, 0°

T (K)	Vacuum		Helium	
	No Rad.	Rad.	No Rad.	Rad.
Major Pore Diameter: 50 μm				
1000	0.6691479	0.6696503	0.8007299	0.8009497
1500	0.6691503	0.6715613	0.8594534	0.8600822
2000	0.6691525	0.6756986	0.8917633	0.8929656
2500	0.6691501	0.6803471	0.8938719	0.8958914
3000	0.6691478	0.6839494	0.8800510	0.8832021
Major Pore Diameter: 25 μm				
1000	0.6691492	0.6694006	0.8007505	0.8008604
1500	0.6691496	0.6703592	0.8594626	0.8597773
2000	0.6691502	0.6724510	0.8917626	0.8923660
2500	0.6691499	0.6748300	0.8938741	0.8948900
3000	0.6691493	0.6766927	0.8800556	0.8816455
Major Pore Diameter: 10 μm				
1000	0.6691493	0.6692499	0.8007563	0.8008001
1500	0.6691495	0.6696341	0.8594650	0.8595910
2000	0.6691496	0.6704768	0.8917627	0.8920047
2500	0.6691498	0.6714424	0.8938750	0.8942828
3000	0.6691495	0.6722031	0.8800570	0.8806966
Major Pore Diameter: 1 μm				
1000	0.6691503	0.6691605	0.8007584	0.8007629
1500	0.6691507	0.6691992	0.8594685	0.8594808
2000	0.6691516	0.6692848	0.8917664	0.8917909
2500	0.6691518	0.6693824	0.8938795	0.8939206
3000	0.6691522	0.6694600	0.8800620	0.8801261

Table C.43 Porosity correction factors for 5 vol-% elliptical porosity; $g=8.0$, 0°

T (K)	Vacuum		Helium	
	No Rad.	Rad.	No Rad.	Rad.
Major Pore Diameter: 50 μm				
1000	0.6316585	0.6322040	0.7877557	0.7879738
1500	0.6316600	0.6342784	0.8531289	0.8537298
2000	0.6316618	0.6387715	0.8880518	0.8891798
2500	0.6316597	0.6438222	0.8903072	0.8921998
3000	0.6316583	0.6477377	0.8754766	0.8784574
Major Pore Diameter: 25 μm				
1000	0.6316593	0.6319324	0.7877757	0.7878848
1500	0.6316599	0.6329735	0.8531377	0.8534387
2000	0.6316602	0.6352452	0.8880519	0.8886178
2500	0.6316597	0.6378289	0.8903094	0.8912610
3000	0.6316594	0.6398531	0.8754808	0.8769836
Major Pore Diameter: 10 μm				
1000	0.6316599	0.6317692	0.7877815	0.7878250
1500	0.6316601	0.6321862	0.8531398	0.8532605
2000	0.6316599	0.6331013	0.8880524	0.8882790
2500	0.6316597	0.6341498	0.8903098	0.8906916
3000	0.6316598	0.6349765	0.8754814	0.8760857
Major Pore Diameter: 1 μm				
1000	0.6316599	0.6316708	0.7877824	0.7877866
1500	0.6316603	0.6317130	0.8531401	0.8531525
2000	0.6316599	0.6318042	0.8880510	0.8880738
2500	0.6316606	0.6319109	0.8903098	0.8903482
3000	0.6316603	0.6319948	0.8754797	0.8755401

Table C.44 Porosity correction factors for 5 vol-% elliptical porosity; $g=9.0$, 0°

T (K)	Vacuum		Helium	
	No Rad.	Rad.	No Rad.	Rad.
Major Pore Diameter: 50 μm				
1000	0.5941656	0.5947544	0.7758481	0.7760640
1500	0.5941674	0.5969939	0.8475254	0.8481019
2000	0.5941694	0.6018448	0.8848257	0.8858884
2500	0.5941674	0.6072991	0.8872119	0.8889933
3000	0.5941651	0.6115274	0.8714710	0.8742997
Major Pore Diameter: 25 μm				
1000	0.5941664	0.5944610	0.7758676	0.7759759
1500	0.5941665	0.5955845	0.8475331	0.8478217
2000	0.5941670	0.5980371	0.8848248	0.8853578
2500	0.5941668	0.6008269	0.8872129	0.8881083
3000	0.5941661	0.6030125	0.8714751	0.8729004
Major Pore Diameter: 10 μm				
1000	0.5941664	0.5942846	0.7758731	0.7759163
1500	0.5941661	0.5947345	0.8475349	0.8476508
2000	0.5941665	0.5957222	0.8848247	0.8850380
2500	0.5941663	0.5968542	0.8872129	0.8875721
3000	0.5941662	0.5977468	0.8714759	0.8720487
Major Pore Diameter: 1 μm				
1000	0.5941657	0.5941775	0.7758730	0.7758771
1500	0.5941654	0.5942222	0.8475329	0.8475446
2000	0.5941633	0.5943195	0.8848208	0.8848424
2500	0.5941644	0.5944349	0.8872104	0.8872463
3000	0.5941639	0.5945248	0.8714736	0.8715312

Table C.45 Porosity correction factors for 5 vol-% elliptical porosity; $g=10.0$, 0°

T (K)	Vacuum		Helium	
	No Rad.	Rad.	No Rad.	Rad.
Major Pore Diameter: 50 μm				
1000	0.5563925	0.5570257	0.7648140	0.7650288
1500	0.5563946	0.5594340	0.8425009	0.8430549
2000	0.5563965	0.5646507	0.8819813	0.8829867
2500	0.5563945	0.5705176	0.8844858	0.8861700
3000	0.5563921	0.5750668	0.8679194	0.8706119
Major Pore Diameter: 25 μm				
1000	0.5563931	0.5567100	0.7648338	0.7649411
1500	0.5563934	0.5579181	0.8425081	0.8427856
2000	0.5563940	0.5605556	0.8819803	0.8824846
2500	0.5563935	0.5635561	0.8844864	0.8853329
3000	0.5563930	0.5659071	0.8679227	0.8692791
Major Pore Diameter: 10 μm				
1000	0.5563930	0.5565200	0.7648391	0.7648820
1500	0.5563929	0.5570039	0.8425101	0.8426208
2000	0.5563933	0.5580662	0.8819798	0.8821818
2500	0.5563932	0.5592832	0.8844863	0.8848257
3000	0.5563926	0.5602437	0.8679229	0.8684681
Major Pore Diameter: 1 μm				
1000	0.5563922	0.5564047	0.7648384	0.7648427
1500	0.5563914	0.5564522	0.8425073	0.8425187
2000	0.5563910	0.5565584	0.8819757	0.8819957
2500	0.5563889	0.5566797	0.8844796	0.8845137
3000	0.5563894	0.5567774	0.8679175	0.8679721

APPENDIX D
PREFP FORTRAN LISTING

PROGRAM PREFP

Steven L. Hayes
 Texas A&M University
 Department of Nuclear Engineering
 College Station, TX 77843-3133

This program is a preprocessor for the thermal conductivity porosity correction factor files used by the routine INTERPOLATE, which has also been integrated into the temperature program TEMP. PREFP reads an input file that contains pore size distribution information, and then integrates that pore size distribution with the porosity correction factor, f, data to obtain f* data that is not dependent upon pore size. The integration procedure employed is shown below:

$$f^*(g, P, T) = \frac{\int_{l=0}^{l=lmax} f(g, P, l, T) p(l) dl}{\int_{l=0}^{l=lmax} p(l) dl}$$

where

f* = porosity correction factor without dependence on pore diameter.
 f = porosity correction factor with dependence on pore diameter.
 g = pore shape factor; diameter perp. to heat flow divided by diameter parallel to heat flow.
 P = volume fraction porosity.
 T = mean pore temperature.
 l = pore diameter perpendicular to heat flow direction.
 p(l) dl = fraction of pores having diameters between l and l+dl.

The integrations are performed numerically using Simpson's extended trapezoid rule.

```

IMPLICIT REAL*8 (A-H,O-Z)
PARAMETER (MAXDAT=2000,MAXBIN=50,MAXOUT=50)
COMMON/FPDATA/ FP0 (2,MAXDAT),G0 (MAXDAT),P0 (MAXDAT),D0 (MAXDAT),
+              T0 (MAXDAT),N0
REAL*8 GOUT (MAXOUT),PMIN (MAXOUT),PMAX (MAXOUT),TMIN (MAXOUT),
+      TMAX (MAXOUT)

```

```

REAL*8 PROB(MAXBIN),BIN(MAXBIN,2)
C
C Request the database to be employed:
WRITE (*,*) 'Which database to be collapsed?'
WRITE (*,*) 'UO2, vacuum-filled pores; (ENTER 1)'
WRITE (*,*) 'UO2, Xe/Kr-filled pores; (ENTER 2)'
WRITE (*,*) 'UO2, He-filled pores; (ENTER 3)'
READ (*,*) IDATA
IF (IDATA.NE.1.AND.IDATA.NE.2.AND.IDATA.NE.3) THEN
    WRITE (*,*) 'DATABASE REQUESTED NOT AVAILABLE!'
    GOTO 599
ENDIF
IF (IDATA.EQ.1) OPEN (UNIT=10,FILE='FP_VACUUM.TABL',STATUS='OLD')
IF (IDATA.EQ.2) OPEN (UNIT=10,FILE='FP_XE-KR.TABL',STATUS='OLD')
IF (IDATA.EQ.3) OPEN (UNIT=10,FILE='FP_HE.TABL',STATUS='OLD')
C
C Read in the f(g,P,l,T) data from external file:
DO I=1,MAXDAT
    READ (10,*,END=26) FP0(1,I),FP0(2,I),G0(I),P0(I),D0(I),T0(I)
END DO
26 N0=I-1
CLOSE (10)
C
C Read in the pore size distribution information:
OPEN (UNIT=13,FILE='PSD.DAT',STATUS='OLD')
DO I=1,MAXBIN
    READ (13,*,END=29) BIN(I,1),BIN(I,2),PROB(I)
END DO
29 NBINS=I-1
CLOSE (13)
C
C Read information on ranges of output file to be generated:
OPEN (UNIT=12,FILE='OUT.FORMAT',STATUS='OLD')
READ (12,*) IRAD
DO I=1,MAXDAT
    READ (12,*,END=39) GOUT(I),PMIN(I),PMAX(I),TMIN(I),TMAX(I)
END DO
39 NG=I-1
CLOSE (12)
C
C Set up some constant value parameters:
DELP=0.5000000D-01
DELT=0.5000000D+03
BUMP=1.0001
NINT=20
C
C Begin generating output data; perform integrations as needed:
OPEN (UNIT=11,FILE='PREFP.OUT',STATUS='NEW')
1000 FORMAT(X,E14.7,2X,E14.7,2X,E14.7,2X,E14.7)
DO 50 IG=1,NG
    G=GOUT(IG)
    IPMAX=IDINT(BUMP*(PMAX(IG)-PMIN(IG))/DELP)+1
    DO 50 IP=1,IPMAX
        P=PMIN(IG)+DELP*REAL(IP-1)
        ITMAX=IDINT(BUMP*(TMAX(IG)-TMIN(IG))/DELT)+1
    
```

```

DO 50 IT=1,ITMAX
  T=TMIN(IG)+DELT*REAL(IT-1)
  SUMINT=0.0
  DO 49 J=1,NBINS
    DEN=BIN(J,2)-BIN(J,1)
    H=(BIN(J,2)-BIN(J,1))/(REAL(NINT-1))
    TOP=0.0
    DO K=1,NINT
      D=BIN(J,1)+H*(REAL(K-1))
      IF (K.EQ.1.OR.K.EQ.NINT) THEN
        TOP=TOP+0.5*H*PCF(IRAD,G,P,D,T)*PROB(J)
      ELSE
        TOP=TOP+H*PCF(IRAD,G,P,D,T)*PROB(J)
      ENDIF
    END DO
    SUMINT=SUMINT+TOP/DEN
49  CONTINUE
    WRITE (11,1000) SUMINT,G,P,T
50  CONTINUE
    CLOSE (11)
C
599 CONTINUE
    STOP
    END
C -----
    FUNCTION PCF(IRAD,GI,PI,DI,TI)
C -----
C   This subroutine returns interpolated values of the porosity
C   correction factor for the thermal conductivity of UO2 using user-
C   supplied external data tables.
C
    IMPLICIT REAL*8(A-H,O-Z)
    PARAMETER (MAXDAT=2000,MAXBIN=50,MAXOUT=50)
    COMMON/FPDATA/ FP0(2,MAXDAT),G0(MAXDAT),P0(MAXDAT),D0(MAXDAT),
+      T0(MAXDAT),N0
    REAL*8 FP(2,MAXDAT),G(MAXDAT),P(MAXDAT),D(MAXDAT),T(MAXDAT)
    REAL*8 YA1(5),YA2(5),XA(5)
C
C   Load original table into working arrays:
    DO I=1,N0
      FP(1,I)=FP0(1,I)
      FP(2,I)=FP0(2,I)
      G(I)=G0(I)
      P(I)=P0(I)
      D(I)=D0(I)
      T(I)=T0(I)
    END DO
C
C   Begin the interpolation and reduction procedure;
C   sort first on the pore shape factor:
    IGEXACT=0
    DO I=1,N0
      IF (IGEXACT.EQ.1) GOTO 5
      IF (IGEXACT.EQ.2) GOTO 6
      IF (GI.EQ.G(I)) THEN

```

```

        IGLO=I
        IGEXACT=1
    ENDIF
    IF (IGEXACT.EQ.0) GOTO 7
5    IF (GI.NE.G(I)) THEN
        IGHI=I-1
        IGEXACT=2
    ENDIF
7    CONTINUE
END DO
IGLO=1
GLO=G(1)
IGHI=N0
GHI=G(N0)
DO 20 I=1,N0
    IF (G(I).GT.GLO.AND.G(I).LE.GI) THEN
        IGLO=I
        GLO=G(I)
    ENDIF
    IF (G(I).LE.GHI.AND.G(I).GT.GI) THEN
        IGH I=I
        GHI=G(I)
    ENDIF
20 CONTINUE
6    J=0
    DO I=IGLO,IGHI
        J=J+1
        FP(1,J)=FP(1,I)
        FP(2,J)=FP(2,I)
        G(J)=G(I)
        P(J)=P(I)
        D(J)=D(I)
        T(J)=T(I)
    END DO
    N=J
C
C    Now interpolate on the pore shape factor to reduce table to a
C    single shape factor table:
    IF (IGEXACT.EQ.0) THEN
        K=0
        GSTART=G(1)
        DO 22 I=1,N
            IF (G(I).NE.GSTART) GOTO 25
            DO 23 J=I+1,N
                IF (P(I).EQ.P(J).AND.D(I).EQ.D(J).AND.T(I).EQ.T(J)) THEN
                    K=K+1
                    FACTOR=(GI-G(I))/(G(J)-G(I))
                    FP(1,K)=FP(1,I)+FACTOR*(FP(1,J)-FP(1,I))
                    FP(2,K)=FP(2,I)+FACTOR*(FP(2,J)-FP(2,I))
                    G(K)=GI
                    P(K)=P(I)
                    D(K)=D(I)
                    T(K)=T(I)
                ENDIF
            ENDIF
23 CONTINUE

```

```

22  CONTINUE
25  N=K
    ENDIF
C
C  Sort next on the volume fraction porosity:
    PEXTRAP=PI
    IF (GI.EQ.1.AND.PI.GT.0.50) PI=0.49
    IF (GI.EQ.2.AND.PI.GT.0.30) PI=0.29
    IF (GI.EQ.3.AND.PI.GT.0.20) PI=0.19
    IF (GI.EQ.4.AND.PI.GT.0.15) PI=0.14
    IF (GI.EQ.5.AND.PI.GT.0.10) PI=0.09
    IF (GI.EQ.6.AND.PI.GT.0.10) PI=0.09
    IF (GI.GT.6.AND.PI.GT.0.05) PI=0.04
    IPEXACT=0
    DO I=1,N
        IF (IPEXACT.EQ.1) GOTO 15
        IF (IPEXACT.EQ.2) GOTO 16
        IF (PI.EQ.P(I)) THEN
            IPLO=I
            IPEXACT=1
        ENDIF
        IF (IPEXACT.EQ.0) GOTO 17
15      IF (PI.NE.P(I)) THEN
            IPHI=I-1
            IPEXACT=2
        ENDIF
17      CONTINUE
    END DO
    IPLO=1
    PLO=P(1)
    IPHI=N
    PHI=P(N)
    DO 30 I=1,N
        IF (P(I).GT.PLO.AND.P(I).LE.PI) THEN
            IPLO=I
            PLO=P(I)
        ENDIF
        IF (P(I).LE.PHI.AND.P(I).GT.PI) THEN
            IPHI=I
            PHI=P(I)
        ENDIF
30    CONTINUE
16    J=0
    DO I=IPLO,IPHI
        J=J+1
        FP(1,J)=FP(1,I)
        FP(2,J)=FP(2,I)
        G(J)=G(I)
        P(J)=P(I)
        D(J)=D(I)
        T(J)=T(I)
    END DO
    N=J
C
C  Now interpolate on the volume fraction porosity to reduce table

```

```

C      to a single porosity table:
      IF (IPEXACT.EQ.0) THEN
        K=0
        PSTART=P(1)
        DO 32 I=1,N
          IF (P(I).NE.PSTART) GOTO 35
          DO 33 J=I+1,N
            IF (D(I).EQ.D(J).AND.T(I).EQ.T(J)) THEN
              K=K+1
              YA1(1)=FP(1,I)
              YA1(2)=FP(1,J)
              YA2(1)=FP(2,I)
              YA2(2)=FP(2,J)
              XA(1)=P(I)
              XA(2)=P(J)
              CALL POLINT(XA,YA1,2,PEXTRAP,FP(1,K))
              CALL POLINT(XA,YA2,2,PEXTRAP,FP(2,K))
              G(K)=GI
              P(K)=PEXTRAP
              D(K)=D(I)
              T(K)=T(I)
            ENDIF
          33      CONTINUE
        32      CONTINUE
        N=K
      ENDIF

C
C      Sort next on the pore diameter perpendicular to heat flow:
      DEXTRAP=DI
      IF (DI.LT.1.0) DI=2.0
      IF (GI.EQ.1.0.AND.DI.GT.200.0) DI=190.0
      IF (GI.GT.1.0.AND.DI.GT.50.0) DI=40.0
      IDEXACT=0
      DO I=1,N
        IF (IDEXACT.EQ.1) GOTO 45
        IF (IDEXACT.EQ.2) GOTO 46
        IF (DI.EQ.D(I)) THEN
          IDLO=I
          IDEXACT=1
        ENDIF
        IF (IDEXACT.EQ.0) GOTO 47
45      IF (DI.NE.D(I)) THEN
          IDHI=I-1
          IDEXACT=2
        ENDIF
47      CONTINUE
      END DO
      IDLO=1
      DLO=D(1)
      IDHI=N
      DHI=D(N)
      DO 50 I=1,N
        IF (D(I).LT.DLO.AND.D(I).GE.DI) THEN
          IDLO=I
          DLO=D(I)

```

```

      ENDIF
      IF (D(I).GE.DHI.AND.D(I).LT.DI) THEN
        IDHI=I
        DHI=D(I)
      ENDIF
50  CONTINUE
46  J=0
      DO I=IDLO,IDHI
        J=J+1
        FP(1,J)=FP(1,I)
        FP(2,J)=FP(2,I)
        G(J)=G(I)
        P(J)=P(I)
        D(J)=D(I)
        T(J)=T(I)
      END DO
      N=J

C
C    Now interpolate on the pore diameter to reduce table to a single
C    diameter table:
      IF (IDEXACT.EQ.0) THEN
        K=0
        DSTART=D(1)
        DO 62 I=1,N
          IF (D(I).NE.DSTART) GOTO 65
          DO 63 J=I+1,N
            IF (T(I).EQ.T(J)) THEN
              K=K+1
              YA1(1)=FP(1,I)
              YA1(2)=FP(1,J)
              YA2(1)=FP(2,I)
              YA2(2)=FP(2,J)
              XA(1)=D(I)
              XA(2)=D(J)
              CALL POLINT(XA,YA1,2,DEXTRAP,FP(1,K))
              CALL POLINT(XA,YA2,2,DEXTRAP,FP(2,K))
              G(K)=GI
              P(K)=PEXTRAP
              D(K)=DEXTRAP
              T(K)=T(I)
            ENDIF
          END DO
        END DO
63  CONTINUE
62  CONTINUE
65  N=K
      ENDIF

C
C    Finally, sort lastly on temperature:
      ITLO=1
      TLO=T(1)
      DO 80 I=1,N
        IF (T(I).GT.TLO.AND.T(I).LE.TI) THEN
          ITLO=I
          TLO=T(I)
        ENDIF
      DO 80 CONTINUE

```



```

      IF (ITLO.EQ.1) THEN
        ITHI=3
      ELSEIF (ITLO.GT.N-2) THEN
        ITLO=N-2
        ITHI=N
      ELSE
        ITHI=ITLO+2
      ENDIF
      J=0
      DO I=ITLO, ITHI
        J=J+1
        YA1(J)=FP(1,I)
        YA2(J)=FP(2,I)
        XA(J)=T(I)
      END DO
      N=J
      CALL POLINT(XA,YA1,N,TI,PCF1)
      CALL POLINT(XA,YA2,N,TI,PCF2)
C
C   Return interpolated f(p) values to the main program:
      IF (IRAD.EQ.0) PCF=PCF1
      IF (IRAD.EQ.1) PCF=PCF2
      RETURN
      END
C -----
      SUBROUTINE POLINT(XA,YA,N,X,Y)
C -----
C   This subroutine taken from Numerical Recipes (1986).
C   Given arrays XA and YA, each of length N, and given a value X,
C   this routine returns a value Y. If P(x) is the polynomial of
C   degree N-1 such that P(XAi)=YAi, i=1,...,N, the the returned
C   value Y=P(X).
C
      IMPLICIT REAL*8 (A-H,O-Z)
      PARAMETER (NMAX=10)
      REAL*8 XA(N),YA(N),C(NMAX),D(NMAX)
C
      NS=1
      DIF=DABS(X-XA(1))
      DO 11 I=1,N
        DIFT=DABS(X-XA(I))
        IF (DIFT.LT.DIF) THEN
          NS=I
          DIF=DIFT
        ENDIF
        C(I)=YA(I)
        D(I)=YA(I)
11    CONTINUE
      Y=YA(NS)
      NS=NS-1
      DO 13 M=1,N-1
        DO 12 I=1,N-M
          HO=XA(I)-X
          HP=XA(I+M)-X
          W=C(I+1)-D(I)

```

```
DEN=HO-HP
IF (DEN.EQ.0.) PAUSE
DEN=W/DEN
D(I)=HP*DEN
C(I)=HO*DEN
12  CONTINUE
    IF (2*NS.LT.N-M) THEN
        DY=C(NS+1)
    ELSE
        DY=D(NS)
        NS=NS-1
    ENDIF
    Y=Y+DY
13  CONTINUE
    RETURN
    END
```

APPENDIX E
TEMP FORTRAN LISTING

PROGRAM TEMP

PROGRAM DESCRIPTION:

This simple program computes the temperature profile in a solid, cylindrical fuel pellet given the linear heat rate of the element and the fuel surface temperature using a 1-D finite difference approximation. Any thermal conductivity correlation can be employed via an external function, COND. Additionally, a variety of options are available for the porosity correction factor used with the thermal conductivity correlation.

A beginning of life (BOL) temperature calculation is always performed. Additionally, the temperature calculation may be performed following densification effects if requested. This computation is done assuming the fuel densifies into three different radial zones: an as-fabricated zone near the fuel surface, a columnar grained zone near the fuel center, and an equiaxed grained zone in between; a central void is also assumed to form.

DISCRETIZATION MODEL:

VARIABLE LIST:

A.....Coefficient array for the temperature calculations.
 B.....Force vector for the temperature calculations.
 D1.....Fraction of theoretical density in columnar grained zone.
 D2.....Fraction of theoretical density in equiaxed grained zone.
 D3.....Fraction of theoretical density in as-fabricated zone.
 DELR.....Node separation distance, or control-volume width (m).
 DIFF.....Current percent change in nodal temperatures since last iteration.
 EIK.....Interface thermal conductivity b/t the $i+1$ and i control volumes (W/m-K).
 EK.....Thermal conductivity of the $i+1$ control volume (W/m-

```

C          K).
C  ERROR.....Percent change in temperatures at which convergence
C             is met.
C  FRADIUS.....Fuel radius (m).
C  G1.....Pore diameter ratio in columnar grained zone.
C  G2.....Pore diameter ratio in equiaxed grained zone.
C  G3.....Pore diameter ratio in as-fabricated zone.
C  H1.....Volumetric heat generation rate in columnar grained
C             zone (W/m^3).
C  H2.....Volumetric heat generation rate in equiaxed grained
C             zone (W/m^3).
C  H3.....Volumetric heat generation rate in as-fabricated
C             zone (W/m^3).
C  HGR.....Linear heat generation rate (W/m).
C  IDENS.....Flag specifying whether to compute temperature
C             profile following fuel densification (0=NO, 1=YES).
C  ITER.....Iteration cycle.
C  KPOR.....Specifier for the porosity correction factor to
C             employ.
C  KTH.....Specifier for the thermal conductivity correlation
C             to employ, used by external function COND.
C  NN.....Number of nodes employed.
C  P1.....Porosity in columnar grained zone.
C  P2.....Porosity in equiaxed grained zone.
C  P3.....Same as as-fabricated porosity.
C  PFAB.....Fuel as fabricated volume fraction porosity.
C  PK.....Thermal conductivity of the i-th control volume
C             (W/m-K).
C  R1.....Radius of onset of columnar grained zone (m).
C  R2.....Radius of onset of equiaxed grained zone (m).
C  RE.....Radial distance from center for the interface
C             between the i and i+1 control volumes (m).
C  RVOID.....Radius of central void (m).
C  RW.....Radial distance from center for the interface
C             between the i and i-1 control volumes (m).
C  T.....Array holding the nodal temperatures (K).
C  T1.....Onset temperature for columnar grained zone (K).
C  T2.....Onset temperature for equiaxed grained zone (K).
C  TOLD.....Array holding nodal temperatures from previous
C             iteration cycle (K).
C  TSURF.....Fuel surface temperature (K).
C  WIK.....Interface thermal conductivity b/t the i-1 and i
C             control volumes (W/m-K).
C  WK.....Thermal conductivity of the i-1 control volume (W/m-
C             K).
C *****
C

```

```

IMPLICIT REAL*8(A-H,O-Z)
PARAMETER (NN=110,ERROR=1.0D-6,ITMAX=20)
PARAMETER (MAXDAT=2000,IN=11)
COMMON/INTERP/ IREAD
COMMON/FPDATA/ FP0(MAXDAT),GO(MAXDAT),PO(MAXDAT),TO(MAXDAT),NO
EXTERNAL COND
REAL*8 T(110),TOLD(110),A(110,3),B(110)

```

```

OPEN (UNIT=10,FILE='TEMP.OUT',STATUS='NEW')
OPEN (UNIT=11,FILE='TEMP.DAT',STATUS='OLD')
C
C   Read input data; units of data as read in are as follows:
C   FRADIUS in mm
C   PFAB in volume percent (unitless)
C   TSURF in K
C   HGR in W/cm
  READ (11,*) KTH,KPOR,IDENS
  IF (KPOR.EQ.6) THEN
    READ (11,*) G3
    IF (IDENS.EQ.1) THEN
      READ (11,*) G1
      READ (11,*) G2
    ENDIF
  ENDIF
  READ (11,*) TSURF,HGR
  READ (11,*) FRADIUS,PFAB
  IF (IDENS.EQ.1) THEN
    READ (11,*) T1,P1
    READ (11,*) T2,P2
  ENDIF
  CLOSE (11)
C
C   Unit conversions to SI as specified in variable descriptions:
  FRADIUS=FRADIUS/1000.0
  HGR=HGR*100.0
C
C   Geometry calculations:
  DELR=FRADIUS/(REAL(NN-1))
  P3=PFAB
C
C   Initialize nodal temperatures:
  IREAD=-1
  DO I=1,NN
    T(I)=TSURF
  END DO
C
C   Volumetric heat generation rate calculation:
  PI=3.141592654
  H3=HGR/(PI*FRADIUS*FRADIUS)
C
C   Begin iteration loop to perform the BOL calculations:
  WRITE (*,*) 'BEGINNING BOL CALCULATION:'
  DO 100 ITER=0,ITMAX
    WRITE (*,*) ITER
C
C   Fill the arrays for the temperature calculation:
    I=1
    A(I,1)=0.0
    A(I,2)=1.0
    A(I,3)=-1.0
    B(I)=0.0
C
    DO I=2,NN-1

```

```

      RE=0.5*DELR+DELR*(REAL(I-1))
      RW=RE-DELR
      EK=PCF(KPOR,P3,T(I),G3)*COND(KTH,T(I+1))
      WK=PCF(KPOR,P3,T(I),G3)*COND(KTH,T(I-1))
      PK=PCF(KPOR,P3,T(I),G3)*COND(KTH,T(I))
      EIK=2.0*EK*PK/(EK+PK)
      WIK=2.0*WK*PK/(WK+PK)
      A(I,1)=WIK*RW/DELR
      A(I,2)=-(EIK*RE+WIK*RW)/DELR
      A(I,3)=EIK*RE/DELR
      B(I)=H3*(RW*RW-RE*RE)/2.0
END DO
C
      I=NN
      A(I,1)=0.0
      A(I,2)=1.0
      A(I,3)=0.0
      B(I)=TSURF
C
C      Call for temperature solution:
      CALL TDMA(NN,A,B)
      DO I=1,NN
        T(I)=B(I)
      END DO
C
C      Check for convergence of the temperatures:
      IF (ITER.GT.0) THEN
        DIFFTOP=0.0
        DIFFBOT=0.0
        DO M=1,NN
          DIFFTOP=DIFFTOP+DABS(T(M)-TOLD(M))
          DIFFBOT=DIFFBOT+DABS(T(M))
        END DO
        DIFF=DIFFTOP/DIFFBOT
        IF (DIFF.LT.ERROR) GOTO 101
      ENDIF
      DO M=1,NN
        TOLD(M)=T(M)
      END DO
C
100 CONTINUE
101 CONTINUE
C
C      Output the computation data and results of the BOL calculations:
      WRITE (10,1000)
      WRITE (10,1002) FRADIUS*1000.0,P3
      IF (KPOR.EQ.6) THEN
        WRITE (10,1003) G3
      ENDIF
      WRITE (10,1004) TSURF,HGR/100.0
      WRITE (10,1006) KTH,KPOR
      WRITE (10,1008) ERROR,ITER
      WRITE (10,1010) T(1),T(11),T(22),T(33),T(44),T(55),T(66),T(77),
+      T(88),T(99),T(110)
C

```

```

C * * If requested, compute the temperature profile after densification
C   has occurred:
C     IF (IDENS.EQ.1) THEN
C
C       Initial calculations:
C       P3=PFAB
C       D1=1.0-P1
C       D2=1.0-P2
C       D3=1.0-P3
C
C       Volumetric heat generation rate calculation:
C       H3=HGR/(PI*FRADIUS*FRADIUS)
C       H2=H3*D2/D3
C       H1=H3*D1/D3
C
C       Begin iteration loop:
C       WRITE (*,*) 'BEGINNING EOL CALCULATION:'
C       DO 200 ITER=0,ITMAX
C         WRITE (*,*) ITER
C
C       Revise geometry:
C       DELR=(FRADIUS-RVOID)/(REAL(NN-1))
C
C       Fill the arrays for the temperature calculation:
C       I=1
C       A(I,1)=0.0
C       A(I,2)=1.0
C       A(I,3)=-1.0
C       B(I)=0.0
C
C       DO I=2,NN-1
C         RE=RVOID+0.5*DELR+DELR*(REAL(I-1))
C         RW=RE-DELR
C         IF (T(I).LT.T2) THEN
C           P=P3
C           H=H3
C           G=G3
C         ELSEIF (T(I).GT.T1) THEN
C           P=P1
C           H=H1
C           G=G1
C         ELSE
C           P=P2
C           H=H2
C           G=G2
C         ENDIF
C         EK=PCF(KPOR,P,T(I),G)*COND(KTH,T(I+1))
C         WK=PCF(KPOR,P,T(I),G)*COND(KTH,T(I-1))
C         PK=PCF(KPOR,P,T(I),G)*COND(KTH,T(I))
C         EIK=2.0*EK*PK/(EK+PK)
C         WIK=2.0*WK*PK/(WK+PK)
C         A(I,1)=WIK*RW/DELR
C         A(I,2)=-(EIK*RE+WIK*RW)/DELR
C         A(I,3)=EIK*RE/DELR
C         B(I)=H*(RW*RW-RE*RE)/2.0

```



```

      END DO
C
      I=NN
      A(I,1)=0.0
      A(I,2)=1.0
      A(I,3)=0.0
      B(I)=TSURF
C
C      Call for temperature solution:
      CALL TDMA(NN,A,B)
      DO I=NN-1,1,-1
        T(I)=B(I)
        IF (T2.LE.T(I).AND.T2.GE.T(I+1)) THEN
          RE=RVOID+0.5*DELR+DELR*(REAL(I-1))
          RW=RE-DELR
          R2=0.5*(RE+RW)
        ENDIF
        IF (T1.LE.T(I).AND.T1.GE.T(I+1)) THEN
          RE=RVOID+0.5*DELR+DELR*(REAL(I-1))
          RW=RE-DELR
          R1=0.5*(RE+RW)
        ENDIF
      END DO
C
C      Determine the central void position:
      RVOID=DSQRT((D1-D2)*R1*R1/D1+(D2-D3)*R2*R2/D1)
C
C      Check for convergence of the temperatures:
      IF (ITER.GT.0) THEN
        DIFFTOP=0.0
        DIFFBOT=0.0
        DO M=1,NN
          DIFFTOP=DIFFTOP+DABS(T(M)-TOLD(M))
          DIFFBOT=DIFFBOT+DABS(T(M))
        END DO
        DIFF=DIFFTOP/DIFFBOT
        IF (DIFF.LT.ERROR) GOTO 201
      ENDIF
      DO M=1,NN
        TOLD(M)=T(M)
      END DO
C
200 CONTINUE
201 CONTINUE
      IF (ITER.EQ.ITMAX+1) THEN
        WRITE (*,*) 'TEMPERATURES FAILED TO CONVERGE!'
        WRITE (*,*) 'DIFF = ', DIFF
      ENDIF
C
C      Output results:
      WRITE (10,2008)
      WRITE (10,2007) ITER
      WRITE (10,2015) T1,P1
      IF (KPOR.EQ.6) WRITE (10,2016) G1
      WRITE (10,2017) T2,P2

```

```

IF (KPOR.EQ.6) WRITE (10,2018) G2
WRITE (10,2009) RVOID*1000.,R1*1000.,R2*1000.
WRITE (10,2010)
DO I=1,11
  IF (I.EQ.1) THEN
    INDEX=1
    RELR=RVOID/FRADIUS
  ELSEIF (I.EQ.11) THEN
    INDEX=110
    RELR=1.0
  ELSE
    INDEX=(I-1)*11
    RELR=(RVOID+DELR*(REAL(INDEX-1)))/FRADIUS
  ENDIF
  WRITE (10,2011) RELR,T(INDEX)
END DO
ENDIF

```

C

```

1000 FORMAT(X,'***** TEMPERATURE PROFILE CALCULATION *****',/,/)
1002 FORMAT(X,'FUEL:',/,
+           X,'Fuel Radius              = ',D14.7,' mm',/,
+           X,'As-Fabricated Porosity   = ',D14.7)
1003 FORMAT(X,'As-Fab. Pore Shape Factor = ',D14.7,/)
1004 FORMAT(X,'BOUNDARY CCNDITIONS:',/,
+           X,'Fuel Surface Temperature = ',D14.7,' K',/,
+           X,'Linear Heat Generation Rate = ',D14.7,' W/cm',/)
1006 FORMAT(X,'CORRELATIONS:',/,
+           X,'Thermal Conductivity     => ',I2,/,
+           X,'Porosity Correction Factor => ',I2)
1008 FORMAT(/,X,'CONVERGENCE INFORMATION:',/,
+           X,'Error Criterion           = ',D14.7,/,/,/,
+           X,'----- TEMPERATURE PROFILE AT STARTUP -----',/,/,
+           X,'Number of Iterations Reqd = ',I14,/)
1010 FORMAT(X,'r/R      Temp. (K)',/,
+           X,'0.0      ',D14.7,/,
+           X,'0.1      ',D14.7,/,
+           X,'0.2      ',D14.7,/,
+           X,'0.3      ',D14.7,/,
+           X,'0.4      ',D14.7,/,
+           X,'0.5      ',D14.7,/,
+           X,'0.6      ',D14.7,/,
+           X,'0.7      ',D14.7,/,
+           X,'0.8      ',D14.7,/,
+           X,'0.9      ',D14.7,/,
+           X,'1.0      ',D14.7)
2007 FORMAT(X,'Number of Iterations Reqd = ',I7,/)
2008 FORMAT(/,/,X,'----- TEMPERATURE PROFILE AFTER DENSIFICATION'
+           ' -----',/,)
2009 FORMAT(/,X,'Radius of Central Void   = ',D14.7,' mm',/,
+           X,'Radius of Columnar Grains = ',D14.7,' mm',/,
+           X,'Radius of Equiaxed Grains = ',D14.7,' mm',/)
2010 FORMAT(X,'r/R      Temp. (K)')
2011 FORMAT(X,F7.5,4X,D14.7)
2015 FORMAT(X,'DENSIFICATION MODEL:',/,
+           X,'Onset Temp. of Columnar Zone = ',D14.7,' K',/,

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```

+      X,'Porosity in Columnar Zone           = ',D14.7)
2016 FORMAT(X,'Pore Shape Factor in Columnar Zone = ',D14.7)
2017 FORMAT(X,'Onset Temp. of Equiaxed Zone     = ',D14.7,' K',/,
+      X,'Porosity in Equiaxed Zone           = ',D14.7)
2018 FORMAT(X,'Pore Shape Factor in Equiaxed Zone = ',D14.7)
      STOP
      END
C -----
      FUNCTION PCF(KPOR,PI,TI,GI)
C -----
C      This subroutine computes the porosity correction factor for the
C      thermal
C      conductivity from any of a variety of porosity correction factor
C      formulas.
C
      IMPLICIT REAL*8(A-H,O-Z)
      PARAMETER (MAXDAT=2000,IN=11)
      COMMON/INTERP/ IREAD
      COMMON/FPDATA/ FP0(MAXDAT),G0(MAXDAT),P0(MAXDAT),T0(MAXDAT),N0
C
C      No correction for porosity:
      IF (KPOR.EQ.1) THEN
         PCF=1.0
      ENDIF
C
C      Generic, material non-specific correction factors:
      IF (KPOR.EQ.2) PCF=1.0-PI**(2.0/3.0)
      IF (KPOR.EQ.3) PCF=DEXP(-2.14*PI)
      IF (KPOR.EQ.4) PCF=(1.0-PI)/(1.0+PI)
      IF (KPOR.EQ.5) PCF=1.0-1.208993965*(PI**(2.0/3.0))
C
C      Interpolate from FE generated PCFs for UO2:
      IF (KPOR.EQ.6) THEN
         CALL INTERPOLATE(GI,PI,TI,PCF)
      ENDIF
C
C      MATPRO PFC for UO2:
      IF (KPOR.EQ.7) THEN
         BETA=6.5-4.69D-03*TI
         IF (TI.GE.1364.647) BETA=15.811308-TI*(.01833647-TI*5.0D-06)
         IF (TI.GT.1833.647) BETA=-1
         PCF=(1-PI)/(1+BETA*PI)
      ENDIF
C
      RETURN
      END
C -----
      SUBROUTINE INTERPOLATE(GI,PI,TI,PCF)
C -----
C
C      Steven L. Hayes
C      Texas A&M University
C      Department of Nuclear Engineering
C      College Station, TX 77843-3133
C      (409) 845-8101

```

```

C
C -----
C   This simple program interpolates values of the porosity
C   correction factor for the thermal conductivity of UO2 from tables
C   organized in the following way:
C
C   COLUMN 1: f(p) without pore radiative transfer.
C   COLUMN 2: f(p) including pore radiative transfer.
C   COLUMN 3: Ratio of pore diameter perp. to heat flow direction
C             to pore diameter parallel to heat flow direction;
C             values should be ordered so that the ratios
C             increase monotonically from top to bottom.
C   COLUMN 4: Volume fraction porosity; values should be ordered
C             so that for constant pore diameter ratio, the
C             porosity values increase monotonically from top to
C             bottom.
C   COLUMN 6: Temperature (K); values should be ordered so that
C             for constant porosity, temperatures increase
C             monotonically from top to bottom.
C
C   Polynomial interpolation is employed; within each independent
C   variable, the following order is used:
C
C   Independent Variable      Interpolation order
C   -----
C   Pore shape factor        1
C   Volume fraction porosity  1
C   Pore diameter            1
C   Temperature              2
C
C -----
C   IMPLICIT REAL*8 (A-H,O-Z)
C   PARAMETER (MAXDAT=2000,IN=11)
C   COMMON/INTERP/ IREAD
C   COMMON/FPDATA/ FP0 (MAXDAT),G0 (MAXDAT),P0 (MAXDAT),T0 (MAXDAT),N0
C   REAL*8 FP (MAXDAT),G (MAXDAT),P (MAXDAT),T (MAXDAT)
C   REAL*8 YA (5),XA (5)
C
C   IREAD=IREAD+1
C   IF (IREAD.EQ.0) THEN
C     OPEN (UNIT=IN,FILE='TEMP_FP.TABL',STATUS='OLD')
C     DO 10 I=1,MAXDAT
C       READ (IN,*,END=11) FP0(I),G0(I),P0(I),T0(I)
10    CONTINUE
11    N0=I-1
C     CLOSE (IN)
C   ENDIF
C
C   Begin the interpolation and reduction procedure; sort first on
C   the pore shape factor:
C   IGEXACT=0
C   DO I=1,N0
C     IF (IGEXACT.EQ.1) GOTO 5
C     IF (IGEXACT.EQ.2) GOTO 6
C     IF (GI.EQ.G0(I)) THEN

```

```

        IGLO=I
        IGEXACT=1
    ENDIF
    IF (IGEXACT.EQ.0) GOTO 7
5    IF (GI.NE.G0(I)) THEN
        IGHI=I-1
        IGEXACT=2
    ENDIF
7    CONTINUE
END DO
IGLO=1
GLO=G0(1)
IGHI=N0
GHI=G0(N0)
DO 20 I=1,N0
    IF (G0(I).GT.GLO.AND.G0(I).LE.GI) THEN
        IGLO=I
        GLO=G0(I)
    ENDIF
    IF (G0(I).LE.GHI.AND.G0(I).GT.GI) THEN
        IGH I=I
        GHI=G0(I)
    ENDIF
20 CONTINUE
6    J=0
    DO I=IGLO,IGHI
        J=J+1
        FP(J)=FP0(I)
        G(J)=G0(I)
        P(J)=P0(I)
        T(J)=T0(I)
    END DO
    N=J
C
C    Now interpolate on the pore shape factor to reduce table to a
C    single shape factor table:
    IF (IGEXACT.EQ.0) THEN
        K=0
        GSTART=G(1)
        DO 22 I=1,N
            IF (G(I).NE.GSTART) GOTO 25
            DO 23 J=I+1,N
                IF (P(I).EQ.P(J).AND.T(I).EQ.T(J)) THEN
                    K=K+1
                    FACTOR=(GI-G(I))/(G(J)-G(I))
                    FP(K)=FP(I)+FACTOR*(FP(J)-FP(I))
                    G(K)=GI
                    P(K)=P(I)
                    T(K)=T(I)
                ENDIF
            ENDIF
23 CONTINUE
22 CONTINUE
25 N=K
    ENDIF
C

```

```

C      Sort next on the volume fraction porosity:
      PEXTRAP=PI
      IF (GI.EQ.1.0.AND.PI.GT.0.50) PI=0.49
      IF (GI.EQ.2.0.AND.PI.GT.0.30) PI=0.29
      IF (GI.EQ.3.0.AND.PI.GT.0.20) PI=0.19
      IF (GI.EQ.4.0.AND.PI.GT.0.15) PI=0.14
      IF (GI.EQ.5.0.AND.PI.GT.0.10) PI=0.09
      IF (GI.EQ.6.0.AND.PI.GT.0.10) PI=0.09
      IF (GI.GT.6.0.AND.PI.GT.0.05) PI=0.04
      IPEXACT=0
      DO I=1,N
        IF (IPEXACT.EQ.1) GOTO 15
        IF (IPEXACT.EQ.2) GOTO 16
        IF (PI.EQ.P(I)) THEN
          IPLO=I
          IPEXACT=1
        ENDIF
        IF (IPEXACT.EQ.0) GOTO 17
15      IF (PI.NE.P(I)) THEN
          IPHI=I-1
          IPEXACT=2
        ENDIF
17      CONTINUE
      END DO
      IPLO=1
      PLO=P(1)
      IPHI=N
      PHI=P(N)
      DO 30 I=1,N
        IF (P(I).GT.PLO.AND.P(I).LE.PI) THEN
          IPLO=I
          PLO=P(I)
        ENDIF
        IF (P(I).LE.PHI.AND.P(I).GT.PI) THEN
          IPHI=I
          PHI=P(I)
        ENDIF
30      CONTINUE
16      J=0
      DO I=IPLO,IPHI
        J=J+1
        FP(J)=FP(I)
        G(J)=G(I)
        P(J)=P(I)
        T(J)=T(I)
      END DO
      N=J
C
C      Now interpolate on the volume fraction porosity to reduce table
C      to a single porosity table:
      IF (IPEXACT.EQ.0) THEN
        K=0
        PSTART=P(1)
        DO 32 I=1,N
          IF (P(I).NE.PSTART) GOTO 35

```

```

DO 33 J=I+1,N
  IF (T(I).EQ.T(J)) THEN
    K=K+1
    YA(1)=FP(I)
    YA(2)=FP(J)
    XA(1)=P(I)
    XA(2)=P(J)
    CALL POLINT(XA,YA,2,PEXTRAP,FP(K))
    G(K)=GI
    P(K)=PEXTRAP
    T(K)=T(I)
  ENDIF
33  CONTINUE
32  CONTINUE
35  N=K
    ENDIF
C
C  Finally, sort lastly on temperature:
ITLO=1
TLO=T(1)
DO 80 I=1,N
  IF (T(I).GT.TLO.AND.T(I).LE.TI) THEN
    ITLO=I
    TLO=T(I)
  ENDIF
80  CONTINUE
  IF (ITLO.EQ.1) THEN
    ITHI=3
  ELSEIF (ITLO.GT.N-2) THEN
    ITLO=N-2
    ITHI=N
  ELSE
    ITHI=ITLO+2
  ENDIF
  J=0
  DO I=ITLO,ITHI
    J=J+1
    YA(J)=FP(I)
    XA(J)=T(I)
  END DO
  N=J
  CALL POLINT(XA,YA,N,TI,PCF)
C
C  Return interpolated f(p) value to the PCF function:
RETURN
END
C -----
SUBROUTINE POLINT(XA,YA,N,X,Y)
C -----
C  This subroutine taken from _Numerical Recipes_ (1986).
C  Given arrays XA and YA, each of length N, and given a value X,
C  this routine returns a value Y. If P(x) is the polynomial of
C  degree N-1 such that P(XAi)=YAi, i=1,...,N, the the returned
C  value Y=P(X).
C

```

```

      IMPLICIT REAL*8 (A-H,O-Z)
      PARAMETER (NMAX=10)
      REAL*8 XA(N),YA(N),C(NMAX),D(NMAX)
C
      NS=1
      DIF=DABS(X-XA(1))
      DO 11 I=1,N
        DIFT=DABS(X-XA(I))
        IF (DIFT.LT.DIF) THEN
          NS=I
          DIF=DIFT
        ENDIF
        C(I)=YA(I)
        D(I)=YA(I)
11     CONTINUE
      Y=YA(NS)
      NS=NS-1
      DO 13 M=1,N-1
        DO 12 I=1,N-M
          HO=XA(I)-X
          HP=XA(I+M)-X
          W=C(I+1)-D(I)
          DEN=HO-HP
          IF (DEN.EQ.0.) THEN
            WRITE (*,*) 'INTERPOLATION ERROR!'
            CALL TERMINATE
          ENDIF
          DEN=W/DEN
          D(I)=HP*DEN
          C(I)=HO*DEN
12     CONTINUE
        IF (2*NS.LT.N-M) THEN
          DY=C(NS+1)
        ELSE
          DY=D(NS)
          NS=NS-1
        ENDIF
        Y=Y+DY
13     CONTINUE
      RETURN
      END
C -----
      SUBROUTINE TERMINATE
C -----
C   This subroutine simply terminates the execution of the program.
C
      STOP
      END
C -----
      SUBROUTINE TDMA(N,A,B)
C -----
C   TDMA IS AN ALGORITHM THAT SOLVES A TRI-DIAGONAL MATRIX BY OPT-
C   IMIZED GAUSSIAN-ELIMINATION; THIS ALGORITHM IS OFTEN CALLED THE
C   THOMAS ALGORITHM.
C

```



```

C      VARIABLE LIST:
C
C      N:  NUMBER OF EQUATIONS.
C      A:  COEFFICIENT MATRIX.
C      B:  SOLUTION VECTOR.
C
C      TO EMPLOY THE FOLLOWING ROUTINE, DIFFERENCE EQUATIONS GENERATING
C      THE TRIDIAGONAL SYSTEM OF EQUATIONS SHOULD BE CAST IN THE FOLLOW-
C      ING FORM:
C
C      
$$A(I,1)*X(I-1) + A(I,2)*X(I) + A(I,3)*X(I+1) = B(I)$$

C
C      WHERE X(I-1), X(I), AND X(I+1) REPRESENT THE UNKNOWN PARAMETER
C      VALUES AT NODES I-1, I, AND I+1, RESPECTIVELY.
C
C      .....
C
C      IMPLICIT REAL*8 (A-H,O-Z)
C      DIMENSION A(110,3), B(110)
C
C      A(1,3)=-A(1,3)/A(1,2)
C      B(1)=B(1)/A(1,2)
C
C      DO I=2,N
C          A(I,3)=-A(I,3)/(A(I,2)+A(I,1)*A(I-1,3))
C          B(I)=(B(I)-A(I,1)*B(I-1))/(A(I,2)+A(I,1)*A(I-1,3))
C      END DO
C
C      DO I=N-1,1,-1
C          B(I)=A(I,3)*B(I+1)+B(I)
C      END DO
C
C      RETURN
C      END

```

VITA

Steven Lowe Hayes, son of John and Katharine Hayes, was born in Abilene, Texas on January 16, 1966. After graduating from Cooper High School in Abilene, he enrolled at Texas A&M University in August 1984; he graduated with Cum Laude honors in May 1988, earning a Bachelor of Science degree in nuclear engineering. On May 28, 1988, Mr. Hayes was married to Eloise Carlton Booker in Abilene, Texas. Immediately following his marriage, Mr. Hayes began graduate work in nuclear engineering at Texas A&M University. With the financial support of a U.S. Department of Energy Nuclear Engineering Fellowship, Mr. Hayes earned a Master of Science degree in nuclear engineering in August 1989 and the Doctor of Philosophy degree in December 1992.

Mr. Hayes is currently employed by Argonne National Laboratory. He can be reached there at P.O. Box 2528, Idaho Falls, Idaho 83403-2528.

END

**DATE
FILMED**

12/15/93