Observation of Nonlinear Neoclassical ∇p -Driven Tearing Modes in TFTR

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Abstract

A quantitative comparison is made between the tearing-type modes observed in TFTR supershot plasmas and the nonlinear, neoclassical pressure gradient (∇p) driven tearing mode theory. Good agreement is found on the nonlinear magnetic island evolution of a single helicity mode ($m/n = 3/2, 4/3$ or $5/4$, where m and n are the poloidal and toroidal mode numbers, respectively). Statistical data on the island width and growth rate are also found to be consistent with this theory. The results imply that the supershot plasmas are stable to the classical current-driven tearing modes.

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Understanding the tearing-type MHD (magnetohydrodynamic) instabilities observed in TFTR neutral-beam (NB) heated supershot [1] plasmas has long been a challenge for plasma theory. These modes typically have low frequency ($f < 50 \text{ kHz}$) and low mode numbers ($m/n = 3/2, 4/3$ and $5/4$). Note that the $m/n = 2/1$ modes are not usually seen in the high performed supershot plasmas [2]. When these modes are large they can cause a strong deterioration in plasma performance [2] as measured by the DD or DT neutron rate, plasma stored energy, energy confinement time, etc. Considerable effort has been expended on the theoretical interpretation and numerical simulation of these modes. These works have been mostly based upon the classical current-driven tearing mode theory [3,4]. However, the results have been unsatisfactory [5,6]. In this Letter, we compare the experimental results with a relatively new theory, the neoclassical *pressure-gradient* (∇p) driven tearing mode theory [7,8]. The results are found to be very encouraging.

The evolution of two typical tearing-type modes in the high power NB heated, high β (plasma pressure/magnetic field pressure) supershot plasmas is shown in Fig. 1. Discharge (a) developed an $m/n = 3/2$ mode. Discharge (b) developed a $4/3$ mode. Detailed analysis of the deterioration effect induced by the MHD modes has been explored previously [2]. Here, we will only concentrate on the nonlinear evolution of the mode. Generally speaking, these modes start at around the peak performance of the plasma, which is typically $\sim 400\text{--}500 \text{ ms}$ after the beam injection. They have a long ($\sim 100\text{--}300 \text{ ms}$) and mostly linear growth phase before the saturation. The modes persist more than 200 ms after turning off of the beam. The mode frequency during the NB phase is nearly constant and decays exponentially after the NB phase (Fig. 1).

The high temperature (typically, $T_i \sim 20\text{--}35 \text{ keV}$, $T_e \sim 10 \text{ keV}$) supershot plasmas are in the low collisionality “banana” regime. The existence of a significant fraction of trapped particles in those plasmas change the plasma physics in many different ways. One of the key effects is the neoclassical bootstrap current which is induced by the damping of the poloidal flow due to the trapped-untrapped particle viscous stresses in these plasmas [9]. Inclusion of the perturbed bootstrap current induced by magnetic islands leads to a nonlinear tearing

mode evolution equation [7,8,10]

$$\frac{dw}{dt} = k_1 \frac{\eta_{nc}}{\mu_0} \left(\Delta' + k_2 \frac{w_c}{w} \right), \quad (1)$$

$$w_c \equiv \sqrt{\epsilon} \beta_p \frac{L_q}{L_p}, \quad (2)$$

where $w(> 0)$ is the magnetic island width, $k_1 = 1.22$, k_2 is a constant of order unity, η_{nc} is the neoclassical plasma resistivity, $\mu_0 \equiv 4\pi \times 10^{-7}$ is the permeability of free space, Δ' is the usual resistive MHD stability parameter [3], $\epsilon \equiv r/R_0 \ll 1$ is the tokamak inverse aspect ratio where r and R_0 are the plasma minor and major radius, $\beta_p \equiv p/(B_p^2/2\mu_0)$ is the local ratio of plasma pressure to the *poloidal* magnetic field pressure, $L_q \equiv q(dq/dr)^{-1}$ is the magnetic shear length (q is the plasma safety factor), and $L_p \equiv -p(dp/dr)^{-1}$ is the unperturbed, plasma pressure gradient scale length. All the quantities are evaluated at the mode rational surface ($r = r_s$, $q(r_s) = m/n$). In the limit of small toroidicity ($\epsilon = 0$), Eq. (1) reduces to the well-known Rutherford equation [4]. The second term in Eq. (1) is a new instability driving term (for normal tokamak plasma profiles with $dp/dr < 0$ and $dq/dr > 0$), and we will refer it as a ∇p -driven term. According to this theory the ∇p -driven term will drastically affect the island evolution. First of all, when the island is small, its evolution will be dominated by the ∇p -driven term:

$$w^2 \simeq 2k_1 k_2 \frac{\eta_{nc}}{\mu_0} \sqrt{\epsilon} \beta_p \frac{L_q}{L_p} t, \quad (3)$$

for $w \ll |k_2 w_c / (-\Delta')|$. Namely, the island grows with $w \sim \sqrt{t}$ in contrast to the Rutherford linear growth ($w \sim \Delta' t$). The second, and more important, feature of Eq. (1) is that when a ∇p -driven saturated island is formed, the plasma has to be stable to the classical Δ' -driven tearing mode, *i.e.*, $\Delta' < 0$. The saturated island width is given by

$$w_{sa.} = k_2 \frac{w_c}{-\Delta'} = \frac{1}{-\Delta'} k_2 \sqrt{\epsilon} \beta_p \frac{L_q}{L_p}. \quad (4)$$

A full quantitative comparison of the island evolution [discharges (a) and (b) in Fig. 1] with Eq. (1) is given in Fig. 2. Here, the “measured” island width (curves labeled a) is from a calculation using the cylindrical island formula [11]:

$$w = 4r_s \sqrt{\frac{1}{m} \frac{\tilde{B}_r}{B_p} \frac{L_q}{r}} \text{ at } r = r_s. \quad (5)$$

The radial component of the magnetic fluctuation $\tilde{B}_r$ in Eq. (5) is obtained by integrating the well-known MHD equation [3,12]:

$$r \frac{d}{dr} r \frac{d\tilde{\psi}}{dr} - m^2 \tilde{\psi} - \frac{q}{1 - nq/m} r \frac{d}{dr} \left(\frac{1}{r} \frac{d}{dr} \frac{r^2}{q} \right) \tilde{\psi} = 0, \quad (6)$$

using the Mirnov coil measured edge $\tilde{B}_p$. In Eq. (6) $\tilde{\psi}$ is the perturbed poloidal magnetic flux, and $\tilde{B}_p = \partial\tilde{\psi}/\partial r$, $\tilde{B}_r = -im\tilde{\psi}/r$. The $q(r, t)$ profile is taken from a transport analysis code (TRANSP [13]), which can be partly justified by comparing the mode rational surface with a local temperature fluctuation measurement using ECE. Curves labeled b in Fig. 2 are obtained by integrating Eq. (1) as an initial value problem beginning at the mode starting time. All the parameters are taken from the TRANSP simulation using the measured T_e , T_i , n_e , Z_{eff} , etc. For simplicity we use $\eta_{nc} = \eta_s/(1 - \sqrt{\epsilon})^2$, where η_s is the Spitzer resistivity. Only the thermal plasma pressure is used in the calculation of w_c (i.e., we exclude the beam component). We adopt an analytic formula from Strauss [14,15] (for large m) to calculate the Δ' :

$$\Delta'_S \equiv -\pi k_\perp \sigma / \tan(\pi\sigma/2), \quad (7)$$

where $k_\perp = m/r_s$, and

$$\sigma \equiv \frac{r_s}{m} \frac{q}{B_p} \frac{\mu_0 |dJ_\parallel/dr|}{dq/dr} = -\frac{1}{m} \frac{q^2}{dq/dr} \frac{d}{dr} \left[\frac{1}{r} \frac{d}{dr} \left(\frac{r^2}{q} \right) \right]. \quad (8)$$

It is found that to within a factor of 2, this formula is in agreement with numerical solutions of Eq. (6).

The time evolution of the three parameters used to obtain curves labeled b in Fig. 2 (η_{nc}/μ_0 , w_c and Δ'_S) is shown in Fig. 3. Good agreement is obtained when we choose the constant $k_2 = 1.20$ for the 3/2 mode case and 1.60 for the 4/3 mode case. The model not only matches the island evolution during the beam phase, but also closely mimics the data in the post-beam phase. If the parameters are fixed (see curve c's in Fig. 2), the model will still

agree with the data in the rising phase, but then it fails to match the saturated island width and the post-beam decay. The difference between curve b's and c's indicates the importance of the nonlinear effect due to the self-consistent changes of the plasma parameters during the island evolution. Similar results are obtained for the $m/n = 5/4$ tearing mode cases. Despite many simplifications and possible uncertainties in both the model and experimental data analysis, we consider the present agreement between the dynamics of the ∇p -driven tearing mode theory and the experimental observation to be a success.

A statistical analysis of the data has also been carried out. First of all, we study the linear rising phase of the $\tilde{B}_p$ (combining Eqs. (3) and (5) leads to $\tilde{B}_p, \tilde{B}_r \propto t$). Note that since the initial island width is around 1 cm, the condition in Eq (3) is usually satisfied. In order to obtain an analytic expression to compare with the measured edge $\tilde{B}_p$, we use the large m asymptotic solution for Eq. (6), i.e., $\tilde{\psi} \sim r^{-m}$, for $r > r_s$. The magnetic perturbations on the mode rational surface become $|\tilde{B}_r| \simeq |\tilde{B}_p| \simeq \tilde{B}_{pc}(r_s/r_c)^{-m-1}$, where r_c is the radial location of the Mirnov coil. The linear slope of the edge poloidal magnetic fluctuation $\tilde{B}_{pc}$ predicted by the ∇p -driven tearing model can thus be written as

$$\frac{d\tilde{B}_{pc}}{dt} \simeq B_t \left(\frac{r_s}{r_c}\right)^{m+1} \frac{n}{8} k_1 k_2 \epsilon^{3/2} \beta_r \frac{\eta_{nc}}{\mu_0} \frac{1}{r_s L_p}. \quad (9)$$

Figure 4(a) shows the comparison of Eq. (9) with the Mirnov coil measured mode growth rate. An equilibrium transport code (SNAP [16]) is used to calculate the parameters in Eq. (9) around the mode starting time (~ 450 ms after the NBI). Good agreement is found within the statistical error bars.

A comparison of the measured and calculated saturated island width is shown in Fig. 4(b). All the plasma parameters used in the calculation are taken from the TRANSP analysis in the mode saturation phase (3.9 sec, the NB phase is from 3 to 4 second). The Strauss' formula [Eq. (7)] is used for Δ' . As we can see that the agreement is within the statistical uncertainties.

Although we have shown that the neoclassical ∇p -driven tearing mode theory provides a very promising explanation of the ($m \geq 3$) tearing-type modes observed in TFTR, there

are still many questions that remain to be solved. According to the theory all the modes should be unstable [17]. This is clearly not the situation in the experiments. The 2/1 mode is very seldom seen in the high performance supershots. The coexistence of multiple modes (with $m \geq 3$) is sometimes seen in the early ($\sim 100 - 200ms$) rising phase of the 3/2 mode, but not often seen in the 4/3 or 5/4 mode cases. Different MHD modes can develop in very similar discharges [2]. It is not clear whether this is due to a different triggering mechanism or subtle changes in the plasma profiles. A companion mode on the $q = 1$ surface is often seen, *e.g.*, a coupled 2/2 mode is often observed with the 3/2 mode and a 3/3 mode is often coupled with the 4/3 mode. These coupled modes could be induced by the effects of toroidal geometry, but have never been worked out in detail.

In conclusion, good agreement has been found in this Letter between the single-helicity magnetic island evolution ($m/n = 3/2, 4/3$ or $5/4$) observed in TFTR supershot plasmas and the nonlinear neoclassical ∇p -driven tearing mode theory. This agreement implies that the plasmas are stable to the classical current-driven tearing modes. Further work needs to be carried out to explore the triggering mechanism and multi-mode interaction in both the experimental and theoretical studies.

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FIG. 1. Two typical supershot discharges with different MHD modes. Here, P_B is the neutral beam power, S_n is the total DD neutron rate and $\langle \beta_p \rangle$ is the volume-averaged poloidal beta. Discharge (a) has a dominant $m/n = 3/2$ mode. Discharge (b) has a $4/3$ mode. The mode amplitude ($\tilde{B}_p$) and frequency (f) are measured by a Mirnov coil.

FIG. 2. Comparison of the "measured" magnetic island (curve a) with the neoclassical ∇p -driven tearing mode theory. (a) $m/n = 3/2$ island. (b) $m/n = 4/3$ island. Curve b uses the time-dependent parameters (see Fig. 3). Curve c uses fixed parameters [for $3/2$ ($4/3$) mode: $\eta_{nc}/\mu_0 = 133$ (120) cm^2/sec , $w_c = 0.32$ (0.38) and $\Delta' = -0.05$ (-0.12) cm^{-1}].

FIG. 3. The time evolution of the parameters used in Eq. (1). (a) Magnetic field diffusivity η_{nc}/μ_0 . (b) Neoclassical driving term w_c . (c) Strauss' Δ' [Eq. (7)]. (d) Mode location. All the quantities are obtained from TRANSP simulations.

FIG. 4. (a) Comparison of the theoretically predicted mode growth rate, Eq. (9), with the experimental measured ones. (b) Comparison of the saturated magnetic island width. The w_{exp} is obtained by integrating Eq. (6) using Mirnov measured edge $\tilde{B}_r$. The w_{sat} is from Eq. (4).

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