1 of 1

DOE/PC/90180-T8 0-2 3

QUARTERLY REPORT
(04/01/93 TO 06/30/93)

NUMERICAL STUDY OF THE FLOW OF GRANULAR MATERIALS DOWN AN INCLINED PLANE USING A MODEL BASED ON A KINETIC THEORY APPROACH

Project Director : K. R. Rajagopal
Department of Mechanical Engineering
University of Pittsburgh
Pittsburgh, PA 15261

Contract No : DE-AC22-91PC90180

Contract Period : 12/13/1990 - 12/13/1993

Student(s) and the degree for which they are registered : Mr. R. Gudhe, (Ph.D)

SUMMARY

In the previous report the linearized stability results for the flow of granular materials down an inclined plane, modeled by a constitutive theory based on the kinetic theory approach [cf. Richman & Marciniec (1990)] were presented. In this report, we derive the governing equations for the flow of granular materials down an inclined plane, modeled by the consitutive theory proposed by Boyle and Massoudi (1990). The governing equations obtained will be solved numerically to obtain the basic solutions.

INTRODUCTION

Kinetic theory approaches in the formulation of rapid flows of granular materials have attracted considerable attention in recent years. There are several models which have been suggested to describe the rapid flow of granular materials which are derived using a statistical approach. Boyle and Massoudi (1989) have written comprehensive survey article of the constitutive equations and the laws governing the flow of granular material. A major difference between the continuum theories discussed previously and the theories based on a statistical approach is the concept of "**granular temperature**" which is introduced in the latter approach. Granular temperature describes the fluctuating velocity of the flow of granular solids and in this sense is similar to the temperature in a gas due to fluctuating motion of the gas particles. However, it is not clear how this quantity can be measured. Several models have been proposed by various investigators using ideas of kinetic theory.

GOVERNING EQUATIONS

Boyle and Massoudi (1990) proposed a model which can exhibit normal-stress effects by including the effects of the gradient of the volume distribution function, which is the noticeable difference between the model proposed by Lun, Savage and co-workers (1984).

The granular stress tensor $\mathbf{T}$ is the sum of the two terms, reflecting that momentun can be transported by the uninterrupted streaming of granules $\mathbf{T_k}$ and by the essentially instantaneous transport from one center to another during a collision $\mathbf{T_c}$. The granular stress tensor $\mathbf{T}$ follows from that of Lun et al. (1984), but there is an additional contibution to $\mathbf{T}$ that is given by

$$\mathbf{T_m} = \frac{4}{5}\,\eta\, g_0\, \rho\, \theta\, \nu \frac{V_0^{*2}\,\bar{\sigma}^2}{(1-\nu V_0)^2}\,(\,2\,\mathbf{M} + tr\mathbf{M}\,\mathbf{1}) \tag{1}$$

The stress tensor for a rapidly sheared granular material is found by summing the above individual contributions, which is given by

$$
\begin{aligned}
\mathbf{T} = \Bigg\{ & \rho\,\theta\,(1 + 4\,\eta\,\nu\,g_0) + \Bigg(\frac{2\,\mu}{3\,\eta\,(2-\eta)\,g_0} (1 + \frac{8}{5}\eta\,\nu\,g_0) \\
& \{ 1 + \frac{8}{5}\eta\,(3\,\eta - 2)\,\nu\,g_0 \} - \frac{3}{5}\mu_b\,\eta \Bigg) \nabla \cdot \mathbf{u} \\
& + \frac{4}{5}\eta\,\nu\,g_0\,\rho\,\theta \frac{V_0^{*\,2}\,\bar{\sigma}^2}{(1-\nu\,V_0^*)^2}\, tr\mathbf{M} \Bigg\} \mathbf{1} \\
& - \Bigg\{ \frac{2\,\mu}{\eta\,(2-\eta)\,g_0} (1 + \frac{8}{5}\eta\,\nu\,g_0)\,[\,1 + \frac{8}{5}\eta(3\eta - 2)\,\nu\,g_0\,] + \frac{6}{5}\mu_b\,\eta \Bigg\} \mathbf{D} \\
& + \frac{8}{5}\eta\,g_0\,\rho\,\theta\,\nu \frac{V_0^{*\,2}\,\bar{\sigma}^2}{(1-\nu\,V_0^*)^2} \mathbf{M}
\end{aligned}
\qquad (2)
$$

where, $\mu_b = \dfrac{256\,\mu\,\nu^2\,g_0}{5\,\pi}$

$$\mu = \frac{5\,m}{16\,\bar{\sigma}^2}$$

$$V_0^* = \frac{6\,V_0}{\pi\,\bar{\sigma}^3}$$

$$\mathbf{M} = \nabla\nu \otimes \nabla\nu$$

$$\eta = \frac{1}{2}(1 + e)$$

$$g_0 = \frac{V_0^*}{4(1-\nu V_0^*)}$$

V_0 repesents the volume per neighboring particle, and $\mathbf{D}$ is the stretching tensor. Also ρ is the bulk density, ν is the volume fraction, θ is the granular temperature, e is the coefficient of restitution, $\bar{\sigma}$ is the diameter of the spherical particles and m is the mass.

Consider the flow of granular materials modeled by the above model down an inclined plane (cf. Figure 1) due to the action of gravity [Savage (1979), Johnson and Jackson (1987), Johnson, et al. (1990), Hui, et al. (1984), Richman and Marciniec (1990), Hutter, et al. (1986a, b)]. In this problem we consider steady one dimensional flow of incompressible granular materials (*i.e.* ρ_p = constant) down an inclined plane, where the angle of inclination is α. The governing equations of motion are the conservation of mass and momentum. We assume that the granular temperature is constant and the volume fraction, velocity to be of the form

$$\begin{aligned} \nu &= \nu(y) \\ u &= U(y) i \end{aligned} \tag{3}$$

For the above flow field, the conservation of mass is automatically satisfied. From the balance of linear momentum we have

$$\frac{6\,E_6\,V_0^*\,\nu^2}{(1-\nu\,V_0^*)^3}\frac{d\nu}{dy}\frac{d^2\nu}{dy^2} + \frac{3\,E_6\,V_0^*\,\nu\,(2+\nu\,V_0^*)}{(1-\nu V_0^*)^4}\left\{\frac{d\nu}{dy}\right\}^3$$

$$\left\{E_1 + \frac{E_2\,V_0^*\,\nu\,(2-\nu V_0^*)}{(1-\nu V_0^*)^2}\right\}\frac{d\nu}{dy} = -\,\gamma\,g\,\nu\,cos\alpha \tag{4}$$

$$\left\{\frac{6\,E_3\,(1-\nu V_0^*)}{V_0^*} + \frac{3\,E_4\,\nu}{2} + \frac{E_7\,V_0^*\,\nu^2}{8\,(1-\nu V_0^*)}\right\}\frac{d^2U}{dy^2} + \left\{\left(\frac{3\,E_4}{2} - 6\,E_3\right)\right.$$

$$\left. + \frac{E_7\,V_0^*\,\nu\,(2-\nu V_0)}{4\,(1-\nu V_0^*)^2}\right\}\frac{d\nu}{dy}\frac{dU}{dy} = -\,\gamma\,g\,\nu\,sin\alpha \tag{5}$$

The equations (4) and (5) are to be solved subject to appropriate boundary conditions. for this problem, they are:

$$U = 0 \qquad at\ y = 0\ (on\ the\ inclined\ plane) \tag{6}$$

$$Q_3 = \int_0^L \nu\,dy \tag{7}$$

and,

$$\frac{dU}{dy} = 0 \tag{8}$$

$$\nu = 0 \qquad at\ y = L\ (at\ the\ free\ surface) \tag{9}$$

The system of equations subject to the boundary conditions are non-dimensionalized by

$$\bar{y} = \frac{y}{L}; \qquad \bar{U} = \frac{U}{u_0} \tag{10}$$

where L is the characteristic length and u_0 is the reference velocity. Now, the above system equations reduces to

$$\frac{2\,D_3\,\nu^2}{(1-\nu V_0^*)^3}\frac{d\nu}{d\bar{y}}\frac{d^2\nu}{d\bar{y}^2} + \frac{D_3\,\nu\,(2+\nu\,V_0^*)}{(1-\nu V_0^*)^4}\left\{\frac{d\nu}{d\bar{y}}\right\}^3$$

$$+\left\{D_1 + \frac{D_2\,\nu\,(2-\nu V_0^*)}{(1-\nu V_0^*)^2}\right\}\frac{d\nu}{d\bar{y}} = -\,\nu\,cos\alpha \tag{11}$$

$$\left\{D_4\,(1-\nu V_0^*) + D_5\,\nu + \frac{D_6\,\nu^2}{(1-\nu V_0^*)}\right\}\frac{d^2\bar{U}}{d\bar{y}^2}$$

$$+\left\{D_5 - D_4\,V_0^* + \frac{D_6\,\nu\,(2-\nu V_0^*)}{(1-\nu V_0^*)^2}\right\}\frac{d\nu}{d\bar{y}}\frac{d\bar{U}}{d\bar{y}} = -\,\nu\,sin\alpha \tag{12}$$

and the boundary conditions become

$$\bar{U} = 0 \qquad at\;\bar{y} = 1\;(on\;the\;inclined\;plane) \tag{13}$$

$$N = \int_0^1 \nu\,d\bar{y} \tag{14}$$

and,

$$\frac{d\bar{U}}{d\bar{y}} = 0 \tag{15}$$

$$\nu = 0 \qquad at\;\bar{y} = 1\;(at\;the\;free\;surface) \tag{16}$$

$$D_1 = \frac{E_1}{L\,\gamma\,g}; \qquad D_2 = \frac{E_2\,V_0^*}{L\,\gamma\,g}; \qquad D_3 = \frac{3\,E_6\,V_0^*}{L^3\,\gamma\,g}$$

$$D_4 = \frac{6\,E_3\,u_0}{L^2\,\gamma\,g\,V_0^*}; \qquad D_5 = \frac{3\,E_4\,u_0}{2\,L^2\,\gamma\,g}; \qquad D_6 = \frac{E_7\,u_0\,V_0^*}{8\,L^2\,\gamma\,g} \tag{17}$$

where,

$$E_1 = \gamma\theta, \qquad E_2 = \gamma\theta\eta$$

$$E_3 = \frac{2\mu}{3\eta(2-\eta)}, \qquad E_4 = \frac{16\mu(3\eta-1)}{15(2-\eta)}$$

$$E_5 = \mu\eta\left\{\frac{128(3\eta-2)}{75(2-\eta)} - \frac{768}{25\pi}\right\}$$

$$E_6 = \frac{4}{5}\gamma\theta\eta\bar{\sigma}^2 V_0^{*2}$$

$$E_7 = \mu\eta\left\{\frac{128(3\eta-2)}{25(2-\eta)} + \frac{1536}{25\pi}\right\} \tag{18}$$

Results and Discussion

The system of equations (11) and (12), with the boundary conditions (13), (14), (15) and (16) will be solved numerically using a collocation code COLSYS [cf. Ascher et al. (1981)]. A parametric study for the volume fraction and velocity profiles will be carried out for the base solution and the results will be presented in the next report.

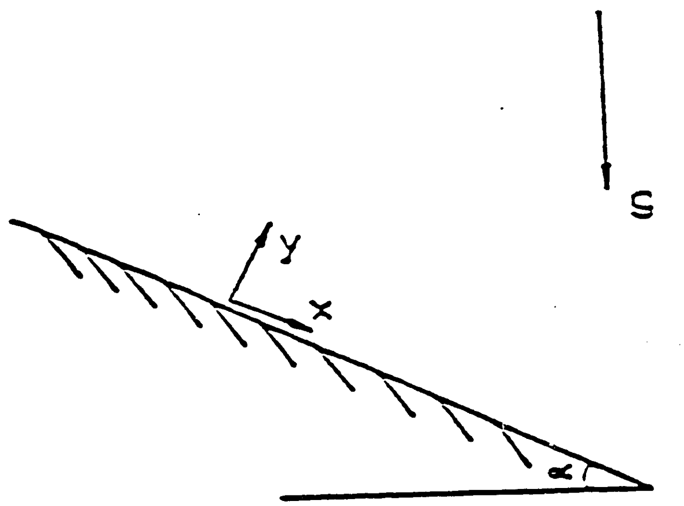

Figure 1. Flow Down An Inclined Plane

REFERENCES

1. **Ascher, U., Christianson, J., and R.D. Russel, 1981**, "Collocation Software for Boundary Value ODE's," *ACM Transactions on Mathematical Software*, 7/2: 209-222.

2. **Boyle, E.J. and M. Massoudi, 1990**, "A Theory for Granular Materials Exhibiting Normal Stress Effects Based on Enskog's Dense Gas Theory," *Int. J. Engng. Sci.*, Vol. 28, No. 12, pp. 1261-1275.

3. **Boyle, E.J. and M. Massoudi, 1989**, "A Kinetic Theory Derivation of the Stress Tensor for Granualr Material That Includes Normal Stress Effects," Technical Note, DOE/METC-90/4093.

4. **Lun, C.K.K., S.B. Savage, D.J. Jeffrey and N. Chepurnity, 1984**, "Kinetic Theories for Granular Flow: Inelastic Particles in Couette Flow and Slightly Inelastic Particles in a General Flow Field," *J. Fluid Mech.*, 140: 223-256.

5. **Hui. K., P.K. Haff, J.E. Unger and R. Jackson, 1984** "Boundary Condition's for High-Shear Grain Flows" *J. of Fluid Mechanics*, 145: 223-233.

6. **Hutter, K., F. Szidarovszky, and S. Yakowitz, 1986a** "Plane Steady Shear Flow of a Cohesionless Granular Material Down an Inclined Plane: A Model for Flow Avalanches Part-I: Theory," *Acta Mechanica*, 63: 87-112.

7. **Hutter, K., F. Szidarovszky, and S. Yakowitz, 1986b** "Plane Steady Shear Flow of a Cohesionless Granular Material Down an Inclined Plane: A Model for Flow Avalanches Part-II: Numerical Results," *Acta Mechanica*, 65: 239-261.

8. **Johnson, P.C. and R. Jackson, 1987** "Frictional-Collisional Constitutive Relations for Granular Materials, with Application to Plane Shear", *J. of Fluid Mechanics*, 176: 67-93.

9. **Johnson, P.C., P. Nott and R. Jackson, 1990** "Frictional-Collisional Equations of Motion for Particular Flows and their Application to Chutes", *J. of Fluid Mechanics*, 210: 501-535.

10. **Richman, M.W., and R.P. Marciniec, 1990** "Gravity-Driven Granular Flows of Smooth, Inelastic Spheres Down Bumpy Inclines", *J. of Applied Mechanics*, 57: 1036-1043.

11. **Savage, S.B., 1979** "Gravity Flow of Cohesionless Granular Materials in Chutes and Channels", *J. Fluid Mech.*, 92: 53-96.

DATE
FILMED
1 / 5 / 94
END