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A SUMMARY OF STUDIES OF PARTICLE STABILITY AT RHIC*

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1 INTRODUCTION

This paper summarizes some studies of particle stability done at RHIC. The topics reported on include

1. Long-term tracking and the dynamic aperture
2. Linear coupling effects and their correction
3. Tune spreads in the beam due to field multipoles in the magnets and their correction.

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2 LONG-TERM TRACKING AND THE DYNAMIC APERTURE

Those working on this problem include J. Claus, G.F. Dell, H. Hahn, M. Harrison, D. Maletić, J. Milutinovic, G. Parzen, S. Peggs, A.G. Ruggiero, S. Tepikian, D. Trbojevic and J. Wei. Tracking programs used included ORBIT, PATRICIA, PATRIS, and TEAPOT.

2.1 Tracking Results without Synchrotron Oscillations

Particles were tracked^{1,2} for about 10^6 turns. Error field multipoles were present in each magnet; random multipoles and systematic multipoles up to the tenth order were included. The momentum of the particle was fixed; no synchrotron oscillations were present. Ten different distributions of field multipoles were studied.

Figure 1 shows a survival plot for a RHIC lattice with six $\beta^* = 6$ insertions. The survival time in turns is plotted against the initial betatron amplitude for $\Delta p/p = 0$ particles and for 10 different distributions of field errors. The particles are started with $x' = y' = 0$ and $\epsilon_y = \epsilon_x$. The figure also shows a rough linear extrapolation to 3×10^9 turns which is about a 10-hour survival time in RHIC. In this case the long-term effects are small. The dynamic aperture, A_{SL} , does not vary much with the survival time.

Figure 2 shows a similar plot for a RHIC lattice with six $\beta^* = 2$ insertions. In this case the dynamic aperture, A_{SL} , depends appreciably on the survival time. It decreases by about 20% as the required survival time increases from 400 turns to 10^6 turns. The linear extrapolation to 3×10^9 turns is not to be trusted very much. One may note that this decrease of A_{SL} with the survival time is present here in the absence of synchrotron oscillations.

The $\beta^* = 2$ results are shown also in Fig. 3, where the survival time is plotted against initial betatron amplitude for four distributions of field errors. In one case, seed No. 8, the particle starting with $x_0 = 6$ mm was tracked for about 8×10^6 turns where it went unstable. In this case, there is some

* Work performed under the auspices of the U.S. Department of Energy.

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Fig. 1:
Survival time in turns versus
the initial betatron amplitude
for 6 $\beta^* = 6$ insertions.

Fig. 2:
Survival time versus the
initial betatron amplitude
for 6 $\beta^* = 2$ insertions.

Fig. 3:
Survival time versus the initial
betatron amplitude for 6 $\beta^* = 2$
insertions for four distributions
of field errors.

indication that the dependence of A_{SL} on the survival time is starting to flatten out.

In the above tracking studies, the particles were started with initial conditions $x' = y' = 0$, $x \neq 0$, $y \neq 0$, $\epsilon_x = \epsilon_y$. A multiparticle approach was explored³ where particles are started with different x, x', y, y' with $\epsilon_x = \epsilon_y$ and $x' \neq 0$, $y' \neq 0$. Figure 4 compares the results for the dynamic aperture found using the one-particle approach with those found using the multiparticle approach. Figure 4 shows histograms which indicate the largest initial x that is stable for each of 10 different distribution of field errors in the magnets. For the multiparticle case, an effective initial x is assigned to each set of starting conditions according to the rule

$$2 x^2 / \beta_x = (\epsilon_x + \epsilon_y)_{\text{initial}} \quad (1)$$

Figure 4 appears to show that the multiparticle approach, tracking 100 particles for 1000 turns for each set of field errors, gives about the same dynamic aperture as the one-particle approach, tracking one particle with $x' = y' = 0$ for 10^6 turns. Studies on the multiparticle approach are continuing.

Long-term tracking studies are being done including the effect of synchrotron oscillations. The ORBIT program has been modified⁴ to use point magnets in order to obtain symplectic tracking when synchrotron oscillations are present. The methods used are similar to those of the TEAPOT program with some differences regarding the choice of the reference orbit. Tracking studies including synchrotron oscillations are also being done⁵ with the TEAPOT program.

Fig. 4: Histograms showing the largest stable initial x for 10 distributions of field errors and 6 $\beta^* = 2$ insertions.

2.2 Some On-Going Long-Term Tracking Studies

The following describes studies on several aspects of long-term tracking. The effects of round-off error on the accuracy were studied.

Using⁶ the PATRIS program and the RHIC AGR lattice, with no sextupoles and no field errors, studies indicate that 14-figure accuracy is reduced to 7-figure accuracy after 10^6 turns.

Using⁷ a one-cell lattice, and tracking runs for up to 10^9 cells, the loss of accuracy was studied using a procedure that allows the round-off error to be varied continuously.

Studies are being done⁸ on understanding the dynamic aperture by doing detailed studies of the motion near the stability boundary and attempting to correlate the results with resonances.

Studies are being done⁹ to estimate the effects of the magnetic field not being Maxwellian by adding the field terms required to make the fields Maxwellian and noting the change in tracking results.

Studies are being done¹⁰ to see the effects of doing tracking using a quantum mechanical description of the particle motion.

2.3 Gradient Ripple Effects

A study was done¹¹ of the effect of a gradient ripple in the quadrupoles of RHIC. The gradients in the quadrupoles were changed with time according to

$$G = G_0 \left(1 + \frac{\Delta G}{G} \sin(2\pi t/T) \right). \quad (2)$$

The corresponding tune ripple is given by

$$\Delta\nu = 80\Delta G/G \quad (3)$$

for a RHIC lattice with 6 $\beta^* = 2$ insertions. In Fig. 5, the dynamic aperture for 10^6 turns is plotted against the magnitude of the ripple, $\Delta G/G$, for a 60-cycle ripple. Limiting the dynamic aperture loss to 5%, one finds a tolerance for $\Delta G/G$ of $\Delta G/G \leq 0.5 \times 10^{-5}$ corresponding to tune ripple of $\Delta\nu \leq 0.4 \times 10^{-3}$. There are no synchrotron oscillations in this study, and $\Delta p/p = 0$.

Fig. 5:
Dynamic aperture versus the gradient ripple for 6 $\beta^* = 2$ insertions.

3 LINEAR COUPLING EFFECTS

Linear coupling effects have become more important for proton colliders. The reasons for this include the following:

1. Superconducting magnets tend to give rise to larger skew quadrupole error fields.
2. The tune is close to the coupling resonance in order to avoid higher-order nonlinear resonances.
3. Low-beta insertions have higher beta magnets which can cause significant coupling effects.

The most important linear coupling effect is probably the tune splitting driven by the nearby difference resonance. This effect is roughly linear in a_1 , the field multipole that describes the skew quadrupole field. For this effect the important harmonic of a_1 is the $\nu_x - \nu_y$ harmonic. This effect can be largely corrected with a 2-family skew quadrupole correction system.

There are also a number of effects driven mostly by the harmonics of a_1 close to $\nu_x + \nu_y$ which include the following:

$\nu_x + \nu_y$ effects

1. Residual higher-order tune shift that remains after correction with a 2-family correction system. This tune shift is roughly quadratic in a_1 .
2. Possible large shifts in the beta-functions.
3. Emittance increase at injection.
4. Normal-mode rotation angle $\neq 0$, after correction with a 2-family correction system.

Those working on these problems include, on theory, V. Garczynski, G. Parzen, S. Peggs, A.G. Ruggiero, and S. Tepikian; and on simulation and correction, G.F. Dell, H. Hahn, M.A. Harrison, S.Y. Lee, G. Parzen, S. Peggs, and S. Tepikian. Programs used include ORBIT and PATRICIA.

Two approaches have been used in the theoretical treatments. These may be described as follows:

1. Find the change in the 4×4 one-turn transfer matrix due to the presence of the a_1 field, using point a_1 errors. Use the result for the transfer matrix to find orbit parameters.^{12,13}
2. Find the 4 eigenfunctions of the transfer matrix in the presence of the a_1 field. Use the results for the eigenfunctions to find the orbit parameters.¹⁴

The theory and its results are fairly complicated. A few results are useful for the insight they provide in designing a correction system for the linear coupling effects. The eigenfunction approach leads to the following driving terms¹⁴ for the linear coupling effects.

The driving terms for the tune splitting due to the nearby difference resonance, for the case when this difference resonance is $\nu_x - \nu_y = 0$, is given by

$$\Delta\nu = \frac{1}{4\pi\rho} \int ds (\beta_x\beta_y)^{\frac{1}{2}} a_1 \exp[i\bar{\nu}(\theta_x - \theta_y)], \quad (4)$$

$$\theta_x = \psi_x/\nu_x, \quad \theta_y = \psi_y/\nu_y, \quad \bar{\nu} = (\nu_x + \nu_y)/2.$$

For the four $\nu_x + \nu_y$ effects, the higher-order tune shift, the shift in the beta-functions, etc., the driving terms are

$$b_n = \frac{1}{4\pi\rho} \int ds (\beta_x\beta_y)^{\frac{1}{2}} a_1 \exp[i((n - \nu_y)\theta_x + \nu_y\theta_y)],$$

$$c_n = \frac{1}{4\pi\rho} \int ds (\beta_x\beta_y)^{\frac{1}{2}} a_1 \exp[i(\nu_x\theta_x + (n - \nu_x)\theta_y)]. \quad (5)$$

The b_n, c_n driving terms usually occur multiplied by the resonance factor $1/(n - \nu_x - \nu_y)$. The b_n, c_n for n near $\nu_x + \nu_y$ are the important driving terms.

The four $\nu_x + \nu_y$ effects listed above have the same driving terms. Correcting one of them, like the residual higher-order tune shift, may be expected to correct the other effects to a considerable extent. This was observed in a simulation study.

A simulation study has been done¹⁵ which computes the effects of linear coupling and the performance of a proposed correction system to correct these effects. Some of the results of this simulation study are given in Table 1 and Table 2. The simulation study indicates that 4 or 5 families of a_1 correctors may be able to correct all the linear coupling effects.

Table 1: Results for the correction of the tune splitting for a $\beta^* = 2$ RHIC lattice using a 2-family tune-splitting correction system set to make $\Delta\nu = 0$; $a_1 = 0$, $\nu_1 = 0.826$, $\nu_2 = 0.821$.

Error Field Dist.	Uncorrected			Corrected		
	ν_1	ν_2	$ \nu_1 - \nu_2 $ / 10^{-3}	ν_1	ν_2	$ \nu_1 - \nu_2 $ / 10^{-3}
1	0.796	0.854	59	0.828	0.823	6
2	0.707	0.935	228	0.838	0.819	19
3	0.869	0.783	86	0.825	0.829	4
4	0.772	0.883	111	0.831	0.823	7
5	0.779	0.872	93	0.836	0.820	16
6	0.848	0.805	43	0.832	0.821	11
7	0.840	0.847	7	0.852	0.834	18
8	0.742	0.895	153	0.838	0.818	20
9	0.785	0.866	81	0.828	0.823	6
10	0.749	0.891	142	0.823	0.827	5

Table 2: Results for the beta-functions for a $\beta^* = 2$ RHIC lattice before and after correction with a 2-family tune-splitting correction system set to make $\Delta\nu = 0$. With $a_1 = 0$, $\beta_{\max} = 50$.

Error Field	Uncorrected		Corrected	
	$\beta_{1,\max}$ at QF(m)	$\beta_{2,\max}$ at QD(m)	$\beta_{1,\max}$ at QF(m)	$\beta_{2,\max}$ at QD(m)
1	68	75	60	58
2	138	95	53	58
3	89	78	55	66
4	83	74	65	62
5	69	65	82	64
6	65	67	58	61
7	76	82	100	80
8	78	79	68	61
9	80	78	57	61
10	77	87	67	74

4 BEAM TUNE SPREAD DUE TO FIELD MULTIPOLES

Field multipoles may be present due to errors, iron saturation, coil design, etc., which produce a tune shift that depends on $\Delta p/p$, ϵ_x and ϵ_y . The resulting beam tune spread may be comparable to the width of the resonance-free box in tune space in which RHIC will operate.

Those working on this problem include J. Claus, G.F. Dell, H. Hahn, M.A. Harrison, G. Parzen, S. Peggs, M. Rhoades-Brown, A.G. Ruggiero, D. Trbojevic, and J. Wei.

The relative importance of the various sources of this tune spread depends on the size of the field multipoles, which changes with time. At this time it appears that a large beam tune spread for Au ions may occur at $\gamma = 30$ that is due primarily to the random b_2, b_3, b_4 in the dipoles. This assumes $\beta^* \geq 2$. Tune spreads for $\beta^* < 2$ require special consideration. In the following, it is usually assumed that $\beta^* \geq 2$.

By random multipoles, one means those multipoles that are not intended to be present in the design. Although they are called "random," they may not have a truly random distribution.

4.1 b_3, b_4, a_3, a_4 Multipoles

It has been found¹⁶⁻¹⁹ that the tune spread due to the random b_3 and b_4 is generated by the average b_3, b_4 in the dipoles. Thus the tune spread depends on how well one can control the average b_3, b_4 in the dipoles. One choice for the achievable average b_3, b_4 would give a worst-case tune spread of about $\Delta\nu = 15 \times 10^{-3}$ in RHIC.

The tune spread due to the random b_3, b_4 can be adequately corrected with b_3 and b_4 correctors at the center and ends of the dipoles. Correctors at the dipole center are essential if the tune spread is large, but this is not possible in RHIC, which has just one dipole in each half-cell. Studies have been done^{19,20} putting the center corrector in the insertion region where β_x, β_y, X_p are about the same as in the center of the dipole. These lumped b_4 correctors raise some concern as they may generate a strong 1/5 resonance. Tracking studies done²⁰ so far show no loss in aperture due to the lumped b_3, b_4 correctors in the insertion region.

4.2 b_2, a_2 Multipoles

The tune spread due to the random b_2, a_2 is quadratic in the strength of the b_2 multipole. Nevertheless, one observes¹⁷ field error distributions that produce an appreciable tune spread. One worse case was seen¹⁷ that produced a tune spread of $\Delta\nu = 6 \times 10^{-3}$. This tune spread may be partly correctable using the chromaticity sextupoles.

4.3 Some Comments

If $\beta^* \leq 1$ then b_5 near the high-beta quadrupoles may become important.²¹ The tune spread due to b_k goes like some high power of β_x, β_y for large k . Thus for $\beta^* \leq 1$, the b_k, a_k in the high-beta quadrupole triplet become important.

It may be helpful that the tune spread due to b_k, a_k and the tune spread due to beam-beam interaction are not directly additive. The first is larger at large betatron amplitudes and the second is larger at small betatron amplitudes. Using analytical results²² for the tune shifts, the tune spreads due to all the sources – field multipoles, beam-beam interaction, space charge – have been added together to produce results like those shown in Fig. 6.

Fig. 6:
The beam tune spread in the beam due to various sources, for Au ions at $\gamma = 100$ with emittances of 40π and for $\Delta p/p = 0, \pm 2 \times 10^{-3}$.

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