Research in Particle Theory

Annual Progress Report
for the period November 1, 1991 - October 10, 1992

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October 1992

N O T I C E

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ABSTRACT

Dynamics of 2+1 dimensional gravity is analyzed by coupling matter to Chern Simons Witten action in two ways and obtaining the exact gravity Hamiltonian for each case. 't Hooft's Hamiltonian is obtained as an approximation. The notion of space-time emerges in the very end as a broken phase of the gauge theory.

We have studied the patterns of discrete and continuous symmetry breaking in 2+1 dimensional field theories. We formulate our analysis in terms of effective composite scalar field theories.

Point-like sources in the Chern-Simons theory of gravity in 2+1 dimensions are described by their Poincaré charges. We have obtained exact solutions of the constraints of Chern-Simons theory with an arbitrary number of isolated point sources in relative motion. We then showed how the spacetime metric is constructed.

A reorganized perturbation expansion with a propagator of soft infrared behavior has been used to study the critical behavior of the mass gap. The condition of relativistic covariance fixes the form of the soft propagator. Approximants to the correlation critical exponent were obtained in two loop order for the two and three dimensional theories.

We proposed a new model of QED exhibiting two phases and a Majorana mass spectrum of single particle states. The model has a new source of coupling constant renormalization which opposes screening and suggests the model may confine. Assuming that the bound states of e^+e^- essentially obey a Majorana spectrum, we obtained a consistent fit of the GSI peaks as well as predicting new peaks and their spin assignments.

1. Non-Perturbative Dynamics in 2+1 Dimensional Gravity

Recent developments in particle physics, early universe cosmology, and general relativity have made the need for a consistent quantum theory of gravity very compelling. The complexity of 3+1 dimensional gravity has motivated the study of the corresponding problem in 2+1 dimensions, the rationale being that some of the features of this theory can be extrapolated to the 3+1 dimensional theory. The classical metric Einstein's theory in 2+1 dimensions has been analyzed by a number of authors, including those of references^{[1] [2] [3]}. It is found to be a locally trivial theory. It becomes less trivial when it is coupled to matter^{[1][2][4]}. Even in this case, the theory is devoid of any propagating gravitational modes. More recently, it was pointed out by Witten^[4] that Einstein's theory in 2+1 dimensions can be viewed as a Chern Simons gauge theory of the Poincare' group, thus connecting it to topological gauge theories^[5]. The main purpose of our contribution was to study how the dynamics of such a gauge theory leads to the emergence of space-time and to obtain an exact expression for the two-particle scattering amplitude in 2+1 dimensional gravity.

Two-particle scattering in 2+1 dimensional Einstein gravity was first studied by 't Hooft^[6] and by Deser and Jackiw^[7]. 't Hooft formulated the problem in terms of the relative coordinates of the particles, which move on a cone with a deficit angle given by the free two-particle Hamiltonian, and he obtained the scattering amplitude for this case. Deser and Jackiw obtained the two-particle scattering amplitude in the test particle approximation. Along with further studies along these lines^[8], two-particle scattering has also been studied by our group in Cincinnati from the point of view of Witten's Chern Simons gauge theory of the Poincare' group^{[4][9]}. In particular, we showed in reference^[9] that in the test particle approximation the scattering amplitudes obtained by the two approaches are identical.

Before we undertook further study of this problem, the 't Hooft approach^[6] represented the only concrete computation of the scattering amplitude which was different from the test particle approximation. It was therefore of great interest to see if his results could be obtained from the gauge theory, and if so under what conditions. One of the main results that we obtained in the last year is the computation of the exact two-body gravity Hamiltonian^{[10] [11]}. It is a highly non-linear function of the masses and momenta of the two particles and it reduces the 't Hooft Hamiltonian only for particles of (nearly) equal mass and low energy.

Another important result that we reported in references^{[10][11]} dealt in detail with how the conical geometry associated with 't Hooft's relative coordinate

emerges from gauge theory. This shows, among other things, that the space-time picture corresponds to a broken phase of the Poincare' gauge theory. In obtaining these results, we relied only on the general properties of the Chern Simons gauge theory and expressed the observables, such as the Hamiltonian, in terms of Wilson loops.

We have also obtained a number of other results in the last year^[12]. As was discussed more fully in references^{[10][11][12]}, we took advantage of the non-uniqueness of the phase space variables to couple point-like sources to the Chern Simons Witten action in two different ways. Until then only one of these matter couplings was known. For each case, we obtained the exact gravity Hamiltonian and the corresponding two-particle scattering amplitude. In each case, a relative coordinate moves on a cone. But the deficit angles of the two cones are different. Since the scattering amplitude depends on this parameter, this provides, in principle, an experimental means of choosing between the two different matter couplings.

2. Spontaneous Violation of Discrete Symmetries in 3-dimensions.

Dynamical symmetry breaking plays a significant role in particle physics research. Starting with the work of Nambu and Jona-Lasinio^[13], it has been investigated as a mechanism for generating fermion masses. The breaking of the chiral symmetry of QCD is now widely accepted as a realization of this phenomenon. It is also proposed as a mechanism for electro-weak symmetry breaking in the standard model^[14]. Among its appealing features are the fact that it requires fewer arbitrary parameters than the alternative spontaneous symmetry breaking with elementary Higgs fields.

The examples discussed in the previous paragraph deal with dynamical breaking of continuous symmetries. It is also important to investigate the possibility of discrete symmetry breaking through dynamical means. Vafa and Witten^[15] have shown that dynamical parity (P) or time reversal (T) breaking cannot happen in vector like gauge theories in 3+1 dimensions. Their arguments also applied to 2+1 dimensional QED if the number of fermion flavors is even and greater than two. Further analysis of dynamical parity breaking in QED3 in the large N flavor approximation was carried out in the thesis of Nash^[16]. It was found that Parity breaking does not happen to the leading order in $1/N$.

Analysis of dynamical symmetry breaking in realistic field theories requires nonperturbative methods. Commonly used are numerical lattice simulations, an-

alytic approximation schemes such as the study of Dyson-Schwinger equations and variational methods. Another useful laboratory where much has been learned is the solution of models in lower dimensions, particularly in 1+1-dimensions where there are numerous solvable interacting quantum field theories with discrete symmetries broken by composite order parameters. We have studied the dynamical breaking of parity and time reversal and a certain discrete flavor symmetry in a strongly-coupled 2+1-dimensional field theory using a conventional effective potential approach and the large N approximation. Our method was especially suited to field theories with four-fermion interactions where the order parameter is the vacuum expectation value of a local composite operator.

We considered a continuum field theory with two kinds of four-fermion couplings defined by the action

$$S = \int d^3x \left\{ \sum_{a=1}^N A \bar{\psi}_a i \gamma_\mu \partial_\mu \psi_a + \frac{\lambda A^2}{2N\Lambda} \left(\sum_{a=1}^N \bar{\psi}_a \psi_a \right)^2 + \frac{\kappa A^2}{2N\Lambda} \left(\sum_{a=1}^N \bar{\psi}_a \tau^3 \psi_a \right)^2 \right\} \quad (2.1)$$

with N flavors of complex four-component fermions. Λ is the ultraviolet cutoff and A , λ , κ and e are dimensionless, cutoff dependent constants. $A^{\frac{1}{2}}$ is the fermion wavefunction renormalization parameter, λ and κ are the dimensionless four-fermi vertices

We write down two kinds of fermion mass operators in this model, a scalar fermion mass $\bar{\psi} \tau^3 \psi$, a pseudoscalar fermion mass $\bar{\psi} \psi$. If this model is gauged with a $U(1)$ gauge field $\mathcal{A}_\mu$ it is also possible to have a Chern-Simons topological mass term for the photon field $\epsilon^{\mu\nu\lambda} \mathcal{A}_\mu \partial_\nu \mathcal{A}_\lambda$. The latter changes by an exact derivative under a gauge transform and therefore has a gauge invariant spacetime integral⁶. These mass terms are distinguished by the way in which they transform under discrete symmetries.

The action (2.1) has $U(N) \times U(N)$ symmetry. It also has a discrete Z_2 symmetry,

Under this transformation the scalar fermion mass operator $\bar{\psi} \tau^3 \psi$ changes sign

$$Z_2 : \bar{\psi} \tau^3 \psi \rightarrow -\bar{\psi} \tau^3 \psi$$

and the mass operators $\bar{\psi} \psi$ and $\epsilon^{\mu\nu\lambda} \mathcal{A}_\mu \partial_\nu \mathcal{A}_\lambda$ are invariant

Since $\bar{\psi}\tau^3\psi$ is not invariant under Z_2 a non-vanishing expectation value, $\langle \bar{\psi}\tau^3\psi \rangle \neq 0$, would indicate that the vacuum of the theory is not Z_2 symmetric. This operator is therefore an order parameter for Z_2 symmetry breaking.

The action in eq.(2.1) is also invariant under the Euclidean three dimensional parity

Under parity transformation the mass operator $\bar{\psi}\tau^3\psi$ transforms like a scalar and $\bar{\psi}\psi$ like a pseudoscalar. We also note that the topological mass term is parity odd⁶

Either of the vacuum expectation values $\langle \bar{\psi}\psi \rangle$ or $\langle \int \epsilon^{\mu\nu\lambda} \mathcal{A}_\mu \partial_\nu \mathcal{A}_\lambda \rangle$ are order parameters for parity breaking. If we have both fermions and gauge fields in a theory if one of these parity odd operators has an expectation value, radiative corrections can induce an expectation value for the other one. In general this occurs when the physical fermion has a pseudoscalar mass larger in magnitude than the scalar mass⁷. Then the physical fermion and the physical photon would have parity violating masses and the Coulomb interaction would be short-ranged. We have only analyzed pure 4-fermi theories. We have indicated how these results could be generalized when gauge interactions are present.

The Z_2 and parity symmetries forbid the appearance of a bare mass for the fermions in the action (2.1). If the physical fermion spectrum is to have a mass gap at least one of these symmetries must be broken dynamically. In the following we shall seek solutions of the model (2.1) which break either Z_2 or parity or both through the generation of fermion masses. We also found that, with the particular choice of four-fermion couplings taken in (2.1) either parity or chiral Z_2 symmetry can be broken but there is no solution where both are simultaneously broken.

To set up the $\frac{1}{N}$ expansion we introduced the Lagrange multiplier fields ϕ and χ and rewrote the continuum action in eq.(1) as

$$S = \int d^3x \left\{ A\bar{\psi}i\gamma_\mu\partial_\mu\psi + iB\phi\bar{\psi}\psi + iC\chi\bar{\psi}\tau^3\psi + \frac{NB^2\Lambda}{2A^2\lambda}\phi^2 + \frac{NC^2\Lambda}{2A^2\kappa}\chi^2 \right\} \quad (2.2)$$

If we solve the equations of motion for the scalar fields ϕ and χ and substitute the solutions into (2.2) yields the four-fermion interaction terms in the action (2.1).

Using the usual manipulations we computed the effective action in terms of

the expectation values of ϕ and χ .

$$\langle \phi \rangle = \frac{\delta}{\delta J_1} W[J_1, J_2] \equiv m_1 \quad (2.3)$$

and

$$\langle \chi \rangle = \frac{\delta}{\delta J_2} W[J_1, J_2] \equiv m_2 \quad (2.4)$$

The computation of the free energy was carried out using the usual functional methods but taking into account the fact that we are employing here a large N resummation of perturbation theory. For the details of this computation see the paper of Semenoff and Wijewardhana^[17]. The leading large N effective action thus obtained is

$$\begin{aligned} \frac{\Gamma[m_1, m_2]}{V} &= \frac{N\Lambda b^2}{2\lambda a^2} m_1^2 + \frac{N\Lambda c^2}{2\kappa a^2} m_2^2 - \frac{N\Lambda}{\pi^2} \frac{b^2}{a^2} (m_1^2 + m_2^2) \\ &+ \frac{N}{6\pi} \frac{b^3}{a^3} (|m_1 + m_2|^3 + |m_1 - m_2|^3) + \dots \end{aligned} \quad (2.5)$$

where, to leading order in $\frac{1}{N}$ we have taken $A = a$ and $B = b = C$ with a and b finite dimensionless constants. For now, it is sufficient to absorb the factors $\frac{b}{a}$ into m_1 and m_2 . Then, if we define the critical coupling constants

$$\lambda_c = \frac{\pi^2}{2} = \kappa_c \quad (2.6)$$

The leading order in large N part of Γ can be written as

$$\begin{aligned} \frac{1}{V} \Gamma[m_1, m_2] &= \frac{N\Lambda}{2} \left(\frac{1}{\lambda} - \frac{1}{\lambda_c} \right) m_1^2 + \frac{N\Lambda}{2} \left(\frac{1}{\kappa} - \frac{1}{\kappa_c} \right) m_2^2 \\ &+ \frac{N}{6\pi} (|m_1 + m_2|^3 + |m_1 - m_2|^3) + \dots \end{aligned} \quad (2.7)$$

It is evident that if either of the four-fermion couplings are sufficiently strong, either $\lambda > \lambda_c$ or $\kappa > \kappa_c$ the free energy is minimized by $m_1 \neq 0$ or $m_2 \neq 0$ and either the parity or Z_2 symmetry are broken dynamically. The completely symmetric phase with $m_1 = m_2 = 0$ is stable only in the region $\lambda < \lambda_c$ and

$\kappa < \kappa_c$. When $\lambda > \lambda_c$ and also $\lambda > \kappa$ the stable minimum of (2.7) has $m_1 \neq 0$ and $m_2 = 0$. When $\kappa > \kappa_c$ and $\kappa > \lambda$ the stable minimum is where $m_2 \neq 0$ and where $m_1 = 0$. There are no stable minima where both m_1 and m_2 are simultaneously nonzero. On the symmetry line $\lambda = \kappa > \lambda_c = \kappa_c$ the symmetry breaking pattern is discontinuous and jumps from $m_1 \neq 0, m_2 = 0$ for $\lambda > \kappa$ to $m_1 = 0, m_2 \neq 0$ for $\kappa > \lambda$. Further details of these calculations can be found in the appropriate references^{[17][18]}. A discussion of a lattice version of this analysis can be found in a further reference^[19].

3. Low Energy Scalars in Extended Technicolor Theories

The recent revival of interest in technicolor theories has been stimulated by the observation that these theories should not necessarily be viewed as scaled up versions of QCD. Momentum components well above the confinement scale Λ_{tc} can play a more important role than they do in QCD, with important consequences such as the generation of large fermion masses. Walking technicolor is one example of this phenomena^[20]. Another possibility is that the higher energy Extended Technicolor (ETC) interactions, which must be present to generate the masses of ordinary fermions, can play an important and direct role, along with the technicolor interactions, in electro-weak breaking, leading to even larger fermion masses^[21]. This can take place only if the combination of the ETC coupling and the technicolor coupling at the ETC scale is sufficiently close to a certain critical value^[22].

It has recently been suggested that this ETC driven enhancement is associated with the appearance of composite scalars that are light compared to the ETC scale^[23]. The enhanced fermion mass arises from an effective Yukawa coupling of the fermion to the scalar, which develops a vacuum expectation value from the technicolor interactions. We have studied the properties of these composite scalar fields. We have concluded that unless the technicolor coupling at the ETC scale is unrealistically weak and the ETC coupling is very close to the critical curve, these light scalars have large widths^[24].

An important question is if the light scalar resonances are able to mediate Flavor Changing neutral current interactions. If we restrict our attention to CP conserving interactions, then the possible off diagonal couplings of ordinary fermions to these resonances will not produce unacceptable Flavor Changing Neutral Currents if the zero momentum scalar masses $M(0)$ are above 1.5 TeV. The contribution of these scalars to Flavor changing neutral processes involving the t quark may be much larger. This could be of immediate interest if the t quark is

discovered in the next few years. In the standard model the top quark does not have direct flavor changing couplings to up or charmed quarks through neutral gauge boson exchanges. But in such technicolor models with extreme fine tunings at the ETC scale light neutral scalars can mediate such flavor changing top decays. We will discuss more about this in our proposal section. We plan to work out the consequences of such flavor changing top decays in concrete technicolor models.

4. Critical Behavior with Composite Scalars

It has been observed that QED in 2+1 dimensions exhibit critical behavior. That is there is dynamical flavor symmetry breaking in this theory only if the number of fermion flavors is below some critical value^[25]. These conclusions were arrived at using an analysis based on the solution of the gap equation in the large N approximation. Recently Pisarski challenged this conclusion by carrying out an alternative analysis using the ϵ -expansion^[26]. We have redone his analysis using the effective scalar field theory technique expounded in the previous paragraph. We disagree with his conclusion and agree with the original assertion of critical behavior^[27].

5. Reconstruction of the Metric in Spacetime Gravity with Point Sources

A description has been given which shows how Einstein's metric theory of gravity arises naturally from Poincaré Chern-Simons gravity as a theory of representation space of the Poincaré group^[28]. Exact solutions of the constraints of the Chern-Simons theory have been obtained for systems with an arbitrary number of isolated point sources in relative motion. The spacetime metric has been reconstructed from these solutions.

Three dimensional gravity, in the absence of local matter fields, is purely topological in character. In 2+1 dimensions it can be described by a gauge theory of the Poincaré group, ISO(2,1) with a Chern-Simons action^[29].

$$I_{\text{C.S.}} = -\frac{1}{2} \text{Tr} \int_M A \wedge (dA + \frac{2}{3} A \wedge A).$$

The integral is defined on a manifold M , and the gauge connection is ex-

panded as

$$A_\mu = e_\mu^a P_a + \omega_\mu^a J_a,$$

where P_a and J_a are the momentum and angular momentum generators of the Poincar group. e_μ^a and ω_μ^a are interpreted as the dreibein and spin connection on M . When e_μ^a is invertible the theory is equivalent to the Einstein metric theory.

Point-like sources (particles) may be introduced in the Chern-Simons theory. However to be consistent with gauge invariance, point-like dynamics must be described, not by the particles coordinates in M , but by internal variables, $q_{(\alpha)}^a$ and $p_{(\alpha)}^a$. ($\alpha = 1, 2, \dots, N$, where N =number of particles.) The $q_{(\alpha)}^a$ form the representation space of the Poincaré group, and can be thought of as living in an internal space, M_q , attached to the trajectories of the particles^[28]. $p_{(\alpha)}^a$ is the momentum conjugate to $q_{(\alpha)}^a$, and M_q is a plane. The $q_{(\alpha)}^a$ obey globally trivial matching conditions, returning to themselves after a rotation by 2π radians. In this way the dynamics of point-like sources in three dimensional Chern-Simons gravity is formulated completely on a flat, globally trivial space. In contrast, the coordinates, x_α^μ , of a particle, in the spacetime which satisfies Einstein's equations in three dimensions, will obey generally complicated matching conditions consistent with a multiconical structure^{[28] [30] [31]}.

Assume that M is globally $R \times \Sigma$, where R is time and Σ is a spatial hypersurface. As we have emphasized, M is not a metric space unless the e_μ^a are constrained to be invertible. On the other hand M_q has a natural Lorentz metric and we should like to connect this internal space to spacetime. To do so, it should be possible to transform M_q into a multiconical space which can be identified with spacetime. This requires, as we have shown^{[28] [30]}, that the Chern-Simons theory be viewed as a gauge theory of the universal covering group of the Poincaré group, rather than the Poincaré group itself.

We have shown that the internal space, M_q , with N punctures, can be turned into the appropriate multiconical space time by gauge transformations living in the covering group. In the process we have been able to solve the constraints of Chern-Simons gravity in the presence of N sources in arbitrary motion. We used our solution algorithm to solve explicitly the constraints in the presence of N spinning sources in arbitrary relative motion and have shown how the standard metrics of Deser, Jackiw and 't Hooft^[31] for a single particle and for N relatively static particles are recovered as special cases of the general N particle solution. A complete discussion and description of the gauge transformations that are used to identify M_q with M is given in^[28]. This paper also describes how to obtain

exact solutions of the constraints of the Chern-Simons theory with an arbitrary number of isolated point sources in relative motion.

6. Field Theories in Schrodinger Representation

The critical behavior of various statistical systems near continuous phase transitions has been the subject of intense investigations for several decades. Continuum quantum field theory turned out to be the most appropriate tool for these investigations. It connects critical phenomena with the fundamental features of renormalization and renormalization group. Nevertheless, the best numerical values for critical parameters are obtained more often then not from lattice methods: high and low temperature expansions, Monte-Carlo investigations, etc. The reason for this dichotomy is obvious. Numerical methods on lattices are convergent procedures, which, as a function of parameters (or computer time), diverge only exactly at the critical point. In contrast to that, the critical behavior in continuum field theories is investigated through the ϵ -expansion of Wilson and Fisher,^[32] which define only an asymptotic series for all values of the expansion parameter, ϵ .^[33]

In a current paper^[34] we have introduced a new method for the investigation of critical behavior in quantum field theories. The method is based on a reorganization of the perturbation expansion series, very similar to mass renormalization, with the intent of extracting key nonperturbative features from field theories. The purpose of the modification of the propagator is to soften the infrared behavior of the propagator, thus providing a finite approximant to the correlation critical exponent in every order of the perturbation expansion. The condition of relativistic covariance fixes the form of the softened propagator up to a scale factor. The form of the propagator is simply related to the correlation exponent, ν . The series of approximants for ν , obtained from successive orders of the soft perturbation expansion, is convergent.

We have added the term

$$\delta H = \frac{1}{2} \int d^D p \Delta(\mathbf{p}) \phi^\dagger(\mathbf{p}) \phi(\mathbf{p}) \quad (6.1)$$

to the kinetic part of the Hamiltonian and subtract it from the potential part. By this rearrangement, we have not been able to make the perturbation series convergent for non-universal quantities, but we have been able to extract convergent approximations for the correlation critical exponent, ν .

The new propagator affects the perturbation series as well, thus, the series expansion for $E(\mathbf{p})$ will not automatically have the correct relativistic form $E(\mathbf{p}) = \sqrt{\mathbf{p}^2 + \mathbf{m}^2}$. The requirement of the correct relativistic form for $E^2(\mathbf{p})$ leads to the integral equation

$$\Delta(\mathbf{q}) + \Sigma(q_0, \mathbf{q}) \Big|_{q_0^2 = \mathbf{q}^2 + m^2 + \Delta(\mathbf{q})} = 0 \quad (6.2)$$

for $\Delta(\mathbf{q})$. Here $\Sigma(q_0, \mathbf{q})$ is the self energy correction. In every order of the perturbation expansion the integral equation has a solution

$$\Delta(\mathbf{q}) \sim |\mathbf{p}|^{2\alpha},$$

where $\alpha < 1$, making the terms of the perturbation expansion are infrared and ultraviolet convergent for $1 \leq D < 3$ (D is the number of spatial dimensions).

The exponent α_n obtained in the n th order of perturbation expansion has a simple relation to the critical exponent ν

$$\frac{1}{\nu_n} = 2 - n(3 - \frac{D}{\alpha_n}). \quad (6.3)$$

In a previous work, we obtained the same results for the critical exponents by a different approach.^[35]

As is shown by (6.3), a smooth limit to the correct value of ν is possible only if we can prove that in large orders of the loop expansion the equation determining α becomes a function of $\rho = n(3 - D/\alpha)$ only, thus making the existence of a nontrivial limit $\rho \rightarrow \rho_c$ for $n \rightarrow \infty$ possible. Then, of course, $\nu = \frac{1}{2-\rho_c}$.

The simplest method for the investigation of large orders of the perturbation expansion is the renormalization group. We investigated the field theory defined by the action

$$\begin{aligned} S(\phi) = & \frac{1}{2} \int dp_0 d^D p [p_0^2 + |\mathbf{p}|^{2\alpha}] \phi(p) \phi(-p) \\ & + \frac{1}{4!} \int dx_0 d^D x \phi(x)^4 + \frac{\delta m^2}{2} \int dp_0 d^D p \phi(p) \phi(-p), \end{aligned} \quad (6.4)$$

by the method of renormalization group.

Similar field theories have been investigated in the past. The closest example is that of Fisher and Grinstein^[36] who found that certain superconducting systems may be described by Hamiltonians similar to (6.4). They, however use a ‘double ϵ -expansion’ in the variables $\epsilon = 3 - D$ and $\epsilon' = 1 - \alpha$. We investigated field theory (6.4), at fixed D near the critical point $\alpha = D/3$. In other words, we performed an expansion in $\delta = 3\alpha - D$. We showed that the propagator at $p_0^2 = -|\mathbf{p}|^{2\alpha}$ can be written in large orders of the perturbation expansion ($n \rightarrow \infty$, $n\delta$ fixed) as

$$\Gamma_n^{(2)}(\mathbf{p}, g, \delta) = p_0^2 \frac{(-\beta_2 g)^n}{\delta^{n-1}} F(x) \exp \left\{ \frac{\beta_3}{\beta_2^2} x \log n \right\}, \quad (6.5)$$

where $x = n\delta$ and where β_i are the perturbative coefficients of the beta function and

$$F(x) = \sum_{m=0}^{n-1} \frac{x^m}{m!} q^m. \quad (6.6)$$

Here q^m is the finite part of the m -loop term in the minimal renormalization scheme.

The zeros of $F(x)$ determine the critical exponent as $\nu = \frac{1}{2-3x/D}$. We calculated ν in the two loop approximation (by truncating $F(x)$). For $D = 2$ we obtained $\nu = .717$, very close to the value we obtained with the perturbative approach, $\nu = .713$.

It is worth underlining that the prescription, we provided for the determination of the correlation exponent in n loop order uses the n loop terms alone. This is the very circumstance which makes the universality of the critical exponent (its independence from the coupling constant) possible. In the ϵ -expansion approach the dependence on the coupling constant is eliminated by the limit $g \rightarrow g_c(\epsilon)$, where $g_c(\epsilon)$ is the fixed point of the renormalization group transformation. Here we approach the critical surface at an arbitrary value of the coupling constant, by adjusting the bare mass. Thus, ν can be independent of g only if in every order it is determined by the highest loop contribution alone.

7. Strongly Coupled Quantum Electrodynamics

The topic of strongly coupled gauge theories was and still the focus of our research. The importance of the strong phase, for example in a theory like quantum electrodynamics (QED), stems from the need to understand the behavior and properties of particles like the electron and the positron at very high energies. This in return will hopefully shed light on explaining the anomalous and mysterious events observed experimentally at GSI. These events are bursts of high energy electron- positron peaks resulting from heavy ion collisions.

The theory of strongly coupled QED is far from being understood even though the weak coupled theory is known to be the most successful particle theory model. The conventional wisdom of understanding the various phases of a gauge theory is to formulate it on the lattice using Wilson loops. However the continuum limit of the lattice theory is not yet successfully achieved partly because some of the conceptual problems associated with the continuum theory are not answered by the lattice theory.

Recently we proposed a new model of strongly coupled QED as an alternative to the lattice model.^[37] The key new element in our proposal is that the confining phase is described by a curvature law rather than an area law as predicted by the lattice model. The consequences were dramatic. The mass spectrum of the electron- positron bound states is a Majorana spectrum (mass goes inversely with the spin) and looks like an inverted hydrogen mass spectrum. In other words both the electron and the positron transmutes into a Majorana particle at very high energies a signature which can be looked for in nature if it is true!. In fact the Majorana mass spectrum we found not only is in very good agreement with the three observed electron- positron peaks at GSI but it also predicts four new resonances and the associated spin assignments. the fine structure constant is predicted to be of the order of 2.5 greater than 1, the value required to have a transition to the strong phase. Our model is therefore consistently strongly coupled.

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