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THE ANGULAR DISTRIBUTION OF DIFFUSE LIGHT
REFLECTED FROM ISOLATED DEFECTS ON
SUPERFINISHED SPHERES

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THE ANGULAR DISTRIBUTION OF DIFFUSE LIGHT REFLECTED FROM ISOLATED DEFECTS ON SUPERFINISHED SPHERES

BDX-613-1363 (Rev.), UNCLASSIFIED Topical Report, Published February 1977

Prepared by P. E. Klingsporn, D/812, under PDO 6984778

A special array of light pipes was constructed and used with a photodetector to measure the angular distribution of light reflected diffusely from small isolated defects on superfinished nickel spheres. Defects in a size range of concern were detected by the system, and angular distributions of the diffuse light generated by the defects correlated with the defect dimensions. At present, a commercial version of the light sensing array used in this study is not available; however, under development is a more desirable optical system for automated surface certification which will use a photodetector array which is available commercially.

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SUMMARY

When light is incident on a superfinished surface, virtually all the reflected light is specularly reflected; a small amount of diffusely reflected light, however, is always present. The amount of diffuse light increases when an isolated surface defect is illuminated by the incident light, and earlier work showed that the total amount of diffuse light was proportional to the defect width or diameter.

In the present study a special array of light pipes was constructed and used with a photodetector to measure the angular distribution of diffuse light reflected from isolated defects. A special feature of the array is the presence of a centrally located hole which provides for the passage of the incident and specularly reflected beams. The hole makes it possible to measure diffuse light under the ideal condition of normally incident light.

A 4-mW helium-neon laser light source was used to illuminate isolated pit and scratch defects on superfinished nickel spheres 0.6 inch (15 mm) in diameter. The scratches ranged in width from 0.0002 to 0.0008 inch (5.1 to 20.3 μm) and in depth from 20 to 70 microinches (0.51 to 1.8 μm). The diffuse light signals at a particular angle off of normal to the surface at the defect point were found to increase linearly with scratch depth for the constant scratch width investigated. The diffuse light reflected into an angular region of space off normal to the surface increased with increasing scratch cross-sectional area for the range of scratch sizes given above. The angular distribution of diffuse light reflected from isolated pit defects ranging from 0.0002 to 0.0008 inch in diameter and 10 to 120 microinches (0.25 to 3.0 μm) in depth was found to increase with increasing pit cross-section.

At the present time a light sensing array of the type used in this study is not commercially available. However, work is in progress to develop an automated surface quality certification system which does not require a hole in the detector array for passage of incident and specular beams. This system has advantages over the light pipe array, and the photodetector array used is available commercially.

DISCUSSION

SCOPE AND PURPOSE

A light pipe array was to be constructed and used with a photo-detector to measure diffusely reflected light from isolated defects on superfinished spheres to determine if the spatial distribution of the diffuse light could be correlated with the defect dimensions. The work was conducted from November, 1973 through June, 1974.

PRIOR WORK

Several techniques were previously investigated as possible candidates for automated surface quality certification of superfinished spheres. A commercial laser interferometric displacement measurement system was used to detect and measure waviness in a size range of concern on superfinished spheres. Diffuse light reflection was used to detect isolated pit and scratch defects on superfinished spheres, and the measured total diffuse light signals were found to increase with increasing defect width or diameter. The root-mean-square heights and slopes of the minute background surface irregularities of a superfinished sphere were determined from measurements of the angular distribution of the background-reflected diffuse light.

A commercial multi-element photodetector was used to measure intensities in the specularly-reflected-light diffraction patterns from isolated pit and scratch defects on superfinished spheres.² The diffraction pattern intensities were found to increase with increasing pit or scratch depth.

ACTIVITY

Surface Background and Defects

Lapping and polishing are used to achieve highly reflective superfinished spherical surfaces consisting of very smooth profiles or "backgrounds" which often contain a few isolated defects in the forms of pits and scratches (Figure 1). The superfinished surfaces obtained by polishing are superior to normally available precision surfaces such as those of ball bearings. The arithmetic average (AA) value of a typical superfinished surface profile is less than a few tenths of one microinch. The small scale background roughness results from minute scratches and undulations remaining on the surface after the final polishing steps. The isolated pit and scratch defects which are often present on the smooth backgrounds

Figure 1. Hypothetical Profile of a Superfinished Spherical Surface Showing the Smooth Background and Cross-Sections of Two Typical Isolated Pit or Scratch Defects

are usually in the range 10 to 200 microinches (0.25 to 5.1 μm) deep and 0.0001 to 0.0015 inch (2.54 to 38 μm) in width or diameter. The defects are truly isolated on the spherical surface in the sense that the average distance between defects is very large compared with the dimensions of the defects.

Specular and Diffuse Light Reflection

When collimated light is incident on a sphere, the specular reflected beam is, in general, divergent because of reflection from the convex spherical surface. The path of the specular reflected light can be calculated on the basis of the simple law of geometrical reflection. Consider the more general case in which a collimated light beam of diameter d is directed toward a sphere with its axis passing through the sphere center. A lens of focal length f is used to converge the light to a virtual focus at point P as shown in Figure 2. The diameter Y of the specular reflected beam at a distance L from the incident spot on the sphere is given by

$$Y = 2\{E + [L + R(1 - \cos \theta)] \tan (2\theta - \alpha)\} \quad (1)$$

where

$$\theta = \arcsin (E/R),$$

$$\alpha = \arctan \left(\frac{E}{X + \sqrt{R^2 - E^2}} \right),$$

$2E$ = the diameter of the illuminated spot on the spherical surface, and

Figure 2. Geometry of Specular Reflection From a Spherical Surface When a Convergent Lens is Used to Focus a Collimated Light Beam to a Virtual Focus at a Point Beyond the Illuminated Spot

X = the distance between the virtual focal point and the sphere center.

Details of the derivation are given in Appendix A.

A special case exists when the alignment is such that the virtual focal point P is at the center of the sphere, because the specular reflected beam is then retroreflected and its diameter in a plane perpendicular to the axis at distance L is the same as that of the convergent incident beam at distance L . A second special case occurs when the virtual focal point P is located midway between the sphere center and the illuminated spot. In this case the specular reflected beam is collimated and has a diameter $2E$.

If the surface is perfectly smooth, all the reflected light is specularly reflected and the rays of the reflected beam obey Equation 1. However, all real surfaces, regardless of how highly polished they may be, contain some degree of small scale background roughness. Therefore, some of the light will be diffusely reflected at angles other than those corresponding to specular reflection. The total amount of diffusely reflected light depends on the arithmetic average of the surface profile. The background diffuse reflected light decreases with increasing angle outward from the specular direction but is axially symmetric about the specular

direction because of the randomly oriented, small scale irregularities of the surface background.

Earlier work demonstrated that the root-mean-square heights and slopes of the minute background irregularities of a superfinished sphere could be determined from measurements of the total diffuse reflected light and its angular distribution in space. When an isolated defect such as a pit or scratch is illuminated by the incident light, the total amount of diffuse reflected light increases. Moreover, light reflected diffusely from an isolated defect such as a pit or scratch is distributed less uniformly in space than the background-reflected diffuse light.

Fixture for Measurement of Angular Distribution of Diffuse Reflected Light

To make measurements of the spatial distribution of diffuse reflected light with a single element detector, it is necessary that the detector be moved about to sample light intensity as a function of position. Spatial distribution measurements of diffuse light can also be made with a special array of many individual detectors located at fixed positions in space. To avoid certain problems associated with these methods, a third approach was taken. This consisted of constructing an array of light pipes in which one end of the pipes serves as a point of detection, and the diffuse light received at the opposite end of each pipe in turn is sampled by a single photodetector.

The light pipe array, shown schematically in Figure 3, was constructed from 0.056-inch-diameter (1.4 mm) Crofon (Valteck, Inc.) light guides. The light pipes are securely positioned in a holder so that one end of each lies on a spherical surface 1.50 inches (38 mm) in diameter. These ends of the light pipes are arranged in six concentric circular rings and the smallest ring has an inside diameter of 0.2 inch (5 mm). The radial thickness of each ring is equal to the light pipe diameter. Each ring is packed fully dense with pipes, and the number of pipes contained in rings 1, 2, 3...6 is 14, 20, 26, 32, 38, and 43, respectively. Although the light pipe ends are packed fully dense, they do not fill all of the space because of their circular cross-sections. However, the combined cross-sectional area of the 173 pipe ends is 78 percent of the annular spherical surface area occupied by the six-ring array. The 0.2-inch-diameter central hole provides for passage of the incident and specularly reflected beams.

Since the purpose of the light pipe array is to detect and measure only diffusely reflected light, the incident light beam, sphere, and light pipe array are so aligned that the specularly reflected beam passes through the central hole. The incident and specular beams are finite in size and some of the diffuse light reflected

Figure 3. Diagrammatic Representation of Light Pipe Array Designed for Measurement of Diffuse Reflected Light as a Function of Angle Off the Direction of Normal Incidence

at small angles off the specular direction escapes through the hole. When the light pipe array is positioned so that the incident beam lies on the axis of the array, each of the circular rings of light pipes intercepts an annular cone of diffuse light having an outer half-angle ϕ given by

$$\phi = \arcsin \left\{ \frac{R' \sin \left[\arcsin \left(\frac{0.1 \text{ inch}}{R'} \right) + N \left(\frac{0.056 \text{ inch}}{R'} \right) \right]}{\sqrt{R'^2 + (R' - D)^2 - 2R'(R' - D) \cos \left[\arcsin \left(\frac{0.1 \text{ inch}}{R'} \right) + N \left(\frac{0.056 \text{ inch}}{R'} \right) \right]}} \right\} \quad (2)$$

where

R' = the radius of curvature of the spherical surface on which the pipe ends lie,

N = the number of the ring (beginning with $N = 1$ for the inner ring), and

D = the distance from the point of incidence on the sphere to the center of the hole in the array.

The derivation of Equation 2 is given in Appendix B.

The amount of diffuse light escaping through the 0.2-inch-diameter hole decreases as the distance between the pipe array and spherical surface increases. However, as the pipe array is moved farther from the surface to reduce the loss through the hole, the cone angle for each individual ring of light pipe detectors decreases. The cone angles of diffuse light received by the array of pipes versus ring number and distance between the array and the spherical surface were calculated from Equation 2. The results are shown in Figure 4. Each ring of pipes intercepts an annular cone of light characterized by inner and outer cone angles because of the diameter of the individual light pipe cross-section.

For example, at a distance $D = 0.13$ inch (3.3 mm), the third ring intercepts a diffuse light cone with a half-angle spread from 64.4 degrees to 72.5 degrees.

Experimental Results

The light pipe array shown in Figure 3 was used to sample the diffusely reflected light intensities from isolated defects on superfinished nickel spheres 0.6 inch (15 mm) in diameter. A 4 mW helium-neon laser beam was used as the source for illuminating the sphere. Preliminary tests showed that a distance D of approximately 0.4 inch (10 mm) was a convenient separation between the pipe array and the illuminated spot on the sphere. If the 0.04-inch-diameter

Figure 4. Calculated Half-Angle Ranges of the Annular Cone of Diffuse Light Received by the Light Pipe Array Versus Pipe Ring Number N and Distance D Between the Illuminated Spot of the Sphere and the Central Hole of the Array

(1.0 mm) laser beam is directed toward the center of the sphere, the corresponding value of X in Figure 2 approaches infinity and hence the angle α approaches zero.

Under these conditions, the diameter Y of the specular reflected beam at a distance $L = 0.43$ inch (11 mm, actual distance used) from the central hole of the pipe array to the illuminated spot can be calculated from Equation 1 by setting $\alpha = 0$. For this case the illuminated spot diameter is 0.04 inch and the calculated specular beam diameter at $L = 0.43$ inch is 0.156 inch (3.96 mm), which is sufficiently small to pass through the 0.20-inch-diameter hole of the pipe array. However, in practice the alignment of the sphere, incident beam, and pipe array is more readily accomplished if the specular reflected beam size at the central hole of the array is smaller than the value 0.156 inch.

A smaller reflected beam size was obtained by using 500-mm-focal-length lens to converge the incident laser beam as indicated in Figure 2. As shown by Figure 2 and the relationship in Equation 1, the use of the lens resulted in a smaller illuminated spot size on the sphere and a smaller specular beam size at the entrance-exit hole of the array.

With the distance D between the illuminated spot on the ball and the central hole of the pipe array set at 0.43 inch, the array was aligned relative to the sphere so that the incident laser beam passed through the central hole of the fixture and the specularly reflected portion of the beam passed back through the hole. Under these alignment conditions only diffuse reflected light was intercepted by the ends of the light pipes. The sphere was mounted in a rotation fixture which provided the capability of rotating the sphere about two perpendicular, intersecting axes while maintaining the sphere center fixed in space to within approximately ± 0.0001 inch ($\pm 2.5 \mu\text{m}$). The accuracy of rotation insured that the alignment of the specular beam would remain unchanged as the sphere was rotated to provide illumination of different regions on the surface.

The sphere was rotated to such a position that the incident beam illuminated a defect-free region of the superfinished nickel surface. A single photodetector was then used to measure the diffuse light signal generated by each of the six rings of light pipes. These signals corresponded to the relatively small amount of light reflected diffusely from the smooth nickel surface background. Next, the sphere was rotated about its center until an isolated scratch defect was illuminated by the incident beam. The photodetector was again used to measure the individual diffuse light intensities transmitted by each of the six light pipe rings. This procedure was repeated until the diffuse light signals generated by several isolated scratches ranging from 0.00020 to 0.00080 inch

(5.1 to 20.3 μm) in width and 20 to 70 microinches (0.51 to 1.8 μm) in depth had been measured.

Four of the scratches investigated were of the same width (0.00030 inch) but varied in depth from 25 to 50 microinches. The diffuse light signals from these scratches were analyzed in several ways, taking the smoothly varying background signals into account, and it was found that the signals from Ring 4 provided the best correlation with defect depth. A plot of the signal difference (difference between the Ring 4 signals produced by the scratches and the Ring 4 background signal) versus the scratch depth is shown in Figure 5. The difference between the scratch and background diffuse light signals is seen to vary approximately linearly with the scratch depth over the range of depth from 25 to 50 microinches. The straight line drawn through the data points passes through the origin, as it should, because as the scratch depth approaches zero the diffuse light generated by the scratch should approach that reflected from the background alone.

According to the analysis given earlier regarding the cone angles of the various light pipe rings, it can be seen that Ring 4 corresponds to an off-axis angle of approximately 37 degrees for a distance of $D = 0.43$ inch between the pipe array and the illuminated spot on the sphere. Therefore, the diffuse light from the scratches which correlates best with scratch depth is that light reflected at an angle of 37 degrees from normal to the spherical surface.

The diffuse light signals from all the rings were also analyzed to determine if a correlation existed between the signals and the scratch cross-sectional area for the range in widths and depths investigated. The diffuse light intensities received by Ring 5 provided the best correlation with the scratch cross-sectional area, which is one-half the product of the scratch width and the scratch depth. A plot of the ratio of the Ring 5 signal to the Ring 5 background signal versus the scratch cross-sectional area is given in Figure 6. The Ring 5 signals are plotted as ratios to the background to show that the diffuse light signals from the scratches represent percentage increases from approximately 30 to 470 percent over the background signal.

The results in Figure 6 show that the scratch diffuse light signals increase with the scratch cross-sectional area for the range investigated. The ratio plotted in Figure 6 approaches unity as the scratch cross-sectional area approaches zero. This is correct because the diffuse light from the region of the scratch should approach that of the background diffuse light as the scratch cross-sectional area vanishes.

Figure 5. Measured Difference Between Isolated Scratch Diffuse Light Signals of Light Pipe Ring 4 and Ring 4 Background Diffuse Light Signal, Versus Scratch Depth for Constant Scratch Width

Figure 6. Measured Ratio of Isolated Scratch Diffuse Light Signals From Light Pipe Ring 5 to Ring 5 Background Diffuse Light Signal, Versus Scratch Cross-Sectional Area in a Plane Perpendicular to Length of Scratch

In addition to isolated defect scratches, three isolated pits of varying sizes were investigated to determine the sensitivity of the system to change in pit size. The same distance between the illuminated spot and the pipe array was maintained as had been used in the work with scratches. Since the pits were of varying size and depth, the signals were analyzed with respect to the pit cross-sectional area, which ranged from $1.0 \times 10^{-9} \text{ in.}^2$ ($0.65 \mu\text{m}^2$) for the 10- $\mu\text{in.}$ -deep pit to $52 \times 10^{-9} \text{ in.}^2$ ($34 \mu\text{m}^2$) for the 120- $\mu\text{in.}$ -deep pit.

The only consistent dependence of signals with pit cross-sectional area was obtained by comparing the difference between Ring 1 and Ring 3 diffuse light intensities. A plot of the difference between Ring 1 and Ring 3 signals versus the pit cross-sectional area is shown in Figure 7. The sensitivity of the diffuse light angular distribution to pit cross-sectional area is not as great as that for scratches because only an 11-percent change in Ring 1 signal over that of Ring 3 was caused by the factor of 52 change in pit cross section.

Conclusions

The light pipe array investigated in this study is sufficiently sensitive to detect the presence of isolated defects and to distinguish among sizes in a size range of concern on superfinished spheres. However, the sensitivity is much greater for scratches than for pits. Instead of using a single photodetector to sample the diffuse light transmitted by each ring of pipes individually, separate photodetectors could be mounted permanently to receive the light intensities simultaneously from the individual rings--or from another appropriate array of light guides. As the sphere is rotated, the simultaneous photodetector signals could be monitored and transferred to a minicomputer programmed in advance to analyze and compare the signals with predetermined calibrations of background quality and of defect sizes versus signals.

At the present time no commercially available photodetector of the type used in the study exists. However, work is in progress to develop an automated surface quality certification system based on a commercial, solid (no central hole), multi-element photodetector.² Since the latter photodetector array is commercially available, a certification system based on it is somewhat more desirable than a system based on a one-of-a-kind array such as that used in this study. Moreover, the optical system using the commercial photodetector array has certain advantages over the light pipe array. A report is being prepared which covers the proposed certification system using the commercial photodetector array.

Figure 7. Check of Sensitivity of Light Pipe Array System to Varying Isolated Pit Size (Measured Difference Between Pit Diffuse Light Signals of Light Pipe Ring 1 and Corresponding Pit Signal of Ring 3, Versus Pit Cross-Sectional Area.)

ACCOMPLISHMENTS

A special array of light pipes was constructed for use with a single photodetector to measure the angular distribution of diffusely reflected light. The pipe array is provided with a central hole for the passage of incident and specularly reflected light, which makes it possible to measure diffuse light distributions under conditions of normal incidence. The array and a photodetector were used to detect and measure angular distributions of diffuse light reflected

by isolated pit and scratch defects in a size range of concern on superfinished nickel spheres. The measurements showed that the diffuse light reflected into certain directions increased with increasing defect dimensions.

FUTURE WORK

The present work combined with that of Reference 1 constitutes an investigation of both the total and angularly partitioned diffuse light signals generated from isolated defects on superfinished spheres. No further work on diffuse light intensities generated from isolated defects is planned at the present time.

REFERENCES

¹Deleted.

²P. E. Klingsporn, *Reflected Light Intensity Distributions From Defects on Highly Reflective Spheres* (Topical Report). UNCLASSIFIED. Bendix Kansas City: BDX-613-1174, November, 1974.

Appendix A

DERIVATION OF EQUATION 1

In Figure A-1 a collimated light beam of diameter d is brought to a virtual focus at point P by the action of a convergent lens of focal length f . It is assumed that the lens, the incident beam, and a diameter of the sphere are coaxial. In this case the specularly reflected beam defined by rays 1' and 2' is symmetric about the axis of the incident beam. Equation 1 can be derived as follows. From Figure A-1, the angle α which the incident rays 1 and 2 make with the axis of the system is

$$\alpha = \arctan (d/2f). \quad (A-1)$$

The normals to the surface at the points of incidence of rays 1 and 2 make an angle with the axis of the system

$$\theta = \arcsin (E/R). \quad (A-2)$$

where

E is the radius of the illuminated spot and

R is the sphere radius.

Therefore, the specularly reflected rays 1' and 2' travel at angles $2(\theta - \alpha)$ relative to the incident rays as depicted in the figure.

From the right triangle ABC of Figure A-1 it can be seen that the radius $Y/2$ of the specularly reflected beam at distance L from the illuminated spot on the sphere is

$$\frac{1}{2}Y = E + (L + \ell) \tan \{2(\theta - \alpha) + \alpha\}. \quad (A-3)$$

But, from the right triangle OBG the distance ℓ is $R - R \cos \theta$, and therefore the diameter of the beam at distance L from the sphere is given by

$$Y = 2E + 2\{L + R(1 - \cos \theta)\} \tan (2\theta - \alpha), \quad (A-4)$$

which is the result given in Equation 1. The angle θ referred to in the text is defined in terms of the illuminated spot radius E and sphere radius R in Equation A-2. In the text, the angle α was expressed in terms of the distance X between the virtual focal point P and the sphere center. Reference to the right triangles PGB and PHJ in Figure A-1 shows that the relation

$$\tan \alpha = \frac{d/2}{f} = \frac{E}{X+R \cos \theta} \quad (A-5)$$

is valid and hence

$$\alpha = \arctan \left(\frac{E}{X+R \cos \theta} \right), \quad (A-6)$$

which is the relation given in the text.

Appendix B

DERIVATION OF EQUATION 2

The light pipe array and sphere are shown in schematic cross-section in Figure B-1. The ends of the light pipes, which serve as small points of light detection, lie on an imaginary spherical surface of radius R' with the center at point O' . The rings of light pipes are arranged symmetrically about the axis of the array defined by the segment OO' . The incident light passes through the small hole of diameter $2r$, centered at point O , and is aligned along the axis of the array, OO' . When the sphere is positioned relative to the array so that a diameter lies along the axis OO' , the light will be normally incident on the sphere and the specularly reflected beam will also be centered on the axis through the center of the hole.

Point B is the illuminated spot on the surface of the sphere. A ray of light reflected diffusely from the illuminated spot and lying in the plane of the paper will emerge at an angle ϕ relative to normal to the surface, and will strike the light pipe array at a point A. The point of incidence A on the pipe array lies at an arc distance S from the axis of the array. From the geometry of the right triangle ACB in Figure B-1 the angle ϕ is given by

$$\phi = \arcsin \left(\frac{AC}{AB} \right). \quad (B-1)$$

The segment AC can be expressed as

$$AC = R' \sin \frac{\hat{S}}{R'}, \quad (B-2)$$

where the angle $\delta = \hat{S}/R'$ is in radians. When the Law of Cosines is applied to triangle $AO'B$, the segment AB can be related to the distance D between the illuminated spot on the sphere and the center of the hole in the pipe array:

$$AB = \left[R'^2 + (R'-D)^2 - 2R'(R'-D) \cos \left(\frac{\hat{S}}{R'} \right) \right]^{1/2}. \quad (B-3)$$

The arc length $\hat{S}$ is the sum of arcs $\hat{S}_1$ and $\hat{S}_2$. The arc $\hat{S}_1$ is the half-arc subtended by the hole, and from Figure B-1 it is seen that

Figure B-1. Schematic Cross Section of Light Pipe Array and Sphere in a Plane Passing Through the Axis of the Array and Center of the Sphere

$$\hat{S}_1 = R' \arcsin \left(\frac{r}{R} \right). \quad (B-4)$$

The arc $\hat{S}_2$ can be expressed in increments of an individual pipe diameter D_p as $S_2 = ND_p$, and therefore the arc distance $\hat{S}$ of point A from the axis of the array is

$$\hat{S} = R' \arcsin \left(\frac{r}{R} \right) + ND_p. \quad (B-5)$$

When Equations B-1 through B-5 are combined, the expression for the angle ϕ in the Equation B-1 becomes

$$\phi = \arcsin \left\{ \frac{R' \sin \left[\arcsin \left(\frac{r}{R'} \right) + \frac{ND_P}{R'} \right]}{\sqrt{R'^2 + (R'-D)^2 - 2R'(R'-D) \cos \left[\arcsin \left(\frac{r}{R'} \right) + \frac{ND_P}{R'} \right]}} \right\} \quad (B-6)$$

The Equation 2 result can be obtained by substituting the values for the hole diameter (0.2 inch) and the light pipe diameter (0.056 inch) into Equation B-6.

It is to be noted that substitution of $N = 0$ into Equation B-6 gives the outer half-angle ϕ of the cone of diffuse light escaping through the central hole of the array. Therefore, substitution of any successive value of N into the expression for ϕ gives the outer half-angle of the annular cone of diffuse light received by the corresponding ring of light pipes.

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