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Predictive Modeling of Particle-Laden, Turbulent Flows

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Notation

Variables

c_μ, c_1, c_2, c_3	turbulence empirical constants
C_D	friction factor
d	solid particle diameter
e	coefficient of restitution for particle-particle collisions
e_w	coefficient of restitution for particle-wall collisions
f_μ, f_2	turbulence functions
g	gravity in the z-direction
g_0	radial distribution function
G_1, G_2	dimensionless functions related to the solid-phase stress
G_3	dimensionless functions related to the pseudo-thermal energy flux
k	gas phase turbulent kinetic energy
p	pressure
q_{PT}	pseudo-thermal energy flux
r	radial coordinate
R	pipe radius
Re	Reynolds number, $Re = \frac{2 R V_{gCL} \rho_g}{\mu}$
Re_p	particle Reynolds number
Re_T	turbulent Reynolds number
T	pseudo-thermal temperature
V'	fluctuating velocity component
V_g	mean axial gas velocity
V_s	mean axial solid velocity
z	axial coordinate

Greek symbols

β	drag coefficient
γ	pseudo-thermal energy rate of dissipation
ϵ	rate of dissipation of gas phase turbulent kinetic energy k
η	dimensionless function the coefficient of restitution e
θ	azimuthal coordinate
λ	pseudo-thermal conductivity associated to the solid assembly

λ_{mfp}	particle mean-free-path
μ	intrinsic gas viscosity
μ_{eg}	effective intrinsic gas viscosity
μ_s	viscosity associated to the solid assembly
μ_t	turbulent or eddy gas viscosity
v	solid volume fraction
v_0	solid volume fraction at random closest packing
v_{eg}	effective intrinsic kinematic gas viscosity
v_t	kinematic eddy gas viscosity
π	numerical constant
ρ_g	gas density
ρ_s	solid density
σ	particle-phase stress tensor, defined in the compressive sense
$\sigma_k, \sigma_\epsilon$	Prandtl turbulent numbers for k and ϵ
τ	gas-phase stress tensor element
ϕ	specularity coefficient
ω	damping function

Subscripts

c	collisional contribution in the granular constitutive relations
CL	pipe centerline
g	gas-phase
k	kinetic contribution in the granular constitutive relations
s	solid-phase

The subscript *next* indicates that the variable is to be evaluated in the grid point next to the wall.

The superscript ' indicates variables related with the fluctuating velocities.

Abstract

An Eulerian "two-fluid" mathematical model is considered for the flow prediction of gas-particle systems. The model expands on the Sinclair and Jackson¹ model by recognizing a turbulent continuous phase. Both the gas- and the particle-phase velocity fields are composed of by a mean and a fluctuating component, and the important physical phenomena arising from the interaction of these components are included in the model. The gas turbulence is modeled by a single-phase closure, namely the k - ϵ low Reynolds model by Myong and Kasagi², modified to account for the presence of a dilute particle phase. The solid phase is considered as a rapid granular flow; hence, a closure based on the kinetic-theory analogy is used for the description of the stresses associated with this phase. Along with the relation between the fluxes of both phases and the pressure drop, the model is capable of predicting features related to the local flow structure, such as the mean and fluctuating velocity components and the concentration of both phases.

In this report, the model is applied to the case of steady, fully developed flow in a vertical cylindrical pipe. An extensive comparison of the model predictions with experimental data is included. The sensitivity of the model predictions to the properties describing the particle collisions is also explored.

¹ Sinclair, J.L. and R. Jackson. 1989. Gas-Particle Flow in a Vertical Pipe with Particle-Particle Interactions. *AIChE J.* 35(9), pp. 1473-1486.

² Myong, H.K. and N. Kasagi. 1990. A New Approach to the Improvement of k - ϵ Turbulence Model for Wall-Bounded Shear Flows. *JSME Int. J. Series II*, 33(1), pp. 63-72.

1. Introduction

Until recently, reliable mathematical models for particle-laden two-phase flows have been rather simple because of the inherent difficulties in predicting the turbulent velocity field for the gas-phase alone (Chen and Wood, 1985). In addition, the mechanism that is responsible for observed particle velocity fluctuations exceeding those of the fluid and for the lateral segregation of particles in vertical ducts was not fundamentally described. Min (1967), Sinclair and Jackson (1989), Pita and Sundaresan (1991), and Louge *et al.* (1991) attribute these interesting flow features to collisions between particles and between the particles and the wall. However, turbulence models have been developed for single phase flows which yield reasonably good predictions in geometries corresponding to engineering applications such as coal injection into entrained-flow gasifiers, solid transport lines, and circulating fluidized beds. These single phase turbulence models are being extended to account for the presence of a particulate phase. Works in this field include those of Genchev and Karpuzov (1980), Pourahmadi and Humphrey (1983), Elghobashi and Abou-Arab (1983), Chen and Wood (1985), Berker and Tulig (1986), Kashiwa and Gore (1991), Ma and Ahmadi (1990), and Louge *et al.* (1991). All but the last two analyses do not include the effects of particle collisions and fluctuation kinetic energy of the particulate phase. Ma and Ahmadi (1990) employ a thermodynamic formulation for dispersed multiphase turbulent flows and apply their model to the relatively simple case of steady shear flows. The dissipation rates for the particulate and fluid phases are given by prescribed functions.

Louge *et al.* (1991) have recently presented a model for vertical gas-solid upflow in a pipe which treats the particle phase as a rapid granular flow. The kinetic theory of dense gases was extended to describe the momentum and energy transferred by the velocity fluctuations of the particles (Lun *et al.*, 1984). This description for the particle

phase is a simpler version of the one used by Sinclair and Jackson (1989) and Pita and Sundaresan (1991). Using this approach, many unusual flow features which match reported behavior, such as the existence of several different states of zero solid flow rate for a given pressure gradient, three possible pressure gradients and solids holdup for a given gas velocity and solid flux, and a pressure gradient which decreases, maintaining the same solid and gas flux, as the tube diameter increases, can be qualitatively predicted. Quantitative comparisons of the predictions of Sinclair and Jackson (1989) and Pita and Sundaresan (1991) with experiments are premature since these two models ignored gas phase turbulence, which should have a marked influence on the particle concentration distribution.

Louge *et al.* (1991), however, have presented a more detailed analysis of dilute gas-solid turbulent flow considering velocity fluctuations in *both* phases. They use a one-equation gas turbulence closure model (a transport equation for the fluctuating energy) with wall functions and compare their predictions with the measurements of Tsuji *et al.* (1984). They also modify an expression proposed by Koch (1990) for the correlation between the velocity fluctuations of the gas and those of the particles. They found the model predictions to be sensitive to the magnitude of the particle shear and flux of granular energy at the wall.

In this work we expand on the effort of Sinclair and Jackson (1989) and Louge *et al.* (1991) by using a two-equation gas turbulence closure model (transport equations for the fluctuation energy and its dissipation rate) which does not require the use of wall functions. Detailed information regarding the local flow structure, such as the particle concentration distribution, is reported. Louge *et al.* (1991) did not show these profiles in their paper. We investigate the description for the correlation of the velocity fluctuations between the two phases proposed by Louge *et al.* (1991). When the coefficient of restitution is decreased, particles lose more fluctuation energy when they collide. In these regions, the predicted solid volume fraction increases in an unrealistic fashion.

Interactions between the fluctuating components of the two velocity fields may sustain the particle fluctuations. It is well known that small particles can dampen gas phase turbulence and massive particles enhance it (Boothroyd, 1967; Owen, 1969; Lee and Durst, 1982; Modares, 1983; Hestroni, 1989; Gore and Crowe, 1989.) Along with the particle concentration profile, predictions are made for the mean and fluctuating velocity profiles for both phases in the case of fully developed flow and compared with the experimental data of Tsuji *et al.* (1984), Lee and Durst (1982), and Maeda *et al.* (1980).

2. Model Summary

Particle-laden, gas-solid flows can be successfully modeled following the Eulerian "two-fluid" method in which the solid *discrete* phase is regarded as a *continuous* phase for which a fundamental equations can be derived by volume-averaging techniques. Thus, the model includes mass, momentum and energy balances for each phase, closed by constitutive relations for the accurate representation of the physics arising from the interaction of both velocity fields. For the case of isothermal, steady, fully developed flow in a cylindrical vertical pipe, the fundamental equations required are summarized in Table 2.1; details regarding the derivation and specifics of these equations are found in Anderson and Jackson (1967), Sinclair and Jackson (1989), and Sinclair (1993).

Gas-Phase Momentum Balance (z component)

$$0 = \frac{1}{r} \frac{\partial}{\partial r} (r \tau_{rz}) - \beta (V_g - V_s) - \frac{\partial p}{\partial z}$$

Solid-Phase Momentum Balance (z component)

$$0 = -\frac{1}{r} \frac{\partial}{\partial r} (r \sigma_{rz}) + \beta (V_g - V_s) - \rho_s v g$$

Solid-Phase Momentum Balance (r component)

$$0 = \frac{1}{r} \frac{\partial}{\partial r} (r \sigma_{rr}) - \frac{\sigma_{\theta\theta}}{r}$$

Pseudo-thermal Energy Balance

$$0 = -\frac{1}{r} \frac{\partial}{\partial r} (r q_{PT}) - \sigma_{rz} \frac{\partial V_s}{\partial r} - \gamma + \beta(1-v)(\overline{v_{gi} v_{si}} - 3T)$$

Table 2.1. Fundamental equations for the description of axisymmetrical, steady, fully developed, particle-laden gas-solid flows

The turbulent stresses of the gas phase are modeled by a single-phase low Reynolds number k - ϵ closure, modified to account for the presence of a dilute solid phase. Among all the existing versions of low Reynolds number k - ϵ closures, the one proposed by Myong and Kasagi (1990) was chosen based on the results of a thorough performance analysis discussed in Sinclair (1993) and Hrenya *et al.* (1993). The gas viscous stresses are described by a modified intrinsic viscosity. Table 2.2. summarizes the gas-phase stresses.

The solid-phase stresses, the pseudo-thermal energy flux and the pseudo-thermal energy dissipation rate are closed with expressions derived by Lun *et al.* (1984) based on the kinetic-theory analogy. Lun *et al.* (1984) treat the particle collisions in an analogous fashion to collisions between molecules in a gas; hence, they build a granular constitutive theory by modifying the Chapman-Enskog kinetic theory of dense gases to take into account the inelastic nature of particle collisions. A summary of these expressions is found in Table 2.3.

Total Gas-Phase Stress

$$\tau_{rz} = (\mu_{eg} + \mu_t) \frac{\partial V_z}{\partial r}$$

$$v_{eg} = \frac{\mu_{eg}}{\rho_g} = \frac{\mu}{\rho_g} (1 + 2.5 v + 7.6 v^2) (1 - v/v_0)$$

$$v_t = \frac{\mu_t}{\rho_g} = \frac{c_\mu f_\mu k^2}{\epsilon}$$

Transport Equations for k and ϵ

$$0 = \frac{1}{r} \frac{\partial}{\partial r} \left[r (1-v) \left(v_{eg} + \frac{v_t}{\sigma_k} \right) \frac{\partial k}{\partial r} \right] + (1-v) v_t \left(\frac{\partial v_z}{\partial r} \right)^2 - (1-v) \epsilon - \beta (1-v) (2k - \overline{v_{gi} v_{si}})$$

$$0 = \frac{1}{r} \frac{\partial}{\partial r} \left[r (1-v) \left(v_{eg} + \frac{v_t}{\sigma_\epsilon} \right) \frac{\partial \epsilon}{\partial r} \right] + (1-v) c_1 \frac{\epsilon}{k} v_t \left(\frac{\partial V_z}{\partial r} \right)^2 - (1-v) c_2 f_2 \frac{\epsilon^2}{k} - (1-v) c_3 f_2 \frac{\epsilon}{k} \beta (2k - \overline{v_{gi} v_{si}})$$

$$f_\mu = \exp \left[\frac{-3.4}{(1 + R_T/50)^2} \right]$$

$$f_2 = 1 - 0.3 \exp(-R_T^2)$$

$$R_T = \frac{\rho_g k^2}{\mu \epsilon}$$

Empirical constants

c_μ	σ_k	σ_ϵ	c_1	c_2	c_3
0.09	1.0	1.3	1.44	1.92	1.2

Table 2.2. Gas-phase stress description

Stress description

$$\sigma_{rz} = -\mu_s (\omega G_{1k} + G_{1c}) \frac{\partial V_s}{\partial r}$$

$$\sigma_{rr} = \sigma_{\theta\theta} = \rho_s (\omega G_{2k} + G_{2c}) T$$

$$G_{1k} = \frac{1}{\eta(2-\eta)g_0} \left[1 + \frac{8}{5} \eta v g_0 (3\eta - 2) \right]$$

$$G_{1c} = \frac{8v}{5(2-\eta)} \left[1 + \frac{8}{5} \eta v g_0 (3\eta - 2) \right] + \frac{768 v^2 g_0 \eta}{25 \pi}$$

$$G_{2k} = v$$

$$G_{2c} = 4\eta v^2 g_0$$

$$\mu_s = \frac{5\sqrt{\pi} d \rho_s \sqrt{T}}{96} \quad g_0 = \frac{v_0^{1/3}}{v_0^{1/3} - v^{1/3}} \quad \eta = \frac{1+\epsilon}{2}$$

Pseudo-thermal energy flux

$$q_{PT} = -\lambda (\omega G_{3k} + G_{3c}) \frac{\partial T}{\partial r}$$

$$G_{3k} = \frac{8}{\eta(41-33\eta)g_0} \left[1 + \frac{12}{5} \eta^2 v g_0 (4\eta - 3) \right]$$

$$G_{3c} = \frac{96v}{5(41-33\eta)} \left[1 + \frac{12}{5} \eta^2 v g_0 (4\eta - 3) + \frac{16}{15\pi} \eta v g_0 (41 - 33\eta) \right]$$

$$\lambda = \frac{25\sqrt{\pi} d \rho_s \sqrt{T}}{128}$$

Kinetic contribution damping function

$$\omega = \frac{1}{1 + 2\lambda_{mfp}/d}$$

$$\lambda_{mfp} = \frac{d}{6\sqrt{2} v}$$

Pseudo-thermal energy dissipation rate

$$\gamma = \frac{48}{\sqrt{\pi}} \eta (1 - \eta) g_0 v^2 \frac{\rho_s}{d} T^{3/2}$$

Table 2.3. Granular constitutive relations

The solid stress and pseudo-thermal energy flux expressions include a kinetic component which, as pointed out by Sinclair (1989) and Louge *et al.* (1991), does not render the correct limit for the empty pipe system where $v \rightarrow 0$. Some modifications have been suggested to correct this behavior. In this investigation we follow the approach of Louge *et al.* (1991) who damped the kinetic contribution with a function proportional to the mean-free path of the particles.

The momentum fluxes across the interface are present at the level of mean and fluctuating velocity components. From the interaction between the mean velocity components of both phases, forces such as the drag force, by which the particles are propelled, result. The drag force is represented in the model with the expression put forth by Ding and Gidaspow (1990). At the fluctuating velocity level, the gas turbulence may be enhanced or dampened by the particle velocity fluctuations, and the reverse effect would be present in the solid phase. This interaction is described following the approach of Louge *et al.* (1991) who use an expression derived rigorously by Koch (1990) modified for high Reynolds numbers. The expressions employed for the description of the interfacial momentum fluxes in the model are summarized in Table 2.4.

Drag Force

$$\beta = \frac{3}{4} \frac{\rho_g}{d} C_D \frac{v}{(1-v)^{2.65}} |V_g - V_s|$$

$$C_D = \frac{24}{Re_p} (1 + 0.15 Re_p^{0.687}) \quad Re_p \leq 1,000$$

$$C_D = 0.44 \quad Re_p > 1,000$$

$$Re_p = \frac{(1-v) \rho_g d |V_g - V_s|}{\mu}$$

Fluctuating-Velocity Correlation

$$\overline{v_{gi}' v_{si}'} = \frac{4}{\sqrt{\pi}} \frac{d}{\rho_s} \beta' \frac{(1-v)}{v} \frac{(V_g - V_s)^2}{\sqrt{T}}$$

$$\beta' = \frac{3}{4} \frac{\rho_g}{d} C_D' \frac{v}{(1-v)^{2.65}} |\sqrt{2k} - \sqrt{3T}|$$

$$C_D' = \frac{24}{Re_p'} (1 + 0.15 Re_p'^{0.687}) \quad Re_p' \leq 1,000$$

$$C_D' = 0.44 \quad Re_p' > 1,000$$

$$Re_p' = \frac{(1-v) \rho_g d |\sqrt{2k} - \sqrt{3T}|}{\mu}$$

Table 2.4. Closures for the momentum fluxes across the interface

Boundary conditions for cylindrical geometry include symmetry at the centerline of the pipe. At the wall, the mean and turbulent kinetic energy k are both set to zero. Boundary conditions for the particle velocity and the pseudo-thermal temperature were derived by Johnson and Jackson (1987). For the solid velocity, the limit of the shear stress in the particle phase, on approaching the wall, is equated to the transfer rate of momentum to the wall by particles that collide with it.

$$\sigma_{rz} = \frac{\pi \rho_s V_s \phi \sqrt{T}}{2\sqrt{3} \left(\frac{v_0}{v} - \frac{v_0^{2/3}}{v^{2/3}} \right)}$$

A boundary condition for the pseudo-thermal temperature at the wall is obtained from an energy balance on a thin region adjacent to the solid surface.

$$q_{PT} + \frac{\pi \rho_s V_s^2 \phi \gamma T}{2\sqrt{3} \left(\frac{v_0}{v} - \frac{v_0^{2/3}}{v^{2/3}} \right)} - \frac{\sqrt{3} \pi \rho_s (1 - c_w^2) T^{3/2}}{4 \left(\frac{v_0}{v} - \frac{v_0^{2/3}}{v^{2/3}} \right)} = 0$$

For the gas turbulent kinetic energy dissipation rate ϵ , the boundary condition given by Myong and Kasagi (1990) is used.

$$\epsilon = \frac{4 \mu k_{next}}{\rho_s y_{next}^2} - \epsilon_{next}$$

The equations summarized previously yield a set of five second-order, non-linear, ordinary differential equations, and a finite-difference marching technique is used to solve them. The particular finite-difference marching technique used is an adaptation of a widely tested method developed by Patankar and Spalding (1967).

3. Comparison of the Model Predictions with Experimental Data

For the comparisons of the model predictions, the experimental data of Tsuji *et al.* (1984), Lee and Durst (1982) and Maeda *et al.* (1980) are selected. In all of these experiments, laser Doppler anemometry techniques were employed, and the experiments were conducted in glass pipes with air as the fluid phase. Tsuji *et al.* (1984) employed polystyrene beads, and Lee and Durst (1982) and Maeda *et al.* (1980) used glass beads.

To make the comparisons, the pressure gradient and the solid volume fraction at the centerline were adjusted in order to match the reported gas velocity at the centerline and the mass loading ratio m or the solid flow rate. Values for the coefficient of restitution for particle-particle and particle-wall collisions were selected based on the

experimental data by Lun and Savage (1986). For the case of glass-glass, polystyrene-polystyrene, and glass-polystyrene collisions, the values of 0.94, 0.9, and 0.7, respectively, were assigned to these coefficients. Since no independent measurement for the specularity factor is available, an approximate value of 0.002 was determined by Sinclair (1993); this value is held constant throughout all the comparisons. An analysis of the sensitivity of the model predictions to the values assigned to these properties is reported in the next section.

The first group of comparisons shows the mean velocity radial profiles for both phases. Figure 3.1 illustrates the lowest and highest loadings investigated by Tsuji *et al.* (1984) for the 200 μm system. Overall, the agreement is good. The model captures the qualitative flow differences observed at different loadings. As the loading increases, the slip or relative velocity decreases, and the crossover point (the point in the radial coordinate where the slip velocity changes sign) moves closer to the wall. The gas velocity profile flattens at the highest loading since the maximum gas velocity occurs away from the pipe centerline. Comparisons for the 500 μm system investigated by Tsuji *et al.* (1984) are shown in Figure 3.2.

Figure 3.3 shows a sequence of predictions for three systems, reported by Lee and Durst (1982), with an increasing particle diameter: 100 μm , 200 μm and 400 μm . The remaining physical properties of the three systems are kept constant; the loading and Re for the three systems are also very similar. The model predicts correctly the increase in slip velocity for larger particle sizes. The same observations hold true for Figure 3.4 which shows two systems investigated by Maeda *et al.* (1980) with different size particles. The particle diameters investigated were 45 μm and 136 μm ; the rest of the physical parameters, the loading, and Re were kept constant.

Figure 3.1. Comparison of model predictions for the mean gas and solid velocities with the experimental data of Tsuji *et al.* (1984). Particle diameter = 200 μm .

Figure 3.2. Comparison of model predictions for the mean gas and solid velocities with the experimental data of Tsuji *et al.* (1984). Particle diameter = 500 μm .

Figure 3.3. Comparison of model predictions for the mean gas and solid velocities with the experimental data of Lee and Durst (1982).

Figure 3.4. Comparison of model predictions for the mean gas and solid velocities with the experimental data of Maeda *et al.* (1980).

The next set of comparisons show predictions for the gas axial fluctuating velocity profile. Although the model predicts the fluctuating kinetic energy k , and not the individual components that make up k , comparisons with experimental data are presented at the level of the axial fluctuating velocity component since the other two components are not reported for two-phase flow. A disagreement between model predictions and experimental data for *one* fluctuating gas velocity component is expected because of the lack of perfect isotropic turbulence for the gas in pipe flow (Sinclair, 1993). The axial fluctuating component is largest near the pipe wall; hence, by employing the isotropic assumption, this component is expected to be underpredicted as is clearly illustrated by Figures 3.5 and 3.6. These figures compare the model prediction, at the no-particle limit, with the single-phase experimental data of Schildknecht *et al.* (1979). Even though both comparisons are done at exactly the same flow conditions, the predicted turbulent kinetic energy k shows a much better agreement with the data than the axial fluctuating gas velocity component, particularly in the near-wall region.

Figure 3.5. Comparison of the turbulent kinetic energy for a clear gas with the experimental data of Schildknecht *et al.* (1979). Pressure drop = -8.2096 Pa/m.

Figure 3.6. Comparison of the axial fluctuating velocity component for a clear gas with the experimental data of Schildknecht *et al.* (1979). Pressure drop = -8.2096 Pa/m.

Figure 3.7 shows comparisons for the axial fluctuating velocity component for the lowest and highest loadings reported by Tsuji *et al.* (1984) for the 200 μm system. The dashed line on this Figure shows the prediction of the model for the case of no particles present in the system. Some degree of turbulence modulation by the particle phase is observed when comparing the single-phase predictions with the two-phase data. The two-phase model predictions, corresponding to the solid lines, underpredict the axial fluctuating velocity throughout the domain; however, at least part of such disagreement is expected as discussed in the previous paragraph. The model predicts correctly the flattening observed at higher loadings, an improvement over simpler models which employ a pre-specified mixing length. The predictions for the 500 μm system by Tsuji *et al.* (1984), shown on Figure 3.8, corroborate this qualitative agreement. Figure 3.9 shows the effect of the particle diameter on the prediction for the fluctuating velocity

component. A fundamental disagreement between the data and the predictions is observed; while the reported centerline axial fluctuating velocity decreases as the particle size increases, the model predictions reflect the contrary. In small particle systems such as these, reported by Maeda *et al.* (1980), the presence of particles tends to suppress the gas turbulence. The particles follow the fluid motion, and the smaller the particles, the closer they follow the fluid eddies, resulting in a larger turbulence modulation (Hestroni, 1989), as the model predicts. However, validity of the model in this particle size range ($< 50 \mu\text{m}$) is questionable. With very small particles, it is conceivable that collisions do not play a very important role and the granular constitutive expressions may not represent correctly the physics of the solid assembly.

Figure 3.7. Comparison of model predictions for the axial fluctuating gas velocity with the experimental data of Tsuji *et al.* (1984). Particle diameter = 200 μm .

Figure 3.8. Comparison of model predictions for the axial fluctuating gas velocity with the experimental data of Tsuji *et al.* (1984). Particle diameter = 500 μm .

Figure 3.9. Comparison of model predictions for the axial fluctuating gas velocity with the experimental data of Maeda *et al.* (1980).

Lastly, Figure 3.10 shows that the model well predicts the pressure drop for the 500 μm system reported by Tsuji *et al.* (1984) over a range of gas mean velocity for a fixed solid flow rate.

Figure 3.10. Comparison of model predictions for the pressure drop with the experimental data of Tsuji *et al.* (1984). Particle diameter = 500 μm .

4. Effect of the Properties Describing the Particle Collisions

Particle collisions play a key role in the physics of two-phase flow and an adequate representation of the collision phenomenon is crucial for the accuracy of the predictions of the model. The physical properties involved in the description of collisions are the coefficients of restitution for particle-particle and particle-wall collisions and the specularity coefficient. In principle, these properties can be measured for different systems and materials; unfortunately, the availability of these measurements is still scarce. Furthermore, the coefficient of restitution is known to be a property that varies with the impact collision. A literature search conducted by our group provided values for the coefficient of restitution of some systems. For example, Lun and Savage (1986) indicated that for collisions between glass beads, the coefficient of restitution varies in the range of 0.85 to 0.97, with 0.94 as the most likely value. Collisions between polystyrene particles are known to be more dissipative than glass bead collisions. Experimental measurements for the specularity coefficient are simply nonexistent. The uncertainty of the numerical values of these properties, and the importance of the collisions motivated the sensitivity analysis which is shown next.

The 400 μm system by Lee and Durst (1982) is taken as the base case, and perturbations to the values of the three properties were made. Profiles for the gas and solid mean velocity, the gas axial fluctuating velocity, and solid volume fraction are shown for each perturbation. Figure 4.1. shows essentially no change in any of the reported profiles as the coefficient of restitution for particle-particle collisions is varied from 0.85 to 0.97. Similarly, Figure 4.2. shows only minor changes in the profiles as the coefficient of restitution for particle-wall collisions is varied from 0.85 to 0.97; however, the changes are larger than in the previous cases, especially in the near-wall region of the solid velocity profile and solid volume fraction. The specularity coefficient sensitivity is

analyzed in Figure 4.3. Due to the lack of experimental data for this property, two different perturbations were done: a "minor" perturbation where the value was changed from 0.002 to 0.001 and 0.003, and a "major" perturbation where the value was increased one and two orders of magnitude, that is to 0.02 and 0.2. The "minor" perturbations show no influence on any of the profiles; however, the "major" perturbations show a significant change in all the profiles, with the largest differences in the solid volume fraction and the axial fluctuating gas velocity.

Figure 4.1. Sensitivity of the model predictions to the value of the coefficient of restitution for particle-particle collisions

Figure 4.2. Sensitivity of the model predictions to the value of the coefficient of restitution for particle-wall collisions

Figure 4.3. Sensitivity of the model predictions to the value of the specular coefficient

7. Conclusions

A dilute gas/solid turbulent flow mathematical model is summarized, and a complete comparison of the model predictions with experimental data for the case of steady, fully developed flow is presented. In general, the agreement with available experimental data is good, particularly for the radial profiles for the mean gas and solid velocity. Flow features such as an off-centerline maximum gas velocity with higher particle loadings, a non-constant slip velocity, and the location of the crossover point of the mean velocities of both phases are correctly predicted. The comparisons indicate the capability of the model to capture the flow fields at different loadings and with different particle densities and sizes. The model predictions indicate an unrealistic degree of turbulence modulation by the particle phase. Our current research efforts are aimed towards improving this aspect of the model by further refining the closure for the interfacial flux of fluctuating kinetic energy.

The sensitivity of the model to the values of the physical properties describing the collisions is also investigated. The choice of values, within previously reported ranges, for the coefficients of restitution for particle-particle and particle-wall collisions does not appreciably affect the model predictions. However, the specular coefficient value has a considerable influence on the predictions, and an independent measurement of this physical property is needed.

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