Analytical Steam Injection Model for Layered Systems

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Abstract

Screening, evaluation and optimization of the steam flooding process in homogeneous reservoirs can be performed by using simple analytical predictive models. In the absence of any analytical model for layered reservoirs, at present, only numerical simulators can be used. And these are expensive.

In this study, an analytical model has been developed considering two isolated layers of differing permeabilities. The principle of equal flow potential is applied across the two layers. Gajdica's (1990) single layer linear steam drive model is extended for the layered system. The formulation accounts for variation of heat loss area in the higher permeability layer, and the development of a hot liquid zone in the lower permeability layer. These calculations also account for effects of viscosity, density, fractional flow curves and pressure drops in the hot liquid zone. Steam injection rate variations in the layers are represented by time weighted average rates. For steam zone calculations, Yortsos and Gavalas's (1981) upper bound method is used with a correction factor.

The results of the model are compared with a numerical simulator. Comparable oil and water flow rates, and breakthrough times were achieved for 100 cp oil. Results

with 10 cp and 1000 cp oils indicate the need to improve the formulation to properly handle differing oil viscosities.

Contents

Acknowledgements	iii
Abstract	v
List of Tables	xi
List of Figures	xiii
1 Introduction	1
2 Literature Survey	4
2.1 Steam Injection Methods	4
2.2 Steam-Drive Predictive Models	5
2.3 Modeling of Stratified Reservoirs	11
2.4 Thermal Numerical Simulators	11
3 Analytical Model	14
3.1 One Dimensional Model	14
3.1.1 Water and Steam Zone Saturations	17
3.1.2 Steam Zone Length	19
3.1.3 Water Zone Length	21
3.1.4 Steam Zone Steam Saturation	21

3.1.5	Pressure Drop in One Dimensional Model	21
3.2	Steam Zone Correction Factor	23
3.3	Variable Injection Rate	27
3.3.1	Time Weighted Average	27
3.4	General Description of Layered System	32
3.5	Model Geometry	35
3.6	Zone Definitions	36
3.7	Boundary Conditions	37
3.8	Heat Losses to Adjacent Formations	37
3.9	Hot Liquid Zone	39
3.10	Pressure Drops Across a Layer	42
3.11	Effects of Higher Temperature in the Hot Liquid Zone	42
3.11.1	Viscosity	43
3.11.2	Fractional Flow Curve	44
3.11.3	Thermal Expansion	46
3.11.4	Relative Permeability	46
3.12	Steam Injectivity in Each Layer	47
3.13	Concluding Remarks	48
4	Results	50
4.1	Base Case Data	50
4.2	Grid Selection for STARS	55
4.3	Results	57

4.4 Concluding Remarks	78
5 Conclusions	81
5.0.1 Recommendations	82
Bibliography	84
Nomenclature	89
Appendix	92
A Model Assumptions	92
A.1 SAM General Assumptions	92
A.2 Phase Relationships	93
A.3 Energy Assumptions	94
A.4 Initial Conditions	94
A.5 Boundary Conditions	95

List of Tables

4.1	Relative Permeability Data for Corey Relation	52
4.2	Base Case Data	53
4.3	Base Case Data (Contd.)	54
4.4	Comparison of Breakthrough Times	79
A.1	Relationships Between Phases and Components	94

List of Figures

3.1	Zone Definition for One Dimensional Model	15
3.2	Fractional Flow Curve: Showing the Tangent Construction	18
3.3	Comparison of Heat Losses in Two Sided System	25
3.4	Comparison of Heat Losses in One Sided System	26
3.5	Case 1: Variable Flow Rate in Single Layer	29
3.6	Case 2: Variable Flow Rate in Single Layer	31
3.7	Case 3: Variable Flow Rate in Single Layer	33
3.8	Zone Definitions for Layered System	36
3.9	Variation of Oil and Water Viscosity with Temperature	43
3.10	Change in Fractional Flow Curve with Temperature Variation	45
3.11	Schematic of Flow Diagram of Analytical Model	49
4.1	Relative Permeability Curves used in the Analytical Model	52
4.2	Effect of Number of Grid Blocks on Production Rate Predictions.	56
4.3	Comparison of Oil and Water Flow Rates, Permeability Ratio, 1:2.5	60
4.4	Comparison of Oil Saturations in Both Layers at Various Times	61
4.5	Comparison of Steam Saturations in Both Layers at Various Times	63

4.6	Comparison of Oil and Water Flow Rates, Permeability Ratio, 1:1.1 .	65
4.7	Comparison of Oil and Water Flow Rates, Permeability Ratio, 1:1.4 .	66
4.8	Comparison of Oil and Water Flow Rates, Permeability Ratio, 1:1.8 .	68
4.9	Comparison of Oil and Water Flow Rates, Permeability Ratio, 1:2.2 .	69
4.10	Comparison of Oil and Water Flow Rates, Permeability Ratio, 1:2.6 .	70
4.11	Comparison of Oil and Water Flow Rates, Permeability Ratio, 1:3.0 .	71
4.12	Comparison of Oil and Water Flow Rates, Permeability Ratio, 1:5.0 .	73
4.13	Comparison of Oil and Water Flow Rates For Low Viscosity Oil, Per- meability Ratio, 1:2.5	75
4.14	Comparison of Oil and Water Flow Ratesfor High Viscosity Oil, Per- meability Ratio, 1:2.5	76

Chapter 1

Introduction

Steam drive, also known as steam flooding, is one of the most widely used methods of enhanced oil recovery. In this process, steam is injected continuously into the reservoir through injection wells. The oil is heated, its mobility is increased, and oil saturation in the steam zone is reduced to residual oil saturation. The displaced oil is driven toward the production wells. Some other mechanisms such as distillation of light components also contribute to increased oil production.

A thermal simulator is generally used for the detailed design of a steam flooding operation. The heat and mass balance equations describing the process are solved numerically by using either a finite difference or a finite element technique. However, these numerical techniques are expensive, and can only be implemented on a mainframe computer having a large computer memory. For homogeneous systems, analytical predictive models have been developed which are much faster than numerical methods. Prior to the initiation of a pilot scale project, or preparation of a design

CHAPTER 1. INTRODUCTION

and plan for field scale steam flooding project, screening and evaluation of a candidate field can be carried out by using these predictive models. The process may also be monitored and optimized by using these models. Some of these analytical steam drive models also take into account phenomena such as gravity override, the build up of an oil bank ahead of the steam zone, and conduction of heat from the steam zone.

No analytical steam drive model, however, has been reported in the literature to predict the response of a layered reservoir. In the absence of a simple method, evaluation, optimization and monitoring of steam drive in such fields can only be carried out by using numerical simulators. The steam drive process in a layered reservoir is more complex than in a homogeneous system. The process involves heat transfer between the layers and steam channeling through the more permeable streaks. Thus, monitoring and optimization of a steam drive in a layered reservoir requires more simulator runs as compared to a homogeneous reservoir. Therefore, there is a need to develop an analytical model for layered reservoirs which may be used as a tool for preliminary evaluation, monitoring and optimization of the process at a reasonable cost.

The purpose of this study is to develop an analytical steam drive model for layered reservoirs. The developed model considers only two isolated layers separated by a thin impermeable medium, so that there is no cross flow of mass between the layers, however, heat transfer is allowed to occur between them. Linear flow is assumed in each layer, and the gravity effects are neglected. The model considers a single producer and injection well completed through both layers at the ends of a linear reservoir. With these simplifications, at any time, the steam injectivity in a layer is

such that the flow potential across the two layers is equal. A technique, similar to that of Dykstra and Parsons's (1950) method for water flooding in layered reservoirs, can be applied to estimate the water and oil flow rates.

The model assumes development of three zones, steam, water and oil, in the high permeability layer. In the low permeability layer, an additional hot liquid zone is included to account for the heat transfer effects between the layers. The zone lengths, the saturation of oil, water and steam in each zone and the pressure drop across these zones are calculated by using a modification of the one dimensional semi-analytical model developed by Gajdica et.al (1990). An overall material balance gives the oil and water flow rates in each layer.

In a steam drive process, the oil production rate decreases after water or steam breakthrough in a layer. So profitability of the operation decreases after breakthrough. Thereafter, the operation may become uneconomical or may require some remedial measure like foam injection. Thus the determination of breakthrough time is important for evaluation of a steam drive prospect. The developed model predicts the water and steam breakthrough times in each layer.

Chapter 2 gives a brief survey of existing analytical models. Chapter 3 presents the development of the analytical model for two isolated linear layers. The results of the developed model are presented in Chapter 4 and compared with the numerical simulator, 'Steam and Additive Reservoir Simulator' (STARS), developed by the Computer Modeling Group. Conclusions and recommendations are included in Chapter 5.

Chapter 2

Literature Survey

This chapter describes some of the presently available steam flooding analytical models. The numerical simulators currently available are also briefly discussed. The chapter also includes the description of the Computer Modelling Group's thermal numerical simulator, STARS. The objective of the development of an analytical model for layered reservoirs is also outlined in this chapter.

2.1 Steam Injection Methods

Two steam injection methods are commonly used: cyclic steam injection, and steam flooding.

In the cyclic steam method, also known as steam stimulation or steam soak, steam injection is carried out for a certain time, then the well is shut in for a short period of time, and afterwards oil is produced from the same wellbore. For cyclic steaming, a good analytical predictive method was developed by Gontijo and Aziz

(1984). Their method has been used by Barua (1990) to optimize steam injection, soak, and production times.

The second method, known as steam flooding or steam drive, refers to the process where steam is injected in wells to drive and displace oil toward production wells. This process is similar to other drive methods such as water flooding. However, steam injection, in addition to providing a pressure gradient to increase the flow through the reservoir, also effects the viscosity of the oil as the temperature of the reservoir is increased. Many factors contribute to oil production from steam-drive. These include the low residual oil saturation in the steam swept zone, distillation of light components in the oil, and a water bank ahead of the steam zone which is formed by condensate. The gravity override and channeling of steam through more permeable streaks, on the other hand, can adversely effect the process.

Analytical modeling of the steam drive process is considered next.

2.2 Steam-Drive Predictive Models

Many analytical solutions are available in the literature for prediction of oil recovery under the steam drive process in homogeneous reservoirs. A brief review of some of these methods is presented in this section.

The first and simplest steam injection model was developed by Marx and Langenheim(1959). The model, based on a simplified heat balance equation, estimates steam invasion rates, and cumulative heated areas. Heat losses are considered in the vertical direction towards the base and cap rocks. A piston-like steam zone growth and a

CHAPTER 2. LITERATURE SURVEY

constant steam zone temperature is assumed, and formation of a water bank ahead of the steam zone is ignored. The horizontal temperature distribution in this model is represented by a step function. The resulting conduction heat transfer equation is solved in time to determine the heat losses from a moving steam front. The steam zone length is, then, estimated based on the remaining heat in the reservoir. After determining the steam zone length, the oil production is estimated by the overall material balance assuming a constant residual oil saturation in the steam zone.

The resulting equation is as follows,

$$N_p = \phi(S_{oi} - S_{or})WL_s \frac{H_n}{H_t} E_c \quad (2.1)$$

where N_p is the oil produced, ϕ is the porosity of the reservoir, S_{oi} is the oil saturation in the reservoir, S_{or} is the estimated residual oil saturation in the steam zone, W is the reservoir width, L_s is the steam zone length. The term, H_n is the net reservoir thickness, H_t is the gross reservoir thickness and E_c is the capture factor defined as the ratio of oil produced to the oil displaced by steam. In Eq. 2.1 the steam zone length, L_s , is the term calculated by the Marx and Langenheim model.

Ramey (1959) showed that the superposition principle may be applied to the Marx and Langenheim model when steam injection rates vary with time. Mandl and Volek (1969) defined a critical time after which the heat transfer by conduction across the condensation zone cannot be ignored. Myhill and Stegemeier(1978) extended the Marx and Langenheim model taking into account this critical time.

Gajdica(1990) observed that the Marx and Langenheim model overpredicts the

steam zone growth rate at later time as well as for low steam injection rates. Jensen et al.'s (1990) study concludes that the Marx-Langenheim and Myhill-Stegemeier models generally over-predict instantaneous oil production rates and oil steam ratios (OSR).

The reasons for higher production rates in the Marx-Langenheim and Myhill-Stegemeier models are embedded in the difference between actual physical phenomena occurring at the field level and the simplifying assumptions made in these models. The formation of a water bank ahead of the steam zone due to steam condensation and the process of steam gravity override is ignored in these models. Similarly the assumption of a step change in temperature and a uniform vertical temperature are over simplifications of the process. These factors cause errors in the calculations.

The effects of some of these phenomena can be taken into account by the capture factor, E_c , in Eq. 2.1 which is included to reflect the fraction of the displaced oil which is recovered. In practice, however, it is used to correct the calculations for any phenomenon. That is not addressed properly in the model. As a result, the estimation of E_c from first principles, prior to a steam flood, is not possible at this time. If the production is known for some time after the start of steam injection, the capture factor may be adjusted to achieve a history match, and thereafter, these models may predict more accurate oil production rates. Using the capture factor approach, Strom (1984) predicted the oil production rates for a pilot study and got a reasonable history match by using the simplest model of Marx and Langenheim. Projections of past history were then successfully used to evaluate the incremental production of a pilot scale surfactant injection project.

CHAPTER 2. LITERATURE SURVEY

Hutchison and Fattahi (1993) showed that, by performing a history match of the actual and calculated steam zone area using any simple predictive method, a conversion factor can be calculated for the steam zone. They showed that the future steam zone area predictions are much improved by using the calculated conversion factor.

Parametric and statistical analyses have also been used to develop correlations for steam flood predictions. Gomaa (1980) developed correlations for predicting steam flood oil recovery based on a parametric sensitivity analysis using the results from a numerical simulator. Parameters studied were porosity, saturation, net and gross thickness, injection rate, pressure, temperature and steam quality. Some effects were correlated through the use of a heat utilizing factor defined as a function of steam quality. Permeability of the reservoir, oil viscosity and gravity were not included as variables.

Jones (1981) extended the Myhill and Stegemeier model by using empirical correlations to account for the affects of oil viscosity, pattern area, and initial gas saturation. This model considers three stages of oil production. The production during the first stage is dominated by oil at the original reservoir temperature, while the second stage considers hot oil mobility. The later time recovery, the third stage, is dominated by the remaining oil in place. The model has advantages over the previous models of Marx and Langenheim and Myhill and Stegemeier in that it includes determination of the time of arrival and magnitude of the peak production rates.

As mentioned earlier, Jensen et al. (1990) performed history matching of four

models including the Gomaa and Jones models. The study concluded that the Gomaa and Jones models predict oil production rate and OSR fairly well for some classes of reservoir but are biased toward certain types of oils: Jones' model is biased toward heavy (14° API) and viscous (2000 cp) oil and high permeability (2000 md) reservoirs; while Gomaa's correlations are biased toward low pressure (60 psia) and low temperature (90° F) shallow reservoirs with oil viscosity of 1000 cp.

Jensen et al. (1991) developed a steam flood predictive model based in part on the Myhill and Stegemeier (1978) and van Lookeren (1983) models. In their model, Myhill-Stegemeier and van Lookeren steam zone expressions are equated, and the resulting equation is solved for time by iteration. The calculated time is called the time of breakthrough. Defining dimensionless groups based on the field parameters as well as the steam injection rate, the time of breakthrough and production rate at that time are statistically correlated using the observed field data. Correlations are also used to improve the post-breakthrough production rates determined by the Myhill-Stegemeier model. The results of this model were compared with field production, and it was observed that the model predictions of sixteen fields were better than those of the other models tested.

Gajdica et al. (1990) developed a semi-analytical model for a linear steam-drive based on steam zone calculations using the Yortsos and Gavalas (1981) upper bound method. In this model the reservoir is divided into three zones; undisturbed oil zone, water zone, and steam zone. The location of oil/water and steam/water fronts were calculated using fractional flow theory. Darcy's law was used to calculate the pressure drops in each zone keeping the production well bottom hole pressure constant. The

CHAPTER 2. LITERATURE SURVEY

model was further extended to two dimensional, linear homogeneous reservoirs. The shape of the steam zone is determined by an exponential function, where the exponent is a ratio of the viscous to the gravity forces. The vertical sweep efficiency was determined by using a 'Modified Craig Ratio', which is an extension of the 'Craig Ratio' used for water flooding. The results of this model for one and two dimensional cases were compared with a numerical thermal simulator, ISCOM, and good matches were achieved. The one dimensional part of this model was selected for further extension to layered reservoirs in the present report.

One last observation is made here regarding analytical modeling of homogeneous reservoirs. In Eq. 2.1 the effect of shales is accounted for by a ratio of net reservoir thickness, H_n , to gross reservoir thickness, H_t . All the models mentioned above account for shales or streaks in a homogeneous reservoir in the same way. Closmann (1967) presented an analytic solution for the growth of steam zones in multiple layers of equal permeabilities and equal thickness separated by uniformly thick impermeable shale layers. The steam injection was considered to be the same in each of the layers so that the steam zones grow at the same rate in each layer. The study concludes that at early times, the growth of any of the steam zones is independent of the others. At later times, the heat fluxes from adjacent layers interact to give larger steam zones. The paper suggests that steam should be injected simultaneously into all the layers, or at least lead into more than one layer, depending upon the steam generating capacity. No predictive model, however, was developed based on this study.

2.3 Modeling of Stratified Reservoirs

All the above analytic models and others in the literature assume a homogeneous isotropic reservoir. In realistic situations, the steam channels through more permeable strata, which leads to low oil recoveries from the less permeable zones. Presently only numerical simulators are used to model layered reservoirs. These numerical simulators are expensive and require much more time to run. The purpose of this study is to develop a simple analytical model for layered systems. The model will be an extension of Gajdica's 1-D model, as it gives more realistic oil production rates than do other models, and it includes pressure drop calculations. However, the model developed herein for the present, will consider only two layers of differing permeabilities separated by a very thin impermeable barrier. The proposed model will not replace the existing numerical simulators, but may be useful for prescreening, optimization, or the quick operative tool for field studies. For comparison of results, a numerical simulator will be used. In the next section, numerical simulation is briefly described.

2.4 Thermal Numerical Simulators

Thermal numerical simulators have been developed to model the oil recovery process. In a thermal simulator, mass, energy and phase equilibrium equations are discretized and solved using some numerical technique. Many models starting from linear, three

CHAPTER 2. LITERATURE SURVEY

phase composition models (Weinstein et al. 1977) to 3-D multi-component, multi-phase simulators (Coats, 1974; Ferrer and Farouq Ali, 1977; Coats, 1976) are available in the literature. Some of these simulators are quite flexible and include options such as rectangular or radial gridding, local grid refinement, variable gridding, or wellbore heat losses (Rubin and Buchanan, 1985). The numerical simulation techniques have advanced rapidly with the advent of fast machines as well as with robust and accurate methods for matrix solutions. For steam flooding, the developed numerical simulators can be applied to include the mechanisms of oil distillation and gas condensation (Coats, 1976).

Any suitable 2-D thermal numerical simulator may be used to study steam flooding in a layered reservoir. The channeling of steam through more permeable layers can be easily modeled in a thermal simulator. Surfactant/foam injection is a remedy that can be partially applied to channeling and gravity override in steam flooding. Its mechanism and the effects of oil on foam are not fully understood yet, however, attempts have been made to simulate this process. The work on two models for surfactant/foam injection, namely the population balance model (Ransohoff and Radke, 1988; Friedmann et al., 1991) and mechanistic model (Fall et al., 1989; Friedmann et al., 1991) have already been reported in the literature.

The results of the analytical model developed in this study are compared with the numerical simulator, STARS. STARS is a three phase, multi-component thermal and steam additive commercial simulator developed by the Computer Modelling Group. A Cartesian or cylindrical with fixed or variable gridding configuration in two or three dimensions may be specified in the simulator. The simulator also includes a facility

for local grid refinement. STARS can be run in the fully implicit or the adaptive implicit mode. It may be used for simulation of hot water injection, steam stimulation, steam flooding, and dry or wet combustion. Naturally fractured reservoirs can also be modeled using STARS. Two options are provided for modeling foam injection, though there is considerable doubt whether either of these options properly match observed foam flow behavior.

Chapter 3

Analytical Model

The analytic model developed for steam flooding in a linear layered reservoir is described in this chapter. Two adjacent isolated layers of differing permeability are considered. The model is an extension of Gajdica's (1990) one dimensional model. That model is described first. The layered system considered in the present study is presented along with the methodology adopted for extension of the one dimensional model to the layered system. The main assumptions and geometry of the layered model are also described. Finally, the basic equations used in the model and the sequence of calculations are explained.

3.1 One Dimensional Model

This section describes Gajdica's one-dimensional analytic model for a linear reservoir of uniform porosity and permeability. In this model, the length is expressed in the x-direction, even if there is a dip to the reservoir, the width is the y-direction, and

Figure 3.1: Zone Definition for One Dimensional Model

the height is the z -direction. All positional functions are assumed to be a function of position along the x -direction only, so the system is one-dimensional. An injection and a production well are located at the ends of the system.

The model considers three zones; the steam, water and oil zones shown in Fig. 3.1. Near the injection well is the steam zone. Its temperature is calculated at the average pressure of the zone, and is assumed constant in the entire zone. This is the only zone where a gas (steam) phase exists. As steam moves away from the injection well, it condenses by losing heat to reservoir fluids and rock, and to the adjacent formations. The point at which the steam is completely condensed is the steam front. This front acts as a boundary between the steam and the water zones. The water supplied by the condensation of steam flows freely in the water zone. This mobile water displaces the mobile oil in the water zone. The zone between the production well and water zone is the oil zone. The water and oil zones are separated by the water front. The water and oil zones are assumed to remain at initial reservoir temperature. Detailed calculations

CHAPTER 3. ANALYTICAL MODEL

by Gajdica and by Wingard and Orr (October 1990) proved these assumptions to be correct.

Wet steam is injected at a constant rate and enthalpy into an injection well, and a constant flowing bottomhole pressure is maintained at the production well. The only mass flow to and from the system occurs at the wells. Heat is allowed to flow to the adjacent formations in the z -direction only.

The one dimensional model determines the location of the steam and water fronts and the average saturations in each of the three zones at any given time. The steam front location is determined by using the Yortsos and Gavalas (1981) upper bound method. Fractional flow calculations are used to determine the steam zone steam saturation, the water saturation in the water zone and the water front location. The steam saturation is corrected for condensation of steam, and the water front location is corrected for the volume of the steam zone.

Pressure drops across the three reservoir zones are calculated using Darcy's law. The process is begun by assuming the pressure at the injection well. The calculations are first carried out for the location of the steam and water fronts, followed by component saturations in the zones, pressure drop across each zone, and then pressure drops at the injection and production wells. Since the pressure at the production well is taken to be constant, the injection well pressure can be calculated from the pressure drops. Comparing the assumed and calculated pressures at the injection well, the process is repeated with a new guess of the injection pressure until convergence is achieved. The production rates can then be calculated by material balance. The main equations and steps used to perform these calculations are outlined next.

3.1.1 Water and Steam Zone Saturations

The water saturation in the water zone and steam saturation in the steam zone are calculated using fractional flow theory. The steam saturations are adjusted for steam condensation after the steam zone length is known.

Fractional flow equations calculate displacement of an in-place fluid by an injected fluid. Steam is the displacing fluid in the steam zone, while the displacing fluid in the water zone is condensate (water). The displaced fluid in the water zone is oil, while in the steam zone, liquid, both oil and water, is the displaced fluid. The concept of two phase relative permeability is used to construct the fractional flow curves.

The equation for the fractional flow of water displacing oil, f_w , neglecting the capillary pressure, is given as,

$$f_w = \frac{1 - [7.8264 * 10^{-6} k_x k_{ro} A (\rho_w - \rho_o) \sin \theta] / [\mu_o q_t]}{1 + [\mu_w k_{ro}] / [\mu_o k_{rw}]} \quad (3.1)$$

where k_x is the permeability in the x direction, k_r is the relative permeability, A is the cross-sectional area, μ is the viscosity, q_t is the total flow rate, ρ is the density and θ is the formation dip. Note that the formation does not need to be horizontal.

The average water saturation behind the water front can be determined by constructing a fractional flow curve using Equation 3.1. A tangent line is constructed, starting from the point of irreducible water saturation and zero fractional flow of water and extending tangent to the fractional flow curve, as shown in Fig. 3.2. The point where the tangent line intersects a water fractional flow value of unity determines the average saturation, $\bar{S}_w$, behind the front. In the model, this tangent line is determined numerically. The unadjusted steam saturation in the steam zone is also

CHAPTER 3. ANALYTICAL MODEL

Figure 3.2: Fractional Flow Curve: Showing the Tangent Construction

derived from the tangent construction using fractional flow calculations.

3.1.2 Steam Zone Length

The most important analytical step is the calculation of the steam zone length, i.e. the steam front location. This step involves the solution of both heat and mass balance equations using the method by Yortsos and Gavalas (1981). Their solution for location of the steam front gives equations for two differing upper bounds. At constant injection rate, the steam zone growth at early time is controlled by the bound based on the total heat balance, and late time growth is controlled by the bound based on the latent heat balance.

Defining the dimensionless steam zone length, L_{sD} , as,

$$L_{sD} = \frac{2a^2 L_s}{[(w_s + w_w)C_w \Delta T + w_s L_v]M_1 \Delta T} \quad (3.2)$$

the dimensionless steam zone length L_{sD} for the early time is given by,

$$L_{sD} = \sqrt{t_D} - 1 + \exp(-\sqrt{t_D}) \quad (3.3)$$

and for late time, the equation is,

$$L_{sD} = F[\sqrt{t_D} - \frac{M_2}{M_1} + \frac{M_2}{M_1} \exp(-\frac{M_1}{M_2} \sqrt{t_D})] \quad (3.4)$$

where M_1 is the reservoir volumetric heat capacity, M_2 is the volumetric latent heat for the steam phase, t_D is the dimensionless time, and F is the ratio of the latent heat to the total heat injected. These terms are defined as,

$$M_1 = \phi(\sum_{i=w,o,g} C_i \rho_i S_i) + (1 - \phi)\rho_R C_R + \phi \frac{L_v \rho_g S_g}{\Delta T} \quad (3.5)$$

CHAPTER 3. ANALYTICAL MODEL

$$M_2 = \phi \frac{L_v \rho_g S_g}{\Delta T} \quad (3.6)$$

$$t_D = t \left(\frac{2a}{M_1 \Delta T} \right)^2 \quad (3.7)$$

$$F = \frac{f_s}{f_s + \frac{C_w \Delta T}{L_v}} = \frac{w_s L_v}{w_s L_v + (w_s + w_w) C_w \Delta T} \quad (3.8)$$

The heat loss parameter, a , in Eq. 3.7 is defined as,

$$a = \frac{2\lambda_{ob} \Delta T}{H \sqrt{\pi \alpha_{ob}}} \quad (3.9)$$

The symbols used in the above equations are as follows: ϕ is the reservoir porosity, ρ_i is the density of component i , the components being water, oil, and steam represented by w, o, s respectively, C_i is heat capacity of component i , and S_i is its saturation, C_R and ρ_R are the reservoir rock heat capacity and density, H is the height of the reservoir, w_s is the mass injection rate of steam, w_w is the mass injection rate of water, L_v is the latent heat of vaporization of steam, C_w is the heat capacity of the liquid phase, ΔT is the temperature difference between the steam zone temperature and initial reservoir temperature, λ_{ob} is the thermal conductivity of the overburden, π is 3.14159 and α_{ob} is the thermal diffusivity of the overburden.

The length of the steam zone by the upper bound method then can be calculated using Eqs. 3.2 to 3.9. Gajdica (1990), however, used a correction factor of 0.79 to reduce the calculated steam zone length. A rational for this factor is presented in Section 3.2. of this study using a comparison of heat losses.

3.1.3 Water Zone Length

The distance to the water front is calculated using the average water saturation behind the water front, $\bar{S}_w$ and the volume of injected fluid. The volume of fluid injected is adjusted for the extra volume of the steam phase in the steam zone.

$$L_{s+w} = \frac{Q_{res}}{\phi A(\bar{S}_w - S_{wi})} \quad (3.10)$$

where L_{s+w} is the sum of the lengths of the steam and water zones and Q_{res} is the volume of the injected fluid. All volumes are expressed at reservoir conditions.

3.1.4 Steam Zone Steam Saturation

The steam zone steam saturation is calculated using the value from fractional flow theory as a initial guess. The saturation is then reduced by the volume of the steam condensed due to heat losses. The heat lost to the adjacent formations and amount of heat used to heat the reservoir are calculated by a heat balance. This amount of lost heat, Q_{loss} is converted to volume of condensed steam by using the following expression.

$$V_{s,con} = \frac{Q_{loss}}{L_v \rho_s} \quad (3.11)$$

3.1.5 Pressure Drop in One Dimensional Model

The pressure drop across the linear system is the sum of the individual pressure drops: the injection well, the steam, water and oil zones, and the production well.

CHAPTER 3. ANALYTICAL MODEL

The pressure drop across the injection well is given by

$$\Delta p_{inj} = \frac{q_{winj} \rho_{wsc}}{I_w \rho_w} \quad (3.12)$$

where I_w is the injectivity index, ρ_{wsc} is the water density at standard conditions, and ρ_w is water density at bottomhole conditions.

The pressure drop in the steam, water, and oil zones is calculated from Darcy's law for multiphase flow. Let L_j be the length of a zone. The pressure drop including gravity effects in the zone is then given by,

$$\Delta p_j = \frac{q_t L_j}{0.001127 k_x A \sum_{i=o,w,g} (k_{ri}/\mu_i)} + \frac{\rho_{avg} L_j \sin \theta}{144} \quad (3.13)$$

where q_t is the total flow rate of the reservoir fluids at the zone conditions and ρ_{avg} is the saturation weighted average density of the fluids in the zone, expressed as,

$$\rho_{avg} = \sum_{i=o,w,g} \rho_i S_i \quad (3.14)$$

The pressure drop into the production well is calculated using the following equation,

$$\Delta p_{prod} = \frac{q_t [\ln(\frac{cc}{r_w} \sqrt{\frac{(\Delta x)^2 + (\Delta y)^2}{\pi}}) + s]}{0.007081 k_x H (\sum_{i=o,w,g} \frac{k_{ri}}{\mu_i})} \quad (3.15)$$

where the term, cc , is a shape factor for the production well grid block, r_w is the wellbore radius, s is the skin factor and Δx and Δy are the dimensions of the production well grid block. The relative permeabilities are evaluated at oil zone saturations.

Equations 3.1 to 3.15 can then be used to develop a predictive steam drive model for a linear single homogeneous reservoir. The model described above has modified for a linear layered system. The modified model is described next.

3.2 Steam Zone Correction Factor

The lower of the two upper bounds, Eq. 3.3 and Eq. 3.4, given by Yortsos and Gavalas (1981) determines the maximum possible steam zone length. A correction factor of 0.79 was used by Gajdica (1990) to estimate the actual steam zone length. In the present study, a comparison has been made between the cumulative heat losses calculated by the analytical method and the losses determined by the numerical simulator, STARS. This comparison provides a possible explanation for the steam zone correction factor he used on the upper bound method.

It was observed that the losses determined by the analytical method were always less than the heat losses determined by STARS, if no correction factor was used. The two losses become comparable by using a factor between 0.76 to 0.83 on the calculated value of the steam zone. In other words, the heat losses needed to be increased by the inverse of that factor. In most cases, a factor of 0.79 gave a reasonable match and was used in this study. Some results of these comparisons are presented here.

For the analytical model, the heat losses are calculated by the overall heat balance. It is assumed that the temperature within the steam zone is constant, and is a function only of the steam zone pressure. Further it is assumed that all the heat in the reservoir is contained in the steam zone only. The heat losses are, then, the difference between the total heat injected and that remaining in the reservoir. Thus the cumulative heat losses, Q_{loss} , are given by

$$Q_{loss} = Q_{tot} - Q_{rem} = Q_{inj}t - L_s \Delta TM_1 \quad (3.16)$$

CHAPTER 3. ANALYTICAL MODEL

where Q_{tot} is the total heat injected, Q_{rem} is the heat gained by the steam zone, M_1 is the reservoir volumetric heat capacity, Eq. 3.5, ΔT is the difference between the steam injection and initial reservoir temperatures, and L_s is the length of the steam zone.

The heat losses to the overburden are assumed to occur only in the vertical direction in both the analytical and the STARS model. In STARS, the heat losses to the adjacent formations are estimated by a semi-analytical method proposed by Vinsome and Westerveid (1980). However, the temperature in the steam zone is assumed to be uniform in the analytical method. In STARS, the steam zone temperature is not constant. There is a slight variation in the steam zone pressure in each grid block, which causes the temperature to vary slightly, with the highest temperature being near the injection well. The analytical method assumes no heat transfer in the horizontal direction, while no such condition is imposed in the numerical simulator. Thus the two methods use different approaches for heat loss calculations. The following paragraphs discuss and compare the heat losses calculated by both methods.

Figure 3.3 shows heat losses calculated as a function of time from a single layer losing heat from both top and bottom. The heat losses calculated by the analytical method are shown as a dotted line, while the solid line indicates losses calculated by STARS. The losses calculated by the analytical model using Eq. 3.16 are considerably lower. The differences increase with time. Figure 3.3 also shows the heat loss calculations using Eq. 3.16 with steam zone length multiplied by a factor of 0.79. These heat loss calculations are now in agreement with the simulator results. In general, this factor was used in all further calculations.

Figure 3.3: Comparison of Heat Losses in Two Sided System

CHAPTER 3. ANALYTICAL MODEL

Figure 3.4: Comparison of Heat Losses in One Sided System

The purpose of this study is to develop an analytical model for a layered system. In a two-layered system, the heat losses from one side of each layer will occur in the same way as they do in a single layer, while between the layers there is a complex interaction. To handle one side analytically, the heat loss parameter defined by Eq. 3.9 can be divided by two. Figure 3.4 indicates the heat losses calculated by STARS, has the uncorrected analytical method and by using a correction factor of 0.81. The corrected calculated heat losses are comparable.

3.3 Variable Injection Rate

The injectivity of steam will vary with time in a layered reservoir. The one layered model was modified for variable steam injection rate. The single layer model determines the steam zone length by the Yortsos and Gavalas upper bound method. The upper bounds are derived by using the Laplace transformation on the total energy balance equation. Two approaches can be adopted for variable steam injection rate. The first approach, time-weighted average injection rate, is simple but somewhat less accurate. The second is the use of superposition to adjust the steam zone length. In this study, a weighted average method was used. The following sections discuss the method and results of variable steam injection rates into a single layer system.

3.3.1 Time Weighted Average

In this study, a time-weighted average steam injection rate is calculated and used as a constant rate until the next change occurs in the steam rate. The one dimensional

CHAPTER 3. ANALYTICAL MODEL

model is modified for the time-weighted average rate as follows. Let the steam injection rate at time t be $(Q_{inj})_{ti}$ and let $(Q_{inj})_{ti+1}$ be the steam injection rate at time t_{i+1} ; then the average injection rate at t_{i+1} for a time step of Δt is given by,

$$(Q_{avg})_{ti+1} = \frac{(Q_{avg})_{ti}t_i + (Q_{inj})_{ti+1} \Delta t}{t_i + \Delta t} \quad (3.17)$$

where $(Q_{avg})_{ti}$ is the average steam injection rate at time t_i .

Three cases of varying steam injection rate are presented using this modification for flowrate calculation in a single homogeneous system.

Case 1

In Case 1, the steam injection rate is kept constant at a level of 300 BPD up to 365 days. All steam rates mentioned in this report are in cold water equivalent. The steam injection rate is then decreased to 150 BPD. Figure 3.5 shows the steam injection rates and the calculated oil and water production rates. The figures also show the results of the thermal simulator, STARS, as solid lines.

The oil flow rates determined by the analytical model closely match with the results of STARS starting from the initial time. As the steam rate is decreased from 300 BPD to 150 BPD after 365 days, the oil production rate starts decreasing for both models. The rate of decrease, however, is more gradual for STARS, while the analytical model indicates a sharp decline. This difference persists for a short time after which both rates again match reasonably well. The steam injection rate after 365 days is kept constant. Water breakthrough in the figure is indicated by the decline in oil flow rate at about 2000 days and the start of water production. The

CASE 1

Figure 3.5: Case 1: Variable Flow Rate in Single Layer

CHAPTER 3. ANALYTICAL MODEL

water breakthrough time determined by the analytic model is slightly delayed. A short time after the water breakthrough, both oil and water flow rates again match fairly well.

Case 2

The steam injection rate is kept constant at a level of 300 BPD for 365 days, as in Case 1. The steam injection rate is then decreased to 150 BPD up to 750 days. Thereafter, the injection rate is returned to 300 BPD and is kept constant for the rest of the run. The steam injection rate and the oil and water production profiles for the analytical model and the simulator for this case are shown as Fig. 3.6.

Figure 3.6 indicates that the oil flow rates determined by the model match very well with the results of STARS for the initial time. After the change in steam injection rate at one year, the trend observed with the change of steam injection rate is similar to that seen in Case 1. The effect of a decrease or an increase in steam injection rate is more gradual in the STARS response compared to abrupt changes in the analytical model. Except for this difference, the oil flow rates calculated by the thermal simulator and the analytical model are close to each other. The water breakthrough time also matches closely at about 1300 days. The water flow rates also closely match in both cases.

Case 3

In this case, the steam injection rate is kept constant at a level of 300 BPD, as in the previous two cases. After 365 days the steam injection rate is increased to 450 BPD

CASE 2

Figure 3.6: Case 2: Variable Flow Rate in Single Layer

CHAPTER 3. ANALYTICAL MODEL

compared to a decrease in earlier cases. This rate is maintained until 750 days and then dropped again to 300 BPD. The steam injection and oil and water production rates are shown in Figure 3.7.

The oil flow rates calculated by the model exhibit the same trend of sharp increase or decrease at the time of steam injection rate variation, while the same gradual changes are observed for STARS. However, the overall oil and water rates, and water breakthrough times calculated by the model fairly well tracks the main trends and magnitudes depicted by STARS. Water breakthrough time, however, is delayed more in the analytic model than it was in the previous cases run.

From the results of the above three cases, it may be concluded that the time-weighted average steam injection rate is adequate for the one dimensional model. The oil rates calculated by the analytical model are quite close to the results from STARS. Further, for a multiple layered system, total steam injection rate is kept constant. Thus the changes of steam injection rates in the individual layers will be quite small for a given time step and these changes will also be quite gradual. Thus the time-weighted average rates can be expected to encompass the trends into each layer.

3.4 General Description of Layered System

The reservoir considered in the layered analytical model consists of two layers of differing permeability. Within each layer, permeability is assumed constant in all directions. The two layers are assumed to be isolated from each other by a thin

CASE 3

Figure 3.7: Case 3: Variable Flow Rate in Single Layer

CHAPTER 3. ANALYTICAL MODEL

impermeable medium. Thus there is no mass cross-flow between the layers. The mass is thus conserved not only within the reservoir but also within each layer. The assumption of no cross flow between layers allows independent flow calculations for each layer. A uniform porosity is assumed for the entire reservoir. Heat transfer is allowed across the layers. All heat transfer is assumed to be only along the vertical axis.

The system consists of an injection well at one end completed through the entire reservoir thickness in both layers, and the production well is also completed in both layers at the other end. Steam is injected into the system at a constant rate and enthalpy, while a constant bottomhole pressure condition is imposed at the production well. A complete list of assumptions made in the analytical layered model is given at Appendix A.

Each layer in the system is continuous from well to well, uniform in properties and is isolated from the other layer except at the wellbores. A method similar to Dykstra-Parsons which was applied to water flooding, can be applied to the system described above for steam flooding. However, the two processes, waterflooding or steamflooding in a stratified reservoir, differ in the following aspects:

1. Waterflooding consists only of one shock wave and thus consists of only an oil/water front. Steam flooding is represented by at least two shock fronts, the steam/water (condensate) front and water/oil front. Some studies even consider more fronts considering the temperature variation in the water zone due to longitudinal conduction. These effects are usually small and may be neglected. However, for steam flooding in a stratified reservoir, the temperature is higher in the liquid

zone of the lower permeability layer in the area adjacent to steam zone of the higher permeability layer. This leads to a need for at least one other zone between the steam and the "cold" liquid zone.

2. Steam zone growth is not only a function of the permeability but also of the mass and energy balance parameters. The heat losses in the system and the heat capacities of the reservoir and its fluids all play a role in determining the lengths of the steam zones.

3. The pressure drops across the production and injection wells need to be calculated in more zones.

The principle, however, remains the same. The injectivity of steam into each layer will vary with time, even for a constant total steam injection rate. The total potential drops, however, will be equal across each layer. This principle may be applied to model steam flooding on layered reservoirs.

In the next section, the model geometry, and the zones considered in each layer are defined.

3.5 Model Geometry

The model herein is based on a Cartesian coordinate system. The length is expressed in x-direction, the width is the y-direction, and the height is the z-direction. Within a layer, all variables which are functions of position change along the x-direction only. Each layer is thus treated as a one-dimensional system. Thus, if one considers a plane that intersects the reservoir at right angles to the x-coordinate, the pressures,

CHAPTER 3. ANALYTICAL MODEL

Figure 3.8: Zone Definitions for Layered System

temperatures, and saturations in this plane are uniform within a layer. The permeability of each layer is assumed constant. A uniform porosity is assumed for the entire reservoir. The reservoir is, thus, vertically stratified in permeability. The system is illustrated in Fig. 3.8.

3.6 Zone Definitions

Each layer is a one-dimensional system, and is divided into three distinct zones, as shown in Fig. 3.8. These zones are similar to the single layer one-dimensional system. In the higher permeability layer, the temperatures in the water and oil zones are assumed to remain at the initial reservoir temperature. In the lower permeability layer, a 'hot' liquid exists. This zone consists of a liquid section adjacent to the steam

zone section in the higher permeability layer. At any time, the extent of the hot liquid zone will depend upon the relative progress of the steam zones in the two layers. This section may include only a small portion of the water zone or the entire water zone or even some or all of the oil zone in the layer. The temperature of the hot liquid zone is calculated using heat losses from the higher permeability layer determined by the heat conduction equation. The temperature in the remaining water or oil zone is assumed to remain at the initial reservoir temperature.

3.7 Boundary Conditions

A constant steam injection rate is applied as a boundary condition at the injection well. The quality of the steam and injection temperature also remain constant for the entire process. The relative amount of steam injection in each layer varies with time. The pressure drop across the reservoir remains consistent in both the layers at any time. At the production well, a constant pressure condition is imposed. For mass, a no flow boundary is taken across the entire reservoir. Heat transfer to the adjacent formations and to adjacent layers is allowed but only in the vertical direction.

3.8 Heat Losses to Adjacent Formations

In Gajdica's (1990) model, as well as in Yortsos and Gavalas's (1981) upper bound method for steam zone calculation, a single layer of homogeneous permeability is considered. The heat losses to the adjacent formations would occur both upward

CHAPTER 3. ANALYTICAL MODEL

and downward. Assuming equal thermal properties for these strata, the parameter, a , related to heat losses was defined by Eq. 3.9. A coefficient of 2.0 appears in the equation to account for losses in both directions. This heat loss constant area, A_{HL} , has been modified for the two-layer model.

In the case of steam injection into two stratified layers, two types of losses will occur: first, from any one layer, heat losses will occur either to the overburden or to the underburden; second, the steam zone growth rates will differ in each layer so the heat transfer rate between them will also differ. The steam zone will grow at a faster rate in the more permeable layer, and be shorter in the less permeable layer. If the steam zone temperatures of the two layers are considered the same, then no heat will transfer across the layers in the area containing both steam zones. Thus, the heat loss in the lower layer will be only to the adjacent formation. The heat loss constant, A_{HL} , will thus be equal to 1.0 for the less permeable layer.

For the higher permeability layer, the heat losses will occur toward both the adjacent impermeable formation and the adjacent flow layer. The losses to the impermeable formation will occur from the entire steam zone length. On the other hand, the area available for heat transfer toward the low permeability layer will depend upon the relative steam growth rates in the two layers. At any time let L_{s1} be the steam zone length in the more permeable layer, and L_{s2} be the length in the less permeable layer. Then the area for heat transfer toward the less permeable layer will be proportional to the difference, $(L_{s1} - L_{s2})$. We can now define the heat loss areas,

A_{HL} , as follows. For the less permeable layer it is,

$$A_{HL} = 1 \quad (3.18)$$

For the more permeable layer it is,

$$A_{HL} = 1 + \frac{L_{s1} - L_{s2}}{L_{s1}} \quad (3.19)$$

With these definitions, the value of A_{HL} will range between 1.0 and 2.0 . It will change with time in the more permeable layer. When the adjacent layer is almost impermeable, the value of A_{HL} will be close to 2.0. With equal permeabilities in the layers, the heat loss area will be equal to 1.0, as the steam zone growth rates will be equal in this case.

The A_{HL} for the high permeability layer must be calculated at each time step. For the low permeability layer, a constant value of unity is used throughout the calculations.

3.9 Hot Liquid Zone

Consider the intermediate zone defined by the length, $L_{s1} - L_{s2}$, in the lower permeability layer. Here heat is transferred from the higher permeability steam zone, raising the temperature of the other layer. This zone is called the hot liquid zone.

In the analytical model, heat conduction is assumed only in the vertical direction. From the steam zone of higher permeability layer, heat first travels towards the lower permeability layer, and then passes on to the adjacent impermeable strata. Thus, the hot liquid zone also loses heat to the surroundings. The problem, thus, consists

CHAPTER 3. ANALYTICAL MODEL

of linear heat transfer through composite layers of differing thermal properties. Assuming that the steam zone temperature is constant, the solution of the linear heat flow equation in such systems is given by Carslaw and Jaeger (1959) in Section 12.8 of their book. In thermal recovery processes, however, the thermal properties of the flowing layer and impermeable surroundings are nearly the same. Thus, for simplification, we can assume that the thermal conduction of the lower permeability layer will be equal to the overburden conduction. Then the temperature in this zone may be calculated using the basic conduction equation.

The temperature in this hot liquid zone is a function of vertical distance as well as the time the steam zone reached a given point along the horizontal axis. The temperature, T , at any point (x, z) at a given time, t , is given by the expression,

$$T(x, z, t) = T_i + (T_s - T_i) \operatorname{erfc}\left(\frac{z}{2\sqrt{\alpha_o(t - \tau)}}\right) \quad (3.20)$$

where τ is the time when the steam zone initially reached at the point x in the high permeability layer. To calculate an average value of temperature, the above expression may be integrated numerically for the entire height and over the entire hot liquid zone length. The required formulation is

$$T_{avg} = \frac{\int_{L_{s2}}^{L_{s1}} \frac{\int_0^H T(z, t) dz}{z} dx}{L_{s1} - L_{s2}} \quad (3.21)$$

or, numerically,

$$T_{avg} = \frac{\sum_{i=1}^n \frac{\sum_{j=1}^n T(z, t) \delta z}{\sum_{j=1}^n \delta z} \delta x}{\sum_{i=1}^n \delta x} \quad (3.22)$$

Equation 3.22, then gives a formulation for the average temperature in the hot liquid zone which is quite close to the exact solution.

In the analytical model the hot liquid temperature calculations are carried out using heat transfer information. The heat losses are determined in each layer to correct the steam zone steam saturations, and to calculate the amount of steam condensed. This already available information is used to calculate the temperature in the hot liquid zone. Let Q_{loss} be the total heat loss from the higher permeability layer. This total heat loss is calculated using Eq. 3.16. The heat transferred to the lower permeability layer, Q , is then given by

$$Q = Q_{loss} * \left[\frac{A_{HL} - 1}{A_{HL}} \right] \quad (3.23)$$

where A_{HL} is given by Eq. 3.19. The temperature in the hot liquid zone, T_{hl} , is then given by,

$$T_{hl} = \frac{Q * X}{M_1 W H (L_{s1} - L_{s2})} + T_i \quad (3.24)$$

where M_1 is the reservoir heat capacity of the hot liquid zone, W is the reservoir width, H is the height of the lower permeability layer, and T_i is the initial reservoir temperature. The factor, X , in Eq. 3.24 is the fraction of heat retained in the lower permeability layer. This factor, then, takes into account the heat lost from the hot liquid zone to the surroundings. However, the value of X is difficult to estimate. At early times almost all of the heat will be retained by the permeable layer, and X will be close to 1.0. Heat transfer to the surroundings will increase with time, thus decreasing the value of X . Further the fraction, X , will also depend upon parameters such as the thickness of the permeable layer, steam injection rate, and permeability

CHAPTER 3. ANALYTICAL MODEL

ratio of the layers. In this study, a factor of 0.8 has been used. For future work it is suggested that the average temperature of the hot liquid zone be calculated by Eq. 3.22 as it does not require such a factor.

3.10 Pressure Drops Across a Layer

The pressure drop within a layer is determined using multi-phase Darcy's Law (Eq. 3.13). In the high permeability layer, the pressure drops are calculated across the three zones; steam, water and oil. The water zone of the low permeability layer is, however, further divided into two sub-zones; hot liquid and cold water. Equation 3.13 indicates that the flow of a phase is directly proportional to its relative permeability and inversely proportional to its viscosity. As these parameters are temperature dependent the effects of higher temperature are discussed next.

3.11 Effects of Higher Temperature in the Hot Liquid Zone

The parameters such as viscosity and relative permeability are functions of temperature of the fluid. Changes in these parameters and their effect on the flow is discussed next.

Figure 3.9: Variation of Oil and Water Viscosity with Temperature

3.11.1 Viscosity

The viscosity of the liquid decreases markedly with the increase in the temperature, with greater changes for the higher viscosity fluids. Steamflooding is usually applied to heavy viscous oils. The decrease in viscosity of such an oil is greater than the change in the viscosity of the water. Figure 3.9 shows a viscosity-temperature graph of the oils and water used in this study. It can be seen, for example that crude oil with a viscosity of 1000 cp at 100° F has a viscosity of about 300 cp at 150° F. By comparison the water viscosity is 0.68 cp at 100° F and reduces only to about 0.44 cp at 150° F. Thus even a small increase in temperature in a formation containing both water and viscous oil, leads to significant reduction in viscosity contrast improving

CHAPTER 3. ANALYTICAL MODEL

the flow rate of oil.

In the analytical model, the viscosity-temperature data for oil and water is entered in the tabular form and exponential interpolation is used to calculate the viscosity at any temperature. For steam, the viscosity temperature relationship is represented by,

$$\mu = AT^n \quad (3.25)$$

where A and n are constants to be specified by the user.

3.11.2 Fractional Flow Curve

The fractional flow curve for water in a water/oil system is given by Eq. 3.1. Due to changes in oil and water viscosities in the hot liquid zone, the fractional flow curve is changed. Figure 3.10 shows two fractional flow curves at two temperatures. The 100 cp oil data is used to draw these curves. The fractional flow curve moves to the right with the increase in temperature because the oil viscosity drops more rapidly. The tangent line indicates the higher water saturation behind the hot water front. This means that a hot water bank is build up behind the cold water bank.

In the analytical model, the location of the water front is first determined by using the average saturation behind the front at initial reservoir temperature. The location is then corrected for water built up in the hot liquid zone.

CHAPTER 3. ANALYTICAL MODEL

Figure 3.10: Change in Fractional Flow Curve with Temperature Variation

CHAPTER 3. ANALYTICAL MODEL

3.11.3 Thermal Expansion

With the increase in temperature, the liquid density decreases, and the rock matrix volume increases. Thus the increase in temperature increases the oil production rate both by liquid expansion and by expulsion. Another effect is an increase in the water zone length.

The overall effects of higher temperature in the hot water zone are modeled as follows. Calculating the hot liquid zone temperature, the respective oil and water viscosities are determined. The average water saturation is then calculated using fractional flow theory. The length of the water zone is, then adjusted as follows,

$$L_w = L_w + L_{hl} * \left[\frac{(\rho_w)_{T_i}}{(\rho_w)_{T_{hl}}} * \frac{(S_w)_{T_i}}{(S_w)_{T_{hl}}} - 1 \right] \quad (3.26)$$

where L_w is the length of the water zone, L_{hl} is the length of hot liquid zone, $(\rho_w)_{T_{hl}}$ and $(\rho_w)_{T_i}$ are water densities in the hot liquid zone and at initial temperatures, and $(S_w)_{T_i}$ and $(S_w)_{T_{hl}}$ are the average water saturations in the cold water and the hot liquid zones.

3.11.4 Relative Permeability

Weinbrandt and Ramey (1972) showed that the relative permeabilities of oil and water systems vary with temperature. It was also shown that with the increase in temperature the irreducible water saturation increases, whereas the residual oil saturation decreases. However, in the model the relative permeabilities and saturation end points are assumed to remain unchanged with the change in temperature.

3.12 Steam Injectivity in Each Layer

Initially the reservoir is considered to be at uniform conditions. The saturations of oil and water are the same everywhere in the reservoir. The initial total steam injection rate, Q_{inj} , is then divided into the two layers at a rate proportional to the layer permeabilities. The initial injection rates are then given by

$$Q_1 = \frac{K_1 H_1}{K_1 H_1 + K_2 H_2} Q_{inj} \quad (3.27)$$

and

$$Q_2 = Q_{inj} - Q_1 \quad (3.28)$$

where Q_1 and Q_2 are the steam injection rates in Layers 1 and 2 respectively, and Q_{inj} is the total steam injection rate.

The principle then applied is that the flow potential across each layer is equal. However, the small differences in pressures at the sand face of the injection well in the two layers is used to correct the steam injection rate into each layer. The steam injection rate is increased or decreased in a layer by using

$$Q_1 = Q_1 + \left[\frac{p_1 - p_2}{p_2} \right] * Q_1 \quad (3.29)$$

and

$$Q_2 = Q_{inj} - Q_1 \quad (3.30)$$

where p_1 and p_2 are the calculated injection pressures in Layer 1 and 2 respectively. The above injection rates are then used to calculate the time-weighted average steam

CHAPTER 3. ANALYTICAL MODEL

injection rates using Eq. 3.17.

3.13 Concluding Remarks

An analytical model for a linear, two layered system has been developed by using the equations given in this chapter. For a constant steam injection rate, the flow of steam is divided into each layer and rates are calculated using Eqs. 3.27 to 3.30. The time weighted average rate, Eq. 3.17, is then used to calculate the steam zone length in each layer by using the Yortsos and Gavalas upper bound method. The calculated steam zone length is corrected to match the heat losses. Water zone lengths and water saturations in the cold and hot water zones, and steam zone steam saturation are calculated using fractional flow theory. Heat losses to the surrounding are used to correct the steam zone steam saturation. The water zone length is corrected for steam zone steam saturation and for expansion and water built up in the hot liquid zone. The heat transferred from the higher permeability layer is used to calculate the average hot liquid zone temperature. The pressure drop in each layer is calculated using Darcy's law. The calculated pressures at the injection well are then used to determine the steam injection rate into each layer for the next time step. A flow chart summarizing these steps is shown in Fig. 3.11.

The model has been tested and results are compared with a numerical simulator, STARS. These results are presented in the next chapter.

Figure 3.11: Schematic of Flow Diagram of Analytic Model

Chapter 4

Results

The results of the analytic model developed for a two layer system are presented in this chapter, and are compared with the output of a thermal simulator. The numerical simulator used is STARS, a steam reservoir simulator written by the Computer Modeling Group. First, the base case reservoir data used in the report are described. Next, are discussed the selection criteria used to decide on the number of grid blocks in the simulator runs. The results of the analytic model are described subsequently; and finally, the results of the model are compared with the results of the simulator.

4.1 Base Case Data

A Cartesian coordinate system is used in the layered analytic model. The thermal simulator, STARS, provides options for selection of the gridding system. A Cartesian coordinate system was selected as it can easily model a linear reservoir. The system consists of only one block along the y-axis, the width of the reservoir. Each layer is

represented by a single block in the z direction. The length of the reservoir is taken as 800 ft, its width as 400 ft, and the heights of the two layers in the z direction are 50 ft each.

The permeabilities in the layers are constant but differ in each layer. For the base cases, the permeability of the higher permeability layer was taken as 5 Darcys and that of lower permeability layer was 2 Darcys; so the ratio of the two permeabilities is 1:2.5. Later, other permeability ratios were taken and compared with the base case. The viscosity of oil used in the base case was 100 cp at 100° F. The variation in viscosity with temperature are shown in Fig. 3.9.

The problem was modeled using a two component, two phase system. The components were water and oil where the water component can exist both as liquid and gas. The oil component was assumed to be a "dead oil" and can only exist as a liquid phase. The oil and water components were assumed to be immiscible. Initial oil saturation was assumed to be 0.75 and water saturation 0.25. The initial reservoir and surrounding temperature was considered to be uniform at 100° F. The steam injection rate was kept constant at 300 BPD (cold water equivalent).

For relative permeability data, the Corey relation was used. Table 4.1 gives the values used for the Corey relationships. STARS uses a linear interpolation method using a tabular input, while the analytic model directly calculates the relative permeabilities from the Corey equation. The relative permeability curves used this study are shown in Figure 4.1. The complete set of data for the base cases is listed in Tables 4.2 and 4.3.

CHAPTER 4. RESULTS

Figure 4.1: Relative Permeability Curves used in the Analytic Model

Table 4.1: Relative Permeability Data for Corey Relation

Water-Oil System	
Variable	Value
Corey exponent for oil	2.0
Corey exponent for water	3.0
Water saturation end points	0.25 , 0.63
Relative permeability end point values for oil	0.00 , 1.00
for water	0.0403 , 0.00
Gas-Liquid System	
Corey exponent for gas	2.0
Corey exponent for liquid	3.0
Liquid saturation end points	0.55 , 1.00
Relative permeability end point values for gas	0.52 , 0.00
for liquid	0.00 , 1.00

Table 4.2: Base Case Data

Variable	Value
i-direction grid blocks	100
j-direction grid blocks	1
k-direction grid blocks	2
number of components	2
maximum newton iteration	10
maximum time step, days	100
Time Step Control	
normal change, pressure, psi	200
normal change, saturation, fraction	0.2
normal change, temperature, degrees F	40
normal change, component mole fraction, fraction	0.2
convergence tolerance, pressure, psi	0.8
convergence tolerance, saturation, fraction	0.01
convergence tolerance, temperature, degree F	0.08
convergence tolerance, mole fraction, fraction	0.01
block size, i-direction, ft	8.0
block size, j-direction, ft	400.0
block size, k-direction, ft	50.0
porosity, fraction	0.3
permeability of layer one, Darcy	2
permeability of layer two, Darcy	5
initial pressure at top of the reservoir, psi	70
initial temperature, Degree F	100
initial water saturation, fraction	0.25
initial oil saturation, fraction	0.75

CHAPTER 4. RESULTS

Table 4.3: Base Case Data (Contd.)

Variable	Value
molecular mass of water, lbm/mole	18.02
molecular mass of oil , lbm/mole	600.0
reference pressure for porosity, psia	14.7
reference pressure for density, psia	14.7
reference temperature for density, degrees F	14.7
standard pressure, psia	14.7
standard temperature, degrees F	14.7
molar density of water at STP, mole/cu ft	3.4628
compressibility of water, 1/psimole/cu ft	4.0e-6
thermal expansion of water, coefficient one, 1/F	3.8e-4
thermal expansion of water, coefficient two, 1/F/F	0.0
molar density of oil at STP, mole/cu ft	0.1
compressibility of oil, 1/psimole/cu ft	5.0e-6
thermal expansion of oil, coefficient one, 1/F	4.0e-4
thermal expansion of oil, coefficient two, 1/F/F	0.0
critical pressure of water, pisa	3206.2
critical temperature of water, degrees F	705.40
gas viscosity coefficient, cp	4.8e-4
gas viscosity exponent	0.593
oil viscosity at initial temperature, cp	100
molar heat capacity of oil, btu/mol-F	300
volumetric heat capacity of rock, btu/cu ft-F	35
volumetric heat capacity of overburden, btu/cu ft-F	35
formation compressibility, 1/psi	0.0e-6
formation thermal expansion	0.0e-6
thermal conductivity of rock, btu/ft-day-F	38.0
thermal conductivity of water, btu/ft-day-F	9.0
thermal conductivity of oil, btu/ft-day-F	2.0
thermal conductivity of gas, btu/ft-day-F	0.5
thermal conductivity of overburden, btu/ft-day-F	24.0

4.2 Grid Selection for STARS

Cases were run taking 20, 100, and 200 grid blocks along the x-axis, while other parameters were kept constant. The results of these runs are shown in Figure 4.2, which shows both oil and water production rates against time. For all cases, when only 20 grid blocks were used, from the start of the run until water breakthrough in the high permeability layer, the oil production rate fluctuations are quite pronounced. Water breakthrough in the high permeability layer is clearly indicated by a decline in the oil production rate and the start of water production.

When only 20 grid blocks were used, the block sizes are larger. When the steam zone expands from one block to another, first the steam condenses and the flow rate declines sharply, because in a numerical simulator the properties within a block are assumed to be uniform and are averaged over the block. This behavior gives rise to the lower oil production rates as seen in Figure 4.2. Later, as the block is heated, most of the steam entering the block remains in the gas phase and production rate rises sharply, as seen in Figure 4.2.

The 20 grid block case also shows oscillations in the water production rates. Steam breakthrough occurred in the high permeability layer at about 3500 days. By contrast, the 100 and 200 grid block results do not have steam breakthrough. The 20 grid block case also did not clearly indicate the time of water breakthrough in the low permeability layer, seen at about 2,600 days in the 100 and 200 grid block runs.

Increasing the number of blocks along the x-axis eliminated almost all these numerical grid effects. With 100 grid blocks, a significant improvement occurred. Except

GRID EFFECT

OIL VISCOSITY = 100 cp
STEAM INJEC RATE = 300 BPD(cwe)
PERM LAYER 1 = 5 D
PERM LAYER 2 = 2 D

Figure 4.2: Effect of Number of Grid Blocks on Production Rate Predictions.

at very early times, the oil production rates were stable. Using 200 grid blocks, almost the same results were seen. This indicates that increasing the number of grid blocks beyond 100 does not greatly improve the results. The water rate histories were also almost the same using 100 or 200 grid blocks. Both cases clearly indicate water breakthrough in the lower permeability layer, at about 2600 days.

These results show that the grid effects in the thermal simulator are for more prominent than they are in isothermal injection processes. In the thermal simulator the mass flow calculations are strongly influenced by the heat balance and the steam zone. Based on these results, in the remaining study 100 grid blocks were used. This size is considered to represent the physics of the system with reasonable accuracy, at a reasonable computer time cost.

4.3 Results

The results of the analytic model for the two layered system are presented in this section. The basic case data used for the study were explained in Section 4.1 and given in Table 4.2 and 4.3. In the base case, the permeability of the higher permeability layer was 2.5 times that of the lower permeability layer. The results of the base case are discussed first. Thereafter, results are presented using 100 cp oil but with differing permeability ratios in the layers. In the last two cases, the oil viscosity is changed to 10 cp and 1000 cp.

Figure 4.3 shows the production rate profile for the base case. Saturation distributions for this case are shown in Figures 4.3 and 4.3. The production profiles

CHAPTER 4. RESULTS

for other 100 cp oil runs are shown in Figs. 4.6 through 4.12. Figure 4.13 shows the profiles for 10 cp oil while the production profile for heavy oil, 1000 cp, are shown in Fig. 4.14. Each of the production rate figures contains two graphs. The upper graphs show the oil production rates with time, and the water production rates are shown on the lower part. The analytic model results are shown by dashed lines, while the solid lines show the results of STARS.

In all these figures, production rates are used for comparison. In most studies of this type, cumulative production are compared. A comparison of total production tends to mask many differences, and may not clearly indicate water or steam breakthrough. Comparison of production rates, on the other hand, magnifies the differences. The rate graphs also allow accurate determination of the water and steam breakthrough times in each layer. Breakthrough times determine the project economic life and overall success or failure of a project. Therefore these breakthrough times are particularly addressed in this study.

In a production rate figures, water breakthrough in the higher permeability layer is indicated by the first sharp decline in oil production rate. At the same time, water production also begins. A sharp second decline in oil production rate may be due to water breakthrough in the lower permeability layer. Alternatively, it may sometimes be caused by steam breakthrough in the higher permeability layer. Water production rate patterns can be used to distinguish between these cases. Water breakthrough in the lower permeability layer will cause an increase in the water production rate, while steam breakthrough in the higher permeability layer will lower the water production rate.

We start this discussion with results from the base case. The permeability contrast in this case is 2.5. The oil production rate graph, given at the upper part of Fig. 4.3 may be divided into three clear sections. Just after the start of steam injection, the oil rates determined by the analytic model are slightly higher than the rates determined by STARS. The production profile traced by the simulator, STARS, shows an increasing trend after some time. Thereafter the two rates are very close, except for the grid effect fluctuations mentioned earlier.

The breakthrough of water in the high permeability layer in both models is indicated by the first decline in oil flow rates. The breakthrough time by the analytic method is delayed by about 50 days compared to STARS. The time determined by the simulator is about 720 days while that of analytic model is about 770 days. So the analytic model shows breakthrough after a delay of about 8 %. The water production rate graph indicates the same time difference.

The second decline in flow rate for this case indicates water breakthrough in the low permeability layer. The decrease in oil rate is coupled with an increase in water rate. This breakthrough time in the analytic model is advanced as compared to STARS, by about 1.2 %. Except for the small differences in the breakthrough times, oil flow rates are near to each other for both models. The analytic model calculates slightly higher water flow rates, and slightly lower oil rates.

Oil saturation distribution graphs for this case are shown in Figure 4.3 at various times. The figure shows the oil distributions in each layer. Along the x-axis, the distance is zero at the injection well and the production well is at the other end, 800 ft away. These graphs also indicate the different zones and their extent at various

CHAPTER 4. RESULTS

Case Permeability Ratio 1:2.5

Figure 4.3: Comparison of Oil and Water Flow Rates, Permeability Ratio, 1:2.5

Comparison of Oil Saturations in Layers 1 & 2

Figure 4.4: Comparison of Oil Saturations in Both Layers at Various Times

CHAPTER 4. RESULTS

times. If we move from the injection well towards the production well, i.e. from the left to the right on a graph, the first increase in the oil saturation indicates the steam zone front. In Layer 1, the high permeability layer, the second jump in the oil saturation indicates the water zone front and the start of the oil zone. In Layer 2, the oil saturation increases more gradually after the end of the steam zone. This behavior is due to the hot liquid zone. The saturation, then stays constant for some distance, showing the cold water zone. Thereafter, the oil saturation is about the same as the initial oil saturation.

The oil saturation distribution in Layer 1, the high permeability layer, match very closely for 365 and 730 days. The profile of 730 days also indicates water breakthrough in the STARS solution, while the analytic model shows the water front at about a distance of 760 ft away from the injection well. The same difference was seen in the production profiles for this case (Fig. 4.3). The saturations calculated by the analytic model are almost the same for the steam zone section even after that time. However the oil saturations in the water zone of this layer are slightly lower in the analytic model than in the simulator. This explains why higher water production rates are calculated by the analytic model after water breakthrough in this layer.

Similar observation can be applied to the second layer. The differences here are also due to the hot liquid zone development. The oil saturation increases gradually ahead of the steam zone in STARS. An average oil zone saturation is used in the analytic model. The areas under the two curves remain about the same, which means the production predictions from the layer will be similar.

Similar to Fig. 4.4, Fig. 4.5 shows steam saturation versus distances at various

Comparison of Steam Saturations in Layer 1 & 2

Figure 4.5: Comparison of Steam Saturations in Both Layers at Various Times

CHAPTER 4. RESULTS

times. These graphs magnify the differences in the steam zone lengths calculated by the two models. The graph shows that steam saturations calculated by the analytic model and the simulator are quite close, but the steam zone length is greater in the analytic model. The differences increase with time. The steam zone lengths shown are corrected steam zone lengths as explained in Section 3.2. The figure shows that at early times the corrected steam zone lengths match with the STARS results. But the correction factor does not work so well at later times. Some further work where the correction factor is correlated with steam injection time, might lead to a better match.

The next case presented considers a permeability ratio of 1:1.1 in the layers. The close layer to layer permeability ratio causes the steam zone growth, and steam injection rates, to remain almost at the same time in each layer. Figure 4.6 shows that this expected trend is observed both in the layered model and in the simulator. The oil rates for the analytic model in this case are observed to be slightly lower before water breakthrough in the layers, and breakthrough of water is slightly delayed in the higher permeability layer. This result is similar to the base case. The water breakthrough timings in the lower permeability layer are also close. The water rates determined by the analytic model are initially low but then cross the STARS result. The earlier observation of slightly higher water flow rates for the base case are also seen in this case. No steam breakthrough was observed in this case.

The layer-permeability ratio in the next case is 1:1.4. The permeability value for the high permeability layer is 2.8 Darcys and for the low permeability layer is 2.0 Darcys. The results of this run are shown in Fig. 4.7. Oil rates are presented in the

Case Permeability Ratio 1:1.1

Figure 4.6: Comparison of Oil and Water Flow Rates, Permeability Ratio, 1:1.1

CHAPTER 4. RESULTS

Case Permeability Ratio 1:1.4

Figure 4.7: Comparison of Oil and Water Flow Rates, Permeability Ratio, 1:1.4

upper section of the figure and water rates in the lower, as before. The oil production rate calculated by the model are slightly lower than STARS from the start of the run. The difference in the two rates decreases gradually. The water breakthrough in the higher permeability layer is earlier in STARS than in the analytic model (900 days compared to 980 days). The water breakthrough in the lower permeability layer, at about 1400 days, matches quite closely.

The oil rates were close to one another over the entire history. However, after water breakthrough in lower permeability layer, the water rate calculated by the model slightly exceeds the rate calculated by the simulator. This trend is similar to that of earlier cases.

Figures 4.8 to 4.10 give the results of permeability ratio of 1:1.8, 1:2.2, and 1:2.6 respectively. Observations similar to the case of permeability ratio of 1:1.4, can be made on these results. The oil flow rates in these runs are close in each model. The water breakthrough times in the high permeability layer are slightly delayed in the analytic model. The water breakthrough in the low permeability layer matches closely for the permeability ratios of 1.8 and 2.2 (Fig. 4.8 and 4.9). The simulator calculated water production rates are slightly lower than the model results. Figure 4.10, for the permeability ratio of 1:2.6, however, shows that, in the second layer, water breakthrough in the analytic model is earlier than the simulator.

The next case presented is conducted with a permeability ratio of 1:3.0. The production profiles for the analytic model and STARS are shown in Figure 4.11. The oil production rates as calculated by the the analytic model are quite close to the simulator results. The water breakthrough time in the high permeability layer

CHAPTER 4. RESULTS

Case Permeability Ratio 1:1.8

Figure 4.8: Comparison of Oil and Water Flow Rates, Permeability Ratio, 1:1.8

Case Permeability Ratio 1:2.2

Figure 4.9: Comparison of Oil and Water Flow Rates, Permeability Ratio, 1:2.2

CHAPTER 4. RESULTS

Case Permeability Ratio 1:2.6

Figure 4.10: Comparison of Oil and Water Flow Rates, Permeability Ratio, 1:2.6

Case Permeability Ratio 1:3.0

Figure 4.11: Comparison of Oil and Water Flow Rates, Permeability Ratio, 1:3.0

CHAPTER 4. RESULTS

differs only by about 40 days. Water breakthrough in the lower permeability layer and steam breakthrough in the higher permeability layer occur almost simultaneously at about 3500 days. First the water breakthrough in the lower permeability layer is indicated by the dip in the oil rate, and then, steam breakthrough in the higher permeability layer is seen from the sharp hike in oil and water flow rates in the simulator. The analytic model, in this case does not indicate water breakthrough in the lower permeability layer but the steam breakthrough in both cases is indicated at about the same time. The rapid rate changes in STARS, after steam breakthrough, are caused by the problems in defining productivity index at steam breakthrough.

Figure 4.11 shows a sharp decrease and rise in oil rate at about 1800 days. The reason for this behavior is as follows. In this case, the hot liquid zone at early time was limited to water zone of the lower permeability layer. The permeability contrast in this case favors more steam injection in the higher permeability layer. The increased steam injection rate in the higher permeability layer increases the steam growth rate in that layer. At about 1800 days the steam zone in the higher permeability layer is so advanced that it starts to heat the oil zone of lower permeability layer. In the analytic model, the hot liquid zone may consist of a small part of water zone or the entire water zone, or may even extend to the oil zone. This production result at 1800 days shows that at the time of the change the model does not handle the position well and needs improvement in coding. However, this effect is quite short lived and can be ignored.

The maximum permeability ratio of 1:5.0 is taken next for the 100 cp oil, Fig. 4.12. In this case, the flow of steam is dominantly in the high permeability layer. So

Case Permeability Ratio 1:5.0

Figure 4.12: Comparison of Oil and Water Flow Rates, Permeability Ratio, 1:5.0

CHAPTER 4. RESULTS

after water breakthrough in the high permeability layer, steam breakthrough occurs in this layer. Both models closely match at these breakthrough times. No water breakthrough occurs from the lower permeability layer. The oil rates determined by the analytic model are about the same as STARS, except at very late times after steam breakthrough. The water flow rates also match closely. Note again the problem with the STARS results at about 3000 days, where steam breakthrough causes problems in the calculation of productivity index.

Like the hump observed in Fig. 4.11, in this case the analytic model shows a large hump in the water production rates at the time of water breakthrough in the higher permeability layer. This was also observed in some other cases. In the analytic model a constant time step is used except at the time of water breakthrough in a layer. At that point, the constant time step is divided into two unequal intervals, pre-breakthrough and post-breakthrough. This allows the exact determination of breakthrough time. The unequal intervals, combined with the saturations related to breakthrough, produce the humps.

Figures 4.13 and 4.14 show the results of different viscosity oils. The data used for these cases is the same as for the base case except for the oil viscosity. The permeability ratio between the layers in both cases is 1:2.5. The results shown in Fig. 4.13 are for the low viscosity oil of 10 cp. Figure 4.14 shows the results with oil viscosity of 1000 cp.

For the 10 cp oil, Figure 4.13, the oil rates determined by the analytic model are lower than the rates calculated by STARS from the initial time. The difference remains about the same until water breakthrough in the high permeability layer.

Case Permeability Ratio 1:2.5

Figure 4.13: Comparison of Oil and Water Flow Rates For Low Viscosity Oil, Permeability Ratio, 1:2.5

Case Permeability Ratio 1:2.5

Figure 4.14: Comparison of Oil and Water Flow Rates for High Viscosity Oil, Permeability Ratio, 1:2.5

After water breakthrough the rates determined by two models become close. The two models indicate different breakthrough time in the high permeability layer. The water breakthrough time in the higher permeability layer has been delayed in the analytic model in almost every case run so far. But in this case, with low viscosity oil, the time determined by the analytic model is delayed by a larger margin (120 days). The error is about 13 %.

As observed in other cases, there is a smaller difference in calculated water breakthrough times in the lower permeability layer. Both the numerical simulator and the analytic model predict water breakthrough at about 2500 days. However, contrary to the trend observed in the 100 cp oil runs, after water breakthrough, water flow rates predicted by the model are lower than STARS. The two rates match again after water breakthrough in the second layer.

STARS results indicate steam breakthrough at about the end of the run. No steam breakthrough is observed in the analytic model. The lower rates and the difference in steam breakthrough in the analytic model indicate that the steam injectivity into the high permeability layer is less than in the simulator. Iteration of sandface pressures in the two layers might improve the results of the model.

The heavy oil results (1000 cp) are presented next (Fig. 4.14). The maximum differences in the oil flow rates are observed in this case. The oil rates determined by the simulator increased to a maximum of 370 BPD at the time of water breakthrough in the higher permeability layer. The oil rates determined by the analytic model are almost constant (about 290 BPD) during the same period. Note also that STARS results show unusual increasing oil flow rate until water breakthrough in the higher

CHAPTER 4. RESULTS

permeability layer. This behavior is contrary to all other cases where either slightly decreasing, or almost constant, oil flow rates were observed during the similar period. Gajdica (1990) also made some heavy oil runs. The oil rate production trend reported by Gajdica for heavy oils in an homogeneous linear reservoir also negate the trend observed in the present work. This matter needs further study to identify the reasons.

The water breakthrough timing in the high permeability layer matched closely in the two models. However, the analytic model predicted water breakthrough in the second layer to be about 300 days earlier than the time determined by STARS. TL is too large a difference to be acceptable.

Both cases of low (10 cp) and high viscosity oil (1000 cp) need further analysis. Why the oil rates differ so widely in the heavy oil case, also needs to be determined.

The breakthrough timings in every case run are shown at Table 4.4. For 100 cp oil, the timings predicted by the analytic model in the higher permeability layer are all within 11 % error. The model predicts the water breakthrough in the lower permeability layer within a 3 % margin.

4.4 Concluding Remarks

The results of an analytic model developed in this study for the layered system are compared with simulator results. The analytic model is based on an equi-potential principle in the two layers. The oil and water rates determined by the analytic model are found to be close to the simulator results for 100 cp oil. For this oil the analytic model predicts water breakthrough in the high permeability layer of the

Table 4.4: Comparison of Breakthrough Times

Case Permeability Ratio	Water Breakthrough Time in High Permeability Layer (Days)			Water Breakthrough Time in Low Permeability Layer (Days)			Steam Breakthrough Time in High Permeability Layer (Days)		
	Ana- lytical Model	STARS	% age Error	Ana- lytical Model	STARS	% age Error	Ana- lytical Model	STARS	% age Error
Oil Viscosity 100 cp									
1:1.1	1080	1000	+7.0	1210	1180	+2.5			
1:1.4	980	900	+8.8	1420	1400	+1.1			
1:1.8	890	800	+11.2	1800	1800	0.0			
1:2.2	800	740	+8.1	2200	2200	0.0			
1:2.5	765	710	+7.7	2510	2555	-1.2			
1:2.6	780	710	+9.8	2600	2700	-3.8	3600	NA	-
1:3.0	750	680	+10.3	-	-	-	3550	3450	+2.8
1:5.0	660	600	+10.0	-	-	-	3100	3000	+3.3
Oil Viscosity 10 cp									
1:2.5	1020	900	+13.3	2450	2500	-2.0			
Oil Viscosity 1000 cp									
1:2.5	500	450	+11.1	1700	2020	-15.0			

CHAPTER 4. RESULTS

system with an accuracy of 11 % error. More accurate timings are achieved with the lower permeability layer, with an error range of 4 %. The results for higher and lower viscosities oils requires further analysis. The low oil viscosity results suggest that iteration of pressures at the injection well may improve the results.

Chapter 5

Conclusions

The following conclusions can be drawn from this study.

1. For the one dimensional homogeneous linear steam drive, the Yortsos and Gavali (1980) model for steam zone upper bounds was used as a basis for calculation. A correction factor to this model is estimated from the heat loss comparisons. This correction factor gives better estimates for the steam zone lengths.
2. The use of a time weighted average steam injection rate in the upper bound calculations is adequate for small changes in steam injection rates.
3. The principle of equal potential drop can be applied to steamflooding in isolated layered reservoirs. A predictive model in such systems can be developed by an approach similar to water flooding in layered reservoirs. It should, however, include temperature variations as needed in the lower permeability layers.

CHAPTER 5. CONCLUSIONS

4. An analytic model has been developed for two isolated differing permeability layers by extending Gajdica's (1990) one dimensional model. The results of the model are comparable with those of the thermal simulator (STARS) for 100 cp oil. The model needs improvement to predict the performance for other viscosities.
5. The analytic model is much simpler than the formulation of a numerical simulator. Certain steps require solution by iteration and update of the fluid properties at each time step. The required calculations can, however, be performed rapidly using a personal computer.

5.0.1 Recommendations

The work performed shows that steam flooding in isolated layered systems can be modeled by using the principle of equal potential drop. The work, thus explores a basic approach and shows partial success. This work, however, is not complete. The following guidelines are suggested for extension of this work to make it more generally useful.

1. The work so far only properly correlates results of 100 cp oil. It needs to be extended to higher and lower viscosities.
2. Improve the hot liquid zone temperature estimates by a formulation similar to the one presented at Section 3.9.
3. Equate the injection pressures at the sandface of both layers by iteration.

4. Test the model for other parameters such as reservoir dip, layer thicknesses, production well bottomhole pressure, and steam injection rate.
5. Include effect of temperature on relative permeabilities.
6. Extend the model for multilayer systems.
7. Extend the model to include gravity override effects; that is, make it truly two-dimensional within each layer.

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Nomenclature

a	=	heat loss parameter
A	=	cross-sectional area, or
	=	constant for viscosity temperature relation
c	=	compressibility
cc	=	shape factor
C	=	specific heat capacity
E	=	efficiency
$erfc$	=	complementary error function
f	=	fractional flow. or
	=	steam quality
F	=	ratio of latent to total heat injected
g	=	acceleration due to gravity
h	=	enthalpy
H	=	height
I	=	injectivity index
k	=	permeability
L	=	length or
	=	latent heat
M	=	volumetric heat capacity
n	=	exponent capacity
N	=	oil produced
p	=	pressure
q	=	flow rate
Q	=	cumulative flow or
	=	cumulative heat
r	=	radius
s	=	skin factor
S	=	saturation

NOMENCLATURE

t	= time
T	= temperature
w	= mass flow rate
W	= width or
	= water injected
z	= distance normal to bedding plane

GREEK

α	= thermal diffusivity
Δ	= difference
θ	= angle of formation dip
λ	= thermal conductivity
μ	= viscosity
ρ	= density
ϕ	= porosity
π	= constant = 3.14159

SUBSCRIPT

av	= average
bt	= breakthrough
con	= condensate
D	= dimensionless
g	= gas
HL	= heat loss
hl	= hot liquid
i	= initial, or
	= time step number
inj	= injection
irr	= irreducible
j	= phase j or
	= zone j
l	= liquid
L	= linear
$loss$	= loss
max	= maximum
n	= net (reservoir thickness)
o	= oil
ob	= overburden
p	= produced
$prod$	= production
r	= relative Or
	= residual
R	= rock

<i>rem</i>	=	remaining
<i>res</i>	=	reservoir
<i>s</i>	=	steam
<i>sc</i>	=	standard condition
<i>t</i>	=	total (reservoir thickness), or
<i>tot</i>	=	total
<i>v</i>	=	vaporization, vertical, volumetric
<i>w</i>	=	water, or
	=	well
<i>x</i>	=	x-direction
<i>z</i>	=	z-direction
1	=	layer 1
2	=	layer 2
SUPERSCRIPT		
<i>n</i>	=	corey exponent, or
	=	exponent for viscosity temperature relation

Appendix A

Model Assumptions

The assumptions made in the semianalytical model (SAM) are described in this appendix . First, the general assumptions are discussed, these are followed by lists of specific assumptions about phase relationships, energy, initial conditions, and boundary conditions.

A.1 SAM General Assumptions

The following general assumptions are made in the model.

1. The Cartesian coordinate system is used to describe the reservoir geometry. Two layers are assumed along the vertical axis. No mass flow is allowed between the layers. Therefore, a one-dimensional linear system is considered for each layer.
2. Each individual layer is homogeneous and has a constant thickness and cross-sectional area.
3. No gravity override of the steam can occur within a layer.

4. Pressure drops across the reservoir system are calculated separately in each layer using Darcy's law.

5. The thermal properties of the overburden and the underburden are identical and are constant.

6. Thermal equilibrium between the fluids and the rock matrix is achieved instantaneously.

7. Capillary pressure is neglected, for steam flooding is usually carried out in the heavy oil reservoirs which typically have high permeabilities where capillary pressures are usually low.

8. The compressibilities of the oil, water, and formation are constant.

9. Oil, water and steam viscosities are temperature dependent but independent of pressure.

10. Temperature-dependent relative permeability, diffusion, dispersion, adsorption, chemical reactions, non-Darcy flow, and inertial effects are neglected.

A.2 Phase Relationships

The computer model is a three phase, two component model. The following phase relationship is assumed between the components and phases.

1. Two components are considered; oil and water. The oil is nonvolatile with no solution gas. The oil and water components are completely immiscible. Water can either be liquid or gas, the gas phase being pure steam. These phase component relationships are summarized in Table A.1.

APPENDIX A. MODEL ASSUMPTIONS

Phase	Component	
	Oil	Water
Oil	x	
Water		x
Gas		x

Table A.1: Relationships Between Phases and Components

2. The solid phase and fluid phases are completely insoluble in each other.

A.3 Energy Assumptions

The conservation of total energy is assumed. The following specific assumptions are made:

1. Thermal energy is conserved.
2. Heat flow to the adjacent layer is proportional to the thermal conductivity and temperature gradient at the interface between the layers. Heat transfer in the adjacent layers is by one-dimensional vertical conduction in the layer.
3. Kinetic energy changes are negligible.
4. Mechanical work done by thermal expansion of the reservoir on its surroundings is negligible.

A.4 Initial Conditions

The reservoir initial conditions are described as follows:

1. The reservoir is uniformly saturated with oil and connate water at irreducible saturation. Only the oil phase is mobile.
2. The initial flow potential of oil in the reservoir is uniform and constant.
3. Initial reservoir temperature is uniform and constant, and equal to the temperature of the adjacent strata.

A.5 Boundary Conditions

The following constraints are imposed along the boundaries of the reservoir:

1. The reservoir boundaries are fixed and do not change with time.
2. There is no mass flow between the reservoir and overburden or underburden.
3. The areal perimeter of the reservoir, except for the wells, is a no flow boundary for both mass and heat.
4. There is an injection well at one end of the system and a production well at the other end. The wells penetrate the reservoir at right angles and are completed in both layers.
5. The production well produces at a constant flowing bottomhole pressure. The temperature at the production well is calculated from the average enthalpy of the produced fluids.
6. Wet steam is injected at the injection well at a constant rate and enthalpy. The temperature at the injection well is the temperature of the wet steam.

END

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