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UNITED STATES ATOMIC ENERGY COMMISSION

GAS COOLED, NATURAL URANIUM, D₂O
MODERATED POWER REACTOR; REACTOR
DESIGN AND FEASIBILITY PROBLEM

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August 1956

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OAK RIDGE SCHOOL OF REACTOR TECHNOLOGY

Reactor Design and Feasibility Problem

"GAS COOLED, NATURAL URANIUM, D₂O MODERATED POWER REACTOR"

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August, 1956

OAK RIDGE NATIONAL LABORATORY
Operated by
Union Carbide Nuclear Company

PREFACE

In September, 1955, a group of men experienced in various scientific and engineering fields embarked on the twelve months of study which culminated in this report. For nine of those months, formal classroom and student laboratory work occupied their time. At the end of that period, these seven students were presented with a problem in reactor design. They studied it for ten weeks, the final period of the school term.

This is a summary report of their effort. It must be realized that, in so short a time, a study of this scope can not be guaranteed complete or free of error. This "thesis" is not offered as a polished engineering report, but rather as a record of the work done by the group under the leadership of the group leader. It is issued for use by those persons competent to assess the uncertainties inherent in the results obtained in terms of the preciseness of the technical data and analytical methods employed in the study. In the opinion of the students and faculty of ORSORT, the problem has served the pedagogical purpose for which it was intended.

The faculty joins the authors in an expression of appreciation for the generous assistance with various members of the Oak Ridge National Laboratory gave. In particular, the guidance of the group consultant, R. A. Charpie, is gratefully acknowledged.

Lewis Nelson

for

The Faculty of ORSORT

ACKNOWLEDGEMENT

We are indebted to R. A. Charpie, the original proponent of this study, whose freely given help was invaluable. Significant contributions were also made by W. E. Kinney, and the ORSORT faculty, particularly R. K. Obsorn, L. G. Alexander, H. S. Pomerance, and M. L. Nelson.

ABSTRACT

The attractiveness of a helium cooled, heavy water moderated, natural uranium central station power plant has been investigated. A fuel element has been devised which allows the D_2O to be kept at a low pressure while the exit gas temperature is high. A preliminary cost analysis indicates that, using currently available materials, competitive nuclear power in foreign countries is possible.

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1.0 INTRODUCTION

The reactor development program for civilian application in the United States has, for the most part, neglected the use of gas as a coolant. In view of the delicate problems which would arise if the incremental basic industry of other nations came to depend on the U. S. diffusion plants for their source of power, it seems important to reconsider all possible power reactor types which make use of natural uranium fuel. This report considers the design and operating characteristics of a gas-cooled natural uranium fueled power reactor station.

The general problems of gas cooling are discussed in references (1), (2), and (3), together with the relevant characteristics of the most important gas coolants. It should be emphasized, however, that the chief advantages of gas cooling derive from the possible use of inexpensive materials of construction and simplicity in design.

In view of the fact that the British have investigated graphite-moderated power piles extensively, it was decided to look at heavy water moderated reactors. The superior nuclear performance of D_2O is most fully realized in a low enrichment uranium reactor where the maximum possible k_{eff} is only about 1.3.

The purpose of this study is to investigate, by means of a preliminary design, the problems of producing power from a D_2O moderated, gas-cooled reactor in order to estimate the probable cost of power, and then to point out ways of improving the design so as to achieve lower power costs.

Two decisions were made at the outset of the study which more or less set a minimum to the cost of electricity which could be generated. These were: (1) the use of zirconium as the fuel element cladding, and (2) the use of helium as the coolant. Zirconium was selected as the cladding material in order

to permit operation of the fuel element at temperatures near the alpha-beta transition temperature of natural uranium. This would lead to higher outlet gas temperatures and relatively higher thermal efficiencies. Helium was chosen for its good heat transfer and nuclear characteristics despite the fact that coolant leakage becomes an important problem and that ample supplies of helium are probably not available outside the United States.

Some general comments are now in order which indicate the direction the study has taken. The limitations of time prevented a thorough examination of anything but the gross behavior of all the variables which affect the cost of power. Many important design points were chosen arbitrarily in order to establish a reference point from which a nuclear and heat transfer analysis could be made. The power level was set at 1000 megawatts and the outlet temperature was chosen to be 1000°F, about 230°F below the alpha-beta transition temperature of uranium metal. In order to minimize the volume fraction of the coolant channels in the pile, the gas velocity was set as high as seemed reasonable from the standpoint of pressure drop--about mach 0.1.

There are several goals to be remembered in order to keep the cost of electricity as low as possible: (1) The inlet gas temperature is a function of the pressure, for a fixed velocity and void space, and the increased advantages of high pressure (i.e., higher inlet temperature) must be balanced against the cost of compressing the gas and of using high pressure equipment and components. (2) Every effort must be made to keep the zirconium inventory to a minimum. (3) As much D₂O as possible should be kept in the reactive region of the pile and there should be a low over-all D₂O inventory per megawatt of electrical output. (4) A high conversion ratio of U²³⁵ to Pu²³⁹ is desirable (contingent, of course, on the criticality requirement) since this permits a long lifetime

from the nuclear point of view.

It has been pointed out (4) that low or medium temperature natural uranium power plants, especially gas-cooled ones, inherently have low thermal efficiencies and these reactors will be able to compete with conventional power plants only if the fuel costs can be kept low. Thus, the most important quantity to optimize is the megawatt days per ton of natural uranium.

2.0 SUMMARY OF RESULTS

Power level

Heat output 1000 MW

Electrical output 214 MW

Moderator Heavy Water

Coolant Helium

Clad Zirconium

Structure Aluminum

Reflector Graphite

Fuel Natural Uranium

Number of Fuel Elements 151

Size

Core Radius 10 feet

Height 20 feet

Reflector thickness 2 feet

Core composition

Natural Uranium 74 tons

Zirconium 5.4 tons

Aluminum 5.6 tons

D₂O (@ 20°C) 171 tons

Nuclear Parameters (clean hot reactor)

η 1.33

ϵ 1.032

p .9363

f .9655

k_{∞}	1.241
$1/(1+M^2B^2)$	.9411
k_{eff}	1.168
Reflector savings	1.11%
Conversion ratio	0.70
Endurance	6000 MWD/ton of natural uranium
Flux (average thermal)	6.2×10^{13} neutron-cm/cm ³ -sec.
Temperature coefficients	
Prompt	$-0.93 \times 10^{-5}/^{\circ}\text{C}$
Delayed	$-19.07 \times 10^{-5}/^{\circ}\text{C}$
Heat transfer parameters	
Coolant	
Inlet temperature	350°F
Outlet temperature	1000°F
Inlet pressure	498 psia
Outlet pressure	464 psia
Minimum spiral velocity (at core center)	308 ft/sec
Maximum spiral velocity (at core center)	585 ft/sec
Fuel	
Maximum fuel temperature	1112°F
Fuel temperature at surface (max.)	1096°F
Average moderator temperature	350°F
Pressure drop	
Core	34 psi

Over-all	44 psi
Net plant efficiency	21.4%
To coolant	88%
D ₂ O	10%
Lost	2%

FIGURE 1
SCHEMATIC OF FUEL ELEMENT

3.0 FUEL ELEMENT

3.a. General Discussion

The fuel element utilized in this reactor was designed to (1) provide a large heat transfer surface per unit mass of fuel and per unit volume, (2) permit high outlet gas temperatures, and (3) give low D_2O temperatures. A large heat transfer surface per unit mass is desirable to compensate for the relatively poor film coefficient and heat capacity of the gas as compared to liquid coolants. A large surface per unit volume is desirable to minimize the coolant channel spaces and hence the neutrons losses due to these spaces. High outlet gas temperatures are desirable from the standpoint of thermal efficiency. Low moderator temperatures result in low system pressures leading to less expensive pressure vessels.

An isometric sketch of the fuel element process tube is shown in Fig. 1. The element is tapered to minimize the volume of the coolant channel and to provide a favorable pressure distribution in the process tube. The cooling gas enters the outer coolant channel at the bottom of the reactor. It then spirals inward through the fuel element to the inner coolant channel, and out the top of the reactor. The heavy water temperature at the hottest point is approximately equal to the inlet gas temperature.

Zirconium was used as the fuel element cladding because of its desirable nuclear and high temperature properties.

Stress, flow distribution, and fabrication problems are reduced by making the element in twenty one-foot sections, as shown in details in Figure 2. This method of construction increases the zirconium content of the reactor but not enough to overbalance the advantages gained by doing so.

The entire fuel element is housed in a process tube constructed of 24S aluminum.

The various dimensions of the fuel element and process tube are listed in the following table.

TABLE 1

Length of fuel element	20 feet
Number of fuel sections/fuel element	20
Height of fuel section	12 inches
Height of uranium in fuel section	11.3 inches
Cladding thickness	0.005 inches
Fuel thickness	0.087 inches
Total length of spiral	73.2 inches
Spacing between spirals	0.08 inches
Inside spiral diameter (central section)	4.13 inches
Outside spiral diameter (central section)	5.67 inches
Process tube inside diameter	6.8 inches
Process tube thickness	0.16 inches

3.b. Fuel Element Fabrication

It is proposed that the fuel element be fabricated by first preparing sheet stock of uranium sandwiched in and edged with zirconium. These would be prepared in sheets one foot wide and about six feet long by a method similar to that used in fabricating individual STR plates. The spiral section would be hot formed from the plate and the proper spacing maintained by a square zirconium wire welded at the plate edges. Thin zirconium washers with inside and outside diameters equal to those of the spiral would then be attached to each end of the section by welding them to the outside turn of the spiral. This leaves the remaining turns free to expand and contract. The twenty individual fuel sections would then be joined into one element by placing them end to end and welding the washers together.

Direct cladding of uranium with zirconium to a minimum thickness of 5 mils is considered feasible (5), (6), (7), although admittedly near the limit of technology.

Thermal stress calculations indicate that heating the element to operating temperature would cause yielding in the zirconium clad due to the greater thermal expansion of uranium. This would require minimum thermal cycling and arrangements to keep the reactor hot during periodic shutdowns of other plant components.

The total nvt to which one element could be exposed is about 1.7×10^{21} . To avoid the effects of radiation damage it is proposed that the uranium be alpha rolled and beta heat treated before incorporation into the zirconium sandwich. All subsequent heat treatments are performed below the transition temperature. It might prove necessary to incorporate 10% niobium (8) or some other element in the uranium to improve radiation stability, since there is no experience

with radiation effects on thin sheets of uranium metal. However, it is hoped that the relative freedom of expansion of a large portion of the fuel spiral would allow for a reasonable amount of dimensional instability.

4.0 NUCLEAR ANALYSIS

4.a. Object and Method

The optimum reactor and lattice parameters from a nuclear standpoint are those which will give a maximum power output per ton of natural uranium; i.e., MWD/ton. Since these parameters will seldom result in the best heat transfer conditions, a reasonable compromise between the best nuclear and best heat transfer parameters must be made. The independent parameters which can be varied are:

1. Overall reactor size.
2. Lattice pitch.
3. Total uranium loading.
4. Total power (heat) output.
5. Dimensions of the fuel region in a unit cell.
6. Coolant channel dimensions.

Some of these parameters can be fixed early in the analysis. A total heat output of 1000 MW was arbitrarily selected and this, with the fuel loading, determines the flux. The size of the reactor core was chosen to be a twenty foot square cylinder. The basis for this selection will be discussed later. With the core volume specified, the lattice pitch depends on the number of unit cells; once the fuel loading and lattice pitch are selected, the heat transfer conditions determine the dimensions of the coolant channels and fuel regions in the unit cell. Hence, the independent parameters which remain to be optimized in the nuclear analysis are the total number of unit cells and the total fuel loading.

The optimization of the reactor from the nuclear standpoint will be pre-

sented in four steps:

1. Determination of the reactor size.
2. Optimization of the number of fuel elements for a given fuel loading.
3. For this number of fuel elements, optimization of the fuel loading and, therefore, the flux.
4. For the new loading, a reconsideration of the total number of fuel elements.

4.b. Formulae

For large reactors the fundamental relationship between the neutron multiplication constant and the nuclear and geometrical properties of the reactor can be well approximated by:

$$k_{\text{eff}} = \frac{K_{\infty}}{1 + M^2 B^2}$$

where $k_{\infty} = \eta f p \epsilon$, and $M^2 = L^2 + \tau$. Each of these quantities will now be considered in detail.

η : The number of fast neutrons released per thermal neutron absorbed in the fuel. In general, for natural uranium thermal reactors, η is given by:

$$\eta = \frac{V_{25} \Sigma_f^{25} + V_{49} \Sigma_f^{49} + V_{41} \Sigma_f^{41}}{\Sigma_a^{25} + \Sigma_a^{28} + \Sigma_a^{49} + \Sigma_a^{41} + \Sigma_a^{40}}$$

where

Σ_f = fission cross section (cm^{-1})

$$25 = {}_{92}\text{U}^{235}$$

$$28 = {}_{92}\text{U}^{238}$$

$$49 = {}_{94}\text{Pu}^{239}$$

$$40 = {}_{94}\text{Pu}^{240}$$

$$41 = {}_{94}\text{Pu}^{241}$$

For a clean reactor, the expression reduces to:

$$\eta = \frac{V_{25} \Sigma_f^{25}}{\Sigma_a^{25} + \Sigma_a^{28}}$$

Eta is usually experimentally determined and the value generally used for a cold reactor (i.e., at 20°C) is 1.34 (40). For higher temperatures, η was determined by (11):

$$\eta(T) = \eta(300^\circ\text{K}) \left[1 - 4.5 \times 10^{-5}(T - 300) \right]$$

where T is in degrees Kelvin. For T = 450°K (350°F), $\eta = 1.33$.

Thermal Utilization, f: The number of thermal absorptions in the fuel per total thermal absorption. The thermal utilization decreases as a result of lumping the fuel due to the self shielding of the interior of the lump from the higher flux in the outer parts of the lump. In general, for natural uranium reactors, f is given by:

$$f = \frac{\bar{\phi}_U \sum_a^U V_U + \bar{\phi}_{Pu} \sum_a^{Pu} V_{Pu}}{\bar{\phi}_U \sum_a^U V_U + \bar{\phi}_{Pu} \sum_a^{Pu} V_{Pu} + \bar{\phi}_M \sum_a^M V_M + \bar{\phi}_S \sum_a^S V_S + \bar{\phi}_P \sum_a^P V_P}$$

where:

$\bar{\phi}$ = average flux

U subscript = natural uranium

M subscript = moderator

S subscript = structure

P subscript = other materials

Pu subscript = plutonium

$\sum_a$ = capture cross section for the pure material, cm⁻¹.

V = volume

For the model used in this analysis, $\bar{\phi}_P = \bar{\phi}_{Pu} = \bar{\phi}_U = \bar{\phi}_S$ and $\frac{\bar{\phi}_M}{\bar{\phi}_U} = S =$ flux disadvantage factor (Appendix A). For the clean reactor, the expression

for f reduces to:

$$f = \frac{1}{1 + \sum \frac{\Sigma_a^M}{\Sigma_a^U} \frac{V_M}{V_U} + \frac{\Sigma_a^S}{\Sigma_a^U} \frac{V_S}{V_U}}$$

Resonance Escape Probability, p : Probability of escaping capture in the U^{238} resonances. The resonance escape probability increases when the fuel is lumped due to the fact that most of the neutrons are thermalized in the moderator away from the fuel. Moreover, the interior of the lump is shielded somewhat from the higher resonance flux at the surface of the lump thereby decreasing the effectiveness of the interior U^{238} atoms as resonance absorbers.

In general, for natural uranium thermal reactors, p is given by:

$$P = \exp \left[- \frac{N_U V_U \bar{\Phi}_U}{\sum_M V_M \sum_S \bar{\Phi}_M} \int (\sigma_a^U)_{\text{eff}} dU \right]$$

where:

N_U = number of uranium atoms per cm^3 .

$\int (\sigma_a^U)_{\text{eff}} dU$ = effective resonance absorption integral

$\sum_S$ = scattering cross section of the moderator (cm^{-1})

It is possible to express the effective resonance integral as the sum of a volume absorption term and a surface absorption term (9); that is,

$$\int (\sigma_a^U)_{\text{eff}} dU = A \left(1 + \mu \frac{S_U}{M_U} \right)$$

where S_U and M_U are, respectively, the surface area of the fuel bearing region directly facing the moderator and the mass of uranium enclosed by the surface. Since zirconium is used as a cladding and is rather more intimately mixed with the uranium than most claddings, it can be crudely considered to dilute the uranium. The effect of the uranium dilution and the effect of zirconium scattering on A and μ is discussed in Reference 9. The result is that A and μ are

functions of the effective scattering cross section per uranium atom in the uranium mixture. If $\overline{\sigma}_s$ = scattering cross section per atom of uranium in the fuel mixture, then:

$$A = 2.40 \sqrt{\overline{\sigma}_s}$$

$$\mu = 5.82 - 0.85 \sqrt{\overline{\sigma}_s}$$

For this analysis, the only scattering material mixed with the uranium is zirconium; hence,

$$\overline{\sigma}_s = 9.5 + 23.6 \frac{M_{Zr}}{M_U}$$

The disadvantage factor, S_{res} , to be used in this calculation is the ratio of the resonance region flux in the moderator (D_2O) to that in the fuel (See Appendix A).

Fast Fission Factor, ϵ : Ratio of the total fissions to the thermal fissions. The fast effect was calculated to be 1.032. This calculation is given in Appendix B.

Non Leakage Probability, $1/(1+M^2B^2)$: The probability that a neutron will not leak out of the reactor before being absorbed as a thermal neutron. The coolant channel leakage of neutrons from a gas cooled reactor is much greater than from a liquid cooled reactor because of the greater mean free path of the neutrons, both fast and thermal. The problem is accentuated by the alignment of coolant channels which gives a decided anisotropy to the migration area compared to what it would be if the coolant voids were uniformly dispersed. A method for calculating the leakage where anisotropy exists has been devised by Behrens (11) which consists of calculating a M^2 in the plane parallel to the channels and a M^2 in the plane perpendicular to the channels. An average M^2

is then defined as:

$$\overline{M}^2 = \frac{2}{3} M_I^2 + \frac{1}{3} M_{II}^2$$

where

$$M_I^2 = L_I^2 + \gamma_I$$

$$M_{II}^2 = L_{II}^2 + \gamma_{II}$$

To calculate the leakage from a cylindrical core, M_I^2 and M_{II}^2 will be used and not the average, as will be shown later.

The results of the analysis presented in Reference 11, modified so as to apply to a heavy water moderated reactor and also slightly altered to simplify the calculations, are:

$$\gamma_I = \gamma \left[1 + 2 \frac{V_U}{V_{D_2O}} \right] \left[1 + \frac{V_{He}}{V_T} (2 + a \sum_{fast}) \right]$$

$$\gamma_{II} = \gamma \left[1 + 2 \frac{V_U}{V_{D_2O}} \right] \left[1 + \frac{V_{He}}{V_T} (2 + 2a \sum_{fast}) \right]$$

$$L_I^2 = L^2 \left[1 + \frac{V_{He}}{V_T} (2 + a \sum_{thermal}) \right]$$

$$L_{II}^2 = L^2 \left[1 + \frac{V_{He}}{V_T} (2 + 2a \sum_{thermal}) \right]$$

The factors in these formulae have the following physical significance:

γ : The age in D_2O natural uranium lattices is about the same as for D_2O alone. This is due to the substantial effect of the inelastic scattering by U^{238} in moderating fission neutrons (12). At elevated temperatures,

$$\gamma(T^\circ K) = \gamma(293^\circ K) \left(\frac{1.1}{\rho(T)} \right)^2$$

where the density of D_2O at $293^\circ K$ is 1.1 gms/cc, and $\gamma(293^\circ K) = 11. \text{ cm}^2$.

$(1 + 2 \frac{V_U}{V_{D_2O}})$: This factor is a correction for the effect of the elastic scattering of uranium which acts to increase the value of γ . $(1 + \frac{V_{He}}{V_T} (2 + \eta a \Sigma))$:

This factor accounts for the presence of the coolant channels, in this report considered void spaces. $\frac{V_{He}}{V_T}$ is the void fraction of the unit cell, "a" is the equivalent radius of the void space, and Σ is the reciprocal of the mean free path in the absence of void spaces and, in this analysis, was taken to be:

$$\Sigma = \frac{1}{\lambda} = \frac{V_U}{V_0} \Sigma_U + \frac{V_{Zr}}{V_0} \Sigma_{Zr} + \frac{V_{D_2O}}{V_0} \Sigma_{D_2O}$$

where $V_0 = V_U + V_{Zr} + V_{D_2O}$. In the formula for the age, the correct cross sections to use are those for the fast region; for L^2 , thermal cross sections should be used. For all practical purposes, $\Sigma \approx \Sigma_{D_2O}$ in both the fast and thermal regions (to about 1%). L^2 : The correct value of L^2 to use in the formulae is:

$$L^2 = f_{D_2O} L_{D_2O}^2 + f_{Zr} L_{Zr}^2 + f_U L_U^2$$

where:

$f(D_2O)$ = fraction of thermal neutrons absorbed in D_2O .

$f(Zr)$ = fraction of thermal neutrons absorbed in zirconium.

$f(U)$ = fraction of thermal neutrons absorbed in uranium
(i.e., the thermal utilization).

L_i^2 = thermal diffusion area in the pure substance.

Typical values of these quantities are (at 293°K):

$$L_{D_2O}^2 = 10,000 \text{ cm}^2, f_{D_2O} = .01$$

$$L_{Zr}^2 = 125 \text{ cm}^2, f_{Zr} = .02$$

$$L_U^2 = 1.4 \text{ cm}^2, f_U = .97$$

Hence, only a small error is introduced in L^2 by using the approximation

$$L^2 \approx f_{D_2O} L_{D_2O}^2$$

In terms of the thermal utilization, f ,

$$f(D_2O) = f \frac{V_{D_2O}}{V_U} \frac{\sum_a^{D_2O}}{\sum_a^U} \int_{Thermal}$$

For cylinders, then, the effective multiplication factor is given by:

$$K_{eff} = \frac{K_{\infty}}{1 + \left(\frac{2.405}{R}\right)^2 M_1^2 + \left(\frac{\pi}{H}\right)^2 M_{II}^2}$$

where:

R = radius of bare reactor.

H = height of bare reactor.

4.c. Cross Sections and Nuclear Constants

The thermal nuclear cross sections and constants used in the analysis are listed below for both the hot and cold conditions. The cold conditions refer to 20°C and the hot to 177°C, the temperature of the moderator. The neutron absorption cross sections correspond to the most probable energy of the thermal Maxwell-Boltzman neutron distribution and should be multiplied by 0.89 to get average cross sections.

TABLE 2

	HOT	COLD
η	1.33	1.34
$\sigma_a (D_2O)$	.0021 b	.0026 b
σ_a (Uranium)	6.12 b	7.68 b
$\sigma_a (Zr)$	.145 b	.18 b
$\sigma_s (D_2O)$	28.7 b	28.7 b
σ_s (Uranium)	8.3 b	8.3 b
$\sigma_s (Zr)$	9.0 b	9.0 b
Uranium density	18.8 g/cc	18.8 g/cc
Zirconium density	6.5 g/cc	6.5 g/cc
D_2O density	0.97 g/cc	1.1 g/cc
Atoms uranium/cc	0.0475×10^{24}	0.0475×10^{24}
Atoms zirconium/cc	0.0430×10^{24}	0.0430×10^{24}
Atoms D_2O /cc	0.0293×10^{24}	0.0332×10^{24}
$\eta (D_2O)$	0.00792 cm^{-1}	0.01 cm^{-1}
$\gamma (D_2O)$	141 cm^2	110 cm^2
$L^2 (D_2O)$	$15,950 \text{ cm}^2$	$10,000 \text{ cm}^2$

For the U^{238} resonance region the nuclear constants which were used were:

TABLE 3

$$\sigma_s (\text{Natural U}) = 9.5 \text{ b}$$

$$\sigma_s (Zr) = 9.0 \text{ b}$$

$$\Sigma_a (D_2O) = 0.0313 \text{ cm}^{-1} \quad (\text{Scattering out of the resonance region is considered as absorption -- Reference 10})$$

$$\eta (D_2O) = 0.155 \text{ cm}^{-1} (10)$$

$$\sigma_a (\text{Natural U}) = \frac{A(1 + \nu \frac{S_v}{M_v})}{\Delta u}$$

where:

$$\Delta u(\text{res}) = 5.63 (10)$$

$$A = 2.40 \sqrt{\sigma_s} (9)$$

$$\nu = 5.82 - 0.85 \sqrt{\sigma_s} (9)$$

$$\sqrt{\sigma_s} = 9.5 \quad 23.6 \frac{M_{Zr}}{M_v}, \text{ where } M \text{ is the mass per unit cell.}$$

$$\sigma_a (Zr) = 0$$

4.d. Determination of reactor and Unit Cell Dimensions; Dependence on Heat Transfer Requirements.

The nuclear calculations used in this analysis were based on one group theory and the model of the unit lattice cell shown below.

FIGURE 3

It was originally planned to use zirconium as both the structural material and cladding, and all of the nuclear calculations were based on this assumption. When it became evident that the zirconium inventory would be very large, it was decided to use aluminum for the process tube. The effect of aluminum on k_{eff} is to reduce it by about 0.008 from the value computed using zirconium.

The helium spaces were treated as voids to keep the calculations simple. This is a conservative simplification in two respects: (1) The helium will not absorb any neutrons but will aid in the moderation of the fast neutrons, increasing the thermal flux at the surface of the fuel. This will probably decrease p but increase both f and the product fp , and hence the reactivity; (2) The presence of helium will decrease the leakage of neutrons out through the

coolant channels.

The net effect of the helium will probably be small. It will tend to increase the reactivity over what it would be if true voids were present.

For simplicity, the reactor is taken to be a square cylinder.

As mentioned before, the determination of the reactor composition and dimensions will be done in four steps:

1. Determination of the reactor size.
2. Optimization of the number of fuel elements for a given fuel loading.
3. Optimization of the fuel loading for this number of fuel elements.
4. Redetermination of the optimum number of fuel elements for the new fuel loading.

The size of the reactor core was established by noting that for a given fuel loading for cores smaller than a 20 foot square cylinder k_{∞} falls off sharply with decreasing radius. Moreover, the leakage also increases. This situation is illustrated graphically in Figure 4. Although the curves are for a fuel loading of 74 tons in 151 fuel elements, the same situation obtains for other fuel loadings and numbers of fuel elements.

On the basis of this curve, it was decided to make the reactor a 20 foot square cylinder.

With the reactor core size determined, the remaining nuclear parameters are the fuel loading and the number of unit cells. The heat transfer requirements determine the sizes of the fuel region and coolant channels for given values of these parameters.

The curves in Figure 5 show k_{eff} as a function of the number of fuel

FIGURE 5

 k_{eff} vs NUMBER OF FUEL ELEMENTS

CONDITIONS:

1. Fuel Loading - 74 Tons
2. Moderator Temp. - 350 F
3. Reactor Geometry - 20 ft. Sq. Cylinder
4. 60 cm. Reflector

elements for a fuel loading of 74 tons. It is obvious that the maximum value of k_{eff} occurs at about 200 fuel elements. However, from the standpoints of heat transfer and cost of core construction, there are advantages to be gained (to be discussed in Section 5) by going to a smaller number of tubes and accepting a 0.6% reduction in k_{eff} .

For a small number of unit cells, the size of the coolant channels is large causing a large leakage, and the thermal utilization is low due to the effect of the flux disadvantage factor. As the number of tubes increases, the leakage decreases. The thermal utilization goes through a maximum and begins to drop as the disadvantage factor decreases; first at a faster and then at a slower rate than the fuel loading per unit cell. The resonance escape probability decreases continuously, and the net effect is to make k_{eff} go through the maximum shown on the curves.

The important quantity in this analysis is the endurance of the fuel loading or the megawatt days per ton which can be extracted, based on nuclear considerations, since this plays the major role in determining the cost of the fuel. The variation in k_{eff} and MWD/ton with the fuel loading is shown in Figures 6 and 7.

The maximum k_{eff} is achieved with a fuel loading of about 60 tons. For small loadings, k_{eff} is low because of the leakage and the low thermal utilization. As the tonnage increases, f increases rapidly at first and then, because of the effect of the increasing disadvantage factor, increases less rapidly; the leakage decreases and p decreases, so that the net result is for k_{eff} to pass through a maximum.

The endurance of a loading depends critically upon the initial conversion ratio (ICR) as well as on k_{eff} . As the fuel loading increases, everything else

FIGURE 6

 k_{eff} vs. FUEL LOADING

CONDITIONS:

1. Moderator Temp. - 350°F
2. 151 Fuel Elements
3. 60 cm. Reflector

FIGURE 7

ENDURANCE AND INITIAL CONVERSION RATIO

VS.

FUEL LOADING

CONDITIONS:

1. Moderator Temp. - 350°F
2. 151 Fuel Elements
3. 60 cm. Reflector

ENDURANCE - MWD/Ton

INITIAL CONVERSION RATIO

6500

6000

5500

5000

4500

4000

.74

.72

.70

.68

.66

.64

.62

.60

Initial Conversion Ratio

MWD/Ton

25

50

75

100

125

FUEL LOADING - tons

remaining constant, the fraction of neutrons captured in the U^{238} resonance increases. The resulting increase in ICR gives a steadily rising endurance curve over the tonnages considered. The ICR as a function of fuel loading is also shown in Figure 7.

For optimum use of the fuel it is desirable to have a high power density. The fuel element designed for this reactor provides a large heat transfer surface per gram of fuel and is therefore particularly useful in a high power density reactor. With a 74 ton fuel loading the power density is 13.5 MW/ton, about three times the power density of the advanced Calder Hall reactors (13) and approaching the power density of liquid cooled piles. A larger loading, therefore, would make the reactor less attractive from the standpoint of material efficiency (MW/ton).

Since most of our calculations had been done using a 74 ton fuel loading, it was decided under the limitations of time to design the reactor for this loading. However, an analysis of the combined effect of MWD/ton and material efficiency on the fuel cost (Section 9) indicates that a larger loading would result in slightly cheaper power.

The average thermal flux in the core of the 74 ton reactor under operation is 6.2×10^{13} neutrons/cm²-sec.

4.e. Reflector Savings

It is worthwhile to consider how the value of k_{eff} for a bare reactor might be increased by using a reflector. If k_{eff} for the bare reactor is designated by k_o , and for the reflected reactor by k_r , then the interesting quantity is k_r/k_o .

As one means of approximating this ratio it can be assumed that the addition of the reflector results in a larger core with the same k_o . The ratio is then, approximately:

$$\frac{k_r}{k_o} = 1 + K \left(\frac{1}{R_o^2} - \frac{1}{R_r^2} \right) + K \left(\frac{1}{H_o^2} - \frac{1}{H_r^2} \right)$$

where:

$$R_r = R_o$$

$$H_r = H_o$$

$$R_o = \text{radius of original core}$$

$$H_o = \text{height of original core}$$

$$K = \text{constant}$$

$$\delta = \frac{D_c}{D_r} L_r \tanh (T/L_r) \quad (\text{Ref. 14, page 235})$$

$$D_c = \text{diffusion coefficient in core}$$

$$D_r = \text{diffusion coefficient in reflector}$$

$$L_r = \text{diffusion length in reflector}$$

$$T = \text{reflector thickness}$$

Since fast leakage is also important, the above reflector savings will be labeled δ_L and a corresponding δ_f will be determined. A mean value will then be obtained by weighting each with the percentage of total leakage and

percentage of reflector seen by the slow and fast neutrons, respectively.

If a reflector other than heavy water is used, it becomes necessary to separate it from the heavy water of the core by a shell of structural material. The reduction in the effectiveness of the reflector will then be proportional to the square of the ratio of neutrons which pass through the shell to those which enter it. A conservative estimate of this ratio is e^{-T/L_1} , where T_1 is the thickness of the shell, and L_1 is its diffusion length. For aluminum this ratio is of the order of .98, hence little is lost in the way of accuracy if the effect of the aluminum is ignored.

About 30% of the leakage through the sides and the ends of the reactor is due to fast neutrons.

Two different cases of a totally reflected cylinder using a 60 cm reflector were considered; (1) Heavy water side reflector and graphite and reflectors, and (2) graphite side and end reflectors.

For the first case,

$$\begin{array}{ll} \delta(\text{sides}) = 45.6 \text{ cm} & (\delta_L = 54 \text{ cm and } \delta_r = 11 \text{ cm}) \\ \delta(\text{ends}) = 31 \text{ cm} & (\delta_L = 38.2 \text{ cm and } \delta_r = 17.2 \text{ cm}) \\ k_r/k_o = 1.0139 & \end{array}$$

For the second case,

$$\begin{array}{ll} \delta(\text{sides}) = 33 \text{ cm} \\ \delta(\text{ends}) = 33 \text{ cm} \\ k_r/k_o = 1.0115 \end{array}$$

It was decided to use graphite as the reflector in order to minimize the D_2O inventory; the reflector savings in this case is about 1.11%.

4.f. Computation of the Endurance; Reactivity Changes During the Prolonged Irradiation of Natural Uranium.

Throughout the life of the reactor core, Pu^{239} is being continually produced. Depending on the initial conversion ratio (ICR), the reactor reactivity initially may rise or fall; as burnup occurs, the fission product poisons and the higher Pu isotopes (excluding Pu^{241} which is fissionable) cause the reactivity to decrease until the reactor will no longer chain react. The ICR is defined as the number of Pu^{239} atoms produced per U^{235} atom destroyed in the clean reactor.

In this analysis it has been assumed that both the U^{238} concentration and flux (and hence the Pu^{239} production rate) are constant. This is a good assumption for limited periods ($\approx 2 \times 10^{21}$ nvt). For extended irradiations this assumption introduces significant errors.

As shown in Section 4.h and in Appendix C, the ICR was determined to be 0.7. The ICR is very sensitive to the resonance escape probability, however, which may vary widely from reactor to reactor. To make this analysis as applicable as possible to other reactors, the results are presented for ICR's of 0.7, 0.8, and 0.9.

The concentrations of U^{235} , U^{238} , Pu^{239} , Pu^{240} , Pu^{241} , and the fission products present after any irradiation time were calculated by using the equations presented in Appendix C. These concentrations were multiplied by the appropriate cross section (Table 4) to give the macroscopic cross sections listed, also, in Appendix C. It should be noted that these cross sections are cm^{-1} per original U^{235} atom, in the manner used in Reference 41. The cross sections have been plotted as a function of time in Figures 8, 9, 10, 11, 12, 13, and

14. From the curves, ηf can be computed as a function of time:

$$\eta f = \frac{\sum_i \nu_i \frac{\sum_f^i}{N_0}}{\sum_i \frac{\sum_a^i}{N_0} + \frac{\sum_a^{\text{Mod \& Structure}}}{N_0} + \frac{\sum_a^{\text{Xenon}}}{N_0}}$$

where:

$\sum_i \nu_i \frac{\sum_f^i}{N_0}$ = summation of the neutron yields per original U^{235} atom due to the fissioning of U^{235} , U^{238} , Pu^{239} , and Pu^{241} (Figure 11).

$\sum_i \frac{\sum_a^i}{N_0}$ = summation of the macroscopic absorption cross section per original U^{235} atom, excluding xenon, D_2O , and structural materials (Fig. 10).

$\frac{\sum_a^{\text{Mod \& Structure}}}{N_0}$ = absorption cross section for moderator and structural materials.

$\frac{\sum_a^{\text{Xe}}}{N_0}$ = xenon absorption cross section (Figures 12, 13, and 14)

Knowing ηf as a function of time, it is possible to calculate k_{eff} as a function of time since $\rho \in (1 + M^2 B^2)^{-1}$ remains constant. The reactor endurance can then be obtained by plotting k_{eff} vs. ϕt (or MWD/ton), and noting where k_{eff} falls below 1.0. At this point the reactor will no longer chain react. Such a curve is shown in Figure 15.

For this reactor;

$$\phi \text{ (2200 m/s)} = 4.46 \times 10^{13} \text{ neutrons/cm}^2\text{-sec.}$$

$$\frac{\sum_a^{\text{Mod \& Structure}}}{N_0} = 40.7 \text{ barns.}$$

$$(\eta f)_{\text{minimum value}} = 1.0869 \text{ (} k_{\text{eff}} = 1.0 \text{)}$$

A burnup of 6000 MWD/ton was achieved with the conversion factor of 0.7. If the initial conversion ratio could somehow be raised to 0.8 or 0.9, all other things remaining constant, the burnup would be increased to 8,200 or 10,900 MWD/ton, respectively. Hence, it appears that the endurance is very sensitive to

the I. C. R. Methods of increasing the I. C. R. have not been extensively investigated in this study, but they include decreasing the resonance escape probability, decreasing the leakage, and using depleted uranium.

TABLE 4

URANIUM AND PLUTONIUM CROSS-SECTIONS AND PARAMETERS*

NUCLIDE	σ_a (barns)	σ_f (barns)	σ_c (barns)	η	ν
U-235	685	580	105	2.08	2.46
U-238	2.75				
Pu-239**	1550	1020	530	1.90	2.88
Pu-240***	1175				
Pu-241	1480	1100	380	2.35	3.16

* All values obtained from BNL-325. Pu-239 and Pu-240 cross-sections have been corrected for resonance absorption.

** The cross-sections are corrected for their non $1/v$ behavior, and 130 barns is added to the fission and capture cross-sections to account for the 0.3 resonance effect. (43)

*** This effective thermal cross-section is good up to concentrations of approximately 13 atomic percent Pu-240, after which the effective cross-section gradually decreased until it reaches the thermal value of 550 barns. (42)

49

-67-

ORNL-LR-DWG.-17780

-50-

-51-

-52-

53

-53-

54

-75-

ORNL-IR-DWG-17785

-55-

56

FIGURE 15
IRRADIATION OF NATURAL URANIUM
vs.
LONG TERM REACTIVITY CHANGES

4.g. Temperature Effects

4.g.1. Temperature Coefficient of Reactivity

A measure of the inherent safety of a reactor can be obtained by calculating the temperature coefficient of reactivity. A negative coefficient means that if there is an increase in the average temperature of the reactor the reactivity will decrease and the reactor will return to its original stable operating condition. A positive coefficient means the reverse. Consequently, it is desirable to have a negative coefficient and it is important to determine both the sign and magnitude of the temperature coefficient at an early stage of design so that the cost and complexity of the control system can be kept to a minimum.

A change in temperature affects the reactivity in 3 ways. (1) It changes the mean energy of the thermal neutrons, thus changing the absorption cross sections; (2) it will change the volume of the reactor and, hence, the density of the materials present; and, (3) the resonance parameters change because of the Doppler broadening effect. It is apparent that a change in reactor temperature, caused by a change in power or in coolant temperature, will not affect all parts of the reactor simultaneously. Due to the rather low heat capacity of D_2O and the large quantity of it present, the moderator temperature will lag significantly behind the fuel element temperature during any transient of the reactor temperature. Therefore, it is possible to calculate a prompt metal coefficient which occurs immediately upon changes in the fuel element temperature, and a delayed moderator coefficient which results from changes in the moderator temperature.

A change in the fuel element temperature affects the reactivity through ρ and β . An increase in temperature causes a decrease in the fuel atom con-

centration, decreasing the thermal utilization, and a broadening of the resonance integral and decreasing the resonance escape probability.

Variations in the moderator temperature affect primarily the migration area, M^2 , although they also produce changes in f , p , and τ . An increase in the moderator temperature causes the moderator to become less dense, reducing its effectiveness in slowing down the neutrons, consequently increasing both L^2 and τ , and, therefore, M^2 . It is important to note that the actual thermal neutron level has been raised, tending to decrease τ , but this is overshadowed by the decrease in moderator atom density. The increase in the average neutron energy lowers both the fuel fission and absorption cross sections. Since the fission cross section falls off faster than $1/v$ while the absorption cross section of U^{238} is taken to be $1/v$, an increase in moderator temperature reduces γ . In addition, the loss of slowing down effectiveness allows the neutrons to remain in the vicinity of the resonance absorption region for longer periods of time and therefore increases the probability of resonance absorption and decreases p . Lastly, increasing the temperature forces more of the moderator out of the core region by expansion, increasing the proportion of fuel atoms present, and hence f .

These variations with temperature which have been evaluated for a hot, clean reactor are summarized in Table 5. A detailed description of the method employed for calculating each of the separate effects appears in Appendix D.

TABLE 5

	<u>METAL (900° F)</u>	<u>MODERATOR (350° F)</u>
$\frac{1}{\eta} \frac{\partial \eta}{\partial T}$		-4.5
$\frac{1}{f} \frac{\partial f}{\partial T}$	-0.06	2.38
$\frac{1}{\rho} \frac{\partial \rho}{\partial T}$	-0.87	-7.20
$-B^2 \frac{\partial M^2}{\partial T}$		-9.75
TOTAL	-0.93	-19.07

4.g.2. Hot to Cold Reactivity Change

The change in reactivity in going from the hot operating condition to the cold shut down condition is important in estimating the endurance and in determining the amount of control necessary. In both cases, the effect is in the wrong direction.

With the exception of the fast effect, all factors in the critical equation are temperature sensitive. The change in the density of the D_2O is responsible for the largest part of the reactivity change. Uranium and zirconium density changes have been neglected.

Listed below are the hot and cold values of the relevant nuclear parameters and the reason for the temperature dependence.

TABLE 6

PARAMETER	HOT VALUE	COLD VALUE	REASON
η	1.33	1.34	U^{238} absorption cross section is $1/v$ and fission cross section of U^{235} is not $1/v$.
f	.9655	.9593	D_2O density changes from .97 to 1.1; disadvantage factor changes from 2.19 to 2.657.
p	.9363	.9433	D_2O density change.
τ (avg.)	215	172	D_2O density change.
L^2 (avg.)	495	431	D_2O density change; D_2O absorption cross section change.
M^2 (avg.)	710	603	
$1/(1+M^2B^2)$	.9411	.9494	
k_{eff}	1.168	1.188	

The effect of temperature on the effectiveness of the reflector was not considered.

4.h. Neutron Balance

The question of the eventual disposition of each fission neutron is basic to the chain reacting process. A neutron balance provides a clear picture of their disposition and enables one to compute the conversion ratio, percent leakage, and effective multiplication constant.

$N = \eta_{25} \Sigma =$ total fast neutrons produced per thermal absorption in U^{235}

$NP_A =$ total fast neutrons absorbed (in U^{238}) per thermal absorption in U^{235} .

$N(1-P_A)P_{FL} =$ total fast neutrons that leak out per thermal absorption in U^{235} .

$N(1-P_A)(1-P_{FL})P_R =$ total neutrons absorbed in the resonance region (U^{238}) per thermal absorption in U^{235} .

$N(1-P_A)(1-P_{FL})(1-P_R)P_{SL} =$ total thermal neutrons that leak out per thermal absorption in U^{235} .

$N(1-P_A)p(1-P_L) =$ total neutrons to be absorbed in the thermal region per thermal absorption in U^{235} ; it should be noted that $(1-P_L) \approx (1-P_{SL})(1-P_{FL})$.

If Σ_a is the total absorption cross section (thermal) for the reactor, then

$\Sigma_a^{25}/\Sigma_a =$ fraction absorbed in U^{235}

$\int \Sigma_a^{D_2O}/\Sigma_a =$ fraction absorbed in D_2O , where $\int$ is the flux disadvantage factor

$\Sigma_a^{28}/\Sigma_a =$ fraction absorbed in U^{238}

$\Sigma_a^{Zr}/\Sigma_a =$ fraction absorbed in zirconium

It should be noted that, for a bare reactor,

$$N(1-P_A)(1-P_L)p \frac{\Sigma_a^{25}}{\Sigma_a} = k_{eff}(1-P_A)$$

Now, in the fast fission energy range for U^{238} , σ_a^{28} and σ_f^{28} are the absorption and fission cross sections, respectively, and

$$P_A = \frac{\sigma_a}{\sigma_f} P_{FF}$$

where

$$\begin{aligned} P_{FF} &= \text{probability of fast fission} \\ &= (1 - \Sigma) \times \text{probability of thermal fission} \\ &= \frac{(1 - \Sigma)(1 - P_A) k_{eff}}{\sqrt{\Sigma}} \end{aligned}$$

Hence, since $\sigma_a^{28} = .33$ and $\sigma_f^{28} = .29$, and $k_{eff} = 1.168$ (bare reactor), then

$$P_A = .01546$$

The product of the fast and slow non leakage probabilities must equal the total non leakage probability which is .9411. If this condition is met, and if the leakage is proportioned according to τ/M^2 and L^2/M^2 , then:

$$P_{FL} = .01840$$

$$P_{SL} = .04127$$

The probability of resonance capture in U^{238} is:

$$P_R = .0637$$

The thermal cross sections have been tabulated previously. The result of the calculation, starting with one fast neutron and putting in enough poison to make the reactor just critical, is:

Absorbed in U^{238} , fast:	.0155
Absorbed in U^{238} , resonance:	.0615
Leakage, fast:	.0181
Slow	.0374
Thermal absorptions:	
U^{235}	.4656

U^{238}	.2647
D_2O	.0148
Zr	.0116
Poison	.1108

The conversion ratio is:

$$C. R. = \frac{.2647 + .0615 + \frac{\sigma_a^{28} - \sigma_f^{28}}{\sigma_a^{28}} .0155}{.4656} = .7$$

If the poison is removed and those neutrons redistributed among the other thermal absorbers, we get:

$$k_{eff} = \frac{\text{neutrons in 2nd generation}}{\text{neutrons in 1st generation}} = 1.168$$

Finally, 1.81% of the neutrons leak out while fast and 3.74% while slow.

4.i. Reactor Control

To provide a long endurance and to enable the reactor to override the effects of poison buildup following a shutdown, a large excess reactivity is desirable. To control this excess and insure the safety of the reactor, moveable neutron absorbers of some sort are incorporated into the reactor design. This is usually done by using control rods but this is by no means the only way of controlling the reactor. Moveable fuel, moveable reflectors, and burnable poisons have also been used effectively. In this section, the problem of calculating the number of rods required to control the reactor as well as other possible means of control will be discussed. No attempt has been made to include a detailed control rod calculation.

Glasstone and Edlund (14) have derived the worth of a single control rod using one group theory and two group theory, the latter being more widely applied. Control rod effectiveness has also been calculated by Wigner, Weinberg, and Williamson (15), subsequently expanded and developed by Wheeler (16), and applied satisfactorily at Oak Ridge (17) and Brookhaven (11). Wheeler's analysis has resulted in the following expression relating the parameters of interest:

$$\left[N_0(\text{br}_{\text{eff}}) - \frac{N_1(\text{br})}{J_1(\text{br})} J_0(\text{br}_{\text{eff}}) \right] = \frac{\tau}{L^2} \left[i H_0^{(1)} \left(i \sqrt{\frac{1}{\tau} + \frac{1}{L^2}} r_{\text{eff}} \right) - H_1^{(1)} \left(i \sqrt{\frac{1}{\tau} + \frac{1}{L^2}} R \right) - i J_1 \left(i \sqrt{\frac{1}{\tau} + \frac{1}{L^2}} R \right) J_0 \left(i \sqrt{\frac{1}{\tau} + \frac{1}{L^2}} r_{\text{eff}} \right) \right]$$

where,

b^2 = abbreviation for the principal term of the buckling.

r_{eff} = effective radius of the control rod.

R = equivalent radius of the zone of the pile over which the rod acts.

$J, N, H^{(1)}$ = bessel functions in the notation of Janhke and Emde (18).

Therefore, knowing all but one of $R, r_{\text{eff}}, \gamma, L^2, M^2$, and δk , (where $\delta k = b^2 M^2 = k_{\infty} - 1$) the remaining unknown may be determined from the above equation or from Figure 22.5.23 of Wheeler's report.

The excess multiplication which must be controlled is $[k_{\text{eff}}(\text{cold}) - 1]$; however, to insure an adequate amount of safety, the reactivity in the scrambled condition should be 0.01 to 0.05 below critical. The additional reactivity contributed by the buildup of fissionable plutonium in the early stages of operation must also be considered. For an ICR of 0.7, this never exceeds the initial effect of equilibrium xenon poisoning in this reactor.

From the size and composition of the reactor it is possible to compute γ, L^2 , and M^2 ; thus b may be calculated. Assuming various r_{eff} , the region of control per rod, R , may be determined. If these rods are assumed to be uniformly distributed over the radial area, the number of rods required for control is:

$$\text{Number} = \frac{(\text{Reactor Radius})^2}{(\text{Radius of Control Region})^2}$$

The number of rods required to control a Δk_{eff} of 0.25 has been plotted as a function of r_{eff} in Figure 16.

The effective rod radius is a function of the actual rod radius, the distance inside the rod at which the thermal neutron flux is assumed to be zero, and the surrounding media. The calculation of r_{eff} for various rod shapes is adequately discussed in the literature (16, 19, 20).

It should be emphasized that this method takes into account the effect of location in the pile on the reactivity value of the rods and also the effect

of the rods on the flux; however, it does not take into account the shadowing effect between rods. Calculations made at Brookhaven indicate that the shadowing effect is not negligible (11).

Since the insertion of a control rod increases the neutron density at a distance from the rod over what it would be without the rod, the relative location of the rods is important. For certain distances apart, the effect of two rods will be greater than the sum of the effects of the two rods separately.

Calculations of the eccentric rod effectiveness and of partial rod insertion have been described in Reference 19 and 21.

In the two group treatment presented by Glasstone and Edlund (14) the following expression is obtained for the reactivity value of a single axial control rod in a cylindrical pile:

$$\Delta k = \frac{7.5 M^2}{R^2} \left[.116 \left(1 + \frac{\tau}{L^2} \right) + \frac{\tau}{L^2} \ln \frac{L \sqrt{\tau}}{M r_{\text{eff}}} + \ln \frac{R}{2.4 r_{\text{eff}}} \right]$$

where

R = reactor radius

r_{eff} = effective rod radius.

For the reactor being considered, a control rod with $r_{\text{eff}} = 2.0$ cm, by the above formula, has a reactivity value of 0.0112. By using statistical weight factors and taking account somehow of the shadowing effects, it is possible to determine the number of such rods necessary to control a Δk of .25. This is very difficult to do for large numbers of control rods, however, since the flux is significantly affected by the presence of the rods and the statistical weight factors would change after the addition of each rod or group of rods. Such an analysis is beyond the scope of this report. However, for an r_{eff} of 2.0 cm, the curve in Figure 16 yields 62 as the number of rods required to con-

trol the reactor.

Considering materials on construction for the control rods, either boron steel or aluminum sheathed cadmium rods may be used. In the case of boron steel, the entire energy of interaction upon absorbing a neutron $[(n, \alpha) \text{ reaction}]$ will be released in the rod probably requiring external cooling of the rod. The use of cadmium would eliminate this problem since an (n, γ) reaction results, although it would cause an increase in the moderator heating. Since the moderator heat is utilized in the feed water heater (see Section 7) this effect is tolerable; hence it appears that aluminum covered cadmium rods would be acceptable for control of this reactor. In addition, these rods can be introduced directly into the moderator proper whereas if cooling of the rods is required, as with boron, thimbles would be necessary.

Since the cost of controlling a reactor is very high if a large number of control rods are necessary, it is worthwhile to consider other means of providing the desired control. For example, if it were determined that the recoverability from a scram afforded by the availability of a large amount of excess k_{eff} did not warrant the control of this excess by expensive control rod systems, then other cheaper control schemes could be used--possibly a poison such as a boron compound dispersed in the moderator or tubes of boric acid running through the reactor.

The use of rods which enter from one side of the core causes a depression of the flux in that region resulting in uneven burnup. Strategically placed poison could flatten the flux and reduce the non-uniformity of the fuel burnup.

In designing control rods, it should be remembered that the rod surface area determines the effectiveness of a rod. The r_{eff} of a cross shaped rod, for example, is much greater than for a cylindrical rod of the same major dimensions.

4.j. Flattening the Thermal Flux

It is desirable to flatten the thermal flux for three reasons: (1) A flat flux results in the most efficient utilization of the fuel; (2) For an average outlet gas temperature of 1000°F (only 230°F below the maximum allowable fuel temperature) the outlet gas temperatures from each fuel element should be essentially the same; and, (3) It was assumed in the heat transfer analysis (Section 5) that, axially and radially, the power density would vary as that portion of the cosine curve between 0.7 and 1.0.

There are several ways in which to flatten the flux:

1. Grouping the fuel elements closer in the center of the core than in the outer core regions, maintaining the fuel loading and number of fuel elements constant.
2. Putting more fuel in the central fuel elements and less in the peripheral fuel elements, maintaining the fuel loading, lattice spacing, and number of fuel elements constant.
3. Using control rods.

It was decided to investigate the first method in detail. The effect on the nuclear analysis of varying the lattice pitch was not determined. The nuclear analysis, however, did assume an average flux over the whole core.

An attempt to determine that fuel distribution which gave a flat flux in this reactor using two group diffusion theory proved unsuccessful. The methods used were those developed by Goertzel (46), Cohen and Estabrook (47), and Balent (48). It appeared that this reactor was "oversized"; that is, the size was greater than that which gave the minimum critical mass. The analysis seemed to indicate that an oscillating fuel distribution was needed with a fictitious

negative fuel concentration in some parts of the core. Lack of time prevented a thorough investigation of the meaning of these results.

The problem was then set up as a three energy group, three spatial region, one dimensional (cylindrical) calculation for the ORACLE. Two of the regions contained fuel and the third was the reflector. The fuel bearing regions were of equal thickness, the first comprising the central 1/4 of the reactor core (radially) and the second containing the peripheral 3/4.

Four cases were run as follows:

	Fuel Concentration		Maldistribution Factor
	$\sum_f(\text{Region 1})$	$\sum_f(\text{Region 2})$	$\left(= \frac{\sum_f(\text{Region 1})}{\sum_f(\text{Region 2})} \right)$
Case 1	1.0	1.0	1.0
Case 2	1.2	0.937	1.28
Case 3	1.5	0.8	1.87
Case 4	1.8	0.7	2.57

The first case is, obviously, for a uniform distribution, while the succeeding cases are for varying fuel concentration accomplished by varying the lattice pitch.

From the ORACLE flux plots, the average fluxes in Regions 1 and 2 were computed and normalized to give the same total power in all four cases. The results are shown in Figure 17. It is apparent that for the reactor model chosen for the calculations, the ratio of the fuel concentrations in the two regions which gives the best flux distribution is about 2.4.

Obviously, the results are only qualitative. In the first place, the homogenization of such a heterogeneous reactor as this one is difficult to justify because of resonance region effects. The resonance region also makes

it difficult to select the three energy groups. And lastly, more spatial regions must be considered to provide an adequate analysis.

It should be noted that the maximum to minimum flux ratios in the four cases in the core are: $1/0.24$, $1/0.30$, $1/0.40$ and $1/0.44$. Hence, the presence of the reflector alone flattens the flux significantly.

5.0 HEAT TRANSFER ANALYSIS

5.a. Basis for Determining Coolant Velocity and Pressure, Number of Unit Cells, and Fuel Loading.

For nuclear reasons it is obviously desirable to reduce to a minimum the amount of structural material in the core. This is also important from the cost standpoint, particularly when zirconium is used.

In an attempt to accomplish this, several parameter studies were undertaken based on heat transfer, fluid flow, and stress calculations. For the purpose of these studies, it was assumed that the process tubes as well as the cladding were fabricated from zirconium. However, the results will be applicable for indicating trends for other tube materials as well.

One study of interest considered the effect of varying the velocity of the coolant passing through the spiral fuel element. Constant parameters and conditions used in this study are listed in Appendix E. The results are shown in Figure 18. These results indicate that coolant velocity through the spiral has a marked effect on the amount of zirconium required for the cladding, but a much smaller effect on the diameter of the process tube and the required amount of tube material. Unfortunately it is not practical to design on the high velocity and low zirconium end of this curve because of pressure drop considerations. For the design finally selected, a helium velocity of about 500 feet per second seemed to a reasonable maximum. In a detailed design study an economic analysis to determine the cheapest combination of pressure drop and coolant velocity would certainly be in order.

A second study considered the effect of varying the number of unit cells or process tubes used in the reactor. Constant parameters and conditions used

FIGURE 18

VARIATION OF ZIRCONIUM REQUIREMENTS
AND PROCESS TUBE DIAMETERS WITH
HELIUM VELOCITY IN FUEL SPIRAL

in this study are indicated in Appendix E. The results of this study are shown in Figure 19. The results indicate that a savings in fuel element cladding and process tube material can be made by using a small number of unit cells. However, the increase in process tube diameter which accompanies the reduction in number of cells affects the neutron leakage and other nuclear parameters so as to establish a practical minimum for the number of cells. This minimum proved to be at about 151 tubes in the design finally selected.

Consideration was also given to the effect of varying the fuel loading. Changes in cladding and process tube metal for uranium loadings of from 30 to 100 tons are shown in Figure 20. Constant parameters and conditions assumed in the study are listed in Appendix E. The results indicate that while the fuel loading has some effect, the variation in tube diameter and zirconium requirements is not drastic. The significance of this is two-fold. First, relatively small material savings are possible by reducing the fuel loading. Second, a reactor of this type with process tubes designed for a given fuel loading could be operated with other loadings differing widely from the original without seriously effecting the flow and heat transfer characteristics of the reactor. Based on considerations other than economy of materials the fuel loading selected for this reactor was 74 tons.

A fourth study involved the effect of changing coolant pressure. The pressure of the moderator was held steady at 200 psia while the helium pressure was varied from 150 psia to 500 psia. Process tubes were designed for bursting under the internal helium pressure or for buckling under the external D_2O pressure, whichever governed. With respect to buckling, ribbed tubes were used whenever they would cause a reduction in weight. Constant parameters and conditions used are listed in Appendix E. The results of the study are shown in

FIGURE 19

VARIATION OF ZIRCONIUM REQUIREMENTS
AND PROCESS TUBE DIAMETERS WITH
NUMBER OF UNIT CELLS

Figure 21. As would be expected, the diameter of the process tubes is drastically reduced by increasing pressure. It should be noted that above 300 psia helium pressure this change in diameter is not accompanied by a correspondingly large reduction in process tube zirconium. This is caused by a change in the governing failure condition from buckling to bursting. The figure also indicates that an increase in pressure produces a very worth-while reduction in fuel-element zirconium. Therefore it is desirable from all points of view to go to as high a pressure as possible within the limits imposed by stress, coolant leakage, cost, and fabrication considerations. An example of a possible limitation imposed by fabrication difficulties would be the problem of rolling a spiral with an adequately uniform clearance between turns when this clearance becomes very small--say several hundredths of an inch. This clearance is of course a function of pressure for given heat transfer conditions.

5.b. Heat Removal From the Reactor Core

As has been pointed out previously, the fuel element for this reactor consists of an assembly of 20 fuel sections connected end to end. Each section is composed of a spirally-coiled fuel plate mounted between two end plates. A spiral fuel element of this type has a number of unique characteristics which must be considered in design. Several of the more important features (some of which have been mentioned before) are:

1. The moderator can be maintained at the inlet temperature of the coolant with very little cooling since only cold gas is in contact with the inner surface of the process tube, and the radiative heat transfer from the fuel element to the tube is negligible.

2. Very large surface area, high gas velocity, good heat transfer properties, and appreciable pressure drop are inherent in the spiral design.

3. The length of the flow path which the coolant must follow in going through the fuel spiral is independent of reactor dimensions and is fixed only by heat transfer and fluid flow considerations. Therefore, design for the highest practical heat transfer film coefficient will lead to economies in the process tube diameter and the amount of cladding and structural material in the core.

4. Any particle of coolant passing through a process tube receives practically all its heat at one axial position in the tube. Thus the performance of this fuel element is very sensitive to both axial and radial variation in power density. To insure the maximum possible outlet gas temperature and optimum performance, fuel spiral sections must be orificed for both radial and axial position in the reactor.

5. At all positions in any process tube the cross sectional area is divided into three parts: cold gas annulus, fuel spiral area, and hot gas space. Since the tube diameter must be kept as small as possible to minimize both neutron leakage and structural material in the core, there is an advantage to varying the diameter of the spiral fuel sections with their axial position in the process tube. For a given tube diameter such an arrangement provides the maximum cold gas area at the end of the tube where it is needed, and the maximum hot gas area at the other.

For the purposes of this study the Dittus-Boelter equation was used to compute the film coefficient of heat transfer. Assuming the perfect gas laws to be applicable, the Dittus-Boelter equation for helium flowing through the spiral reduces to:

$$h = \frac{0.00818 K^{0.6} \text{ PU } (\Delta Z)^{0.2}}{W^{0.2} \mu^{0.4} T}$$

(See Appendix F for definition of symbols)

Heat transfer design to determine the thickness and length of fuel plate in any spiral section was based on the fuel section located at the center foot of the central tube. This point was used as a design basis since it must handle both the highest heat flux and the highest mass rate of flow of coolant. Modifications to this standard unit to adapt it to other positions in the reactor are accomplished by:

1. Varying the diameter of the spiral into which the fuel plate is rolled in accordance with the requirements of the axial position in the process tube.
2. Orificing to pass the amount of coolant required so as to give the desired outlet gas temperature for the sections assigned position in the

reactor.

For the purpose of the heat removal calculations it was estimated that the energy of fission would eventually be removed from the reactor in the following manner: ⁽¹⁾

Heat to helium coolant - - - - -	88%
Heat to D ₂ O moderator - - - - -	10%
Heat losses from reactor - - - - -	<u>2%</u>
Total	100%

Reasonable assumptions were also necessary with respect to the variation of heat release throughout the reactor. In general, the geometry of the reactor is a right cylinder with unit cells arranged in a hexagonal lattice. For heat removal calculations it was assumed that both axially and radially the power density would vary as that portion of a cosine curve having values between 0.7 and 1.0.

The maximum allowable metal temperature in the central fuel section was taken as 1112°F. This is 118°F below the δ - β phase transformation temperature of uranium, and this margin in the central fuel spiral with its maximum flow of coolant provides some leeway for fuel units near the edge of the reactor where gas velocities and film coefficients are lower. It also allows some margin against temperature transients. For the desired mixed mean outlet gas temperature of 1000°F, this maximum metal temperature of 600°C allows a temperature drop of 112°F from the center of the fuel plate at its hot end to the temperature of the outlet gas.

An important assumption made in the heat transfer calculations was that the film coefficient at the hot end of the fuel spiral is the same on both the inner and outer surfaces of the fuel plate. That is, the film coefficient around the surface of the hot gas space is the same as the coefficient in the last turn

of the fuel spiral. It was also assumed that the effect of flux reduction toward the center of the spiral could be adequately accounted for by taking the average generation rate to be 1.1 times the generation rate at the hot end of the fuel spiral. Later nuclear calculations indicate that this is very conservative.

The final reactor design utilized 151 process tubes, 74 tons of uranium, 20 foot fuel elements, 500 psi gas pressure, a 5 mil cladding of zirconium on the fuel, and a clear space of 0.08 inches between adjacent turns of the fuel spiral. For these conditions the important numerical results applicable to the central fuel section are listed below:

Film coefficient at hot end	
of spiral - - - - -	1090 BTU/hr ft ²
Spiral fuel plate thickness - - - - -	0.097 inches
Spiral fuel plate length - - - - -	73.2 inches
Allowable heat flux at hot end	
of spiral - - - - -	99,500 BTU/hr ft ²
Gas velocity at hot end of	
spiral - - - - -	585 ft/sec
Gas velocity at cold end of	
spiral - - - - -	308 ft/sec
Radiative heat transfer to	
process tube wall - - - - -	852 BTU/hr ft ²

With respect to the amount of thermal radiation to the process tube wall and moderator, the figure of 852 BTU/hr ft² is based on the surface area around the perimeter of the coiled fuel spiral. This amounts to less than 0.1% of the total heat generated in the fuel spiral and contributed little to moderator heating.

With the dimensions of the fuel spiral known, the design of the process tube can be completed. However, it is first necessary to fix the maximum permissible coolant velocity in the hot and cold gas space. For this study it was assumed that an axial velocity of about 500 feet per second would be permissible at the entrance and exit to the process tube. It was also assumed that the process tube should have an inside diameter one-half inch larger than the fuel spiral. On this basis the inside diameter of the tube used in the final reactor design was 6.80 inches. The wall thickness required to resist bursting by the interior helium pressure of 500 psi was computed. This was compared with the thickness required to resist collapse under the exterior D₂O pressure of 150 psia. This thickness was computed by the formula (49):

$$t^3 = \frac{4 \text{ Per } (1 - \sqrt{2}) r_o^3}{E}$$

Assuming the tube material to be 24S aluminum at 400°F and using a factor of safety of 1.5, the buckling consideration determines the required wall thickness which is 0.162 inches.

The ends of the process tubes must be equipped with fittings which minimize neutron leakage. On the hot end this fitting should also incorporate a means of converting the velocity head to pressure and it should minimize the transfer of thermal energy to the moderator. Sketches of a possible design for these end fittings are shown in Figure 22.

6.0 POWER PLANT

6.a. Choice of Cycle

The requirements of the power cycle are that: (1) The cycle have the highest possible net plant efficiency; (2) The equipment be of reasonable size and cost; and, (3) The work ratio (22) be as large as possible. The cycle must be designed for a helium coolant, a coolant outlet temperature of 1000°F , a coolant inlet temperature of 350°F , and a total power of 1000 megawatts. To obtain a reasonable comparison between possible power cycles, a helium pressure loss of 10% was assumed (exclusive of the gas turbine). Three cycles were investigated; a gas turbine cycle using the reactor coolant, a vapor cycle using steam, and a binary cycle using the reactor coolant in a gas turbine with a steam vapor cycle operating on the gas turbine exhaust.

A simple gas turbine cycle (Figure 23) was investigated first because of the simplicity obtained with one working medium and a small number of equipment components. The net plant efficiency versus pressure ratio is plotted in Figure 28. This plot assumes a compressor efficiency of 80%, turbine efficiency of 83%, turbine inlet temperature of 1000°F , and a compressor inlet temperature of 100°F . A maximum efficiency of about 9.3% was obtained with a pressure ratio of 2.25. This efficiency could be boosted slightly by adding an inter-cooler and regenerator to the system, although a regenerator cannot be used to full effectiveness because of the 350°F limit on the gas entering the reactor.

The gas turbine cycle has the added disadvantage that approximately 10% of all the heat generated is dissipated in the moderator due to fast neutrons and gamma rays. There was no apparent way to use this energy since it was at a temperature of 350°F .

A vapor cycle using steam as a working medium was also investigated. The equipment for this cycle was more complex than a gas cycle but the increase in thermal efficiency tends to off-set this disadvantage. The first vapor cycle investigated was a simple Rankine cycle with superheat to 900°F (See Figure 24). In order to keep the helium entering the reactor at a temperature less than 350°F it was necessary to set the steam pressure at about 125 psi. This system yielded a net plant efficiency of about 15%. The work ratio was about 70% which was too low to be desirable.

The simple Rankine cycle was revised by adding a helium regenerator and pre-cooler allow one to compress the gas at a lower temperature, leading to smaller compressors, due to increased density, and less work of compression. The regenerator and pre-cooler complicated the system somewhat, but they are standard pieces of equipment and do not require any expensive materials.

With this modified Rankine cycle, the variation of net plant efficiency versus steam pressure was studied. The assumptions used in making this study were (1) a minimum temperature difference of 100°F except in the pre-cooler, (2) heat to within 15°F of saturated temperature in the economizer, (3) no pressure drop in the steam system, (4) a turbogenerator efficiency of 80%, and (5) a compressor efficiency of 83%.

The results are shown in Figure 29. The best efficiency was 18.5% at 500 psia steam pressure. This cycle does not use any of the heat produced in the moderator by the neutrons and gamma rays.

The next step was to add a D₂O feed water heater to salvage the 10% of the total heat which is in the moderator. Since the D₂O must be circulated and cooled in order to keep the temperature at 350°F, it is possible to circulate it through a feed water heater and recover this energy. This cycle is shown in

Figure 30. There is no additional cost of equipment since a circulating pump and heat exchanger are required even if this energy is wasted. For the case of 500 psia steam, 10% helium pressure loss, and 350°F inlet coolant temperature, this modification of the previous cycle increased the efficiency to 20.6%. The final efficiency of the plant is a function of pressure loss (See Figure 27) and was found to be 21.4% (see section 6.d.).

There is an added advantage realized from this modification. If the steam pressure is higher than the moderator pressure, the light water will leak into the moderator where it can be removed. When the D₂O pressure is higher, the D₂O is lost from the system and would be expensive to recover.

A binary cycle using a helium gas turbine and a steam turbine was also discussed. It was felt that with the maximum temperature limit of 1000°F the increased efficiency, if any, would be over-balanced by the increased complexity of the system. If the maximum helium temperature could be raised to about 1300°F it appears that a binary cycle might become attractive for this reactor.

The optimum system is obviously the Rankine cycle with the helium regenerator, pre-cooler and D₂O feed water heater.

6.b. Cycle Parameters

6.b.1. Fluid Properties

The properties of helium were taken from Reference 24. The density, enthalpy and entropy above 600°F were calculated from the perfect gas law. The properties of heavy water were taken from Reference 24. The properties of light water were taken from Reference 25 (See Figure 26).

6.b.2. Flow Rates

With a 21.4% efficient plant this system will produce 214 megawatts of electricity (see Appendix H); 214 megawatts is slightly large for one turbo-generator with 900°F and 500 psi throttle conditions. Two loops would put the turbo-generator size well within present technology, and, in addition, two loops were easier to arrange from a plant layout standpoint with the added advantage of smaller helium passages through the shield.

As previously mentioned, 10% of all the power generated was assumed to be generated in the moderator. With the total power fixed at 1000 megawatts and the inlet and exit helium temperature fixed at 350°F and 100°F, the helium flow rate was found to be 3.78×10^6 lbs./hour per reactor or 1.89×10^6 lbs./hour per loop.

A heat balance and the assumption of 100°F minimum temperature difference for a helium to light water exchanger yielded a flow rate of 0.523 lbs. of steam per lb. of helium or a flow rate of 9.90×10^5 lbs. of steam per hour for each loops.

The temperature change of heavy water in the feed water heater was set as high as practical to minimize heavy water hold up. An inlet temperature

of 350°F and an arbitrary exit temperature of 150°F gave a flow rate of 8.3×10^6 lbs. of heavy water per hour.

6.b.3. Helium Pressure Loss

Figure 27 shows a plot of net plant efficiency versus helium pressure loss. The compressor efficiency was assumed to be 84%.

Pressure Drop Through the Reactor Core

The accuracy of a pressure drop calculation for this reactor which was based on elementary theory may leave much to be desired. This is due to the unique features of the fuel element and the complicated flow pattern. In a detailed design program for a reactor of this type, an accurate determination of pressure drop and the balancing of the flow pattern would be a major experimental project.

For lack of a better approach, approximate pressure drops through the reactor were computed using elementary theory and the following approximations:

1. Flow through the cold gas annulus was treated as isothermal.
2. Flow through the hot gas space was treated as isothermal after the coolant has made one turn around the inner perimeter of the fuel spiral.
3. The step-tapered and variable flow characteristics of the cold gas annulus and the hot gas space was approximated by considering separately, (1) short sections of constant area and constant flow wherein all friction losses occur, and (2) sections incorporating both change in area and mass flow rate but with no friction loss.
4. The friction factor for helium was approximated from the von Karman-Nikuradse relation (24) by:

$$f = \frac{0.25}{(Re)^{0.23}}$$

5. All heat generated in the outer turn of the fuel spiral is transferred to the helium passing through the first outer turn of the spiral, and none flows to the helium in the cold gas annulus.

6. Half of the heat generated in the inner turn of the fuel spiral is transferred to the helium making its first turn around the hot gas space - the remainder goes to the gas in the inner turn of the fuel spiral.

7. The effect of tangential velocity was neglected in the cold gas space.

8. In making its first turn around the hot gas area, the helium maintains about the same cross section of flow area as it had on leaving the fuel spiral.

9. All friction losses due to tangential velocity in the hot gas space were computed on the basis of the gas passing through the fuel unit at the center of the process tube. These losses occurred in the first turn of the gas about the hot gas space and were reflected in a reduction in tangential velocity. All losses are on the gas-to-metal side of the stream--none on the gas to gas side. After reduction by this friction loss, the tangential velocity remains constant in the remainder of the process tube.

10. The pressure loss in the hot gas annulus due to axial velocity was treated in the normal manner, ignoring the tangential velocity.

11. In the hot gas space, the vector sum of axial and tangential velocities determine the velocity head.

12. Of the final velocity head in the hot gas space, 80% is recoverable.

Entrance losses (24) were approximated by:

$$\Delta P = 1.07 \times 10^{-10} \left[1 - \left(\frac{A_2}{A_1} \right)^2 \right] \frac{\rho U^2}{g}$$

In passing through the spiral where heat is being added, the pressure drop and temperature change was calculated from the following combination of basic flow equations:

$$\Delta T = \frac{4g}{C_p} - \frac{1.86 \times 10^{-12} W^2 R^2}{g J A^2 C_{pm}^2} \Delta \left(\frac{T}{P} \right)^2$$

$$- \Delta P = \frac{4.56 \times 10^{-13} W^{1.77} R}{g A^{1.77} D_e^{1.23} m} \left(\frac{A^{2.3} T}{P} \right) \Delta x + \frac{3.72 \times 10^{-12} W^2 R}{A^2 g m} \Delta \left(\frac{T}{P} \right)$$

In the pressure equation D_e is equal to 2 times the clear distance between adjacent turns of the fuel spiral.

After passing through the fuel spiral the gas makes one turn around the hot gas space in contact with the metal surface. For calculating the temperature and velocity changes which occur as the gas makes this turn, the following equations were used:

$$\Delta T = \frac{4g}{C_p} - \frac{3.86 \times 10^{-8} \Delta U^2}{g J C_p}$$

$$- \Delta U^2 = \frac{0.0798 \bar{U}^{1.77}}{D_e^{1.23}} \left(\frac{\bar{U} R T}{P m} \right) \Delta x$$

These were developed from the basic fluid flow equations and the assumptions listed above. The value of D_e to be used in the velocity change equation is taken to be 4 times the thickness of the gas space at the exit of the fuel spiral.

This hydraulic diameter is applicable to the case of a thin layer of fluid flowing over one side of a single flat plate.

A plot of the computed pressure variation through the center process tube for the final reactor design is shown in Figure 32. It will be noted that the total drop is 34 psi. In this design the fuel element had a clear distance between fuel spirals of 0.08 inches. Pressure loss through the spiral is based on the fuel units at the center of the tube since this is the point of maximum rate of flow. Presumably, the remaining fuel units would be orificed to pass the smaller required flows and give the indicated pressure drops.

Consideration of the power cycle indicates that marked savings in cycle efficiency are possible by minimizing the pressure drop through the reactor. On the other hand, reducing the pressure drop causes a corresponding reduction in gas velocity and heat transfer efficiency. This leads to an increase in process tube diameter and to a requirement for more fuel cladding material. An indication of where the best design lies may be had by examining Figure 33. This is a plot of pressure drops in the center process tube for fuel spirals with various clear distances between adjacent turns. The very fast rise in pressure losses for spacings less than about 0.08 inch indicates that it would probably be uneconomical to design in that region. Above about 0.10 inch the heat transfer film coefficient becomes very ordinary. Although a detailed economic analysis would be required to determine the best design, these values indicate the proper range of fuel spiral spacing for efficient operation.

Pressure Drop Through the External System

To estimate the pressure drop in the external system it was necessary to know the sizes of tubes, the lengths of tubes, and the heat exchanger arrangement.

After determining the amount of heat to be transferred in each exchanger, a tube size or plate spacing, a wall thickness, and the material for each exchanger was assumed (see Appendix G). Film coefficients were calculated by methods outlined in McAdams (26). Then the heat transfer area was computed and the velocity from the film coefficient formulae was used to determine the number of tubes or spaces and the individual tube lengths.

The pressure loss through each piece of equipment was calculated using methods outlined in McAdams (26). The loss was found to be 10 psi giving a total system loss of 44 psi. A temperature entropy diagram for the helium cycle is shown in Figure 31.

6.c. Equipment

A short study of equipment size and arrangement was made. A steam generator similar to the conventional generator with super-heater, boiler, and economizer was used. A plate type exchanger (23) similar to the type used as air pre-heaters on conventional boilers appears to be the most practical for the helium regenerator. This type of unit was recommended because moderate velocities give good film coefficients and a small pressure drop compared to small tube or compact heat exchangers. A regenerator similar to a Ljungström type air pre-heater would be very attractive except for the large leakage from the high pressure to the low pressure side of the system. Four U-tube heat exchangers with helium on the tube side were used for the pre-coolers.

The helium compressors required about 24,000 horse power for each loop (See Appendix H). The pressure ratio was 1.1. Motor driven axial flow compressors are best suited for this type of service. The number of compressors per loop was not determined.

Figures 34 and 35 show a plan and cross section of the power plant.

6.d. Net Plant Efficiency

The cycle shown in Figure 30 was analyzed for net plant efficiency. Westinghouse (27) gives a heat rate of 11,180 BTU/kwh or an efficiency of 81% for a turbo-generator in this size and throttle conditions. Obert (22) suggests compression efficiencies of 84% and motor efficiencies of 97% for large units (i.e., an overall compressor efficiency of 81.5%).

The net plant efficiency was calculated to be 21.4% (see Appendix H).

FIGURE 23
GAS TURBINE CYCLE

FIGURE 24
STEAM CYCLE

FIGURE 25
STEAM CYCLE

FIGURE 26
DESIGN PROPERTIES - STEAM CYCLE

FIGURE 30
SYSTEM FLOW SHEET

FIGURE 31

T-S DIAGRAM FOR HELIUM

SWITCH YARD, HELIUM TRAILER,
MAINTENANCE SHOPS & OFFICE BUILDING

FIGURE 34
PLANT LAYOUT

SECTION "A-A"
FIGURE 35
PLANT LAYOUT

7.0 AUXILIARY SYSTEMS

7.a. Heavy Water Purification Cycle

To maintain the heavy water purity, a system to remove such undesirables as light water, ionic impurities, and suspended materials was incorporated into the plant design. The high purity requirements are necessary to 1) reduce process tube corrosion effects to a minimum, 2) reduce moderator radiolytic decomposition to a minimum, and 3) minimize parasitic capture of neutrons by foreign materials.

The heavy water moderator is circulated at a rate of about 1570 gpm or once every 20 minutes. From this main cycle stream, a smaller stream (30 gpm) is then circulated through the purification cycle. The magnitude of this stream was chosen assuming that a 12 hour period was required to completely clean the moderator following the injection of a poisonous salt which acts as a final reactor safety device.

This stream first passes through a cooler, where it is cooled from 150°F to about 100°F, in order to protect the anion resin bed which becomes highly ineffective at elevated temperatures. Following this the let-down valve reduces the pressure from 145 psi to about atmospheric (16 psi). The water then enters a large surge tank and is pumped through a filter to remove suspended particles prior to entering the deionizers.

The deionizers contain a mixture of anion and cation exchange resin which has been deuterized previous to use. Deuterization is the process which replaces

the ionic hydrogen content of the resin with deuterium and is accomplished by slowly contacting the bed with heavy water. An after-filter is also provided to retain any particulate matter which may be acquired in passing through the ion exchanger. Although there are three parallel lines (containing a pump, filters, and deionizer) from the surge tank, it is anticipated that only one will be employed during normal operation, with the remaining two on standby for added capacity (90 gpm) during cleanup operations following poison salt injection into the moderator. Continual monitoring for conductivity of the effluent deionized stream will indicate the condition of the bed.

This effluent stream from the deionizer is further separated, a small portion being fed to a distillation column and the remainder being pumped back into the main heavy water cycle stream. Separation of light from heavy water is effected in the distillation column. The tails from the column, richer in heavy water, is then mixed with a make-up stream from the heavy water storage tank and fed back to the surge tank. The overhead or product stream from the column which contains a mixture of light and heavy water is then either sent to storage, to await future reprocessing, or else is fed directly to an intermediate stage of the processing cycle.

The flow sheet for the D_2O purification and circulation cycle is presented in Figure 36.

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FIGURE 36 D_2O CIRCULATION & PURIFICATION SYSTEM

7.b. Heavy Water Recombination Cycle

It is estimated that in the HRT, 30 lbs./min. of deuterium and 120 lb./min. of oxygen will be produced in the core by the radiolytic decomposition of the moderator (30). This problem will also be present in this reactor, although its magnitude has not been determined, and provisions have been made for recombining D_2 and O_2 and returning it to the system as heavy water.

The space above the moderator in the reactor will be continuously swept with a stream of helium to remove such gases as D_2 , O_2 , and D_2O vapor. These gases will then be fed to a bubble plate column where they will be contacted with the heavy water return stream from the cooler-condenser. In this manner some of the heavy water will be vaporized and thereby act as a diluent for both D_2 and O_2 . To prevent the possibility of an explosive mixture of D_2 and O_2 it is necessary to insure that the concentration of these gases be kept below the explosive limit by diluting the stream with, in this case, helium and the additional vaporized D_2O from the bubble plate column. The quantity of helium required is determined by the amount of D_2O vapor formed in the column and the minimum explosive concentration that can be tolerated.

These gases then go to the catalytic recombiner which is a bed of cylindrical platinized alumina pellets containing about 0.03-0.3 weight percent platinum, which if present in sufficient quantity will insure complete recombination of the deuterium and oxygen. Catalytic recombiners have an advantage over flame recombiners in that they are capable of recombining the component gases, D_2 and O_2 , in concentrations below the explosive limit (31). Following the recombiner, the gases enter a cooler-condenser where the recombined heavy water vapor is condensed out. The liquid heavy water stream is then either

returned to the top of the bubble plate column or piped directly to storage. The effluent liquid stream from the bubble plate column is in turn pumped back into the main heavy water stream.

The gas stream from the condenser containing helium and uncondensed D_2O vapor is sent back across the top of the moderator after passing through the helium blower to repeat the cycle, or else is sent to the helium reservoir. If the latter path is chosen the gas stream first passes into a combination freon cold trap and regenerator to insure that any uncondensed D_2O vapor left by the cooler will be removed.

This cycle is shown in Figure 37.

7.c. Helium Processing Cycle

To detect the rupture or cracking of a fuel element with its subsequent release of radioactivity, a device must be included for continually monitoring the coolant stream. A very small fraction of the coolant is tapped and sent through the monitoring channel. This channel consists of a small chamber, sufficient to provide some gas holdup, and a thin-walled G-M tube attached to the container wall to act as the sensing element. The G-M tube in turn actuates four motor driven, quick acting, cut-off valves as shown in Figure 38.

When the activity of the monitored coolant stream ($1000^{\circ}F$) exceeds a predetermined value, a signal is delivered to close the mainline cut-off valves and open the by-pass line. The contaminated gas first passes through a main cooler which removes about 75 MW of heat (7.5% delayed heating). After the cooler the gases pass through a filter which removes any solid matter present. Following the filter, but prior to separation, the gases are fed to a blower to maintain the desired circulation rate. Part of the separated stream is recycled directly

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FIGURE 37
D₂O RECOMBINATION SYSTEM

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FIGURE 38
HELIUM CYCLE

to the reactor while the rest of the gas goes to the pre-cooler. In the pre-cooler the gases are cooled, by the cold effluent absorption bed stream, to about -297°F , the operating temperature of the bed.

The absorption bed contains Columbia CXA carbon as the active material. Experiments have shown that when this material is at -297°F (-183°C) complete removal of xenon and krypton from a nitrogen carrier result (32), (33). Since helium is more inert than nitrogen it is expected that these results will also hold for helium. The cold effluent stream from the bed goes to the after-filter which is provided to remove any particles that may have been picked up in the bed. Then it is passed back through the pre-cooler where it acts as the coolant. After leaving the pre-cooler, the clean deactivated helium is reintroduced to the coolant system at a point between the pre-filter and the blower.

During normal operation with the main stream valve open, the helium passes through a particulate filter after leaving the compressor.

Shielding will have to be provided for the main cooler, pre-filter, blower, pre-cooler, absorption bed, and after-filter. In addition, remote handling equipment will be needed to do all the necessary maintenance and equipment associated with the helium clean-up cycle.

8.0 COST ANALYSIS

An attempt must be made to estimate the power costs in a preliminary design such as this since this is the final criterion for determining its attractiveness. A cost analysis will usually indicate the most fruitful lines of future endeavor for lowering the costs.

In this brief cost analysis, the net cost will be subdivided into fuel costs, fixed charges, and operating and maintenance costs. The Evaluation Staff of the AEC Division of Reactor Development has established a convention for determining fixed charges (50); 15% is assessed against the capital investment and 12% is assessed against inventory items such as fuel and heavy water. However, 4% has been used in recent analyses for the fuel and D₂O inventory charges. Both 12% and 4% have been used in this report. Operating and maintenance costs are difficult to evaluate, but for the purposes of this study these costs were estimated to be 20% of the fixed charges.

The fuel costs result from fuel element material and fabrication costs, chemical processing costs, and inventory charges. D₂O charges (inventory and losses) will be included in the fuel costs, and helium charges will be included in the operating costs.

The design parameters of this reactor that are pertinent to fuel costs are listed below:

Endurance	6000 MWD/ton
Net Plant Efficiency	21.4%
Net Electrical Output	1.30 MW/ton of D ₂ O

Since fissionable material such as Pu²³⁹ and Pu²⁴¹ is built up during reactor operation, an analysis of the chemical processing costs was made to

determine the advisability of recovering this material and salvaging the zirconium. After 6000 MWD/ton of irradiation, the amounts of Pu isotopes built up are:

Pu²³⁹: 141,200 gms.

Pu²⁴⁰: 45,400 gms.

Pu²⁴¹: 14,100 gms.

With 23% Pu²⁴⁰ present, the value of the plutonium is \$12 per gram (53). The cost of processing was taken to be \$18,000 per ton of uranium (52). The result of the computation indicates that a credit of 0.2 mils/kwh of electricity could be realized by processing the spent fuel elements.

Apart from the D₂O charges and the savings realized from the fuel element processing, the fuel element costs may be analyzed as follows.

The material efficiency can be defined as the MW/ton of fuel. The fuel inventory costs will be inversely proportional to this quantity. The total fuel cost may then be estimated as follows:

Definitions:

X = cost of fabricated fuel element in mils/ton of fuel

Y = MWD/ton of fuel

T = tons of fuel

M = material efficiency in MW/ton of fuel

Z = burnup cost in mils/kwh of electricity

W = fuel inventory cost in mils/kwh of electricity

a = kwh/MWD = 2.4×10^4

i = inventory charge in fraction per annum

P = total power in MW = 1000

b = MWD/year = $360P = 3.6 \times 10^5$

C = total fuel cost in mills/kwh of electricity

e = thermal efficiency (assumed to be 0.2)

From these definitions it will now be obvious that:

$$Z = \frac{X}{aYe}$$

$$W = iXT/abe$$

$$M = P/T$$

$$W = iPX/abM$$

$$C = Z + W = \frac{X}{ea} \left[\frac{1}{Y} + \frac{iP}{bM} \right]$$

Since, for a constant power, Y versus T passes through a maximum and M versus T decreases monotonically, C versus T will have a minimum. In the past it has been customary to take $i = 0.12$. More recent practice has been to use a value of 0.04. In Figure 39, (Cae/X) is plotted versus T for $i = 0.12$ and $i = 0.04$ over the range of T for which Y has been determined. It will be noted that for $i = 0.12$, the curve passes through a minimum which is relatively near the design point, while for $i = 0.04$, a cost decrease of about 6% could be obtained by going to the optimum point.

If the price of zirconium is taken to be \$13.5/lb. and that of uranium fabricated in fuel elements to be \$49/lb., then it can be easily shown that:

$$X = 10^8 \text{ mils/ton of uranium}$$

$$C = .208 \frac{eac}{X} \times 10^5$$

Hence, for $i = 0.12$ and 0.04, at the design point of 74 tons, the cost of fuel is, respectively, 4.00 and 3.65 mills/kwh of electricity.

Annual D_2O losses of 5% are recommended for evaluation studies for most

recent reactor plant. Because of the low circulation rate in this reactor, a lower value of 2% will be assumed. For a net output of 1.30 MW of electricity per ton of D_2O , and a plant factor of 0.8, the D_2O charges are 0.74 and 0.25 mills/kwh for inventory charges of 12% and 4%, respectively, and 0.12 mills/kwh for losses.

The fixed charges will consist of a 15% charge against the capital investment of the power plant and the reactor plant. Reference 51 indicates that the cost of power plants exclusive of boilers is \$100/kw of gross power for throttle temperatures of the order of 900°F. It is estimated that four 200-MW boilers would be required in the plant and it was assumed that boiler costs are approximately \$10/kw. With a plant factor of 0.8, the fixed charges due to the power plant are about 3.0 mills/kwh.

A detailed design is necessary to determine the costs of the reactor plant. Since this is not available for this analysis, the electrical costs were plotted as a function of the reactor plant costs. This is shown in Figure 40.

It is apparent that the fuel costs vary from 50% to about 30% of the total cost over the range of reactor plant costs considered. This allows significant room for improving the total cost by paying attention to the fuel costs.

9.0 CONCLUSION

The reactor design developed here cannot be considered the optimum from the standpoint of power costs. The cost per kilowatt was found to depend very sensitively on several parameters which were not considered in detail. It is the purpose of this section to point these out, to summarize the results of the analysis which was done, to list a number of experiments which must be done prior to completion of a final design, and to comment in general on the applicability of D_2O , gas cooled, natural uranium reactors for power production.

The initial conversion ratio has a profound effect on the fuel cost in a natural uranium reactor. It was pointed out in Section 4.d that if the ICR were raised somehow from 0.7 to 0.9, the MWD/ton available would almost double. The most obvious way of increasing the ICR is to decrease p , the resonance escape probability. Since this obviously results in a reduction in the excess k_{eff} available for burnup, the best combination of p and ICR must be found. The resonance escape probability is influenced by the surface-to-mass ratio of the fuel lumps, the amount of D_2O per unit cell, and the amount of fuel per unit cell. This will also affect the leakage, but f is relatively insensitive to the changes in the amounts of D_2O and fuel per unit cell. Hence, the problem is to optimize the obtainable MWD/ton by varying the fuel and D_2O per unit cell and noting the effect on k_{eff} through the leakage and p .

It is also possible to raise the ICR by using depleted uranium, decreasing the leakage, or using a less efficient moderator.

Another factor which is important is the inventory charge for fuel.

Although this is only 4% it makes the optimum fuel load a little less than that which gives the maximum MWD/ton (See Section 8).

The over-all efficiency directly affects the cost of electricity. In medium temperature reactors cooled by He, air, CO₂, or N₂, low efficiency is inherent. There are several ways to increase the efficiency, some of which could be easily incorporated in this reactor. The most apparent way would be to raise the outlet gas temperature. This could be done by alloying the uranium with an element such as Nb which alleviates the problem created by the α - β transition temperature of pure uranium metal. Provided the inlet temperature remained unchanged, an increase in outlet temperature would not increase the D₂O temperature appreciably due to the fuel element design.

Other interesting possibilities for improving efficiency are the possible use of hydrogen as a coolant and the use of graphite dust in the coolant stream. In both cases, the heat transfer properties would be very much improved.

The pressure drop through the reactor has a large effect on the cost of power. Pressure drop controls the pumping power requirement with strongly affects the over-all reactor efficiency. However, reducing the pressure drop also lowers the heat transfer efficiency which in turn raises the requirements for cladding and structural materials and increases neutron leakage through the larger process tubes. Determination of the most economical pressure drop would require a more detailed analysis.

It is also probable that some adjustment of the moderator temperature and pressure would alter the cost picture. Raising the moderator temperature would make a higher inlet gas temperature feasible. This raises the average temperature of the heat available to the steam cycle from the helium and increases the efficiency and possibly simplifies the power cycle. On the other hand, raising

the moderator temperature would have adverse nuclear effects and would render the problem of containing the D_2O more difficult.

An experimental investigation might show that a higher maximum gas velocity could be used in the hot and cold gas areas of the process tube without disrupting the desired flows of helium to the fuel spiral sections. A higher gas velocity would reduce the process tube diameter, neutron leakage, and structural material in the core without drastically increasing the pressure drop.

In addition, it is possible that a detailed economic study would show that an adjustment of the number of process tubes or in the size of the reactor would be desirable.

In general, one can conclude that the most significant decreases in power cost would be obtained by lowering the total fuel costs and that this would have to be done through three factors: 1) increasing MWD/ton, 2) decreasing fabrication costs, 3) increasing thermal efficiency.

Some of the obvious experimental work which would need to accompany the development of this reactor is summarized as follows:

1. Balancing of coolant flow through fuel elements
2. Fuel element pressure drop measurements
3. Dimensional stability of fuel element under irradiation
4. Thermal cycling limit of fuel element

It is felt that this study supports the following conclusions:

1. D_2O is an economically feasible moderator for natural uranium reactors.
2. Gas cooling can be made competitive with liquid cooling using present technology and will be even more attractive when it is possible to go to gas temperatures well above $1000^{\circ}F$.

3. An important factor in determining the feasibility of a natural uranium reactor is the cost of fabricating the fuel elements. It is hoped that the conservative estimate used in this study can be significantly improved.

10.0 APPENDIX

10.a. Appendix A: Calculation of the Flux Disadvantage Factor

The ratio of the average flux in the moderator to the average flux in the fuel region is called the disadvantage factor. It is important in the calculation of the thermal utilization and the resonance escape probability.

For this analysis, the disadvantage factor, S , was calculated using one group theory on the unit cell model presented in Section 4.d. The accuracy of such a calculation is probably poor although it should allow reasonably accurate comparisons to be made between the unit cells which were considered. For graphite lattices, one group theory underestimates the disadvantage factor (11) and it is reasonable to expect a similar behavior in D_2O lattices.

The one group equations applicable to the model of the unit cell are:

$$D_1 \nabla^2 \phi_1 - \sum_{a_1} \phi_1 = 0$$

$$D_2 \nabla^2 \phi_2 - \sum_{a_2} \phi_2 + S_0 = 0$$

where the subscript "1" refers to the fuel region and "2" refers to the moderator region. The other terms are defined as follows:

D = thermal diffusion coefficient

$\sum_a$ = average thermal absorption cross section (cm^{-1})

S_0 = source of thermal neutrons--it has been assumed that thermal neutrons are produced only in the D_2O .

The helium spaces have been treated as void spaces.

The solution to these equations are:

$$\phi_1 = AI_0(\kappa_1, \rho) + BK_0(\kappa_1, \rho)$$

$$\phi_2 = CI_0(\kappa_2, \rho) + DK_0(\kappa_2, \rho) + \frac{S_0}{\sum_{a_2}}$$

The boundary conditions are: 1) the gradient of the flux at the helium-fuel interface must be zero; and 2) the gradient of the flux at the unit cell boundary must be zero. Application of these conditions yields:

$$\phi_1 = A \left[\frac{K_1(\kappa_1 \rho_1)}{I_1(\kappa_1 \rho_1)} I_0(\kappa_1 \rho) - K_0(\kappa_1 \rho) \right]$$

$$\phi_2 = C \left[\frac{K_1(\kappa_2 \rho_2)}{I_1(\kappa_2 \rho_2)} I_0(\kappa_2 \rho) - K_0(\kappa_2 \rho) \right] + \frac{S_0}{\Sigma_{a_2}}$$

The two remaining conditions apply to the interface and are: 1) the neutron density must be the same on both sides of the helium annulus (i.e., $\phi_1(\rho_1) = \phi_2(\rho_2)$); and, 2) the neutrons which leave one surface must arrive at the other surface (i.e., $\rho_2 J_1(\rho_2) = \rho_1 J_2(\rho_1)$, where J = the net current). The latter is an assumption and is valid if the helium channel is small compared to the other cell dimensions.

If ϕ is averaged over the fuel volume (after the application of the interface conditions), the following result is obtained:

$$\bar{\phi} = \frac{2 S_0 \rho_2 \frac{\rho_3}{\rho_2} \alpha_2 \beta_2}{\Sigma_{a_2} (\rho_2^2 - \rho_1^2) D, \kappa_2^2 (\alpha, \beta_2 \frac{\rho_3}{\rho_2} - \alpha_2 \beta_1)}$$

where α_1 , α_2 , β_1 , and β_2 will be defined later.

Since all the thermal neutrons which are produced in the moderator must be absorbed somewhere in the unit cell, the following neutron balance condition is necessary:

$$S_0 V_0 = \bar{\phi}_1 V_1 \Sigma_{a_1} + \bar{\phi}_2 V_2 \Sigma_{a_2}$$

where $\bar{\phi}$ is the volume averaged flux.

This yields for the disadvantage factor:

$$S = \frac{\bar{\phi}_2}{\bar{\phi}_1} = \frac{\Sigma_{a_1}}{\Sigma_{a_2}} \left[\frac{S_0}{\Sigma_{a_1} \bar{\phi}_1} - \frac{V_1}{V_2} \right]$$

and, upon substituting for $\bar{\phi}_1$, we get:

$$S = \frac{P_2^2 - P_1^2}{2P_2} \kappa_1 A - \left(\frac{P_2}{P_3}\right) \frac{P_2^2 - P_1^2}{2P_2} \kappa_2 \frac{\sum a_1}{\sum a_2} B - \frac{P_2^2 - P_1^2}{P_4^2 - P_3^2} \frac{\sum a_1}{\sum a_2}$$

where

$$B = D_2 \kappa_2 \frac{B_1}{B_2} = \frac{K_1(\kappa_2 P_4) I_0(\kappa_2 P_3) + I_1(\kappa_2 P_4) K_0(\kappa_2 P_3)}{K_1(\kappa_2 P_4) I_1(\kappa_2 P_3) - I_1(\kappa_2 P_4) K_1(\kappa_2 P_3)}$$

$$A = D_1 \kappa_1 \frac{A_1}{A_2} = \frac{K_1(\kappa_1 P_1) I_0(\kappa_1 P_2) + I_1(\kappa_1 P_1) K_0(\kappa_1 P_2)}{K_1(\kappa_1 P_1) I_1(\kappa_1 P_2) - I_1(\kappa_1 P_1) K_1(\kappa_1 P_2)}$$

The above expression for the disadvantage factor applies to thermal neutrons using one group theory. The cross sections to be used are appropriately averaged cross sections, the flux is the total track length per cm³, and the volumes are those of the fuel region (cladding included) and moderator. The same expression can be used to calculate the disadvantage factor in the resonance region, for use in the formula for the resonance escape probability, under the following rules:

1. The applicable energy group is that containing the resonances of U²³⁸. In lethargy units its width is 5.63 (10).
2. The average microscopic absorption cross section to be used for the uranium is $A \frac{(1 + A \frac{S_0}{M_0})}{5.63}$ (10).
3. $\sum a_2$, the absorption cross section for the moderator, must include a fictitious cross section to account for scattering out of the energy interval.
4. Resonance neutrons are also assumed to be born only in the moderator and not in the fuel region.

In the application of the formula for S , care must be taken to determine accurately the values of the Bessel functions, particularly the modified function of the second kind.

10.b. Appendix B: Determination of the Fast Effect

In addition to the fissions resulting from the absorption of thermal neutrons in U^{235} there will be fissions due to absorption by U^{238} of neutrons with energies above the fission threshold (about 0.9 mev). Allowance for this is provided by including in the simple one group formula for k_{eff} a factor, Σ , which has the following definition:

$$\Sigma = \frac{\text{total fast neutrons produced by all fissions}}{\text{fast neutrons produced by thermal fissions}}$$

A mathematical expression for Σ in terms of the geometry and physical constants of a heterogeneous chain reacting system is given by Weinberg and Wigner (34) as:

$$\Sigma = 1 + \frac{(\nu - 1 - \frac{\Sigma_a}{\Sigma_f}) P \frac{\Sigma_f}{\Sigma}}{1 - \frac{\nu \Sigma_f + \Sigma_e}{\Sigma} P}$$

where

ν = mean number of neutrons emitted per fission

Σ_a = non-fission capture cross section, cm^{-1}

Σ_f = fission cross section, cm^{-1}

Σ_i = inelastic scattering cross section, cm^{-1}

Σ_e = elastic scattering cross section, cm^{-1}

Σ = total cross section, cm^{-1}

P = probability that a neutron produced in the fuel will make a collision before escaping from the fuel.

The geometry and physical constants of the particular system being con-

sidered are given below:

where the shaded region (volume, V) contains a homogeneous mixture of natural uranium, zirconium, and helium and corresponds to the fuel region in the unit cell model described in Section 4.d. Also,

$$\frac{V_u}{V} = 0.520$$

$$\frac{V_{Zr}}{V} = 0.385$$

$$\text{uranium atoms/cc} = 0.0475 \times 10^{24}$$

$$\text{zirconium atoms/cc} = 0.043 \times 10^{24}$$

$$p_2 - p_1 = 1.73 \text{ cm}$$

All cross sections are flux averages over the applicable volume and energy ranges. The following values were obtained from the Weinberg-Wigner discussion and from an examination of BNL-325:

$$\sigma_t(Zr) = 5.5 \text{ b}$$

$$\sigma_l(Zr) = .8 \text{ b}$$

$$\sigma_e(Zr) = 4.7 \text{ b}$$

$$\sigma_a(Zr) = 0$$

$$\sigma_t(U) = 4.3 \text{ b}$$

$$\sigma_f(v) = .29 \text{ b}$$

$$\sigma_a(v) = .04 \text{ b}$$

$$\sigma_i(v) = 2.47 \text{ b}$$

$$\sigma_e(v) = 1.5 \text{ b}$$

Assuming a homogeneous distribution of uranium, zirconium, and helium in the fuel region, one obtains:

$$\Sigma = 1 + \frac{.049 P}{1 - .673 P}$$

The calculation of P for a hollow cylinder geometry is outlined by Guggenheim and Pryce (35). Their equations are given as follows:

$$1 - P = \frac{4a}{\pi(a^2 - e^2)} \int_0^{\sqrt{a^2 - e^2}} dx (\sin \phi_1 - \sin \phi_0) K_{12}(2 \Sigma x)$$

where:

$$\sin \phi_1 = \sqrt{1 - x^2/a^2}$$

$$\sin \phi_0 = \begin{cases} 0, & 0 \leq x \leq a - e \\ \frac{2\sqrt{(s-a)(s-e)(s-x)}}{ax}, & a - e \leq x \leq \sqrt{a^2 - e^2} \end{cases}$$

$$2s = a + e + x$$

$$K_{12} = \int_t^\infty (z - t) K_0(z) dz = \int_0^\infty \frac{e^{-2 \Sigma x \cosh u}}{\cosh^2 u} du$$

K_{12} is tabulated by Bickley and Nayler (36).

Performing the indicated integration numerically for the special case at hand, one obtains:

$$P = .455$$

Substituting, for $\nu = 2.5$

$$\Sigma = 1.032$$

10.c. Appendix C: Endurance Calculation; Formulae and Constants

The method of calculating the reactivity effects due to burnup in this report is essentially the same as outlined in Reference 41. However, more recent cross sections were used and more general equations were developed. Since the formulae, method, and constants are generally applicable to any natural uranium reactor, they are presented here in detail.

Initial Conversion Ratio

The all important quantity in determining the endurance is the initial conversion ratio (ICR); i.e., the atoms of fissionable material produced per atom of fissionable material consumed in the clean reactor. In the thermal region, the capture cross section of U^{238} is 382 barns per U^{235} atom (without the resonance contribution); therefore, the number of Pu^{239} atoms produced per U^{235} atom destroyed at the original concentration is 0.558. In the resonance region, Pu^{239} is produced at the rate of $(1-p)$ per fast neutron, where p is the resonance escape probability. Assuming a fast fission factor of 1.03, a ν_{25} of 2.46, and a p of 0.9363, there are 0.137 atoms of Pu^{239} produced per U^{235} atom destroyed at the original concentration. Hence, the ICR is 0.695.

Uranium and Plutonium Concentrations as a Function of Time

The formulae giving the variation of the fuel concentration with time will now be presented.

U^{235} :

N_0 = original concentration of U^{235} atoms

$$N = N_0 e^{-\sigma_a^{25} \phi t}, \quad \frac{dN}{dt} = -\sigma_a^{25} \phi N$$

Pu²³⁹: The concentration of U²³⁸ was assumed to be constant over the life of the fuel load; hence, in a constant flux, the production of Pu²³⁹ will be at a constant rate. Then:

$$\frac{dP}{dt} = (ICR) N_0 \sigma_a^{25} \phi - \sigma_a^{49} \phi P$$

where P is the concentration of Pu²³⁹ at any time.

Integrating, we get:

$$P = P_0 e^{-\sigma_a^{49} \phi t} + \frac{(ICR) \sigma_a^{25}}{\sigma_a^{49}} N_0 \left[1 - e^{-\sigma_a^{49} \phi t} \right]$$

where P₀ is the initial Pu²³⁹ concentration.

Pu²⁴⁰:

$$Q = Q_0 e^{-\sigma_a^{40} \phi t} + (ICR) N_0 \frac{\sigma_a^{25}}{\sigma_a^{40}} \frac{\sigma_c^{49}}{\sigma_a^{49}} \left\{ 1 - e^{-\sigma_a^{40} \phi t} \right.$$

$$\left. + \frac{\sigma_a^{40}}{\sigma_a^{40} - \sigma_a^{49}} \left(\frac{P_0}{(ICR) N_0 \frac{\sigma_a^{25}}{\sigma_a^{49}}} - 1 \right) \left(e^{-\sigma_a^{49} \phi t} - e^{-\sigma_a^{40} \phi t} \right) \right\}$$

where: $\sigma_c = \sigma_a - \sigma_f$

Q = concentration of Pu²⁴¹ at any time.

Q₀ = initial Pu²⁴¹ concentration.

Pu²⁴¹:

$$R = R_0 e^{-\sigma_a^{41} \phi t} + \frac{(ICR) N_0 \sigma_a^{25} \sigma_a^{40} \sigma_c^{49}}{\sigma_a^{49} (\sigma_a^{40} - \sigma_a^{49}) (\sigma_a^{41} - \sigma_a^{49})} \left(\frac{P_0}{(ICR) N_0 \frac{\sigma_a^{25}}{\sigma_a^{49}}} - 1 \right) \left(e^{-\sigma_a^{49} \phi t} - e^{-\sigma_a^{41} \phi t} \right)$$

$$+ Q_0 \sigma_a^{40} - \frac{0.8 N_0 \sigma_a^{25} \sigma_c^{49}}{\sigma_a^{49}} - \frac{(ICR) N_0 \sigma_a^{25} \sigma_a^{40} \sigma_c^{49}}{\sigma_a^{49} (\sigma_a^{40} - \sigma_a^{49})} \left(\frac{P_0}{(ICR) N_0 \frac{\sigma_a^{25}}{\sigma_a^{49}}} - 1 \right)$$

$$\left(e^{-\sigma_a^{40} \phi t} - e^{-\sigma_a^{41} \phi t} \right) + \frac{(I.C.R) N_0 \sigma_a^{25} \sigma_c^{49}}{\sigma_a^{49} \sigma_a^{41}} \left(1 - e^{-\sigma_a^{41} \phi t} \right)$$

where:

R = Pu^{241} concentration at any time

R_0 = initial Pu^{241} concentration

$$\sigma_c = \sigma_a - \sigma_f$$

Fission Product Concentration as a Function of Time

U^{235} , U^{238} (fast), Pu^{239} , and Pu^{241} all fission and their fission products contribute to the total poisoning of the pile. With the exception of xenon and samarium, which saturate, the effect of the fission products on reactivity is a function of time. The fission products of each of the above materials is considered separately below.

U^{235} Fission Products:

Let Y = concentration of fission product atoms of yield y at any time.

Y_1 = initial concentration

N = concentration of U^{235} atoms at any time.

σ_y = abs. cross section of the fission product

Then:

$$Y = y N_0 \frac{\sigma_f^{25}}{\sigma_a^{25} - \sigma_y} \left[e^{-\sigma_y \phi t} - e^{-\sigma_a^{25} \phi t} \right] + Y_1 e^{-\sigma_y \phi t}$$

and the total absorption cross section of the fission product per original atom of U^{235} is:

$$\frac{\sigma_y}{N_0} Y = y \frac{\sigma_f^{25} \sigma_y}{\sigma_a^{25} - \sigma_y} \left[e^{-\sigma_y \phi t} - e^{-\sigma_a^{25} \phi t} \right] + \frac{\sigma_y Y_1}{N_0} e^{-\sigma_y \phi t}$$

Substituting the numerical values for all U^{235} fission products (xeno. excluded) given in Tables 7 and 8, we obtain the curve given in Figure 9, assuming that no fission products are present at time $t = 0$.

Pu^{239} Fission Products: Using the notation given above, the total absorption cross section of a fission product per original fissionable atom is:

$$\frac{\sigma_y Y}{N_0} = (ICR) \frac{\sigma_a^{25}}{\sigma_a^{49}} \frac{\sigma_f^{49} \sigma_y}{\sigma_a^{49} - \sigma_y} y \left[\frac{\sigma_a^{49}}{\sigma_y} (1 - e^{-\sigma_y \phi t}) - (1 - e^{-\sigma_a^{49} \phi t}) \right] + \frac{Y_1 \sigma_y}{N_0} e^{-\sigma_y \phi t}$$

The result of substituting numerical values from Tables 7 and 8, and plotting the result as a function of time is shown in Figure 9, assuming Y_1 is zero at time zero.

Pu^{241} Fission Products: It has been assumed that the fission product yields for Pu^{241} are the same as those for Pu^{239} (41). Using the above notation, we get for the total absorption cross section per original fissionable atom:

$$\begin{aligned} \frac{\sigma_y Y}{N_0} = & (ICR) \frac{\sigma_a^{25} \sigma_c^{49} \sigma_f^{41} Y}{\sigma_a^{49} \sigma_a^{41}} \left[1 - \sigma_y \left(\frac{\sigma_a^{40} \sigma_a^{41} e^{-\sigma_a^{41} \phi t}}{(\sigma_a^{40} - \sigma_a^{49})(\sigma_a^{41} - \sigma_a^{49})(\sigma_y - \sigma_a^{49})} \right. \right. \\ & + \frac{\sigma_a^{49} \sigma_a^{41} e^{-\sigma_a^{40} \phi t}}{(\sigma_a^{49} - \sigma_a^{40})(\sigma_a^{41} - \sigma_a^{40})(\sigma_y - \sigma_a^{40})} + \frac{\sigma_a^{49} \sigma_a^{40} e^{-\sigma_a^{41} \phi t}}{(\sigma_a^{49} - \sigma_a^{41})(\sigma_a^{40} - \sigma_a^{41})(\sigma_y - \sigma_a^{41})} \\ & \left. \left. + \frac{\sigma_a^{49} \sigma_a^{40} \sigma_a^{41} e^{-\sigma_y \phi t}}{\sigma_y (\sigma_a^{49} - \sigma_y)(\sigma_a^{40} - \sigma_y)(\sigma_a^{41} - \sigma_y)} \right) \right] \end{aligned}$$

Substituting values from Tables 7 and 8, we obtain the curve in Figure 9.

U^{238} Fission Products: The contribution to fission product poisoning due to the fast fission of U^{238} is relatively small and can be approximated by adding an appropriate percentage to that from other fission products. This was performed by the method outlined in Reference 41 and the results have been plotted in Figure 9.

Xenon Poisoning

The steady state concentration of Xe^{135} for an arbitrary fuel is:

$$X_{\text{e}}^{135} = \frac{y \sum_f \phi}{\lambda_x + \sigma_a^x \phi}$$

where $y = y(\text{I}^{135}) + y(\text{Xe}^{135})$, and λ decay constant. The total xenon absorption cross section per original U^{235} atom for the four fissionable isotopes is then:

$$\text{U}^{235}: \frac{y \sigma_a^x \sigma_f^{25} \phi}{\lambda_x + \sigma_a^x \phi}$$

$$\text{U}^{238}: \text{Negligible}$$

$$\text{Pu}^{239}: \frac{y \sigma_a^x \sigma_f^{49} \phi P}{\lambda_x + \sigma_a^x \phi}$$

$$\text{Pu}^{241}: \frac{y \sigma_a^x \sigma_f^{41} \phi R}{\lambda_x + \sigma_a^x \phi}$$

The relevant constants are:

$$\sigma_a^x = 3.5 \times 10^6 \text{ barns}$$

$$\lambda_x = 2.1 \times 10^{-5} \text{ sec}^{-1}$$

$$y = 0.059 (\text{U}^{235})$$

$$= 0.055 (\text{Pu}^{239})$$

$$= 0.05 (\text{U}^{238})$$

TABLE 7

ABSORPTION CROSS SECTIONS AND NEUTRON YIELD PER ORIGINAL U^{235} ATOM

ϕt	HEAVY NUCLIDE ABSORPTION					FISSION PRODUCT ABSORPTION				NEUTRON YIELD			TOTALS	
	Σ_a^{25}/N_0	Σ_a^{28}/N_0	Σ_a^{49}/N_0	Σ_a^{40}/N_0	Σ_a^{41}/N_0	$U^{235}FP_5$	$U^{238}FP_5$	$Pu^{239}FP_5$	$Pu^{241}FP_5$	$\nu_{25}\frac{\Sigma_f^{25}}{N_0}$	$\nu_{49}\frac{\Sigma_f^{49}}{N_0}$	$\nu_{41}\frac{\Sigma_f^{41}}{N_0}$	$\Sigma \frac{\nu \Sigma_f}{N_0}$	$\Sigma \frac{\Sigma_a}{N_0}$
						INITIAL	CONVERSION	RATIO	0.7					
0	685	382	0	0	0	0	0	0	0	1427	0	0	1427	1067
8.65×10^{19}	646	382	60.5	3.6	0	12.0	0.5	1.239	0	1346	115	0	1461	1106
1.73×10^{20}	608	382	113	4.4	0	13.2	0.53	2.455	0	1266	214	0	1480	1124
3.46×10^{20}	540	382	199	14.1	1.4	15.4	0.62	5.0	0.03	1125	376	3.3	1504	1157
5.19×10^{20}	480	382	265	26.2	6.5	17.0	.71	7.196	.18	999	503	15.3	1517	1185
6.92×10^{20}	427	382	315	40.2	11.7	18.4	.82	9.485	.33	889	597	27.5	1514	1205
8.65×10^{20}	378	382	354	52.6	19.8	19.7	.89	13.0	.90	787	671	46.5	1505	1221
1.73×10^{21}	209	382	446	109	70.5	23.0	1.26	22.71	3.3	435	845	166	1446	1267
3.46×10^{21}	63.7	382	477	155	140	22.3	1.82	38.51	9.3	133	904	329	1366	1290
5.19×10^{21}	19.2	382	480	163	160	18.8	2.24	51.12	15.3	40	909	376	1325	1292
						INITIAL	CONVERSION	RATIO	0.8					
8.65×10^{19}	646	382	69	4.1	0	12.0	.5	1.416	0	1346	131	0	1477	1115
1.73×10^{20}	608	382	129	5.1	0	13.2	.53	2.80	0	1266	244	0	1510	1141
3.46×10^{20}	540	382	227	16.1	1.6	15.4	.62	6.0	.03	1125	430	3.76	1559	1188
5.19×10^{20}	480	382	303	30.0	7.5	17.0	.71	8.226	.21	999	574	17.6	1591	1229
6.92×10^{20}	427	382	360	45.9	13.4	18.4	.82	10.837	.38	889	682	31.5	1603	1259
8.65×10^{20}	378	382	404	60.2	22.6	19.7	.89	14.0	1.03	787	766	53.1	1606	1282
1.73×10^{21}	209	382	510	125	80.6	23.0	1.26	25.92	3.8	435	967	189	1591	1361
3.46×10^{21}	63.7	382	546	177	160	22.3	1.82	43.985	10.6	133	1035	376	1544	1407
5.19×10^{21}	19.2	382	548	186	182	18.8	2.24	58.429	17.4	40	1039	427	1506	1414
						INITIAL	CONVERSION	RATIO	0.9					
8.65×10^{19}	646	382	77.7	4.6	0	12.0	.5	1.593	0	1346	147	0	1493	1124
1.73×10^{20}	608	382	145	5.7	0	13.2	.53	3.156	0	1266	275	0	1541	1157
3.46×10^{20}	540	382	255	18.1	1.8	15.4	.62	7.0	.04	1125	483	4.2	1612	1220
5.19×10^{20}	480	382	341	33.7	8.4	17.0	.71	9.257	.23	999	646	19.7	1665	1272
6.92×10^{20}	427	382	405	51.7	15.0	18.4	.82	12.196	.43	889	768	35.2	1692	1313
8.65×10^{20}	378	382	455	67.7	25.4	19.7	.89	16.0	1.16	787	862	59.7	1709	1346
1.73×10^{21}	209	382	574	141	90.6	23.0	1.26	29.21	4.2	435	1088	213	1736	1454
3.46×10^{21}	63.7	382	613	199	180.1	22.3	1.82	49.49	11.9	133	1162	423	1718	1523
5.19×10^{21}	19.2	382	616	209	205	18.8	2.24	65.75	19.6	40	1167	481	1688	1538

TABLE 8

FISSION PRODUCT CROSS SECTIONSLONG LIVED FISSION PRODUCTS WITH LARGE CROSS SECTIONS

NUCLIDE	CROSS SECTION	HALF LIFE	FISSION YIELDS		
			U ²³⁵	Pu ²³⁹	U ²³⁸ (fast)
Sm ¹⁴⁹	50,000 b	stable	.013	.014	.028
Eu ¹⁵⁵	14,000 b	2 yrs.	.00031	.0021	.001
Gd ¹⁵⁵	70,000 b	stable	.00031	.0021	.001
Gd ¹⁵⁷	160,000 b	stable	.00015	.0006	.0001
Cd ¹¹³	20,800 b	stable	.00012	.0006	.0005
Sm ¹⁵¹	7,000 b	1000 yrs.	.0045	.01	.0446

Notes: 1. All nuclide cross sections obtained from BNL-325.

2. All fission yields obtained from Reference 41.

OTHER FISSION PRODUCTS OF SIGNIFICANT CROSS SECTIONS

Eu ¹⁵³	420 b	stable	.0015	.0039	.0039
Nd ¹⁴³	280 b	stable	.054	.051	.05
Kr ⁸³	205 b	stable	.00586	.0008	.00018
Sm ¹⁵²	150 b (44)	stable	.00279	.007	.009
Rh ¹⁰³	150 b	stable	.037	.055	.074
Xe ¹³¹	120 b	stable	.028 (44)	.036	.02
Tc ⁹⁹	100 b	10 ⁴ yrs.	.062	.061	.059
Pm ¹⁴⁷	60 b	4.4 yrs.	.026	.026	.037
Cd ¹¹²	30 b	stable	.00011	.001	.00055
Cs ¹³³	29 b	stable	.045 (44)	.05	.04
Cs ¹³⁵	15 b	2 x 10 ⁴ yrs.	.056 (44)	.055	.05
Mo ⁹⁵	13.4 b	stable	.064 (44)	.056	.073
Pr ¹⁴¹	11.2 b	stable	.046 (44)	.049	.053
La ¹³⁹	9 b	stable	.063	.054	.051
U ²³⁶	6 b	long	.1835	0	0

Notes: 1. All fission fields obtained from Reference 41 unless otherwise specified.

2. All nuclide cross section obtained from BNL-325 unless otherwise specified.

FISSION PRODUCTS OF UNKNOWN CROSS SECTION--POSSIBLY SIGNIFICANT

Zr ⁹³	very long	.06	.04	.05
Sm ¹⁴⁷	10 ¹¹ yrs.	.026	.026	.037
Pd ¹⁰⁷	very long	.0015	.035	.010
Cd ¹¹¹	stable	.00018	.0027	.00072
Gd ¹⁵⁶	stable	.00014	.0012	.00063

- Notes: 1. Fission yields obtained from Reference 41.
2. Gd¹⁵⁶ forms Gd¹⁵⁷ ($\sigma_a = 160,000$ b) upon capture of a neutron, and therefore a second neutron will be quickly captured.
3. Either of the first three could have a large effect if their cross section were large; the others are relatively unimportant because of their small yields.

10.d. Appendix D: Derivation of Temperature Coefficient Formulae

Consider the following equation;

$$k_{\text{eff}} = k_{\infty} (1 - B^2 M^2)^{-1}$$

For small values of $B^2 M^2$, it may be written:

$$k_{\text{eff}} = k_{\infty} (1 - B^2 M^2)$$

Then,

$$\frac{\partial k_{\text{eff}}}{\partial T} = \frac{\partial k_{\infty}}{\partial T} (1 - B^2 M^2) - k_{\infty} B^2 \frac{\partial M^2}{\partial T} = \frac{k_{\text{eff}}}{k_{\infty}} \frac{\partial k_{\infty}}{\partial T} - k_{\infty} B^2 \frac{\partial M^2}{\partial T}$$

considering changes in B^2 , the buckling with temperature to be negligible.

The infinite multiplication factor, k_{∞} , may be expressed as:

$$k_{\infty} = \eta f p \epsilon$$

and therefore,

$$\frac{1}{k_{\infty}} \frac{\partial k_{\infty}}{\partial T} = \frac{1}{\eta} \frac{\partial \eta}{\partial T} + \frac{1}{f} \frac{\partial f}{\partial T} + \frac{1}{p} \frac{\partial p}{\partial T} + \frac{1}{\epsilon} \frac{\partial \epsilon}{\partial T}$$

Since the buckling can be expressed as,

$$B^2 = \frac{k_{\infty} - k_{\text{eff}}}{k_{\infty} M^2}$$

and assuming the reactor is near critical (i.e., $k_{\text{eff}} \approx 1$), the following basic equation is obtained:

$$\frac{\partial k_{\text{eff}}}{\partial T} = \frac{1}{\epsilon} \frac{\partial \epsilon}{\partial T} + \frac{1}{\eta} \frac{\partial \eta}{\partial T} + \frac{1}{f} \frac{\partial f}{\partial T} - \frac{(k_{\infty} - 1)}{M^2} \frac{\partial M^2}{\partial T}$$

Let us now consider each term separately.

Variation of ϵ with Temperature: This term is usually neglected since it is of the order of $10^{-6} / ^\circ\text{C}$ (11), (19) which is about a factor of 10 smaller than others of interest.

Variation of η with Temperature: This term is affected by changes in moderator temperature. Experimental results and analytical treatments in graphite piles have shown the term $\left(-\frac{1}{\eta} \frac{\partial \eta}{\partial T}\right)$ to vary from $4.5 \times 10^{-5} / ^\circ\text{C}$ (11), (37), (38), (39). The chief cause for the uncertainty arises in the evaluation of $\frac{\partial \alpha}{\partial T}$ where α is defined as follows:

$$\eta = \frac{\nu}{1 + \alpha + \beta \frac{N_a^{28}}{N^{25}}}$$

where:

$$\alpha \equiv \frac{\sigma_c^{25}}{\sigma_f^{25}} \text{ and } \beta \equiv \frac{\sigma_a^{28}}{\sigma_f^{25}}$$

It is sometimes assumed that $\frac{\partial \alpha}{\partial T} \approx 0$ (38), (39); but for a conservative calculation, the lower value of $\frac{1}{\eta} \frac{\partial \eta}{\partial T}$ is usually used (19). Moreover, theory agrees with experiment in the Oak Ridge graphite reactor if the lower range of values for $\frac{1}{\eta} \frac{\partial \eta}{\partial T}$ are used (11); hence, for this calculation,

$$\frac{1}{\eta} \frac{\partial \eta}{\partial T} = -4.5 \times 10^{-5} / ^\circ\text{C}$$

The variation in η with temperature is mainly attributable to β . This is due to the fact that σ_f^{25} is not a $1/v$ cross section while σ_a^{28} is $1/v$.

Variation in f with Temperature: The term $\frac{1}{f} \frac{\partial f}{\partial T}$ is affected by both changes in the fuel element and moderator temperatures. An increase in fuel element temperature causes a decrease in the density of the fuel atoms, reducing the amount of absorption in the fuel. An increase in moderator temperature reduces the absorption cross sections and the moderator density, hence increasing the relative number of absorptions in the fuel.

The thermal utilization, f , may be written as:

$$f = \frac{1}{1 + \frac{V_{O_2O}}{V_u} \int \frac{\sigma_a^{O_2O}}{\sigma_a^u} \frac{N_{O_2O}}{N_u} + \frac{N_{Zr}}{N_u} \frac{\sigma_a^{Zr}}{\sigma_a^u}}$$

where S is the flux disadvantage factor. For the temperature range of interest, it was assumed that S did not change significantly. All absorption cross sections were assumed to be $1/v$.

In the case of a fuel element temperature change only, f can be expressed as

$$f = \frac{1}{B + C \frac{\rho_{zr}}{\rho_u}}$$

where

$$B = 1 + \frac{M_{D_2O}}{M_u} \frac{A_u}{A_{D_2O}} S \frac{\sigma_a^{D_2O}}{\sigma_a^u}$$

$$C = \frac{A_u}{A_{zr}} \frac{\sigma_a^{zr}}{\sigma_a^u}$$

Taking the derivative and simplifying, the following expression is obtained:

$$\frac{1}{f} \frac{\partial f}{\partial T} = \frac{-C \frac{\rho_{zr}}{\rho_u} \left(\frac{1}{V_u} \frac{\partial V_u}{\partial T} - \frac{1}{V_{zr}} \frac{\partial V_{zr}}{\partial T} \right)}{(B + C \frac{\rho_{zr}}{\rho_u})}$$

which yields, at the operating metal temperature, a value of $-0.06 \times 10^{-5} / ^\circ\text{C}$.

If only the moderator temperature changes, the derivative of the expression for f yields:

$$\frac{1}{f} \frac{\partial f}{\partial T} = \frac{B V_{D_2O} \frac{\partial \rho_{D_2O}}{\partial T}}{B M_{D_2O} + D}$$

where:

$$B = \frac{A_u}{A_{D_2O}} \frac{S}{M_u} \frac{\sigma_a^{D_2O}}{\sigma_a^u}$$

$$D = 1 + \frac{A_u}{A_{zr}} \frac{\rho_{zr}}{\rho_u} \frac{\sigma_a^{zr}}{\sigma_a^u}$$

Evaluated at the operating moderator temperature, $\frac{1}{f} \frac{\partial f}{\partial T} = +2.38 \times 10^{-5} / ^\circ\text{C}$.

Variation in p with temperature: The term $\frac{1}{p} \frac{\partial p}{\partial T}$ is affected by changes in the fuel temperature, which affects the width of the resonances, and changes in the moderator temperature, which affects the moderator density.

It has been shown (Section 4.b) that p can be expressed as follows:

$$p = \exp - \left[\frac{M_u A_{0.20}}{A_u V_{0.20} \Sigma_{0.20} S_{res} \sigma_s^{0.20} \rho_{0.20}} A \left(1 + \mu \frac{S_u}{M_u} \right) \right]$$

If a change in the fuel element temperature only is considered, the principal change in p is due to the temperature dependence of A. Hence, the expression for p becomes:

$$p = \exp - (KA)$$

where

$$K = \frac{M_u A_{0.20}}{A_u V_{0.20} \Sigma_{0.20} S_{res} \sigma_s^{0.20} \rho_{0.20}} \left(1 + \mu \frac{S_u}{M_u} \right)$$

The quantity of interest then becomes:

$$\left(\frac{1}{p} \frac{\partial p}{\partial T} \right) = -K \left(\frac{1}{A} \frac{\partial A}{\partial T} \right)$$

The coefficient $\left(\frac{1}{A} \frac{\partial A}{\partial T} \right)$ has been evaluated experimentally at Brookhaven National Lab (11), the result agreeing with a theoretical prediction by Dancoff (CP-1589); this is:

$$\frac{1}{A} \frac{\partial A}{\partial T} = 1.5 \times 10^{-4} / ^\circ\text{C}, \text{ whence } \left(\frac{1}{p} \frac{\partial p}{\partial T} \right) = -1.87 \times 10^{-5} / ^\circ\text{C}$$

If the moderator temperature changes, the resonance escape probability is affected significantly only by the density. The expression for p can then be written:

$$p = \exp - \left[K_1 \frac{1}{\rho_{0.20}} \right] \text{ where } K_1 = \frac{A_{0.20} M_u}{A_u \Sigma_{0.20} S \sigma_s^{0.20} V_{0.20}} A \left(1 + \mu \frac{S_u}{M_u} \right)$$

If the derivative is taken and evaluated, the following result is obtained:

$$\left(\frac{1}{p} \frac{\partial p}{\partial T}\right) = \frac{k_1}{p_{D_2O}^2} \frac{\partial p_{D_2O}}{\partial T} = -7.2 \times 10^{-5}/^{\circ}\text{C}$$

Variation of M^2 with Temperature: The variation in the migration area with temperature can be expressed as the sum of the variations in L^2 and τ with temperature.

The diffusion length, L^2 , may be expressed as:

$$L^2 = f_{mod} L_{mod}^2 + f_{fuel} L_{fuel}^2 + f_{structure} L_{structure}^2$$

It has been previously shown, however, that the latter two terms contribute only about 1% to the total L^2 and, hence, can be ignored. Since

$$f_{mod} = \frac{\frac{\sigma_a^{D_2O}}{\sigma_a^u} \sum \frac{M_{D_2O}}{M_u} \frac{A_u}{A_{D_2O}}}{\frac{\sigma_a^{D_2O}}{\sigma_a^u} \sum \frac{M_{D_2O}}{M_u} \frac{A_u}{A_{D_2O}} + \frac{\sigma_a^{Zr}}{\sigma_a^u} \frac{A_u}{A_{Zr}} \frac{M_{Zr}}{M_u} + 1}$$

then

$$L^2 = \frac{K M_{D_2O} L_{D_2O}^2}{K M_{D_2O} E}$$

where:

$$E = \frac{\sigma_a^{Zr}}{\sigma_a^u} \frac{A_u}{A_{Zr}} \frac{M_{Zr}}{M_u}$$

$$K = \frac{\sigma_a^{D_2O}}{\sigma_a^u} \frac{A_u}{A_{D_2O}} \sum \frac{1}{M_u}$$

$$L_{D_2O}^2 = \left(\frac{\rho_{D_2O_0}}{\rho_{D_2O}}\right)^2 \sqrt{\frac{T}{T_0}} L_{D_2O_0}^2$$

the subscript zero referring to the initial temperature.

Hence,

$$\frac{\partial L_{D_2O}^2}{\partial T} = \frac{1}{2} \left(\frac{\rho_{D_2O_0}}{\rho_{D_2O}}\right)^2 \frac{L_{D_2O_0}^2}{\sqrt{TT_0}} \left(1 - \frac{4T}{\rho_{D_2O}} \frac{\partial \rho_{D_2O}}{\partial T}\right)$$

Finally,

$$\frac{\partial L^2}{\partial T} = \left(\frac{A M_{D_2O}}{A M_{D_2O} + E} \right) \left[\frac{1}{2} \left(\frac{\rho_{D_2O}}{\rho_{D_2O}} \right)^2 \frac{L_{D_2O}^2}{\sqrt{T_0 T}} \right] \left[1 + \frac{4T}{\rho_{D_2O}} \frac{\partial \rho_{D_2O}}{\partial T} \left(\frac{E}{2(A M_{D_2O} + E)} \right) \right]$$

Evaluating the above, one obtains

$$\frac{\partial L^2}{\partial T} = + 0.737 / ^\circ C$$

The expression for τ can be written:

$$\tau = K \int_{E_{th}}^{E_f} \frac{D_{D_2O}}{\sum_{D_2O} \sum_t D_{D_2O}} \frac{dE}{E}$$

where K allows for elastic scattering by the fuel atoms and is assumed to be temperature independent. Therefore, τ depends only on changes in the moderator temperature.

Since $E = kT$, $\sum_t D_{D_2O} \approx \sum_s D_{D_2O} \approx \sum_{s_0} \left(\frac{\rho}{\rho_0} \right)$, and $D = \frac{1}{3 \left(\frac{f}{\rho_0} \right) \sum_{s_0}}$, the equation becomes,

$$\tau = K \left(\frac{\rho_0}{\rho} \right)^2 \int_{T(E_{th})}^{T(E_f)} \frac{D_{D_2O}^0}{\sum_{D_2O} \sum_{s_0}} \frac{dT}{T}$$

Finally,

$$\frac{\partial \tau}{\partial T} = -2K \left(\frac{\rho_0}{\rho} \right)^2 \tau_{D_2O}^0 \frac{1}{\rho_{D_2O}} \frac{\partial \rho_{D_2O}}{\partial T} - K \left(\frac{\rho_0}{\rho} \right)^2 \frac{D_{D_2O}^0}{\sum_{D_2O} \sum_{s_0}} \frac{1}{T_0}$$

and, evaluating,

$$\frac{\partial \tau}{\partial T} = + 0.368 / ^\circ C$$

Therefore,

$$\frac{\partial M^2}{\partial T} = \frac{\partial L^2}{\partial T} + \frac{\partial \tau}{\partial T} = 1.105 / ^\circ C$$

10.e. Appendix E: Conditions Used in Heat Transfer and Fluid Flow Studies

	Parameter Study			
	Gas Velocity in Fuel Spiral	Number of Process Tubes	Coolant Pressure	Fuel Loading
Reactor Power	1000 MW	1000MW	1000MW	1000MW
Coolant Pressure	150 psia	300 psia	Variable	500 psia
Reactor Inlet Temp.	370°F	370°F	370°F	370°F
Reactor Outlet Temp.	1000°F	1000°F	1000°F	1000°F
Quantity of Fuel	Variable	74 tons	200 gm/cm cell	Variable
Fuel Plate Thickness with Cladding	30 mils	Variable	Variable	Variable
Cladding Thickness	5 mils	5 mils	5 mils	5 mils
Nr of Process Tubes	271	Variable	301	151
Maximum Gas Velocity in Hot and Cold Gas Spaces	500 ft/sec	500 ft/sec	500 ft/sec	500 ft/sec
Maximum Gas Velocity in Fuel Spiral	Variable	1000 ft/sec	1000 ft/sec	1000 ft/sec
Axial Variation of Reactor Power	Center 2/3 of cosine	0.7 to 1.0 of cosine	0.7 to 1.0 of cosine	0.7 to 1.0 of cosine
Radial Variation of Reactor Power	Center 2/3 of cosine	0.7 to 1.0 of cosine	0.7 to 1.0 of cosine	0.7 to 1.0 of cosine
Maximum Metal Temp.	1112°F	1112°F	1112°F	1112°F
Clear Space Between Spiral Turns	Variable	Variable	Variable	0.05 inch
Process Tube Length	25 ft	20 ft	20 ft	20 ft
Moderator Pressure	150 psia	200 psia	200 psia	200 psia
Ratio of Average Power Density to Power Density at Hot End of Spiral	1.0	1.1	1.1	1.1

10.f. Appendix F: Definition of Symbols Used in Heat Transfer, Pressure Drop,
and Power Cycle Calculations

A - Area - ft^2
 c_p - Specific heat - $\text{BTU/lb } ^\circ\text{F}$
 D_e - Equivalent hydraulic diameter - ft
E - Modulus of elasticity - psi
g - Acceleration of gravity - 32 ft/sec^2
H - Enthalpy - BTU/lb
h - Film coefficient of heat transfer - $\text{BTU/hr ft}^2 ^\circ\text{F}$
J - Conversion factor, ft lbs to BTU - 778 ft lb/BTU
k - Thermal conductivity - $\text{BTU/hr ft } ^\circ\text{F}$
m - Molecular weight
P - Pressure - psia
 P_{cr} - Pressure of D_2O moderator - psia
 Δq - Heat added to coolant in increment of fuel element passage having length
x - BTU/lb
R - Perfect gas constant - $1545 \text{ mole}^{\circ}\text{R ft}$
 r_o - Radius to outside of process tube - in
S - Entropy - $\text{BTU/lb } ^\circ\text{F}$
T - Temperature - $^{\circ}\text{R}$
t - Temperature - $^{\circ}\text{F}$
U - Velocity - ft/hr
W - Mass rate of flow - lb/hr
 Δx - Increment of distance along curved passage of spiral - ft

Δz - Length along axis of process tube of gas area in any one section of
fuel spiral - ft

ρ - Density - lb/ft³

μ - Viscosity - lb/ft hr

ν - Poisson's ratio

10.g. Appendix G: Equipment Data for Power Plant

1. SUPERHEATER

Heat transferred	1.73×10^8 Btu/hr
Log mean temperature difference	224 °F
360 tubes, $1\frac{1}{2}$ " schedule 40, 60 ft long made of low alloy steel using cross flow, 1 helium pass, 6 steam passes	
Film coefficient (helium)	1030 Btu/(hr)(ft ²)(°F)
Film coefficient	935 Btu/(hr)(ft ²)(°F)
Mass velocity (helium)	4.5×10^4 #/(hr)(ft ²)
Pressure loss	1.0 psi

2. BOILER

Heat transferred	5.1×10^8 Btu/hr
Log mean temperature difference	236 °F
630 tubes - $1\frac{1}{2}$ " schedule 40, 15.7 ft. long, made of low alloy steel using cross flow, 2 helium passes, 1 water pass	
Film coefficient (helium)	768 Btu/(hr)(ft ²)(°F)
Film coefficient (water)	2200 Btu/(hr)(ft ²)(°F)
Mass velocity (helium)	5.2×10^4 #/(hr)(ft ²)
Pressure loss	1.0 psi

3. ECONOMIZER

Heat transferred	1.32×10^8 Btu/hr
Log mean temperature difference	162 °F
90 tubes $1\frac{1}{2}$ " schedule 40, 43' long made of carbon steel, using cross flow, 1 helium pass, 7 water passes.	
Film coefficient (helium)	835 Btu/(hr)(ft ²)(°F)

Film coefficient (water)

2100 Btu/(hr)(ft²)(°F)

Mass velocity

6.3×10^4 #/(ft²)(hr)

Pressure loss

1.3 psi

4. REGENERATOR

Heat transferred

3.5×10^8 Btu/hr

Log mean temperature difference

133 °F

360 parallel plates 2.7' x 19' long
plates spaced on $\frac{1}{2}$ " centers, using
counter flow.

Film coefficient (high pressure and
low pressure helium)

280 Btu/(hr)(ft²)(°F)

Mass velocity

9.3×10^4 Btu/(hr)(ft²)(°F)

Pressure loss - low pressure side
high pressure side

.6 psi

.5

1.1 psi

5. PRECOOLER

Heat transferred

2.6×10^8 Btu/hr

Log mean temperature difference

84.5 °F

4 u-tube heat exchangers, helium
in tube, cooling water in shell,
412 tubes per exchanger, 1" OD
27' long made of admiralty metal.

Film coefficient (helium)

550 Btu/(hr)(ft²)(°F)

Film coefficient (water)

1000 Btu/(hr)(ft²)(°F)

Mass velocity helium

1.8×10^5 #/(hr)(ft²)

Pressure loss

3.1 psi

SUMMARY

Total pressure loss in reactor

34.0 psi

Total pressure loss in equipment

7.5 psi

Total pressure loss in ducts, etc.

2.5 psi

Total pressure loss

44.0 psi

Pressure loss = 9%

10.h Appendix H: Calculation of Net Plant Efficiency

1. Assumptions

Turbo-generator	81%
Turbine	83%
Generator	97.5%
Compressor efficiency	81.5%
Compression	84%
Motor	97%
Pressure leaving compressor	500 psi
Pressure entering compressor	456 psi
Helium flow rate	1.89×10^6 #/hr/loop

2. Calculations

Enthalpy entering condenser	1466 - (1466 - 933) .83
	1023 Btu/# steam
Work from generator per pound of helium	$443 \times .975 \times .523$
	225.5 Btu/# helium
$\frac{P_o}{P_i} = 1.095$	
Compression work	$\frac{H_o - H_i}{\text{Compression efficiency}}$
	$\frac{c_p (T_o - T_i)}{\text{Compression efficiency}}$
From perfect gas relation $T_o = 125^\circ F$	
Work of Compression	1.248×25
	31.2 Btu/# Helium

Compressor work	=	$\frac{31.2}{.97} = 32.2 \text{ Btu/\# He}$
Net work	=	$225.5 - 32.2$
	=	$193.3 \text{ Btu/\# He}$
Net plant efficiency	=	$\frac{193.3}{811} = 90.4$
	=	21.4%
Size of turbo-generator	=	$225.5 \times 1.89 \times 10^6$
	=	$4.26 \times 10^8 \text{ Btu/hr}$
	=	125 MW/loop
Size of compressor	=	$32.2 \times 1.89 \times 10^6$
	=	$6.09 \times 10^7 \text{ Btu/hr}$
	=	24000 Hp/loop

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