Crystalline Beams: The Horizontal Zigzag

A.F. Haffmans, D. Maletić, and A.G. Ruggiero

May 8, 1994

Department of Advanced Technology

Informal Report

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Abstract *

We continue our investigation of "Crystalline Beams " in the ground state. In this paper we study the possibility of storing an intense beam of charged particles in a storage ring where the particles acquire a *horizontal zigzag* configuration. We define the equilibrium configuration, and examine the confinement and stability conditions. The problem of "shear" between particles circulating side by side caused by the curvature of the trajectories is examined and solved.

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1 Introduction

The study of the horizontal zigzag adds an other level of complexity to our study of specific examples of “Crystalline Beams”, which started with the *string* [2], and continued with the *vertical zigzag* [3]. The string does not need external forces [1, 2] to confine it, and it suffices to derive the stability conditions. The vertical zigzag [3] needs external forces to confine it in the vertical direction, and this turns out to be, because of the absence of curvature, a fairly simple one-degree-of-freedom problem.

In the lattices we consider here, the curvature is in the horizontal plane, and this requires a more elaborate confinement analysis. The horizontal zigzag is a two-dimensional configuration, with two degrees of freedom: the radial and longitudinal motion, coupled to each other by the curvature of the storage ring. In this paper we investigate and solve the fundamental problem of “shear”. The ions in the two substrings making up the horizontal zigzag experience shear, because in order to confine them, we have to allow them to drift with respect to each other. In other words, the ions in the substrings preceed and regress periodically with respect to each other.

The notation used in this report is the same as introduced in [1, 2, 3]. The reader should familiarize him/herself with these references, since they contain the derivation of the equations used in this paper.

2 Equilibrium Configuration

The horizontal zigzag is a “Crystalline Beam” in equilibrium given by two identical substrings placed side by side on the $y = 0$ mid-plane. One of the substrings is located at $x = a$, the other one at

$x = -a$, and the radial separation is $2a$. The longitudinal spacing between the ions in both substrings is λ . All the considerations made for the string and the vertical zigzag concerning the magnitude of $2a$ and λ still apply [2, 3]. The curvature of the beam taking place in the bending magnets causes dispersion, and the particles on the outer and inner substring have resultant different momenta. For stability the beam energy has to be below the transition energy of the storage ring – that is $\gamma < \gamma_T$. Therefore, the particles on the outer substring have a momentum error $\delta > 0$, and those on the inner string the momentum error $-\delta$.

Using the same procedure as for the vertical zigzag [3], we find that the two substrings of the horizontal zigzag are shifted by $\lambda/2$ with respect to each other, which corresponds to a minimum in the single-particle potential. Consequently, there is no net force in the longitudinal direction. Obviously, there is also no net force in the vertical direction. A nonzero net force appears in the radial direction, which needs to be balanced by the external focussing forces of the storage ring.

3 Confinement of the Horizontal Zigzag

Confinement is required in the radial direction everywhere. Moreover, the longitudinal force appearing in the bending magnets and caused by the curvature of the trajectory needs to balance the longitudinal force by the repulsion of the two substrings.

3.1 The Effect of Curvature

This effect is similar to the one already studied for the string [2] and for the vertical zigzag [3]. The radial force has the same magnitude

for all particles, and can be derived from the potential

$$\phi_{sc} = \nu_1 x. \quad (1)$$

ν_1 is given in Ref. [3], provided that the vertical separation $2b$ is replaced by the horizontal separation $2a$. In particular, it can be shown that in the straight sections where the curvature $h = 0$, $\nu_1 = 0$. This term is generally very small and can be ignored, though, formally it should be included in the generalized Lorentz condition [1, 2].

3.2 Confinement in the Radial Direction

For the horizontal zigzag in its equilibrium configuration, the interparticle repulsion needs to be balanced by the focussing of the storage ring. As discussed in [1], the radial component of the space charge force is given by the first-order derivative of ϕ_{sc} with respect to the radial coordinate x , evaluated at the equilibrium. Ignoring the curvature in the bending magnets we find:

$$\frac{\partial \phi_{sc}}{\partial x} \Big|_{eq.} = -4Qe\gamma \sum_{i=1}^{\infty} \frac{a}{[4a^2 + \gamma^2 \lambda^2 (i - \frac{1}{2})^2]^{3/2}}. \quad (2)$$

The horizontal repulsive force is present everywhere, even in the straight sections, where $h = 0$. We introduce the functions [1]:

$$f_n(\alpha) = \sum_{i=1}^{\infty} \frac{1}{[\alpha^2 + (i - \frac{1}{2})^2]^{n/2}}, \quad (3)$$

and the horizontal zigzag aspect ratio $\alpha = 2a/\lambda\gamma$. The equations for the equilibrium configuration on the $y = 0$ midplane are [1]:

$$a'' + K_h(s)a - K_{sc} \frac{2f_3}{g_0} a = h(s) \delta, \quad (4)$$

$$\sigma' = h(s)a - \frac{\delta}{\gamma^2}, \quad (5)$$

$$\delta' = 2K_{sc}\gamma^2(2\eta_e)\sigma, \quad (6)$$

where

$$K_{sc} = \frac{2Q^2 r_0 g_0}{A\beta^2 \gamma^5 \lambda^3} = h_{sc}^2, \quad (7)$$

$$\eta_e = 1 + \frac{f_3 - 1.5\alpha^2 f_5}{g_0}, \quad (8)$$

and $g_0 \approx 1.2$. The equation of motion in the vertical direction is

$$b'' + K_v(s)b = 0 \quad (9)$$

which is obviously satisfied by $b = 0$.

It is seen that if a , σ , and δ are solution of Eqs. (4–6), then so are $-a$, $-\sigma$, and $-\delta$. That is, if the particles on the outer substring satisfy the equations of motion, so do those on the inner substring. The longitudinal position, expressed by σ , remains invariant under a change of sign. The two substrings move relative to each other. This “shear” effect is caused by the curvature of the trajectories in the bending magnets.

Thus, particles move longitudinally with respect to each other in the equilibrium configuration. The net variation of σ over one period of the storage ring has to be zero.

For a circular magnet like a betatron, the equations of the motion Eqs. (4–6) are easily solved by taking the gradient to be constant.

Denoting the field index by n , the motion of a single particle is stable provided that $0 < n < 1$. For the radial confinement of the horizontal zigzag the following condition has to be satisfied:

$$1 - n = \gamma^2 + \frac{4Q^2 r_0 f_3(\alpha)}{A\beta^2 \gamma^5 \lambda^3 h^2} \quad (10)$$

This equation can be compared with the equivalent one derived for the vertical zigzag in [3]. The main difference between the kinematic terms from the horizontal and vertical zigzag is the additional γ^2 -term in Eq. (10). This is a result of the one-to-one correspondence of the particle position a and momentum deviation δ through

Eq. (5), after taking $\sigma' = 0$. This is *isochronism*: All particles move together rigidly in the longitudinal direction. As we shall see later, this configuration is unstable against individual particle perturbation. We have, therefore, to consider storage rings with alternating gradient magnets.

3.3 Radial Confinement with Alternating Gradient Focusing

The width of the zigzag varies around the storage ring. The variation will be small compared to a and λ and will take place over lengths comparable to those of the magnets. We look for a solution to the envelope equations Eqs. (4 – 6) with the same fundamental periodicity of the storage ring.

Similar to the case of the vertical zigzag, an approximate solution to the system Eqs. (4 – 6) can be found using simple and intuitive physical arguments. Basically, a *bifurcation* occurs when the space-charge term depresses the horizontal betatron tune ν_h until a horizontal resonance is encountered[†]. We require that a , σ and δ are periodic and closed. Integration of Eq. (5) over one period gives:

$$\int \sigma' ds = \int h a ds - \frac{1}{\gamma^2} \int \delta ds = 0, \quad (11)$$

which relates the momentum spread δ to the width a . On average,

$$\delta \approx \gamma^2 \frac{a}{R}. \quad (12)$$

Inserting this in Eq. (4) gives

$$a'' + K_h(s)a - \gamma^2 \frac{h}{R}a - K_{sc} \frac{2f_3}{g_0}a = 0. \quad (13)$$

[†]Approaching a coupling resonance is not sufficient to trigger the formation of a zigzag, though it obviously destroys the beam stability.

Eq. (13) is the equation of the beam envelope, and the kinematic term proportional to γ^2 in Eq. (13) has no equivalent in the ordinary equation describing the motion of the single particle. It is a result of *isochronism* of the envelope across one superperiod: the beam width a is proportional to the momentum spread δ ; see Eq. (11) and Eq. (12).

The tune depression is caused by both the space-charge term, and the kinematic term in γ^2 . In order to reach the nearest integral stopband $2\delta\nu_h$ away from the original betatron tune, the following relation must hold:

$$F(\alpha) = \frac{1}{f_3(\alpha)} = \frac{2Q^2 r_0 R}{A\beta^2 \gamma^5 \lambda^3 \delta \nu_h} \beta_h. \quad (14)$$

Eq. (14) determines the beam width $2a$ as a function of the spacing λ , and is similar to the one derived for the vertical zigzag [3]. The momentum separation δ is estimated using Eq. (12). $F(\alpha)$ has a non-vanishing value for $\alpha = 0$, namely:

$$F(0) = \frac{1}{7g_0}. \quad (15)$$

Eq. (15) is equivalent to the condition for *bifurcation*:

$$\lambda < 2.4\lambda_c \bar{c}_h = \lambda_u. \quad (16)$$

The critical lattice constant is

$$\lambda_c = \left(\frac{Q^2 r_0 g_0 R^2}{A\beta^2 \gamma^5} \right)^{1/3}. \quad (17)$$

The stability coefficient

$$\bar{c}_h = (\nu_h \overline{\delta\nu_h})^{-1/3} \quad (18)$$

takes the kinematic correction into account, viz.:

$$\overline{\delta\nu_h} = \delta\nu_h - \frac{\gamma^2}{2\nu_h}. \quad (19)$$

As Eq. (16) indicates, the maximum lattice spacing λ in the horizontal zigzag is roughly 2.4 times the lattice spacing at which the string becomes unstable radially.

All this is essentially the same as derived for the vertical zigzag, except for the interchange between vertical and horizontal variables. A term in γ^2 is subtracted from the actual distance of the working tune to the nearest stopband. The horizontal zigzag therefore appears, when the following condition is satisfied:

$$\gamma < \sqrt{2\nu_h \delta \nu_h} \quad (20)$$

This generalizes a result obtained earlier by Wei, Li and Sessler [4]. Eq. (20) means that the energy of the beam has to be below the transition energy of the storage ring.

Combining Eqs. (5) and (6) yields the equation for the longitudinal motion, viz.:

$$\delta'' + 4K_{sc}\eta_e\delta - 4K_{sc}\gamma^2\eta_e h(s)a = 0. \quad (21)$$

Inserting Eq. (12) simplifies it to:

$$\delta'' + 4K_{sc}\eta_e(1 - hR)\delta = 0. \quad (22)$$

The frequency of the longitudinal oscillations averaged over one superperiod is zero, and the second term vanishes. Hence, a horizontal zigzag may appear only when a radial stopband is crossed. For $\lambda \leq \lambda_u$ we can derive the relation between the aspect ratio α and the ion spacing λ from Eqs. (14) and (17), viz.:

$$F(\alpha)\lambda^3 = \frac{2}{g_0}\lambda_c^3\overline{c_h}^3. \quad (23)$$

3.4 Using the Transfer Matrix to Determine Radial Confinement

The exact solution of the nonlinear system Eqs. (4 – 6) is not trivial. It can be found numerically using the same approach described

for the vertical zigzag [3]. Each magnet component of the storage ring can be subdivided into a sufficiently large number of sections where the zigzag dimension a can be approximated as a constant. With each of these sections we can associate a 4×4 transfer matrix, with elements that are in general nonlinear functions of a . The transfer matrices are similar to the ones derived previously [2].

To find a closed periodic solution, we start at a waist. After guessing a , we propagate the vector (a, a') by multiplying it with the sequence of the transfer matrices of the sections in the storage ring. Before each multiplication, we replace the value of a in the elements of the matrix with the one obtained at the end of the previous multiplication. In this way we obtain the total transfer matrix $\mathbf{M}$ for a superperiod, which is a function of the original value of a . We change a until the determinant of the matrix $\mathbf{M}-\mathbf{I}$ vanishes.

4 Stability of the Test-Particle Motion

4.1 Stability of the Vertical Motion

The only possible force in the vertical direction is the one caused by the vertical displacement of the particles. From [1] we derive the following equations for a particle displaced vertically by y as

$$y'' + K_v(s)y + \frac{2Qr_0\nu_3}{eA\beta^2\gamma^3}y = 0, \quad (24)$$

where [1]

$$\nu_3 = -\nu_o\left(1 + \frac{f_3}{g_0}\right) = -\nu_o\eta_v, \quad (25)$$

and

$$\nu_0 = \frac{Qeg_0}{\gamma^2\lambda^3}. \quad (26)$$

Eq. (24) is similar to the one derived for the vertical zigzag [3], except for the interchange of the horizontal and vertical parameters. Aside from this important difference, all the results and conclusions in [3] still apply. In particular the stability condition is:

$$F(\alpha)\eta_v(\alpha) < \frac{2}{g_0}, \left(\frac{\bar{c}_h}{c_v}\right)^3 \quad (27)$$

where

$$c_v = (\nu_v \delta \nu_v)^{-\frac{1}{3}}. \quad (28)$$

4.2 Stability of the Radial – Longitudinal Motion

Let x , σ , and δ denote a perturbation from the equilibrium configuration. The equations governing the stability in the horizontal plane are [1]:

$$x'' + (K_h - K_{sc}\eta_h)x = h\delta, \quad (29)$$

$$\sigma' = hx - \frac{\delta}{\gamma^2}, \quad (30)$$

$$\delta' = 2\gamma^2 K_{sc}\eta_e \sigma, \quad (31)$$

where

$$\eta_h = 1 + \frac{f_3 - 3\alpha^2 f_5}{g_0}, \quad (32)$$

$$\eta_e = 1 + \frac{f_3 - 1.5\alpha^2 f_5}{g_0}. \quad (33)$$

These equations are similar to those used to determine the stability of the planar motion for the vertical zigzag [3]. The main difference is that the stability form factor η_h is equal the vertical one, η_v , for the vertical zigzag [3]. The same results and conclusions therefore apply. The following stability condition is derived

$$\lambda > \lambda_c c_i \eta_i^{1/3} = \lambda_l. \quad (34)$$

$\eta_s = \eta_c = \eta_e$ and c_i are coefficients which depend on the distance from the original working tunes to the nearest resonance:

$$c_h = (\nu_h \delta \nu_h)^{-1/3} \quad (35)$$

$$c_s = \left(\frac{4}{N_p}\right)^{2/3} \quad (36)$$

$$c_c = \left(\frac{\eta_h}{\nu_h q \eta_e}\right)^{2/3} \quad (37)$$

where q is the smallest positive value of

$$q = 1 \pm \sqrt{1 + \frac{\eta_h}{\eta_e} \frac{\nu_h - m_+ N_p}{\nu_h}}, \quad (38)$$

where m_+ is an integer. From Eq. (23), the stability condition Eq. (34) can also be written as

$$F(\alpha)\eta_i(\alpha) < \frac{2}{g_0} \left(\frac{\bar{c}_h}{c_i}\right)^3. \quad (39)$$

This equation was discussed in [3] for the vertical zigzag, and the same conclusions apply here. A plot of the functions $F(\alpha)\eta_i(\alpha)$ was provided in [3], except that η_v and η_h are now interchanged. For the horizontal zigzag to appear, and to be stable, requires:

$$g_0 \bar{c}_h > c_i, \quad i = v, s, c. \quad (40)$$

That is, the horizontal stopband is closer to the original (dilute beam) working point than any other resonance.

5 Stability of Motion in a Betatron Magnet

The equations of motion needed to determine the stability of motion in a Betatron are Eq. (24) and Eqs. (29–31), with the curvature h constant and $K_v = nh^2$, $K_h = (1-n)h^2$. The conventional betatron condition for the stability of single-particle motion is $0 < n < 1$.

For the case of the horizontal zigzag the procedure is the same as one used for the string [2] and the vertical zigzag [3]. The stability of the vertical motion requires:

$$n > \xi^2 \eta_v > 0, \quad (41)$$

where $\xi = h_{sc}/h$ is the space-charge parameter. Stability of the planar motion requires

$$n < 1 - \gamma^2 - \xi^2 \eta_h < 0. \quad (42)$$

Clearly the two conditions, Eq. (41) and Eq. (42) cannot be satisfied simultaneously. Therefore, the motion of the horizontal zigzag is not stable in a betatron storage ring.

6 Applications

We have applied the foregoing analysis to four storage-ring lattices, with a FODO structure [5]. We have shown previously, that the string bifurcates into a horizontal zigzag in all four of them [2]. Each lattice may have several tuning modes shown in Table 1. As the reference case we take a beam of partially stripped gold ions ($Q = 51, A = 197$). The lattices have a magnetic rigidity of $B\rho = 1.2$ Tm. In Table 2 we show our estimate of the tuning coefficients. The coefficients $\overline{c}_h$ and $\overline{c}_v$ have been calculated using Eq. (18) and Eq. (28), respectively, taking the distance from the integral resonances $\nu_{v,h} = m_{v,h}N_p$ with $m_{v,h}$ integers. The larger of the two values determines in which direction, vertical or horizontal, the string bifurcates into the zigzag. In all the cases listed in Table 1 $\overline{c}_h > \overline{c}_v$.

We use Eq. (28) and Eqs. (35 – 37) to determine the other coefficients c_v, c_h, c_s , and c_c . They are used to estimate the stability limit of the horizontal zigzag. The zigzag become unstable, when

one of the major half-integral resonances located at $2\nu_{v,h} = m_{v,h}N_p$ and $2\nu_s = N_p$ or a coupling resonance with $\nu_h + \nu_s = m_+N_p$ are encountered ($m_{v,h,+}$ are integers). An asterisk in Table 2 shows those cases for which Eq. (40) is not satisfied, and those lattices don't support the horizontal zigzag for an extended range of ion spacing λ .

Table 6 illustrates this further. The first four columns list the smallest of the horizontal aspect ratio obtained using the stability condition Eq. (27) and Eq. (39) for the four stability coefficients. The upper limit λ_u is obtained from Eq. (16), and the lower limit λ_l – if it exists – from Eq. (34). According to our analytical estimate only the FODO16 lattices, and one of the FODO4 lattices are able to contain a horizontal zigzag. In all the others, the horizontal zigzag exhibits some kind of instability, even before the point of bifurcation.

Exact results are obtained numerically using transfer matrices, where we look for periodic orbits by inspecting the eigenvalues. The analytical and numerical results are compared in Table 4. We see, that the FODO8 and FODO4R lattices cannot maintain a horizontal zigzag, according to both the numerical and analytical estimates. For the FODO4 lattice we see differences between the analytical and numerical results. For $\nu_h = 4.8$ we find analytically but not numerically that the lattice supports a horizontal zigzag. For $\nu_h = 5.8$ it is the other way around. The explanation for this discrepancy is that the condition for isochronism Eq. (12) really assumes a smooth dispersion function. This is the case in all the FODO16 lattices, and the FODO4 lattice with $\nu_h = 5.8$, but not in the other ones.

As an example we display in Fig. 1 the dependency of the ion spacing λ and the momentum separation δ on the aspect ratio α , for the FODO16 lattice with $\nu_h = 4.8$.

name	ν_v	ν_h	N_p	λ_c μm	$2\pi R$ [m]
FODO16	4.8	4.8	16	46.4	40.6
	4.0	3.9	16	46.4	40.6
	3.2	3.2	16	46.4	40.6
	2.4	2.4	16	46.4	40.6
FODO4	5.8	5.8	4	60.8	60.9
	4.8	4.8	4	60.8	60.9
	3.6	3.6	4	60.8	60.9
FODO8	7.2	7.2	8	60.8	60.9
	6.0	6.0	8	60.8	60.9
FODO4R	7.3	7.5	4	60.8	60.9

Table 1: Summary of the four FODO storage ring lattices and their tuning modes.

name	ν_v	$\overline{c}_v$	$\overline{c}_h$	c_v	c_h	c_s	c_t
FODO16	4.8	0.443	0.449	0.443	0.443	0.397	0.195
	4.0	0.511	0.515	0.511	0.504	0.397	0.222
	3.2	0.580	0.601	0.580	0.580	0.397	0.256
	2.4	0.703	0.750	0.703	0.703	0.397	0.310
FODO4	5.8	0.576	0.596	0.576	0.576	*1.000	*0.939
	4.8	0.805	0.891	0.805	0.805	1.000	0.731
	3.6	0.536	0.554	*0.703	0.708	*1.000	*2.809
FODO8	7.2	0.338	0.340	*0.443	0.443	*0.630	*1.842
	6.0	0.382	0.385	*0.550	0.550	*0.630	*1.000
FODO4R	7.3	0.424	0.443	*0.562	0.596	*1.000	*2.008

Table 2: Summary of tuning coefficients for the different lattices in Table 1

name	ν_v	α^v	α^h	α^s	α^c	λ_u	λ_l
FODO16	4.8	0.86	1.66	1.56	4.51	50.1	25.7
	4.0	0.84	1.67	1.91	4.56	57.4	29.7
	3.2	0.92	1.69	2.40	4.64	66.9	32.3
	2.4	1.01	1.74	3.35	4.85	83.5	39.2
FODO4	5.8	0.92	1.69	—	—	87.1	—
	4.8	1.12	1.81	1.09	1.74	130.0	60.8
	3.6	—	1.34	—	—	80.9	—
FODO8	7.2	—	1.32	—	—	49.7	—
	6.0	—	1.25	—	—	56.2	—
FODO4R	7.3	—	1.30	—	—	64.6	—

Table 3: Summary of aspect ratios at the limit of stability, and the extremes of the *existence range* of the ion spacing for the lattices shown in Table 1.

analytical results						
name	ν_v	λ_u μm	λ_l μm	α	δ (10^{-6})	
FODO16	4.8	50.1	25.7	0.86	2.94	Vertical
	4.0	57.4	29.7	0.84	3.03	Vertical
	3.2	66.9	32.3	0.92	3.42	Vertical
	2.4	83.5	39.2	1.01	4.08	Vertical
FODO4	5.8	87.1	—	—	—	
	4.8	130.0	60.8	1.09	3.47	Longitudinal
	3.6	80.9	—	—	—	
FODO8	7.2	49.7	—	—	—	
	6.0	56.2	—	—	—	
FODO4R	7.3	64.6	—	—	—	
numerical results						
name	ν_v	λ_u μm	λ_l μm	α	δ (10^{-6})	
FODO16	4.8	56	28.2	1.33	2.14	Vertical
	4.0	61	31.9	1.21	2.29	Vertical
	3.2	70	33.8	1.29	2.70	Vertical
	2.4	86	38.5	1.35	3.41	Vertical
FODO4	5.8	92	62.2	5.3	139	Coupling
	4.8	—	—	—	—	
	3.6	—	—	—	—	
FODO8	7.2	—	—	—	—	
	6.0	—	—	—	—	
FODO4R	7.3	—	—	—	—	

Table 4: Comparison of the analytical and numerical results. The aspect ratio α and the momentum separation δ correspond to the limit of stability at $\lambda = \lambda_l$. The direction of the instability is shown in the last columns.

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Figure 1. Plot of the aspect ratio α and of the half of the momentum separation δ for the FODO16 lattice with $v_h = 4.8$ versus ion spacing λ .