240
12-20-77
UC-19 h Plus
UK Japan

1687

AI-DOE-13216

MASTER

EVALUATIONS
OF THE
PIPING SYSTEM INELASTIC ANALYSIS
COMPUTER PROGRAM PIRAX2

DOE Research and Development Report

*Prepared for the United States
Department of Energy,
Division of Reactor Research and Development,
under Contract Number EY-76-C-03-0824*

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

Rockwell International

Atomics International Division
8900 DeSoto Avenue
Canoga Park, California 91304

~~For Announcement in Energy
Research Abstracts. Distribution Limited
to Participants in the LMR Program.
(Request from TIC)~~

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Department of Energy nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

Printed in the United States of America
Available from
National Technical Information Service
U.S. Department of Commerce
5285 Port Royal Road
Springfield, Virginia 22161
Price: Printed Copy \$5.00 Microfiche \$3.00

EVALUATIONS
OF THE
PIPING SYSTEM INELASTIC ANALYSIS
COMPUTER PROGRAM PIRAX2

By
E. C. RODABAUGH
Battelle Columbus Laboratories

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Department of Energy, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

Rockwell International

Atomics International Division
8900 DeSoto Avenue
Canoga Park, California 91304

CONTRACT: EY-76-C-03-0824
ISSUED: OCTOBER 24, 1977

~~Released For Announcement in Energy
Research Abstracts. Distribution Limited
to Participants in the LMFBR Program
On Request from PAC.~~

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

DISTRIBUTION

This report has been distributed according to the category "LMFBR Structural Materials and Design Engineering," as given in the Standard Distribution for Unclassified Scientific and Technical Reports, TID-4500.

TABLE OF CONTENTS

	<u>Page</u>
ABSTRACT	5
NOMENCLATURE	6
INTRODUCTION	7
ORNL BEAM TESTS	9
Load Controlled Tests, Short-Time Loading	9
Load Controlled Tests, Long-Time Loading	12
Deflection Controlled Tests, Beam B7	19
ORNL LIMIT LOAD TESTS ON ELBOWS	21
Deflections	23
Strains	24
Combined Pressure and Moment Loading	26
ANALYSIS OF A 3-ANCHOR PIPING SYSTEM	28
Piping System Description	28
Input Data	29
Output Data	29
Comparison With Other Analysis Methods	29
Example Analysis Results	33
Problem/Improvement Areas	35
SUMMARY AND CONCLUSIONS	37
REFERENCES	40
Figures 1 through 33 in numerical order	41-75

THIS PAGE
WAS INTENTIONALLY
LEFT BLANK

ABSTRACT

This report contains two sets of comparisons of inelastic test data with PIRAX2-Theory; i.e., ORNL beam tests and ORNL elbow tests. The purpose of these comparisons is to evaluate the accuracy of the simplified analytical techniques used in PIRAX2. The test data are on structures that are much simpler than piping systems but provide a fundamental basis for comparison. The report includes an analysis of a 3-anchor piping system to illustrate the relative simplicity of PIRAX2 input/output data and relatively small computer running time. Some areas of needed improvements in PIRAX2 are discussed.

NOMENCLATURE

A = constant in Equation $e_p = A\sigma^n$
 b = beam width
 E = modulus of elasticity
 e = strain
 e_p = inelastic strain
 e_c = creep strain
 $\dot{e}_c$ = creep strain rate
 F = loading force
 F_t = force as a function of time, t
 F_i = initial force
 h = one-half of beam depth
 I = moment of inertia
 $K = EA(1/Z_p)^{n-1}$
 L = half beam span (Fig. 1) or elbow test length (Fig. 17)
 M = moment
 n = exponent in Equation $e_p = A\sigma^n$
 R = elbow bend radius
 S_y = material yield strength, 0.2% offset
 t = time
 Z_p = plastic section modulus for beam, $= [2n/(2n+1)]bh^2$
 δ = displacement
 δ_p = plastic displacement
 σ = stress

INTRODUCTION

The computer program PIRAX2⁽⁸⁾ was developed to provide a simple efficient method for evaluating the inelastic behavior of piping systems. The need for such a method becomes apparent when we consider the number of piping systems used in nuclear power plants (several hundred) and the complexity* of each piping system.

PIRAX2 was not developed to incorporate the degree of analytical accuracy available in some general-purpose, elastic-plastic finite-element computer programs. Rather, in its present stage of development, it is deemed suitable for preliminary layout of piping systems and evaluation of strain concentrations due to elastic follow-up in piping systems. Several improvements in PIRAX2 can and should be made; these will extend its scope of application. However, there are three aspects of high-temperature structural design which make it difficult to obtain exact analytical evaluations, even with the most sophisticated of analytical techniques. These are:

(1) Material Variability.

In any inelastic analysis, the "yield strength" of the material is an important parameter. Data from Reference (9) gives an example of the variability of the yield strength of type 304. At room temperature, yield strengths vary from 25000 to 69000 psi; at 1100F from 10000 to 43000 psi. As will become obvious in theory versus test comparisons made in this report, an inaccurate description of the inelastic stress-strain curve, corresponding to 25% inaccurate yield strength, could lead to errors in predicted deflection and strain of an order of magnitude. Similar variations exist in creep rates of materials of a given nominal description.

It should be noted that often the critical area of structures is at welds; not in the base material. Weld material is often significantly anisotropic and little is known about the inelastic characteristics as a function of direction

* The complexity of a piping system, when analyzed in detail, is illustrated by an example from Reference (9). The equivalent of a branch connection or tee (there could be the equivalent of several such tees in a typical piping system) was analyzed for thermal gradient loadings using the 3-D MARC general purpose, elastic plastic finite element computer program. An estimated 3600 hours of manpower was associated with the modeling, analysis, evaluation of results and reporting. The results were printed on a stack of computer output paper about 10 feet high.

in the weld.

(2) Long-time Extrapolation.

Most data on creep and rupture of materials at elevated temperatures are based on tests of 10000 hours and less. Extrapolations to needed lifetimes of 30 years (263000 hours) is necessary.

(3) History Dependence.

For elastic analysis, history dependence is significant only in the fatigue analysis and this aspect, at least in a relative sense, is easy to evaluate. However, for inelastic analysis, the material properties can change significantly, depending upon the exact prior history of strain, stress, and time. There are constitutive equations that do take account of how the material properties depend upon prior history. However, it seems unlikely that we will, a priori, know the history that will be imposed on a piping system. For example, it might be assumed for analysis that a piping system will be heated to 1100F at a uniform rate over a 24-hour period. Actually, something may go wrong so that, when halfway up to temperature, it becomes necessary to hold for several days or perhaps return to ambient temperature.

It was consideration of these aspects, along with the number and complexity of piping systems, which motivated the relatively simple evaluation methods in PIRAX2. The analysis is intended to provide useful engineering evaluations of inelastic characteristics of piping systems in a relatively fast and efficient manner.

This report contains two sets of comparisons of inelastic test data with PIRAX2-theory; i.e. ORNL beam tests and ORNL elbow tests. The purpose of these comparisons is to evaluate the accuracy of the simplified analytical techniques used in PIRAX2. The test data are on structures that are much simpler than typical piping systems but never-the-less provide a fundamental basis for comparison. This is followed by a chapter, "Analysis of a 3-Anchor Piping System" to illustrate the relative simplicity of PIRAX2 input/outdata and relatively small computer running time. The "Summary" then indicates significant features of the comparisons and the example analysis, and lists areas of needed improvements in PIRAX2.

ORNL BEAM TESTS

Tests on rectangular cross section beams have been conducted at Oak Ridge National Laboratory. The test beam with its loadings, deflection-measurement dial gages and strain gages are shown in Figure 1.*

In this report, we will compare the test results obtained during the initial loading of the beams with theory. Results for four beam tests will be presented.

- Beam B2: Tested at room temperature with load control
- Beam B3: Tested at 1200F with load control
- Beam B9: Tested at 1100F with load control
- Beam B7: Tested at 1100F with deflection control.

Load-Controlled Tests, Short-Time Loading

In these tests (B2, B3, B9), a load was applied to the center of the beam at a loading rate such that the deflection increased at a rate of 0.2in./min. Deflections at the full load (~2000 lb) did not exceed 0.15 in. hence, the time to apply the load did not exceed 45 seconds. This part of the test is considered to represent time-independent characteristics.

Theory

PIRAX2, at present, is programmed to handle circular pipe cross sections. However, for evaluation of the ORNL beam tests, the basic theory used in PIRAX2 reduces to the following simple equations.

For loads applied a short time period, such that creep during that time is negligible, the displacement at the center of the beam is given by:

$$\delta = \frac{L^3 F}{4Ebh^3} + \frac{AL^{n+2}}{hZ_p^{n(n+2)}} \left(\frac{F}{2}\right)^n \quad (1)$$

and strain at the top and bottom beam surfaces is given by

$$e = \frac{3}{2} \frac{M}{Ebh^2} + A \left(\frac{M}{Z_p}\right)^n \quad (2)$$

* Figures are included following the text.

where

L = half of beam span (12")

F = load at center of beam

b = beam width (1")

h = one-half of beam depth (1")

E = modulus of elasticity

A, n = constants in material constitutive equation

$$e_p = A\sigma^n \quad (3)$$

e_p = plastic strain

σ = stress

Z_p = plastic section modulus, $= [2n/(2n+1)]bh^2$

M = Moment acting on the beam at the section under consideration; e.g. for a strain gage located 0.667" from the center of the beam, $M = (F/2)(12-.667)$.

Material Properties

Before the theory can be used, it is necessary to establish an appropriate value for the modulus of elasticity, E, and the constants A and n used in Equation (3). Tensile test stress/strain curves are given in Reference (1) for the material and temperatures used in beams B2 and B3. The material is identified as type 304, Heat 8043813; temperatures were room and 1200F for B2 and B3, respectively. These were matched to equation

$$e = \frac{\sigma}{E} + A\sigma^n, \quad (4)$$

using the given values of E and selecting A and n so as to match the given stress-strain curve as closely as possible. Figure 2 shows the tensile test data and points representing Equation (4); it can be seen that a good match was made. Reference (2) gives analogous data for beam B9. The material is also type 304, but Heat 9T2796. Figure 3 shows the tensile test data and points representing Equation (4); again, a good match was made.

Deflections

Having established appropriate values of E, A, and n, the deflection can be calculated from Equation (1). After going through the arithmetic, we obtain:

$$\begin{aligned} \text{For beam B2: } E &= 2.9 \times 10^7, A = 6.753 \times 10^{-69}, n = 14.705 \\ \delta &= 1.490 \times 10^{-5} F + 2.639 \times 10^{-56} F^{14.705} \end{aligned} \quad (5)$$

$$\begin{aligned} \text{For beam B3: } E &= 2.09 \times 10^7, A = 8.79 \times 10^{-30}, n = 6.37 \\ \delta &= 2.067 \times 10^{-5} F + 2.216 \times 10^{-23} F^{6.37} \end{aligned} \quad (6)$$

$$\begin{aligned} \text{For beam B9: } E &= 2.17 \times 10^7, A = 4.59 \times 10^{-41}, n = 9.3377 \\ \delta &= 1.991 \times 10^{-5} F + 1.751 \times 10^{-32} F^{9.3377} \end{aligned} \quad (7)$$

Comparison of Equations (1), (2), and (3) with the measured deflections are shown in Figures 4(a), 4(b), and 5(a) for beams B2, B3, and B9 respectively. It can be seen that the calculated and measured deflections are in good agreement.

Strains

Equation (2) gives:

$$\begin{aligned} \text{For beam B2, at gages 0.667 from center of beam (see Fig. 1(a)).} \\ e &= 2.931 \times 10^{-7} F + 1.322 \times 10^{-57} F^{14.705} \end{aligned} \quad (8)$$

Figure 6(a) shows comparisons of measured* strains and those calculated by Equation (8). Agreement is deemed to be very good even though, at the maximum load of 5145 lb, the strain gages indicated about* 0.045 to 0.048 percent whereas Equation (8) gives 0.065 percent at that load.

Equation (2) can also be used to calculate strains along the axis of the beam. Figure 6(b) shows comparisons of averaged, measured strains with strains computed by Equation (2). Agreement between theory and measured strains is good. It may be observed that even the gages closest to the beam center

* Strain gages T4, T5, and B5 (strain gage B4 malfunctioned). These gages gave about the same strain magnitudes (signs, - on top, + on bottom) except at the maximum load of 5145 lb, where they gave strains ranging from 0.045 to 0.048.

(at 0.667" from the center) were far from the location of maximum strains; which theoretically is at the center of the beam and is about 2.5 times as high as at 0.667" from the center.

Strains were not measured on beam B-3. Equation (2), for beam B-9 at gages 1.5" from the center of the beam [see Fig. 1(b)], gives:

$$e = 3.629 \times 10^{-7} F + 3.962 \times 10^{-34} F^{9.3377} \quad (9)$$

Figure 5(b) shows comparisons of measured strains with those calculated by Equation (9). It is pertinent to note the scatter in the test data for the 4 strain gages. Theoretically, these 4 gages would give identical strains. Even in this very carefully conducted test, the maximum strain is 1.55 times the minimum strain. This illustrates an aspect of the plastic characteristics of structures mentioned in the introduction; i.e. they are very difficult to accurately predict or measure.

Load-Controlled Tests, Long-Time Loading

Beam B2 (tested at room temperature) was apparently stable after applying the load; no time-dependent deflections or strains were reported. However, in beams B3 and B9, the maximum load was held for time periods up to 625 hours. We will now discuss the time-dependent deflections and strains for beams B3 and B9.

Theory

In principle, Equation (1) could be used to calculate deflections but the constitutive equation, instead of Equation (3), would be:

$$\dot{e}_c = A\sigma^n \quad (10)$$

where $\dot{e}_c$ is the creep rate and, under the time-hardening hypothesis,

$$e_c = A\sigma^n t \quad (11)$$

However, Equation (10) involves the assumption that first stage creep is

negligible compared to second stage (steady-state) creep and hence Equation (10) is not applicable to the ORNL beam tests. There are two ways to solve the problem:

- (1) Use isochronous stress-strain curves
- (2) Develop a constitutive equation which is a function of time so that first stage creep can be included; i.e.

$$\dot{\epsilon}_c = A\sigma^n \times f(t) \quad (12)$$

where $f(t)$ is an appropriate function of time.

We will apply the first method to beam B3, using isochronous stress-strain curves published in the ASME Boiler Code Case 1592, and both methods to beam B9.

Beam B3

Isochronous stress-strain curves for Type 304 material at 1200F are shown in Figure 7. Each of these curves can be approximated by Equation (4), using a suitable choice of E, A, and n. Table 1 shows the values of A and n so chosen; $E = 2.09 \times 10^7$ psi in accordance with Table I-14.7 of Code Case 1592. These values of E, A, and n can then be used in Equation (1) to calculate δ at the corresponding time. However, the test data gives the increase in deflection as a function of time; accordingly we use only the plastic portion of δ ; i.e.

$$\delta_p = \frac{AL^{n+2}}{h Z_p^n (n+2)} \left(\frac{F}{2}\right)^n \quad (13)$$

Equation (13) was used to calculate δ_p at times corresponding to Figure 7. For example:

$$\delta_p = 3.258 \times 10^{-20} F^{5.45} \quad \text{at } t = 10 \text{ hours} \quad (14)$$

The results of these calculations are shown in Figure 8 for a load $F = 1900$ lb, which was maintained in the test for the first 312 hours, and for $F = 2250$ lb which was applied (quickly) at 312 hours and held for the following 313 hours.

TABLE 1: CONSTANTS A AND n FOR MATCHING ISOCHRONOUS STRESS
VS. PLASTIC-STRAIN CURVES WITH $e_p = A\sigma^n$

Type 304 at 1200F ⁽¹⁾			Heat 9T2796 at 1100F ⁽²⁾		
Time, Hrs.	A	n	Time, Hrs.	A	n
10	6.00×10^{-26}	5.45	10	2.937×10^{-21}	4.132
30	1.64×10^{-21}	4.44	20	1.129×10^{-18}	3.572
100	3.14×10^{-19}	3.94	50	1.958×10^{-17}	3.315
300	6.69×10^{-19}	3.89	100	5.615×10^{-18}	3.481
1000	2.94×10^{-19}	4.03	200	6.606×10^{-18}	3.496
			500	1.592×10^{-17}	3.441
			1000	1.160×10^{-16}	3.255

- (1) Curves from ASME Boiler and Pressure Vessel Code Case 1592-10, Figure 7 herein. (Beam B3).
- (2) Curves plotted from data given in Reference (2), Table A-2, Figure 10. herein, (Beams B7 and B9).

To calculate deflections for the loading of 2250 lb, a suitable hypothesis of the effect of prior history on subsequent creep rates must be assumed. Two widely used hypotheses are called "strain hardening" and "time hardening". With strain hardening, it is assumed that the effect of prior history is dependent only on the amount of accumulated strain. In the simple theory used herein, deflections are proportional to strains (both are proportional to F^n) hence, in Figure 8, under the strain hardening hypothesis, the predicted deflection for the last 313 hours of test is obtained by moving the segment of curve A-B (see Fig. 8) to the right so that point A coincides with point P. With time hardening, it is assumed that the effect of prior history is dependent only on the amount of accumulated time. Accordingly, the prediction deflections for the last 313 hours of the test is obtained by moving the segment of curve C-D (see Fig. 8) down so that point C coincides with point P.

Figure 9 shows comparisons of measured and calculated deflections. It can be seen that agreement is poor. Some or all of this discrepancy may be due to the use of isochronous stress strain curves applicable to the general class of material identified as type 304, not those for the specific heat of material. As will be seen in the subsequent discussion of beam B-9, where we use data for the specific heat of type 304 material, we get better agreement between test and theory. As remarked in the Introduction, one of the problems involved in accurate plastic analysis is the variability of materials within presently defined classes. Presumably, Figure 9 illustrates the kind of inaccuracies that may occur for this reason.

Beam B9

Beam B9 was made of type 304 material, ORNL Heat 9T2796. Reference (2), Table A-2, gives a matrix of uniaxial creep strain data for Heat 9T2796 at 1100 F; covering the range of stresses from 1 to 35 ksi, times from 1 to 10,000 hours. From this data, isochronous stress-creep-strain curves were constructed as shown in Figure 10. Each of these curves can be approximated by Equation (4), except that we have creep-strain, not total strain, hence we do not use the elastic term σ/E in Equation (4). Table 1 shows the values of A and n chosen and the $t = 10$ and $t = 500$ curves in Figure 10 show how the Equation $\epsilon_p = A\sigma^n$ correlate with Reference (2) data. These values of A and n are then

used in Equation (13) to calculate δ_p at the corresponding time. For example,

$$\delta_p = 1.815 \times 10^{-16} F^{4.132} \quad \text{at } t = 10 \text{ hours} \quad (15)$$

Proceeding as discussed under beam B3, curves of δ_p versus time were developed for $F = 2000$ lb (held for the first 312 hours) and $F = 2250$ lb (held for the subsequent 313 hours); these curves are shown in Figure 11.

Figure 12 shows comparisons of measured and calculated deflections. It can be seen that agreement at 625 hours is good. The agreement at 312 hours, while better than for beam B3 (40% over for B9, 80% over for B3) is still not good and suggests that actual beam behavior is more complex than represented by the simple theory used herein. However, for piping purposes, if we can predict creep deflections within 40% we are in reasonably good shape.

An alternative method, as previously mentioned, consists of the establishment of the constants A and n and the function f(t) in Equation (12) so that, for the present purpose, the set of curves shown in Fig. 10 are approximated by a single equation. The equation developed is:

$$e_p = 7.963 \times 10^{-20} \sigma^{3.6816} t^{.5} \quad (16)$$

Table 2 shows how values of e_p calculated by Equation (16) compare with the basic creep strains from Ref. (2), Table A.2. In the beam tests, the stress is highly variable, ranging from about 16000 psi at the top and bottom surfaces of the center of the beam to zero at the support points of the beam. However, the creep deflection/strain characteristics are mostly dependent upon the higher stresses. As can be seen in Table 2, the match is good for stresses from 15,000 to 23,000 psi (exact for $t = 50$ hours, $\sigma = 15,000$ and 20,000 psi). The agreement is poor for low stresses but this is deemed to be insignificant.

The creep deflection, using equation (16) is:

$$\delta_p = \frac{AL^{n+2}}{hZ_p^n(n+2)} \left(\frac{F}{2}\right)^n \cdot t^{.5} = 2.363 \times 10^{-15} F^{3.6816} t^{.5} \quad (17)$$

Points calculated by Equation (17) are shown in Figure 12. Equation (17) agrees closely with the isochronous stress-creep-strain approach and reasonably

TABLE 2. COMPARISON OF UNIAXIAL CREEP STRAINS FROM REF. (2)
WITH CREEP STRAINS CALCULATED BY EQUATION (16)

σ	Source	Strain (%) for Time (hrs.) of:				
		10	20	50	100	200
5000	(1)	.00410	.00570	.00700	.00830	.0101
	(2)	.00104	.00147	.00329	.00329	.00465
10000	(1)	.0181	.0240	.0360	.0470	.0600
	(2)	.0134	.0190	.0300	.0424	.0600
15000	(1)	.0510	.0850	.1335	.191	.260
	(2)	.0597	.0844	.1335	.189	.267
20000	(1)	.160	.240	.385	.575	.820
	(2)	.172	.243	.385	.544	.770
23000	(1)	.310	.480	.775	1.111	1.685
	(2)	.288	.407	.644	.910	1.287
25000	(1)	.470	.725	1.200	1.775	2.800
	(2)	.391	.553	.875	1.237	1.750

(1) Table A.2, Reference (2)

(2) Equation (16): $e_c = 7.963 \times 10^{-20} \sigma^{3.6816} t^{.5}$

well with the test data although predicted deflections are high compared to the test data for the first 312 hours.

Creep strains, using the constitutive Equation (16), can be calculated by the equation:

$$e_p = 7.963 \times 10^{-20} \left(\frac{M}{Z_p} \right)^{3.6816} t^{.5} \quad (18)$$

For gages located 1.5" from the center of the beam (see Fig. 1), $M = (F/2) \times 10.5$, and for the sustained load of 2,000 lb, Equation (18) gives:

$$e_p = 8.112 \times 10^{-5} t^{.5} \quad (19)$$

Figure 13 shows comparisons of measured and calculated strains. It is apparent that no agreement exists. The theory predicts that strains would increase, parallel to the increase in deflection shown in Figure 12. The test data indicates that, immediately after the loading to 2,000 lb was completed and the load of 2,000 lb was being maintained, the strains began to change direction (those on the top of the beam, which were negative during loading, went towards positive; those on the bottom of the beam, which were positive during loading, went towards negative). After a few (less than 10) hours, the strains began to go in the direction indicated by theory; but much less than indicated by the theory.

Measurements of strain at 1100F is difficult. Reference (2) discusses the "drift" of the strain gages used in the tests. They checked the drift by placing gages on unstrained blocks, held at 1100F for time periods of around 800 hours. Any strain indicated by the gages on the unstrained blocks was attributed to drift. These tests indicated that drift was erratic, it could be positive or negative, but in 300 hours would not exceed about 0.02 percent. This is quite inadequate to explain the discrepancy shown in Figure 13

It can be seen in Figure 12 that the deflection at the center of the beam did increase as predicted by the theory, but the theory overpredicted the deflection by some 40%; possibly due to inaccuracies in Equation (16). We might expect that strains would also be over-predicted by the theory by about the same amount; but the discrepancy in Figure 13 is far greater than 40%.

It might be noted that there is an inconsistency in the simple theory used herein in that, for short time loading, the stress distribution through the thickness of the beam is established by the value of $n = 9.3372$; for creep, by the value of $n = 3.6816$. The stress distribution needed to balance the moment at the cross section containing the strain gages, load of 2000 lb, is shown in Figure 14. The theory involves the assumption that the stress distribution for $n = 9.3377$ exists at time $t = 0$; the stress distribution for $n = 3.6816$ exists at all times, $t > 0$. This would imply an instantaneous change in stress distribution which, of course, cannot actually be true. There must be some transient time during which the stress distribution changes. This crude model suggests that strains during this transient period, $t > 0$, would be somewhat less than predicted by theory because that period (of unknown time duration) during which shifting occurred would be from a stronger stress distribution (less strain per unit moment) to the weaker stress distribution assumed for time-dependent conditions. However, this model would not lead to a prediction of strain-direction reversal shown by the test data, Figure 13.

This discrepancy shown in Figure 13 cannot be ignored because it may well carry over into piping systems evaluations and we might be over-predicting creep strains by a significant amount. Further study of the test data and theory is needed but is beyond the scope of this report.

Deflection Controlled Test, Beam B7

In this test, a controlled deflection of 0.10" was applied to the beam at a rate of 0.2 in./min; a loading time of 30 seconds during which time loads were recorded. The deflection was maintained for 144 hours during which time the loads were recorded.

For the short-time loading period, we can obtain deflections by Equation (1), and strains by Equation (2) even though deflection is the controlled variable rather than load. Because beam B7 was made of the same material (Heat 9T2796) as beam B9, and tested at the same temperature (1100F), Equation (7) gives the deflections, Equation (9) gives the strains during the short-time loading. Figure 15 shows comparisons of measured and calculated deflections and strains; agreement is good.

The force on the beam, during the deflection-constant period of 144

hours, is obtained by the equation:

$$F_t = F_i \left\{ \left[(n-1) (F_i/2)^{n-1} \right] \left[3KL^{n-1}/(n+2) \right] t^{.5} + 1 \right\}^{-1/(n-1)} \quad (20)$$

where

F_t = force as a function of time, t

F_i = initial force (1900 lb in the test of beam B7)

$K = EA(1/Z_p)^{n-1}$

E = modulus of elasticity (2.17×10^7 psi)

Z_p = plastic section modulus, $= [2n/(2n+1)] bh^2$

Equation (20) is based on the constitutive Equation (16); and a time-hardening hypothesis. Figure 16 shows comparisons of measured and calculated forces; the agreement is good.

By the simple theory used herein, the strains during the time the deflection is held constant do not change; the creep strains gradually replace the elastic strains with no change in total strain. Test Data are shown in Figure 13 along with that for beam B9, because it is of interest to compare the strains following the application and holding of 2000 lb load with the strains following the application and holding of 0.10" deflection. For both load-control and deflection control, the measured strains during the hold times are of the same order of magnitude; that order of magnitude is in the same ball park as strain gage drift. For this reason, we cannot say that the measured strains are necessarily contradictory to the theory for beam B7 even though they are for beam B9.

ORNL LIMIT LOAD TESTS ON ELBOWS

Bolt and Greenstreet (3) report results of tests on elbows using a test arrangement as shown in Figure 17. Load-deflection curves and load strain curves are presented for a portion of the tests; the elbow identification, dimensions, and elbow material yield strengths (0.2% offset) are summarized in Table 3.

In order to obtain theoretical predictions, using PIRAX2, it is necessary to establish a material descriptive equation of the form:

$$e = \frac{\sigma}{E} + A\sigma^n \quad (21)$$

where e = total strain, σ = stress, E = modulus of elasticity, A and n are constants. Reference (3) gives only the yield strength of the elbow-material. However, representative stress-strain curves were furnished by the authors of Reference (3). The stress-strain curves were obtained from tensile test coupons cut out from the extrados ($\phi = 90^\circ$) portion of the elbows after completion of the elbow tests. Accordingly, the stress-strain curves obtained from the coupons may be affected by the plastic straining that occurred during the tests of the elbows.

Total strains measured during the tests, as indicated by Figures 13 through 17 of Reference (3), probably did not exceed 5000 $\mu\text{in/in}$, which is equivalent to 0.5 percent total strain. Accordingly, we need to match Equation (21) to the stress-strain curves for total strains of up to about 0.5 percent.

Figure 18 shows an example of the match for carbon-steel test specimen PE-2, coupon No. 2. This is perhaps the best of the carbon-steel material matches. Some of the tensile tests showed an upper and lower yield point characteristic; e.g. test specimen PE-2, coupon No. 1, which had an upper yield point of 52,100 psi, lower yield of 50,000, 50,500 psi at 0.2% strain offset and essentially 50,500 psi up to 1.0% total strain. Accordingly, the match of Equation (21) to the carbon-steel materials is a rough approximation.

Figure 19 shows the stress-strain curve for the stainless steel test specimen PE-15, coupon No. 2. For this material, it was possible to match Equation (21) to the tensile test data very closely up to 0.5% total strain.

The constants used in Equation (21) were:

TABLE 3: ORNL ELBOW TESTS, SUMMARY OF DIMENSIONS⁽¹⁾,
MATERIALS AND MOMENTS APPLIED IN TESTS.

Test Number	Wall Thick., in. (2)	R, in. (1)	L, in. (1)	Material		Test Loading (5)
				Nominal Type (3)	Yield Strength, psi (4)	
PE-1	0.280	9	28-1/2	A-106B	50,000	M_z
PE-2	0.280	9	28-1/4	↓	50,000	$-M_z$
PE-3	0.280	9	28-3/4		50,000	M_y
PE-8	0.432	9	25		37,800	$-M_z$
PE-11	0.280	6	30		39,600	$-M_z$
PE-13	0.280	6	27-1/2		39,600	$M_z + P$
PE-15	0.280	9	27-1/2	A-312, 304L	37,700	$-M_z$

- (1) All tests were made on 6" nominal size (6.625" nominal outside diameter) elbows and pipe. See Figure 17 for definition of R and L.
- (2) Nominal wall thickness, 0.280 is Sch. 40, 0.432 is Sch. 80. The actual wall thicknesses are not given in Ref. (3).
- (3) Gives ASTM (or SA) classification. A-106B is carbon-steel, A312, 304L is type 304 austenitic stainless steel, low carbon.
- (4) Yield strength by 0.2% offset.
- (5) See Figure 17. All except PE-13 were run without internal pressure. PE-3 was loaded with M_z while an internal pressure of 1500 psi was maintained.

Test No.	E	A	n
1,2,3	3.00×10^7	3.68×10^{-81}	16.543
8	3.00×10^7	1.60×10^{-74}	15.532
11	3.00×10^7	8.00×10^{-49}	9.874
15	2.83×10^7	1.22×10^{-33}	6.606

For all materials, the constants are such that the nominal yield strength at 0.2% (see Table 3) is a point matched by Equation (21).

Deflections

In-Plane Moment, M_1

Comparisons of calculated and measured deflections are shown in Figures 20, 21, 22, 23, and 24 for tests PE-1, PE-2, PE-8, PE-11, and PE-15, respectively.

The theoretical elastic deflections are shown on the figures. In general, the test elastic deflections are somewhat less than the theoretical elastic deflections. This is a usually encountered relationship; it is presumably due to "end-effects" of the attached pipe legs which restrain the ovalization of the elbow cross section and thus reduce the effective elbow flexibility factor. The displacements are largely due to the elbow, with little contribution from the attached straight pipe. For example, in Test No. PE-1 at Point 2, the elbow contribution is 81% of the total.

Figures 20 through 24 show the theoretical load, F_y , corresponding to first yield of the elbow as calculated by the equation:

$$C_2 M_r / I = S_y \quad (22)$$

where

C_2 = theoretical elastic stress index (C_2 multiplied by the nominal stress, M_r / I , gives the maximum stress in the elbow)

M = moment at middle of elbow, $M = F_y(L + 0.707R)$

F_y = yield load

r_o = outside radius of cross section (3.3125 in.)

I = moment of inertia of cross section

S_y = elbow material yield strength

It can be noted in Figures 20 through 23 that, for carbon steel elbows, there is very little plastic displacement until the load is 2 or 3 times the initial yield load. The stainless steel elbow, Figure 24, is different in that plastic deformations are significant at about 1.5 to 2.0 times the yield load.

For carbon steel elbow tests, where the material description equation is a rough approximation, the calculated results over-predict plastic deflections, although Figure 21 agrees fairly well with test results. For the stainless steel elbow test, Figure 24, where the material descriptive equation is deemed to be fairly accurate, the agreement between calculated and test deflection is good.

Out-of-Plane, M_o

Comparisons of calculated and measured deflections are shown in Figure 25 for test PE-3. The theoretical-elastic deflections agree almost exactly with the test elastic deflections. This is somewhat anomalous because end effects would be expected to reduce the flexibility of the elbow. Calculated and test plastic deflections are in reasonable agreement.

Strains

Reference (3) includes the results of strain measurements during tests of PE-1, PE-2, PE-3, PE-8, and PE-11. Supplementary information was obtained from the author's of Reference (3) concerning details of the strain gage locations and directions.

- (a) Gage locations on all but PE-3 were at ϕ (see Figure 17) of 90° for gage 00, 180° for gage 01, 270° for gage 02 and 0° for gage 03. Gages 01 and 03 were on the sides of the elbow, gage 00 was on the extrados, gage 02 was on the intrados. All four gages measured strains in the circumferential direction.

- (b) In Figure 13 of Reference (3), for Test No. PE-1, the gage identified as 02 should be 03.
- (c) The absolute magnitude of the measured strain was plotted in all cases.

In-Plane Moment

Comparisons of calculated and measured strains are shown in Figures 26, 27, 28, and 29 for tests PE-1, PE-2, PE-8, and PE-11, respectively.

Figure 26, test No. PE-1, shows a single line representing the test strains from both gages 01 and 03. There was some scatter between the two gages but the variation is minor. Because of symmetry, these two strains are theoretically identical. The theoretical elastic strains at $\phi = 0, 180^\circ$ are shown in Figure 26. These are larger than the measured elastic strains which is consistent with the deflection relationships shown in Figure 4. This is presumably at least partially due to end-effects. However, strain measurements are very sensitive to exact gage location; strains are averaged over the gage length and strains may be more sensitive than deflections to variations in wall thickness and/or initial out-of-roundness. Figure 27, Test No. PE-2, is similar since both gages were at the sides; i.e. at $\phi = 0$ and 180° .

Figure 28, Test No. PE-8, gives data for strains at the intrados ($\phi = 270^\circ$) and at the side ($\phi = 0$). The calculated elastic strains are shown in Figure 28; strains at $\phi = 0$ theoretically should be higher* than at $\phi = 270^\circ$ and the test data agrees with this. The theoretical strains are larger than the measured elastic strains, consistent with deflection relationships shown in Figure 6. At present, PYRAX2 is programmed to calculate plastic strains at $\phi = 0$ only; accordingly comparisons of plastic strains in Figure 28 are limited to that location (gage 03).

Figure 29, Test No. PE-11, gives data for strains at the intrados ($\phi = 270^\circ$) and extrados ($\phi = 90^\circ$). The calculated elastic strains are shown in Figure 29. Theoretically, the elastic strains at $\phi = 90^\circ$ and 270° are essentially the same; the test strains agree reasonably well with this aspect of the theory but, as for the other in-plane moment test models, the test

* Actually, the elastic theory predicts negative strains at $\phi = 270^\circ$ however, only absolute magnitudes of strains are given in Reference (3).

elastic strains are substantially less than the theoretical elastic strains. While no direct comparison can be made, Figure 29 also shows the theoretical strains at $\phi = 0$; as mentioned previously, PIRAX2 is not programmed to calculate other than maximum strains in elbows.

It would be of interest to compare calculated and measured strains for the stainless steel elbow, PE-15, but the test results are not given in Reference (3).

Out-of-Plane Moment

Figure 30, Test No. PE-3, gives data for strains $\phi = 330^\circ$, circumferential direction. This is essentially the location and direction of maximum elastic strains. The calculated elastic strains at this location are shown in Figure 30 as for the in-plane models, we note that measured elastic strains are significantly less than theoretical elastic strains. Figure 30 also shows the maximum strains (elastic-plus-plastic) from PIRAX2. However, PIRAX2 is not presently programmed for strains due to out-of-plane moments; it uses the same strain concentration factor as for in-plane moments.

Combined Pressure & Moment Loading

Reference (3) includes tests in which an internal pressure of 1500 psi was maintained inside the test assembly (Fig. 17) while the external load was applied. A load-deflection curve was given for test PE-13, shown herein as Figure 31. Figure 31 also shows the load-deflection results for test PE-11; which is nominally the same as PE-13 (see Table 3).

It can be seen that the internal pressure of 1500 psi (nominal hoop stress of 17000 psi) significantly increased the resistance to load. The stiffening effect of internal pressure in the elastic regime is well known; e.g. Reference (10). The elastic theory indicates that the deflections should be about 11% less with 1500 psi than with zero pressure and roughly, the test data agrees. However, there is no available theory for the effect of pressure in the inelastic regime.

An applicable inelastic theory would have to include the effects of small displacements. The usual finite-element, elastic plastic analysis methods would be expected to predict a decrease in resistance to load because of the

combined stresses due to internal pressure and the external load whereas the test data shows that, because the pressure tends to keep the cross section circular, the resistance to load increases.

ANALYSIS OF A 3-ANCHOR PIPING SYSTEM

The computer program PIRAX2 was used in the evaluation of ORNL beam tests. The PIRAX User's Manual⁽⁴⁾ gives an example of its application and References (5) and (6) give some results using the program. However, in-so-far as the writer is aware, there are no published data concerning the use of PIRAX2 for three-anchor piping systems; a significantly more complex problem than two-anchor systems or the one-anchor test arrangement used in the ORNL beam tests.

The primary purpose of this portion of the report is to illustrate the relative simplicity of the input/output data and relatively small computer time for use of PIRAX2. However, the numerical results from the example are presented and several salient points concerning the results are discussed.

Piping System Description

The details of the model of the three-anchor piping system are shown in Figure 32. The branch (Nodes 2, 3, 5, 7) was made smaller than the run (Nodes 1, 2, 4, 6, 8, 9) and elbows were included so that, quoting from -3138 of Code Case 1592, "a small portion of the structure undergoes inelastic strains while the major portion of the structural system behaves in an elastic manner... In these cases, certain areas may be subjected to strain concentrations due to the elastic follow-up of the rest of the connected structure."

In conducting the analysis, it was assumed that $\alpha\Delta T = 0.0024$, where α = mean coefficient of thermal expansion, ΔT = temperature change of piping system. This value of $\alpha\Delta T$ is such that, if Nodes 7 and 9 were free (see Fig. 32), Node 7 would move up 0.31", to the right 0.89", Node 9 would move down 1.20", to the right 1.20". The restraint of these "free thermal expansions" is evaluated by the PIRAX2 program.

In this particular study, we used the equation:

$$e_p = 4.59 \times 10^{-41} \sigma^{9.3377} \quad (23)$$

to describe the short-time plastic characteristics of the material. This is the same as used earlier in conjunction with ORNL beam tests, however that

is not significant to the main objectives of this portion of the report. The modulus of elasticity was assumed to be 2.79×10^7 psi.

Input Data

The input data is shown in Table 4. It consists of 23 punched cards; about the same amount of input as is used for an elastic analysis of a piping system. A detailed description of the input data is contained in Reference (4); our purpose here is only to illustrate the simplicity of the input data hence we only briefly describe the input data in the following.

Card 1 is a title or comment card. Card 2 gives the number of nodes and elements, the number of specified zero displacements, number of materials steps in the plastic analysis (10) and the value of $\alpha\Delta T$ (.0024). The other three entries are iteration and error limits. Cards 3 and 4 identify the specified zero displacements. Card 5 gives the material properties; modulus of elasticity, shear modulus, A and n in the equation $e_p = A\sigma^n$. The next 9 cards give the node locations. The next 8 cards give the properties of the elements. The last card indicates that there are no specified non-zero forces or displacements.

Output Data

The output data consists of the input data followed by a set of forces, stresses, strains and displacements for each step of the analysis. In this example, 10 steps in the short-time plasticity analysis were used, hence there are 10 sets of data. Table 5 shows the output data for the last step, "Plasticity Step 10". Reference (4) describes the output data in detail. For the present purpose, we note only that the output is comparable in simplicity to that obtained from a conventional elastic analysis of a piping system.

Comparison With Other Analysis Methods

The running time (central processor on a CDC 6400 computer) for the example was 19.9 seconds. The total time was 25 seconds.

Reference (6) gives comparisons using the MARC⁽⁷⁾ program and PIRAX2. The "piping system" was like the ORNL elbow tests; i.e. one anchor, two straight

TABLE 4: PIRAX2 INPUT DATA FOR EXAMPLE 3-ANCHOR PIPING SYSTEM ANALYSIS

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	
1	EXAMPLE ANALYSIS OF A 3-ARMCHOR PIPING SYSTEM USING PIRAX2																																																																																																			
2	9	8	2.1	0	1	10	0	50	0.024	10	1																																																																																									
3	-1	-7	-9	9	10	11	15	16	17	21	22	23	27	28	29	33	34	35	45	46	47																																																																															
4	47																																																																																																			
5	2.179	E+711	0.731	E+74	.59	E-41	1	9	3377																																																																																											
6	1	0	0	0																																																																																																
7	2	240	0	0																																																																																																
8	3	240	120	0																																																																																																
9	4	480	0	0																																																																																																
10	5	249	129	0																																																																																																
11	6	498	-18	0																																																																																																
12	7	369	129	0																																																																																																
13	8	498	-258	0																																																																																																
14	9	498	-498	0																																																																																																
15	1	2	1	1	0	1	6	375	6																																																																																											
16	2	3	1	1	0	1	3	3125	3	0325																																																																																										
17	3	5	2	1	0	1	3	3125	3	0325	9	90																																																																																								
18	5	7	1	1	0	1	3	3125	3	0325																																																																																										
19	2	4	1	1	0	1	6	375	6																																																																																											
20	4	6	2	1	0	1	6	375	6	18	90																																																																																									
21	6	8	1	1	0	1	6	375	6																																																																																											
22	8	9	1	1	0	1	6	375	6																																																																																											
23	0																																																																																																			
24																																																																																																				
25																																																																																																				

TABLE 5: PIRAX2 OUTPUT DATA FOR PLASTICITY STEP 10,
EXAMPLE PIPING SYSTEM SHOWN IN FIGURE 32

PLASTICITY STEP 10						
DISPLACEMENT VECTOR						
NODE	X	Y	Z	THETA-X	THETA-Y	THETA-Z
1	.E+00	.E+00	.E+00	.E+00	.E+00	.E+00
2	5.7407E-01	3.4207E-01	.E+00	.E+00	.E+00	1.2700E-03
3	-2.8554E-01	6.3004E-01	.E+00	.E+00	.E+00	7.2202E-03
4	1.1566E+00	1.1761E+00	.E+00	.E+00	.E+00	2.0059E-03
5	-2.8714E-01	6.4053E-01	.E+00	.E+00	.E+00	-5.0007E-03
6	1.1900E+00	1.1511E+00	.E+00	.E+00	.E+00	-1.0235E-03
7	.E+00	.E+00	.E+00	.E+00	.E+00	.E+00
8	4.8657E-01	5.7553E-01	.E+00	.E+00	.E+00	-3.2627E-03
9	.E+00	.E+00	.E+00	.E+00	.E+00	.E+00

MEMBER FORCES, STRESSES, AND STRAINS									
MEMBER I	FX	FY	FZ	MX	MY	MZ			
MEMBER J	FX	FY	FZ	MX	MY	MZ			
STRESS	AXIAL-I	AXIAL-J	BENDING-YI	BENDING-YJ	BENDING-ZI	BENDING-ZJ	TORSION-I	TORSION-J	CIRCUMFERENTIAL
STRAIN	AXIAL-I	AXIAL-J	BENDING-YI	BENDING-YJ	BENDING-ZI	BENDING-ZJ	TORSION-I	TORSION-J	CIRCUMFERENTIAL
1	-4.1627E+02	1.7505E+03	.E+00	.E+00	.E+00	-5.9508E+04			
2	4.1627E+02	-1.7505E+03	.E+00	.E+00	.E+00	1.5941E+05			
	-1.2007E+02	-1.2007E+02	.E+00	-E+00	1.3541E+03	3.6381E+03	.E+00	.E+00	
	2.3957E-03	2.3957E-03	.E+00	-E+00	4.8678E-05	1.3041E-04	.E+00	.E+00	
2	1.1143E+03	1.2087E+03	.E+00	.E+00	.E+00	-8.9491E+04			
3	-1.1143E+03	-1.2087E+03	.E+00	.E+00	.E+00	-4.6225E+04			
	-2.1656E+02	-2.1656E+02	.E+00	-E+00	1.0526E+04	-5.2056E+03	.E+00	.E+00	
	2.3922E-03	2.3922E-03	.E+00	-E+00	5.2611E-04	-1.8681E-04	.E+00	.E+00	
3	1.1143E+03	1.2087E+03	.E+00	.E+00	.E+00	4.2229E+04			
5	1.2087E+03	-1.1143E+03	.E+00	.E+00	.E+00	-4.2229E+04			
	-2.1656E+02	-1.0665E+02	.E+00	-E+00	-5.2056E+03	-5.1056E+03	.E+00	.E+00	2.1249E+04
	2.3922E-03	2.3922E-03	.E+00	-E+00	-1.8681E-04	-1.8319E-04	.E+00	.E+00	1.3815E-03
5	-1.2087E+03	1.1143E+03	.E+00	.E+00	.E+00	4.2229E+04			
7	1.2087E+03	-1.1143E+03	.E+00	.E+00	.E+00	1.8157E+05			
	-1.0965E+02	-1.0965E+02	.E+00	-E+00	-5.1056E+03	1.1957E+04	.E+00	.E+00	
	2.3922E-03	2.3922E-03	.E+00	-E+00	-1.8319E-04	9.7702E-04	.E+00	.E+00	

TABLE 5: PIRAX2 OUTPUT DATA FOR PLASTICITY STEP 10,
EXAMPLE PIPING SYSTEM SHOWN IN FIGURE 32
(Continued)

2	7.9242E+02	6.3615E+02	.F+00	.F+00	.F+00	-6.5522E+04		
4	-7.9242E+02	-6.3615E+02	.F+00	.F+00	.F+00	-1.2026E+05		
	-6.3635E+01	-6.3635E+01	.F+00	.F+00	1.5458E+03	-2.7445E+03	-.F+00	-.F+00
	2.3904E-03	2.3904E-03	.F+00	.F+00	5.7196E-05	-9.8371E-05	-.F+00	-.F+00
4	7.9242E+02	6.3615E+02	.F+00	.F+00	.F+00	1.2026E+05		
6	6.3615E+02	-7.9242E+02	.F+00	.F+00	.F+00	-1.2367E+05		
	-6.3635E+01	-6.3635E+01	.F+00	.F+00	-2.7445E+03	-2.4007E+03	-.F+00	.F+00
	2.3904E-03	2.3904E-03	.F+00	.F+00	-9.8371E-05	-1.0057E-04	-.F+00	.F+00
								1.4442E+04
								5.4140E-04
6	-6.3615E+02	7.9242E+02	.F+00	.F+00	.F+00	1.2367E+05		
8	6.3615E+02	-7.9242E+02	.F+00	.F+00	.F+00	-2.9606E+04		
	-6.4353E+01	-6.4353E+01	.F+00	.F+00	-2.4007E+03	6.7567E+02	-.F+00	-.F+00
	2.3901E-03	2.3901E-03	.F+00	.F+00	-1.0667E-04	2.4218E-05	-.F+00	-.F+00
8	-6.3615E+02	7.9242E+02	.F+00	.F+00	.F+00	-2.9606E+04		
9	6.3615E+02	-7.9242E+02	.F+00	.F+00	.F+00	1.8228E+05		
	-6.4353E+01	-6.4353E+01	.F+00	.F+00	6.7567E+02	4.1601E+03	-.F+00	-.F+00
	2.3901E-03	2.3901E-03	.F+00	.F+00	2.4218E-05	1.4512E-04	-.F+00	-.F+00
EQUILIBRATED APPLIED FORCE VECTOR								
NODE	FX	FY	FZ	UX	UY	UZ		
1	1.7505E+03	4.1827E+02	.F+00	.F+00	.F+00	-5.9508E+04		
2	-2.2012E-00	-7.1886E-09	.F+00	.F+00	.F+00	4.6566E-10		
3	6.2630E-08	-5.2627E-08	.F+00	.F+00	.F+00	-1.3735E-05		
4	-2.5200E-07	-1.5155E-07	.F+00	.F+00	.F+00	-4.0956E-06		
5	4.3627E-08	-7.7511E-08	.F+00	.F+00	.F+00	7.0616E-07		
6	-1.2685E-07	-2.5046E-07	.F+00	.F+00	.F+00	4.8256E-06		
7	-1.1143E+03	-1.2087E+03	.F+00	.F+00	.F+00	1.0167E+05		
8	-7.2760E-12	.F+00	.F+00	.F+00	.F+00	5.8208E-10		
9	-6.3615E+02	7.9242E+02	.F+00	.F+00	.F+00	1.8228E+05		
NUMBER OF ITERATIONS 36								

pipes and an elbow. They remark that:

"The MARC input data is difficult to prepare relative to the PIRAX data. Further, MARC is more costly to operate and the output is more difficult to interpret. The relative complexity of the MARC model provides the principal motivation for this evaluation of the simplified model contained in the PIRAX computer program."

Reference (6) gives computer execution times for 16 different cases; the MARC times ranged from 91.4 to 1400 seconds; PIRAX2 from 1.32 to 6.35 seconds. The average ratio of MARC to PIRAX2 time was 133.

Example Analysis Results

The most significant aspects of the results are summarized in Table 6. The second column shows the stresses that were obtained assuming the material remained elastic. The third column shows the stresses given by the final (10th) plasticity step. It can be seen that stresses by plastic analysis are lower than those by elastic analysis for all nodes except 1 and 8. The fourth and fifth column shows analogous data for strains.

It may be noted that Node 1 shows a major increase in stress and strain as compared to elastic results. However, no plastic strain occurred at Node 1. This is shown by the last column of Table 6, which shows the elastic strain at the plastic-analysis stress.

The analysis indicates that significant plastic strains occurred at Node 7, in both elbows, and in the branch (Node 3) side of Node 2. The complex behavior involved is partially illustrated by Figure 33. Figure 33 shows moments 1, 3, 7, and 9 plotted against the fraction of total "free end displacement". It can be seen that when that fraction reaches about 0.45, the piping system begins to act in a non-elastic manner. The elbow, which is the weakest point, undergoes plastic straining and sheds part of its load. However, other parts of the piping system, in particular at and near Node 1, are still in the elastic regime and are capable of and do pick up part of the load.

In terms of strain concentration due to elastic follow-up, we note that Node 1 has about 9 times the strain predicted by an elastic analysis.

TABLE 6: SUMMARY OF STRESSES AND STRAINS, EXAMPLE
PIPING SYSTEM SHOWN IN FIGURE 32

Node	Stress, psi		Strain, x 10 ⁶		
	Elastic Analysis	Plastic Analysis	Elastic Analysis	Plastic Analysis	Elastic-PS (1)
1	147	1358	5.261	48.68	48.68
7	15945	11967	571.5	977.0	428.9
9	4283	4160	153.5	149.1	149.1
3/5 Elbow	39409	21249	1413	1382	761.9
4/6 Elbow	15836	14642	567.6	541.0	524.8
2/1	4863	3638	174.3	130.4	130.4
2/3	13623	10534	488.3	544.1	377.6
2/4	2222	1596	79.63	57.20	57.20
8	622	676	22.31	24.22	24.22

(1) This column gives the elastic strain at the stress obtained from the plastic analysis. The difference between this column and the preceding column is the plastic strain.

However, this is still elastic strain. The largest plastic strains occurred at the 6" elbow (0.062 percent) and the 6" branch anchor, Node 7, (0.055 percent), hence, in this particular example, the strain due to elastic follow-up is negligible.

Problem/Improvement Areas

Convergence of Solutions

It was initially anticipated that the example problem would be run using a value of $\alpha\Delta T = 10.4 \times 10^{-6} \times 1030 = 0.0107$. This represents the mean coefficient of expansion of 304 material from 70 to 1100F and ΔT from 70F to 1100F. However, the solution diverged and led to numbers too large for the computer to handle.

Reference (8) describes the technique used for the initial plasticity solution and discusses the convergence problem. There are some choices available as part of the input data to obtain convergence, however the program was not checked for a 3-anchor piping system. Available time and budget limited the investigation that could be made and, after several trials without success, the value of $\alpha\Delta T$ was decreased to 0.0024 and convergent results were obtained. Further investigation of the conditions under which convergence can be obtained is needed.

The initial choice of $\alpha\Delta T = 0.0107$ is rather extreme, it leads to an elastic-theory maximum stress of 176000 psi in the 6" elbow along with a material with plastic-characteristics as shown in Figure 3; i.e. material with a 0.2% offset yield strength of about 10,000 psi. It is perhaps too much to expect that this extreme case can be handled, but further guidance on just what can be handled without divergence is needed.

Characteristics of Branch Connections/Tees

It may be noted in Figure 32 that the 12 x 6 branch connection is modeled as an intersection of three beams; the intersection point is assumed to be a rigid juncture. In a real piping system, of course, the branch connection or tee is an extremely complex shell structure. Appropriate parameters to more closely describe the actual plastic behavior of branch connections are

needed. Limit load tests on branch connections have been run and possibly could be used to develop such parameters. It is believed that these could be introduced into PIRAX2, with little or no sacrifice of its economic aspects, and would provide better engineering guidance for piping systems containing branch connections or tees.

Prior History Effects

In this report, we have limited our evaluations/comparisons to initial loading or displacement conditions. However, there is no inherent reason why PIRAX2 can not be used for cycles of loadings. Conceptually, we would need to introduce constitutive equations which represent the strain-hardening (or softening) effects of prior strains and hold times. Also, creep equations which involve a time function, such as equation (16), could be used. These additions would probably increase the complexity of PIRAX2 but the economics would still be relatively very attractive.

SUMMARY AND CONCLUSIONS

Summary

ORNL Beam Tests

PIRAX2, at present, is programmed to handle circular pipe cross sections. However, for evaluation of ORNL beam tests, the basic theory used in PIRAX2 reduces to a few simple equations which can be solved by hand. The equations involve the material plastic properties approximated by the equation $\epsilon_p = A\sigma^n$.

Results from initial loading of ORNL Beams B2, B3, B9, and B7 are presented and compared with theory. For short-time loadings, material properties were obtained from short-time uniaxial stress strain curves. For all four beams, the test and theory agreed very well. For long-time loadings, material properties were based on isochronous stress strain curves. When these curves were taken from Code Case 1592 (Beam B3), the agreement between test and theory deflections was poor, but when more directly representative of the material, (Beams B9 and B7) the agreement was satisfactory. This is deemed to illustrate the effect of material variability.

For Beam B7, which was a deflection-controlled test, the change in force as a function of time was predicted satisfactorily by the theory.

The only major area of disagreement between test and theory was the measured strains in the test of Beam B9. In this test, strains decreased immediately following load application, then increased, but not as much as predicted by theory.

ORNL Elbow Tests

PIRAX2 was used to calculate deflections and strains for comparison with the test data. The agreement was satisfactory, considering the necessary approximations of the material constitutive equations. The comparisons illustrate that PIRAX2 can be used to identify and quantify, in an engineering sense, the inelastic behavior of elbows in piping systems under sustained loads.

Analysis of a 3-Anchor Piping System

The input/output data for this piping system is shown to be relatively simple. The running time (CDC 6400) was 19.9 seconds; total time of 25 seconds.

The results illustrate the complex inelastic behavior of a piping system; the loads shift; stresses and strains may increase or decrease. In this particular example, the largest plastic strains occurred at the 6" elbow and at the anchor of the 6" branch. However, these were quite small (less than 0.06 percent) and are deemed to be insignificant. This illustrates how PIRAX2 helps to make an engineering judgement on the inelastic characteristics of piping systems.

Problems/Improvements in PIRAX2

- (a) In evaluating the 3-anchor piping system, problems were encountered in obtaining convergence of the solution for large "free end displacements". Further investigation is needed to establish the conditions under which convergence can be obtained.
- (b) Appropriate parameters for branch connections or tees need to be developed and incorporated into PIRAX2.
- (c) Improvements in the ability of PIRAX2 to handle "history effects" is needed.

Conclusions

The comparisons made herein with ORNL beam tests and ORNL elbow tests indicate that PIRAX2 is sufficiently accurate to give adequate engineering guidance for such purposes as preliminary piping layout and evaluation of the significance of strain concentration due to elastic follow-up in piping systems. More sophisticated and potentially more accurate evaluation methods exist, however these appear to be much more expensive to use and, considering the number and complexity of piping systems, may be not feasible to use from an economic standpoint.

A number of improvements can be made in PIRAX2, probably with relatively little sacrifice of the main advantage of PIRAX2; i.e. its simplicity and low cost.

More sophisticated analysis methods, as well as PIRAX2, are at present limited in accuracy by variations in material properties and by large and uncertain extrapolations of relative short-time material test data. Further, and perhaps for piping systems in particular, the use of more sophisticated methods is difficult to justify in view of the probability that the detailed operating history cannot be accurately predicted.

REFERENCES

- (1) Corum, J. M., and Richardson, M., "Elastic-Plastic-Creep Tests of Type 304 Stainless Steel (Heat 8043813) Beams", ORNL-TM-3856, April 1973.
- (2) Corum, J. M., Richardson, M., and Clinard, J. A., "Elevated Temperature Benchmark Tests of Simply-Supported Beams and Circular Plates Subjected to Time-Varying Loads", ORNL-5301, September 1977.
- (3) Bolt, S. E., and Greenstreet, W. L., "Experimental Determinations of Plastic Collapse Loads for Pipe Elbows", ASME Paper No. 71-PVP-37.
- (4) Workman, G. H., and Rodabaugh, E. C., "User's Manual for PTRAX2, Inelastic Analysis of 3-Dimensional Piping Systems", ORNL-SUB-3651-3, December 10, 1974.
- (5) Rodabaugh, E. C., and Workman, G. H., "Strain Concentration in an Elbow in a Piping System Under High Temperature Relaxation Conditions", in Second Int'l Conf. on Pressure Vessel Technology, San Antonio, Texas, Oct. 1973, Part 1, pp 375-392, ASME, New York, 1973.
- (6) Scheller, J. D., Mallett, R. H., and Nair, B. R., "Numerical Evaluation of an Elbow Element for Inelastic Flexibility Analysis of Piping", WARD-D-0184, June, 1977, Westinghouse Advanced Reactors Div., Box 158, Madison, PA, 15663.
- (7) MARC is a proprietary computer program of the MARC Analysis Corp. Available from Control Data Corporation, Minneapolis, Minn. 55440.
- (8) Workman, G. H., and Rodabaugh, E. C., "Development of a Simplified Analytical Procedure for Determining the Inelastic Response of Piping Systems, ORNL SUB-3651-1, Sept. 1974.
- (9) Staff, ORNL High-Temperature Structural Design Program, "Status Report on Inelastic Analysis Capability", Prepared for ASME Working Group on Creep Analysis, Sept. 13, 1977.
- (10) Rodabaugh, E. C., and George, H. H., "Effect of Internal Pressure on Flexibility and Stress-Intensification Factors of Curved Pipe or Welding Elbows", ASME Trans., 1957.

D1-D5: DEFLECTION DIAL GAGES
 L1-L5: DEFLECTION LVDT'S
 T1-T8: AXIAL STRAIN GAGES, TOP SURFACE (B2 ONLY)
 B1-B8: AXIAL STRAIN GAGES, BOTTOM SURFACE (B2 ONLY)
 TC1-TC10: THERMOCOUPLES (B3 AND B4 ONLY)

DT: DCDT
 DG: DIAL GAGE
 TC: THERMOCOUPLE
 SG: WELDABLE RESISTANCE STRAIN GAGE

FIGURE 1: ORNL BEAMS AND MEASUREMENT DEVICES

(a) At temperature of 100 F, for beam B2

(b) At temperature of 1200 F, for beam B3

FIGURE 2. MATCHING OF EQUATION (4) TO TENSILE TEST DATA, TYPE 304 STAINLESS STEEL, HEAT 8043813

FIGURE 3. MATCHING OF EQUATION (4) TO TENSILE TEST DATA, TYPE 304 STAINLESS STEEL, HEAT 9T2796, AT 1100F, FOR BEAM B9

(a) Beam B2, tested at 100 F

(b) Beam B3, tested at 1200 F

FIGURE 4. COMPARISONS OF MEASURED AND CALCULATED TIME-INDEPENDENT DEFLECTIONS, BEAMS B2 AND B3

(a) Deflections

(b) Strains at gages 1.5 in. from center

FIGURE 5. COMPARISON OF MEASURED AND CALCULATED TIME-INDEPENDENT DEFLECTIONS AND STRAINS, BEAM B9

(a) Strain at gages 0.667 (in.) from center

(b) Strains along length of beam at $F = 5000$ lb

FIGURE 6. COMPARISON OF MEASURED AND CALCULATED STRAINS, BEAM B2

FIGURE 7: ISOCHRONOUS STRESS-STRAIN CURVES FOR TYPE 304 STAINLESS STEEL AT 1200F, FROM FIGURE T-1800-A-9 OF ASME BOILER AND PRESSURE VESSEL CODE CASE 1592-10

FIGURE 8. CALCULATED DEFLECTIONS USING EQUATION (13) AND A AND n VALUES FROM TABLE 1, BEAM B3

FIGURE 9. COMPARISON OF MEASURED AND CALCULATED TIME-DEPENDENT DEFLECTIONS, BEAM B-3

FIGURE 10. ISOCHRONOUS STRESS-PLASTIC-STRAIN CURVES FOR HEAT 9T2796 (TYPE 304) AT 1100F AND MATCHING BY $e_p = A\sigma^n$

FIGURE 11. CALCULATED DEFLECTIONS USING EQUATION (13) AND A AND n VALUES FROM TABLE 1, BEAM B9

FIGURE 12. COMPARISON OF MEASURED AND CALCULATED TIME-DEPENDENT DEFLECTIONS, BEAM B9

(a) Beam B9, Strains during 312 hour creep time

(b) Beam B7, Strains during 144 hour relaxation time

TABLE 13. CHANGES IN STRAIN DURING LOAD-HOLD (BEAM B-9) AND DEFLECTION HOLD (BEAM B7)

Theory Assumptions

Planes remain planes,
plastic strength in tension
and compression is equal

TABLE 14. STRESS DISTRIBUTION THROUGH BEAM SECTION 1.5" FROM BEAM CENTER AT LOAD OF 2000 LB

(a) Deflection at beam center

(b) Strains at 1.5 (in.) from center of beam, Top and bottom surfaces

FIGURE 15. COMPARISONS OF MEASURED AND CALCULATED TIME-INDEPENDENT DEFLECTIONS AND STRAINS, BEAM B7

FIGURE 16. COMPARISON OF MEASURED AND CALCULATED TIME-DEPENDENT FORCES, BEAM B7

FIGURE 17. SCHEMATIC OF TEST ARRANGEMENT AND DEFINITION OF COORDINATE ANGLE, ϕ , ORNL ELBOW TESTS

FIGURE 18. STRESS-STRAIN CURVE FOR MATERIAL OF ELBOW PE-2, COUPON NO. 2.

FIGURE 19. STRESS-STRAIN CURVE FOR MATERIAL OF ELBOW PE-15, COUPON NO. 2

FIGURE 20. MODEL PE-1, TEST (HEAVY SOLID LINES) AND CALCULATED DEFLECTIONS.

FIGURE 21. MODEL PE-2, TEST (HEAVY SOLID LINES) AND CALCULATED DEFLECTIONS

FIGURE 22. MODEL PE-8, TEST (HEAVY SOLID LINES) AND CALCULATED DEFLECTIONS

FIGURE 23. MODEL PE-11, TEST (HEAVY SOLID LINES) AND CALCULATED DEFLECTIONS

FIGURE 24. MODEL PE-15, TEST (HEAVY SOLID LINES) AND CALCULATED DEFLECTIONS

FIGURE 25. MODEL PE-3, TEST (HEAVY SOLID LINES) AND CALCULATED DEFLECTIONS

FIGURE 26. MODEL PE-1, TEST (HEAVY SOLID LINES) AND CALCULATED STRAINS

FIGURE 27. MODEL PE-2, TEST (HEAVY SOLID LINES) AND CALCULATED STRAINS

FIGURE 28. MODEL PE-8, TEST (HEAVY SOLID LINES) AND CALCULATED STRAINS

FIGURE 29. MODEL PE-11, TEST (HEAVY SOLID LINES) AND CALCULATED STRAINS

FIGURE 30. MODEL PE-3, TEST (HEAVY SOLID LINES) AND CALCULATED STRAINS

FIGURE 31: COMPARISON OF MODEL PE-13 (1500 PSI INTERNAL PRESSURE) WITH NOMINALLY IDENTICAL MODEL PE-11 (0 INTERNAL PRESSURE)

FIGURE 32: MODEL OF EXAMPLE 3-ANCHOR PIPING SYSTEM

FIGURE 33: MOMENTS AT NODES 1, 3, 7, AND 9, EXAMPLE PIPING SYSTEM SHOWN IN FIGURE 32

Rockwell International

Atomics International Division
8900 DeSoto Avenue
Canoga Park, California 91304