Novel fringe scanning/Fourier transform
method of synthetic imaging

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ABSTRACT

We have developed a one-dimensional theory and a computer model for synthetically imaging scenes using a novel fringe scanning/Fourier transform technique. Our method probes a scene using two interfering beams of slightly different frequency. These beams form a moving fringe pattern which scans the scene and resonates with any spatial frequency components having the same spatial frequency as the scanning fringe pattern. A simple, non-imaging detector above the scene observes any scattered radiation from the scene falling onto it. If a resonance occurs between the scanning fringe pattern and the scene, then the scattered radiation will be modulated at the difference frequency between the two probing beams. By changing the spatial period of the fringe pattern and then measuring the amplitude and phase of the modulated radiation that is scattered from the scene, the Fourier amplitudes and phases of the different spatial frequency components making up the scene can be measured. A synthetic image of the scene being probed can be generated from this Fourier amplitude and phase data by taking the inverse Fourier transform of this information. This technique could be used to image objects using light, ultrasonic, or other electromagnetic or acoustic waves.

1. INTRODUCTION

An important concept in the field of optical imaging is the fact that all scenes can be viewed as being made up of an infinite number of two-dimensional spatial frequency components.^{1,2,3,4} Lower spatial frequencies make up the gross details of a scene while higher spatial frequencies produce the finer details. Optical designers use this fact on a regular basis to analyze optical systems for diffraction effects and to determine what the diffraction limited resolution of the system is. And since the aperture of an optical system acts as a low pass spatial filter with a cutoff frequency related to the aperture's size, larger apertures produce greater spatial resolutions than do smaller apertures. Since we can know everything there is to know about a scene if we measure and record the amplitude and phase of each of its two-dimensional spatial frequency components (i.e. - measure all the scene's two-dimensional spatial frequency Fourier components), we have enough information to synthetically generate an image of the original scene. This is done by taking the inverse Fourier transform of this

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information.

The above approach is the basis for our fringe scanning/Fourier transform method of synthetic imaging. In this method, we measure the amplitudes and phases of the spatial frequency components of a scene by scanning a fringe pattern over the scene. The amplitude of the AC signal received by a detector observing the scanning fringe pattern is proportional to the amplitude of the spatial frequency component in the scene having the same spatial frequency as that of the fringe pattern. The phase of the AC signal is also related to the phase of the spatial frequency component being measured. By changing the fringe spacing, we can probe different spatial frequencies within the scene. We have developed a one-dimensional theory for this method and developed a computer model from this theory which can synthetically generate images of one-dimensional scenes. Only a slight extension of concept is needed to do the same for imaging two-dimensional scenes.

2. ONE DIMENSIONAL THEORY

Consider the situation where two plane waves having a wavelength, λ , illuminate a surface at an angle, θ , from opposite sides of the surface's normal. For simplicity, we assume that each of the two beams has the same intensity, I_0 . Simple physical optics teaches us that the two beams will interfere and produce a non-scanning fringe pattern along the surface of the form

$$I_{\text{surface}} = 2I_0\{1 + \cos[2kz \cdot \sin(\theta)]\} \quad (1)$$

where k is the wave number and z is the coordinate along the surface in the direction perpendicular to the fringes that are formed. Notice that for convenience, the origin was defined to make any additional phase within the cosine function vanish.

Since the fringe spacing is a function of both λ and θ , we can see that one way to probe different spatial frequencies in a fringe scanning system is to vary the angle at which the two beams are falling onto the surface being imaged while leaving the wavelength of the incident beams constant. The resolution of such a system will be one-half the wavelength of the illuminating radiation and will occur when $\theta = 90^\circ$. A second way of probing different spatial frequencies in the scene is to leave θ constant (say 30° for a resolution of one wavelength) and vary the wavelength over several octaves by chirping to probe the different spatial frequencies. In this case, the resolution will be based on the shortest wavelength used. This second method would probably find more usefulness in acoustic, ultrasonic, and microwave types of imaging, and the first method would be more useful for optical imaging using lasers.

The non-scanning fringe pattern shown in equation (1) can be converted into a scanning fringe pattern by offsetting the frequencies of the two beams without changing the wavelengths significantly. In this case,

the fringe pattern set up by the two plane waves has the form

$$\begin{aligned}
 I_{\text{surface}} &= 2I_0\{1 + \cos(\delta\omega*t)*\cos[2kz*\sin(\theta)] \\
 &\quad + \sin(\delta\omega*t)*\sin[2kz*\sin(\theta)]\} \\
 &= 2I_0\{1 + \cos[\delta\omega*t - 2kz*\sin(\theta)]\}
 \end{aligned}
 \tag{2}$$

where $\delta\omega$ is the difference in the angular frequencies of the two beams and t is the time. As can be seen, the fringe pattern produced is now a scanning fringe pattern which can be used to probe spatial frequency components in a scene having a periodicity of

$$\lambda_{\text{spatial}} = \lambda/2 \sin(\theta) \tag{3}$$

We can intuitively illustrate the principle of spatial frequency measurement using a scanning fringe pattern by the following simple example. Suppose that our scanning fringe pattern falls onto a picket fence, and that we detect the light scattered off of the fence by a simple, non-imaging detector which sees the whole scene being imaged. If the picket fence and our fringe pattern have the same spatial period, then when the fringes fall on top of the pickets, our detector will have a large amount of light scattered into it and when the fringes fall between the pickets our detector will not detect any scattered light. Our detector will therefore detect a modulated signal at the same temporal frequency as the difference frequency between the two beams setting up the scanning fringe pattern. On the other hand, if the picket fence and the scanning fringe pattern do not have the same spatial period, then the light being scattered into the detector will remain constant in time, assuming both a fringe pattern and a picket fence of infinite extent. The AC component of the signal detected by our detector will therefore be related to the spatial frequency being probed by the scanning fringe pattern, and we can measure the Fourier spatial frequency coefficients by measuring the amplitude and phase (with respect to some predefined point on the surface) of the AC signal being received by the detector.

The theory which we developed shows that the above intuitive example provides a plausible answer. Suppose we illuminate a one-dimensional scene with a scanning fringe pattern of the form given in equation (2). Also assume that the surface of our scene scatters light in all directions with a scattering reflectance, $R(z)$, which is a function of position along the surface of our one-dimensional scene. For mathematical simplicity we will also define $R(z)=0$ for regions outside the particular area over which our detector can receive light from the scene. This will take into account the fact that we may have a lens system looking only at a particular area, that the illuminated spot of light only has a finite size, or that the object being studied only has a finite size. The light received from a small section of the scene between z and $z+dz$ will therefore have an intensity of

$$dI_{\text{rcvd}} = CR(z)I(z)dz \quad (4)$$

where C is a constant which accounts for geometrical factors of the situation that do not concern us presently. Note also that C will have a predictable θ dependence which can be modeled out in the end and can therefore be ignored in our present analysis.

By integrating equation (4), we find that our detector will receive a time dependent intensity of the form

$$\begin{aligned} I_{\text{rcvd}} &= C \int_{-\infty}^{\infty} R(z)I(z)dz \\ &= C \int_{-\infty}^{\infty} R(z)dz + C \cos(\delta\omega \cdot t) \int_{-\infty}^{\infty} R(z) \cos[2kz \sin(\theta)] dz \\ &\quad + C \sin(\delta\omega \cdot t) \int_{-\infty}^{\infty} R(z) \sin[2kz \sin(\theta)] dz \end{aligned} \quad (5)$$

which consists of a DC term and two orthogonal time dependent terms.

We can compare the AC terms from equation (5) to the Fourier integral theorem which states⁵

$$R(z) = \int_{-\infty}^{\infty} \{A(k') \cos(k'z) + B(k') \sin(k'z)\} dk' \quad (6a)$$

$$\text{where} \quad A(k') = 1/\pi \int_{-\infty}^{\infty} R(z) \cos(k'z) dz \quad (6b)$$

$$B(k') = 1/\pi \int_{-\infty}^{\infty} R(z) \sin(k'z) dz$$

From this comparison, we can see directly that if

$$k' = 2k \sin(\theta) \quad (7)$$

for the system where the angle is changed in order to change the fringe spacing, then the two time dependent terms in equation (5) can be used to measure the $A(k')$ and $B(k')$ coefficients of the scene's spatial frequencies, and we can then synthetically generate the scene by using equation (6a).

3. ANALYTICAL COMPUTER MODELING

As part of our work, we developed a computer model to analytically demonstrate our fringe scanning/Fourier transform method of synthetic imaging. In this model, the expected values of $A(k')$ and $B(k')$ from a one-dimensional scene were calculated, and 1000 separate, evenly spaced k' values were chosen which corresponded to non-evenly spaced θ values between 90° and approximately 0.06° . This gave fringe spacings of between $1/2$ wavelength and 500 wavelengths with which to probe the scene. The one-dimensional scene was then recreated using equation (6a) and compared to the original scene. Figures 1 and 2 show sample runs from our computer model. In each figure, various original scenes are compared with synthetically generated images of the scenes.

Figure 1 shows a series of "picket fences" of different spatial period and the synthetic images which were produced by our model. The "picket fence" is considered to have a square wave reflectance function which varies between 0 and some constant maximum value. The period of the square wave pattern was varied from two wavelengths at the top to just less than one-half wavelength on the bottom. As can be seen from this series of figures, the fringe scanning/Fourier transform method has the potential of recreating the original scene down to a resolution of one-half wavelength, but it is incapable of measuring spatial frequencies finer than one-half wavelength in resolution. This is consistent with the minimum resolvable spatial wavelength given by equation (3) when θ is 90° .

Figure 2 shows a second series of scenes and images for a highly scattering object with two closely spaced slits in it. In these figures, the width of each slit and the width of the section between the two slits are equal. The width between the outside edges of the slits is varied from 1.5λ down to 0.5λ . We are again able to reconstruct an image of the original scene down to a resolution of about one-half wavelength.

4. CONCLUSION

We have developed the theory and a computer model for imaging one-dimensional scenes using a fringe-scanning/Fourier transform method of synthetic imaging. Both the theory and the computer model show that it is possible to image scenes with a resolution down to about $1/2$ wavelength using this method. This theory is easily extendable to two dimensions and we have plans for doing this. We also have plans to demonstrate this method in the lab using both acoustic and optical waves.

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FIGURE 1 One-dimensional objects and synthetically generated images for "picket fences" having different spatial frequencies. This sequence demonstrates a minimum resolution of $\lambda/2$.

FIGURE 2 Objects and synthetically generated images of two slits in a block. Distance across is the distance between the outside edges of the slits. The width of each slit and the width of the post between the slits are equal.

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