DYNAMIC MODELS OF FALLING FILM EVAPORATORS
FOR
OCEAN THERMAL ENERGY CONVERSION PLANTS

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DYNAMIC MODELS OF FALLING FILM EVAPORATORS

FOR

OTEC POWER PLANTS

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DYNAMIC MODELS OF FALLING FILM EVAPORATORS
FOR OTEC POWER PLANTS

SUMMARY

In [1], a methodology is developed for modeling the dynamic and steady-state behavior of Ocean Thermal Energy Conversion (OTEC) Power Plants. Because of the complexity of an OTEC Plant, dynamic modeling is an essential step concurrent with the design and prerequisite to the construction of a prototype plant.

In this report, the pertinent physical laws of heat transfer and mass balance are applied to develop lumped parameter, dynamic and steady-state models for shell-and-tube evaporators incorporating falling films. The dynamic evaporator model is specified by three ordinary differential equations and one algebraic equation with two time-delays. It is shown that, in the steady-state, this dynamic model reduces to the steady-state evaporator model developed in [2]. In Appendix A, it is demonstrated that the dynamic evaporator model is stable for perturbations around an equilibrium operating point. Provision is made for incorporating the data emerging from the CMU/OTEC Laboratory in the evaporator models formulated in this report. The effect of the recycle ratio

$$\sigma = \frac{\text{mass rate of liquid flow from bottom of tube}}{\text{mass rate of liquid flow into top of tube}}$$

on the heat transfer coefficient of the evaporating working fluid (ammonia) is noted.

In the current phase of the research, the dynamic evaporator model is being incorporated with the component models developed in [1] and [3] to assemble a simulation model of the power cycle of an OTEC plant. The power

cycle model will be simulated on the digital computer to insure that the component models adequately characterize experimentally observed data. The objective of this research is to develop simulation models of the OTEC power cycle as tools of power cycle and control system design and optimization.

DYNAMIC MODELS OF FALLING FILM EVAPORATORS

FOR

OTEC POWER PLANTS

1. INTRODUCTION

The Ocean Thermal Energy Conversion (OTEC) Power Plant conceptual design of Lavi and Zener [4] employs falling film evaporators and condensers (Figure 1). In this OTEC layout scheme, the warm water flows down through the evaporator and the cold water flows up through the condenser. The working fluid, in the form of a falling film, flows vertically downward along the outside of the tubes. The water flows inside the tubes, counter-current in the condenser and co-current in the evaporator.

In Heteyi and Neuman [1], both steady-state and transient models are developed for shell-and-tube condensers and pre-heaters. In Heteyi and Neuman [2], steady-state models are developed for smooth-tube evaporators. Upon analyzing this steady-state model of the evaporator operated in the turbulent flow regime, it is concluded that:

the local heat transfer coefficient $h_{tv}(z)$ on the working fluid side of the evaporator tube can be approximated by a constant average heat transfer coefficient for both a constant inlet working fluid flow rate $W_{F1}(0)$ and $\pm 10\%$ changes in $W_{F1}(0)$.

In this report, dynamic models are developed to characterize the transient behavior of falling film smooth-tube evaporators.* The operation of the evaporator can be illustrated by the CMU Argonne National Laboratory

* Dynamic models of falling film evaporators employing fluted surfaces [5] will be developed in a subsequent report.

Figure 1. Schematic of an OTEC Power Plant System

Accelerated Core Test Unit. The water makes one pass through the tubes. The water passes directly from the supply nozzle to the header, thence through the tubes, the other header and the discharge nozzle.

In the evaporator (Figure 2), liquid ammonia is supplied through two ports in the upper tube sheet to a plenum above the vapor space of the tube bundle. Pressure in the plenum causes the liquid to flow through applicators onto the fluted outer surfaces of the individual tubes. Evaporation occurs as the liquid flows down the tube in a thin layer under the action of gravity. At the bottom of the vapor space, the remaining liquid is halted by the lower tube sheet and passes out of the unit through the discharge nozzle leading to the recirculation sump. The generated vapor travels radially outward through the tube bundle to free-space adjacent to the shell, thence past an impingement baffle and out of the unit through the vapor discharge nozzle.

Thus, apart from engineering considerations of design and fabrication, the CMU falling film evaporator can be understood and simulated on the basis of a single tube model. Furthermore, in OTEC power plants, there can be hundreds of thousands and even millions of tubes. At the outset it is impractical to analyze such a configuration to obtain dynamic models. For these reasons, a single tube falling film evaporator is modeled initially. Later the analysis may be extended to include banks of tubes and/or evaporator modules. Even though the model is designed to characterize the transient behavior of falling film smooth-tube evaporators, provision has been made to incorporate both the CMU and Argonne core-test data as it becomes available. Dynamic models will then be developed to characterize the transient behavior of falling film fluted-tube evaporators. In the sequel, furthermore, all heat transfer coefficients and material properties are considered to be constant, independent of both time and position along the tube.

CARNEGIE - MELLON
FALLING FILM EVAPORATOR

FIGURE 2

The evaporator modeled consists of a single, smooth, vertical tube embedded in a saturated homogeneous vapor enclosed by a shell which is perfectly insulated on the outside. Water flows downward inside the tube conducting heat through the tube to the liquid working fluid film flowing co-currently along the outside of the tube. Some of the liquid film, which enters the evaporator at the evaporation temperature, is evaporated by the heat conduction process. The vapor thus produced enters the region between the shell and the tube, and thence flows upward and exits at the top of the evaporator.

The modeling effort commences with the application of the pertinent physical laws. Heat balances are written for the water, the tube, and the vapor, and a mass balance is written for the vapor. Simplifying assumptions are then made to extract a realistic, albeit tractable, dynamic model of the falling film evaporator. Later, the assumptions will be evaluated and relaxed, if necessary, to extract models which characterize the operational behavior of falling film evaporators.

The nonlinear evaporator model is linearized around an equilibrium operating point. The Bode method of control engineering is applied to the open-loop transfer function of the linearized evaporator model to show that the nonlinear evaporator model is stable for perturbations around an equilibrium operating point. The DC gains of the model are calculated. A DC gain is the gain of the transfer function (between an input and an output) at zero frequency. The DC gain is the ratio of the output change in the steady-state to a step-change in the input.

The symbols employed throughout this report are listed in Table 1 along with their dimensions of measurement.

Table 1
SYMBOL TABLE

Symbol	Definition	Dimensions
A_s	Cross-Sectional Area of Shell Material	ft^2
A_t	Cross-Sectional Area of Tube Material	ft^2
A_v	Cross-Sectional Area Between Shell and Tube	ft^2
A_w	Cross-Sectional Area of Water	ft^2
C_s	Heat Capacity of Shell Material	$\text{Btu/lb/}^\circ\text{F}$
C_t	Heat Capacity of Tube Material	$\text{Btu/lb/}^\circ\text{F}$
C_w	Heat Capacity of Water	$\text{Btu/lb/}^\circ\text{F}$
D_s	Inside Diameter of Shell	ft
D_t	Outside Diameter of Tube	ft
D_t'	Inside Diameter of Tube	ft
$G(s)$	Transfer Function	
H_L	Specific Enthalpy of Working Fluid Liquid	Btu/lb
H_v	Specific Enthalpy of Vapor	Btu/lb
h_{wt}	Heat Transfer Coefficient, Water to Tube	$\text{Btu/}^\circ\text{F/ft}^2/\text{sec}$
h_{tv}	Heat Transfer Coefficient, Tube to Vapor	$\text{Btu/}^\circ\text{F/ft}^2/\text{sec}$
L_e	Length of Evaporator Tube	ft
Th_s	Thickness of Shell	ft
P_v	Pressure of Saturated Vapor	lb/in^2
U_v	Specific Internal Energy of Vapor	Btu/lb
V_w	Velocity of Water	ft/sec
V_v	Specific Volume of Saturated Vapor	ft^3/lb
W_v	Mass Flow Rate of Working Fluid Vapor	lb/sec
$W_{F1}(z=0)$	Inlet Working Fluid Flow Rate	lb/sec
$W_{F1}(z=L_e)$	Outlet Working Fluid Flow Rate	lb/sec
W_w	Mass Flow Rate of Water	lb/sec

Table 1 (continued)

Symbol	Definition	Dimensions
ρ_s	Density of Shell Material	lb/ft ³
ρ_t	Density of Tube Material	lb/ft ³
ρ_v	Density of Working Fluid Vapor	lb/ft ³
ρ_w	Density of Water	lb/ft ³
λ	Latent Heat of Evaporation	Btu/lb
$T_t(z,t)$	Temperature of Tube	°F
$T_w(z,t)$	Temperature of Water	°F
$T_w(z=0)$	Temperature of Inlet Water at $z = 0$	°F
$T_w(z=L_e)$	Temperature of Outlet Water at $z = L_e$	°F
$T_v(t)$	Vapor Temperature	°F
$X_e(z,t)$	Local Rate of Evaporation	lb/ft/sec
$X_v(z,t)$	Local Rate of Exiting Vapor	lb/ft/sec
γ	Transit Time of Water through the Evaporator Tube	sec
γ_{ave}	Transit Time of the Water from Point along the Tube where the Local Water and Tube Temperatures Attain their Average Values	sec
s	Laplace Transform Variable	sec ⁻¹
t	Time	sec
z	Distance along Evaporator Tube	ft
ω	Radian Frequency	sec ⁻¹
σ	Recycle Ratio = $\frac{\text{liquid out}}{\text{liquid in}} = \frac{\text{liquid at bottom}}{\text{total liquid at top}}$	-
ζ	Length Constant $\zeta = \left(\frac{W_w C_w}{\pi} \right) \left[\frac{1}{D_t' h_{wt}} + \frac{1}{D_t h_{tv}} \right]$	ft
$\langle \square \rangle$	Length Average Value of $\square$	
$\square^*$	Steady-State Value of $\square$	

2. DYNAMIC EVAPORATOR MODELS

The development begins by modeling the dynamics of the single cylindrical shell-and-tube evaporator depicted in Figure 3. Dynamic models of the evaporator are formulated by writing heat and mass balances on an incremental section of the evaporator Δz long (Figure 4). The z -axis, which coincides with the centerline of the tube, starts at the top of the tube (at $z = 0$) and ends at the bottom (at $z = L_e$). In this modeling activity, the dynamics of the vapor flow are not considered. Therefore, it is possible to conceptualize the vapor as being produced from the falling liquid film, flowing into the region between the tube and the shell, and then exiting radially out of the evaporator through a porous shell.

The dynamic behavior of the single-tube evaporator is modeled by writing:

- (i) the heat balance for the water, the tube, and the vapor, and
- (ii) the mass balance for the vapor.

To derive an additional relationship between the evaporator variables, the outlet water temperature is approximated in terms of the inlet water temperature and the vapor temperature.

2.1 THE WATER HEAT BALANCE

The heat balance on the water states that

$$\left\{ \begin{array}{l} \text{the rate of heat accumulation} \\ \text{in the water control volume} \end{array} \right\} = - \left\{ \begin{array}{l} \text{the heat conducted from the} \\ \text{water to the tube wall} \end{array} \right\} + \left\{ \begin{array}{l} \text{the enthalpy of the} \\ \text{entering water} \end{array} \right\} - \left\{ \begin{array}{l} \text{the enthalpy of the} \\ \text{exiting water} \end{array} \right\}$$

Figure 3
Evaporator

Figure 4

Evaporator Section

That is,

$$\Delta z \left(\rho_w A_w C_w \right) \frac{\Delta T_w(z, t)}{\Delta t} \bigg|_{(z+\frac{1}{2}\Delta z)} = - \pi D_t' h_{wt} \left[T_w(z+\frac{1}{2}\Delta z, t) - T_t(z+\frac{1}{2}\Delta z, t) \right] \Delta z + W_w C_w T_w(z, t) - W_w C_w T_w(z+\Delta z, t) \quad (1)$$

Since the change in the water temperature between the entrance and exit of the control volume at time t is

$$\Delta T_w(z+\frac{1}{2}\Delta z, t) \bigg|_t = T_w(z+\Delta z, t) - T_w(z, t),$$

the water heat balance is rewritten, upon dividing by Δz ,

$$\rho_w A_w C_w \frac{\Delta T_w(z, t)}{\Delta t} \bigg|_{(z+\frac{1}{2}\Delta z)} = - \pi D_t' h_{wt} \left[T_w(z+\frac{1}{2}\Delta z, t) - T_t(z+\frac{1}{2}\Delta z, t) \right] - W_w C_w \frac{\Delta T_w(z+\frac{1}{2}\Delta z, t)}{\Delta z} \bigg|_t \quad (2)$$

2.2 THE TUBE HEAT BALANCE

In writing the heat balance for the tube, the tube is considered to be thin with a finite heat capacity. Since the tube is thin, negligible heat is conducted along the length of the tube. The heat balance on the tube states that

$$\left\{ \begin{array}{l} \text{the rate of heat accumulation} \\ \text{in the tube control volume} \end{array} \right\} = \left\{ \begin{array}{l} \text{the heat conducted from the} \\ \text{water to the tube} \end{array} \right\} - \left\{ \begin{array}{l} \text{the heat conducted from the} \\ \text{tube to the working fluid} \end{array} \right\}$$

That is,

$$\Delta z \left(\rho_t A_t C_t \right) \frac{\Delta T_t(z, t)}{\Delta t} \bigg|_{(z+\frac{1}{2}\Delta z)} = \pi D_t' h_{wt} \left[T_w(z+\frac{1}{2}\Delta z, t) - T_t(z+\frac{1}{2}\Delta z, t) \right] \Delta z - \pi D_t h_{tv} \left[T_t(z+\frac{1}{2}\Delta z, t) - T_v(t) \right] \Delta z$$

or

$$\left. \rho_t A_t C_t \frac{\Delta T_t(z, t)}{\Delta t} \right|_{(z+\frac{1}{2}\Delta z)} = \pi D_t' h_{wt} [T_w(z+\frac{1}{2}\Delta z, t) - T_t(z+\frac{1}{2}\Delta z, t)] - \pi D_t h_{tv} [T_t(z+\frac{1}{2}\Delta z, t) - T_v(t)] \quad (3)$$

2.3 THE VAPOR HEAT BALANCE

The heat balance on the vapor states that

$$\begin{aligned} \left\{ \begin{array}{l} \text{the heat conducted from} \\ \text{the tube to the vapor} \end{array} \right\} &= \left\{ \begin{array}{l} \text{the enthalpy of the vapor} \\ \text{exiting through the porous shell} \end{array} \right\} \\ &- \left\{ \begin{array}{l} \text{the enthalpy of the liquid} \\ \text{being evaporated} \end{array} \right\} \\ &+ \left\{ \begin{array}{l} \text{the time-rate of change of the} \\ \text{energy of the vapor contained} \\ \text{between the tube and the shell} \end{array} \right\} \\ &+ \left\{ \begin{array}{l} \text{the heat conducted to the shell} \end{array} \right\} \end{aligned}$$

The shell is assumed to be at the vapor temperature $T_v(t)$. Thus, the heat balance on the vapor is

$$\Delta z \pi D_t h_{tv} [T_t(z+\frac{1}{2}\Delta z, t) - T_v(t)] = X_v(z+\frac{1}{2}\Delta z, t) H_v \Delta z - X_e(z+\frac{1}{2}\Delta z, t) H_L \Delta z + \Delta z A_v \frac{\Delta(\rho_v U_v)}{\Delta t} + \Delta z \rho_s A_s C_s \frac{\Delta T_v(t)}{\Delta t} \quad (4)$$

2.4 THE VAPOR MASS BALANCE

The mass balance on the working fluid side of the evaporator states that

$$\left\{ \begin{array}{l} \text{the amount of} \\ \text{liquid being} \\ \text{vaporized} \end{array} \right\} = \left\{ \begin{array}{l} \text{the amount of vapor} \\ \text{leaving through the} \\ \text{porous shell} \end{array} \right\} + \left\{ \begin{array}{l} \text{the change in the amount} \\ \text{of vapor between the} \\ \text{tube and the shell} \end{array} \right\}$$

That is,

$$X_e(z+\frac{1}{2}\Delta z, t)\Delta z = X_v(z+\frac{1}{2}\Delta z, t)\Delta z + \Delta z A_v \frac{\Delta \rho_v}{\Delta t} \quad (5)$$

Substitution of the vapor mass balance (5) into the vapor heat balance (4) to eliminate the local rate of evaporation X_e (lb/ft/sec) yields (per unit length):

$$\begin{aligned} \left\{ \begin{array}{l} \text{the heat conducted from} \\ \text{the tube to the vapor} \end{array} \right\} &= \left\{ \begin{array}{l} \text{the gain in the enthalpy of the} \\ \text{liquid being evaporated that} \\ \text{exits through the porous shell} \end{array} \right\} \\ &- \left\{ \begin{array}{l} \text{the enthalpy of the liquid which} \\ \text{is evaporated and causes a change} \\ \text{in the density of the vapor} \end{array} \right\} \\ &+ \left\{ \begin{array}{l} \text{the time-rate of change of the} \\ \text{energy of the vapor contained} \\ \text{between the tube and the shell} \end{array} \right\} \\ &+ \left\{ \text{the heat conducted to the shell} \right\} \end{aligned}$$

That is,

$$\begin{aligned} \pi D_t h_{tv} [T_t(z+\frac{1}{2}\Delta z, t) - T_v(t)] &= (H_v - H_L) X_v(z+\frac{1}{2}\Delta z, t) - A_v H_L \frac{\Delta \rho_v}{\Delta t} \\ &+ A_v \frac{\Delta(\rho_v U_v)}{\Delta t} + \rho_s A_s C_s \frac{\Delta T_v(t)}{\Delta t} \quad (6) \end{aligned}$$

Equations (2), (3), and (6) constitute the basic equations describing the evaporator. Following Hettyei and Neuman [2], the heat transfer coefficient h_{tv} on the working fluid side of the evaporator tube, along with all heat transfer coefficients and material properties, are considered to be constant, independent of both time and position along the tube. Upon taking the limit as Δt and Δz tend to zero, and noting that the partial-derivatives can be replaced by total derivatives [1, p.13], equations (2), (3), and (6) become, respectively:

Water heat balance

$$\rho_w A_w C_w \frac{dT_w(z,t)}{dt} = - \pi D_t' h_{wt} [T_w(z,t) - T_t(z,t)] - W_w C_w \frac{dT_w(z,t)}{dz} \quad (7)$$

Tube heat balance

$$\rho_t A_t C_t \frac{dT_t(z,t)}{dt} = \pi D_t' h_{wt} [T_w(z,t) - T_t(z,t)] - \pi D_t' h_{tv} [T_t(z,t) - T_v(t)] \quad (8)$$

Vapor heat balance

$$\rho_s A_s C_s \frac{dT_v(t)}{dt} + A_v \frac{d(\rho_v U_v)}{dt} - A_v H_L \frac{d\rho_v}{dt} = \pi D_t' h_{tv} [T_t(z,t) - T_v(t)] - (H_v - H_L) X_v(z,t) \quad (9)$$

The system of partial-differential equations (7-9) characterizes the dynamic behavior of the evaporator.

For OTEC plant control, lumped parameter (ordinary differential equation) models are sought to describe the input-output characteristics of the system components. To extract such a model of the evaporator, the heat balances (7-9) are averaged over the length of the tube. Throughout this report, the average value of a quantity $\square$ is denoted by the bracket symbol $\langle \square \rangle$. To average these heat balances, equations (7-9) are integrated over the length of the tube, from $z = 0$ to $z = L_e$, and the results are divided by the tube length L_e . Thereby,

Water heat balance

$$\rho_w A_w C_w \langle \dot{T}_w \rangle = - \pi D_t' h_{wt} [\langle T_w \rangle - \langle T_t \rangle] + \frac{W_w C_w}{L_e} [T_w(0) - T_w(L_e)] \quad (10)$$

Tube heat balance

$$\rho_t A_t C_t \langle \dot{T}_t \rangle = \pi D_t' h_{wt} [\langle T_w \rangle - \langle T_t \rangle] - \pi D_t' h_{tv} [\langle T_t \rangle - T_v] \quad (11)$$

Vapor heat balance

$$\rho_s A_s C_s \dot{T}_v + A_v \frac{d(\rho_v U_v)}{dt} - A_v H_L \dot{\rho}_v = \pi D_t h_{tv} [\langle T_t \rangle - T_v] - (H_v - H_L) \frac{W_v}{L_e} \quad (12)$$

In (12), the difference between the specific enthalpies of the vapor (H_v) and the condensate (H_L) is the latent heat of evaporation (Btu/lb):

$$\lambda \triangleq H_v - H_L \quad (13)$$

3. THE SINGLE-TUBE EVAPORATOR MODEL

Equations (10-12) constitute the basic lumped-parameter evaporator model. At first glance, there appear to be six dependent variables: namely, $\langle T_w(t) \rangle$, $T_w(L_e)$, $\langle T_t(t) \rangle$, $T_v(t)$, $U_v(t)$, and $\rho_v(t)$, and only three equations. $\dot{\rho}_v$ and $\dot{U}_v$, however, can be expressed in terms of $\dot{T}_v$. The enthalpy (H_v) and the internal energy (U_v) of the vapor are related thermodynamically by

$$U_v = H_v - P_v V_v.$$

P_v is the pressure of the saturated vapor and V_v is the volume of one pound of saturated vapor. The enthalpy, the pressure, and the specific volume are determined by the vapor temperature. Therefore, U_v is specified by the temperature. A change in the vapor temperature produces a corresponding change in the enthalpy, pressure, and specific volume, and thus a change in the internal energy. The pressure in the volume of the evaporator is assumed constant. With this assumption, the vapor density is directly related to the vapor temperature through the (ideal gas law) equation of state. Increasing (decreasing) the vapor temperature increases (decreases) the vapor volume and decreases (increases) the vapor density.

What remains is to relate the outlet water temperature $T_w(L_e)$ to the inlet water temperature $T_w(0)$ and the vapor temperature $T_v(t)$. This task is accomplished by writing the steady-state water and tube heat balances. Upon rearranging (7-8), these steady-state heat balances are, respectively:

Water heat balance

$$\left(\frac{W_w C_w}{\pi D_t' h_{wt}} \right) \frac{dT_w(z)}{dz} + T_w(z) = T_t(z) \quad (14)$$

Tube heat balance

$$0 = D_t' h_{wt} [T_w(z) - T_t(z)] - D_t h_{tv} [T_t(z) - T_v] \quad (15)$$

Upon substituting (15) into (14) to eliminate the tube temperature $T_t(z)$, there results the first-order linear differential equation in $T_w(z)$:

$$\zeta \frac{dT_w(z)}{dz} + T_w(z) = T_v, \quad (16)$$

with the length constant (ft)

$$\zeta \triangleq \left(\frac{W_w C_w}{\pi} \right) \left[\frac{1}{D_t' h_{wt}} + \frac{1}{D_t h_{tv}} \right] \quad (17)$$

For the evaporator parameter values listed in Table A1, $\zeta \approx 28.07$ ft corresponds to $h_{tv} = 0.59$ Btu/ft²/°F/sec. The solution of (16) with the constant vapor temperature T_v is

$$T_w(z) = T_w(0)e^{-z/\zeta} + [1 - e^{-z/\zeta}]T_v. \quad (18)$$

At the bottom of the evaporator tube (at $z = L_e$), therefore,

$$T_w(L_e) = T_w(0)e^{-L_e/\zeta} + [1 - e^{-L_e/\zeta}]T_v. \quad (19)$$

Equation (19) relates the outlet water temperature $T_w(L_e)$ to the inlet water temperature $T_w(0)$ and the vapor temperature T_v in the steady-state. To approximate the dynamic behavior of $T_w(L_e, t)$, the transit time of the water is introduced as time delays on the inlet water and vapor temperatures.

Let

γ = the transit time of the water through the evaporator tube, from $z = 0$ to $z = L_e$. The inlet water time delay

$$\gamma \triangleq \left(\frac{L_e}{V_w} \right)$$

is the transit time of the water through the evaporator.

V_w is the average velocity of the water through the tube (ft/sec), and

γ_{ave} = the transit time of the water in the steady-state [2]
 from the point z_{ave} along the tube, where the local
 temperatures $T_w(z)$ and $T_t(z)$ attain their average
 values $\langle T_w \rangle$ and $\langle T_t \rangle$, to $z = L_e$.

That is, z_{ave} is defined implicitly by

$$T_w(z=z_{ave}) = \langle T_w \rangle \quad \text{for} \quad \gamma_{ave}.$$

For the outlet water temperature equation (19), the time delay

$$\gamma_{ave} = \left(1 - \frac{z_{ave}}{L_e} \right) \gamma.$$

For the exemplary operating conditions and material properties listed in Table A1,

$$L_e = 14.3 \text{ ft}$$

$$z_{ave} = 6.84 \text{ ft}$$

$$\frac{z_{ave}}{L_e} = 0.477$$

$$\gamma = 2.39 \text{ sec}$$

$$\gamma_{ave} = 0.523 \gamma = 1.25 \text{ sec}.$$

To incorporate the dynamic behavior of the outlet water temperature $T_w(L_e, t)$, therefore, equation (19) is replaced by

$$T_w(L_e, t) = T_w(0, t - \gamma) \exp(-L_e/\zeta) + [1 - \exp(-L_e/\zeta)] T_v(t - \gamma_{ave}). \quad (20)$$

Equation (20) specifies $T_w(L_e, t)$ as a time-delayed function of the inlet water temperature and the vapor temperature, delayed by their respective transit times. In the simulation studies, the transient response of the evaporator will be examined in detail, along with the behavior of the model in the vicinity of operating points. The approximations leading to (20) will be evaluated and the need for introducing variable time-delays and length parameters assessed.

The lumped parameter single tube evaporator model is summarized in Table 2 and illustrated in Figure 5. This model is specified by the three coupled nonlinear differential equations (10-12) and the algebraic equation (20). The system of differential equations (10-12) is nonlinear because of the water flow-rate and outlet water temperature product $[W_w T_w(L_e, t)]$ appearing in (10) and the two equation of state terms $[\dot{U}_v$ and $\dot{\rho}_v]$ in (12). Upon substituting (20) into (10), the dynamic behavior of the evaporator is characterized by three coupled nonlinear differential equations with the two time-delays γ and γ_{ave} . The three inputs or control variables are the inlet water temperature $T_w(0, t)$, the water flow-rate $W_w(t)$, and the vapor flow rate $W_v(t)$. The dependent variables are the water temperature $\langle T_w(t) \rangle$, tube temperature $\langle T_t(t) \rangle$, and vapor temperature $T_v(t)$. The two outputs are the outlet water temperature $T_w(L_e, t)$ and the vapor temperature $T_v(t)$.

Table 2

SINGLE TUBE EVAPORATOR MODEL

Water heat balance

$$\rho_w A_w C_w \dot{\langle T_w \rangle} = - \pi D_t' h_{wt} [\langle T_w \rangle - \langle T_t \rangle] + \frac{W_w C_w}{L_e} [T_w(0) - T_w(L_e)] \quad (10)$$

Tube heat balance

$$\rho_t A_t C_t \dot{\langle T_t \rangle} = \pi D_t' h_{wt} [\langle T_w \rangle - \langle T_t \rangle] - \pi D_t h_{tv} [\langle T_t \rangle - T_v] \quad (11)$$

Vapor heat balance

$$\begin{aligned} \rho_s A_s C_s \dot{T}_v + A_v \frac{d(\rho_v U_v)}{dt} \\ - A_v H_L \dot{\rho}_v = \pi D_t h_{tv} [\langle T_t \rangle - T_v] - (H_v - H_L) \frac{W_v}{L_e} \end{aligned} \quad (12)$$

Outlet water temperature

$$T_w(L_e, t) = T_w(0, t - \gamma) \exp(-L_e/\zeta) + [1 - \exp(-L_e/\zeta)] T_v(t - \gamma_{ave}) \quad (20)$$

Figure 5

SCHEMATIC REPRESENTATION OF EVAPORATOR MODEL

4. STEADY-STATE EVAPORATOR MODELS

The steady-state evaporator model is obtained by setting the time-derivatives in (10-12) equal to zero and replacing (20) by (19). In summarizing this model in Table 3, the asterisk (*) is employed to signify the steady-state value of the indicated quantity.

Table 3

STEADY-STATE MODEL OF SINGLE-TUBE EVAPORATOR

Water heat balance

$$-\pi D_t' h_{wt} [T_w^* - T_t^*] + \left(\frac{W_w^* C_w}{L_e} \right) [T_w^*(0) - T_w^*(L_e)] = 0 \quad (10)^*$$

Tube heat balance

$$D_t' h_{wt} [T_w^* - T_t^*] - D_t h_{tv} [T_t^* - T_v^*] = 0 \quad (11)^*$$

Vapor heat balance

$$\pi D_t h_{tv} [T_t^* - T_v^*] - \left(\frac{\lambda}{L_e} \right) W_v^* = 0 \quad (12)^*$$

Outlet water temperature

$$T_w^*(L_e) = T_w^*(0) e^{-L_e/\zeta^*} + \left[1 - e^{-L_e/\zeta^*} \right] T_v^* \quad (20)^*$$

Length constant

$$\zeta^* = \left(\frac{W_w^* C_w}{\pi} \right) \left[\frac{1}{D_t' h_{wt}} + \frac{1}{D_t h_{tv}} \right] \quad (17)$$

Upon substituting (20)* into (10)*, the steady-state evaporator model becomes

$$\begin{bmatrix}
 1 & -1 & \frac{W_w^* C_w}{\pi D_t h_{wt} L_e} \left[1 - e^{-L_e / \zeta^*} \right] \\
 D_t' h_{wt} & -(D_t' h_{wt} + D_t h_{tv}) & D_t h_{tv} \\
 0 & 1 & -1
 \end{bmatrix}
 \begin{bmatrix}
 T_w^* \\
 T_t^* \\
 T_v^*
 \end{bmatrix}
 =
 \begin{bmatrix}
 \frac{W_w^* C_w}{\pi D_t h_{wt} L_e} \left[1 - e^{-L_e / \zeta^*} \right] T_w^*(0) \\
 0 \\
 \left(\frac{\lambda}{\pi D_t h_{tv} L_e} \right) W_v^*
 \end{bmatrix}
 \quad (21)$$

The static model (21) specifies three algebraic equations in the three steady-state temperatures T_w^* , T_t^* , and T_v^* . The inputs are the inlet water temperature $T_w^*(0)$, the water flow rate W_w^* , and the vapor flow rate W_v^* . It is observed that W_w^* and the length constant ζ^* , which depends upon W_w^* , appear in the coefficient matrix of (21). In the steady-state, W_w^* and hence ζ^* are constant. Therefore, once an operating point and a set of evaporator parameters are selected, (21) can be solved for the steady-state evaporator temperatures. This model (or an improvement thereon) will be utilized in developing an optimization algorithm for OTEC plant design.

In Heteyi and Neuman [2], steady-state models of the evaporator are developed by writing heat and mass balances on a section of the evaporator tube Δz long (Figure 4). The z -axis starts at the top of the tube (at $z=0$) and ends at the bottom (at $z=L_e$). The saturated vapor is maintained at the constant temperature T_v . The tube is thin with negligible heat conduction in the z direction. At this point, it is deemed appropriate to summarize this development and relate the steady-state evaporator models of [2] to

the steady-state model of the single-tube evaporator presented in Table 3.

The steady-state heat balance on the water states that [2]

$$\frac{dT_w(z)}{dz} = - \left(\frac{\pi D_t' h_{wt}}{W_w^* C_w} \right) [T_w(z) - T_t(z)]. \quad (22)$$

The steady-state heat balance on the tube states that

$$D_t' h_{wt} [T_w(z) - T_t(z)] = D_t h_{tv} [T_t(z) - T_v^*]. \quad (23)$$

The working fluid enters the Δz length of tube with flow rate $W_{F1}(z)$ and leaves at this flow rate minus the amount of fluid evaporated. The steady-state heat balance on the working fluid then states

$$h_{tv} [T_t(z) - T_v^*] = - \left(\frac{\lambda_v}{\pi D_t} \right) \frac{dW_{F1}(z)}{dz} \quad (24)$$

Combining (22-24) to eliminate $T_t(z)$ and T_v yields

$$\frac{dT_w(z)}{dz} = \left(\frac{\lambda_v}{W_w^* C_w} \right) \frac{dW_{F1}(z)}{dz}, \quad (25)$$

and upon integrating (25),

$$T_w(z) = T_w^*(0) + \left(\frac{\lambda_v}{W_w^* C_w} \right) [W_{F1}(z) - W_{F1}^*(0)]. \quad (26)$$

Substitution of (23) into (22) to eliminate $T_t(z)$ yields the first-order differential equation in $T_w(z)$:

$$\zeta \frac{dT_w(z)}{dz} + T_w(z) = T_v^*. \quad (27)$$

And substitution of (26) into (27) yields the first-order differential equation in $W_{F1}(z)$:

$$\zeta \frac{dW_{F1}(z)}{dz} + W_{F1}(z) = W_{F1}^*(0) - \left(\frac{W_w^* C_w}{\lambda_v} \right) [T_w^*(0) - T_v^*] \quad (28)$$

In the steady-state [2], therefore, the evaporator is characterized by the algebraic equations (23) and (26) and the differential equation (27) or (28).

Averaging[†] the water heat balance (22) over the length of the evaporator tube (L_e) yields (10)*. Similarly, averaging the tube heat balance (23) over the length of the evaporator tube yields (11)*. Averaging the working fluid heat balance (24) over the length of the evaporator tube and noting that the amount of working fluid evaporated

$$W_v^* = W_{F1}^*(0) - W_{F1}(L_e)$$

yields the vapor heat balance (12)*. Finally, evaluating the solution of the first-order differential equation (27) at the bottom of the evaporator tube provides the outlet water temperature (20)*.

[†] To average the water, tube, and working fluid heat balances, equations (22-24) are integrated over the length of the evaporator tube, from $z=0$ to $z=L_e$, and the results are divided by the tube length L_e .

5. STABILITY ANALYSIS OF THE SINGLE-TUBE EVAPORATOR MODEL

The stability of the single-tube model is analyzed prior to designing feedback control schemes for the evaporator. The details of the analysis are contained in Appendix A.

In this analysis, the model of Table 2 is perturbed around a stable operating point. After subtracting the steady-state equations (Table 3), the open-loop transfer function $G(s)$ of the resulting linearized evaporator model is derived. The stability of this linearized model is determined by applying the Bode method of classical control engineering. It is demonstrated that the magnitude of the open-loop transfer function is less than unity for all angles of the open-loop transfer function less than -180° . Consequently, the evaporator model is stable for perturbations around this equilibrium operating point.

What remains is to evaluate the dynamic behavior of the single-tube evaporator model. A salient aspect of current research involves developing simulation models and control strategies for OTEC power plants. The evaporator model (Tables 2 and 3) constitute integral components of the OTEC power cycle model.

6. REFERENCES

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APPENDIX A

STABILITY ANALYSIS OF THE SINGLE-TUBE EVAPORATOR MODEL

A.1 LINEARIZATION OF THE EVAPORATOR MODEL

To investigate the stability of the single-tube evaporator model (Table 2) in the vicinity of a stable operating point, each of the time-varying model variables is replaced by its equilibrium steady-state value (denoted by an asterisk) plus a time-varying deviation from that equilibrium value. That is,

$$\begin{array}{lcl} \text{Inputs} & \left\{ \begin{array}{l} T_w(0,t) \\ W_w(t) \\ W_v(t) \end{array} \right. & = \begin{array}{l} T_w^*(z=0) \\ W_w^* \\ W_v^* \end{array} + \begin{array}{l} \Delta T_w(0,t) \\ \Delta W_w(t) \\ \Delta W_v(t) \end{array} \end{array} \quad \begin{array}{l} \text{(A1)} \\ \text{(A2)} \\ \text{(A3)} \end{array}$$

$$\begin{array}{lcl} \text{State Variables} & \left\{ \begin{array}{l} \langle T_w(t) \rangle \\ \langle T_t(t) \rangle \\ T_v(t) \end{array} \right. & = \begin{array}{l} T_w^* \\ T_t^* \\ T_v^* \end{array} + \begin{array}{l} \Delta T_w(t) \\ \Delta T_t(t) \\ \Delta T_v(t) \end{array} \end{array} \quad \begin{array}{l} \text{(A4)} \\ \text{(A5)} \\ \text{(A6)} \end{array}$$

$$\begin{array}{lcl} \text{Parameters} & \left\{ \begin{array}{l} U_v(t) \\ \rho_v(t) \end{array} \right. & = \begin{array}{l} U_v^* \\ \rho_v^* \end{array} + \begin{array}{l} \Delta U_v(t) \\ \Delta \rho_v(t) \end{array} \end{array} \quad \begin{array}{l} \text{(A7)} \\ \text{(A8)} \end{array}$$

$$\text{Output} \quad T_w(L_e, t) = T_w^*(z=L_e) + \Delta T_w(L_e, t) \quad \text{(A9)}$$

Substitution of equations (A1) through (A9) into equations (10-12) and (20) of Table 2 yields, respectively:

$$\begin{aligned} \rho_w A_w C_w \frac{d}{dt} (T_w^* + \Delta T_w) &= \\ &= -\pi D_t' h_{wt} [(T_w^* + \Delta T_w) - (T_t^* + \Delta T_t)] + \frac{C_w}{L_e} (W_w^* + \Delta W_w) \\ &\times \{ [T_w^*(0) + \Delta T_w(0)] - [T_w^*(L_e) + \Delta T_w(L_e)] \} \end{aligned} \quad \text{(A10)}$$

$$\begin{aligned} & \left(\rho_t A_t C_t \right) \frac{d}{dt} (T_t^* + \Delta T_t) = \\ & = \pi D_t' h_{tv} (T_v^* + \Delta T_v) + \pi D_t' h_{wt} (T_w^* + \Delta T_w) - (\pi D_t' h_{wt} + \pi D_t' h_{tv}) (T_t^* + \Delta T_t) \end{aligned} \quad (A11)$$

$$\begin{aligned} & - \frac{\lambda}{L_e} (W_v^* + \Delta W_v) + A_v H_L \frac{d}{dt} (\rho_v^* + \Delta \rho_v) = \\ & = \pi D_t' h_{tv} (T_v^* + \Delta T_v) - \pi D_t' h_{tv} (T_t^* + \Delta T_t) + A_v \frac{d}{dt} [(U_v^* + \Delta U_v) (\rho_v^* + \Delta \rho_v)] \\ & + \rho_s A_s C_s \frac{d}{dt} (T_v^* + \Delta T_v) \end{aligned} \quad (A12)$$

$$\begin{aligned} [T_w^*(L_e) + \Delta T_w(L_e, t)] & = \left[T_w^*(0) + \Delta T_w(0, t-\gamma) \right] e^{-L_e/\zeta (W_w^* + \Delta W_w)} \\ & + \left[T_v^* + \Delta T_v(t-\gamma_{ave}) \right] \left[1 - e^{-L_e/\zeta (W_w^* + \Delta W_w)} \right], \end{aligned} \quad (A13)$$

where the length constant

$$\begin{aligned} \zeta (W_w^* + \Delta W_w) & = \left\{ \frac{(W_w^* + \Delta W_w) C_w}{\pi} \right\} \left[\frac{1}{D_t' h_{wt}} + \frac{1}{D_t' h_{tv}} \right] \\ & = \underbrace{\left(\frac{W_w^* C_w}{\pi} \right) \left[\frac{1}{D_t' h_{wt}} + \frac{1}{D_t' h_{tv}} \right]}_{\zeta^*} + \underbrace{\left(\frac{\Delta W_w C_w}{\pi D_t} \right) \left[\frac{1}{D_t' h_{wt}} + \frac{1}{D_t' h_{tv}} \right]}_{\Delta \zeta} \end{aligned}$$

In the steady-state, equations (10-12) and (20) become, respectively

(Table 3):

$$- \pi D_t' h_{wt} (T_w^* - T_t^*) + \left(\frac{W_w^* C_w}{L_e} \right) [T_w^*(0) - T_w^*(L_e)] = 0 \quad (A14)$$

$$D_t' h_{wt} [T_w^* - T_t^*] - D_t' h_{tv} (T_t^* - T_v^*) = 0 \quad (A15)$$

$$\pi D_t' h_{tv} (T_t^* - T_v^*) - \left(\frac{\lambda}{L_e} \right) W_v^* = 0 \quad (A16)$$

$$T_w^*(L_e) = T_w^*(0) e^{-L_e/\zeta^*} + T_v^* (1 - e^{-L_e/\zeta^*}), \quad (A17)$$

where $\zeta^* = \left(\frac{W_w^* C_w}{\pi} \right) \left[\frac{1}{D_t' h_{wt}} + \frac{1}{D_t' h_{tv}} \right]$

Upon subtracting (A14 - A17) from (A10 - A13) and noting that the time-derivatives of the equilibrium values are zero:

$$\begin{aligned} \left(\rho_w A_w C_w \right) \frac{d(\Delta T_w)}{dt} = & - \pi D'_t h_{wt} [\Delta T_w - \Delta T_t] + \frac{W_w^* C_w}{L_e} [\Delta T_w(0) - \Delta T_w(L_e)] \\ & + \frac{C_w}{L_e} \left\{ [T_w^*(0) + \Delta T_w(0)] + [T_w^*(L_e) - \Delta T_w(L_e)] \right\} \Delta W_w \end{aligned} \quad (A18)$$

$$\left(\rho_t A_t C_t \right) \frac{d(\Delta T_t)}{dt} = - \pi D_t h_{tv} [\Delta T_t - \Delta T_v] + \pi D'_t h_{wt} [\Delta T_w - \Delta T_t] \quad (A19)$$

$$\begin{aligned} - \frac{\lambda}{L_e} \Delta W_v + A_v H_L \frac{d(\Delta \rho_v)}{dt} = & - \pi D_t h_{tv} [\Delta T_t - \Delta T_v] \\ & + A_v \frac{d(U_v^* \Delta \rho_v + \Delta U_v \rho_v^* + \Delta U_v \Delta \rho_v)}{dt} \\ & + \left(\rho_s A_s C_s \right) \frac{d(\Delta T_v)}{dt} \end{aligned} \quad (A20)$$

$$\begin{aligned} \Delta T_w(L_e, t) = & \Delta T_w(0, t-\gamma) e^{-L_e/\zeta(W_w^* + \Delta W_w)} + \Delta T_v(t-\gamma_{ave}) \left[1 - e^{-L_e/\zeta(W_w^* + \Delta W_w)} \right] \\ & + T_w^*(0, t-\gamma) \left[- e^{-L_e/\zeta^*} + e^{-L_e/\zeta(W_w^* + \Delta W_w)} \right] \\ & + T_v^*(t-\gamma_{ave}) \left[- e^{-L_e/\zeta(W_w^* + \Delta W_w)} + e^{-L_e/\zeta^*} \right] \end{aligned} \quad (A21)$$

Let

$$T_w^*(0) \left[e^{-L_e/\zeta(W_w^* + \Delta W_w)} - e^{-L_e/\zeta^*} \right] \triangleq \alpha_{T_w W_w} \Delta W_w(t-\gamma)$$

and

$$T_v^* \left[e^{-L_e/\zeta(W_w^* + \Delta W_w)} - e^{-L_e/\zeta^*} \right] \triangleq \alpha_{T_v W_w} \Delta W_w(t-\gamma_{ave})$$

A.2 THE LINEARIZED EVAPORATOR MODEL

Upon neglecting the second-order terms $\Delta T_w(0)\Delta W_w$, $\Delta T_w(L_e)\Delta W_w$, and $\Delta U_v\Delta\rho_v$ in (A18) and (A20), the linearized evaporator model is:

Water heat balance

$$\begin{aligned} \left(\rho_w A_w C_w\right) \frac{d(\Delta T_w)}{dt} = & -\pi D_t h_{wt} [\Delta T_w - \Delta T_t] + \frac{C_w}{L_e} W_w^* [\Delta T_w(0) - \Delta T_w(L_e)] \\ & + \frac{C_w}{L_e} [T_w^*(0) - T_w^*(L_e)] \Delta W_w \end{aligned} \quad (A22)$$

Tube heat balance

$$\left(\rho_t A_t C_t\right) \frac{d(\Delta T_t)}{dt} = -\pi D_t h_{tv} [\Delta T_t - \Delta T_v] + \pi D_t' [\Delta T_w - \Delta T_t] \quad (A23)$$

Vapor heat balance

$$\begin{aligned} \frac{-\lambda}{L_e} \Delta W_v + A_v H_L \frac{d(\Delta\rho_v)}{dt} = & -\pi D_t h_{tv} [\Delta T_t - \Delta T_v] + A_v \frac{d[U_v^* \Delta\rho_v + \rho_v^* \Delta U_v]}{dt} \\ & + \left(\rho_s A_s C_s\right) \frac{d(\Delta T_v)}{dt} \end{aligned} \quad (A24)$$

Outlet water temperature

$$\begin{aligned} \Delta T_w(L_e, t) = & \Delta T_w(0, t-\gamma) e^{-L_e/\zeta^*} + \Delta T_v(t-\gamma_{ave}) (1 - e^{-L_e/\zeta^*}) \\ & + \alpha_{T_w W_w} \Delta W_w(t-\gamma) - \alpha_{T_v W_w} \Delta W_w(t-\gamma_{ave}) \end{aligned} \quad (A25)$$

The linearized evaporator model is characterized by the three first-order linear differential equations (A22 - A24) and the algebraic equation (A25) with two time-delays.

Having linearized the evaporator model, the relationships between $\dot{\rho}_v$ and $\dot{T}_v$, and between $\dot{U}_v$ and $\dot{T}_v$ are derived. The pressure, density, specific enthalpy, and specific volume of the working fluid (ammonia) are obtained at various temperatures from thermodynamic tables [A1].

The data are plotted, and the local slopes are calculated to approximate the derivatives. In the vicinity of 68°F , for example,

$$R \triangleq \frac{\Delta \dot{\rho}_v}{\Delta \dot{T}_v} = 0.00719 \text{ lb/ft}^3/\text{°F} \quad (\text{A26})$$

and

$$S = \frac{\Delta \dot{U}_v}{\Delta \dot{T}_v} = 0.144 \text{ Btu/lb/°F} , \quad (\text{A27})$$

where

$$U = H - \frac{144}{778} PV = H - 0.185 PV .$$

Thereby, (A24) becomes, upon collecting terms,

$$\left\{ A_v [U_v^* R + \rho_v^* S - H_L R] + \rho_s A_s C_s \right\} \frac{d(\Delta T_v)}{dt} =$$

$$- \pi D_t h_{tv} [\Delta T_v - \Delta T_t] - \frac{\lambda}{L_e} \Delta W_v . \quad (\text{A28})$$

Upon substituting (A25) into (A22), the linearized evaporator model becomes, in vector form:

$$\begin{aligned}
 \frac{d}{dt} \begin{bmatrix} \Delta T_w(t) \\ \Delta T_t(t) \\ \Delta T_v(t) \end{bmatrix} &= \underbrace{\begin{bmatrix} -f_{11} & f_{11} & 0 \\ f_{21} & -(f_{21}+f_{23}) & f_{23} \\ 0 & f_{33} & -f_{33} \end{bmatrix}}_{F_1} \underbrace{\begin{bmatrix} \Delta T_w(t) \\ \Delta T_t(t) \\ \Delta T_v(t) \end{bmatrix}}_{x(t)} + \underbrace{\begin{bmatrix} 0 & 0 & -h_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{F_2} \underbrace{\begin{bmatrix} \Delta T_w(t-\gamma_{ave}) \\ \Delta T_t(t-\gamma_{ave}) \\ \Delta T_v(t-\gamma_{ave}) \end{bmatrix}}_{x(t-\gamma_{ave})} \\
 &+ \underbrace{\begin{bmatrix} h_2 & h_3 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -h_4 \end{bmatrix}}_{H_1} \underbrace{\begin{bmatrix} \Delta T_w(0,t) \\ \Delta W_w(t) \\ \Delta W_v(t) \end{bmatrix}}_{U(t)} + \underbrace{\begin{bmatrix} 0 & h_5 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{H_2} \underbrace{\begin{bmatrix} \Delta T_w(0,t-\gamma_{ave}) \\ \Delta W_w(t-\gamma_{ave}) \\ \Delta W_v(t-\gamma_{ave}) \end{bmatrix}}_{U(t-\gamma_{ave})} \\
 &+ \underbrace{\begin{bmatrix} -h_6 & -h_7 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{H_3} \underbrace{\begin{bmatrix} \Delta T_w(0,t-\gamma) \\ \Delta W_w(t-\gamma) \\ \Delta W_v(t-\gamma) \end{bmatrix}}_{U(t-\gamma)}, \tag{A29}
 \end{aligned}$$

or

$$\dot{x}(t) = F_1 x(t) + F_2 x(t-\gamma_{ave}) + H_1 U(t) + H_2 U(t-\gamma_{ave}) + H_3 U(t-\gamma)$$

The three state-variables of the linearized model (A29) are:

$\Delta T_w(t)$ = Deviation of the water temperature from the equilibrium value T_w^*

$\Delta T_t(t)$ = Deviation of the tube temperature from the equilibrium value T_t^*

$\Delta T_v(t)$ = Deviation of the vapor temperature from the equilibrium value T_v^* .

The three inputs (which are the elements of the input vector U) are:

$$\Delta T_w(0,t) = \text{Deviation of the inlet water temperature from the equilibrium value } T_w^*(0)$$

$$\Delta W_w(t) = \text{Deviation of the water flow rate from the equilibrium value } W_w^*$$

$$\Delta W_v(t) = \text{Deviation of the vapor flow rate from the equilibrium value } W_v^* .$$

The two outputs are:

$$\Delta T_w(z=L_e,t) = \text{Deviation of the outlet water temperature from the equilibrium value } T_w^*(z=L_e)$$

$$\Delta T_v(t) = \text{Deviation of the vapor temperature from the equilibrium value } T_v^* .$$

The eleven parameters in (A28) are:

$$f_{11} = \left(\frac{\pi D_t' h_{wt}}{\rho_w A_w C_w} \right)$$

$$f_{21} = \left(\frac{\pi D_t' h_{wt}}{\rho_t A_t C_t} \right)$$

$$f_{23} = \left(\frac{\pi D_t h_{tv}}{\rho_t A_t C_t} \right)$$

$$f_{33} = \pi D_t h_{tv} \eta$$

$$h_1 = \frac{W_w^*}{\rho_w A_w L_e} \left(1 - e^{-L_e/\zeta^*} \right)$$

$$h_2 = \left(\frac{W_w^*}{\rho_w A_w L_e} \right)$$

$$h_3 = \frac{T_w^*(0) - T_w^*(L_e)}{\rho_w A_w L_e}$$

$$h_4 = \left(\frac{\lambda}{L_e} \right) \eta$$

$$h_5 = \left(\frac{W_w^*}{\rho_w A_w L_e} \right) \alpha_{T_v W_w}$$

$$h_6 = \left(\frac{W_w^*}{\rho_w A_w L_e} \right) e^{-L_e/\zeta^*}$$

$$h_7 = \left(\frac{W_w^*}{\rho_w A_w L_e} \right) \alpha_{T_w W_w}$$

where:

$$\eta \triangleq \left[A_v (U_v^* R + \rho_v^* S - H_L R) + \rho_s A_s C_s \right]^{-1}.$$

A.3 THE OPEN-LOOP TRANSFER FUNCTION OF THE EVAPORATOR MODEL

By applying the Laplace transform to (A29) and neglecting initial conditions,

$$X(s) = \left\{ \left[sI - F_1 - F_2 e^{-\gamma_{ave}s} \right]^{-1} \left[H_1 + H_2 e^{-\gamma_{ave}s} + H_3 e^{-\gamma s} \right] \right\} U(s) \quad (A30)$$

Consequently, the open-loop transfer function matrix of the evaporator model is

$$G(s) = \left[sI - F_1 - F_2 e^{-\gamma_{ave}s} \right]^{-1} \left[H_1 + H_2 e^{-\gamma_{ave}s} + H_3 e^{-\gamma s} \right] \quad (A31)$$

The transfer function matrix $G(s)$ relates the Laplace transform of the evaporator model inputs

$$U(s) = \begin{bmatrix} \Delta T_w(z=0, s) \\ \Delta W_w(s) \\ \Delta W_v(s) \end{bmatrix}$$

to the Laplace transform of the evaporator model state variables

$$X(s) = \begin{bmatrix} \Delta T_w(s) \\ \Delta T_t(s) \\ \Delta T_v(s) \end{bmatrix}$$

That is,

$$\begin{bmatrix} \Delta T_w(s) \\ \Delta T_t(s) \\ \Delta T_v(s) \end{bmatrix} = G(s) \begin{bmatrix} \Delta T_w(z=0,s) \\ \Delta W_w(s) \\ \Delta W_v(s) \end{bmatrix} \quad (A32)$$

$$\underbrace{\begin{bmatrix} \Delta T_w(s) \\ \Delta T_t(s) \\ \Delta T_v(s) \end{bmatrix}}_{X(s)} = G(s) \underbrace{\begin{bmatrix} \Delta T_w(z=0,s) \\ \Delta W_w(s) \\ \Delta W_v(s) \end{bmatrix}}_{U(s)}$$

Since there are three inputs and three outputs, $G(s)$ has three rows and three columns. Symbolically, the evaporator model may be depicted as illustrated in Figure A1.

Figure A1

Block Diagram of the Linearized Evaporator Model

In turn, from (A25), the output water temperature is

$$\Delta T_w(L_e, s) = \Delta T_w(0, s) e^{-\gamma s} e^{-L_e/\zeta^*} + \Delta T_v(s) e^{-\gamma_{ave} s} \left[1 - e^{-L_e/\zeta^*} \right]$$

$$+ \alpha_{TW} \Delta W_w(s) e^{-\gamma s} - \alpha_{TV} \Delta W_v(s) e^{-\gamma_{ave} s}$$

Substitution of (A29) into (A31) yields the explicit open-loop transfer function matrix:

$$\begin{aligned}
 G(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \\ g_{31}(s) & g_{32}(s) \end{bmatrix} &= \begin{bmatrix} \left[h_2 - h_6 e^{-\gamma s} \right] \left\{ \frac{s^2 + (f_{21} + f_{23} + f_{33})s + f_{21}f_{33}}{\Delta(s)} \right\} & \left[h_3 + h_5 e^{-\gamma_{ave}s} - h_7 e^{-\gamma s} \right] \left\{ \frac{s^2 + (f_{21} + f_{23} + f_{33})s + f_{21}f_{33}}{\Delta(s)} \right\} \\ \left[h_2 - h_6 e^{-\gamma s} \right] \left\{ \frac{f_{21}s + f_{21}f_{33}}{\Delta(s)} \right\} & \left[h_3 + h_5 e^{-\gamma_{ave}s} - h_7 e^{-\gamma s} \right] \left\{ \frac{f_{21}s + f_{21}f_{33}}{\Delta(s)} \right\} \\ \left[h_2 - h_6 e^{-\gamma s} \right] \left\{ \frac{f_{21}f_{33}}{\Delta(s)} \right\} & \left[h_3 + h_5 e^{-\gamma_{ave}s} - h_7 e^{-\gamma s} \right] \left\{ \frac{f_{21}f_{33}}{\Delta(s)} \right\} \end{bmatrix} \\
 g_{13}(s) &= -h_4 \left\{ \frac{-h_1 e^{-\gamma_{ave}s} s - (f_{21} + f_{23})h_1 e^{-\gamma_{ave}s} + f_{11}f_{23}}{\Delta(s)} \right\} \\
 g_{23}(s) &= -h_4 \left\{ \frac{f_{23}s - h_1 f_{21} e^{-\gamma_{ave}s} + f_{11}f_{23}}{\Delta(s)} \right\} \\
 g_{33}(s) &= -h_4 \left\{ \frac{s^2 + (f_{11} + f_{21} + f_{23})s + f_{11}f_{23}}{\Delta(s)} \right\}
 \end{aligned} \tag{A33}$$

In (A33),

$$\begin{aligned}\Delta(s) &= \text{determinant} \left[sI - F_1 - F_2 e^{-\gamma_{ave}s} \right] \\ &= s^3 + (f_{11} + f_{21} + f_{23} + f_{33})s^2 + (f_{11}f_{23} + f_{11}f_{33} + f_{21}f_{33})s + h_1 f_{21} f_{33} e^{-\gamma_{ave}s} \quad (A34)\end{aligned}$$

is the characteristic polynomial of the linearized evaporator model (A29).

The transfer functions appearing in (A33) are:

$$g_{11}(s) = \left. \frac{\Delta T_w(s)}{\Delta T_w(0,s)} \right|_{\Delta W_w(s) = \Delta W_v(s) = 0}$$

$$g_{12}(s) = \left. \frac{\Delta T_w(s)}{\Delta W_w(s)} \right|_{\Delta T_w(0,s) = \Delta W_v(s) = 0}$$

$$g_{13}(s) = \left. \frac{\Delta T_w(s)}{\Delta W_v(s)} \right|_{\Delta T_w(0,s) = \Delta W_w(s) = 0}$$

$$g_{21}(s) = \left. \frac{\Delta T_t(s)}{\Delta T_w(0,s)} \right|_{\Delta W_w(s) = \Delta W_v(s) = 0}$$

$$g_{22}(s) = \left. \frac{\Delta T_t(s)}{\Delta W_w(s)} \right|_{\Delta T_w(0,s) = \Delta W_v(s) = 0}$$

$$g_{23}(s) = \left. \frac{\Delta T_t(s)}{\Delta W_v(s)} \right|_{\Delta T_w(0,s) = \Delta W_w(s) = 0}$$

$$g_{31}(s) = \frac{\Delta T_v(s)}{\Delta T_w(0,s)} \bigg|_{\Delta W_w(s)=\Delta W_v(s)=0}$$

$$g_{32}(s) = \frac{\Delta T_v(s)}{\Delta W_w(s)} \bigg|_{\Delta T_w(0,s)=\Delta W_v(s)=0}$$

$$g_{33}(s) = \frac{\Delta T_v(s)}{\Delta W_v(s)} \bigg|_{\Delta T_w(0,s)=\Delta W_w(s)=0}$$

A.4 STABILITY ANALYSIS OF THE OPEN-LOOP EVAPORATOR MODEL

To analyze the stability of $G(s)$, the characteristic equation (A34) of the evaporator model; that is, $\Delta(s) = 0$, is rewritten as follows:

$$1 + M(s) = 0 ,$$

$$\text{where } M(s) = \frac{h_1 f_{21} f_{33} e^{-\gamma_{ave} s}}{s[s^2 + (f_{11} + f_{21} + f_{23} + f_{33})s + (f_{11}f_{23} + f_{11}f_{33} + f_{21}f_{33})]} . \quad (A35)$$

$M(s)$ is the open-loop transfer function of a unity negative feedback system with characteristic equation $\Delta(s) = 0$.

The Bode method of classical control engineering is applied to demonstrate the stability of the open-loop model for the set of evaporator parameters listed in Section A.5. In Figure A2, it is shown that the magnitude of $M(s) \Big|_{s=j\omega}$ is less than unity for all arguments $\angle M(j\omega)$ less than -180° , for all frequencies ω , $0 \leq \omega < +\infty$.

A.5 A NUMERICAL EXAMPLE

An equilibrium operating point is assumed to obtain the numerical values of the model parameters. The working fluid is ammonia and the temperature of the saturated vapor leaving the evaporator is 68°F . The tube and shell are made of aluminum. The cylindrical shell is 1" thick with an inside diameter such that the cross-sectional area of the vapor A_v equals the cross-sectional area of the water A_w . The temperature of the incoming water is 77°F and has a velocity of 6 ft/sec based upon a 0.87 inch ID tube

In OTEC power plant evaporators, only partial evaporation of the falling film is obtained on each tube. The inlet working fluid flow rate is several times higher than the evaporation rate in order to take

advantage of the higher heat transfer coefficients available in the turbulent flow regime. This results in the recirculation of the liquid not evaporated after passing through the evaporator. The recycle ratio*

$$\sigma \triangleq \frac{\text{liquid out}}{\text{liquid in}}$$

Corresponding to each value of σ is a constant heat transfer coefficient h_{tv} . Actually, the heat transfer coefficient h_{tv} is not constant over the length of the evaporator tube. As shown in Heteyi and Neuman [2], however, the heat transfer coefficient can be considered a constant with less than 1% error in prediction of the water temperature at each point along the tube. Accordingly, in the sequel, several heat transfer coefficients h_{tv} are considered corresponding to recycle ratios of $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$ and $\frac{5}{6}$.

The parameter values are summarized in Table A1 and the model parameters are listed in Table A2 for a range of vapor-tube heat transfer coefficients h_{tv} .

$$* \quad \sigma = \frac{\text{liquid at bottom}}{\text{total liquid at top}} = \left[\frac{W_{F1}(L_e)}{W_{F1}(0)} \right]$$

TABLE A1
EVAPORATOR PARAMETER VALUES

Water Velocity $V_w = 6$ ft/sec (on a 0.87" inside diameter tube)

Densities

Water	$\rho_w = 62.24$	lb/ft ³
Tube and shell	$\rho_t = \rho_s = 168.48$	lb/ft ³

Heat Capacities

Water	$C_w = 1.0$	Btu/lb/°F
Tube and shell	$C_t = C_s = 0.2151$	Btu/lb/°F

Heat Transfer Coefficients

Tube-water	$h_{wt} = 0.34$	Btu/ft ² /°F/sec
Vapor-tube	$h_{tv} = 0.50, 0.55,$	
	$0.59, 0.62,$	
	0.65	

Specific Enthalpy

Vapor	$H_v = 628.8$	Btu/lb
Liquid	$H_L = 118.3$	Btu/lb

Heat of Vaporization of Ammonia

λ	$= 510.5$	Btu/lb
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Steady State Values

Vapor density	$\rho_v^* = 0.4179$	lb/ft ³
Vapor internal energy	$U_v^* = 573.745$	Btu/lb

Dimensions

Tube diameter (OD)	$D_t = 1$ inch	$= 0.0833$ ft
Tube diameter (ID)	$D_t' = 0.87$ inch	$= 0.0725$ ft
Shell diameter (ID)	$D_s = \sqrt{(D_t)^2 + (D_t')^2}$	$= 0.1105$ ft
Tube length	$L_e = 15.2, 14.7, 14.3, 14.1, 13.9$	ft

Shell Thickness

$Th_s = 1$	inch
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Water Flow Rate

$$W_w = \rho_w \pi \frac{(D_t')^2}{4} v_w = 1.542 \text{ lb/sec}$$

Areas

Water cross-sectional area	$A_w = \frac{\pi}{4} D_t'^2$	$= 0.004128$ ft ²
Tube cross-sectional area	$A_t = \frac{\pi}{4} [(D_t)^2 - (D_t')^2]$	$= 0.00133$ ft ²
Shell cross-sectional area	$A_s = \frac{\pi}{4} [(D_s + Th_s)^2 - D_s^2]$	$= 0.0199$ ft ²
Vapor cross-sectional area	$A_v = \frac{\pi}{4} [D_s^2 - D_t^2]$	$= 0.004128$ ft ²

TABLE A2
MODEL PARAMETER VALUES

h_{tv}	0.50	0.55	0.59	0.62	0.65	Btu/ft ² /°F/sec
σ	1/2	2/3	3/4	4/5	5/6	-
L_e	15.2	14.7	14.3	14.1	13.9	ft
f_{11}	0.3211	0.3211	0.3211	0.3211	0.3211	sec ⁻¹
f_{21}	1.6088	1.6088	1.6088	1.6088	1.6088	sec ⁻¹
f_{23}	2.7158	2.9874	3.2046	3.3676	3.5305	sec ⁻¹
f_{33}	0.1817	0.1999	0.2145	0.2254	0.2363	sec ⁻¹
h_1	0.1579	0.1634	0.1674	0.1701	0.1727	sec ⁻¹
h_2	0.3947	0.4085	0.4185	0.4254	0.4318	sec ⁻¹
h_3	0.9217	0.9540	0.9772	0.9932	1.0083	°F/lb
h_4	46.62	48.26	49.43	50.24	51.00	°F/lb
h_5	5.3648	5.5525	5.6875	5.7810	5.8686	°F/lb
h_6	0.2368	0.2451	0.2511	0.2552	0.2591	sec ⁻¹
h_7	6.0748	6.2874	6.4403	6.5461	6.6453	°F/lb
$\gamma = \left(\frac{L_e}{V_w} \right)$	2.5336	2.4479	2.3898	2.3512	2.3161	sec
$\gamma_{ave} = \left(1 - \frac{z_{ave}}{L_e} \right) \gamma$	1.3327	1.2827	1.2499	1.2273	1.2090	sec
$\frac{\gamma_{ave}}{\gamma} = \left(1 - \frac{z_{ave}}{L_e} \right)$	0.526	0.524	0.523	0.522	0.522	-
ζ	29.76	28.75	28.07	27.62	27.20	ft
* $\begin{cases} \alpha_{T_w W_w} \\ \alpha_{T_v W_w} \end{cases}$	15.39	15.39	15.39	15.39	15.39	°F/(lb/sec)
	13.59	13.59	13.59	13.59	13.59	°F/(lb/sec)

* Corresponding to $\Delta W_w = \pm 10\% W_w$

For $h_{tv} = 0.59 \text{ Btu/ft}^2/\text{°F/sec}$, the magnitude and phase of $M(s)$ are displayed in Figure A2. It is observed that $|M(j\omega)|$ is always less than unity when the phase of $M(j\omega)$ is less than -180° for all frequencies ω , $0 \leq \omega < +\infty$. Furthermore, it is noted that both the magnitude and phase of $M(s)$ decrease monotonically as ω increases from zero to $+\infty$. Consequently, the single-tube evaporator model is stable for perturbations about an equilibrium operating point.

The static (or DC) gains of the linearized evaporator model are listed in Table A3. A DC gain is the gain of the transfer function (between an input and an output) at zero frequency ($s=0$) . The DC gain, therefore, is the ratio of the output change in the steady-state to a step-change in the input.

TABLE A3
STATIC GAINS OF THE EVAPORATOR
 $h_{tv} = 0.59 \text{ Btu/ft}^2/\text{°F/sec}$

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Output		Input		
	Water Temperature	Tube Temperature	Vapor Temperature	
Inlet Water Temperature	$g_{11}(0) = \frac{\Delta T_w}{\Delta T_w(z=0)}$ $= \left(\frac{h_2 - h_6}{h_1} \right) = 1.0$	$g_{21}(0) = \frac{\Delta T_t}{\Delta T_w(z=0)}$ $= \left(\frac{h_2 - h_6}{h_1} \right) = 1.0$	$g_{31}(0) = \frac{\Delta T_v}{\Delta T_w(z=0)}$ $= \left(\frac{h_2 - h_6}{h_1} \right) = 1.0$	
Water Flow Rate	$g_{12}(0) = \frac{\Delta T_w}{\Delta W_w}$ $= \left(\frac{h_3 + h_5 - h_7}{h_1} \right) = 1.341 \text{ } ^\circ\text{F}/(\text{lb}/\text{sec})$	$g_{22}(0) = \frac{\Delta T_t}{\Delta W_w}$ $= \left(\frac{h_3 + h_5 - h_7}{h_1} \right) = 1.341 \text{ } ^\circ\text{F}/(\text{lb}/\text{sec})$	$g_{32}(0) = \frac{\Delta T_v}{\Delta W_w}$ $= \left(\frac{h_3 + h_5 - h_7}{h_1} \right) = 1.341 \text{ } ^\circ\text{F}/(\text{lb}/\text{sec})$	
Vapor Flow Rate	$g_{13}(0) = \frac{\Delta T_w}{\Delta W_v}$ $= - \left(\frac{f_{11}f_{23} - (f_{21} + f_{23})h_1}{f_{21}f_{33}} \right) \left(\frac{h_4}{h_1} \right) = -191.0 \text{ } ^\circ\text{F}/(\text{lb}/\text{sec})$	$g_{23}(0) = \frac{\Delta T_t}{\Delta W_v}$ $= - \left(\frac{f_{11}f_{23} - h_1f_{21}}{f_{21}f_{33}} \right) \left(\frac{h_4}{h_1} \right) = -650.1 \text{ } ^\circ\text{F}/(\text{lb}/\text{sec})$	$g_{33}(0) = \frac{\Delta T_v}{\Delta W_v}$ $= - \left(\frac{f_{11}f_{23}}{f_{21}f_{33}} \right) \left(\frac{h_4}{h_1} \right) = -880.6 \text{ } ^\circ\text{F}/(\text{lb}/\text{sec})$	

REFERENCE FOR

APPENDIX A

- [A1] Baumeister, T. (editor) and Marks, L.S., Standard Handbook for Mechanical Engineers (seventh edition), McGraw-Hill Book Company, New York, 1967.