**Analysis of Magnetic Measurement Data by
Least Squares Fit to Series Expansion Solution
of 3-D Laplace Equation***

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ANALYSIS OF MAGNETIC MEASUREMENT DATA BY LEAST SQUARES FIT TO SERIES EXPANSION SOLUTION OF 3-D LAPLACE EQUATION

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SUMMARY

We have analysed simulated magnetic measurements data for the SXLS bending magnet in a plane perpendicular to the reference axis at the magnet midpoint ($\theta=90^\circ$) by fitting the data to an expansion solution of the 3-dimensional Laplace Equation in curvilinear coordinates as proposed by Brown and Servranckx. The method of Least Squares is used to evaluate the expansion coefficients and their uncertainties, and compared to results from an FFT fit of 128 simulated data points on a 12-mm radius circle about the reference axis. We find that the FFT method gives smaller coefficient uncertainties (RMS deviations) than the Least Squares method when the data are within similar areas. The Least Squares method compares more favorably when a larger number of data points (1891) are used within a rectangular area of 30-mm vertical by 60-mm horizontal --perhaps the largest area within the 35-mm x 75-mm vacuum chamber for which data could be obtained. For a grid with 0.5-mm spacing within the 30 X 60 mm area (7381 points) the Least Squares fit gives much smaller uncertainties than the FFT. We are therefore in the favorable position of having two methods which can determine the multipole coefficients to much better accuracy than the tolerances specified to General Dynamics. The FFT method may be preferable since it requires only one Hall probe rather than the four envisioned for the Least Squares grid data. However Least Squares can attain better accuracy with fewer probe movements (less data-taking time). The time factor in acquiring the data (60 days for 200 planes per magnet for the FFT data as opposed to 22 days per magnet for the 1891 point 30-mm X 60-mm Least Squares grid) will likely be the determining factor in choice of method. We should further explore Least Squares analysis of a Fourier expansion of data on a circle or arc of a circle since that method gives coefficient uncertainties without need for multiple independent sets of data as needed by the FFT method.

1. FORMULATION OF BROWN-SERVANCKX EXPANSION AND LEAST SQUARES ANALYSIS.

The Least Squares method is well-documented in standard texts and the present program NLLS (nonlinear least squares) has been described[1]. The present version is a straight-forward extension to the case of two independent variables and is appended. The program requires the user to specify the function to which the data are to be fitted in a subroutine CALC. The present version pertains to two-dimensional magnetic field data represented by power-series expansions for the normal and skew multipole components which satisfy the 3-dimensional Laplace Equation as summarized by Brown and Servranckx[2], hereafter referred to as BS, and discussed in a previous publication[3]. The expansion of the major component, B_z , for the normal solution which is symmetric about the median plane $z=0$ is

$$B_z = \sum_{m=0}^{NR/2} \sum_{n=0}^{NR-2m} C_{2m+1,n} X^n Z^{2m} \quad (1)$$

where the infinite series given by BS is truncated to NR-th order and factorials have been subsumed into the C-coefficients. A series for the other two components Bx and Bs in terms of the same coefficients and their first derivatives is given by BS. It is instructive to write out the series explicitly, say to 6-th order, as

$$\begin{aligned} B_z = & \overset{(NRH)}{C_{10}} + C_{11}x + C_{12}x^2 + C_{13}x^3 + C_{14}x^4 + C_{15}x^5 + C_{16}x^6 \\ & + z^2(C_{30} + C_{31}x + C_{32}x^2 + C_{33}x^3 + C_{34}x^4) \\ & + z^4(C_{50} + C_{51}x + C_{52}x^2) + z^6 C_{70}. \end{aligned} \quad (2)$$

C_{10} and C_{11} are the dipole and quadrupole strengths, respectively. The sextupole component is $C_{12}x^2 + C_{30}z^2$ which differs from the more familiar form of this term from the two-dimensional Laplace Equation normal solution in polar coordinates

$$\phi_{2D} = \sum_{n=1}^{\infty} a_n r^n \sin(n\theta) \quad \text{with } x=r\cos\theta \text{ and } z=r\sin\theta \quad (3)$$

which, with $\vec{B} = \nabla\phi$ leads to the expression for the vertical component

$$B_z = \sum_{n=1}^{\infty} n a_n r^{n-1} \cos(n-1)\theta. \quad (4)$$

dipole: $B_z = a_1$

quadrupole: $B_z = 2a_2x$

sextupole: $B_z = 3a_3(x^2 - z^2)$

octupole: $B_z = 4a_4(x^3 - 3xz^2)$

decapole: $B_z = 5a_5(x^4 - 6x^2z^2 + z^4)$

In the BS formulation $C_{30} \rightarrow C_{12}$. Indeed the coefficients $C_{2m+1,n}$ for $m > 0$ are related to the $m=0$ coefficients by a recursion relation, viz. $A_{30} = -A''_{10} - A_{12} - hA_{11}$ with $A_{2m+1,n} = n!(2m)!C_{2m+1,n}$, $h(s) = 1/\rho(s)$ where $\rho(s)$ is the radius of curvature and s is the longitudinal coordinate along the curvilinear axis, and $A''_{10} = \partial^2 A_{10} / \partial s^2$. In the two-dimensional limit where $h=0$ and $A''_{10}=0$ it is readily seen that the BS solution goes to the familiar 2-D multipoles. Since Eq.(1) and Eq.(4) differ, it is clear that Eq.(4) is not the correct solution for the B_z component in the plane normal to the SXLS reference axis, which we take as the axis of the curvilinear coordinate system. However, Reusch has worked out a procedure for modifying the coefficients given by a Fourier analysis of B_z on a circle in the transverse plane where B_z is a periodic function and therefore well-represented by the Fourier expansion so that the curvature-corrected coefficients correspond to the median-plane C_{1n} coefficients of Eq.(1). Reusch has also worked out an iteration for obtaining the field components out of the median plane by the requirement that $\nabla \times \vec{B} = 0$ be satisfied. This procedure should be equivalent to use of the recursion relations for the $m > 0$ coefficients.

We initially felt that use of Eq.(1) was the most convenient method for obtaining the entire 3-D field without recourse to recursion relations which involve higher order longitudinal derivatives. This assumption may still be correct if one envisions further use of the mag measurements data beyond the acceptance test, such as tracking in the as-built

machine. For the acceptance test alone, only the median plane coefficients were specified to General Dynamics (GD) and it is clear from inspection of Eq. (1) and Eq. (4) that both reduce to a simple Taylor series for $z=0$.

The 3-dimensional Laplace Equation also has a solution for which B_z is antisymmetric about the median plane, the skew multipole solution. It is

$$B_z^{(SKEW)} = \sum_{m=0}^{NS/2} \sum_{n=0}^{NS-2m} C_{2m+2,n}(s) x^n z^{2m+1} \quad (5)$$

A recursion relation has also been obtained for this solution[3]. NS is the maximum order of the skew multipole for the truncated series of Eq. (5), which of course is actually an infinite series. Again, the factorials are subsumed into the C -coefficients. We write out the above series to 6-th order as

$$B_z^{(SKEW)} = z (C_{20} + C_{21}x + C_{22}x^2 + C_{23}x^3 + C_{24}x^4 + C_{25}x^5) + z^3 (C_{40} + C_{41}x + C_{42}x^2 + C_{43}x^3) + z^5 (C_{60} + C_{61}x). \quad (6)$$

Note that the skew solution has no dipole term, i.e. there are no transverse components in the median plane by definition of that plane. This agrees with the 2-dimensional skew multipoles which are obtained from the $\cos(n\theta)$ terms of the 2-D Laplace Eq. in polar coordinates, viz.

$$\phi_{2D}^{(SKEW)} = \sum_{n=1}^{\infty} b_n r^n \cos(n\theta) \quad (7)$$

which leads, again from $\vec{B} = \nabla\phi$ to the 2-D skew multipoles

dipole:	$B_z=0$	
quadrupole:	$B_z=-2b_2 z$	
sextupole:	$B_z=-6b_3 xz$	(8)
octupole:	$B_z=-4b_4 (3x^2 z - z^3)$	
decapole:	$B_z=-20b_5 (x^3 z - xz^3)$	
dodecapole:	$B_z=-6b_6 (5x^4 z - 10x^2 z^3 + z^5)$	

Note that when compared to the 3-D solution of Eq. (6) the form of the skew solutions also differs, in this case for octupole and higher order since, in Eq. (6), $C_{22} \neq -3C_{40}$, etc. We have included the skew series in the Least Squares fit subroutine CALC. Presumably the skew terms can also be included in the FFT fits. The important point regarding the skew solution for our purposes is that it gives $B_z=0$ in the median plane and therefore could never be determined from median plane data alone. Indeed we will need 2-D data in the plane normal to the reference axis to locate the median plane, which need not be the horizontal plane when skew terms are present. Since there are also skew resonances in the vicinity of the operating point of the SXLS, it is of importance that we know the strength of these terms in the as-built machine. We have not previously included skew terms in tracking calculations. Further, we will want to know the equilibrium orbit, the tune for small amplitude oscillations about that orbit as well as momentum-displaced closed orbits and tunes to determine the machine chromaticity and other optical

properties. In general, the closed orbits will be deformed due to magnet and placement imperfections and will not even be in the median plane. We should perform these calculations before the machine is turned on to have a high degree of confidence that the as-built machine will work. It is therefore essential that the complete 3-dimensional properties of the magnet be known from the mag measurements. We should also mention that, in addition to the multipole specs at the magnet center given to GD, an additional tolerance on the coil length was specified, $\pm 1/32$ -inch at either end, to assure that the orbit circumference is within the ± 500 KHz spec of the RF cavity tunability. We will then have to integrate B_z data longitudinally along the reference axis to infer the coil length spec from the integrated dipole field and we will thus have to take data at many azimuthal points, say 100 locations as was done for the Phase I magnet per quadrant.

The Least Squares fitting procedure has been well-checked by two methods. First, we computed "exact" fields from a double precision version of TOSCA supplied by Reusch for the ideal conductor configuration of a design designated COILNOV29, and compared the Least Squares fitted coefficients to those obtained by a Differential Algebra (DA) method also performed by Reusch. The DA method is essentially an evaluation of the Biot-Savart Law integral and its derivatives over the conductor configuration. The second method was to generate fields given by Eq.(1) with known coefficients and then fitting that "data" with the Least Squares code NLLS. We also included skew fields from Eq.(5) in the data. In all cases the Least Squares analysis gives very reasonable results for the known coefficients. The agreement of a 14-th order fit of the TOSCA-generated data (shown in Table VI) to the DA results of the first six coefficients is quite good. A third check on the least squares program NLLS used here was to compare it to the linear least squares program LFIT published in "Numerical Recipes" [4]. Both programs gave the same results on numerous test cases.

The simulated data for the least squares fits of the BS expansion as well as the FFT fits were obtained by first generating values of the vertical component B_z using a special double precision version of TOSCA prepared by Reusch in the appended program MULTIPOLDP. We then added random errors to these numbers using the appended Gaussian random number generator RANGAUSS. The standard deviation of the Gaussian was somewhat arbitrarily taken as $\sigma = 0.5$ Gauss. The vendor specifications[5] for the Hall probe to be used in the measurements states an accuracy of one part in 60,000 and, if this number is interpreted as an RMS error, we should use the somewhat larger value of $\sigma = 0.645$ G in the $B_z = 3.87$ T SXLS central field. On the other hand, if the errors are uniformly distributed in the interval ± 0.645 G, then we are justified in using a somewhat smaller σ for the Gaussian error approximation since it will produce errors $> 1\sigma$ in about 1/3 of the cases. We are also aware that other errors will be present in the data such as knowledge of the absolute probe position and, perhaps more important, the probe temperature for which a correction will have to be applied using a probe temperature measurement of accuracy $\pm 0.1^\circ\text{C}$ and the known temperature coefficient of the probe of $\sim 4\text{G}/^\circ\text{C}$ [5]. Moreover the probe will be in an inhomogeneous field of gradient $\sim 10\text{G/mm}$ and will read some average value over its $\sim 1\text{mm}$ diameter active area[5]. The correct error distribution for the data will have to be established experimentally by multiple independent measurements at a given point in a more uniform field than the SXLS magnet and compared to an NMR field measurement. There should be little error in reproducing the probe position since the shaft encoders for the probe holder movement will give a position accuracy of ~ 1 -micron. For our purposes, it is important to use Gaussian errors since the parameter uncertainty calculation of Least Squares assumes that the error bars are RMS values.

To select the best fit, we typically perform fits as a function of order NR. A goodness-of-fit quantity VAR (variance) is evaluated; it is essentially the average value of χ^2 for the N data point ensemble, i.e. the mean square deviation of a data point from the fitted function, defined as

$$\text{VAR} = \chi^2 / \text{NDF} = \sum_{j=1}^N \left(\frac{y_j - f(x_j)}{\sigma_j} \right)^2 / (N - \text{NP}) \quad (9)$$

where NP is the number of free parameters and NDF is the number of degrees of freedom for the fit. Including NP essentially prevents one from using as many parameters as data points and thereby getting an exact fit--it penalizes the user for using more parameters unless they are justified by the data. We find that VAR decreases rapidly with increasing order NR until it reaches a plateau where the minimum is often ill-defined. We choose the lowest order fit on the plateau as the best fit because there is no apparent benefit in going to higher order than necessary. Indeed, the method adds additional parameters generally at the expense of reduced accuracy of the lower order parameters.

2. RESULTS

A. One-Dimensional Least Squares Fits.

Since the tolerance specs given to GD are for the first five median-plane multipole coefficients (C_{10} to C_{14} in the BS series of Eq.(1)) at the magnet midpoint ($\theta=90^\circ$, see Fig.1) it is reasonable to ask whether we can obtain these numbers from fits to data only in the median-plane ($z=0$), i.e. Bz vs. x data where x is the curvilinear coordinate in the median plane normal to the reference axis as in Fig.1. A version of NLLS (nonlinear least squares) adapted to the linear case with only one independent variable was used. In Table I we show first the fits to simulated data from $x=-30\text{mm}$ to $+30\text{mm}$ in $DX=0.5\text{mm}$ steps (121 points). Note that the present probe holder is not capable of covering that range, but we assume that one could be designed for this interval within the $x=\pm 37.5\text{mm}$ vacuum chamber. The resulting octupole strength C_{13} overlaps the GD tolerance but is not well within the tolerance range. The decapole, C_{14} coefficient is within the tolerance but the error bars (1σ) are somewhat larger than we would prefer, i.e. the 2σ (95% confidence interval) is at the outer limit of the $1-\sigma$ GD tolerance. On the other hand, if we push the outer limits to $x=\pm 32\text{-mm}$ and go to $DX=0.25\text{-mm}$ steps (257 points) we are well within the tolerance range for all coefficients and could obtain better than a $3-\sigma$ (99%) confidence even for the decapole strength. Thus, it is possible only with median plane data to verify the tolerances specified in the acceptance test. We assume here that there is no inherent problem in taking data in steps smaller than the sensitive area (1-mm dia.) of the probe since the measured field will be an average over the sensitive area and correspond to the field at some point near the center of the probe. A third comparison was made over the interval $x=\pm 18\text{-mm}$ with $DX=0.25\text{-mm}$ (145 points) which we believe can be attained with the present probe holder. Here the decapole coefficient C_{14} overlaps the GD tolerance but the uncertainty is larger than we would prefer. Smaller step sizes, say 0.1-mm, would improve the fit but we want to keep the step size large compared to the uncertainty in absolute position of the probe center which, for normal machining tolerances, would be $\pm 0.05\text{-mm}$.

B. BS LEAST SQUARES COEFFICIENTS vs FFT RESULTS WITH PRESENT PROBE HOLDER

The present probe holder, shown in Fig.3, holds four Hall probes located symmetrically at $\pm 6\text{-mm}$ about the holder center. The quoted nominal motion capability of the probe [6] is $\pm 10\text{-mm}$ horizontally; thus, the data can be obtained within $\pm 16\text{-mm}$ horizontally, and vertically we

can attain ± 12 -mm nominal. In Table II we show a comparison in which 128 points on the circumference of a circle of radius $R=12$ -mm (0.6-mm spacing between points) were fitted by the FFT program and a grid with a comparable number of field points (221) in a rectangular grid of spacing $DX=2$ -mm from $x=-16$ mm to $+16$ mm and $z=-12$ mm to $+12$ mm were fitted by Least Squares. It should be pointed out that the FFT fits require multiple independent sets of 128-point data. In this case we quote results from 10 sets which means 1280 data points with 1280 movements of the probe holder in order to obtain the mean values and estimates of RMS uncertainties of the coefficients. For the grid data we require only about 56 movements of the probe holder since 4 data points are measured per probe position. The FFT results are remarkably good, with coefficient uncertainties well within the GD tolerances. We are extremely impressed with the sensitivity of the FFT since, for the decapole component of nominal strength $C_{14}=44.3$ T/m, the field contribution at $x=12$ -mm is only $B_z = C_{14} x^4 = 9.2 \cdot 10^{-7}$ T. This is about 2% of the $\sigma = 5 \cdot 10^{-5}$ T RMS noise! The RMS uncertainties in the least squares fit to the BS solution are much larger and, for the octupole and decapole, larger than the GD tolerance range. The Least Squares fit is unacceptable.

In Table III we have extended the range of the grid to $x=\pm 21$ -mm horizontal and $z=\pm 14$ -mm vertical, perhaps the largest range which could be attained with the present probe holder if the engineering clearances from the probe holder to the chamber wall are reduced to zero. Also we reduced the data spacing to $DX=1$ -mm so that the probe sensitive area is just touching for adjacent points. This increases the number of data points to $N=1247$ which is comparable to the 1280 pt. FFT, although for this grid only about 312 probe holder movements are required. The Least Squares fit has improved over the results of Table II but are still not as good as the FFT fit. The octupole and decapole uncertainties are now well within the GD tolerances, however, so the Least Squares method could adequately resolve the coefficients in this case with fewer probe movements (less data-taking time) than the FFT.

In Table IV we show a Least Squares fit with the present probe holder with data points in the nominal rectangle $x=\pm 16$ mm, $z=\pm 12$ mm but with the point spacing of $DX=0.5$ mm. (This seems fair since the 128 pt. FFT data spacing is 0.6-mm.) The resulting $N=3185$ point fit again gives results worse than the 128-point FFT fit--in particular, larger error bars for the octupole and decapole coefficients and, for the decapole, larger than the GD tolerances. Again, the Least Squares fit is unacceptable.

We could consider again the extreme grid range $x=\pm 21$ mm, $z=\pm 14$ mm but reduce the point spacing to $DX=0.5$ -mm to give $N=4845$ points. In Table V we summarize the Least Squares fits to the BS expansion for this case. The coefficient uncertainties have decreased relative to the Table III results but the decapole error is larger than for the 128-pt FFT fit. Thus the FFT still seems better. It is conceivable that we could go to a very small point spacing, say $DX=0.25$ mm over this ± 21 mm $\times$ ± 14 mm grid (19097 points, about 4775 probe holder movements). But, by the same token, we could also increase the number of FFT points to 256 on the 12-mm radius circle (0.3-mm spacing) for 2560 probe movements and achieve a better FFT result also.

C. BS Least Squares Coefficients with new 4-probe Holder, 15mm Vertical, 30mm Horizontal Grid.

If a new 4-probe holder is built to cover the range $x=\pm 30$ -mm, $z=\pm 15$ -mm, we show in Table VI the results of a Least Squares fit for $\Delta X=1$ -mm spacing (1891 data points, about 473 probe movements). The octupole and decapole coefficient uncertainties here are distinctly better than the 10 set FFT fits, and with only 1/3 of the probe movements. And of course

we obtain the full set of 16 coefficients with the Least Squares run and thus the field representation within the 30mm X 60mm rectangle directly. This would presumably weigh the balance in favor of Least Squares, but Reusch points out that, rather than data taken on a full circle, he can analyse data on arcs of circles at larger than 12mm radius which would presumably give better results. So we still cannot say with certainty that a least squares fit of grid data will do better than an FFT for the same number of probe movements.

In Table VII we go one step further, an $x=\pm 30\text{-mm}$ X $z=\pm 15\text{-mm}$ grid with $\Delta X = 0.5\text{-mm}$ spacing (7381 points, about 1846 probe movements). Here, the results for the coefficient uncertainties are much better than the 10-set FFT fit. But we also have about 50% more probe movements. At 20-sec per probe movement we would require about 10 hours to measure the complete grid, which would not be excessive for the $\theta=90\text{-deg}$ plane alone but would be too long for the ~ 200 planes needed to map the entire magnet (86 days). We would thus have to accept the grid of Table VI (2.63 hours/plane, 22 days for the entire magnet). The 10-set FFT data, on the other hand, at 1280 probe movements per plane, 201 planes per magnet, would require ~ 60 days which would also be excessive. In terms of data-taking time, the Least Squares 1891 point, $X=\pm 30\text{mm}$, $z=\pm 15\text{mm}$ data seems the best choice.

3. CONCLUSIONS

Both the 128 point FFT fits for 10-sets of data on an $R=12\text{mm}$ radius circle, and the Least Squares fit of the BS expansion to data on a rectangular grid of $\pm 30\text{mm}$ X $\pm 15\text{mm}$ with either $\Delta X=1\text{-mm}$ (1891 point) or $\Delta X=.5\text{-mm}$ (7381 point) are capable of sufficient accuracy to determine whether the GD magnet meets the multipole strength tolerances up to decapole. The Least Squares method may be better in that it gives better accuracy for fewer probe holder movements if a new probe holder is built.

4. ACKNOWLEDGEMENTS

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TABLE I

8/24/91

COMPARISON OF 1-DIMENSIONAL LEAST SQUARES FITS TO
BROWN-SERVANCKX EXPANSION IN MEDIAN PLANE. $\theta = 90^\circ$ COIL NO V29
COIL NO V29 $\theta = 90^\circ$

Midplane COEFF	$X = -30 \text{ mm} \rightarrow +30$ mm, $\Delta X = .5 \text{ mm}$ 121 PTS	$X = -18 \text{ mm} \rightarrow +18 \text{ mm}$ $\Delta X = .25 \text{ mm}$ 145 PTS (4th order fit)	$X = -32 \text{ mm} \rightarrow +32 \text{ mm}$ $\Delta X = .25$ 257 PTS	GD TOLERANCES
$C_{10}(\text{dip})$ (Tesla)	$3.870013 \pm 9.6 \cdot 10^{-6}$	$3.870020 \pm 7.0 \cdot 10^{-6}$	$3.870021 \pm 6.2 \cdot 10^{-6}$	$3.8700 \pm .009$
$C_{11}(\text{quad})$ (T/m)	$-1.0491 \pm 1.7 \cdot 10^{-3}$	$-1.0476 \pm 9.0 \cdot 10^{-4}$	$-1.0480 \pm 6.7 \cdot 10^{-4}$	$-1.0457 \pm .14$
$C_{12}(\text{sext})$ (T/m ²)	$-4.5360 \pm .126$	$-4.4016 \pm .132$	$-4.4663 \pm .072$	-4.453 ± 1.7
$C_{13}(\text{oct})$ (T/m ³)	23.5 ± 13.0	7.38 ± 4.16	5.11 ± 2.53	1.767 ± 17.2
$C_{14}(\text{deca})$ (T/m ⁴)	121.5 ± 380.5	-629.5 ± 454.5	-68.0 ± 191.0	49.9 ± 833
$C_{15}(\text{dodec})$ (T/m ⁵)	-28794 ± 29604		21990 ± 2154	
$C_{16}(\text{14 pole})$ (T/m ⁶)	$-1.24 \cdot 10^6 \pm 3.0 \cdot 10^5$		$-1.12 \cdot 10^6 \pm 1.33 \cdot 10^5$	

TABLE II

8/2/91

NOV29 COIL

COMPARISON OF MEDIAN PLANE MULTIPOLE
COEFFICIENTS FROM CURVATURE-CORRECTED FFT
FIT AND LEAST SQUARES FIT TO POWER-SERIES
TO TOSCA-GENERATED DATA WITH RANDOM ERRORS
FROM GAUSSIAN WITH STANDARD DEVIATION = 0.5 GAUSS
SIMULATED DATA ARE IN PLANE AT MAGNET CENTER ($\theta = 90^\circ$)

	FOURIER		GAUSS		GENERAL DYNAMICS TOLERANCES AT $\theta = 90^\circ$	DA "GOLD"
	FFT (128 POINTS) R=1.2 CM CIRCLE *		LEAST SQUARES (221 POINTS) $\pm 12 \times$ ± 16 MM GRID $\Delta X = 2$ MM			
C_{10} (dipole) (Tesla)	$3.870024 \pm 3.0 \cdot 10^{-6}$		$3.870020 \pm 1.3 \cdot 10^{-5}$		$3.8700 \pm .009$	3.870011
C_{11} (quad) (T/M)	-1.0470	$4.3 \cdot 10^{-4}$	-1.0483	$1.9 \cdot 10^{-3}$	$-1.0457 .14$	-1.0469
C_{12} (sext) (T/M ²)	-4.47	$.02$	-4.89	$.38$	$-4.453 1.7$	-4.50
C_{13} (octup) (T/M ³)	2.51	2.18	$.13$	23	$1.767 17.2$	$.83$
C_{14} (deca) (T/M ⁴)	35.1	138	3717	3393	$49.9 833$	44.3
C_{15} (dodeca) (T/M ⁵)	$19,985$	16799	73070	68991		25969
C_{16} (14-pole) (T/M ⁶)	$-8.8 \cdot 10^5$	$1.2 \cdot 10^6$	$-1.01 \cdot 10^7$	$8.6 \cdot 10^6$		

* FFT MEAN VALUES ARE FROM FIT TO 10 SETS OF RANDOMIZED
TOSCA DATA WITH GAUSSIAN NOISE. FOR ANALYSIS OF THE
ACTUAL DATA, PARAMETER UNCERTAINTIES WILL BE
OBTAINED FROM MULTIPLE INDEPENDENT DATA SETS.
UNCERTAINTIES ARE STANDARD DEVIATIONS.

TABLE III

8/4/91

NOV29 COIL

$\theta = 90^\circ$

COMPARISON OF MEDIAN-PLANE MULTIPOLE COEFFICIENTS FROM CURVATURE-CORRECTED FFT FIT OF 128 TOSCA-GENERATED B_z VALUES ON $R=12\text{mm}$ CIRCLE AND LEAST SQUARES FIT OF 1247 TOSCA-GENERATED B_z VALUES ON $\pm 14\text{mm} \times \pm 21\text{mm}$ GRID. "DATA" RANDOMIZED IN BOTH CASES BY ADDING RANDOM NOISE FROM GAUSSIAN WITH STANDARD DEVIATION $\sigma = .5\%$. SIMULATED DATA ARE IN PLANE NORMAL TO SXLS REFERENCE AXIS AT MAGNET CENTER ($\theta = 90^\circ$, $\phi = 90^\circ$) FOR NOV29 CONDUCTOR CONFIGURATION.

	* FOURIER FFT (128 pt.) $R = 1.2\text{cm circle}$	** $\Delta X = 1\text{mm}$ GAUSS 6 th -order LEAST SQUARES 1247 pts $\pm 14 \times \pm 21\text{mm}$	GENERAL DYNAMICS TOLERANCES	DA "GOLD"
$C_{10}(\text{dipole})$ (Tesla)	$3.870024 \pm 3 \cdot 10^{-6}$	$3.870010 \pm 5 \cdot 10^{-6}$	$3.8700 \pm .009$	3.870012
$C_{11}(\text{quad})$ (T/M)	-1.0470 $4 \cdot 10^{-4}$	-1.0475 $6 \cdot 10^{-4}$	-1.0457 .14	-1.0469
$C_{12}(\text{sext})$ (T/M ²)	-4.47 .02	-4.48 .09	-4.453 1.7	-4.50
$C_{13}(\text{octup})$ (T/M ³)	2.5 2.2	6.6 4.3	1.767 17.2	.83
$C_{14}(\text{deca})$ (T/M ⁴)	35 138	-104 488	49.9 833	44.3
$C_{15}(\text{dodeca})$ (T/M ⁵)	19,985 16799	16,449 8012		25,969
$C_{16}(\text{14-pole})$ (T/M ⁶)	$-8.8 \cdot 10^5$ $1.2 \cdot 10^6$	$-7.8 \cdot 10^5$ $7.5 \cdot 10^5$		

* FFT MEAN VALUES AND RMS UNCERTAINTIES FROM 10 SETS OF 128 PTS. EACH ON RANDOMIZED TOSCA DATA WITH GAUSSIAN NOISE.

** $B_z(x, z) = C_{10} + C_{11}x + C_{12}x^2 + C_{13}x^3 + C_{14}x^4 + C_{15}x^5 + C_{16}x^6$
 $+ z^2(C_{30} + C_{31}x + C_{32}x^2 + C_{33}x^3 + C_{34}x^4)$
 $+ z^4(C_{50}x + C_{51}x^2 + C_{52}x^3) + z^6 C_{70}$

TABLE IV

8/22/91

COIL NOV 29

 $\theta = 90^\circ$

COMPARISON OF 128 PT FFT RUNS (10 AND 20 SETS) WITH
 LEAST SQUARES FIT OF BROWN-SERVANCKX (BS) EXPANSION ON
 3185 PT. GRID, $-16 \text{ mm} \leq X \leq 16 \text{ mm}$, $-12 \text{ mm} \leq Z \leq 12 \text{ mm}$, $DX = .5 \text{ mm}$
 TOSCA DATA RANDOMIZED WITH GAUSSIAN NOISE, $\sigma = 0.5 \text{ Gauss}$

MID-PLANE COEFF.	128 PT FFT (10 SETS) $R = 12 \text{ mm}$	128 PT FFT (20 SETS) $R = 12 \text{ mm}$	3185 PT $DX = .5 \text{ mm}$ BS LEAST SQ. $\pm 16 X \pm 12 \text{ mm}$	GENERAL DYNAMICS TOLERANCES
C_{10} (dip.) (Tesla)	$3.870024 \pm 3.0 \cdot 10^{-6}$	$3.870024 \pm 2.3 \cdot 10^{-6}$	$3.870017 \pm 3.2 \cdot 10^{-6}$	$3.8700 \pm .00^\circ$
C_{11} (quad) (T/m)	$-1.0470 \pm 4.3 \cdot 10^{-4}$	$-1.0469 \pm 2.9 \cdot 10^{-4}$	$-1.0465 \pm 5.0 \cdot 10^{-4}$	$-1.0457 \pm .14$
C_{12} (sext) (T/m ²)	$-4.473 \pm .021$	$-4.480 \pm .037$	$-4.512 \pm .101$	-4.453 ± 1.7
C_{13} (oct) (T/m ³)	2.51 ± 2.18	$.92 \pm 2.11$	$\pm .94 \pm 6.39$	1.767 ± 17.2
C_{14} (deca) (T/m ⁴)	35.1 ± 138.0	48.5 ± 164.5	$\pm 135 \pm 94.7$	49.9 ± 833
C_{15} (dodec) (T/m ⁵)	19985 ± 16799	29918 ± 16717	22982 ± 20639	
C_{16} (14-pole) (T/m ⁶)	$-8.82 \cdot 10^5 \pm 1.2 \cdot 10^6$	$-9.41 \cdot 10^5 \pm 1.4 \cdot 10^6$	$-5.61 \cdot 10^5 \pm 2.5 \cdot 10^6$	

TABLE V

8/28/91

COMPARISON OF 128 POINT FFT FIT (10 DATA SETS) WITH
 LEAST SQUARES FIT OF BROWN-SERURANCKX (BS) EXPANSION ON
 4845 POINT, $-21 \leq x \leq 21$ mm, $-14 \leq z \leq 14$ mm GRID, $\Delta x = 0.5$ mm.
 TOSCA DATA RANDOMIZED WITH $\sigma = 0.5$ G GAUSSIAN NOISE

MIDPLANE COEFF.	128 PT FFT 10 SETS, $R=12$ mm	4845 PT LEAST SQ. $x = \pm 21$ mm, $z = \pm 14$ mm $\Delta x = .5$ mm	GENERAL DYNAMICS TOLERANCES	DIFFERENTIAL ALGEBRA (DA, "GOLD")
$C_{10}(\text{dip})$ (T/sla)	$3.870024 \pm 3.0 \cdot 10^{-8}$	$3.870016 \pm 2.6 \cdot 10^{-6}$	$3.8700 \pm .009$	3.870012
$C_{11}(\text{quad})$ (T/m)	$-1.0470 \pm 4.3 \cdot 10^{-4}$	$-1.0474 \pm 3.1 \cdot 10^{-4}$	$-1.0457 \pm .14$	-1.0469
$C_{12}(\text{sext})$ (T/m ²)	$-4.473 \pm .021$	$-4.578 \pm .048$	-4.453 ± 1.7	-4.50
$C_{13}(\text{oct})$ (T/m ²)	2.51 ± 2.18	1.91 ± 2.31	1.767 ± 17.2	$.83$
$C_{14}(\text{deca})$ (T/m ⁴)	35.1 ± 138.0	427 ± 261	49.9 ± 833	44.3
$C_{15}(\text{dodeca})$ (T/m ⁵)	19985 ± 16799	25552 ± 4359		25969
$C_{16}(\text{14-pole})$ (T/m ⁶)	$-8.82 \cdot 10^5 \pm 1.2 \cdot 10^6$	$-1.74 \cdot 10^6 \pm 4.1 \cdot 10^5$		

TABLE Va

8/4/91
COIL NOV29
 $\theta = 90^\circ$

COMPARISON OF MEDIAN PLANE COEFFICIENTS FOR
128 point 10-SET FFT AND BROWN-SERVANCEX LEAST SQUARES
FIT TO 19097 POINTS IN $X = \pm 21 \text{ mm}$ $Y = \pm 14 \text{ mm}$ grid, $\Delta X = .25 \text{ mm}$
GAUSSIAN NOISE WITH $\sigma = 0.5 \text{ G}$

6th order

	FOURIER FFT 128 PT 10 SET R=12 mm	BS LST SQE 19097 PTS $\Delta X = .25 \text{ mm}$ $X = \pm 21, Y = \pm 14 \text{ mm}$	GD TOLERANCE	DA "GOLD"
$C_{10}(\text{dip})$ (Tesla)	3.870024 ± 3.010^{-6}	$3.870012 \pm 1.3 \cdot 10^{-6}$	$3.8700 \pm .009$	3.870012
$C_{11}(\text{quad})$ (T/m)	-1.0470 ± 4.010^{-4}	$-1.0464 \pm 1.6 \cdot 10^{-4}$	$-1.0457 \pm .14$	-1.0469
$C_{12}(\text{sext})$ (T/m ²)	$-4.47 \pm .02$	$-4.485 \pm .024$	-4.453 ± 1.7	-4.50
$C_{13}(\text{oct})$ (T/m ³)	2.5 ± 2.2	-3.43 ± 1.17	1.767 ± 17.2	.83
$C_{14}(\text{dec})$ (T/m ⁴)	35 ± 138	-99.8 ± 133	49.9 ± 833	44.3
$C_{15}(\text{dodec})$ (T/m ⁵)	19985 ± 16799	33039 ± 2240		25969
$C_{16}(\text{14 pole})$ (T/m ⁶)	$-8.810^5 \pm 1.210^6$	$-8.7310^5 \pm 2.110^5$		

TABLE VI

6/20/91 Roy B

$\phi = 90^\circ$

COMPARISON OF FITS BY NLLSF LEAST SQUARES CODE
TO NOV29 SXLS COIL $\theta = 90^\circ$ TO SCA WITH VARYING NUMBERS OF
DATA POINTS IN INTERVAL $-30 \leq x \leq 30$ mm, $-15 \leq z \leq 15$ mm

$$B_z = \sum_{m=0}^{NR/2} \sum_{n=0}^{NR-2m} C_{2m+1,n} x^m y^{2m}$$

NR=14 ± 4 ORDER
1891 POINTS NP=64
 $\sigma = 3.10^{-10}$ T $\Delta = 1$ mm
VAR = .786 "GOLD"

NR=6 ± 4 ORDER
1891 POINTS NP=16
 $\sigma = .5$ Gauss $\Delta = 1$ mm
VAR = 1.012

NR=6 ± 4 ORDER NP=16
527 POINTS ($\Delta = 2$ mm)
 $\sigma = .5$ Gauss
VAR = .993

NR=6 ± 4 ORDER
153 POINTS NP=1
 $\sigma = .5$ Gauss
VAR = .854
 $\Delta = 4$ mm

C_{10}	$3.8700125892 \pm 4.73 \cdot 10^{-11}$	$3.87001051 \pm 4.23 \cdot 10^{-6}$	$3.87000612 \pm 7.50 \cdot 10^{-6}$	3.8700044 ± 1.261
C_{11}	$-1.046887107 \quad 8.93 \cdot 10^{-9}$	$-1.047053 \quad 3.50 \cdot 10^{-4}$	$-1.047711 \quad 6.36 \cdot 10^{-4}$	$-1.04856 \quad 1.13 \cdot 10$
C_{12}	$-4.4962531 \quad 1.70 \cdot 10^{-6}$	$-4.477134 \quad 3.72 \cdot 10^{-2}$	$-4.443029 \quad 6.76 \cdot 10^{-2}$	$-4.3973 \quad .112$
C_{13}	$.829903 \quad 1.67 \cdot 10^{-4}$	$2.0145 \quad 1.26$	$3.7611 \quad 2.27$	$8.0175 \quad 3.89$
C_{14}	$44.1445 \quad 2.21 \cdot 10^{-2}$	$-5.7725 \quad 99.4$	$-4.0850 \quad 179$	$-22.011 \quad 297$
C_{15}	$25973.99 \quad 1.23$	$24,976.0 \quad 1153.4$	$24,017.7 \quad 2038$	$21,112 \quad 3350$
C_{16}	$-1,178,103 \quad 129.1$	$-1.1661 \cdot 10^6 \quad 7.57 \cdot 10^4$	$-1.1907 \cdot 10^6 \quad 1.33 \cdot 10^5$	$-1.2366 \cdot 10^6 \quad 2.18$
C_{17}	$-286,748 \quad 4325$			
C_{18}	$-2.0004 \cdot 10^7 \quad 3.87 \cdot 10^5$			
C_{30}	$5.290584 \pm 7.03 \cdot 10^{-6}$	$5.4534 \pm .146$	$5.3803 \pm .26$	$5.3663 \pm .521$
C_{31}	$3.93720 \quad 5.05 \cdot 10^{-4}$	$-.6791 \quad 3.87$	$3.0649 \quad 6.75$	$-1.7951 \quad 11.3$
C_{32}	$-278.4877 \quad 6.04 \cdot 10^{-2}$	$-572.4 \quad 285$	$-1014.7 \quad 490$	$-1763.2 \quad 758$
C_{33}	$-2.59872 \cdot 10^5 \quad 3.19$	$-2.5704 \cdot 10^5 \quad 3.78 \cdot 10^3$	$-2.6390 \cdot 10^5 \quad 6388$	$-2.6567 \cdot 10^5 \quad 9551$
C_{34}	$1.7567 \cdot 10^7 \quad 324.4$	$1.8257 \cdot 10^7 \quad 2.47 \cdot 10^5$	$1.8659 \cdot 10^7 \quad 4.12 \cdot 10^5$	$1.9107 \cdot 10^7 \quad 5.911$
C_{35}	$1.2139 \cdot 10^7 \quad 1.08 \cdot 10^4$			
C_{36}	$5.3534 \cdot 10^8 \quad 9.58 \cdot 10^5$			
C_{50}	$46.815 \pm .379$	-1633.7 ± 1530	-483.7 ± 2766	$547.8 \quad 5976$
C_{51}	$1.30023 \cdot 10^5 \quad 14.52$	$1.4527 \cdot 10^5 \quad 1.52 \cdot 10^4$	$1.4583 \cdot 10^5 \quad 2.57 \cdot 10^4$	$1.6082 \cdot 10^5 \quad 4.1011$
C_{52}	$-1.7459 \cdot 10^7 \quad 1329$	$-1.8076 \cdot 10^7 \quad 9.44 \cdot 10^5$	$-1.8631 \cdot 10^7 \quad 1.61 \cdot 10^6$	$-1.6644 \cdot 10^7 \quad 2.411$
C_{53}	$-3.0148 \cdot 10^7 \quad 4.12 \cdot 10^4$			
C_{54}	$-1.3314 \cdot 10^9 \quad 3.51 \cdot 10^6$			
C_{70}	$1.1317 \cdot 10^6 \quad 9168$	$6.717 \cdot 10^6 \pm 4.56 \cdot 10^6$	$4.504 \cdot 10^6 \quad 8.09 \cdot 10^6$	$4.520 \cdot 10^5 \quad 1.761$
C_{71}	$7.8263 \cdot 10^6 \quad 2.09 \cdot 10^5$			
C_{72}	$5.4632 \cdot 10^8 \quad 1.61 \cdot 10^7$			

TABLE VII

8/16/91

COMPARISON OF 128 PT FFT FIT (10 SETS) WITH LEAST SQUARES FIT OF BROWN-SERVANKKX (BS) EXPANSION ON 7381 PT GRID, $-30\text{mm} \leq x \leq 30\text{mm}$, $-15\text{mm} \leq z \leq 15\text{mm}$, $\Delta x = .5\text{mm}$. TOSCA DATA RANDOMIZED WITH $\sigma = 0.5$ GAUSS NOISE FROM GAUSSIAN. $\theta = 90^\circ$, COILNOV2

Midplane COEFF	128 PT FFT 10 SETS $R=12\text{mm}$	$\Delta x = .5\text{mm}$ 7381 PT BS LEAST SQUARES $\pm 30x \pm 15\text{mm}$	GENERAL DYNAMICS TOLERANCES	DIFFERENTIAL ALGEBRA DA "GOLD"
$C_{10}(\text{dip})$ (Tesk)	$3.870024 \pm 3.0 \cdot 10^{-6}$	$3.870013 \pm 2.1 \cdot 10^{-6}$	$3.8700 \pm .009$	3.870012
$C_{11}(\text{quad})$ (T/m)	$-1.0470 \pm 4.3 \cdot 10^{-4}$	$-1.0472 \pm 1.8 \cdot 10^{-4}$	$-1.0457 \pm .14$	-1.0469
$C_{12}(\text{sext})$ (T/m ²)	$-4.473 \pm .021$	$-4.501 \pm .019$	-4.453 ± 1.7	-4.50
$C_{13}(\text{oct})$ (T/m ³)	2.51 ± 2.18	$2.07 \pm .64$	1.767 ± 17.2	$.83$
$C_{14}(\text{deca})$ (T/m ⁴)	35.1 ± 138.0	53.0 ± 50.7	49.9 ± 833	44.3
$C_{15}(\text{dodec})$ (T/m ⁵)	19985 ± 16799	25080 ± 594		25969
$C_{16}(\text{14-pole})$ (T/m ⁶)	$-8.82 \cdot 10^5 \pm 1.2 \cdot 10^6$	$-1.200 \cdot 10^6 \pm 3.92 \cdot 10^4$

TYPICAL CHAMBER CROSS-SECTION

Fig. 1 Cross section of SXLS vacuum chamber at $\theta = 90^\circ$ showing 4-probe Hall Probe holder

Fig. 2 Plan view of section of SXLS vacuum chamber

Fig.3 Detail of 4-probe Hall Probe holder for magnetic measurements of SXL5 magnet.

PROGRAM LSTSQR2

C GENERAL NONLINEAR LEAST SQUARES FITTING PROGRAM USING GAUSS METHOD OF
 C SUBROUTINE FIT TO FIT A FUNCTION $F(X; P(1), \dots, P(NP))$ TO N DATA POINTS
 C $YD(1), \dots, YD(N)$ AT ABSCISSAS $X(1), \dots, X(N)$ WHICH ARE READ IN BY LSTSQR OR,
 C IF N IS NEGATIVE, FROM FILE INDATA. WEIGHTS $W(1), \dots, W(N)$ ARE SUPPLIED
 C BY SUBROUTINE PRELIM. FIT USES SUBROUTINE MATINV FOR MATRIX INVERSION
 C AND SUBROUTINE CALC TO EVALUATE THE FUNCTION AND ITS DERIVATIVES WITH
 C RESPECT TO THE NFP FREE PARAMETERS $P(1), \dots, P(NFP)$ WHERE $NFP \leq NP$. ANY
 C COMBINATION OF THE NP PARAMETERS CAN BE HELD FIXED. PROGRAM COMPUTES
 C GOODNESS OF FIT $CHISQR = \sum (W(I) * (YC(I) - YD(I))^2)$ AND VARIANCE $VAR =$
 C $CHISQR / (N - NFP)$. CC=CONVERGENCE CRITERION AND NPR CONTROLS VARIOUS OUT
 C PUT OPTIONS. SUBROUTINE POST IS CALLED AFTER FIT TO DETERMINE USER-
 C CONTROLLED ACTION. FILENAMES OF INDATA AND OUTDATA ARE USER-SPECIFIED.
 C 12/4/90 VERSION: F IS A FUNCTION OF IV INDEPENDENT VARIABLES, DATA
 C ARRAY DIMENSIONS INCREASED TO 2000. THIS VERSION ASSUMES $IV=2$. SUBROU-
 C TIME CALC EVALUATES 2-DIMENSIONAL MAGNETIC MULTIPOLE EXPANSION $By = \dots$
 C 1/6/91 VERSION: NR AND NS ARE LIMITS OF DOUBLE SUMMATION FOR By FROM
 C BROWN AND SERVIRANCKX. 1/10/91: corrections made for fixed parameters.
 C 2/1/91 version=REAL*16(n11s6). 2/3/91=BS sum to order NR(n11s7). 2/12/91
 C VERSION(NLLS8): INCLUDE ODD-PARITY SERIES(SKEW) $By = \sum_{m=0}^{NS/2} \sum_{n=0}^{NS} \{C(2m+2, n) x^{2m+1} y^n\}$ where $C(20)=P(NPR+1), \dots, C(NS+2)=P(NP)$. $NP =$
 C $NPR + NPS$, $NPR = (NR+2) ** 2 / 4$, $NPS = (NS+2) ** 2 / 4$. NR=ORDER OF REGULAR SERIES,
 C NS=ORDER OF SKEW SERIES. 7/3/91 increased NPMAX to 260, NMAX=8000.

IMPLICIT REAL*16(A-H, O-Z)

PARAMETER (IV=2)

DIMENSION X(IV, 8000), YD(8000), W(8000), YC(8000), P(260), S(260),

1 DY(8000), PI(260), DP(260)

CHARACTER*10 DAY, HOUR

CHARACTER*12 INDATA, OUTDATA

CHARACTER*80 HEADER

CALL DATE(DAY)

CALL TIME(HOUR)

WRITE(*, *) ' ENTER HEADER(ABO)='

READ(*, *) HEADER

10 WRITE(*, *) DAY, ' PROGRAM LSTSQR ENTER N, NR, NS, NP, NPR, CC='

READ(*, *) N, NR, NS, NP, NPR, CC

IF(N.EQ.0) STOP ' USER-INITIATED PROGRAM TERMINATION'

IF(N.LT.0) GOTO 20

WRITE(*, *) ' ENTER YD(I), X(1, I), X(2, I), ... I=1, N'

READ(*, *) (YD(I), I=1, N), ((X(J, I), J=1, IV), I=1, N)

WRITE(*, *) ' ENTER PARAMETER GUESSES P(I), I=1, NP='

READ(*, *) (P(I), I=1, NP)

GOTO 30

C N is negative. Read data from an input file.

20 N=-N

WRITE(*, *) ' PLEASE ENTER NAME OF INPUT DATA FILE, INDATA='

READ(*, *) INDATA

OPEN(1, FILE=INDATA, STATUS='OLD', FORM='FORMATTED')

READ(1, 25) (YD(I), (X(J, I), J=1, IV), I=1, N)

25 FORMAT(3(1PE17.9))

READ(1, 26) (P(I), I=1, NP)

26 FORMAT(1PE17.10)

CLOSE(1)

30 CALL PRELIM(N, X, YD, P, NP, W)

DO 35 I=1, NP

35 PI(I)=P(I)

40 CALL FIT(NR, NS, NP, P, N, X, YD, W, NPR, CC, VAR, YC, S, NC)

CALL POST(N, X, YD, NP, P, NPR, NC, VAR, NTEST)

IF(NTEST.EQ.1) GOTO 40

C WRITE RESULTS ON OUTPUT FILE

```

DO 50 I=1,NP
50 DP(I)=PI(I)-P(I)
DO 55 I=1,N
55 DY(I)=YD(I)-YC(I)
WRITE(*,*) ' PLEASE ENTER NAME OF OUTPUT DATA FILE, OUTDATA='
READ(*,*) OUTDATA
OPEN(2,FILE=OUTDATA,STATUS='NEW',FORM='FORMATTED')
WRITE(2,57) DAY,HOUR,HEADER
57 FORMAT(' PROGRAM LSTSQR',2A10,1X,A80)
WRITE(2,60) VAR,NC
60 FORMAT(' PARAMETERS AND UNCERTAINTIES   VAR=',1PE10.4,'   NC=',12/
1 9X,'FINAL PARAM',8X,'SIGMA P   INITIAL PARAM   DELTA PARAM'//)
WRITE(2,70) (I,P(I),S(I),PI(I),DP(I),I=1,NP)
70 FORMAT(I3,1PE17.10,1PE15.7,1PE15.7,1PE15.7)
WRITE(2,80)
80 FORMAT(' POINT',13X,'X1',13X,'X2',13X,'YD',13X,'YC',13X,'DY')
WRITE(2,90) (I,(X(J,I),J=1,IV),YD(I),YC(I),DY(I),I=1,N)
90 FORMAT(I6,5F15.6)
CLOSE(2)
GOTO 10
END
SUBROUTINE PRELIM(N,X,YD,P,NP,W)
C TEST CASE FOR PROGRAM LSTSQR. READS CONSTANT WEIGHT WIN.2/1/91=REAL*16
IMPLICIT REAL*16(A-H,O-Z)
DIMENSION X(2,8000),YD(8000),P(260),W(8000)
WRITE(*,*) ' PLEASE ENTER CONSTANT WEIGHT WIN='
READ(*,*) WIN
DO 10 J=1,N
10 W(J)=WIN
RETURN
END
SUBROUTINE FIT(NR,NS,NP,P,N,X,YD,W,NPR,CC,VAR,YC,S,NC)
C GENERAL NON-LINEAR LEAST SQUARES FIT BY GAUSS METHOD. SEE LOS
C ALAMOS REPORT LA-2367, MOORE AND ZEIGLER (1960). FITS FUNCTION
C F(X;P(1),...P(NP)) SPECIFIED BY SUBROUTINE CALC TO N DATA POINTS
C (X(1),YD(1)),... (X(N),YD(N)). CALC ALSO EVALUATES DERIVATIVES OF
C F WITH RESPECT TO FREE PARAMETERS AND FUNCTION VALUE YC(I),I=1,N.
C NPR=0,1,2 OR 3 GIVES NO LEVENBERG DAMPING. NPR=1 OR 4 GIVES A-MATRIX
C PRINT, NPR=2 OR 5 GIVES ITERATION PRINT, NPR=3 OR 6 GIVES CORRELATION
C MATRIX PRINT. ANY PARAMETER CAN BE FIXED IF NFIXP.GT.0, IN WHICH CASE
C PARAMETER NUMBERS NFIX(I) WILL BE DELETED FROM P(I) TO GIVE ARRAY OF
C FREE PARAMETERS PF(I). CC IS CONVERGENCE CRITERION, VAR=CHISQR/NDF
C IS THE VARIANCE OF THE FIT, S(I) ARE THE UNCERTAINTIES IN THE FITTED
C PARAMETERS, AND NC IS THE NUMBER OF ITERATION CYCLES REQUIRED.12/4/90
C VAX VERSION: F is a function of IV independent variables contained in
C array X(IV,N). This version uses IV=2 to evaluate 2-dimensional mag-
C netic multipole data. NMAX=2000. 1/6/91 VERSION. NR AND NS ARE LIMITS
C OF BROWN AND SERVIRANCKX DOUBLE SUMATION FOR By. USED IN CALC.1/10/91
C CORRECTIONS MADE FOR FIXED PARAMETER CASE. 2/1/91 version=REAL*16.
C 2/12/91 VERSION:REGULAR & SKEW SERIES TO NR-th AND NS-th ORDER, RESPEC-
C TIVELY,NP=((NR+2)**2+(NS+2)**2)/4. NLLS8.7/3/91 NP to 260, N to 8000.
IMPLICIT REAL*16(A-H,O-Z)
PARAMETER (IV=2)
DIMENSION P(260),X(IV,8000),YD(8000),YC(8000),W(8000),DF(260),
1 V(260),DP(260),S(260),DIAG(260),ROW(260),A(33930),NFIX(260),
2 PF(260),SF(260)
DO 2 J=1,NP
2 PF(J)=P(J)
NFP=NP
WRITE(*,*) ' NUMBER OF FIXED PARAMETERS NFIXP='

```

```

      READ(*,*) NFIXP
      IF(NFIXP.EQ.0) GOTO 7
      IF(NFIXP.LT.0) GOTO 41
C NFIXP>0. Read in fixed parameter numbers NFIX(1),.....etc.
      WRITE(*,*) ' FIXED PARAMETERS NFIX(J),J=1,NFIXP ='
      READ(*,*) (NFIX(J),J=1,NFIXP)
      GOTO 42
C NFIXP<0. There are fixed parameters. Use existing values NFIX(I).
      41 NFIXP=-NFIXP
      42 NFP=NP-NFIXP
      L=0
      DO 6 J=1,NFIXP
      M=NFIX(J)
      DO 5 K=M,NP
      5 PF(K-L)=P(K+1)
      L=L+1
      6 CONTINUE
      7 NA=NFP*(NFP+1)/2
C      ENTER LOOP TO MODIFY FREE PARAMETERS PF(I).
      DO 260 I=1,70
      NC=I
      CHISQR=0.0Q0
      DO 10 J=1,NA
      10 A(J)=0.0Q0
      DO 20 J=1,NP
      20 V(J)=0.0Q0
C      LOOP TO FORM MATRIX A AND VECTOR V FOR N DATA POINTS.
      DO 60 K=1,N
      CALL CALC(NR,NS,NP,P,X(1,K),X(2,K),YC(K),DF)
      IF(NFIXP.EQ.0) GOTO 25
      DO 22 J=1,NP
      22 ROW(J)=DF(J)
      LL=0
      DO 24 J=1,NFIXP
      M=NFIX(J)
      DO 23 L=M,NP
      23 DF(L-LL)=ROW(L+1)
      LL=LL+1
      24 CONTINUE
      25 DY=YD(K)-YC(K)
      L=1
      DO 50 J=1,NFP
      IF(DF(J).NE.0.0Q0) GOTO 30
      L=L+NFP-J+1
      GOTO 50
      30 DO 40 M=J,NFP
      A(L)=A(L)+DF(J)*DF(M)*W(K)
      40 L=L+1
      V(J)=V(J)+DF(J)*DY*W(K)
      50 CONTINUE
      60 CHISQR=CHISQR+DY*DY*W(K)
      VAR=CHISQR/QFLOAT(N-NFP)
C      APPLY LEVENBERG DAMPING FACTOR FDL IF NPR.GT.3
      IF(NPR.LT.4) GOTO 82
      SUM=0.0Q0
      J=1
      L=1
      LL=NFP
      DO 70 K=1,NFP
      IF(A(L).EQ.0.0Q0) GOTO 310

```

```

SUM=SUM+V(J)*V(J)/A(L)
J=J+1
L=L+LL
70 LL=LL-1
FDL=1.0Q0+2.0Q0*SUM/CHISQR
L=1
LL=NFP
DO 80 K=1,NFP
A(L)=A(L)*FDL
L=L+LL
80 LL=LL-1
C PRINT A-MATRIX IF NPR=1 OR 4
82 IF (NPR.NE.1.AND.NPR.NE.4) GOTO 100
IF (I.NE.1) GOTO 90
WRITE(*,*) ' MATRIX PRINT OF SUBROUTINE FIT'
90 WRITE(*,95) I,NFP,(A(K),K=1,NA)
95 FORMAT(' NC=',I2,' NFP=',I2/(7(1X,1PE10.3)))
C TEST FOR DIAGONAL ELEMENT OF A-MATRIX EQUAL ZERO.
100 L=1
LL=NFP
DO 96 K=1,NFP
IF (A(L).EQ.0.0Q0) GOTO 310
L=L+LL
96 LL=LL-1
C REPLACE A-MATRIX BY ITS INVERSE AND TEST IF SINGULAR.
CALL MATINV(A,NFP,NTEST)
IF (NTEST.NE.0) GOTO 280
C EVALUATE PARAMETER INCREMENTS DP(I) AND NEW PARAMETERS PF(I).
DO 120 K=1,NFP
DP(K)=0.0Q0
L=K
LL=NFP-1
DO 120 J=1,NFP
DP(K)=DP(K)+A(L)*V(J)
IF (J.GE.K) GOTO 110
L=L+LL
LL=LL-1
GOTO 120
110 L=L+1
120 CONTINUE
DO 130 K=1,NFP
130 PF(K)=PF(K)+DP(K)
C PRINT ITERATION RESULTS IF NPR=2 OR 5
IF (NPR.NE.2.AND.NPR.NE.5) GOTO 165
IF (I.NE.1) GOTO 150
WRITE(*,*) ' ITERATION PRINT OF SUBROUTINE FIT'
WRITE(*,*) ' NC PARAMETERS
1 VARIANCE'
150 WRITE(*,160) I,(PF(J),J=1,5),VAR
160 FORMAT(1X,I2,1X,5(1PE12.5,1X),1PE10.4)
C FILL IN P ARRAY WITH NEW PF VALUES AND TEST FOR CONVERGENCE.
165 IF (NFXP.EQ.0) GOTO 166
L=1
DO 164 J=1,NP
DO 162 K=1,NFXP
IF (NFX(K).EQ.J) GOTO 164
162 CONTINUE
C PARAMETER J IS FREE PARAMETER.REPLACE IT WITH PF(L).
P(J)=PF(L)
L=L+1

```

```

164 CONTINUE
    GOTO 169
166 DO 167 J=1,NP
167 P(J)=PF(J)
169 DO 170 J=1,NFP
    IF (PF(J).EQ.0.0Q0) GOTO 170
    IF (QABS(DP(J)/PF(J)).GT.CC) GOTO 260
170 CONTINUE
C   CALCULATION CONVERGED. COMPUTE PARAMETER UNCERTAINTIES S(I)
175 CHISQR=0.0Q0
    DO 176 K=1,N
    CALL CALC(NR,NS,NP,P,X(1,K),X(2,K),YC(K),DF)
    DY=YD(K)-YC(K)
176 CHISQR=CHISQR+DY*DY*W(K)
    VAR=CHISQR/QFLOAT(N-NFP)
    L=1
    LL=NFP
    DO 180 J=1,NFP
    DIAG(J)=A(L)
    SF(J)=QSQRT(QABS(DIAG(J)*VAR))
    L=L+LL
180 LL=LL-1
    IF(NFIXP.EQ.0) GOTO 187
C   FILL IN S ARRAY WITH NEW UNCERTAINTIES IF THERE ARE FIXED PARAMS
    L=1
    DO 186 J=1,NP
    DO 182 K=1,NFIXP
    IF(NFIX(K).EQ.J) GOTO 184
182 CONTINUE
    S(J)=SF(L)
    L=L+1
    GOTO 186
184 S(J)=0.0Q0
186 CONTINUE
    GOTO 190
187 DO 188 J=1,NP
188 S(J)=SF(J)
C   EVALUATE AND PRINT CORRELATION MATRIX IF NPR=3 OR 6.
190 IF(NPR.NE.3.AND.NPR.NE.6) GOTO 270
    IF(I.NE.1) GOTO 200
    WRITE(*,*) ' CORRELATION MATRIX PRINT OF SUBROUTINE FIT'
200 DO 210 J=1,NFP
210 DIAG(J)=1.0Q0/QSQRT(QABS(DIAG(J)))
    L=1
    DO 250 J=1,NFP
    DO 220 K=1,NFP
220 ROW(K)=0.0Q0
    DO 230 K=J,NFP
    ROW(K)=A(L)*DIAG(J)*DIAG(K)
230 L=L+1
    WRITE(*,240) J,(ROW(K),K=1,NFP)
240 FORMAT(1X,I2,7(1X,1PE10.3))
250 CONTINUE
    GOTO 270
C   NO CONVERGENCE. CONTINUE ITERATIONS.
260 CONTINUE
270 RETURN
C   ERROR PRINTS.
280 WRITE(*,*) ' SUBROUTINE FIT ERROR IN MATRIX INVERSION'
    WRITE(*,290) NC,(A(M),M=1,NA)

```

```

290 FORMAT(' NC=', I2//('1X, 7(1PE10.3, 1X)))
300 NC=70
    GOTO 270
310 WRITE(*,*) ' SUBROUTINE FIT DIAGONAL A(L)=0 ERROR PRINT'
    WRITE(*, 320) NC, (A(K), K=1, NA)
320 FORMAT(' NC=', I2//('1X, 7(1PE10.3, 1X)))
    GOTO 300
    END
    SUBROUTINE MATINV(A, N, NFAIL)
C INVERSION OF REAL SYMMETRIC NxN MATRIX BY GAUSS-JORDAN ELLIMINATION.
C INPUT MATRIX IS A11=A(1), A12=A(2), ... A1N=A(N), A22=A(N+1), ... A2N=A(2N-1)
C ... ANN=A(N*(N+1)/2). ELEMENTS OF INVERSE MATRIX ARE IN CORRESPONDING
C POSITIONS, VIZ. AI11=A(1), ... AI1N=A(N), AI22=A(N+1), ... AI2N=A(2N-1), ...
C AINN=A(N*(N+1)/2). NFAIL NON-ZERO IS ERROR RETURN. 2/1/91=REAL*16.
    IMPLICIT REAL*16(A-H, O-Z)
    DIMENSION A(33930)
    K=1
    IF(N.NE.1) GOTO 10
    A(1)=1.0Q0/A(1)
    GOTO 130
10 DO 60 M=1, N
    IMAX=M-1
    DO 50 L=M, N
    SUMA=0.0Q0
    KL=L
    KM=M
    IF(IMAX.LE.0) GOTO 30
    DO 20 I=1, IMAX
    SUMA=SUMA+A(KL)*A(KM)
    KL=KL+N-I
20 KM=KM+N-I
30 TERM=A(K)-SUMA
    IF(L.GT.M) GOTO 40
    IF(TERM.LE.0.0Q0) GOTO 150
    DENOM=QSQRT(TERM)
    A(K)=DENOM
    GOTO 50
40 A(K)=TERM/DENOM
50 K=K+1
60 CONTINUE
C INVERSION OF TRIANGULAR MATRIX
    A(1)=1.0Q0/A(1)
    KD=1
    DO 90 L=2, N
    KD=KD+N-L+2
    A(KD)=1.0Q0/A(KD)
    KM=0
    KL=L
    IMAX=L-1
    DO 80 M=1, IMAX
    K=KL
    SUMA=0.0Q0
    DO 70 I=M, IMAX
    J=KM+I
    SUMA=SUMA-A(KL)*A(J)
70 KL=KL+N-I
    A(K)=SUMA+A(KD)
    KL=K+N-M
80 KM=KM+N-M
90 CONTINUE

```

C PREMULTIPLY LOWER TRIANGLE BY TRANSPOSE

```

K=1
DO 120 M=1,N
KL=K
DO 110 L=M,N
KM=K
IMAX=N-L+1
SUMA=0.0Q0
DO 100 I=1,IMAX
SUMA=SUMA+A(KL)*A(KM)
KL=KL+1
100 KM=KM+1
A(K)=SUMA
110 K=K+1
120 CONTINUE
130 NFAIL=0
140 RETURN
150 NFAIL=K
GOTO 140
END

```

SUBROUTINE POST(ND, XD, YD, NP, P, NPR, NC, VAR, NTEST)

C TEST CASE FOR PROGRAM LSTSQR. IF FIT WAS SUCCESSFUL, DO NOTHING.

C IF FIT DID NOT CONVERGE, OPTIONS ARE PROVIDED TO CHANGE INITIAL DATA,

C PARAMETERS, AND PRINTOUT MODE NPR OF SUBROUTINE FIT, OR JUST QUIT.

C 12/4/90 VAX VERSION: F is a function of IV independent variables in

C data array XD(IV,N). NMAX increased to 2000. 2/1/91 version=REAL*16.

IMPLICIT REAL*16(A-H,Q-Z)

PARAMETER (IV=2)

DIMENSION XD(IV,8000), YD(8000), P(260)

CHARACTER*1 Y

CHARACTER*1 N

CHARACTER*1 L

IF(NC.LT.70) GOTO 170

C NC=70. CALCULATION DID NOT CONVERGE. QUESTION USER.

WRITE(*,*) 'FIT DID NOT CONVERGE. WANT TO TRY NEW FIT? (Y/N)'

READ(*,*) L

IF(L.EQ.N) GOTO 190

C USER WANTS TO TRY NEW FIT. WHAT CHANGES ARE DESIRED?

WRITE(*,*) 'DO YOU WANT TO MODIFY OR ELLIMINATE DATA POINTS?(Y/N)'

READ(*,*) L

IF(L.EQ.N) GOTO 70

C USER WANTS TO MODIFY DATA POINTS. READ NEW VALUES.

30 WRITE(*,*) 'ENTER POINT NUMBER I AND NEW YD(I). I=0 IS END, YD(I)=

10 IS ELLIMINATE'

READ(*,*) I, YD(I)

IF(I.EQ.0) GOTO 70

IF(YD(I).EQ.0.0Q0) GOTO 50

GOTO 30

C ELLIMINATE I-TH DATA POINT AND DECREMENT ND.

50 DO 60 J=1,ND

60 YD(J)=YD(J+1)

ND=ND-1

GOTO 30

70 WRITE(*,*) 'DO YOU WANT TO MODIFY OR ELLIMINATE PARAMETERS? (Y/N)'

READ(*,*) L

IF(L.EQ.N) GOTO 130

C USER WANTS TO MODIFY OR ELLIMINATE PARAMETERS. READ NEW P(I).

90 WRITE(*,*) 'ENTER PARAM NUMBER I AND NEW P(I). I=0 IS END, P(I)=0 IS

1 ELLIMINATE'

READ(*,*) I, P(I)


```

      IF(I.EQ.0) GOTO 130
      IF(P(I).EQ.0.0Q0) GOTO 110
      GOTO 90
C     ELLIMINATE I-TH PARAMETER AND DECREMENT NP.
110 DO 120 J=1,NP
120 P(J)=P(J+1)
      NP=NP-1
      GOTO 90
130 WRITE(*,*) ' DO YOU WANT TO CHANGE FIT PRINTOUT WITH NEW NPR?(Y/N)'
      READ(*,*) L
      IF(L.EQ.N) GOTO 160
C     USER WANTS NEW PRINTOUT FORMAT FROM SUBROUTINE FIT. READ NPR.
      WRITE(*,*) ' PLEASE ENTER NEW PRINT NUMBER NPR='
      READ(*,*) NPR
160 NTEST=1
      GOTO 200
170 WRITE(*,*) ' CALCULATION CONVERGED.PRINT RESULTS AND GOTO NEW CASE'
190 NTEST=0
200 RETURN
      END
      SUBROUTINE CALC (NR,NS,NP,P,X1,X2,F,DF)
C  CALCULATES VERTICAL COMPONENT OF MAGNETIC FIELD By=F FROM 2-DIMEN-
C  SIONAL EXPANSION OF BROWN AND SERVIRANCKX (SLAC-PUB-3381,July,1984)
C  By=SUM(m=0,NR/2)SUM(n=0,NR)B(2m+1,n)*X**n*Y**2m=B10+(B11*X+...B1N*X**NR)
C  +Y**2*(B30+B31*X+...B3N*X**NR-2)+Y**4*(B50+B51*X+...B5N*X**NR-4)+..
C  +Y**NR/2*B(NR+2,0).m=0,1,2,..NR/2. NR=even integer is specified order
C  of fit with NP=(NR+2)**2/4.2/1/91 version,REAL*16.(n11s7). 2/12/91
C  version(NLLS8):include odd-parity(skew)series By=SUM(m=0,NS/2)SUM(n=0,
C  NS)B(2m+2,n)*X**n*Y**(2m+1) where B20=P(NPR+1),C21=P(NPR+2),..C(NS+2,0)=
C  P(NP).NR=order of even-parity series,NPR=(NR+2)**2/4.NS=order of odd-
C  parity series,NPS=(NS+2)**2/4.NP=NPR+NPS.
      IMPLICIT REAL*16 (A-H,O-Z)
      DIMENSION P(260),DF(260)
      F=0.0Q0
      NPR=(NR+2)**2/4
      DO 10 I=1,NP
10 DF(I)=0.0Q0
      K=NR+1
      M=0
C  First,evaluate terms of even-parity series to NR-th order.
20 DO 30 N=1,K
      MR=M*(K+M+1)+N
      IF(X1.EQ.0.0Q0) THEN
        IF(X2.EQ.0.0Q0) THEN
          DF(1)=1.0Q0
          F=F+P(1)
          GOTO 50
        ELSE
          DF(MR)=X2**(2*M)
          F=F+P(MR)*DF(MR)
          GOTO 40
        ENDIF
      ELSE
        IF(X2.EQ.0.0Q0) THEN
          IF(M.GT.0) GOTO 50
          DF(MR)=X1**(N-1)
          GOTO 30
        ELSE
          DF(MR)=X1**(N-1)*X2**(2*M)
          GOTO 30
        ENDIF
      ENDIF
30
50

```

```
      ENDIF
    ENDIF
30 F=F+P(MR)*DF(MR)
40 K=K-2
   M=M+1
   IF(M.LE.NR/2) GOTO 20
C Next, evaluate terms of odd-parity series to NS-th order.
50 M=0
   L=NS+1
60 DO 70 N=1,L
   MS=NPR+M*(L+M+1)+N
   IF(X1.EQ.0.0Q0) THEN
     DF(MS)=X2**(2*M+1)
     F=F+P(MS)*DF(MS)
     GOTO 80
   ELSE
     DF(MS)=X1**(N-1)*X2**(2*M+1)
     F=F+P(MS)*DF(MS)
   ENDIF
70 CONTINUE
80 L=L-2
   M=M+1
   IF(M.LE.NS/2) GOTO 60
RETURN
END
```

PROGRAM MULTIPOL

C Obtains the coefficients of the Mth order expansion $B=B_0+B_1*Xt+B_2*Xt**2+B_3*Xt**3+....B_m*Xt**m$ along a straight line perpendicular to orbit at N points $S_1, S_2, S_3, ... S_n$ along the orbit by least squares fit of multipole expansion in curvilinear coordinates to calculated values of B produced by TOSCA at $2L+1$ points along the perpendicular. The orbit coordinates $THETA_i, R_i, R'_i$ and S_i are obtained from file calculated by PROGRAM ORBIT. Also obtained is the integrated multipole strengths and the tabulated multipole coefficients B_{ji} at S_i . The fit is performed by subroutine LLSFIT. For $NOPT=1$ the coefficients are along actual orbit; for $NOPT=2$ along the reference axis. A file of XT vs. BZ is written in DIAGFILE for offline analysis by PROGRAM LLS if $NDIAG>0$. If $NDIAG=-N$, A 2-DIMENSIONAL ARRAY OF BZ VALUES WILL BE COMPUTED AT $(2L+1) \times (L+1)$ POINTS ABOUT THE REFERENCE AXIS AT $THETA=PI/2$ AND WRITTEN TO GRIDFILE FOR SUBSEQUENT ANALYSIS BY PROGRAM LSTSQR2. Modified 8/23/91 to do vertical grid from $-K*DX$ to $+K*DX$.

IMPLICIT REAL*8 (A-H,O-Z)

CHARACTER*10 HOUR, DAY

CHARACTER*12 ORBFILE, OUTFILE, DIAGFILE, GRIDFILE

CHARACTER*80 HEADER

PARAMETER (LMAX=300, MMAX=8, NMAX=1001)

DIMENSION ORBDATA(7, NMAX), XT(LMAX), BZ(LMAX), SIGB(LMAX), P(MMAX),

1 COEFF(MMAX, NMAX), LP(MMAX), COVAR(MMAX, MMAX), SIGP(MMAX, NMAX),

2 SUM(MMAX), CHISQR(NMAX)

COMMON/PARAM/RIG, DEL, RO, YIN, YC

EXTERNAL FUNC

PIOVER2=1.570796326795D0

C Read control data

WRITE(*,*) ' PLEASE ENTER HEADER (A80)= '

READ(*,*) HEADER

WRITE(*,*) ' PLEASE ENTER K, L, M, N, NOPT, NDIAG, DX, XIN, YIN, YC= '

READ(*,*) K, L, M, N, NOPT, NDIAG, DX, XIN, YIN, YC

WRITE(*,*) ' ENTER NAMES OF ORBFILE, OUTFILE, DIAGFILE, GRIDFILE= '

READ(*,*) ORBFILE, OUTFILE, DIAGFILE, GRIDFILE

CALL TIME (HOUR)

CALL DATE (DAY)

OPEN (2, FILE=OUTFILE, STATUS='NEW', FORM='FORMATTED')

WRITE(2, 5) HEADER, DAY, HOUR, L, M, N, NOPT, NDIAG, DX, XIN, YIN, YC, OUTFILE,

1 ORBFILE, DIAGFILE

5 FORMAT(1x, A80, ' PROGRAM MULTIPOL', 2A10/ ' L=', I2, ' M=', I1, ' N=', I3,

1 ' NOPT=', I1, ' NDIAG=', I3, ' DX=', F5.3, ' XIN=', F7.6, ' YIN=', F4.3,

2 ' YC=', F6.4, ' OUTFILE=', A12, ' ORBFILE=', A12, ' DFIL=', A12/

3 ' THETA(R) S(METER) B0(TESLA) B1(T/m**1) B2(T/m**2) ',

4 ' B3(T/m**3) B4(T/m**4) B5(T/m**5) B6(T/m**6) B7(T/m**7)',

5 ' CHISQR')

GOTO (20, 100) NOPT

C NOPT=1. Read in orbit data from ORBFILE.

20 OPEN (3, FILE=ORBFILE, STATUS='OLD', FORM='FORMATTED')

READ(3, 30) ((ORBDATA(I, J), I=1, 7), J=1, N)

30 FORMAT(7(F12.9, 1x))

CLOSE (3)

C Main loop. Evaluate multipoles at N positions S_i on ORBDATA orbit.

DT=ORBDATA(1, 2)-ORBDATA(1, 1)

40 CALL DATAIN(7)

DO 90 I=1, N

C=DCOS(ORBDATA(1, I))

S=DSIN(ORBDATA(1, I))

XI=C*ORBDATA(2, I)

YI=S*ORBDATA(2, I)

IF (DABS(ORBDATA(1, I)-PIOVER2).LE.1.0D-10) GOTO 60

```

RN=ORBDATA(3, I)/ORBDATA(2, I)
B=(-C*RN+S)/(S*RN+C)
ALPHA=DATAN(B)
CA=DCOS(ALPHA)
C      Evaluate BZ vs. XT at points Si using TOSCA.
      DO 50 J=-L, L
      LL=J+L+1
      XT(LL)=DFLOTJ(J)*DX
      XD=XI+XT(LL)*CA
      YD=YI+XT(LL)*CA*B
      R=DSQRT(XD**2+YD**2)
      THETA=DACOS(XD/R)
      CALL ROY (R, THETA, 0.0DO, BR, BT, BPERP)
      BZ(LL)=BPERP
50     SIGB(LL)=1.0DO
      GOTO 70
C      THETA is pi/2. Evaluate BZ along radial.
60     DO 65 J=-L, L
      LL=J+L+1
      XT(LL)=DFLOTJ(J)*DX
      R=ORBDATA(2, I)+XT(LL)
      CALL ROY (R, PIOVER2, 0.0DO, BR, BT, BPERP)
      BZ(LL)=BPERP
65     SIGB(LL)=1.0DO
C      Least squares fit of BZ vs. transverse position XT to polynomial
70     ND=2*L+1
      NP=M+1
      DO 75 J=1, MMAX
      LP(J)=J
      CALL LLSFIT(XT, BZ, SIGB, ND, P, NP, LP, NP, COVAR, MMAX, CHISQ, FUNC)
      CHISQ(I)=CHISQ
      DO 80 J=1, NP
      SIGP(LP(J), I)=DSQRT(COVAR(LP(J), LP(J)))
80     COEFF(J, I)=P(J)
      IF (NDIAG.NE.1) GOTO 90
      OPEN (4, FILE=DIAGFILE, STATUS='NEW', FORM='FORMATTED')
      WRITE(4, 85) (XT(J), BZ(J), J=1, ND)
85     FORMAT (F8.4, 1x, F7.5)
      CLOSE (4)
90     CONTINUE
C      If NDIAG=-N evaluate BZ over a 2-dimensional grid (2L+1)x(L+1).
      IF (NDIAG.EQ.-N.AND.NOPT.EQ.2) THEN
      OPEN(5, FILE=GRIDFILE, STATUS='NEW', FORM='FORMATTED')
C      L2=L/2
      DO 96 I=-K, K
      ZT=DFLOTJ(I)*DX
      DO 92 J=-L, L
      LL=J+L+1
      XT(LL)=DFLOTJ(J)*DX
      R=ORBDATA(2, N)+XT(LL)
      CALL ROY(R, PIOVER2, ZT, BR, BT, BPERP)
92     BZ(LL)=BPERP
      WRITE(5, 94) (BZ(J), XT(J), ZT, J=1, ND)
94     FORMAT (3(1PE17.9))
96     CONTINUE
      CLOSE(5)
      GOTO 200
      ELSE
      GOTO 200
      ENDIF

```

```

C      NOPT=2. Evaluate multipoles along the reference axis starting
C      at X=XIN,Y=YIN. Reference circle with RO=XIN starts at Y=YC.
C      Take N equal increments DT from T=TI to T=pi/2.
100  TI=DATAN(YIN/XIN)
      DT=(PIOVER2-TI)/DFLOTJ(N-1)
      TC=DATAN(YC/XIN)
C Compute theta,r,r' and S at N positions. Store in ORBDATA array.
      T=TI
      DO 110 J=1,N
        ORBDATA(1,J)=T
        IF (DABS(T-PIOVER2).LE.1.0D-10) THEN
          ORBDATA(2,J)=YC+XIN
          ORBDATA(3,J)=0.0D0
          ORBDATA(6,J)=YC-YIN+PIOVER2*XIN
          GOTO 110
        ELSE
          GOTO 105
        ENDIF
105   C=DCOS(T)
      S=DSIN(T)
      IF (T.LE.TC) THEN
        ORBDATA(2,J)=XIN/C
        ORBDATA(3,J)=XIN*S/C**2
        ORBDATA(6,J)=S*ORBDATA(2,J)-YIN
      ELSE
        BETA=DASIN(C*YC/XIN)
        TP=T-BETA
        ORBDATA(2,J)=DSQRT(XIN**2+YC**2+2.0D0*XIN*YC*DSIN(TP))
        ORBDATA(3,J)=XIN*YC*DCOS(TP)/ORBDATA(2,J)*(1.0D0+YC*S/XIN/
1      DSQRT(1.0D0-(YC*C/XIN)**2))
        ORBDATA(6,J)=YC-YIN+TP*XIN
      ENDIF
110   T=T+DT
      GOTO 40
C Integrate multipoles over S with Simpson's Rule. N=odd integer.
200  DO 220 J=1,NP
      SUM(J)=0.0D0
      DO 210 I=1,N
        IF (JMOD(I,2).EQ.0) CONST=4.0D0
        IF (JMOD(I,2).EQ.1) CONST=2.0D0
        IF (I.EQ.1.OR.I.EQ.N) CONST=1.0D0
210   SUM(J)=SUM(J)+CONST*COEFF(J,I)*DSQRT(ORBDATA(2,I)**2+
1      ORBDATA(3,I)**2)
220  SUM(J)=SUM(J)*DT/3.0D0
C      Write coefficients to output file.
      DO 305 J=1,N
        WRITE(2,300) ORBDATA(1,J),ORBDATA(6,J),(COEFF(I,J),I=1,MMAX),
1      CHISQR(J)
300  FORMAT(2(1X,F8.6),8(1X,1PE12.5),1PE10.3)
305  CONTINUE
      WRITE(2,310) (SUM(J), J=1,NP)
310  FORMAT(10X,'INT(B0)=' ,1PE15.8,' TESLA-METERS' /10X,' INT(B1)=' ,
1 1PE15.8,' TESLA' /10X,' INT(B2)=' ,1PE15.8,' TESLA/METER' /10X,
2 'INT(B3)=' ,1PE15.8,' TESLA/METER**2' /10X,' INT(B4)=' ,1PE15.8,
3 ' TESLA/METER**3' /10X,' INT(B5)=' ,1PE15.8,' TESLA/METER**4' /
4 10X,' INT(B6)=' ,1PE15.8,' TESLA/METER**5' /10X,' INT(B7)=' ,1PE15.8,
5 ' TESLA/METER**6')
      CLOSE (2)
      CALL EXIT
      END

```

```

C *****
SUBROUTINE ROY(R, THETA, Z, BR, BTHETA, BZ)
C INPUTS: R, THETA and Z in meters and radians, origin at machine center
C OUTPUTS: BR, BTHETA, BZ in Tesla
C REQUIRED FILE: conin.dat (Conductor description, in a form
C suitable for TOSCO, a collection of routines from TOSCA program)
      IMPLICIT REAL*8 (A-H, O-Z)
      COMMON/PARAM/RIG, DEL, RO, YI, YC
C Convert to CENTIMETRES, and a coordinate system matching the one
C used in (conin.dat)
      XX = (R * dsin(THETA) - YC) * 1.0D2
      YY = - R * dcos(THETA) * 1.0D2
      ZZ = Z * 1.0D2
      CALL SOURCE(XX, YY, ZZ, HXOUT, HYOUT, HZOUT)
C      Compute RADIAL and AZIMUTHAL components
      BR = (HXOUT * dsin(THETA) - HYOUT * dcos(THETA)) * 1.0D-4
      BTHETA = (HXOUT * dcos(THETA) + HYOUT * dsin(THETA)) * 1.0D-4
      BZ = HZOUT * 1.0D-4
      RETURN
      END
C *****
SUBROUTINE LLSFIT(X, Y, SIG, ND, A, MA, LISTA, MFIT, COVAR, NCM, CHISQ, FUNC)
C Linear least squares from Press, et.al., "Numerical Recipes", p.513. Given
C a set of ND points X(I), Y(I) with individual standard deviations SIG(I)
C use chi-square minimization to determine MFIT of MA coefficients A of
C a function that depends linearly on A,  $y = \sum(A \cdot AFUNC(x))$ . The array
C LISTA rennumbers the parameters so that the first MFIT elements corresp
C ond to the parameters actually being determined; the remaining MA-MFIT
C elements are held fixed at their input values. The program returns val
C ues for the MA fit parameters A, CHISQ, and the covariance matrix
C COVAR(I, J). NCM is the physical dimension of COVAR(NCM, NCM) in the
C calling routine. The user supplies a subroutine FUNC(X, AFUNC, MA) that
C returns the MA basis functions evaluated at  $x=X$  in the array AFUNC. The
C program also uses subroutine GAUSSJ for matrix inversion by Gauss-Jord
C an elimination. The variances sigJ2 of parameters A(J) are the diag-
C onal elements CJJ of COVAR.
      IMPLICIT REAL*8 (A-H, O-Z)
      PARAMETER (NMAX=50)
      DIMENSION X(ND), Y(ND), SIG(ND), A(MA), LISTA(MA), COVAR(NCM, NCM),
      1 BETA(NMAX), AFUNC(NMAX)
C Check that LISTA contains proper permutation of coefficients and fill.
      KK=MFIT+1
      DO 12 J=1, MA
        IHIT=0
        DO 11 K=1, MFIT
          IF (LISTA(K).EQ.J) IHIT=IHIT+1
11      CONTINUE
          IF (IHIT.EQ.0) THEN
            LISTA(KK)=J
            KK=KK+1
          ELSE IF (IHIT.GT.1) THEN
            STOP 'Improper set in LISTA'
          ENDIF
12      CONTINUE
          IF (KK.NE.(MA+1)) STOP 'Improper set in LISTA'
C Initialize the symmetric matrix COVAR.
      DO 14 J=1, MFIT
        DO 13 K=1, MFIT
          COVAR(J, K)=0.0D0
13      CONTINUE
14      CONTINUE

```

```

      BETA(J)=0.0D0
14  CONTINUE
C Loop over data to accumulate coefficients of the normal equations.
  DO 18 I=1,ND
    CALL FUNC (X(I),AFUNC,MA)
    YM=Y(I)
    IF (MFIT.LT.MA) THEN
C Subtract off dependences on known pieces of the fitting function.
      DO 15 J=MFIT+1,MA
        YM=YM-A(LISTA(J))*AFUNC(LISTA(J))
15      CONTINUE
      ENDIF
      SIG2I=1.0D0/SIG(I)**2
      DO 17 J=1,MFIT
        WT=AFUNC(LISTA(J))*SIG2I
        DO 16 K=1,J
          COVAR(J,K)=COVAR(J,K)+WT*AFUNC(LISTA(K))
16      CONTINUE
        BETA(J)=BETA(J)+YM*WT
17      CONTINUE
18  CONTINUE
    IF (MFIT.GT.1) THEN
C Fill in above the diagonal from symmetry.
      DO 21 J=2,MFIT
        DO 19 K=1,J-1
          COVAR(K,J)=COVAR(J,K)
19      CONTINUE
21      CONTINUE
      ENDIF
C Solve normal equations by matrix inversion method.
      CALL GAUSSJ(COVAR,MFIT,NCM,BETA,1,1)
      DO 22 J=1,MFIT
C Partial solution to appropriate coefficients A.
        A(LISTA(J))=BETA(J)
22  CONTINUE
C Evaluate goodness of fit CHISQ.
      CHISQ=0.0D0
      DO 24 I=1,ND
        CALL FUNC (X(I),AFUNC,MA)
        SUM=0.0D0
        DO 23 J=1,MA
          SUM=SUM+A(J)*AFUNC(J)
23      CONTINUE
        CHISQ=CHISQ+((Y(I)-SUM)/SIG(I))**2
24  CONTINUE
      RETURN
      END
C ++++++
      SUBROUTINE GAUSSJ (A,N,NP,B,M,MP)
C Linear equation solution by Gauss-Jordan elimination from Press,et.
C al., "Numerical Recipes", p.28. A is an input matrix of N by N elements
C stored in an array of physical dimensions NP by NP. B is an input mat
C rix of N by M containing the M right hand side vectors stored in an
C array of NP by MP. On output A is replaced by its matrix inverse and B
C is replaced by the corresponding set of solution vectors.
      IMPLICIT REAL*8 (A-H,O-Z)
      PARAMETER (NMAX=50)
      DIMENSION A(NP,NP),B(NP,MP),IPIV(NMAX),INDXR(NMAX),INDXC(NMAX)
      DO 11 J=1,N
        IPIV(J)=0

```

```

11 CONTINUE
C Main loop over the columns to be reduced.
DO 22 I=1,N
  BIG=0.0D0
  DO 13 J=1,N
    IF (IPIV(J).NE.1) THEN
      DO 12 K=1,N
        IF (IPIV(K).EQ.0) THEN
          IF (DABS(A(J,K)).GE.BIG) THEN
            BIG=DABS(A(J,K))
            IROW=J
            ICOL=K
          ENDIF
        ELSE IF (IPIV(K).GT.1) THEN
          STOP 'SINGULAR MATRIX'
        ENDIF
      CONTINUE
    ENDIF
  CONTINUE
  IPIV(ICOL)=IPIV(ICOL)+1
C Interchange rows to put the pivot element on the diagonal.
  IF (IROW.NE.ICOL) THEN
    DO 14 L=1,N
      DUM=A(IROW,L)
      A(IROW,L)=A(ICOL,L)
      A(ICOL,L)=DUM
    CONTINUE
    DO 15 L=1,M
      DUM=B(IROW,L)
      B(IROW,L)=B(ICOL,L)
      B(ICOL,L)=DUM
    CONTINUE
  ENDIF
C Divide the pivot row by the pivot element located at IROW and ICOL.
  INDXR(I)=IROW
  INDXC(I)=ICOL
  IF (A(ICOL,ICOL).EQ.0.0D0) STOP 'SINGULAR MATRIX'
  PIVINV=1.0D0/A(ICOL,ICOL)
  A(ICOL,ICOL)=1.0D0
  DO 16 L=1,N
    A(ICOL,L)=A(ICOL,L)*PIVINV
  CONTINUE
  DO 17 L=1,M
    B(ICOL,L)=B(ICOL,L)*PIVINV
  CONTINUE
C Reduce the rows except for the pivot one.
  DO 21 LL=1,N
    IF (LL.NE.ICOL) THEN
      DUM=A(LL,ICOL)
      A(LL,ICOL)=0.0D0
      DO 18 L=1,N
        A(LL,L)=A(LL,L)-A(ICOL,L)*DUM
      CONTINUE
      DO 19 L=1,M
        B(LL,L)=B(LL,L)-B(ICOL,L)*DUM
      CONTINUE
    ENDIF
  CONTINUE
  DO 22 CONTINUE
C End main loop. Interchange pairs of columns in the reverse order.

```



```

      DO 24 L=N,1,-1
        IF (INDXR(L).NE.INDXC(L)) THEN
          DO 23 K=1,N
            DUM=A(K,INDXR(L))
            A(K,INDXR(L))=A(K,INDXC(L))
            A(K,INDXC(L))=DUM
          23 CONTINUE
        ENDIF
      24 CONTINUE
      RETURN
      END
C *****
      SUBROUTINE FUNC (X,P,NP)
C Fitting routine used by linear least squares subroutine LLSFIT to
C evaluate a polynomial of degree NP-1 with NP adjustable coefficients
C A(1),...A(NP).  $y(x)=a(1)+a(2)*x+a(3)*x**2+.....a(NP)*x**(NP-1)$ .
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION P(NP)
      P(1)=1.0D0
      DO 11 J=2,NP
        P(J)=P(J-1)*X
      11 CONTINUE
      RETURN
      END
C *****

```

PROGRAM RANDBZER

C Adds a random error of standard deviation SIGMA to mag field data BZ
 C generated by TOSCADP in INFILE and writes modified data to OUTFILE.
 C NSEED is an integer which starts the random generator RANGAUSS. The
 C data consists of NANGLES blocks of BZ(X,Z) values with NROWS of Z-data
 C arranged in NCOLS of X-values.

```

    IMPLICIT REAL*8 (A-H,O-Z)
    REAL*4 R
    CHARACTER*12 INFILE,OUTFILE
    DIMENSION BZ(8000),X(8000),Z(8000)
    WRITE(*,*) 'Enter NANGLES,NROWS,NCOLS,NSEED,SIGMA='
    READ(*,*) NANGLES,NROWS,NCOLS,NSEED,SIGMA
    WRITE(*,*) 'Enter names of INFILE and OUTFILE='
    READ(*,*) INFILE,OUTFILE
    NLINES=NROWS*NCOLS
    OPEN(1,FILE=INFILE,STATUS='OLD',FORM='FORMATTED')
    OPEN(2,FILE=OUTFILE,STATUS='NEW',FORM='FORMATTED')
  
```

```

C      Loop to process data at each angle
      DO 60 I=1,NANGLES
        READ(1,10) ANGLE
10     FORMAT(7x,F12.10)
        READ(1,20) (BZ(J),X(J),Z(J), J=1,NLINES)
20     FORMAT(3(1PE17.9))
        DO 30 J=1,NLINES
          RG=DBLE(RANGAUSS(NSEED))
          R=RAND(NSEED)
          IF(R.LE.0.5) RG=-RG
30     BZ(J)=BZ(J)+SIGMA*RG
          WRITE(2,40) ANGLE
40     FORMAT(1x,'THETA=',F12.10)
          WRITE(2,20) (BZ(J),X(J),Z(J), J=1,NLINES)
60    CONTINUE
        CLOSE(1)
        CLOSE(2)
        CALL EXIT
      END
      FUNCTION RAND(NSEED)
  
```

C Generates random numbers from uniform distribution in interval
 C 0 to 1. See "Oh! Pascal", D.Cooper & M.Clancy, W.W.Norton & Co.
 C New York,1982.p.225. REAL*4 VAX version.

```

      DATA MULT/25173/,INC/13849/,MODULUS/65536/,FMOD/65536./
      NSEED=JMOD((MULT*NSEED+INC),MODULUS)
      RAND=FLOATJ(NSEED)/FMOD
      RETURN
      END
  
```

FUNCTION RANGAUSS(NSEED)

C Generates random numbers from a normalized Gaussian distribution of
 C rms width=1. The normalized probability function S(X) is the integral
 C (0 to X) P(T)DT where $P(T)=2/\sqrt{2\pi} \cdot \exp(-T^2/2)$. REAL*4 VAX version
 C Random numbers are in interval 0 to 3.9. Use is made of random gener-
 C ator from uniform distribution in interval 0 to 1 from "Oh! Pascal",
 C D.Cooper & M.Clancy,W.W.Norton & Co.(NY,1982) p.225.

```

      DIMENSION X(128),S(128)
      DATA X/0.,.02,.04,.06,.08,.1,.12,.14,.16,.18,.2,.22,.24,.26,.28,
1     .3,.32,.34,.36,.38,.4,.42,.44,.46,.48,.5,.52,.54,.56,.58,.6,.62,
2     .64,.66,.68,.7,.72,.74,.76,.78,.8,.82,.84,.86,.88,.9,.92,.94,.96,
3     .98,1.,1.02,1.04,1.06,1.08,1.1,1.12,1.14,1.16,1.18,1.2,1.22,1.24,
4     1.26,1.28,1.3,1.32,1.34,1.36,1.38,1.4,1.42,1.44,1.46,1.48,1.5,
5     1.52,1.54,1.56,1.58,1.6,1.62,1.64,1.66,1.68,1.7,1.72,1.74,1.76,
6     1.78,1.8,1.82,1.84,1.86,1.88,1.9,1.92,1.94,1.96,1.98,2.,2.05,2.1,
  
```

```
7 2.15,2.2,2.25,2.3,2.35,2.4,2.45,2.5,2.55,2.6,2.65,2.7,2.75,2.8,
8 2.85,2.9,2.95,3.,3.1,3.2,3.3,3.4,3.5,3.7,3.9/
  DATA S/0.,.016,.032,.0478,.0638,.0796,.0956,.1114,.1272,.1428,
1  .1586,.1742,.1896,.2052,.2206,.2358,.251,.2662,.2812,.296,.3108,
2  .3256,.34,.3544,.3688,.383,.397,.4108,.4246,.438,.4514,.4648,
3  .4778,.4908,.5034,.516,.5284,.5408,.5528,.5646,.5762,.5878,.599,
4  .6102,.6212,.6318,.6424,.6528,.663,.673,.6826,.6922,.7016,.7108,
5  .7198,.7286,.7372,.7458,.754,.762,.7698,.7776,.785,.7924,.7994,
6  .8064,.8132,.8198,.8262,.8324,.8384,.8444,.8502,.8558,.8612,
7  .8664,.8714,.8764,.8812,.8858,.8904,.8948,.899,.903,.907,.9108,
8  .9146,.9182,.9216,.925,.9282,.9312,.9342,.9372,.9398,.9426,.9452,
9  .9476,.95,.9522,.9544,.9596,.9642,.9684,.9722,.9756,.9786,.9812,
1  .9836,.9858,.9876,.9892,.9906,.992,.993,.994,.9948,.9956,.9962,
2  .9968,.9974,.998,.9986,.999,.9994,.9996,.9998,1./
  DATA MULT/25173/, INC/13849/, MODULUS/65536/, FMOD/65536./
  NSEED=JMOD((MULT*NSEED+INC),MODULUS)
  R=FLOATJ(NSEED)/FMOD
  DO 10 I=2,128
    IF (R.LE.S(I)) THEN
      RANGAUSS=X(I)-(S(I)-R)*(X(I)-X(I-1))/(S(I)-S(I-1))
      GOTO 20
    ENDIF
10  CONTINUE
    RANGAUSS=X(128)
20  RETURN
  END
```