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Wall Stabilization Action on MHD Instabilities

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MASTER

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WALL STABILIZATION ACTION ON MHD INSTABILITIES

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ABSTRACT

This paper presents the effects of a perfectly conducting wall on the MHD stability behavior of high beta tokamak equilibria. Toroidal wave numbers $N = 0, 1$, and 2 have been considered. For a reasonable wall-plasma distance, a range of stable operation corresponding to beta values greater than 10% has been found.

1. INTRODUCTION

The FCT concept¹ has considerably changed our understanding of tokamak limitations. Equilibria characterized by an average beta up to 40% have been obtained², well above the value in current experiments. If such equilibria are stable against gross instabilities, the impact on the design of an electric power reactor is considerable, decreasing its size by a factor of three and increasing its output power by a factor of ten.

Magnetohydrodynamic (MHD) stability computations on ballooning modes have shown critical beta values up to 10% for equilibria characterized by flat current profiles.^{3,4} To fulfill this condition, the gradient of the toroidal current at the plasma edge induces a strong kink instability. The balance between these two effects leads to an optimization of the current profile and gives rise to a beta limit in the range of 2-4% for an elongated plasma cross section^{4,5} surrounded by an infinite vacuum.

The optimal beta can be increased if a perfectly conducting wall lies at some distance around the plasma to stabilize surface modes.

The purpose of this letter is to study the critical wall position required to avoid the kink instability. We have chosen one of the worst equilibria with respect to kink behavior, namely the Solov'ev equilibrium.⁶ The toroidal wave numbers $N = 0, 1$, and 2 have been considered.

This study also tests the vacuum module implemented in the ERATO code.⁷ The results of previous computations with both no wall and with a wall tight against the plasma surface⁸ are nicely reproduced. Only a careful treatment of the singularities⁹ linked with the Green's function technique used can lead to such a good agreement.

For low- N modes ($N = 0, 1$) the shell position constraint is very flexible. However, for larger N , the wall-plasma distance required can vary between 5% and 15% of the plasma radius.

Wall stabilization, supplied by a feedback system acting on the time scale of the field diffusion across the shell, may have to be designed in high beta tokamak experiments ($\beta > 5\%$).

2. EQUILIBRIUM PARAMETERS

We have chosen the Solov'ev equilibrium⁶ given by $\psi(r, Z) = \frac{E}{2q_0} \frac{a^2}{R^2} \left[\frac{r^2 Z^2}{E^2} + \frac{(r^2 - R^2)^2}{4} \right]$ and characterized by three parameters: the aspect ratio R/a , the vertical elongation E , and the safety factor on axis $q(0)$. The poloidal beta is equal to 1 and the aspect ratio is kept fixed at 3; note that the current is linear in r and that there is a substantial amount of shear since $q(\psi_s)/q_0(0) = 1.74$. We assume also uniform mass density ρ ; the index s refers to the plasma surface.

The wall is defined as

$$\rho_l(\theta) = \rho_p(\theta) [\Lambda + f(\Lambda, \theta)]$$

where ρ_p and ρ_w are the polar radii from the magnetic axis to the wall and the plasma, respectively. The polar angle θ is defined by the magnetic axis and the midplane of symmetry. Λ defines the distance, plasma wall, and $F(\Lambda, \theta)$ equals

$$F(\Lambda, \theta) = 0$$

$$\Lambda \leq \frac{0.99}{\rho_p(\pi)} \quad (1)$$

$$F(\Lambda, \theta) = \left[\frac{0.99}{\rho_p(\pi)} - 1 \right] \left(\sin^2 \frac{\theta}{2} - \frac{1}{2} \sin^2 \theta \right) \quad \Lambda > \frac{0.99}{\rho_p(\pi)} \quad (2)$$

Equation (2) simply means that the inner point of the wall on the midplane of up-down symmetry is kept fixed for $\Lambda > 0.99/\rho_p(\pi)$.

The stability analysis is performed with the spectral ERATO code.⁷ Growth rates are normalized with respect to the poloidal Alfvén frequency $\omega_A = B_T/\sqrt{\rho} q(0)R$ where B_T is the toroidal magnetic field on axis.

3. RESULTS

3.1. AXISYMMETRIC MODE

In Fig. 1, the square of the normalized growth rate of the axisymmetric instability is plotted versus the shell position Λ . The elongation of the plasma cross section is equal to 2 and the safety factor on the axis is kept fixed at 1. $E = 2$ gives an elongated, D-shaped cross section very close to the JET design; the corresponding Λ value equals 1.5. Note that the variation of the growth rate is proportional to the volume between the plasma and the wall $1 - 1/\Lambda^3$. By extrapolation, we determine that the critical value for Λ for stability is $\Lambda_c = 1.8$. Such a big value reflects the strong effect of the wall in stabilizing axisymmetric modes.

3.2. $N = 1, N = 2$

Figure 2 represents the growth rate of the most unstable mode versus Λ for toroidal mode numbers $N = 1$ and $N = 2$. Dashed lines refer to a circular plasma cross section ($E = 1$) and the solid line to an elongated D-shape ($E = 2$). On each curve the safety factor is kept fixed. The

variation is proportional to $1 - 1/\Lambda^2$ over a wide range of values of Λ , and both limits ($\Lambda \rightarrow 1, \Lambda \rightarrow \infty$) are well reproduced.⁹ As previously found there is no internal mode for $N = 1, E = 1$. In that case the critical value of Λ to completely stabilize the external kink is $\Lambda_c \approx 1.25$. For other cases we see that the critical value of Λ decreases as N increases ($\Lambda_c = 1.11$ for $N = 2$); also the wall has to be closer for higher elongations ($\Lambda_c = 1.11$ for $E = 2$).

The technological constraints induced by these Λ values are severe but do not jeopardize the use of wall stabilization to increase the total average beta. $E = 1, q_0 = 1$ correspond to an average beta of 10%.

3.3. q_s Dependence

We now investigate the variations of the growth rate of the most unstable mode versus q_0 for different wall locations. Results are shown in Fig. 3. Two important points have to be noted. At high q_0 the three curves do not tend together as expected for a cylindrical plasma discharge. This phenomenon is due to the toroidal mode coupling. The $M = 1$ cylindrical mode is always present, even if $q_s = 4$ at the plasma surface. For a reasonable wall location, $\Lambda = 1.55$, we obtain stable ranges of operation for a high beta tokamak device, for example $0.7 \leq q_0 \leq 1$ or $q_0 > 1.2$; this latter result is in qualitative agreement with a recent analytical model.¹⁰ The critical distance of the stabilizing shell is plotted versus q_0 in Fig. 4. Each oscillation corresponds to an increment of one in the dominant poloidal Fourier component of the mode.

4. CONCLUSION

For a toroidal, low aspect ratio discharge surrounded by an infinite vacuum region, beta limitations imposed by MHD stability considerations lie in the range of 2-3%. In order to increase this value up to 5-10%, other factors such as force-free equilibrium current, cold plasma blanket, or wall stabilization have to be included. We have studied the effect of wall stabilization on a high beta, flat current equilibrium. Parallel computations have shown that this equilibrium is ballooning stable.³ For a stabilizing shell at a reasonable distance from the plasma, a range of operation has been found with a corresponding beta value greater than 10%.

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FIGURE CAPTIONS

- Fig. 1. Growth rate Γ^2 of the most stable axisymmetric mode versus the shell position Λ . $q_0 = 1$.
- Fig. 2. Growth rate Γ^2 of the most unstable $N = 1$ and $N = 2$ modes versus the shell position Λ (— $E = 2$, --- $E = 1$).
- Fig. 3. Plot of the normalized growth rate Γ^2 of the most unstable mode versus q_0 . Four different shell positions (Λ) are represented.
- Fig. 4. Critical shell position for stability versus q_0 . The dashed region corresponds to the stability requirements ($E = 1$, $N = 2$).

Fig. 1.

Fig. 2.

Fig. 3.

Fig. 4.

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