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THE MULTICOMPONENT LAMINAR
BOUNDARY LAYER OVER GRAPHITE SPHERE
CONES: SOLUTIONS FOR QUASISTEADY
ABLATION AND APPLICATION TO
TRANSIENT REENTRY TRAJECTORIES

Prepared by
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AND APPLICATION TO TRANSIENT REENTRY TRAJECTORIES

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ABSTRACT

Nonsimilar laminar boundary layer solutions are presented for sphere-cone body shapes with and without the addition of graphite ablation products for a matrix of ballistic reentry conditions. The diffusive heat and mass fluxes for these solutions are correlated, to some extent, in terms of convective transfer coefficients consistent with the definitions employed in the Aerotherm Charring Material Ablation (CMA) and Aerotherm Chemical Equilibrium (ACE) computer programs.

Finally, a procedure is outlined for utilizing these correlations and programs to perform transient reentry predictions for graphite in the vicinity of the stagnation point.

SUMMARY

Results obtained with the Boundary Layer Integral Matrix Procedure (BLIMP) are presented for nonsimilar laminar boundary layers over sphere-cone body shapes with and without the addition of graphite ablation products. A range of flight conditions corresponding to moderate-to-severe ballistic reentry is considered. The solutions consider homogeneous equilibrium throughout the boundary layer and consider unequal diffusion and thermal diffusion coefficients for all species through the use of convenient approximations. The air boundary layer solutions are for various uniform wall temperatures, whereas the graphite solutions consider surface equilibrium and satisfy mass and energy balances at the wall with quasisteady conduction into the solid. It is shown that the quasisteady assumption is quite good during regions of peak heating. The results are directly applicable for estimating graphite ablation rates and surface temperatures under those conditions.

The diffusive heat and mass fluxes are presented in terms of convective transfer coefficients consistent with the definitions employed in the Charring Material Ablation (CMA) and Aerotherm Chemical Equilibrium (ACE) computer programs. A procedure is outlined for utilizing these data and these codes to perform transient reentry predictions for graphite in the vicinity of the stagnation point. Results obtained using this procedure should be nearly as accurate as those which would be obtained by direct application of the BLIMP procedure since it was found possible to obtain a blowing correction relation which, although based on results from a wide range of flight conditions, was seen not to depend upon those conditions and, hence, would not be expected to depend on surface temperature.

Although a number of nonsimilar boundary-layer solutions were generated for stations around the body, the level of effort was not sufficient to permit the development of correlation procedures to duplicate BLIMP results with CMA/ACE. With regard to the nonsimilar boundary-layer solutions, a significant nonsimilar effect was predicted to occur. Under reentry conditions sufficiently severe such that extensive sublimation occurs in the stagnation region, ablation rates downstream of the sphere-cone juncture are reduced well below that which would be predicted on the basis of a similar boundary-layer procedure, even becoming negative (i.e., deposition) in the most severe cases.

Finally, the present effort represents, to some extent, a validity check on the transfer-coefficient correlation approach for unequal diffusion coefficients proposed in previous publications and used in the CMA/ACE programs (but never substantiated by comparison to boundary-layer solutions). In particular, one of the basic requirements of the correlation procedure, that the mass fluxes be expressible in terms of a single mass-transfer coefficient, was seen to be satisfied, at least for the three-element chemical system considered herein.

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It is with sincere appreciation that the authors acknowledge the outstanding efforts of Mrs. Violet Nicholson in preparing the auxiliary computer programs and plot routines utilized in this study and of Mrs. Beverly Moore and Mrs. Faye Baxter for their careful typing of the text and numerous tables. Also, we wish to recognize the valuable contributions from several members of the Aerotherm staff, in particular, Thomas J. Dahm, Robert M. Kendall, Carl B. Moyer, William E. Nicolet, and Roald A. Rindal, and to thank Sandia Corporation for making this study possible.

TABLE OF CONTENTS

	Page
LIST OF FIGURES	vii
LIST OF TABLES	viii
LIST OF SYMBOLS	xiii
1. INTRODUCTION	1
2. AIR BOUNDARY LAYER SOLUTIONS	7
2.1 Stagnation Point	7
2.1.1 Solutions for Equal and Unequal Diffusion Coefficients	7
2.1.2 Effect of Nose Radius	17
2.1.3 Effect of Surface Temperature	20
2.2 Downstream Stations	20
2.2.1 Nonsimilar Solutions for Unequal Diffusion Coefficients	25
2.2.2 Effect of Nose Radius	33
2.2.3 Effect of Cone Angle	33
2.2.4 Effect of Unequal Diffusion Coefficients	39
2.2.5 Effect of Nonsimilar Pressure Gradient	39
3. BOUNDARY LAYER SOLUTIONS FOR QUASISTEADY GRAPHITE ABLATION	44
3.1 Accuracy Considerations	44
3.1.1 Effect of Number of Nodal Points	44
3.1.2 Effect of Assumed Profile Shape	49
3.2 Stagnation Point	49
3.2.1 Solutions for Equal and Unequal Diffusion Coefficients	53
3.2.1.1 Ablation Rate and Surface Temperature	53
3.2.1.2 Mass-Transfer Coefficient	59
3.2.1.3 Blowing Parameter	60
3.2.1.4 Heat-Transfer Coefficient for Equal Diffusion Coefficients	65
3.2.1.5 Diffusive Heat Flux	65
3.2.1.6 Other Boundary-Layer Parameters	67
3.2.1.7 Boundary-Layer Profiles	67

TABLE OF CONTENTS (concluded)

	Page
3.2.2 Comparison to Nonablating (Air) Solutions	70
3.2.2.1 Diffusive Heat Flux	70
3.2.2.2 Heat-Transfer Coefficient for Equal Diffusion Coefficients	72
3.2.2.3 Skin-Friction Coefficient	75
3.2.2.4 Momentum and Displacement Thicknesses	77
3.3 Downstream Stations	77
3.3.1 Ablation Rate	78
3.3.2 Mass-Transfer Coefficient and Blowing Parameter	88
3.3.3 Surface Temperature	91
3.3.4 Additional Boundary-Layer Parameters	94
3.4 Applicability of Quasisteady Ablation Approximation	94
4. CORRELATION OF BOUNDARY-LAYER SOLUTIONS WITH CONVECTIVE TRANSFER-COEFFICIENT APPROACH	100
4.1 General Remarks	100
4.2 Review of Transfer-Coefficient Formulation	101
4.2.1 Mass Transport	101
4.2.2 Energy Transport	104
4.3 Application of Transfer-Coefficient Formulation to Graphite Boundary-Layer Solutions	106
4.4 Compatibility Between BLIMP and CMA/ACE Programs	110
5. PROCEDURE FOR APPLYING QUASISTEADY GRAPHITE ABLATION RESULTS TO TRANSIENT PROBLEMS	115
6. SUMMARY OF CONCLUSIONS AND RECOMMENDATIONS	117
APPENDIX A - Elemental Mass Transfer Coefficients Versus γ for Quasisteady Graphite Stagnation-Point Ablation	121
APPENDIX B - Heat and Mass Transfer Coefficients versus γ for Quasisteady Graphite Stagnation-Point Ablation	153

LIST OF FIGURES

1. Pressure Distribution Around a Sphere Cone: Cone Half Angle = 5° (After Chushkin and Shulishnina as Reported by Ellett)	5
2. Normalized Heat-Transfer Coefficient in a Stagnation Point Air Boundary Layer with Equal Diffusion Coefficients	
a. Wall Temperature = $1,000^\circ\text{R}$	8
b. Wall Temperature = $4,000^\circ\text{R}$	9
c. Wall Temperature = $6,000^\circ\text{R}$	10
d. Wall Temperature = $8,000^\circ\text{R}$	11
3. Normalized Heat-Transfer Coefficient in a Stagnation Point Air Boundary Layer with Unequal Diffusion Coefficients	
a. Wall Temperature = $1,000^\circ\text{R}$	12
b. Wall Temperature = $4,000^\circ\text{R}$	13
c. Wall Temperature = $6,000^\circ\text{R}$	14
d. Wall Temperature = $8,000^\circ\text{R}$	15
4. Ratio of Ablation Rates for Unequal and Equal Diffusion Coefficients for Quasisteady Stagnation Point Graphite Ablation: $R_{\text{eff}} = 1 \text{ inch}$, $\epsilon_w = 1.0$	58
5. Quasisteady Stagnation Point Blowing Parameters for Graphite versus Stagnation Pressure for Various Stagnation Enthalpies: $R_{\text{eff}} = 1 \text{ inch}$, $\epsilon_w = 1.0$	61
6. Relative Importance of Carbon Sublimation and Combustion Products as a Function of Total Pressure for Quasisteady Stagnation-Point Graphite Ablation: $H_T = 13,000 \text{ Btu/lb}$, $R_{\text{eff}} = 1 \text{ Inch}$, $\epsilon_w = 1.0$	63
7. Blowing Parameter for Graphite Ablation in Air for Equal and Unequal Diffusion Coefficients in Terms of Wall Temperature and Pressure	64
8. Ratio of Ablating to Nonablating Diffusive Heat Flux for Quasisteady Stagnation Point Graphite Ablation (Unequal Diffusion, $R_{\text{eff}} = 1 \text{ inch}$, $\epsilon_w = 1.0$)	71
9. Heat-Transfer Coefficient for Quasisteady Stagnation Point Graphite Ablation Normalized by Nonablating Values as a Function of B' (Equal Diffusion, $R_{\text{eff}} = 1 \text{ inch}$, $\epsilon_w = 1.0$)	73
10. Heat-Transfer Coefficient for Quasisteady Stagnation Point Graphite Ablation Normalized by Nonablating Values as a Function of $B'C_H/C_{H_0}$ (Equal Diffusion, $R_{\text{eff}} = 1 \text{ inch}$, $\epsilon_w = 1.0$)	74

LIST OF FIGURES (concluded)

11.	Skin-Friction Coefficient for Quasisteady Stagnation Point Graphite Ablation Normalized by Nonablating Values as a Function of $B'C_f/C_{f_o}$ (Unequal Diffusion, $R_{eff} = 1$ inch, $\epsilon_w = 1.0$)	76
12.	Blowing Parameter Around a Sphere Cone for Quasisteady Graphite Ablation: $R_N = 1$ inch, $\theta_c = 5^\circ$, $\epsilon_w = 1.0$	90
13.	Wall Temperatures Normalized by Stagnation Values for Quasisteady Graphite Ablation Around a Sphere Cone: $R_N = 1$ inch, $\theta_c = 5^\circ$, $\epsilon_w = 1.0$	
	a. Total Enthalpy = 11,500 Btu/lb	92
	b. Total Pressure = 30 atm	93
14.	Heat Conducted into Graphite Stagnation Point During Quasisteady Ablation: $R_{eff} = 1$ inch, $\epsilon_w = 1.0$	95
15.	Typical Ballistic Reentry Trajectory	97
16.	Comparison of Graphite Stagnation Point Quasisteady and Transient Ablation Rates and Surface Temperatures for Typical Ballistic Reentry Trajectory	98

LIST OF TABLES

I	Matrix of Flight Conditions Considered in this Study	2
II	Effect of Unequal Diffusion Coefficients on Diffusive Heat Flux, Skin Friction Coefficient, Momentum Thickness, and Displacement Thickness for a Stagnation Point Air Boundary	
	a. High Total Enthalpy	18
	b. Moderate to Low Total Enthalpy	19
III	Effect of Surface Temperature on the Stagnation Point Air Boundary Layer: Unequal Diffusion Coefficients, Effective Nose Radius = 1 Inch	
	a. Diffusive Heat Flux	21
	b. Skin Friction Coefficient	22
	c. Momentum Thickness	23
	d. Displacement Thickness	24

LIST OF TABLES (continued)

IV	<p>Nonsimilar Air Boundary Layer Over A Sphere Cone with Unequal Diffusion Coefficients: Nose Radius = 1 inch, Cone Half Angle = 5°, Wall Temperature = $1,000^\circ\text{R}$</p> <p>a. Reynolds Number per Foot 26</p> <p>b. Diffusive Heat Flux 27</p> <p>c. Wall Shear 28</p> <p>d. Heat Transfer Coefficient 29</p> <p>e. Skin Friction Coefficient 30</p> <p>f. Momentum Thickness 31</p> <p>g. Displacement Thickness 32</p>
V	<p>Effect of Nose Radius on Diffusive Heat Flux, Skin Friction Coefficient, Momentum Thickness and Displacement Thickness for a Nonsimilar Air Boundary Layer Over a Sphere Cone with Unequal Diffusion Coefficients: Cone Half Angle = 5°, Wall Temperature = $1,000^\circ\text{R}$</p> <p>a. Total Enthalpy = 11,500 Btu/lb, Total Pressure = 1 atm 34</p> <p>b. Total Enthalpy = 11,500 Btu/lb, Total Pressure = 10 atm 35</p> <p>c. Total Enthalpy = 11,500 Btu/lb, Total Pressure = 30 atm 36</p> <p>d. Total Enthalpy = 10,000 Btu/lb, Total Pressure = 30 atm 37</p>
VI	<p>Effect of Cone Half Angle on Diffusive Heat Flux, Skin Friction Coefficient, Momentum Thickness, and Displacement Thickness for a Nonsimilar Air Boundary Layer over a Sphere Cone with Unequal Diffusion Coefficients: Nose Radius = 1 inch, Wall Temperature = $1,000^\circ\text{R}$, Total Enthalpy = 11,500 Btu/lb, Total Pressure = 30 atm 38</p>
VII	<p>Effect of Unequal Diffusion Coefficients on Diffusive Heat Flux, Skin Friction Coefficient, Momentum Thickness, and Displacement Thickness for a Nonsimilar Air Boundary Layer Over a Sphere Cone: Nose Radius = 1 inch, Cone Half Angle = 10°, Wall Temperature = $1,000^\circ\text{R}$, Total Enthalpy = 11,500 Btu/lb, Total Pressure = 30 atm 40</p>

LIST OF TABLES (continued)

VIII	<p>Nonsimilar Effects on Diffusive Heat Flux, Skin Friction Coefficient, Momentum Thickness and Displacement Thickness for an Air Boundary Layer Over a Sphere Cone: Nose Radius = 1 inch, Wall Temperature = 1,000°R, Total Enthalpy = 11,500 Btu/lb, Total Pressure = 30 atm</p> <p>a. Unequal Diffusion Coefficients, Cone Angle = 10°</p> <p>b. Equal Diffusion Coefficients, Cone Angle = 5°</p>	<p>41</p> <p>42</p>
IX	<p>Effect of Numbers of Nodal Points on Stagnation Point Boundary Layer Parameters with Quasisteady Ablation of Graphite: Unequal Diffusion Coefficients, Effective Nose Radius = 1 inch, Surface Emissivity = 1</p>	<p>46</p>
X	<p>Effect of Number of Nodal Points on Stagnation Point Boundary Layer Profiles with Quasisteady Ablation of Graphite: Unequal Diffusion Coefficients, Effective Nose Radius = 1 inch, Surface Emissivity = 1, Total Enthalpy = 13,000 Btu/lb, Total Pressure = 30 atm</p> <p>a. Stream Function, Velocity Ratio, Shear Function, and Thermodynamic Properties</p> <p>b. Mole Fractions of Principal Species</p>	<p>47</p> <p>48</p>
XI	<p>Effect of Assumed Profile Shape on a Stagnation Point Boundary Layer With Quasisteady Ablation of Graphite: Equal Diffusion Coefficients, Effective Nose Radius = 1 inch, Surface Emissivity = 1, Total Enthalpy = 11,500 Btu/lb, Total Pressure = 30 atm</p> <p>a. Boundary Layer Parameters</p> <p>b. Profiles of Stream Function, Velocity Ratio, Shear Function, and Thermodynamic Properties</p> <p>c. Concentration Profiles of Principal Species</p>	<p>50</p> <p>51</p> <p>52</p>
XII	<p>Effect of Unequal Diffusion Coefficients on Ablation Rate, Surface Temperature, Mass Transfer Coefficient, and Blowing Parameter for a Stagnation Point Boundary Layer with Quasisteady Ablation of Graphite: Effective Nose Radius = 1 inch, Surface Emissivity = 1</p> <p>a. High Total Enthalpy</p> <p>b. Moderate to Low Total Enthalpy</p>	<p>54</p> <p>55</p>

LIST OF TABLES (continued)

XIII	Effect of Unequal Diffusion Coefficients on Diffusive Heat Flux, Skin Friction Coefficient, Momentum Thickness, and Displacement Thickness for a Stagnation Point Boundary Layer with Quasisteady Ablation of Graphite: Effective Nose Radius = 1 inch, Surface Emissivity = 1	
	a. High Total Enthalpy	56
	b. Moderate to Low Total Enthalpy	57
XIV	Comparison of Heat and Mass Transfer Coefficients for a Stagnation Point Boundary Layer with Quasisteady Ablation of Graphite: Effective Nose Radius = 1 inch, Surface Emissivity = 1, Equal Diffusion Coefficients	66
XV	Effect of Unequal Diffusion on Stagnation Point Boundary Layer Profiles with Quasisteady Ablation of Graphite: Effective Nose Radius = 1 inch, Surface Emissivity = 1, Total Enthalpy = 11,500 Btu/lb, Total Pressure = 30 atm	
	a. Stream Function, Velocity Ratio, Shear Function, and Thermodynamic Properties	68
	b. Mole Fractions of Principal Species	69
XVI	Nonsimilar Boundary Layer Over a Sphere Cone with Quasisteady Ablation of Graphite: Nose Radius = 1 inch, Cone Half Angle = 5° , Surface Emissivity = 1, Unequal Diffusion Coefficients	
	a. Ablation Rate	79
	b. Surface Temperature	80
	c. Diffusive Heat Flux	81
	d. Wall Shear	82
	e. Skin Friction Coefficient	83
	f. Mass Transfer Coefficient	84
	g. Blowing Parameter	85
	h. Momentum Thickness	86
	i. Displacement Thickness	87

LIST OF TABLES (concluded)

XVII	Heat and Mass Transfer Coefficients for Quasi-steady Stagnation Point Graphite Ablation with Unequal Diffusion Consistent with Aerotherm Transfer-Coefficient Model	111
XVIII	Comparison of BLIMP and CMA/ACE Predictions for Quasisteady Graphite Stagnation-Point Ablation	114

LIST OF SYMBOLS

B'	blowing parameter, defined by Equation (5)
c_s	heat capacity of the solid
$\bar{D}$	a reference binary diffusion coefficient introduced by the approximation for D_{ij} embodied in Equation (14)
D_{ij}	binary diffusion coefficient for species i and j
f	stream function
F_i	diffusion factor for species i introduced by the approximation for D_{ij} embodied in Equation (14)
h_{c_w}	enthalpy of the surface material evaluated at wall temperature
h_i	static enthalpy of species i
h_i^o	heat of formation of species i
h_s	sensible enthalpy
H_{s_r}	recovery enthalpy
H_T	total enthalpy
$\tilde{j}_{k_w}$	diffusional mass flux of element k per unit area away from the surface
K	number of elements in chemical system of interest

LIST OF SYMBOLS (continued)

K_i	mass fraction of species i
$\bar{K}_k$	mass fraction of element k irrespective of molecular configuration, defined by Equation (3)
Le	Lewis number
M	Mach number
m_i	molecular weight of species i
$\dot{m}_w$	surface mass ablation rate
N	number of species in chemical system of interest
P	pressure
P_{T_2}	total pressure in the stagnation region of the boundary layer
q_a	diffusive heat flux per unit area towards the surface
q_{cond}	heat conducted per unit area into the solid
R_{eff}	effective nose radius
R_N	actual nose radius
Re_s	Reynolds number based on wetted distance
s	wetted distance along body from stagnation point
$\dot{s}$	surface recession rate
t	time
T	temperature
u	velocity in the streamwise direction s

LIST OF SYMBOLS. (continued)

x_i	mole fraction of species i
y	distance from surface measured into boundary layer
Z_i	a quantity for species i intermediate between K_i and x_i which is introduced as a result of the approximation for δ_{ij} and which reduces to K_i when all δ_{ij} are assumed equal, defined by Equation (13)
Z_i^*	a quantity for species i intermediate between Z_i and K_i which is introduced to achieve a satisfactory transfer-coefficient formulation considering unequal δ_{ij} and which reduces to K_i when all δ_{ij} are assumed equal, defined by Equation (18)
$\tilde{Z}_k$	a quantity for element k which is introduced as a result of the approximation for δ_{ij} and which reduces to $\tilde{K}_k$ when all δ_{ij} are assumed equal, defined by Equation (12)
$\tilde{Z}_k^*$	a quantity for element k which is introduced to achieve a satisfactory transfer-coefficient formulation considering unequal δ_{ij} and which reduces to $\tilde{K}_k$ when all δ_{ij} are assumed equal, defined by Equation (17)
α_H	normalizing parameter used in definition of η defined implicitly by the constraint that the velocity ratio at a given boundary-layer nodal point take on a particular value
α_{ki}	mass fraction of element k in species i
γ	a parameter introduced by the definition of Z_i^*
δ^*	boundary-layer displacement thickness
Δl	effective thickness of a slab of surface material within the context of a lumped capacitance transient heat conduction approximation
ϵ_w	surface emittance
η	transformed boundary-layer coordinate in a direction normal to the surface

LIST OF SYMBOLS (concluded)

λ	blowing reduction parameter, defined implicitly by Equations (7) and (8)
μ	viscosity
ρ	density
$\rho_e^u C_f/2$	skin-friction coefficient
$\rho_e^u C_H$	heat-transfer coefficient, defined in general by Equation (25)
$\rho_e^u C_M$	system mass-transfer coefficient (can be defined only for equal diffusion coefficients or if transfer-coefficient formulation is successful in causing the $\rho_e^u C_{M_k}$ to all be equal)
$\rho_e^u \bar{C}_M$	system mass-transfer coefficient defined arbitrarily (e.g., by Eq. (24))
$\rho_e^u C_{M_k}$	mass-transfer coefficient for element k, defined in general by Equation (23)
τ_w	wall shear per unit area
φ	parameter utilized in blowing reduction relationship, defined by Equation (8)

SUBSCRIPTS

c	refers to solid (wall) material
e	evaluated at boundary-layer edge
i	refers to species i
j	refers to species j
k	refers to element k
o	nonablating value, evaluated at ablation temperature
w	evaluated at the wall

SUPERSCRIPTS

' (prime)	represents partial differentiation with respect to η
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1. INTRODUCTION

The purpose of this report is threefold: 1) to present solutions for the nonsimilar multicomponent chemically-reacting boundary layer over ablating graphite sphere cones considering coupled quasisteady wall boundary conditions, 2) to express these results in terms of a convective transfer coefficient formulation proposed previously and thus to attempt to substantiate that formulation, and 3) to outline an approach by which the boundary-layer results can be applied through use of these transfer coefficients to obtain transient graphite ablation predictions.

The boundary layer solutions and correlation procedures have been conducted for a matrix of conditions of interest in moderate to severe ballistic reentry trajectories. The matrix of stagnation enthalpies and stagnation pressures which are considered are presented in Table I. At least three enthalpies were considered for each pressure, and vice versa, to provide a means for nonlinear interpolation. The bulk of the solutions were performed for a nose radius of 1 inch and a cone half angle of 5 degrees, but some other results are presented which should permit consideration of other nose radii and cone angles.

The boundary layer solutions were generated with the Boundary Layer Integral Matrix Procedure (BLIMP) computer code described in Reference 1. The boundary layer was considered to be in equilibrium and unequal diffusion and thermal diffusion coefficients for all species were considered through use of the convenient bifurcation approximations described in Reference 2. Heterogeneous equilibrium between the boundary layer and the graphite surface was also considered. The boundary layer was coupled to relations for conservation of mass, species, and energy at the surface. Thus, there were no assignments of such quantities as heat transfer rates, surface temperatures, or ablation rates; rather, these and all other quantities of interest

TABLE I

MATRIX OF FLIGHT CONDITIONS CONSIDERED IN THIS STUDY

Stagnation Enthalpy (Btu/lb)	Stagnation Pressure, atm					
	0.01	0.1	1.0	10.0	30.0	100.0
13,000	x	x	x	x	x	x
11,500	x	x	x	x	x	x
10,000	x	x	x	x	x	x
8,000			x	x	x	x
6,000				x	x	x
4,000				x	x	x
2,000				x	x	x

were calculated given only boundary-layer edge conditions, body shape, position on the body, and material properties. Conduction into the body was considered to be one-dimensional and quasisteady. It is shown that the quasisteady assumption is quite adequate over most of the trajectory if ablation rates are of primary concern.

The BLIMP program uses a newly developed integral matrix solution procedure. This procedure is, in the limit, capable of yielding exact solutions for the particular boundary-layer equations which are considered. In this approach, the boundary layer is divided into a number of segments and the primary dependent variables are considered to vary as either cubics or quadratics (this option is left up to the user) with continuous derivatives at the junction points (so called spline fits). Cubic spline fits between 7 nodal points were employed in the present calculations. A stagnation point solution for quasisteady graphite ablation was performed using quadratic spline fits which showed differences in quantities of interest at the wall of the order of only one-tenth of one percent. A matrix of stagnation-point solutions for quasisteady graphite ablation

was generated using 13 nodal points which demonstrated differences when compared to the 7 point solutions of as much as one percent in surface conditions in some cases. Therefore, although the solutions are presented to four significant figures, the accuracy with which the equations are solved is more like three significant figures. However, as will be seen, the results are very precise (i.e., results tend to vary smoothly to four significant figures).

The BLIMP program treats streamwise derivatives with three-point finite-difference relations. No accuracy checks were conducted relative to the step sizes which were employed. However, based on past experience with similar problems, errors in solutions for downstream stations could be of the order of possibly as much as 2 to 5 percent for some problems when nonsimilar effects are strong.

With regard to accuracy, the reader should also keep the following considerations in mind. Although the present mathematical model includes most of the physical phenomena known to be important it is itself not precise nor is the physical model free from uncertainties. First, the approximations for multi-component transport properties described in Reference 2 are employed and curve fits are used for representing species thermodynamic data. Secondly, the present results are for laminar flow whereas transition to turbulent flow may occur in some of the cases considered. Thirdly, only thermochemical ablation is considered in the graphite calculations, whereas mechanical erosion may be important. Fourthly, an isentropic expansion around the body from the stagnation region has been assumed, whereas the streamlines which enter the boundary layer over the cone have actually crossed a weaker (oblique) shock and thus have lower entropies and higher velocities. Finally, the solutions for stations around the body are highly dependent on the pressure distribution and, therefore, are only as good as the pressure data employed. The pressure distributions used in the present

effort were obtained from the calculations of Chushkin and Shulishnina (Ref. 3) as reported by Ellett (Ref. 4). Pressures normalized by the stagnation values are shown in Figure 1 versus s/R_N (where s is the streamwise dimension measured along the surface and R_N is the nose radius) for various values of H_T (the $M = \infty$ curves of Ref. 4 have been utilized for H_T of 8,000 Btu/lb and above).

Although the quasisteady BLIMP solutions are in themselves useful for many applications, it is often desirable to perform transient calculations. A procedure for accomplishing this is presented herein which makes use of the Aerotherm Chemical Equilibrium (ACE) and Charring Material Ablation (CMA) computer programs.

The ACE program (Ref. 5) generates surface boundary condition information for the CMA program (Ref. 6) which, in turn, computes the transient in-depth material response. These programs employ a convective transfer coefficient formulation, described in Reference 7, which considers unequal diffusion coefficients for all species. It was found possible for the present problem to express the quasisteady stagnation-point boundary-layer results in terms of this transfer-coefficient approach in a precise way. That is, it was found that transfer coefficients calculated from the boundary layer results, when utilized as input to ACE/CMA, yield identical results for quasisteady problems. This preciseness is not retained in applying the quasisteady BLIMP solutions to transient problems; however, it is believed that the approach which is recommended is not too inaccurate for stagnation point problems.

As a consequence of the success of the correlation of the quasisteady stagnation-point boundary-layer solutions in terms of the transfer coefficient formulation of Reference 7, some degree of confidence in that formulation is established (the formulation is based on an analogy with approaches employed

Figure 1. Pressure Distribution Around A Sphere Cone:
Cone Half Angle = 5° (After Chushkin and
Shulishnina as Reported by Ellett)

for equal diffusion coefficients and had never to date been validated). The present success, although encouraging, should not be considered as a final and complete authentication, however, because the unequal diffusion effects in the present problem are relatively small.

While a number of nonsimilar boundary layer solutions have been generated and are reported and discussed in this report, the development of an approach for expressing these results in terms of a transfer coefficient formulation is incomplete. The correlation of the nonsimilar boundary layer results for positions around the body in terms of transfer coefficients must be somewhat artificial and mechanical since transfer coefficient approaches are predicated upon similarity concepts. This can be seen from the fact that transfer coefficients relate the surface condition directly to the boundary-layer-edge condition with no consideration for upstream effects. Nevertheless, it is believed that such a correlation is practical. It is incomplete herein only because the level of effort did not permit more detailed consideration.

Solutions for air boundary layers and for quasisteady graphite ablation are presented in Sections 2 and 3, respectively. The correlation of the quasisteady graphite boundary-layer solutions with the transfer-coefficient formulation of Reference 7 and the comparison of quasisteady CMA and BLIMP solutions are presented in Section 4. The approach for applying the results of this report to transient reentry problems is outlined in Section 5. Conclusions and recommendations are presented in Section 6.

2. AIR BOUNDARY LAYER SOLUTIONS

A rather extensive set of air boundary-layer solutions were generated to serve as a framework for correlating the graphite quasisteady ablation results in terms of transfer coefficients and blowing corrections and for studying the effects of various parameters of the solutions. Some of the more significant aspects of these solutions are presented and discussed in this section. Solutions for the stagnation point and for positions around the body are presented in Sections 2.1 and 2.2, respectively.

2.1 Stagnation Point

A matrix of stagnation-point air boundary-layer solutions were generated over the range of total pressure and total enthalpy of interest in moderate to severe ballistic reentry. The 31 conditions listed in Table I were considered for equal and for unequal diffusion coefficients, both for uniform surface temperatures of 1,000, 4,000, 6,000 and 8,000°R. The solutions were performed for an effective nose radius of 1 inch, although a few solutions were generated for a second nose radius of 1/2 inch. It is obviously not possible to present the results of all of these calculations in detail. Therefore, results are presented only for heat transfer coefficient, needed for the correlation procedure of Section 5, and for a few additional boundary layer parameters of general interest. The effects of unequal diffusion, nose radius, and surface temperature are discussed in the following subsections.

2.1.1 Solutions for Equal and Unequal Diffusion Coefficients

Results for heat transfer coefficient, $\rho_e u_e C_H$, normalized by the square root of the ratio of total pressure behind the shock, P_{T_2} , to effective nose radius, R_{eff} , is presented in Figure 2 for assumed equal diffusion coefficients and in Figure 3 for unequal diffusion coefficients. Here, the heat transfer coefficient has been defined simply as

Figure 2. Normalized Heat-Transfer Coefficient in a Stagnation Point Air Boundary Layer with Equal Diffusion Coefficients
a. Wall Temperature = 1000°R

Figure 2 (continued)
b. Wall Temperature = 4000°R

Figure 2 (continued)
 c. Wall Temperature = 6000° R

Figure 2 (concluded)
 d. Wall Temperature = 8000° R

Figure 3. Normalized Heat-Transfer Coefficient in a Stagnation Point Air Boundary Layer with Unequal Diffusion Coefficients
a. Wall Temperature = $1000^\circ R$

Figure 3 (continued)
b. Wall Temperature = 4000°R

Figure 3 (continued)
c. Wall Temperature = 6000°R

Figure 3 (concluded)
d. Wall Temperature = 8000°R

$$\rho_e u_e C_H = \frac{q_a}{H_{T_e} - H_{T_w}} \quad (1)$$

where q_a is the total heat flux to the wall due to convective and diffusive processes in the boundary layer* and H_{T_e} and H_{T_w} are the total enthalpies at the edge of the boundary layer and at the wall, respectively. This definition is consistent with the formulation of Reference 7 only for the case of equal diffusion and equal heat and mass transfer coefficients. Nevertheless, it is a convenient reference quantity for correlating ablation solutions and is used for that purpose herein.

Looking first at the equal diffusion results of Figure 2 it can be seen that the parameter $\rho_e u_e C_H / (P_{T_2} / R_{eff})^{1/2}$ differs by no more than about ± 5 percent over the total range of pressure (0.01 to 100 atm) and enthalpy (2,000 to 13,000 Btu/lb) considered. The effect of surface temperature is to decrease the heat transfer coefficient, an increase of T_w from 1,000 to 8,000°R introducing a nominal decrease of heat transfer coefficient of 7 percent.

The effect of unequal diffusion coefficients is to increase the heat-transfer coefficient (when defined by Equation (1)) by about 3 to 4 percent except at low enthalpies or low pressures where the effect is less. This is primarily the result of an increase in diffusive heat flux, the enthalpy of the gas at the wall, at a given T_w , being relatively insensitive to the assumption of equal or unequal diffusion coefficients in air boundary layers (differing nominally by 1 Btu/lb and by 10 Btu/lb at surface temperatures of 1,000 and 8,000°R, respectively).

The smoothness of the curves of Figure 2 show a precision in the results to four significant figures. However, as mentioned in Section 1 and as will be shown in Section 3.1, the solutions are accurate to only three significant figures.

* See Reference 1 for mathematical relation defining q_a .

In Table II values of diffusive heat flux, skin friction coefficient, momentum thickness, and displacement thickness are presented for stagnation-point air boundary layers for equal and for unequal diffusion coefficients, all for a wall temperature of $1,000^{\circ}\text{R}$. The effect of unequal diffusion coefficients on the diffusive heat flux is similar to that discussed previously for the heat transfer coefficient since the change in enthalpy potential between equal and unequal diffusion coefficients is negligibly small. The effect of unequal diffusion coefficients on skin friction coefficient and on momentum thickness is a few hundreds of a percent and a few tenths of a percent, respectively. The effect on displacement thickness is much more striking, unequal diffusion markedly decreasing the displacement thickness (more negative or less positive). However, this is probably not too important in any practical sense since under the conditions considered the stagnation point displacement thickness never exceeds about five mils (the lowest pressure, 0.01 atm, cases).

2.1.2 Effect of Nose Radius

The results of Figures 2 and 3 and Table II were all generated for an effective nose radius of 1 inch and thus are strictly applicable only to that case. However, the results of a submatrix of seven combinations of total pressure and total enthalpy for 1/2 inch nose radius considering unequal diffusion coefficients and a wall temperature of $1,000^{\circ}\text{R}$ showed the $\rho_e u_e C_H / (P_{T_2}/R_{\text{eff}})^{1/2}$ to be invariant to at least 4 significant figures. Therefore, the results of Figures 2 and 3 are probably applicable with little error over a relatively large range of R_{eff} . The effect of the nose radius on other parameters will be discussed in Section 2.2.2.

TABLE II

EFFECT OF UNEQUAL DIFFUSION COEFFICIENTS ON DIFFUSIVE HEAT FLUX, SKIN FRICTION
COEFFICIENT, MOMENTUM THICKNESS, AND DISPLACEMENT THICKNESS FOR
A STAGNATION POINT AIR BOUNDARY LAYER:

Effective Nose Radius = 1 inch, Wall Temperature = 1000°R

(a) High Total Enthalpy

Total Enthalpy (Btu/lb)	Total Pressure (Atm)	Diffusive Heat Flux, q_a (Btu/sec ft ²)		Skin Friction Coefficient, (lb/sec ft ²)		Momentum Thickness, θ (ft)		Displacement Thickness, δ^* (ft.)	
		Equal δ_{ij}	Unequal δ_{ij}	Equal δ_{ij}	Unequal δ_{ij}	Equal δ_{ij}	Unequal δ_{ij}	Equal δ_{ij}	Unequal δ_{ij}
13000	.01	2.052 + 2	2.142 + 2	1.714 - 2	1.710 - 2	2.431 - 3	2.467 - 3	3.404 - 4	2.121 - 4
13000	.1	6.587 + 2	6.837 + 2	5.440 - 2	5.425 - 2	8.222 - 4	8.326 - 4	6.973 - 5	3.152 - 5
13000	1	2.119 + 3	2.194 + 3	1.729 - 1	1.724 - 1	2.796 - 4	2.824 - 4	7.099 - 6	-4.758 - 6
13000	10	6.839 + 3	7.063 + 3	5.507 - 1	5.490 - 1	9.566 - 5	9.649 - 5	-3.766 - 6	-7.210 - 6
13000	30	1.197 + 4	1.235 + 4	9.574 - 1	9.549 - 1	5.743 - 5	5.788 - 5	-4.178 - 6	-6.048 - 6
13000	100	2.213 + 4	2.280 + 4	1.756	1.752	3.292 - 5	3.319 - 5	-3.658 - 6	-4.608 - 6
11500	.01	1.817 + 2	1.896 + 2	1.718 - 2	1.713 - 2	2.311 - 3	2.343 - 3	3.938 - 4	2.765 - 4
11500	.1	5.831 + 2	6.085 + 2	5.460 - 2	5.441 - 2	7.820 - 4	7.914 - 4	9.022 - 5	5.100 - 5
11500	1	1.877 + 3	1.952 + 3	1.737 - 1	1.731 - 1	2.659 - 4	2.688 - 4	1.443 - 5	3.004 - 6
11500	10	6.054 + 3	6.269 + 3	5.526 - 1	5.513 - 1	9.096 - 5	9.174 - 5	-1.481 - 6	-4.450 - 6
11500	30	1.059 + 4	1.095 + 4	9.603 - 1	9.584 - 1	5.463 - 5	5.500 - 5	-2.897 - 6	-4.436 - 6
11500	100	1.958 + 4	2.018 + 4	1.760	1.758	3.129 - 5	3.152 - 5	-2.975 - 6	-3.730 - 6
10000	.01	1.582 + 2	1.671 + 2	1.725 - 2	1.719 - 2	2.205 - 3	2.238 - 3	4.277 - 4	2.969 - 4
10000	.1	5.079 + 2	5.335 + 2	5.484 - 2	5.465 - 2	7.447 - 4	7.547 - 4	1.032 - 4	6.593 - 5
10000	1	1.635 + 3	1.704 + 3	1.742 - 1	1.738 - 1	2.531 - 4	2.556 - 4	1.796 - 5	8.756 - 6
10000	10	5.270 + 3	5.453 + 3	5.537 - 1	5.531 - 1	8.648 - 5	8.697 - 5	-5.903 - 7	-2.622 - 6
10000	30	9.218 + 3	9.510 + 3	9.618 - 1	9.611 - 1	5.189 - 5	5.225 - 5	-2.404 - 6	-3.399 - 6
10000	100	1.702 + 4	1.752 + 4	1.762	1.761	2.970 - 5	2.984 - 5	-2.689 - 6	-3.189 - 6

TABLE II (concluded)
(b) Moderate to Low Total Enthalpy

Total Enthalpy (Btu/lb)	Total Pressure (Atm)	Diffusive Heat Flux, q_a (Btu/sec ft ²)		Skin Friction Coefficient, (lb/sec ft ²)		Momentum Thickness, θ (ft)		Displacement Thickness, δ^* (ft)	
		Equal δ_{ij}	Unequal δ_{ij}	Equal δ_{ij}	Unequal δ_{ij}	Equal δ_{ij}	Unequal δ_{ij}	Equal δ_{ij}	Unequal δ_{ij}
8,000	1	1.311 + 3	1.355 + 3	1.742 - 1	1.743 - 1	2.372 - 4	2.381 - 4	1.257 - 5	1.007 - 5
8,000	10	4.217 + 3	4.339 + 3	5.534 - 1	5.538 - 1	8.075 - 5	8.110 - 5	-2.006 - 6	-2.691 - 6
8,000	30	7.369 + 3	7.576 + 3	9.612 - 1	9.616 - 1	4.834 - 5	4.853 - 5	-3.045 - 6	-3.536 - 6
8,000	100	1.359 + 4	1.396 + 4	1.761	1.761	2.756 - 5	2.768 - 5	-2.911 - 6	-3.256 - 6
6,000	10	3.146 + 3	3.247 + 3	5.507 - 1	5.505 - 1	7.463 - 5	7.505 - 5	-5.917 - 6	-7.275 - 6
6,000	30	5.490 + 3	5.659 + 3	9.563 - 1	9.558 - 1	4.436 - 5	4.462 - 5	-4.877 - 6	-5.723 - 6
6,000	100	1.011 + 4	1.040 + 4	1.751	1.750	2.505 - 5	2.520 - 5	-3.542 - 6	-4.014 - 6
4,000	10	2.050 + 3	2.111 + 3	5.519 - 1	5.518 - 1	6.239 - 5	6.278 - 5	-1.548 - 6	-2.468 - 6
4,000	30	3.573 + 3	3.666 + 3	9.579 - 1	9.578 - 1	3.668 - 5	3.689 - 5	-1.332 - 6	-1.809 - 6
4,000	100	6.569 + 3	6.711 + 3	1.751	1.751	2.054 - 5	2.064 - 5	-1.182 - 6	-1.396 - 6
2,000	10	9.706 + 2	9.781 + 2	5.480 - 1	5.484 - 1	4.574 - 5	4.576 - 5	6.921 - 6	6.937 - 6
2,000	30	1.686 + 3	1.696 + 3	9.463 - 1	9.468 - 1	2.693 - 5	2.694 - 5	2.962 - 6	2.963 - 6
2,000	100	3.087 + 3	3.098 + 3	1.723	1.724	1.500 - 5	1.501 - 5	1.094 - 6	1.093 - 6

2.1.3 Effect of Surface Temperature

The effect of surface temperature on stagnation point air boundary layers with unequal diffusion coefficients and an effective nose radius of 1 inch is summarized in Table III. The diffusive heat flux, skin-friction coefficient, momentum thickness, and displacement thickness are presented in Tables III(a) through (d), respectively, for surface temperatures of 1,000, 4,000, 6,000 and 8,000°R. The diffusive heat flux is appreciably decreased by an increase of surface temperature. This is primarily due to the decrease in enthalpy potential across the boundary layer, the heat-transfer coefficient having been seen to decrease only a few percent as the wall temperature is elevated (see Fig. 3). Unlike the heat-transfer coefficient, the skin-friction coefficient is seen to increase with increase in surface temperature. Finally, the momentum thickness decreases rather substantially with surface temperature, whereas the displacement thickness increases dramatically with surface temperature.

2.2 Downstream Stations

Nonsimilar air boundary layer solutions were obtained for 8 total enthalpy, total pressure combinations, all for a one-inch nose radius, five-degree cone half angle, and 1,000°R wall temperature and with unequal diffusion coefficients. Five total pressures were considered for an enthalpy of 11,500 Btu/lb, while four enthalpies were considered for a total pressure of 30 atmospheres. Auxiliary calculations were performed to study the effect of nose radius, cone half angle, unequal-versus-equal diffusion coefficients, and similar-versus-nonsimilar boundary layers. The results are tabulated in this section for a number of boundary-layer parameters of general interest.

TABLE III
EFFECT OF SURFACE TEMPERATURE ON THE STAGNATION POINT
AIR BOUNDARY LAYER: UNEQUAL DIFFUSION COEFFICIENTS,
EFFECTIVE NOSE RADIUS = 1 INCH

(a) Diffusive Heat Flux

Total Enthalpy, (Btu/lb)	Total Pressure (Atm)	Diffusive Heat Flux, (Btu/sec ft ²) for Surface Temperature, T _w , of			
		1000°R	4000°R	6000°R	8000°R
13000	.01	2.142 + 2	1.947 + 2	1.590 + 2	1.362 + 2
13000	.1	6.837 + 2	6.288 + 2	5.303 + 2	4.612 + 2
13000	1	2.194 + 3	2.026 + 3	1.783 + 3	1.520 + 3
13000	10	7.063 + 3	6.528 + 3	5.957 + 3	5.107 + 3
13000	30	1.235 + 4	1.141 + 4	1.051 + 4	9.162 + 3
13000	100	2.280 + 4	2.106 + 4	1.952 + 4	1.736 + 4
11500	.01	1.896 + 2	1.722 + 2	1.360 + 2	1.136 + 2
11500	.1	6.085 + 2	5.545 + 2	4.552 + 2	3.877 + 2
11500	1	1.952 + 3	1.782 + 3	1.546 + 3	1.282 + 3
11500	10	6.269 + 3	5.733 + 3	5.188 + 3	4.329 + 3
11500	30	1.095 + 4	1.002 + 4	9.150 + 3	7.799 + 3
11500	100	2.018 + 4	1.847 + 4	1.698 + 4	1.484 + 4
10000	.01	1.671 + 2	1.487 + 2	1.126 + 2	9.058 + 1
10000	.1	5.335 + 2	4.778 + 2	3.788 + 2	3.129 + 2
10000	1	1.704 + 3	1.535 + 3	1.307 + 3	1.039 + 3
10000	10	5.453 + 3	4.932 + 3	4.403 + 3	3.541 + 3
10000	30	9.510 + 3	8.607 + 3	7.765 + 3	6.423 + 3
10000	100	1.752 + 4	1.584 + 4	1.440 + 4	1.230 + 4
8000	1	1.355 + 3	1.199 + 3	9.804 + 2	7.092 + 2
8000	10	4.339 + 3	3.837 + 3	3.330 + 3	2.476 + 3
8000	30	7.576 + 3	6.686 + 3	5.881 + 3	4.567 + 3
8000	100	1.396 + 4	1.229 + 4	1.090 + 4	8.870 + 3
6000	10	3.247 + 3	2.732 + 3	2.238 + 3	1.400 + 3
6000	30	5.659 + 3	4.753 + 3	3.970 + 3	2.691 + 3
6000	100	1.040 + 4	8.718 + 3	7.373 + 3	5.401 + 3
4000	10	2.111 + 3	1.614 + 3	1.151 + 3	3.555 + 2
4000	30	3.666 + 3	2.799 + 3	2.064 + 3	8.617 + 2
4000	100	6.711 + 3	5.117 + 3	3.849 + 3	2.001 + 3
2000	10	9.781 + 2	5.157 + 2	8.912 + 1	-5.990 + 2
2000	30	1.696 + 3	8.912 + 2	2.090 + 2	-8.487 + 2
2000	100	3.098 + 3	1.624 + 3	4.432 + 2	-1.215 + 3

TABLE III (continued)
(b) Skin Friction Coefficient

Total Enthalpy (Btu/lb)	Total Pressure (Atm)	Skin Friction Coefficient, (lb/sec ft ²) for Surface Temperature, T _w , of			
		1000°R	4000°R	6000°R	8000°R
13000	.01	1.710 - 2	1.741 - 2	1.795 - 2	1.828 - 2
13000	.1	5.425 - 2	5.540 - 2	5.671 - 2	5.788 - 2
13000	1	1.724 - 1	1.763 - 1	1.791 - 1	1.836 - 1
13000	10	5.490 - 1	5.617 - 1	5.692 - 1	5.817 - 1
13000	30	9.549 - 1	9.768 - 1	9.899 - 1	1.009
13000	100	1.752	1.792	1.817	1.847
11500	.01	1.713 - 2	1.757 - 2	1.817 - 2	1.852 - 2
11500	.1	5.441 - 2	5.585 - 2	5.731 - 2	5.862 - 2
11500	1	1.731 - 1	1.776 - 1	1.807 - 1	1.858 - 1
11500	10	5.513 - 1	5.651 - 1	5.740 - 1	5.881 - 1
11500	30	9.584 - 1	9.826 - 1	9.980 - 1	1.019
11500	100	1.758	1.802	1.831	1.866
10000	.01	1.719 - 2	1.770 - 2	1.838 - 2	1.878 - 2
10000	.1	5.465 - 2	5.620 - 2	5.791 - 2	5.942 - 2
10000	1	1.738 - 1	1.786 - 1	1.824 - 1	1.882 - 1
10000	10	5.531 - 1	5.685 - 1	5.789 - 1	5.950 - 1
10000	30	9.611 - 1	9.883 - 1	1.006	1.031
10000	100	1.761	1.813	1.846	1.885
8000	1	1.743 - 1	1.799 - 1	1.846 - 1	1.919 - 1
8000	10	5.538 - 1	5.729 - 1	5.855 - 1	6.054 - 1
8000	30	9.616 - 1	9.959 - 1	1.018	1.048
8000	100	1.761	1.826	1.867	1.916
6000	10	5.505 - 1	5.770 - 1	5.930 - 1	6.179 - 1
6000	30	9.558 - 1	1.003	1.032	1.069
6000	100	1.750	1.840	1.895	1.957
4000	10	5.518 - 1	5.934 - 1	6.171 - 1	6.482 - 1
4000	30	9.578 - 1	1.032	1.075	1.124
4000	100	1.751	1.890	1.973	2.059
2000	10	5.484 - 1	6.274 - 1	6.742 - 1	7.195 - 1
2000	30	9.468 - 1	1.085	1.168	1.248
2000	100	1.724	1.976	2.132	2.283

TABLE III (continued)
(c) Momentum Thickness

Total Enthalpy, (Btu/lb)	Total Pressure, (Atm)	Momentum Thickness, θ (ft) for Surface Temperature, T_w , of			
		1000°R	4000°R	6000°R	8000°R
13000	.01	2.467 - 3	2.354 - 3	2.187 - 3	2.104 - 3
13000	.1	8.326 - 4	7.965 - 4	7.453 - 4	7.162 - 4
13000	1	2.824 - 4	2.707 - 4	2.566 - 4	2.436 - 4
13000	10	9.649 - 5	9.253 - 5	8.888 - 5	8.402 - 5
13000	30	5.788 - 5	5.560 - 5	5.357 - 5	5.086 - 5
13000	100	3.319 - 5	3.184 - 5	3.076 - 5	2.937 - 5
11500	.01	2.343 - 3	2.228 - 3	2.051 - 3	1.966 - 3
11500	.1	7.914 - 4	7.539 - 4	6.997 - 4	6.700 - 4
11500	1	2.688 - 4	2.565 - 4	2.418 - 4	2.282 - 4
11500	10	9.174 - 5	8.772 - 5	8.395 - 5	7.878 - 5
11500	30	5.500 - 5	5.267 - 5	5.062 - 5	4.773 - 5
11500	100	3.152 - 5	3.011 - 5	2.906 - 5	2.761 - 5
10000	.01	2.238 - 3	2.114 - 3	1.923 - 3	1.835 - 3
10000	.1	7.547 - 4	7.152 - 4	6.561 - 4	6.254 - 4
10000	1	2.556 - 4	2.429 - 4	2.278 - 4	2.129 - 4
10000	10	8.697 - 5	8.294 - 5	7.916 - 5	7.362 - 5
10000	30	5.225 - 5	4.976 - 5	4.768 - 5	4.464 - 5
10000	100	2.984 - 5	2.848 - 5	2.738 - 5	2.584 - 5
8000	1	2.381 - 4	2.256 - 4	2.095 - 4	1.922 - 4
8000	10	8.110 - 5	7.664 - 5	7.281 - 5	6.654 - 5
8000	30	4.853 - 5	4.590 - 5	4.370 - 5	4.040 - 5
8000	100	2.768 - 5	2.616 - 5	2.503 - 5	2.339 - 5
6000	10	7.505 - 5	6.994 - 5	6.567 - 5	5.866 - 5
6000	30	4.462 - 5	4.158 - 5	3.926 - 5	3.559 - 5
6000	100	2.520 - 5	2.348 - 5	2.226 - 5	2.052 - 5
4000	10	6.278 - 5	5.732 - 5	5.294 - 5	4.558 - 5
4000	30	3.689 - 5	3.366 - 5	3.131 - 5	2.758 - 5
4000	100	2.064 - 5	1.882 - 5	1.759 - 5	1.584 - 5
2000	10	4.576 - 5	3.925 - 5	3.415 - 5	2.826 - 5
2000	30	2.694 - 5	2.307 - 5	2.025 - 5	1.709 - 5
2000	100	1.501 - 5	1.284 - 5	1.133 - 5	9.772 - 6

TABLE III (concluded)
(d) Displacement Thickness

Total Enthalpy, (Btu/lb)	Total Pressure, (Atm)	Displacement Thickness, δ^* (ft) for Surface Temperature, T_w , of			
		1000°R	4000°R	6000°R	8000°R
13000	.01	2.121 - 4	6.908 - 4	1.436 - 3	1.833 - 3
13000	.1	3.152 - 5	1.922 - 4	4.071 - 4	5.492 - 4
13000	1	-4.758 - 6	4.980 - 5	1.053 - 4	1.662 - 4
13000	10	-7.210 - 6	1.099 - 5	2.583 - 5	4.678 - 5
13000	30	-6.048 - 6	4.755 - 6	1.310 - 5	2.460 - 5
13000	100	-4.608 - 6	1.526 - 6	6.092 - 6	1.189 - 5
11500	.01	2.765 - 4	8.091 - 4	1.597 - 3	2.006 - 3
11500	.1	5.100 - 5	2.284 - 4	4.573 - 4	6.050 - 4
11500	1	3.004 - 6	6.063 - 5	1.192 - 4	1.839 - 4
11500	10	-4.450 - 6	1.436 - 5	2.991 - 5	5.227 - 5
11500	30	-4.436 - 6	6.734 - 6	1.545 - 5	2.765 - 5
11500	100	-3.730 - 6	2.635 - 6	7.382 - 6	1.349 - 5
10000	.01	2.969 - 4	8.859 - 4	1.736 - 3	2.165 - 3
10000	.1	6.593 - 5	2.496 - 4	4.997 - 4	6.563 - 4
10000	1	8.756 - 6	6.718 - 5	1.299 - 4	2.003 - 4
10000	10	-2.622 - 6	1.678 - 5	3.312 - 5	5.726 - 5
10000	30	-3.399 - 6	8.229 - 6	1.732 - 5	3.039 - 5
10000	100	-3.189 - 6	3.514 - 6	8.459 - 6	1.492 - 5
8000	1	1.007 - 5	7.189 - 5	1.397 - 4	2.209 - 4
8000	10	-2.691 - 6	1.867 - 5	3.610 - 5	6.319 - 5
8000	30	-3.536 - 6	9.435 - 6	1.918 - 5	3.356 - 5
8000	100	-3.256 - 6	4.267 - 6	9.596 - 6	1.662 - 5
6000	10	-7.275 - 6	1.830 - 5	3.778 - 5	6.858 - 5
6000	30	-5.723 - 6	9.692 - 6	2.057 - 5	3.661 - 5
6000	100	-4.014 - 6	4.797 - 6	1.073 - 5	1.848 - 5
4000	10	-2.468 - 6	2.779 - 5	4.890 - 5	8.134 - 5
4000	30	-1.809 - 6	1.625 - 5	2.816 - 5	4.492 - 5
4000	100	-1.396 - 6	8.815 - 6	1.539 - 5	2.357 - 5
2000	10	6.937 - 6	4.743 - 5	7.340 - 5	1.049 - 4
2000	30	2.963 - 6	2.683 - 5	4.181 - 5	5.940 - 5
2000	100	1.093 - 6	1.439 - 5	2.267 - 5	3.182 - 5

2.2.1 Nonsimilar Solutions for Unequal Diffusion Coefficients

The solutions for the nonsimilar boundary layer with unequal diffusion coefficients, nose radius of one inch, and cone half angle of five degrees are presented first, in Table IV. In particular, Reynolds number per foot, diffusive heat flux, wall shear, heat transfer coefficient, skin friction coefficient, momentum thickness, and displacement thickness are presented in Tables IV(a) through IV(g), respectively, for 20 streamwise stations, these being the same stations considered in the calculations.

The unit Reynolds number (Table IV(a)) is of interest as it provides a means of assessing the validity of the assumption of laminar flow. The prediction of transition is always approached with a certain amount of trepidation because of the large number of parameters which can influence transition. In Reference 8, Moulds and Young have presented criteria for sphere-cone reentry bodies which are reported to correlate with experimental data. In particular, they utilize an Re_s of 1.5×10^5 on the spherical nose, an Re_s of 5.0×10^5 on the conical portion aft of the spherical nose for a wetted length of up to 5 times the nose radius, and an Re_s of 2.0×10^6 for a wetted length of one foot to the extreme aft end of the conical vehicle. Applying these criteria to the present results, transition would be predicted to occur on the spherical nose for the 6,000 Btu/lb, 30 atmosphere case, and to be marginal for the 8,000 Btu/lb, 30 atmosphere case, but not to occur for the reported s/R_N for the other six cases considered in Table IV.

The other boundary layer parameters presented in Table IV(b) through IV(g) are primarily presented for the record and for later reference when studying the importance of certain effects and for comparing to graphite ablation results. Therefore, no more will be said about these results here.

TABLE IV

NONSIMILAR AIR BOUNDARY LAYER OVER A SPHERE CONE
WITH UNEQUAL DIFFUSION COEFFICIENTS:

Nose Radius = 1 Inch, Cone Half Angle = 5° , Wall Temperature = $1,000^\circ\text{R}$

(a) Reynolds Number per Foot

Stream- wise Dimen- sion, ft.	S/R_N	Total Enthalpy, Btu/lb							
		11,500	11,500	11,500	11,500	11,500	10,000	8,000	6,000
		Total Pressure, atm							
		.01	.1	1	10	30	30	30	30
		Reynolds Number per foot, $\rho_e u_e / \mu_e$							
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.01561	0.1873	3.540 + 2	3.175 + 3	2.828 + 4	2.502 + 5	7.062 + 5	7.810 + 5	9.053 + 5	1.082 + 6
0.03125	0.3750	6.356 + 2	5.712 + 3	5.099 + 4	4.522 + 5	1.278 + 6	1.413 + 6	1.636 + 6	1.959 + 6
0.04686	0.5623	7.922 + 2	7.143 + 3	6.402 + 4	5.703 + 5	1.616 + 6	1.784 + 6	2.063 + 6	2.479 + 6
0.05727	0.6872	8.167 + 2	7.386 + 3	6.642 + 4	5.940 + 5	1.686 + 6	1.860 + 6	2.149 + 6	2.594 + 6
0.07292	0.8750	7.465 + 2	6.789 + 3	6.144 + 4	5.535 + 5	1.577 + 6	1.736 + 6	2.003 + 6	2.448 + 6
0.08366	1.0039	6.466 + 2	5.906 + 3	5.372 + 4	4.868 + 5	1.391 + 6	1.529 + 6	1.764 + 6	2.188 + 6
0.10092	1.2110	4.508 + 2	4.151 + 3	3.811 + 4	3.490 + 5	1.003 + 6	1.100 + 6	1.272 + 6	1.641 + 6
0.12363	1.4836	2.256 + 2	2.102 + 3	1.956 + 4	1.819 + 5	5.273 + 5	5.774 + 5	6.804 + 5	9.452 + 5
0.13209	1.5851	2.216 + 2	2.065 + 3	1.922 + 4	1.788 + 5	5.183 + 5	5.676 + 5	6.695 + 5	9.320 + 5
0.14196	1.7035	2.163 + 2	2.016 + 3	1.878 + 4	1.748 + 5	5.067 + 5	5.549 + 5	6.553 + 5	9.149 + 5
0.16204	1.9445	2.061 + 2	1.923 + 3	1.792 + 4	1.670 + 5	4.844 + 5	5.305 + 5	6.278 + 5	8.808 + 5
0.20587	2.4704	1.849 + 2	1.728 + 3	1.613 + 4	1.506 + 5	4.375 + 5	4.794 + 5	5.706 + 5	8.078 + 5
0.25305	3.0366	1.661 + 2	1.554 + 3	1.454 + 4	1.360 + 5	3.954 + 5	4.335 + 5	5.194 + 5	7.408 + 5
0.33586	4.0303	1.402 + 2	1.315 + 3	1.234 + 4	1.158 + 5	3.373 + 5	3.702 + 5	4.489 + 5	6.453 + 5
0.46803	5.6164	1.130 + 2	1.063 + 3	9.999 + 3	9.423 + 4	2.752 + 5	3.026 + 5	3.741 + 5	5.409 + 5
0.60690	7.2828	9.498 + 1	8.954 + 2	8.445 + 3	7.985 + 4	2.336 + 5	2.576 + 5	3.242 + 5	4.721 + 5
1.0795	12.954	6.569 + 1	6.220 + 2	5.898 + 3	5.616 + 4	1.650 + 5	1.833 + 5	2.413 + 5	3.584 + 5
1.7136	20.563	5.102 + 1	4.845 + 2	4.611 + 3	4.412 + 4	1.301 + 5	1.457 + 5	1.975 + 5	2.981 + 5
2.7994	33.593	4.086 + 1	3.890 + 2	3.714 + 3	3.570 + 4	1.056 + 5	1.194 + 5	1.651 + 5	2.653 + 5

TABLE IV (continued)
(b) Diffusive Heat Flux

Stream- wise Dimen- sion, ft.	S/R _N	Total Enthalpy, Btu/lb							
		11,500	11,500	11,500	11,500	11,500	10,000	8,000	6,000
		Total Pressure, atm							
		.01	.1	1	10	30	30	30	30
		Diffusive Heat Flux, q _a (Btu/sec ft ²)							
0.	0.	1.896 + 2	6.085 + 2	1.952 + 3	6.269 + 3	1.095 + 4	9.510 + 3	7.576 + 3	5.659 + 3
0.01561	0.1873	1.838 + 2	5.896 + 2	1.891 + 3	6.069 + 3	1.059 + 4	9.203 + 3	7.332 + 3	5.475 + 3
0.03125	0.3750	1.706 + 2	5.471 + 2	1.754 + 3	5.625 + 3	9.817 + 3	8.523 + 3	6.789 + 3	5.069 + 3
0.04686	0.5623	1.430 + 2	4.580 + 2	1.466 + 3	4.692 + 3	8.180 + 3	7.097 + 3	5.656 + 3	4.219 + 3
0.05727	0.6872	1.246 + 2	3.987 + 2	1.274 + 3	4.074 + 3	7.099 + 3	6.154 + 3	4.906 + 3	3.658 + 3
0.07292	0.8750	9.453 + 1	3.018 + 2	9.626 + 2	3.070 + 3	5.342 + 3	4.625 + 3	3.691 + 3	2.749 + 3
0.08366	1.0039	7.430 + 1	2.368 + 2	7.538 + 2	2.399 + 3	4.171 + 3	3.610 + 3	2.884 + 3	2.147 + 3
0.10092	1.2110	4.587 + 1	1.457 + 2	4.624 + 2	1.467 + 3	2.548 + 3	2.209 + 3	1.764 + 3	1.312 + 3
0.12363	1.4836	2.027 + 1	6.405 + 1	2.026 + 2	6.440 + 2	1.120 + 3	9.761 + 2	7.743 + 2	5.773 + 2
0.13209	1.5851	1.796 + 1	5.672 + 1	1.793 + 2	5.698 + 2	9.916 + 2	8.628 + 2	6.854 + 2	5.103 + 2
0.14196	1.7035	1.663 + 1	5.263 + 1	1.663 + 2	5.279 + 2	9.179 + 2	7.967 + 2	6.311 + 2	4.704 + 2
0.16204	1.9445	1.494 + 1	4.731 + 1	1.496 + 2	4.742 + 2	8.239 + 2	7.144 + 2	5.654 + 2	4.217 + 2
0.20587	2.4704	1.233 + 1	3.915 + 1	1.240 + 2	3.930 + 2	6.822 + 2	5.911 + 2	4.684 + 2	3.508 + 2
0.25305	3.0366	1.044 + 1	3.320 + 1	1.053 + 2	3.335 + 2	5.789 + 2	5.011 + 2	3.973 + 2	2.991 + 2
0.33586	4.0303	8.255	2.627 + 1	8.335 + 1	2.640 + 2	4.580 + 2	3.961 + 2	3.139 + 2	2.382 + 2
0.46803	5.6164	6.255	1.991 + 1	6.321 + 1	2.002 + 2	3.472 + 2	3.002 + 2	2.375 + 2	1.823 + 2
0.60690	7.2828	5.071	1.615 + 1	5.125 + 1	1.623 + 2	2.815 + 2	2.434 + 2	1.924 + 2	1.503 + 2
1.0795	12.954	3.336	1.427 + 1	3.375 + 1	1.069 + 2	1.854 + 2	1.603 + 2	1.265 + 2	1.041 + 2
1.7136	20.563	2.521	8.041	2.555 + 1	8.097 + 1	1.404 + 2	1.213 + 2	9.575 + 1	8.159 + 1
2.7994	33.593	1.931	6.171	1.963 + 1	6.226 + 1	1.079 + 2	9.325 + 1	7.362 + 1	6.703 + 1

TABLE IV (continued)

(c) Wall Shear

Stream- wise Dimen- sion, ft.	S/R _N	Total Enthalpy, Btu/lb							
		11,500	11,500	11,500	11,500	11,500	10,000	8,000	6,000
		Total Pressure, atm							
		.01	.1	1	10	30	30	30	50
		Wall Shear, τ_w (lb/ft ²)							
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
.01561	.1873	8.262 - 1	2.743	9.144	3.058 + 1	5.445 + 1	5.194 + 1	4.815 + 1	4.328 + 1
.03125	.3750	1.548	5.138	1.712 + 1	5.724 + 1	1.019 + 2	9.718 + 1	9.006 + 1	8.091 + 1
.04686	.5623	2.012	6.668	2.218 + 1	7.401 + 1	1.316 + 2	1.255 + 2	1.163 + 2	1.044 + 2
.05727	.6872	2.188	7.248	2.409 + 1	8.028 + 1	1.426 + 2	1.361 + 2	1.260 + 2	1.131 + 2
.07292	.8750	2.199	7.272	2.413 + 1	8.020 + 1	1.423 + 2	1.358 + 2	1.257 + 2	1.127 + 2
.08366	1.0039	2.053	6.781	2.246 + 1	7.450 + 1	1.321 + 2	1.260 + 2	1.165 + 2	1.047 + 2
.10092	1.2110	1.652	5.445	1.797 + 1	5.938 + 1	1.050 + 2	1.002 + 2	9.259 + 1	8.356 + 1
.12363	1.4836	1.039	3.406	1.118 + 1	3.673 + 1	6.477 + 1	6.184 + 1	5.714 + 1	5.187 + 1
.13209	1.5851	7.674 - 1	2.527	8.335	2.755 + 1	4.877 + 1	4.644 + 1	4.283 + 1	3.879 + 1
.14196	1.7035	5.663 - 1	2.195	7.246	2.396 + 1	4.242 + 1	4.032 + 1	3.710 + 1	3.361 + 1
.16204	1.9445	5.576 - 1	1.873	6.188	2.048 + 1	3.628 + 1	3.449 + 1	3.167 + 1	2.865 + 1
.20587	2.4704	4.573 - 1	1.511	4.998	1.657 + 1	2.936 + 1	2.796 + 1	2.571 + 1	2.323 + 1
.25305	3.0366	3.885 - 1	1.283	4.248	1.409 + 1	2.497 + 1	2.381 + 1	2.192 + 1	1.985 + 1
.33586	4.0303	3.118 - 1	1.030	3.411	1.131 + 1	2.005 + 1	1.912 + 1	1.762 + 1	1.603 + 1
.46803	5.6164	2.406 - 1	7.950 - 1	2.632	8.724	1.546 + 1	1.474 + 1	1.359 + 1	1.243 + 1
.60690	7.2828	1.969 - 1	6.507 - 1	2.154	7.141	1.266 + 1	1.206 + 1	1.111 + 1	1.027 + 1
1.0795	12.954	1.299 - 1	4.296 - 1	1.423	4.719	8.365	7.970	7.324	7.033
1.7136	20.563	9.664 - 2	3.198 - 1	1.060	3.519	6.238	5.944	5.447	5.383
2.7994	33.593	7.224 - 2	2.393 - 1	7.941 - 1	2.638	4.678	4.459	4.075	4.216

TABLE IV (continued)
(d) Heat Transfer Coefficient

Stream- wise Dimen- sion, ft.	S/R _N	Total Enthalpy, Btu/lb							
		11,500	11,500	11,500	11,500	11,500	10,000	8,000	6,000
		Total Pressure, atm							
		.01	.1	1	10	30	30	30	30
		Heat Transfer Coefficient, $\rho_e U_e C_H$ (lb/sec ft ²) as defined by Eq. (1)							
0.	0.	1.666 - 2	5.345 - 2	1.715 - 1	5.507 - 1	9.616 - 1	9.622 - 1	9.610 - 1	9.618 - 1
.01561	.1873	1.615 - 2	5.180 - 2	1.661 - 1	5.331 - 1	9.307 - 1	9.312 - 1	9.301 - 1	9.305 - 1
.03125	.3750	1.498 - 2	4.806 - 2	1.541 - 1	4.942 - 1	8.624 - 1	8.624 - 1	8.612 - 1	8.615 - 1
.04686	.5623	1.256 - 2	4.024 - 2	1.287 - 1	4.122 - 1	7.186 - 1	7.181 - 1	7.174 - 1	7.172 - 1
.05727	.6872	1.094 - 2	3.503 - 2	1.119 - 1	3.579 - 1	6.236 - 1	6.226 - 1	6.223 - 1	6.218 - 1
.07292	.8750	8.304 - 3	2.651 - 2	8.456 - 2	2.697 - 1	4.692 - 1	4.680 - 1	4.682 - 1	4.672 - 1
.08366	1.0039	6.527 - 3	2.080 - 2	6.622 - 2	2.108 - 1	3.664 - 1	3.653 - 1	3.658 - 1	3.650 - 1
.10092	1.2110	4.029 - 3	1.280 - 2	4.062 - 2	1.289 - 1	2.238 - 1	2.235 - 1	2.238 - 1	2.230 - 1
.12363	1.4836	1.781 - 3	5.627 - 3	1.780 - 2	5.657 - 2	9.839 - 2	9.876 - 2	9.822 - 2	9.812 - 2
.13209	1.5851	1.577 - 3	4.982 - 3	1.575 - 2	5.006 - 2	8.711 - 2	8.730 - 2	8.695 - 2	8.674 - 2
.14196	1.7035	1.462 - 3	4.623 - 3	1.461 - 2	4.638 - 2	8.063 - 2	8.060 - 2	8.005 - 2	7.995 - 2
.16204	1.9445	1.313 - 3	4.156 - 3	1.314 - 2	4.166 - 2	7.237 - 2	7.229 - 2	7.171 - 2	7.167 - 2
.20587	2.4704	1.083 - 3	3.439 - 3	1.091 - 2	3.452 - 2	5.993 - 2	5.981 - 2	5.941 - 2	5.962 - 2
.25305	3.0366	9.175 - 4	2.917 - 3	9.249 - 3	2.930 - 2	5.085 - 2	5.070 - 2	5.040 - 2	5.085 - 2
.33586	4.0303	7.251 - 4	2.308 - 3	7.322 - 3	2.319 - 2	4.023 - 2	4.008 - 2	3.982 - 2	4.048 - 2
.46803	5.6164	5.495 - 4	1.749 - 3	5.552 - 3	1.759 - 2	3.050 - 2	3.037 - 2	3.013 - 2	3.099 - 2
.60690	7.2828	4.455 - 4	1.418 - 3	4.502 - 3	1.426 - 2	2.473 - 2	2.462 - 2	2.440 - 2	2.554 - 2
1.0795	12.954	2.931 - 4	9.337 - 4	2.965 - 3	9.391 - 3	1.629 - 2	1.621 - 2	1.604 - 2	1.770 - 2
1.7136	20.563	2.215 - 4	7.064 - 4	2.244 - 3	7.113 - 3	1.233 - 2	1.228 - 2	1.215 - 2	1.387 - 2
2.7994	33.593	1.696 - 4	5.421 - 4	1.725 - 3	5.469 - 3	9.482 - 3	9.435 - 3	9.339 - 3	1.139 - 2

TABLE IV (continued)
(e) Skin Friction Coefficient

Stream- wise Dimen- sion, ft.	S/R _N	Total Enthalpy, Btu/lb							
		11,500	11,500	11,500	11,500	11,500	10,000	8,000	6,000
		Total Pressure, atm							
		.01	.1	1	10	30	30	30	30
		Skin Friction Coefficient, $\rho_e u_e C_f / 2$ (lb/sec ft ²)							
0	0	1.713 - 2	5.441 - 2	1.731 - 1	5.513 - 1	9.584 - 1	9.611 - 1	9.616 - 1	9.558 - 1
0.01561	0.1873	1.669 - 2	5.300 - 2	1.685 - 1	5.366 - 1	9.327 - 1	9.354 - 1	9.358 - 1	9.299 - 1
0.03125	0.3750	1.562 - 2	4.958 - 2	1.577 - 1	5.020 - 1	8.725 - 1	8.750 - 1	8.751 - 1	8.697 - 1
0.04686	0.5623	1.349 - 2	4.281 - 2	1.360 - 1	4.325 - 1	7.512 - 1	7.533 - 1	7.532 - 1	7.486 - 1
0.05727	0.6872	1.199 - 2	3.805 - 2	1.209 - 1	3.840 - 1	6.668 - 1	6.686 - 1	6.683 - 1	6.646 - 1
0.07292	0.8750	9.465 - 3	3.001 - 2	9.525 - 2	3.024 - 1	5.246 - 1	5.259 - 1	5.253 - 1	5.230 - 1
0.08366	1.0039	7.722 - 3	2.447 - 2	7.761 - 2	2.461 - 1	4.267 - 1	4.277 - 1	4.271 - 1	4.267 - 1
0.10092	1.2110	5.185 - 3	1.642 - 2	5.197 - 2	1.644 - 1	2.848 - 1	2.584 - 1	2.848 - 1	2.869 - 1
0.12363	1.4836	2.740 - 3	8.536 - 3	2.694 - 2	8.501 - 2	1.470 - 1	1.474 - 1	1.473 - 1	1.504 - 1
0.13209	1.5851	1.990 - 3	6.310 - 3	2.001 - 2	6.355 - 2	1.103 - 1	1.103 - 1	1.100 - 1	1.121 - 1
0.14196	1.7035	1.720 - 3	5.456 - 3	1.732 - 2	5.501 - 2	9.553 - 2	9.536 - 2	9.490 - 2	9.676 - 2
0.16204	1.9445	1.451 - 3	4.611 - 3	1.466 - 2	4.661 - 2	8.098 - 2	8.086 - 2	8.033 - 2	8.185 - 2
0.20587	2.4704	1.146 - 3	3.647 - 3	1.161 - 2	3.698 - 2	6.432 - 2	6.434 - 2	6.401 - 2	6.525 - 2
0.25305	3.0366	9.554 - 4	3.042 - 3	9.689 - 3	3.089 - 2	5.375 - 2	5.383 - 2	5.366 - 2	5.489 - 2
0.33586	4.0303	7.463 - 4	2.377 - 3	7.577 - 3	2.417 - 2	4.207 - 2	4.214 - 2	4.208 - 2	4.334 - 2
0.46803	5.6164	5.577 - 4	1.778 - 3	5.670 - 3	1.810 - 2	3.150 - 2	3.155 - 2	3.154 - 2	3.276 - 2
0.60690	7.2828	4.459 - 4	1.422 - 3	4.539 - 3	1.450 - 2	2.524 - 2	2.527 - 2	2.527 - 2	2.659 - 2
1.0795	12.954	2.815 - 4	8.992 - 4	2.874 - 3	9.194 - 3	1.602 - 2	1.604 - 2	1.604 - 2	1.765 - 2
1.7136	20.563	2.039 - 4	6.520 - 4	2.087 - 3	6.686 - 3	1.166 - 2	1.168 - 2	1.167 - 2	1.326 - 2
2.7994	33.593	1.491 - 4	4.776 - 4	1.531 - 3	4.913 - 3	8.574 - 3	8.595 - 3	8.576 - 3	1.027 - 2

TABLE IV (continued)
(f) Momentum Thickness

Stream- wise Dimen- sion, ft.	S/R _N	Total Enthalpy, Btu/lb							
		11,500	11,500	11,500	11,500	11,500	10,000	8,000	6,000
		Total Pressure, atm							
		.01	.1	1	10	30	30	30	30
		Momentum Thickness, θ (ft)							
0	0	2.345 - 3	7.914 - 4	2.688 - 4	9.179 - 5	5.500 - 5	5.214 - 5	4.853 - 5	4.462 - 5
0.01561	0.1873	2.409 - 3	8.137 - 4	2.760 - 4	9.409 - 5	5.647 - 5	5.359 - 5	4.981 - 5	4.583 - 5
0.03125	0.3750	2.565 - 3	8.646 - 4	2.928 - 4	9.986 - 5	5.986 - 5	5.676 - 5	5.286 - 5	4.858 - 5
0.04686	0.5623	3.000 - 3	1.010 - 3	3.417 - 4	1.162 - 4	6.967 - 5	6.617 - 5	6.160 - 5	5.652 - 5
0.05727	0.6872	3.379 - 3	1.137 - 3	3.838 - 4	1.304 - 4	7.800 - 5	7.410 - 5	6.911 - 5	6.325 - 5
0.07292	0.8750	4.308 - 3	1.445 - 3	4.867 - 4	1.645 - 4	9.827 - 5	9.358 - 5	8.738 - 5	7.942 - 5
0.08366	1.0039	5.336 - 3	1.787 - 3	6.000 - 4	2.023 - 4	1.207 - 4	1.150 - 4	1.075 - 4	9.681 - 5
0.10092	1.2110	8.213 - 3	2.740 - 3	9.166 - 4	3.067 - 4	1.829 - 4	1.747 - 4	1.635 - 4	1.435 - 4
0.12363	1.4836	1.718 - 2	5.705 - 3	1.899 - 3	6.333 - 4	3.753 - 4	3.594 - 4	3.343 - 4	2.757 - 4
0.13209	1.5851	1.822 - 2	6.046 - 3	2.010 - 3	6.693 - 4	3.963 - 4	3.795 - 4	3.524 - 4	2.904 - 4
0.14196	1.7035	1.938 - 2	6.422 - 3	2.132 - 3	7.093 - 4	4.198 - 4	4.018 - 4	3.726 - 4	3.064 - 4
0.16204	1.9445	2.156 - 2	7.133 - 3	2.364 - 3	7.847 - 4	4.638 - 4	4.438 - 4	4.103 - 4	3.363 - 4
0.20587	2.4704	2.615 - 2	8.633 - 3	2.854 - 3	9.442 - 4	5.571 - 4	5.327 - 4	4.899 - 4	3.987 - 4
0.25305	3.0366	3.085 - 2	1.016 - 2	3.353 - 3	1.107 - 3	6.521 - 4	6.232 - 4	5.704 - 4	4.610 - 4
0.33586	4.0303	3.880 - 2	1.276 - 2	4.196 - 3	1.381 - 3	8.123 - 4	7.756 - 4	7.043 - 4	5.643 - 4
0.46803	5.6164	5.054 - 2	1.657 - 2	5.436 - 3	1.782 - 3	1.046 - 3	9.977 - 4	8.953 - 4	7.109 - 4
0.60690	7.2828	6.152 - 2	2.014 - 2	6.589 - 3	2.155 - 3	1.263 - 3	1.202 - 3	1.067 - 3	8.386 - 4
1.0795	12.954	9.079 - 2	2.960 - 2	9.641 - 3	3.133 - 3	1.830 - 3	1.733 - 3	1.497 - 3	1.148 - 3
1.7136	20.563	1.178 - 1	3.830 - 2	1.243 - 2	4.023 - 3	2.342 - 3	2.208 - 3	1.870 - 3	1.422 - 3
2.7994	33.593	1.514 - 1	4.911 - 2	1.589 - 2	5.121 - 3	2.973 - 3	2.785 - 3	2.327 - 3	1.705 - 3

TABLE IV (concluded)
(g) Displacement Thickness

Stream- wise Dimen- sion, ft.	S/R _N	Total Enthalpy, Btu/lb									
		11,500	11,500	11,500	11,500	11,500	10,000	8,000	6,000		
		Total Pressure, atm									
		.01	.1	1	10	30	30	30	30		
		Displacement Thickness, δ* (ft)									
0	0	2.767 - 4	5.100 - 5	3.004 - 6	4.455 - 6	4.436 - 6	3.408 - 6	3.536 - 6	5.723 - 6		
0.01561	0.1873	2.802 - 4	5.169 - 5	3.016 - 6	4.539 - 6	4.513 - 6	3.479 - 6	3.658 - 6	5.904 - 6		
0.03125	0.3750	3.095 - 4	6.019 - 5	5.473 - 6	3.859 - 6	4.166 - 6	3.203 - 6	3.577 - 6	6.022 - 6		
0.04686	0.5623	3.278 - 4	6.268 - 5	4.899 - 6	4.661 - 6	4.840 - 6	3.905 - 6	4.652 - 6	7.420 - 6		
0.05727	0.6872	3.559 - 4	6.965 - 5	6.142 - 6	4.698 - 6	5.023 - 6	4.218 - 6	5.392 - 6	8.341 - 6		
0.07292	0.8750	4.161 - 4	8.327 - 5	8.156 - 6	5.117 - 6	5.686 - 6	5.235 - 6	7.528 - 6	1.028 - 5		
0.08366	1.0039	4.549 - 4	8.938 - 5	7.134 - 6	6.772 - 6	7.037 - 6	7.116 - 6	1.069 - 5	1.218 - 5		
0.10092	1.2110	4.122 - 4	5.384 - 5	1.368 - 5	1.740 - 5	1.448 - 5	1.626 - 5	2.336 - 5	1.563 - 5		
0.12363	1.4836	6.790 - 4	3.970 - 4	1.985 - 4	9.027 - 5	6.046 - 5	6.867 - 5	7.933 - 5	9.889 - 6		
0.13209	1.5851	9.994 - 4	1.510 - 4	2.120 - 5	3.365 - 5	2.781 - 5	3.796 - 5	5.110 - 5	2.000 - 5		
0.14196	1.7035	2.434 - 3	6.215 - 4	1.319 - 5	1.511 - 5	3.822 - 7	1.194 - 5	2.734 - 5	4.709 - 5		
0.16204	1.9445	4.607 - 3	1.336 - 3	3.652 - 4	8.940 - 5	4.310 - 5	2.688 - 5	7.450 - 6	8.967 - 5		
0.20587	2.4704	8.255 - 3	2.526 - 3	7.533 - 4	2.133 - 4	1.143 - 4	9.042 - 5	6.360 - 5	1.647 - 4		
0.25305	3.0366	1.150 - 2	3.577 - 3	1.094 - 3	3.223 - 4	1.771 - 4	1.461 - 4	1.139 - 4	2.347 - 4		
0.33586	4.0303	1.673 - 2	5.257 - 3	1.636 - 3	4.951 - 4	2.768 - 4	2.345 - 4	1.985 - 4	3.518 - 4		
0.46803	5.6164	2.474 - 2	7.830 - 3	2.463 - 3	7.593 - 4	4.297 - 4	3.702 - 4	3.409 - 4	5.332 - 4		
0.60690	7.2828	3.290 - 2	1.045 - 2	3.307 - 3	1.030 - 3	5.867 - 4	5.105 - 4	5.011 - 4	7.105 - 4		
1.0795	12.954	5.866 - 2	1.875 - 2	5.999 - 3	1.901 - 3	1.096 - 3	9.732 - 4	1.070 - 3	1.220 - 3		
1.7136	20.563	8.820 - 2	2.828 - 2	9.115 - 3	2.925 - 3	1.698 - 3	1.536 - 3	1.758 - 3	1.755 - 3		
2.7994	33.593	1.311 - 1	4.197 - 2	1.361 - 2	4.418 - 3	2.583 - 3	2.382 - 3	2.731 - 3	2.402 - 3		

2.2.2 Effect of Nose Radius

The effects of nose radius on diffusive heat flux, skin friction coefficient, momentum thickness, and displacement thickness are considered in Table V for four flight conditions. The boundary layers were considered to be nonsimilar and unequal diffusion coefficients were considered in these calculations. Nose radii of one-half inch and one inch were considered for a cone half angle of 5 degrees and a wall temperature of 1,000°R. It can be seen by examination of these results that the diffusive heat flux and skin friction coefficient vary as $R_N^{-1/2}$ while the momentum thickness and displacement thickness vary as $R_N^{1/2}$, give or take one or two digits in the fourth significant figure. Of course, this ability to scale accurately with nose radius is lost in quasisteady ablation problems since the surface temperature will be elevated as the nose radius is decreased due to the increase in the diffusive heat flux.

2.2.3 Effect of Cone Angle

The effect of cone half angle on the nonsimilar air boundary layer with unequal diffusion coefficients and a uniform wall temperature of 1,000°R is presented in Table VI for a flight condition of 11,500 Btu/lb and 30 atmospheres. As one would expect, the effects of increasing the cone half angle from 5 to 10 degrees is to increase the diffusive heat flux and skin friction coefficient and generally to decrease the momentum and displacement thicknesses. Although, further calculations are certainly required to substantiate the following, it is tentatively suggested that the results presented in Table IV for other flight conditions with a 5 degree cone half angle could be corrected to 10 degree cone half angle by applying the ratios for the single case of Table VI and, furthermore, that it would not be too inaccurate to linearly interpolate for cone half angles between 5 and 10 degrees. Extrapolation to much larger or smaller cone angles should be viewed with caution, however.

TABLE V

EFFECT OF NOSE RADIUS ON DIFFUSIVE HEAT FLUX, SKIN FRICTION COEFFICIENT, MOMENTUM THICKNESS AND DISPLACEMENT THICKNESS FOR A NONSIMILAR AIR BOUNDARY LAYER OVER A SPHERE CONE WITH UNEQUAL DIFFUSION COEFFICIENTS:

Cone Half Angle = 5° , Wall Temperature = 1000°R

(a) Total Enthalpy = 11,500 Btu/lb, Total Pressure = 1 atm

S/R_N	Diffusive Heat Flux, q_a (Btu/sec ft ²)		Skin Friction Coefficient, (lb/sec ft ²)		Momentum Thickness, θ (ft)		Displacement Thickness, δ^* (ft)	
	$R_N = \frac{1}{2}"$	$R_N = 1"$	$R_N = \frac{1}{2}"$	$R_N = 1"$	$R_N = \frac{1}{2}"$	$R_N = 1"$	$R_N = \frac{1}{2}"$	$R_N = 1"$
0	2.761 + 3	1.952 + 3	2.448 - 1	1.731 - 1	1.899 - 4	2.688 - 4	2.105 - 6	3.004 - 6
.1873	2.674 + 3	1.891 + 3	2.384 - 1	1.685 - 1	1.950 - 4	2.760 - 4	2.118 - 6	3.016 - 6
.3750	2.480 + 3	1.754 + 3	2.230 - 1	1.577 - 1	2.071 - 4	2.928 - 4	3.871 - 6	5.473 - 6
.5623	2.073 + 3	1.466 + 3	1.924 - 1	1.360 - 1	2.416 - 4	3.417 - 4	3.464 - 6	4.899 - 6
.6872	1.802 + 3	1.274 + 3	1.709 - 1	1.209 - 1	2.714 - 4	3.838 - 4	4.343 - 6	6.142 - 6
.8750	1.361 + 3	9.626 + 2	1.347 - 1	9.525 - 2	3.442 - 4	4.867 - 4	5.768 - 6	8.156 - 6
1.0039	1.066 + 3	7.538 + 2	1.098 - 1	7.761 - 2	4.243 - 4	6.000 - 4	5.045 - 6	7.134 - 6
1.2110	6.539 + 2	4.624 + 2	7.350 - 2	5.197 - 2	6.482 - 4	9.166 - 4	9.675 - 6	1.368 - 5
1.4836	2.865 + 2	2.026 + 2	3.810 - 2	2.694 - 2	1.343 - 3	1.899 - 3	1.404 - 4	1.985 - 4
1.5851	2.535 + 2	1.793 + 2	2.830 - 2	2.001 - 2	1.421 - 3	2.010 - 3	1.499 - 5	2.120 - 5
1.7035	2.353 + 2	1.663 + 2	2.449 - 2	1.732 - 2	1.508 - 3	2.132 - 3	9.324 - 5	1.319 - 5
1.9445	2.116 + 2	1.496 + 2	2.073 - 2	1.466 - 2	1.672 - 3	2.364 - 3	2.582 - 4	3.652 - 4
2.4704	1.754 + 2	1.240 + 2	1.642 - 2	1.161 - 2	2.018 - 3	2.854 - 3	5.327 - 4	7.533 - 4
3.0366	1.489 + 2	1.053 + 2	1.370 - 2	9.689 - 3	2.317 - 3	3.353 - 3	7.737 - 4	1.094 - 3
4.0303	1.179 + 2	8.335 + 1	1.072 - 2	7.577 - 3	2.967 - 3	4.196 - 3	1.157 - 3	1.636 - 3
5.6164	8.939 + 1	6.321 + 1	8.019 - 3	5.670 - 3	3.844 - 3	5.436 - 3	1.742 - 3	2.463 - 3
7.2828	7.248 + 1	5.125 + 1	6.419 - 3	4.539 - 3	4.659 - 3	6.589 - 3	2.339 - 3	3.307 - 3
12.954	4.773 + 1	3.375 + 1	4.064 - 3	2.874 - 3	6.817 - 3	9.641 - 3	4.242 - 3	5.999 - 3
20.563	3.613 + 1	2.555 + 1	2.951 - 3	2.087 - 3	8.791 - 3	1.243 - 2	6.445 - 3	9.115 - 3
33.593	2.776 + 1	1.963 + 1	2.165 - 3	1.531 - 3	1.124 - 2	1.589 - 2	9.623 - 3	1.361 - 2

TABLE V (continued)

(b) Total Enthalpy = 11,500 Btu/lb, Total Pressure = 10 atm

S/R _N	Diffusive Heat Flux, q (Btu/sec ft ²)		Skin Friction Coefficient, (lb/sec ft ²)		Momentum Thickness, θ (ft)		Displacement Thickness, δ* (ft)	
	R _N = ½"	R _N = 1"	R _N = ½"	R _N = 1"	R _N = ½"	R _N = 1"	R _N = ½"	R _N = 1"
0	8.886 + 3	6.269 + 3	7.796 - 1	5.513 - 1	6.489 - 5	9.179 - 5	3.145 - 6	4.455 - 6
.1873	8.583 + 3	6.069 + 3	7.588 - 1	5.366 - 1	6.653 - 5	9.409 - 5	3.209 - 6	4.539 - 6
.3750	7.956 + 3	5.625 + 3	7.099 - 1	5.020 - 1	7.061 - 5	9.986 - 5	2.728 - 6	3.859 - 6
.5623	6.635 + 3	4.692 + 3	6.116 - 1	4.325 - 1	8.219 - 5	1.162 - 4	3.296 - 6	4.661 - 6
.6872	5.762 + 3	4.074 + 3	5.431 - 1	3.840 - 1	9.220 - 5	1.304 - 4	3.322 - 6	4.698 - 6
.8750	4.341 + 3	3.070 + 3	4.276 - 1	3.024 - 1	1.163 - 4	1.645 - 4	3.661 - 6	5.117 - 6
1.0039	3.393 + 3	2.399 + 3	3.480 - 1	2.461 - 1	1.431 - 4	2.023 - 4	4.789 - 6	6.772 - 6
1.2110	2.075 + 3	1.467 + 3	2.326 - 1	1.644 - 1	2.175 - 4	3.067 - 4	1.230 - 5	1.740 - 5
1.4836	9.107 + 2	6.440 + 2	1.202 - 1	8.501 - 2	4.478 - 4	6.333 - 4	6.383 - 5	9.027 - 5
1.5851	8.059 + 2	5.698 + 2	8.987 - 2	6.355 - 2	4.733 - 4	6.693 - 4	2.380 - 5	3.365 - 5
1.7035	7.466 + 2	5.279 + 2	7.779 - 2	5.501 - 2	5.016 - 4	7.093 - 4	1.068 - 5	1.511 - 5
1.9445	6.707 + 2	4.742 + 2	6.591 - 2	4.661 - 2	5.549 - 4	7.847 - 4	6.322 - 5	8.940 - 5
2.4704	5.557 + 2	3.930 + 2	5.230 - 2	3.698 - 2	6.677 - 4	9.442 - 4	1.508 - 4	2.133 - 4
3.0366	4.717 + 2	3.335 + 2	4.369 - 2	3.089 - 2	7.825 - 4	1.107 - 3	2.279 - 4	3.223 - 4
4.0303	3.734 + 2	2.640 + 2	3.418 - 2	2.417 - 2	9.763 - 4	1.381 - 3	3.501 - 4	4.951 - 4
5.6164	2.831 + 2	2.002 + 2	2.559 - 2	1.810 - 2	1.260 - 3	1.782 - 3	5.369 - 4	7.593 - 4
7.2828	2.296 + 2	1.623 + 2	2.050 - 2	1.450 - 2	1.524 - 3	2.155 - 3	7.283 - 4	1.030 - 3
12.954	1.512 + 2	1.069 + 2	1.300 - 2	9.194 - 3	2.215 - 3	3.133 - 3	1.344 - 3	1.901 - 3
20.563	1.145 + 2	8.097 + 1	9.456 - 3	6.686 - 3	2.845 - 3	4.023 - 3	2.068 - 3	2.925 - 3
33.593	8.804 + 1	6.226 + 1	6.948 - 3	4.913 - 3	3.621 - 3	5.121 - 3	3.124 - 3	4.418 - 3

TABLE V (continued)

(c) Total Enthalpy = 11,500 Btu/lb, Total Pressure = 30 atm

S/R_N	Diffusive Heat Flux, q_{a_2} (Btu/sec ft ²)		Skin Friction Coefficient (lb/sec ft ²)		Momentum Thickness, θ (ft)		Displacement Thickness, δ^* (ft)	
	$R_N = \frac{1}{2}$ "	$R_N = 1$ "	$R_N = \frac{1}{2}$ "	$R_N = 1$ "	$R_N = \frac{1}{2}$ "	$R_N = 1$ "	$R_N = \frac{1}{2}$ "	$R_N = 1$ "
0	1.548 + 4	1.095 + 4	1.355	9.584 - 1	3.889 - 5	5.500 - 5	3.136 - 6	4.436 - 6
.1873	1.498 + 4	1.059 + 4	1.319	9.327 - 1	3.993 - 5	5.647 - 5	3.191 - 6	4.513 - 6
.3750	1.388 + 4	9.817 + 3	1.234	8.725 - 1	4.233 - 5	5.986 - 5	2.946 - 6	4.166 - 6
.5623	1.157 + 4	8.180 + 3	1.062	7.512 - 1	4.926 - 5	6.967 - 5	3.422 - 6	4.840 - 6
.6872	1.004 + 4	7.099 + 3	9.430 - 1	6.668 - 1	5.515 - 5	7.800 - 5	3.552 - 6	5.023 - 6
.8750	7.554 + 3	5.342 + 3	7.419 - 1	5.246 - 1	6.948 - 5	9.827 - 5	4.021 - 6	5.686 - 6
1.0039	5.899 + 3	4.171 + 3	6.035 - 1	4.267 - 1	8.535 - 5	1.207 - 4	4.976 - 6	7.037 - 6
1.2110	3.603 + 3	2.548 + 3	4.028 - 1	2.848 - 1	1.293 - 4	1.829 - 4	1.024 - 5	1.448 - 5
1.4836	1.584 + 3	1.120 + 3	2.079 - 1	1.470 - 1	2.654 - 4	3.753 - 4	4.275 - 5	6.046 - 5
1.5851	1.402 + 3	9.916 + 2	1.560 - 1	1.103 - 1	2.802 - 4	3.963 - 4	1.966 - 5	2.781 - 5
1.7035	1.298 + 3	9.179 + 2	1.351 - 1	9.553 - 2	2.968 - 4	4.198 - 4	2.703 - 7	3.822 - 7
1.9445	1.165 + 3	8.239 + 2	1.145 - 1	8.098 - 2	3.279 - 4	4.638 - 4	3.048 - 5	4.310 - 5
2.4704	9.648 + 2	6.822 + 2	9.096 - 2	6.432 - 2	3.939 - 4	5.571 - 4	8.081 - 5	1.143 - 4
3.0366	8.186 + 2	5.789 + 2	7.602 - 2	5.375 - 5	4.611 - 4	6.521 - 4	1.252 - 4	1.771 - 4
4.0303	6.477 + 2	4.580 + 2	5.949 - 2	4.207 - 2	5.744 - 4	8.123 - 4	1.958 - 4	2.768 - 4
5.6164	4.910 + 2	3.472 + 2	4.455 - 2	3.150 - 2	7.398 - 4	1.046 - 3	3.038 - 4	4.297 - 4
7.2828	3.981 + 2	2.815 + 2	3.570 - 2	2.524 - 2	8.930 - 4	1.263 - 3	4.148 - 4	5.867 - 4
12.954	2.622 + 2	1.854 + 2	2.266 - 2	1.602 - 2	1.294 - 3	1.830 - 3	7.747 - 4	1.096 - 3
20.563	1.986 + 2	1.404 + 2	1.649 - 2	1.166 - 2	1.656 - 3	2.342 - 3	1.201 - 3	1.698 - 3
33.593	1.527 + 2	1.079 + 2	1.213 - 2	8.574 - 3	2.102 - 3	2.973 - 3	1.826 - 3	2.583 - 3

TABLE V (concluded)

(d) Total Enthalpy = 10,000 Btu/lb, Total Pressure = 30 atm

S/R _N	Diffusive Heat Flux, q _a (Btu/sec ft ²)		Skin Friction Coefficient, (lb/sec ft ²)		Momentum Thickness, θ (ft)		Displacement Thickness, δ* (ft)	
	R _N = ½"	R _N = 1"	R _N = ½"	R _N = 1"	R _N = ½"	R _N = 1"	R _N = ½"	R _N = 1"
0	1.345 + 4	9.510 + 3	1.359	9.611 - 1	3.694 - 5	5.214 - 5	-2.404 - 6	-3.408 - 6
.1873	1.302 + 4	9.203 + 3	1.323	9.354 - 1	3.790 - 5	5.359 - 5	-2.460 - 6	-3.479 - 6
.3750	1.205 + 4	8.523 + 3	1.237	8.750 - 1	4.014 - 5	5.676 - 5	-2.265 - 6	-3.203 - 6
.5623	1.004 + 4	7.097 + 3	1.065	7.533 - 1	4.679 - 5	6.617 - 5	-2.761 - 6	-3.905 - 6
.6872	8.702 + 3	6.154 + 3	9.455 - 1	6.686 - 1	5.239 - 5	7.410 - 5	-2.982 - 6	-4.218 - 6
.8750	6.541 + 3	4.625 + 3	7.437 - 1	5.259 - 1	6.617 - 5	9.358 - 5	-3.702 - 6	-5.235 - 6
1.0039	5.105 + 3	3.610 + 3	6.048 - 1	4.277 - 1	8.130 - 5	1.150 - 4	-5.032 - 6	-7.116 - 6
1.2110	3.123 + 3	2.209 + 3	4.036 - 1	2.584 - 1	1.236 - 4	1.747 - 4	-1.150 - 5	-1.626 - 5
1.4836	1.380 + 3	9.761 + 2	2.085 - 1	1.474 - 1	2.541 - 4	3.594 - 4	-4.856 - 5	-6.867 - 5
1.5851	1.220 + 3	8.628 + 2	1.560 - 1	1.103 - 1	2.683 - 4	3.795 - 4	-2.684 - 5	-3.796 - 5
1.7035	1.127 + 3	7.967 + 2	1.349 - 1	9.536 - 2	2.841 - 4	4.018 - 4	-8.440 - 6	-1.194 - 5
1.9445	1.010 + 3	7.144 + 2	1.143 - 1	8.086 - 2	3.138 - 4	4.438 - 4	1.901 - 5	2.688 - 5
2.4704	8.359 + 2	5.911 + 2	9.099 - 2	6.434 - 2	3.767 - 4	5.327 - 4	6.394 - 5	9.042 - 5
3.0366	7.086 + 2	5.011 + 2	7.612 - 2	5.383 - 2	4.407 - 4	6.232 - 4	1.033 - 4	1.461 - 4
4.0303	5.602 + 2	3.961 + 2	5.959 - 2	4.214 - 2	5.485 - 4	7.756 - 4	1.658 - 4	2.345 - 4
5.6164	4.245 + 2	3.002 + 2	4.462 - 2	3.155 - 2	7.055 - 4	9.977 - 4	2.618 - 4	3.702 - 4
7.2828	3.442 + 2	2.434 + 2	3.574 - 2	2.527 - 2	8.502 - 4	1.202 - 3	3.610 - 4	5.105 - 4
12.954	2.266 + 2	1.603 + 2	2.269 - 2	1.604 - 2	1.226 - 3	1.733 - 3	6.881 - 4	9.732 - 4
20.563	1.716 + 2	1.213 + 2	1.652 - 2	1.168 - 2	1.561 - 3	2.208 - 3	1.086 - 3	1.536 - 3
33.593	1.319 + 2	9.325 + 1	1.215 - 2	8.595 - 3	1.969 - 3	2.785 - 3	1.685 - 3	2.382 - 3

TABLE VI

EFFECT OF CONE HALF ANGLE ON DIFFUSIVE HEAT FLUX, SKIN FRICTION COEFFICIENT,
MOMENTUM THICKNESS, AND DISPLACEMENT THICKNESS FOR A NONSIMILAR AIR
BOUNDARY LAYER OVER A SPHERE CONE WITH UNEQUAL DIFFUSION COEFFICIENTS:

Nose Radius = 1 Inch, Wall Temperature = 1,000°R,
Total Enthalpy = 11,500 Btu/lb, Total Pressure = 30 atm

S/R_N	S/R_N	Diffusive Heat Flux, q_a (Btu/sec ft ²)		Skin Friction Coefficient, (lb/sec ft ²)		Momentum Thickness, θ (ft)		Displacement Thickness, δ^* (ft)	
		$\theta_c = 5^\circ$	$\theta_c = 10^\circ$	$\theta_c = 5^\circ$	$\theta_c = 10^\circ$	$\theta_c = 5^\circ$	$\theta_c = 10^\circ$	$\theta_c = 5^\circ$	$\theta_c = 10^\circ$
0	0	1.095 + 4	1.095 + 4	9.584 - 1	9.584 - 1	5.500 - 5	5.500 - 5	-4.436 - 6	-4.436 - 6
0.1873	0.1873	1.059 + 4	1.059 + 4	9.327 - 1	9.327 - 1	5.647 - 5	5.647 - 5	-4.513 - 6	-4.513 - 6
0.3750	0.3750	9.817 + 3	9.817 + 3	8.725 - 1	8.725 - 1	5.986 - 5	5.986 - 5	-4.166 - 6	-4.166 - 6
0.5623	0.5623	8.180 + 3	8.180 + 3	7.512 - 1	7.512 - 1	6.967 - 5	6.967 - 5	-4.840 - 6	-4.840 - 6
0.6872	0.6872	7.099 + 3	7.099 + 3	6.668 - 1	6.668 - 1	7.800 - 5	7.800 - 5	-5.023 - 6	-5.023 - 6
0.8750	0.8750	5.342 + 3	5.342 + 3	5.246 - 1	5.246 - 1	9.827 - 5	9.827 - 5	-5.686 - 6	-5.686 - 6
1.0039	1.0039	4.171 + 3	4.171 + 3	4.267 - 1	4.267 - 1	1.207 - 4	1.207 - 4	-7.037 - 6	-7.037 - 6
1.2110	1.2110	2.548 + 3	2.549 + 3	2.848 - 1	2.849 - 1	1.829 - 4	1.829 - 4	-1.448 - 5	-1.444 - 5
1.4836	1.3963	1.120 + 3	1.484 + 3	1.470 - 1	1.839 - 1	3.753 - 4	2.924 - 4	-6.046 - 5	-3.666 - 5
1.5851	1.5198	9.916 + 2	1.285 + 3	1.103 - 1	1.344 - 1	3.963 - 4	3.166 - 4	-2.781 - 5	-5.244 - 6
1.7035	1.6234	9.179 + 2	1.199 + 3	9.553 - 2	1.190 - 1	4.198 - 4	3.350 - 4	3.822 - 7	1.505 - 5
1.9445	1.8264	8.239 + 2	1.082 + 3	8.098 - 2	1.027 - 1	4.638 - 4	3.698 - 4	4.310 - 5	4.538 - 5
2.4704	2.2428	6.822 + 2	9.099 + 2	6.432 - 2	8.353 - 2	5.571 - 4	4.385 - 4	1.143 - 4	9.347 - 5
3.0366	2.9638	5.789 + 2	7.220 + 2	5.375 - 2	6.501 - 2	6.521 - 4	5.474 - 4	1.771 - 4	1.612 - 4
4.0303	3.9182	4.580 + 2	5.733 + 2	4.207 - 2	5.100 - 2	8.123 - 4	6.791 - 4	2.768 - 4	2.416 - 4
5.6164	5.2789	3.472 + 2	4.578 + 2	3.150 - 2	3.998 - 2	1.046 - 3	8.339 - 4	4.297 - 4	3.489 - 4
7.2828	7.2184	2.815 + 2	3.737 + 2	2.524 - 2	3.176 - 2	1.263 - 3	1.002 - 3	5.867 - 4	4.902 - 4
12.954	10.173	1.854 + 2	3.162 + 2	1.602 - 2	2.581 - 2	1.830 - 3	1.167 - 3	1.096 - 3	6.723 - 4
20.563	19.241	1.404 + 2	2.765 + 2	1.166 - 2	2.037 - 2	2.342 - 3	1.339 - 3	1.698 - 3	1.009 - 3
33.593	31.630	1.079 + 2	2.972 + 2	8.574 - 3	1.954 - 2	2.973 - 3	1.301 - 3	2.583 - 3	1.149 - 3

2.2.4 Effect of Unequal Diffusion Coefficients

The effect of unequal diffusion coefficients on the non-similar air boundary layer is investigated in Table VII for a constant wall temperature of $1,000^{\circ}\text{R}$, a nose radius of 1 inch and a cone half-angle of 10 degrees for a flight condition of 11,500 Btu/lb and 30 atmospheres. In the case of diffusive heat flux and momentum thickness, the effect of unequal diffusion coefficients (to increase both of these quantities) is maintained at about the same percentage correction at positions around the body. The displacement thickness is always reduced (in the positive sense) by unequal diffusion coefficients but it is not possible to make any comparison of percentage changes because it is negative over the spherical nose and the frontal portion of the conical afterbody and then becomes positive. On the other hand, the skin-friction coefficient is slightly reduced by unequal diffusion over most of the body stations but the influence is reversed for distances greater than s/R_N of 5 or so.

2.2.5 Effect of Nonsimilar Pressure Gradient

Nonsimilar and similar air boundary layer solutions are compared in Table VIII for a total enthalpy of 11,500 Btu/lb, a total pressure of 30 atmospheres, a nose radius of 1 inch, and a uniform wall temperature of $1,000^{\circ}\text{R}$. Table VIII(a) considers unequal diffusion coefficients and a cone angle of 10 degrees, while Table VIII(b) considers equal diffusion coefficients and a cone angle of five degrees. The principal non-similar effect in these problems is the body shape which imposes a nonsimilar pressure gradient. Mass addition and variable wall temperature, not considered here, can also produce substantial nonsimilar effects.

The nonsimilar pressure gradient causes the diffusive heat flux to be slightly decreased on the spherical portion of the

TABLE VII

EFFECT OF UNEQUAL DIFFUSION COEFFICIENTS ON DIFFUSIVE HEAT FLUX, SKIN FRICTION COEFFICIENT, MOMENTUM THICKNESS, AND DISPLACEMENT THICKNESS FOR A NONSIMILAR AIR BOUNDARY LAYER OVER A SPHERE CONE:

Nose Radius = 1 Inch, Cone Half Angle = 10° , Wall Temperature = $1,000^\circ\text{R}$,
Total Enthalpy = 11,500 Btu/lb, Total Pressure = 30 atm

Stream- wise Dimen- sion, ft.	S/R_N	Diffusive Heat Flux, q_2 (Btu/sec ft ²)		Skin Friction Coefficient, (lb/sec ft ²)		Momentum Thickness, θ (ft)		Displacement Thickness, δ^* (ft)	
		Equal δ_{ij}	Unequal δ_{ij}	Equal δ_{ij}	Unequal δ_{ij}	Equal δ_{ij}	Unequal δ_{ij}	Equal δ_{ij}	Unequal δ_{ij}
0	0	1.059 + 4	1.095 + 4	9.603 - 1	9.584 - 1	5.462 - 5	5.500 - 5	-2.883 - 6	-4.436 - 6
0.01561	0.1873	1.025 + 4	1.059 + 4	9.346 - 1	9.327 - 1	5.607 - 5	5.647 - 5	-2.930 - 6	-4.513 - 6
0.03125	0.3750	9.503 + 3	9.817 + 3	8.742 - 1	8.725 - 1	5.940 - 5	5.986 - 5	-2.496 - 6	-4.166 - 6
0.04686	0.5623	7.922 + 3	8.180 + 3	7.526 - 1	7.512 - 1	6.904 - 5	6.967 - 5	-2.890 - 6	-4.840 - 6
0.05727	0.6872	6.878 + 3	7.099 + 3	6.681 - 1	6.668 - 1	7.724 - 5	7.800 - 5	-2.879 - 6	-5.023 - 6
0.07292	0.8750	5.182 + 3	5.342 + 3	5.255 - 1	5.246 - 1	9.737 - 5	9.827 - 5	-3.002 - 6	-5.686 - 6
0.08366	1.0039	4.052 + 3	4.171 + 3	4.274 - 1	4.267 - 1	1.195 - 4	1.207 - 4	-3.830 - 6	-7.040 - 6
0.10092	1.2110	2.486 + 3	2.549 + 3	2.851 - 1	2.849 - 1	1.809 - 4	1.829 - 4	-9.893 - 6	-1.444 - 5
0.11636	1.3963	1.454 + 3	1.484 + 3	1.839 - 1	1.839 - 1	2.889 - 4	2.924 - 4	-2.942 - 5	-3.666 - 5
0.12665	1.5198	1.258 + 3	1.285 + 3	1.349 - 1	1.344 - 1	3.131 - 4	3.166 - 4	2.162 - 6	-5.244 - 6
0.13528	1.6234	1.169 + 3	1.199 + 3	1.193 - 1	1.190 - 1	3.315 - 4	3.350 - 4	2.300 - 5	1.505 - 5
0.15220	1.8264	1.053 + 3	1.082 + 3	1.029 - 1	1.027 - 1	3.662 - 4	3.698 - 4	5.466 - 5	4.538 - 5
0.18690	2.2428	8.825 + 2	9.099 + 2	8.367 - 2	8.353 - 2	4.347 - 4	4.385 - 4	1.057 - 4	9.347 - 5
0.24698	2.9638	6.984 + 2	7.220 + 2	6.508 - 2	6.501 - 2	5.431 - 4	5.474 - 4	1.786 - 4	1.612 - 4
0.32652	3.9182	5.538 + 2	5.733 + 2	5.103 - 2	5.100 - 2	6.742 - 4	6.791 - 4	2.659 - 4	2.416 - 4
0.43991	5.2789	4.418 + 2	4.578 + 2	3.997 - 2	3.998 - 2	8.281 - 4	8.339 - 4	3.822 - 4	3.489 - 4
0.60153	7.2184	3.602 + 2	3.737 + 2	3.173 - 2	3.176 - 2	9.953 - 4	1.002 - 3	5.351 - 4	4.902 - 4
0.84777	10.173	3.042 + 2	3.162 + 2	2.575 - 2	2.581 - 2	1.159 - 3	1.167 - 3	7.308 - 4	6.723 - 4
1.6034	19.241	2.649 + 2	2.765 + 2	2.025 - 2	2.037 - 2	1.331 - 3	1.339 - 3	1.092 - 3	1.009 - 3
2.6358	31.630	2.839 + 2	2.972 + 2	1.933 - 2	1.954 - 2	1.296 - 3	1.301 - 3	1.242 - 3	1.149 - 3

TABLE VIII

NONSIMILAR EFFECTS ON DIFFUSIVE HEAT FLUX, SKIN FRICTION
COEFFICIENT, MOMENTUM THICKNESS AND DISPLACEMENT THICKNESS FOR
AN AIR BOUNDARY LAYER OVER A SPHERE CONE:

Nose Radius = 1 Inch, Wall Temperature = 1000°R ,
Total Enthalpy = 11,500 Btu/lb, Total Pressure = 30 atm.

(a) Unequal Diffusion Coefficients, Cone Angle = 10°

Stream- wise Dimen- sion, ft.	S/R_N	Diffusive Heat Flux, q (Btu/sec ft^2)		Skin Friction Coefficient, (lb/sec ft^2)		Momentum Thickness, θ		Displacement Thickness, δ^* (ft.)	
		Similar	Non- similar	Similar	Non- similar	Similar	Non- similar	Similar	Non- similar
0	0	1.095 + 4	1.095 + 4	9.584 - 1	9.584 - 1	5.501 - 5	5.500 - 5	-4.434 - 6	-4.434 - 6
.01561	.1873	1.059 + 4	1.059 + 4	9.327 - 1	9.327 - 1	5.647 - 5	5.647 - 5	-4.513 - 6	-4.513 - 6
.03125	.3750	9.826 + 3	9.817 + 3	8.734 - 1	8.725 - 1	5.982 - 5	5.986 - 5	-4.179 - 6	-4.166 - 6
.04686	.5623	8.194 + 3	8.180 + 3	7.554 - 1	7.512 - 1	6.943 - 5	6.967 - 5	-5.334 - 6	-4.840 - 6
.05727	.6872	7.122 + 3	7.099 + 3	6.719 - 1	6.668 - 1	7.763 - 5	7.800 - 5	-5.658 - 6	-5.023 - 6
.07292	.8750	5.375 + 3	5.342 + 3	5.319 - 1	5.246 - 1	9.728 - 5	9.827 - 5	-7.138 - 6	-5.686 - 6
.08366	1.0039	4.205 + 3	4.171 + 3	4.358 - 1	4.267 - 1	1.191 - 4	1.207 - 4	-1.006 - 5	-7.040 - 6
.10092	1.2110	2.576 + 3	2.549 + 3	2.951 - 1	2.849 - 1	1.786 - 4	1.829 - 4	-2.351 - 5	-1.444 - 5
.11636	1.3963	1.501 + 3	1.484 + 3	1.937 - 1	1.839 - 1	2.837 - 4	2.924 - 4	-6.101 - 5	-3.666 - 5
.12665	1.5198	1.272 + 3	1.285 + 3	1.027 - 1	1.344 - 1	3.391 - 4	3.166 - 4	1.549 - 4	-5.244 - 6
.13528	1.6234	1.217 + 3	1.199 + 3	9.913 - 2	1.190 - 1	3.535 - 4	3.350 - 4	1.583 - 4	1.505 - 5
.15220	1.8264	1.120 + 3	1.082 + 3	9.309 - 2	1.027 - 1	3.808 - 4	3.698 - 4	1.626 - 4	4.538 - 5
.18690	2.2428	9.518 + 2	9.099 + 2	8.060 - 2	8.353 - 2	4.405 - 4	4.385 - 4	1.849 - 4	9.347 - 5
.24698	2.9638	7.566 + 2	7.220 + 2	6.463 - 2	6.501 - 2	5.421 - 4	5.474 - 4	2.365 - 4	1.612 - 4
.32652	3.9182	5.990 + 2	5.733 + 2	5.107 - 2	5.100 - 2	6.691 - 4	6.791 - 4	3.158 - 4	2.416 - 4
.43991	5.2789	4.762 + 2	4.578 + 2	3.969 - 2	3.998 - 2	8.244 - 4	8.339 - 4	4.461 - 4	3.489 - 4
.60153	7.2184	3.870 + 2	3.737 + 2	3.120 - 2	3.176 - 2	9.955 - 4	1.002 - 3	6.245 - 4	4.902 - 4
.84777	10.173	3.256 + 2	3.162 + 2	2.503 - 2	2.581 - 2	1.166 - 3	1.167 - 3	8.539 - 4	6.723 - 4
1.6034	19.241	2.808 + 2	2.765 + 2	1.935 - 2	2.037 - 2	1.355 - 3	1.339 - 3	1.243 - 3	1.009 - 3
2.6358	31.630	2.976 + 2	2.972 + 2	1.825 - 2	1.954 - 2	1.328 - 3	1.301 - 3	1.371 - 3	1.149 - 3

TABLE VIII (Concluded)
 (b) Equal Diffusion Coefficients, Cone Angle = 5°

Stream- wise Dimen- sion, ft.	S/R _N	Diffusive Heat Flux, q_a (ft ²) (Btu/sec ft ²)		Skin Friction Coefficient, (lb/sec ft ²)		Momentum Thickness, θ		Displacement Thickness, δ^* (ft.)	
		Similar	Non- similar	Similar	Non- similar	Similar	Non- similar	Similar	Non- similar
0	0	1.059 + 4	1.059 + 4	9.603 - 1	9.603 - 1	5.462 - 5	5.462 - 5	-2.896 - 6	-2.896 - 6
.01561	.1873	1.026 + 4	1.026 + 4	9.346 - 1	9.346 - 1	5.607 - 5	5.607 - 5	-2.929 - 6	-2.929 - 6
.03125	.3750	9.511 + 3	9.503 + 3	8.751 - 1	8.742 - 1	5.933 - 5	5.940 - 5	-2.510 - 6	-2.496 - 4
.04686	.5623	7.933 + 3	7.922 + 3	7.569 - 1	7.526 - 1	6.882 - 5	6.906 - 5	-3.480 - 6	-2.932 - 6
.05727	.6872	6.897 + 3	6.878 + 3	6.731 - 1	6.680 - 1	7.688 - 5	7.732 - 5	-3.630 - 6	-2.911 - 6
.07292	.8750	5.210 + 3	5.182 + 3	5.327 - 1	5.255 - 1	9.643 - 5	9.739 - 5	-4.751 - 6	-3.078 - 6
.08366	1.0039	4.081 + 3	4.052 + 3	4.363 - 1	4.274 - 1	1.178 - 4	1.195 - 4	-7.400 - 6	-3.901 - 6
.10092	1.2110	2.506 + 3	2.485 + 3	2.950 - 1	2.851 - 1	1.770 - 4	1.810 - 4	-2.043 - 5	-9.908 - 6
.12363	1.4836	1.104 + 3	1.098 + 3	1.556 - 1	1.469 - 1	3.591 - 4	3.706 - 4	-9.235 - 5	-5.083 - 5
.13209	1.5851	9.343 + 2	9.725 + 2	7.966 - 2	1.107 - 1	4.249 - 4	3.916 - 4	2.277 - 4	-1.810 - 5
.14196	1.7035	8.949 + 2	8.958 + 2	7.705 - 2	9.576 - 2	4.417 - 4	4.150 - 4	2.321 - 4	1.070 - 5
.16204	1.9445	8.224 + 2	8.026 + 2	7.154 - 2	8.114 - 2	4.769 - 4	4.590 - 4	2.476 - 4	5.511 - 5
.20587	2.4704	6.925 + 2	6.629 + 2	6.155 - 2	6.441 - 2	5.564 - 4	5.520 - 4	2.820 - 4	1.303 - 4
.25305	3.0366	5.895 + 2	5.613 + 2	5.282 - 2	5.381 - 2	6.430 - 4	6.464 - 4	3.318 - 4	1.974 - 4
.33586	4.0303	4.663 + 2	4.433 + 2	4.213 - 2	4.209 - 2	7.925 - 4	8.056 - 4	4.263 - 4	3.053 - 4
.46803	5.6164	3.521 + 2	3.356 + 2	3.164 - 2	3.151 - 2	1.018 - 3	1.038 - 3	6.040 - 4	4.713 - 4
.60690	7.2828	2.844 + 2	2.719 + 2	2.525 - 2	2.523 - 2	1.230 - 3	1.253 - 3	8.047 - 4	6.421 - 4
1.0795	12.954	1.858 + 2	1.788 + 2	1.585 - 2	1.559 - 2	1.790 - 3	1.816 - 3	1.460 - 3	1.195 - 3
1.7136	20.563	1.396 + 2	1.351 + 2	1.144 - 2	1.162 - 2	2.304 - 3	2.325 - 3	2.210 - 3	1.847 - 3
2.7994	33.593	1.064 + 2	1.037 + 2	8.397 - 3	8.525 - 3	2.934 - 3	2.952 - 3	3.214 - 3	2.805 - 3

body, to be increased by a few percent just downstream of the cone-sphere junction, and then to be reduced by several percent back over the cone. Nonsimilarity also slightly decreases the skin friction coefficient over the spherical region. However, it is increased just downstream of the sphere-cone juncture by a much more sizeable amount (as much as a 40 percent increase) and this effect persists much further back over the cone. The nonsimilar pressure gradient affects the momentum thickness inversely as it affects the diffusive heat flux, but with about the same magnitude of effect; that is, the momentum thickness is increased slightly on the sphere, decreased a few percent for a short distance just beyond the tangent point, and then slightly increased again back over the cone. The displacement thickness is increased (i.e., less negative) on the spherical portion and back past the tangent point by nonsimilar effects, but then is markedly decreased over the remainder of the cone surface (by a factor of 4 or 5 for an appreciable distance).

3. BOUNDARY LAYER SOLUTIONS FOR QUASISTEADY GRAPHITE ABLATION

In this section solutions are presented for the laminar boundary layer over ablating graphite sphere cones at conditions of interest in moderate to severe ballistic reentry problems. The ablating graphite surface is considered to be in equilibrium with the boundary layer gases, and surface species, mass, and energy balances are satisfied with consideration of quasisteady conduction.

The accuracy of the numerical solutions is considered in Section 3.1. Boundary layer solutions for the stagnation point and for positions around the body are presented in Sections 3.2 and 3.3, respectively. Finally, the applicability of the quasisteady assumption is discussed in Section 3.4.

3.1 Accuracy Considerations

The integral matrix procedure which is employed to solve the boundary layer equations divides the boundary layer up into a number of segments. The primary dependent variables (velocity ratio, elemental mass fractions, and total enthalpy) are then represented by polynomials (cubics or quadratics) which have continuous derivatives at the junction points. The bulk of the calculations presented in this report were obtained considering seven nodal points and using connected cubic curve fits. The errors associated with these assumed profile shapes are discussed in this section.

3.1.1 Effect of Number of Nodal Points

A matrix of nine stagnation-point boundary-layer solutions for quasisteady graphite ablation were performed considering 13 nodal points to assess the accuracy of the 7-point solutions. These solutions covered a range of conditions from 2,000 to 13,000 Btu/lb and from 10 to 100 atmospheres. The six additional points in the 13-point solutions were placed midway between

(and were in addition to) the points used in the 7-point solutions. The effect on boundary-layer parameters of interest are shown for the matrix of nine conditions in Table IX. Boundary layer profiles for one of the flight conditions are presented in Table X.

Looking at Table IX, the three quantities of primary interest in ablation problems, surface temperature, ablation rate, and diffusive heat flux, agree nominally within one-fourth of one percent, three-fourths of one percent, and one-half of one percent, respectively. The skin-friction coefficient is within one-fourth of one percent, while the momentum thickness and displacement thickness are typically within one-half of one percent or better. The mole fraction of C_3 at the wall, representative of the degree of sublimation, is seen to differ by as much as two percent in some cases. Finally, variations in the boundary-layer stretching parameter (α_H), indicative of changes in profile shape, are about one-half of one percent. Assuming the 13-point boundary-layer solutions are relatively precise, it can be seen that the use of 7 nodal points typically introduces an error of the order of one percent or less in boundary-layer parameters of interest.

Profiles of distance, stream function, velocity ratio, shear function, total enthalpy, total enthalpy derivative, temperature, density, and mole fractions of principal species are presented in terms of the transverse boundary-layer coordinate, η , in Table X for the flight condition of 13,000 Btu/lb and 30 atmospheres. Comparing values at a given η , differences in some quantities of up to 10 percent are observed. However, when one considers values at a given physical distance from the wall and normalizes quantities relative to maximum values encountered somewhere in the profile, the errors are considerably less but still greater than one percent in some cases. This demonstrates that quantities of interest at the surface are not affected nearly as

TABLE IX

EFFECT OF NUMBERS OF NODAL POINTS ON STAGNATION POINT BOUNDARY
LAYER PARAMETERS WITH QUASISTEADY ABLATION OF GRAPHITE:

Unequal Diffusion Coefficients, Effective Nose Radius = 1 Inch,
Surface Emissivity = 1

Total Enthalpy (Btu/lb)	Total Pressure (atm)	7-pt	13-pt	7-pt	13-pt	7-pt	13-pt	7-pt	13-pt
		Stretching Parameter, α_H		Ablation Rate, $\dot{m}_w$ (lb/sec ft ²)		Surface Temp, T_w (°R)		Diffusive Heat Flux, (Btu/sec ft ²)	
13,000	10	1.211	1.207	2.419 - 1	2.410 - 1	7753	7750	3.111 + 3	3.097 + 3
13,000	30	1.236	1.230	4.562 - 1	4.532 - 1	8193	8195	4.972 + 3	4.964 + 3
13,000	100	1.262	1.250	8.896 - 1	8.786 - 1	8708	8717	8.545 + 3	8.543 + 3
8,000	10	1.004	1.001	1.634 - 1	1.629 - 1	7496	7486	2.117 + 3	2.097 + 3
8,000	30	1.013	1.009	3.145 - 1	3.129 - 1	7936	7937	3.228 + 3	3.211 + 3
8,000	100	1.023	1.015	6.223 - 1	6.144 - 1	8432	8446	5.294 + 3	5.274 + 3
2,000	10	0.7108	0.7099	7.591 - 2	7.661 - 2	5542	5543	5.209 + 2	5.253 + 2
2,000	30	0.6886	0.6876	1.317 - 1	1.333 - 1	6024	6021	7.706 + 2	7.772 + 2
2,000	100	0.6645	0.6633	2.427 - 1	2.459 - 1	6497	6492	1.157 + 3	1.166 + 3
Total Enthalpy (Btu/lb)	Total Pressure (atm)	Skin Friction Coefficient, (lb/sec ft ²)		Mole Fraction of C ₃ at Wall		Momentum Thickness, θ (ft)		Displacement Thickness, δ^* (ft)	
13,000	10	4.476 - 1	4.465 - 1	1.448 - 1	1.439 - 1	1.013 - 4	1.011 - 4	5.052 - 5	5.022 - 5
13,000	30	7.615 - 1	7.605 - 1	1.473 - 1	1.481 - 1	6.131 - 5	6.111 - 5	2.986 - 5	2.985 - 5
13,000	100	1.375	1.376	1.402 - 1	1.429 - 1	3.533 - 5	3.513 - 5	1.648 - 5	1.669 - 5
8,000	10	5.259 - 1	5.257 - 1	7.087 - 2	6.901 - 2	7.471 - 5	7.454 - 5	7.510 - 5	7.532 - 5
8,000	30	8.999 - 1	8.998 - 1	7.818 - 2	7.833 - 2	4.455 - 5	4.442 - 5	4.578 - 5	4.588 - 5
8,000	100	1.632	1.635	7.699 - 2	7.935 - 2	2.531 - 5	2.517 - 5	2.620 - 5	2.640 - 5
2,000	10	6.363 - 1	6.357 - 1	3.158 - 5	3.182 - 5	3.611 - 5	3.599 - 5	7.938 - 5	7.925 - 5
2,000	30	1.121	1.120	1.143 - 4	1.128 - 4	2.064 - 5	2.044 - 5	4.937 - 5	4.931 - 5
2,000	100	2.086	2.084	2.504 - 4	2.460 - 4	1.109 - 5	1.100 - 5	2.926 - 5	2.927 - 5

TABLE A

EFFECT OF NUMBER OF NODAL POINTS ON STAGNATION POINT BOUNDARY
LAYER PROFILES WITH QUASISTEADY ABLATION OF GRAPHITE:

Unequal Diffusion Coefficients, Effective Nose Radius = 1 Inch,
Surface Emissivity = 1, Total Enthalpy = 13,000 Btu/lb, Total Pressure = 30 atm
(a) Stream Function, Velocity Ratio, Shear Function, and Thermodynamic Properties

η	7-pt	13-pt*	7-pt	13-pt*	7-pt	13-pt*	7-pt	13-pt*
η	Distance from Wall, y (ft)		Stream Function, f		Velocity Ratio, $f' = u/u_e$		Shear Function, f''	
0	0	0	-3.697 - 1	-3.673 - 1	0	0	2.917 - 1	2.954 - 1
0.5	2.275 - 5	2.298 - 5	-3.134 - 1	-3.090 - 1	0.1863	0.1935	3.292 - 1	3.337 - 1
1.0	5.298 - 5	5.333 - 5	-1.315 - 1	-1.249 - 1	0.4055	0.4083	3.605 - 1	3.601 - 1
1.5	9.342 - 5	9.386 - 5	1.859 - 1	1.934 - 1	0.6187	0.6235	3.284 - 1	3.276 - 1
2.0	1.423 - 4	1.427 - 4	6.269 - 1	6.338 - 1	0.8000	0.8000	2.416 - 1	2.404 - 1
3.0	2.597 - 4	2.600 - 4	1.742	1.748	0.9677	0.9728	6.171 - 2	5.699 - 2
5.0	5.254 - 4	5.242 - 4	4.205	4.196	1.0000	1.0000	0	0
η	Total Enthalpy, H_T (Btu/lb)		Total Enthalpy Derivative, H_T' (Btu/lb)		Temperature, T (°R)		Density, ρ (lb/ft ³)	
0	6,149	6,165	735.7	673.8	8,193	8,195	1.507 - 1	1.489 - 1
0.5	6,511	6,590	715.2	718.2	9,310	9,398	1.175 - 1	1.154 - 1
1.0	7,218	7,196	1,590	1,619	11,520	11,570	8.527 - 2	8.487 - 2
1.5	8,487	8,545	2,519	2,525	13,530	13,580	6.733 - 2	6.685 - 2
2.0	10,160	10,150	2,600	2,592	14,570	14,570	5.735 - 2	5.735 - 2
3.0	12,430	12,510	1,091	1,011	15,670	15,710	4.759 - 2	4.731 - 2
5.0	13,000	13,000	0	0	16,030	16,030	4.517 - 2	4.517 - 2

* Results for 13-point solutions are presented only for alternate (every second) node.

TABLE X (concluded)
(b) Mole Fractions of Principal Species

η	7-pt	13-pt*	7-pt	13-pt*	7-pt	13-pt*	7-pt	13-pt*
	Mole Fractions							
	C		C ₂		C ₃		CN	
0	2.921 - 2	2.935 - 2	6.460 - 2	6.495 - 2	1.473 - 1	1.481 - 1	8.642 - 2	8.728 - 2
0.5	9.073 - 2	9.737 - 2	8.979 - 2	9.048 - 2	5.590 - 2	5.110 - 2	1.201 - 1	1.213 - 1
1.0	1.951 - 1	1.940 - 1	2.686 - 2	2.524 - 2	1.151 - 3	1.008 - 3	8.392 - 2	8.192 - 2
1.5	1.652 - 1	1.628 - 1	3.421 - 3	3.193 - 3	1.407 - 5	1.232 - 5	3.370 - 2	3.262 - 2
2.0	1.138 - 1	1.131 - 1	7.941 - 4	7.851 - 4	9.151 - 7	8.994 - 7	1.638 - 2	1.628 - 2
3.0	3.429 - 2	3.191 - 2	3.736 - 5	3.167 - 5	5.666 - 9	4.351 - 9	3.401 - 3	3.123 - 3
5.0	0	0	0	0	0	0	0	0
η	Mole Fractions							
	CO		N		N ₂		O	
0	2.394 - 1	2.392 - 1	1.152 - 3	1.164 - 3	3.714 - 1	3.765 - 1	2.145 - 6	2.149 - 6
0.5	2.446 - 1	2.456 - 1	5.492 - 3	6.085 - 3	3.820 - 1	3.785 - 1	2.349 - 5	2.800 - 5
1.0	2.528 - 1	2.535 - 1	4.969 - 2	5.188 - 2	3.883 - 1	3.900 - 1	1.604 - 3	1.775 - 3
1.5	2.200 - 1	2.171 - 1	1.888 - 1	1.944 - 1	3.479 - 1	3.460 - 1	3.750 - 2	4.030 - 2
2.0	1.304 - 1	1.304 - 1	3.126 - 1	3.127 - 1	3.018 - 1	3.016 - 1	1.168 - 1	1.175 - 1
3.0	2.295 - 2	2.072 - 2	4.702 - 1	4.758 - 1	2.362 - 1	2.336 - 1	2.221 - 1	2.240 - 1
5.0	0	0	5.242 - 1	5.242 - 1	2.147 - 1	2.147 - 1	2.502 - 1	2.502 - 1

* Results from 13-point solutions are presented only for alternate (every second) node.

appreciably as boundary layer profiles. (This has also been seen in problems with unequal diffusion coefficients: whereas unequal diffusion coefficients can appreciably affect boundary layer profiles, the effect on wall quantities is usually relatively small for carbon ablation problems.)

In conclusion, in view of other uncertainties (e.g., transport and thermodynamic properties) the seven-point boundary-layer solutions are considered adequate for determining such quantities as surface temperatures and ablation rates. However, more points would be desirable if the detailed structure of the boundary layer were desired.

3.1.2 Effect of Assumed Profile Shape

A quasisteady graphite stagnation point ablation solution was generated for a single flight condition considering connected quadratic profiles in order to judge the effect of assumed profile shape (the more flexible and presumably more accurate cubic profiles were utilized throughout the remainder of this effort). The relatively severe condition of 11,500 Btu/lb and 30 atmospheres was chosen for this study.

Various boundary-layer parameters of interest are compared in Table XI(a), whereas boundary layer profiles are compared in Table XI(b) and (c). The ablation rate and diffusive heat flux differ by less than one-fourth of 1 percent while the surface temperature did not change to four significant figures. On the whole, other parameters also varied less than the variation that was seen in Section 3.1.1 in comparing 7 and 13 nodal point solutions. Therefore, it can be concluded, at least for the one flight condition considered, that the solution is not sensitive to the choice of assumed profile shape.

3.2 Stagnation Point

Solutions for quasisteady graphite ablation at the stagnation point have been generated for the complete range of stagnation

TABLE XI

EFFECT OF ASSUMED PROFILE SHAPE ON A STAGNATION
POINT BOUNDARY LAYER WITH QUASISTEADY ABLATION OF GRAPHITE:

Equal Diffusion Coefficients, Effective Nose Radius = 1 Inch,
Surface Emissivity = 1, Total Enthalpy = 11,500 Btu/lb,
Total Pressure = 30 atm

(a) Boundary Layer Parameters

Boundary Layer Parameter	Cubic Profiles	Quadratic Profiles
Boundary layer stretching parameter, α_H	1.156	1.167
Ablation rate, $\dot{m}_w$ (lb/sec-ft ²)	0.4021	0.4027
Surface temperature, T_w (°R)	8071	8071
Diffusive heat flux, q_a (Btu/sec ft ²)	4097	4101
Skin friction coefficient (lb/sec-ft ²)	0.8055	0.8053
Heat transfer coefficient (lb/sec-ft ²)	0.6417	0.6428
Mass transfer coefficient (lb/sec-ft ²)	0.6318	0.6320
Blowing parameter, B'	0.6364	0.6372
Momentum thickness, θ (ft)	5.558×10^{-5}	5.559×10^{-5}
Displacement thickness, δ^* (ft)	3.531×10^{-5}	3.536×10^{-5}

TABLE XI (continued)

(b) Profiles of Stream Function, Velocity Ratio,
Shear Function, and Thermodynamic Properties

η	Cubics	Quadratics	Cubics	Quadratics	Cubics	Quadratics	Cubics	Quadratics
	Distance from Wall, y (ft)		Stream Function, f		Velocity Ratio, $f' = u/u_e$		Shear Function, f''	
0	0	0	-3.258 - 1	-3.263 - 1	0	0	3.248 - 1	3.247 - 1
0.5	2.213 - 5	2.247 - 5	-2.709 - 1	-2.689 - 1	0.1935	0.2003	3.589 - 1	3.619 - 1
1.0	5.117 - 5	5.213 - 5	-9.637 - 2	-8.915 - 2	0.4130	0.4180	3.840 - 1	3.845 - 1
1.5	8.977 - 5	9.148 - 5	2.045 - 1	2.177 - 1	0.6247	0.6297	3.443 - 1	3.412 - 1
2.0	1.357 - 4	1.381 - 4	6.188 - 1	6.376 - 1	0.8000	0.8000	2.484 - 1	2.424 - 1
3.0	2.421 - 4	2.459 - 4	1.658	1.693	0.9641	0.9718	6.694 - 2	5.195 - 2
5.0	4.783 - 4	4.846 - 4	3.958	4.017	1.000	1.000	0	0
η	Total Enthalpy, H_T (Btu/lb)		Total Enthalpy Derivative, H_T' (Btu/lb)		Temperature, T (°R)		Density, ρ (lb/ft³)	
0	5,116	5,120	1,200	1,201	8,071	8,071	1.510 - 1	1.510 - 1
0.5	5,872	5,908	1,485	1,500	9,250	9,312	1.189 - 1	1.176 - 1
1.0	6,840	6,877	1,811	1,823	11,380	11,470	8.754 - 2	8.661 - 2
1.5	7,948	8,010	2,049	2,059	13,400	13,460	6.934 - 2	6.877 - 2
2.0	9,147	9,184	1,970	1,967	14,290	14,310	6.065 - 2	6.044 - 2
3.0	10,850	10,880	959	934	15,080	15,100	5.232 - 2	5.221 - 2
5.0	11,500	11,500	0	0	15,390	15,390	4.958 - 2	4.958 - 2

TABLE XI (concluded)
(c) Concentration Profiles of Principal Species

η	Cubics	Quadratics	Cubics	Quadratics	Cubics	Quadratics	Cubics	Quadratics
	Mole Fractions							
	C		C ₂		C ₃		CN	
0	2.202 - 2	2.205 - 2	4.688 - 2	4.694 - 2	1.095 - 1	1.096 - 1	7.748 - 2	7.752 - 2
0.5	7.908 - 2	8.287 - 2	7.475 - 2	7.470 - 2	4.550 - 2	4.232 - 2	1.132 - 1	1.140 - 1
1.0	1.787 - 1	1.801 - 1	2.606 - 2	2.412 - 2	1.227 - 3	1.019 - 3	8.372 - 2	8.121 - 2
1.5	1.445 - 1	1.415 - 1	2.879 - 3	2.634 - 3	1.168 - 5	9.867 - 6	3.186 - 2	3.054 - 2
2.0	8.955 - 2	8.817 - 2	5.899 - 4	5.644 - 4	6.725 - 7	6.232 - 7	1.469 - 2	1.437 - 2
3.0	2.559 - 2	2.460 - 2	2.922 - 5	2.681 - 5	5.072 - 9	4.434 - 9	3.237 - 3	3.100 - 3
5.0	0	0	0	0	0	0	0	0

η	Mole Fractions							
	CO		N		N ₂		O	
0	2.663 - 1	2.662 - 1	1.016 - 3	1.017 - 3	4.266 - 1	4.264 - 1	2.035 - 6	2.036 - 6
0.5	2.581 - 1	2.578 - 1	5.315 - 3	5.727 - 3	4.140 - 1	4.135 - 1	2.410 - 5	2.725 - 5
1.0	2.536 - 1	2.535 - 1	4.539 - 2	4.890 - 2	4.094 - 1	4.090 - 1	1.351 - 3	1.586 - 3
1.5	2.221 - 1	2.187 - 1	1.820 - 1	1.885 - 1	3.768 - 1	3.747 - 1	3.644 - 2	3.988 - 2
2.0	1.426 - 1	1.401 - 1	2.870 - 1	2.896 - 1	3.407 - 1	3.397 - 1	1.171 - 1	1.196 - 1
3.0	3.239 - 2	3.095 - 2	4.011 - 1	4.029 - 1	2.972 - 1	2.965 - 1	2.286 - 1	2.300 - 1
5.0	0	0	4.480 - 1	4.480 - 1	2.778 - 1	2.778 - 1	2.618 - 1	2.618 - 1

pressures and temperatures listed in Table I. Solutions were obtained for equal as well as unequal diffusion coefficients. The effects of equal-versus-unequal diffusion coefficients are discussed first, in Section 3.2.1. The results are then compared to the nonablating (air) solutions in Section 3.2.2.

3.2.1 Solutions for Equal and Unequal Diffusion Coefficients

Solutions for quasisteady graphite ablation at the stagnation point for a 1-inch nose radius are presented for the complete range of flight conditions in Tables XII and XIII. Ablation rate, surface temperature, mass transfer coefficient, and blowing parameter are presented in Table XII, whereas Table XIII contains values for diffusive heat flux, skin-friction coefficient, momentum thickness, and displacement thickness. The primary purpose for presenting these data is for their direct application in the prediction of quasisteady ablation and for later reference when establishing procedures for obtaining transient solutions. The validity of the quasisteady approximation is discussed in Section 3.4. The procedure for applying to transient problems is developed in Section 4 and summarized in Section 5. Therefore, the present discussion is directed toward an assessment of the significance of unequal diffusion coefficients, while the subsequent section discusses the effect of quasisteady ablation on the air boundary-layer solutions of Section 2.

3.2.1.1 Ablation rate and surface temperature

As seen in Table XII and shown graphically in Figure 4, the effect of unequal diffusion is to typically increase the ablation rate by 2 to 8 percent. However, for intermediate pressures 1 to 10 atmospheres, the increase can be as high as 8 to 12 percent, while for low enthalpies (2,000 Btu/lb) unequal diffusion is predicted to decrease the ablation rate by 1 to 2 percent. Unequal diffusion typically increases the surface temperature by 50 to 75°R, although for some conditions the increase is as small as 30° or as large as 100°R (see Table XII).

TABLE XII

EFFECT OF UNEQUAL DIFFUSION COEFFICIENTS ON ABLATION RATE,
 SURFACE TEMPERATURE, MASS TRANSFER COEFFICIENT, AND
 BLOWING PARAMETER FOR A STAGNATION POINT
 BOUNDARY LAYER WITH QUASISTEADY ABLATION OF GRAPHITE:
 Effective Nose Radius = 1 Inch; Surface Emissivity = 1

(a) High Total Enthalpy

Total Enthalpy (Btu/lb)	Total Pressure (atm)	Ablation Rate, $\dot{m}_w$ (lb/sec ft ²)		Surface Temperature, T_w (°R)		Mass Transfer Coefficient, (lb/sec ft ²)		Blowing Parameter, B'	
		Equal B_{ij}	Unequal B_{ij}	Equal B_{ij}	Unequal B_{ij}	Equal B_{ij}	Unequal B_{ij}	Equal B_{ij}	Unequal B_{ij}
13,000	.01	2.485 - 3	2.554 - 3	4370	4432	1.407 - 2	1.430 - 2	.1766	.1786
13,000	.1	8.808 - 3	9.329 - 3	5716	5794	4.417 - 2	4.44 - 2	.1994	.2101
13,000	1	5.178 - 2	5.476 - 2	6808	6877	1.253 - 1	1.131 - 1	.4132	.4842
13,000	10	2.332 - 1	2.419 - 1	7693	7753	3.603 - 1	3.217 - 1	.6472	.7519
13,000	30	4.428 - 1	4.562 - 1	8131	8193	6.085 - 1	5.40 - 1	.7276	.8448
13,000	100	8.706 - 1	8.896 - 1	8639	8708	1.093	9.616 - 1	.7965	.9251
11,500	.01	2.486 - 3	2.581 - 3	4238	4300	1.408 - 2	1.426 - 2	.1766	.1810
11,500	.1	8.370 - 3	8.826 - 3	5562	5649	4.452 - 2	4.407 - 2	.1880	.2005
11,500	1	4.551 - 2	4.873 - 2	6729	6792	1.294 - 1	1.216 - 1	.3517	.4007
11,500	10	2.102 - 1	2.199 - 1	7633	7698	3.739 - 1	3.367 - 1	.5621	.6531
11,500	30	4.021 - 1	4.170 - 1	8071	8137	6.318 - 1	5.665 - 1	.6364	.7360
11,500	100	7.949 - 1	8.162 - 1	8573	8648	1.135	1.007	.7003	.8105
10,000	.01	2.489 - 3	2.610 - 3	4092	4153	1.410 - 2	1.421 - 2	.1765	.1837
10,000	.1	8.124 - 3	8.589 - 3	5378	5466	4.474 - 2	4.455 - 2	.1816	.1928
10,000	1	3.940 - 2	4.281 - 2	6626	6671	1.334 - 1	1.332 - 1	.2953	.3214
10,000	10	1.861 - 1	1.966 - 1	7758	7629	3.885 - 1	3.531 - 1	.4790	.5567
10,000	30	3.592 - 1	3.751 - 1	7995	8067	6.569 - 1	5.951 - 1	.5468	.6303
10,000	100	7.147 - 1	7.375 - 1	8492	8572	1.180	1.059	.6057	.6964

TABLE XII (concluded)
(b) Moderate to Low Total Enthalpy

Total Enthalpy (Btu/lb)	Total Pressure (atm)	Ablation Rate $\dot{m}_w$ (lb/sec ft ²)		Surface Temperature, T_w (°R)		Mass Transfer Coefficient, (lb/sec ft ²)		Blowing Parameter B'	
		Equal $\bar{F}_{ij}$	Unequal $\bar{F}_{ij}$	Equal $\bar{F}_{ij}$	Unequal $\bar{F}_{ij}$	Equal $\bar{F}_{ij}$	Unequal $\bar{F}_{ij}$	Equal $\bar{F}_{ij}$	Unequal $\bar{F}_{ij}$
8,000	1	3.204 - 2	3.563 - 2	6417	6465	1.382 - 1	1.427 - 1	.2318	.2497
8,000	10	1.526 - 1	1.634 - 1	7417	7496	4.088 - 1	3.795 - 1	.3732	.4305
8,000	30	2.981 - 1	3.145 - 1	7857	7936	6.924 - 1	6.377 - 1	.4305	.4931
8,000	100	5.991 - 1	5.223 - 1	8345	8432	1.244	1.135	.4816	.5483
6,000	10	1.184 - 1	1.284 - 1	7186	7254	4.277 - 1	4.19 - 1	.2768	.3064
6,000	30	2.330 - 1	2.486 - 1	7639	7721	7.268 - 1	6.90 - 1	.3205	.3603
6,000	100	4.726 - 1	4.942 - 1	8118	8209	1.306	1.222	.3619	.4044
4,000	10	8.794 - 2	9.740 - 2	6689	6788	4.363 - 1	4.30 - 1	.2016	.2265
4,000	30	1.687 - 1	1.801 - 1	7202	7234	7.474 - 1	7.53 - 1	.2257	.2392
4,000	100	3.396 - 1	3.562 - 1	7683	7768	1.350	1.319	.2516	.2700
2,000	10	7.648 - 2	7.591 - 2	5465	5542	4.311 - 1	4.28 - 1	.1774	.1774
2,000	30	1.335 - 1	1.317 - 1	5931	6024	7.460 - 1	7.430 - 1	.1789	.1773
2,000	100	2.477 - 1	2.427 - 1	6395	6497	1.360	1.358	.1821	.1787

TABLE XIII

EFFECT OF UNEQUAL DIFFUSION COEFFICIENTS ON DIFFUSIVE HEAT FLUX, SKIN FRICTION COEFFICIENT, MOMENTUM THICKNESS, AND DISPLACEMENT THICKNESS FOR A STAGNATION POINT BOUNDARY LAYER WITH QUASISTEADY ABLATION OF GRAPHITE:

Effective Nose Radius = 1 Inch, Surface Emissivity = 1

(a) High Total Enthalpy

Total Enthalpy (Btu/lb)	Total Pressure (atm)	Diffusive Heat Flux, q_a (Btu/sec-ft ²)		Skin Friction Coefficient (lb/sec-ft ²)		Momentum Thickness, θ (ft)		Displacement Thickness, δ^* (ft)	
		Equal δ_{ij}	Unequal δ_{ij}	Equal δ_{ij}	Unequal δ_{ij}	Equal δ_{ij}	Unequal δ_{ij}	Equal δ_{ij}	Unequal δ_{ij}
13,000	0.01	1.767 + 2	1.869 + 2	1.663 - 2	1.665 - 2	2.484 - 3	2.531 - 3	1.059 - 3	9.371 - 4
13,000	0.1	5.242 + 2	5.538 + 2	5.285 - 2	5.242 - 2	8.253 - 4	8.409 - 4	3.976 - 4	3.571 - 4
13,000	1	1.211 + 3	1.295 + 3	1.536 - 1	1.516 - 1	2.845 - 4	2.895 - 4	1.514 - 4	1.407 - 4
13,000	10	2.882 + 3	3.111 + 3	4.526 - 1	4.476 - 1	1.005 - 4	1.013 - 4	5.240 - 5	5.052 - 5
13,000	30	4.582 + 3	4.972 + 3	7.688 - 1	7.615 - 1	6.067 - 5	6.131 - 5	3.056 - 5	2.986 - 5
13,000	100	7.859 + 3	8.545 + 3	1.384	1.375	3.513 - 5	3.533 - 5	1.661 - 5	1.648 - 5
11,500	0.01	1.562 + 2	1.657 + 2	1.681 - 2	1.673 - 2	2.340 - 3	2.377 - 3	1.214 - 3	1.105 - 3
11,500	0.1	4.689 + 2	4.995 + 2	5.377 - 2	5.338 - 2	7.756 - 4	7.884 - 4	4.445 - 4	4.050 - 4
11,500	1	1.120 + 3	1.185 + 3	1.600 - 1	1.579 - 1	2.633 - 4	2.678 - 4	1.715 - 4	1.624 - 4
11,500	10	2.606 + 3	2.824 + 3	4.738 - 1	4.682 - 1	9.198 - 5	9.318 - 5	6.004 - 5	5.830 - 5
11,500	30	4.097 + 3	4.465 + 3	8.055 - 1	7.972 - 1	5.558 - 5	5.628 - 5	3.531 - 5	3.470 - 5
11,500	100	6.954 + 3	7.600 + 3	1.451	1.440	3.214 - 5	3.239 - 5	1.941 - 5	1.934 - 5
10,000	0.01	1.359 + 2	1.442 + 2	1.696 - 2	1.688 - 2	2.206 - 3	2.241 - 3	1.321 - 3	1.236 - 3
10,000	0.1	4.099 + 2	4.375 + 2	5.448 - 2	5.412 - 2	7.295 - 4	7.410 - 4	4.783 - 4	4.450 - 4
10,000	1	1.022 + 3	1.064 + 3	1.668 - 1	1.647 - 1	2.432 - 4	2.473 - 4	1.890 - 4	1.832 - 4
10,000	10	2.328 + 3	2.528 + 3	4.971 - 1	4.910 - 1	8.405 - 5	8.523 - 5	6.722 - 5	6.570 - 5
10,000	30	3.610 + 3	3.942 + 3	8.461 - 1	8.374 - 1	5.069 - 5	5.127 - 5	3.992 - 5	3.946 - 5
10,000	100	6.046 + 3	6.627 + 3	1.526	1.515	2.922 - 5	2.941 - 5	2.220 - 5	2.222 - 5

TABLE XIII (concluded)
(b) Moderate to Low Total Enthalpy

Total Enthalpy (Btu/lb)	Total Pressure (atm)	Diffusive Heat Flux, q_a (Btu/sec-ft ²)		Skin Friction Coefficient (lb/sec-ft ²)		Momentum Thickness, θ (ft)		Displacement Thickness, δ^* (ft)	
		Equal δ_{ij}	Unequal δ_{ij}	Equal δ_{ij}	Unequal δ_{ij}	Equal δ_{ij}	Unequal δ_{ij}	Equal δ_{ij}	Unequal δ_{ij}
8,000	1	8.712 + 2	9.063 + 2	1.759 - 1	1.737 - 1	2.195 - 4	2.223 - 4	2.064 - 4	2.030 - 4
8,000	10	1.955 + 3	2.117 + 3	5.322 - 1	5.259 - 1	7.375 - 5	7.471 - 5	7.613 - 5	7.510 - 5
8,000	30	2.961 + 3	3.228 + 3	9.087 - 1	8.999 - 1	4.415 - 5	4.455 - 5	4.595 - 5	4.578 - 5
8,000	100	4.835 + 3	5.294 + 3	1.642	1.632	2.520 - 5	2.531 - 5	2.602 - 5	2.620 - 5
6,000	10	1.565 + 3	1.661 + 3	5.715 - 1	5.663 - 1	6.327 - 5	6.383 - 5	8.315 - 5	8.335 - 5
6,000	30	2.306 + 3	2.487 + 3	9.834 - 1	9.758 - 1	3.711 - 5	3.740 - 5	5.169 - 5	5.196 - 5
6,000	100	3.632 + 3	3.945 + 3	1.787	1.779	2.078 - 5	2.085 - 5	3.000 - 5	3.038 - 5
4,000	10	1.096 + 3	1.171 + 3	6.037 - 1	6.010 - 1	5.066 - 5	5.125 - 5	8.052 - 5	8.195 - 5
4,000	30	1.612 + 3	1.660 + 3	1.062	1.062	2.884 - 5	2.878 - 5	5.323 - 5	5.457 - 5
4,000	100	2.432 + 3	2.586 + 3	1.960	1.956	1.572 - 5	1.574 - 5	3.226 - 5	3.267 - 5
2,000	10	4.951 + 2	5.209 + 2	6.361 - 1	6.363 - 1	3.582 - 5	3.611 - 5	8.008 - 5	7.938 - 5
2,000	30	7.346 + 2	7.706 + 2	1.120	1.121	2.040 - 5	2.064 - 5	4.969 - 5	4.937 - 5
2,000	100	1.103 + 3	1.157 + 3	2.083	2.086	1.098 - 5	1.109 - 5	2.944 - 5	2.926 - 5

Figure 4. Ratio of Ablation Rates for Unequal and Equal Diffusion Coefficients for Quasisteady Stagnation Point Graphite Ablation:
 $R_{EFF} = 1$ inch, $\epsilon_w = 1.0$

3.2.1.2 Mass-transfer coefficient

For assumed equal diffusion coefficients, one can define a mass-transfer coefficient for the k^{th} element as

$$\rho_e u_e C_{M_k} \equiv \frac{(-\tilde{j}_{k_w})}{(\tilde{K}_{k_e} - \tilde{K}_{k_w})} \quad (2)$$

where $\tilde{K}_k$ is the mass fraction of element k irrespective of molecular configuration,

$$\tilde{K}_k \equiv \sum_{j=1}^N \alpha_{kj} K_j \quad (3)$$

K_j is the mass fraction of species j , α_{kj} is the mass fraction of element k in species j , N is the number of species in the chemical system of interest, $\tilde{j}_k$ is the diffusive mass flux of element k , and the subscripts w and e refer to the wall and the boundary-layer edge, respectively. With this definition it can be shown for equal diffusion coefficients that the $\rho_e u_e C_{M_k}$ are the same for all elements k for a given problem.

In the case of unequal diffusion coefficients the definition of Equation (2) is not entirely satisfying since in general the $\rho_e u_e C_{M_k}$ will be different for each of the k elements. In Reference 7, the correlation Equation (2) was generalized for the case of unequal diffusion coefficients to the form

$$\rho_e u_e C_{M_k} \equiv \frac{(-\tilde{j}_{k_w})}{(\tilde{Z}_{k_e}^* - \tilde{Z}_{k_w}^*)} \quad (4)$$

where $\tilde{Z}_k^*$ is an elemental fraction which, for unequal diffusion coefficients, is intermediate between a mass and a mole

fraction and which reduces to $\tilde{K}_k$ for assumed equal diffusion coefficients.* One of the hopes of this correlation procedure was that the $\rho_e u_e C_{M_k}$ would be nearly the same for all elements k in unequal diffusion problems. It will be shown in Section 4.3 that the $\tilde{Z}_k^*$ can indeed be defined such that the $\rho_e u_e C_{M_k}$ are identically the same for the present problem for most of the flight conditions considered.

The values for mass-transfer coefficient presented in Table XII were obtained using the definition of Equation (4) which, for assumed equal diffusion coefficients, is identical to Equation (2). It is seen that the effect of unequal diffusion coefficients is to slightly decrease the mass-transfer coefficient, defined in this way, except at the lower pressures considered.

3.2.1.3 Blowing parameter

Given the ablation rate, $\dot{m}_w$, and the mass-transfer coefficient,** $\rho_e u_e C_M$, one can define a blowing parameter in the conventional manner

$$B' \equiv \frac{\dot{m}_w}{\rho_e u_e C_M} \quad (5)$$

The blowing parameters for equal and unequal diffusion coefficients are also presented in Table XII and are shown graphically in Figure 5. For the case of equal diffusion coefficients, a B' of 0.174 is representative of the diffusion-controlled surface combustion plateau. This behavior, which occurs for graphite (or carbon) over a several-thousand-degree surface-temperature

* This approach will be reviewed in Section 4.2 at which time $\tilde{Z}_k^*$ will be defined and its meaning will be discussed.

** The subscript k is dropped from C_{M_k} in the remainder of this section since the C_{M_k} are the same for all elements k .

Figure 5. Quasisteady Stagnation-Point Blowing Parameters for Graphite versus Total Pressure for Various Total Enthalpies:
 $R_{EFF} = 1$ inch, $\epsilon_w = 1.0$

range, is characterized by the stoichiometric combustion of the oxygen which diffuses across the boundary layer to form CO gas. Values of B' above this plateau value are due primarily to sublimation to form C, C_2 , C_3 , C_4 and C_5 to reaction with nitrogen to form CN, C_2N_2 , and $C_4N_2^*$. The ratio of the atoms of carbon in the wall gas associated with sublimation and nitrogen-reaction products to the number of atoms contained in the wall concentration of CO is also shown in Figure 5. It can be seen that appreciable sublimation is occurring for all of the flight conditions considered with the exception of the lowest pressures (0.01 atmospheres) and the lowest enthalpies (2,000 Btu/lb).

For a given flight condition, the B' for unequal diffusion coefficients are greater than those for equal diffusion coefficients. However, a more meaningful comparison of B' for equal and unequal diffusion coefficients is to compare curves of B' versus T_w with pressure as a parameter since the surface temperatures differ in the two cases. This is done in Figure 7. It can be seen that the B' are equivalent when strong sublimation occurs, that the B' for unequal diffusion are slightly higher when weak sublimation is occurring, and that the B' on the plateau are within a few percent of each other for equal and unequal diffusion coefficients. Of course, this result is dependent upon the particular definition of $\rho_e u_e C_M$ used in the definition of B' .

Taking another point of view, one can look upon the consistency of the present results as somewhat of a check on the definition of $\rho_e u_e C_M$ which was employed. It is therefore of interest to speculate that the B' would be expected to be relatively unchanged in the strong sublimation regime where the ablation is controlled more by graphite vapor pressure relations than by diffusional processes; whereas, the surface combustion (plateau) values would be expected to be influenced by unequal diffusion coefficients since the amount of oxygen which can diffuse across the boundary layer and thus react with the surface

*The relative importance of these carbon-containing species is illustrated on Figure 6 for a range of P_{T_2} with an H_T of 13,000 Btu/lb.

Figure 6. Relative Importance of Carbon Sublimation and Combustion Products as a Function of Total Pressure for Quasisteady Stagnation-Point Graphite Ablation: $H_T = 13,000$ BTU/lb, $R_{EFF} = 1$ inch, $\epsilon_w = 1.0$.

Figure 7. Blowing Parameter for Graphite Ablation in Air for Equal and Unequal Diffusion Coefficients in Terms of Wall Temperature and Pressure

depends upon the relative mobility of oxygen. Thus, the present results tend to confirm that the definition of $\rho_e u_e C_M$ given by Equation (4) is consistent with physical intuition.

3.2.1.4 Heat-transfer coefficient for equal diffusion coefficients

For assumed equal diffusion coefficients the definition for heat-transfer coefficient, $\rho_e u_e C_H$, given as Equation (1) is valid in the special case that $\rho_e u_e C_H$ is equal to $\rho_e u_e C_M$. Therefore, Equation (1) has been utilized to evaluate $\rho_e u_e C_H$ for the equal diffusion cases, and these are compared to the $\rho_e u_e C_M$ in Table XIV. It can be seen that the $\rho_e u_e C_H$ are indeed nearly equal to $\rho_e u_e C_M$, being always greater but never by more than two percent. (This result is consistent with the fact that the Lewis number at the wall, also shown in Table XIV, is always slightly less than unity.) Thus the use of Equation (1) is justified in the present case, and the equal diffusion values for $\rho_e u_e C_H$ presented in Table XIV are meaningful.

In the case of unequal diffusion coefficients, Equation (1) should not be utilized for the calculation of heat-transfer coefficients even if $\rho_e u_e C_H$ is equal to $\rho_e u_e C_M$. A correlation equation for diffusive heat flux, involving $\rho_e u_e C_H$ and $\rho_e u_e C_M$ and thus serving as a definition for $\rho_e u_e C_H$, was proposed in Reference 7. This equation is introduced in Section 4.2 and is subjected to the present data in Section 4.3. As will be discussed therein, the results are not conclusive, the $\rho_e u_e C_H$ being strongly dependent upon a parameter the value of which is not well defined by the present data. Therefore, further discussion of unequal diffusion values of $\rho_e u_e C_H$ is relegated to Section 4.

3.2.1.5 Diffusive heat flux

The diffusive heat flux is determined directly from boundary-layer profiles computed by the BLJMP program and is not clouded

TABLE XIV

COMPARISON OF HEAT AND MASS TRANSFER COEFFICIENTS FOR A
STAGNATION POINT BOUNDARY LAYER WITH QUASISTEADY
ABLATION OF GRAPHITE:

Effective Nose Radius = 1 Inch, Surface Emissivity = 1,
Equal Diffusion Coefficients

Total Enthalpy, (Btu/lb)	Total Pressure, (atm)	Mass Transfer Coefficient, (lb/sec ft ²)	Heat Transfer Coefficient, (lb/sec ft ²)	Ratio of Heat to Mass Transfer Coefficients	Lewis Number Evaluated at the Wall
13,000	0.01	1.407 - 2	1.415 - 2	1.006	0.965
13,000	0.1	4.417 - 2	4.440 - 2	1.005	0.967
13,000	1	1.253 - 1	1.266 - 1	1.010	0.975
13,000	10	3.603 - 1	3.660 - 1	1.016	0.981
13,000	30	6.085 - 1	6.193 - 1	1.018	0.983
13,000	100	1.093	1.115	1.020	0.984
11,500	0.01	1.408 - 2	1.416 - 2	1.006	0.965
11,500	0.1	4.452 - 2	4.475 - 2	1.005	0.966
11,500	1	1.294 - 1	1.306 - 1	1.009	0.973
11,500	10	3.739 - 1	3.790 - 1	1.014	0.979
11,500	30	6.318 - 1	6.417 - 1	1.016	0.981
11,500	100	1.135	1.155	1.018	0.983
10,000	0.01	1.410 - 2	1.419 - 2	1.006	0.965
10,000	0.1	4.474 - 2	4.499 - 2	1.006	0.966
10,000	1	1.334 - 1	1.347 - 1	1.010	0.971
10,000	10	3.885 - 1	3.931 - 1	1.012	0.977
10,000	30	6.569 - 1	6.663 - 1	1.014	0.979
10,000	100	1.180	1.200	1.017	0.981
8,000	1	1.382 - 1	1.393 - 1	1.008	0.969
8,000	10	4.088 - 1	4.134 - 1	1.011	0.974
8,000	30	6.924 - 1	7.018 - 1	1.014	0.976
8,000	100	1.244	1.264	1.016	0.977
6,000	10	4.277 - 1	4.332 - 1	1.013	0.971
6,000	30	7.268 - 1	7.369 - 1	1.014	0.972
6,000	100	1.306	1.328	1.017	0.973
4,000	10	4.363 - 1	4.390 - 1	1.006	0.967
4,000	30	7.474 - 1	7.624 - 1	1.020	0.968
4,000	100	1.350	1.373	1.017	0.969
2,000	10	4.311 - 1	4.357 - 1	1.011	0.966
2,000	30	7.460 - 1	7.543 - 1	1.011	0.966
2,000	100	1.360	1.374	1.010	0.966

with mysticism as is the heat-transfer coefficient.* Therefore, diffusive heat fluxes are presented in Table XIII for unequal as well as equal diffusion coefficients. Unequal diffusion is seen to typically increase the diffusive heat flux to the body by 3 to 10 percent with the biggest effect occurring at high pressures (10 atmospheres and above) coupled with moderate to high enthalpies (6,000 Btu/lb and above).

3.2.1.6 Other boundary-layer parameters

The skin-friction coefficient is typically decreased by unequal diffusion but the effect is small, being less than one percent. The displacement thickness is decreased by as much as 12 percent for total enthalpies above 8,000 Btu/lb and below 2,000 Btu/lb (the effect being greatest at low pressures) but is increased by unequal diffusion at intermediate total enthalpies by as much as 3 percent. Finally, the momentum thickness is increased by unequal diffusion coefficients, the increase being typically less than one percent except at low pressures where it is about two percent (see Table XIII).

3.2.1.7 Boundary-layer profiles

The influence of unequal diffusion coefficients on boundary-layer profiles is illustrated in table XV for the flight condition of 11,500 Btu/lb and 30 atmospheres. The stream function, velocity ratio, shear function, enthalpy, enthalpy derivative, temperature and density are shown in Table XV(a) while mole fractions of the principal species are presented in Table XV(b). Of the quantities shown, appreciable differences are seen only in the total enthalpy, total enthalpy derivative, and mole

* It should be mentioned, however, that the diffusive heat flux does depend upon the chemical base state, the JANAF conditions being employed in the present results.

TABLE XV

EFFECT OF UNEQUAL DIFFUSION ON STAGNATION POINT BOUNDARY
LAYER PROFILES WITH QUASISTEADY ABLATION OF GRAPHITE:

Effective Nose Radius = 1 Inch, Surface Emissivity = 1, Total Enthalpy = 11,500 Btu/lb,
Total Pressure = 30 atm

(a) Stream function, velocity ratio, shear function, and thermodynamic properties

η	Equal B_{ij}	Unequal B_{ij}	Equal B_{ij}	Unequal B_{ij}	Equal B_{ij}	Unequal B_{ij}	Equal B_{ij}	Unequal B_{ij}
	Distance from Wall, y (ft)		Stream Function, f		Velocity Ratio, $f' = u/u_e$		Shear Function, f''	
0	0	0	-3.258 - 1	-3.379 - 1	0.	0	3.248 - 1	3.208 - 1
0.5	2.213 - 5	2.251 - 5	-2.709 - 1	-2.822 - 1	.1935	.1939	3.589 - 1	3.563 - 1
1.0	5.117 - 5	5.205 - 5	-9.637 - 2	-1.046 - 1	.4130	.4147	3.840 - 1	3.786 - 1
1.5	8.977 - 5	9.075 - 5	2.045 - 1	2.010 - 1	.6247	.6249	3.443 - 1	3.373 - 1
2.0	1.357 - 4	1.366 - 4	6.188 - 1	6.208 - 1	.8000	.8000	2.484 - 1	2.461 - 1
3.0	2.421 - 4	2.439 - 4	1.658	1.675	.9641	.9652	6.694 - 2	6.642 - 2
5.0	4.783 - 4	4.841 - 4	3.958	4.007	1.0000	1.0000	0.	0.
η	Total Enthalpy, H_T (Btu/lb)		Total Enthalpy Derivative, H_T (Btu/lb)		Temperature, T ($^{\circ}$ R)		Density, ρ (lb/ft ³)	
0	5116	5649	1200	649.7	8071	8137	1.510 - 1	1.507 - 1
0.5	5872	5937	1485	620.6	9250	9274	1.189 - 1	1.183 - 1
1.0	6840	6548	1811	1472	11380	11450	8.754 - 2	8.758 - 2
1.5	7948	7636	2049	2199	13400	13320	6.934 - 2	7.057 - 2
2.0	9147	8986	1970	2228	14290	14220	6.065 - 2	6.144 - 2
3.0	10850	10920	959	1035	15080	15110	5.232 - 2	5.217 - 2
5.0	11500	11500	0.	0.	15390	15390	4.958 - 2	4.958 - 2

TABLE XV (concluded)
(b) Mole Fractions of Principal Species

η	Equal D_{ij}	Unequal D_{ij}	Equal D_{ij}	Unequal D_{ij}	Equal D_{ij}	Unequal D_{ij}	Equal D_{ij}	Unequal D_{ij}
	Mole Fractions							
	C		C ₂		C ₃		CN	
0	2.202 - 2	2.573 - 2	4.688 - 2	5.593 - 2	1.095 - 1	1.289 - 1	7.748 - 2	8.233 - 2
0.5	7.908 - 2	8.133 - 2	7.475 - 2	7.620 - 2	4.550 - 2	4.555 - 2	1.132 - 1	1.138 - 1
1.0	1.787 - 1	1.673 - 1	2.606 - 2	2.123 - 2	1.227 - 3	8.537 - 4	8.372 - 2	7.662 - 2
1.5	1.445 - 1	1.363 - 1	2.879 - 3	2.729 - 3	1.168 - 5	1.132 - 5	3.186 - 2	3.095 - 2
2.0	8.955 - 2	9.221 - 2	5.899 - 4	6.564 - 4	6.725 - 7	8.187 - 7	1.469 - 2	1.550 - 2
3.0	2.559 - 2	2.847 - 2	2.922 - 5	3.564 - 5	5.072 - 9	6.754 - 9	3.237 - 3	3.570 - 3
5.0	0.	0.	0.	0.	0.	0.	0.	0.

η	Mole Fractions							
	CO		N		N ₂		O	
0	2.663 - 1	2.547 - 1	1.016 - 3	1.090 - 3	4.266 - 1	3.958 - 1	2.035 - 6	2.126 - 6
0.5	2.581 - 1	2.601 - 1	5.315 - 3	5.432 - 3	4.140 - 1	4.078 - 1	2.410 - 5	2.524 - 5
1.0	2.536 - 1	2.681 - 1	4.539 - 2	4.848 - 2	4.094 - 1	4.152 - 1	1.351 - 3	1.742 - 3
1.5	2.221 - 1	2.361 - 1	1.820 - 1	1.737 - 1	3.768 - 1	3.804 - 1	3.644 - 2	3.659 - 2
2.0	1.426 - 1	1.515 - 1	2.870 - 1	2.776 - 1	3.407 - 1	3.444 - 1	1.171 - 1	1.108 - 1
3.0	3.239 - 2	3.413 - 2	4.011 - 1	4.046 - 1	2.972 - 1	2.953 - 1	2.286 - 1	2.223 - 1
5.0	0.	0.	4.480 - 1	4.480 - 1	2.778 - 1	2.778 - 1	2.618 - 1	2.618 - 1

fractions. The total enthalpy derivative is considerably reduced near the wall and, consistently, the total enthalpy is higher with unequal diffusion coefficients. The primary changes seen in the mole fractions are that the sublimation products C_3 , C_2 , and C and the product of reaction with nitrogen, CN , are present in larger quantities near the wall due to the higher surface temperatures which, in turn, are the result of the higher diffusive heat fluxes in the unequal diffusion problems.

3.2.2 Comparison to Nonablating (Air) Solutions

In this section stagnation-point quasisteady graphite ablation results presented in Section 3.2.1 are compared to the results for the air boundary layer of Section 2.1. Results are compared at the same surface temperature in order to minimize the effect of different surface temperatures on the results.

3.2.2.1 Diffusive heat flux

The unequal diffusion values for diffusive heat flux with ablation (Table XIII) are normalized in Figure 8 by the values for the air boundary layer (Table III) interpolated for quasisteady ablation surface temperatures. Values of B' are also shown in Figure 8 (see Table XII). The diffusive heat flux is seen to be increased by ablation for those flight conditions where lower values of B' are achieved. The magnitude of the increase is greatest at low stagnation enthalpies, increasing from about 190 to 1,157 Btu/sec-ft² for the 2,000 Btu/lb, 100 atmosphere case. The reason for this is that the exothermic carbon combustion reaction more than offsets the reduction in heat-transfer coefficient due to mass addition to the boundary layer. As sublimation becomes important (high B') the diffusive heat flux is decreased because of the endothermicity of the sublimation reactions, the decrease in importance of the oxidation since it occurs within the interior of the boundary layer, and the increase in the blowing reduction effect. The largest effect

Figure 8. Ratio of Ablating to Nonablating Diffusive Heat Flux for Quasisteady Stagnation Point Graphite Ablation (Unequal Diffusion, $R_{EFV} = 1$ inch, $\epsilon_w = 1.0$)

is seen for the 13,000 Btu/lb, 100 atmosphere case where the diffusive heat flux is reduced from about 16,500 Btu/sec-ft² to 8,545 Btu/sec-ft².

3.2.2.2 Heat-transfer coefficient for equal diffusion coefficients

The heat-transfer coefficients for quasisteady graphite ablation with equal diffusion coefficients,* C_H , normalized by the nonablating values (i.e., for air boundary layers at the ablation wall temperatures), C_{H_0} , are presented in Figure 9 as a function of the blowing parameter, B' . Results for all flight conditions of Table I (total enthalpies of 2,000 to 13,000 Btu/lb and total pressures from 0.01 to 100 atmospheres) are presented in this figure. It can be seen that nearly all of the values are represented quite accurately by a single smooth curve. A few low B' values fall below the curve: these are all for the low total enthalpy cases, H_T of 2,000 and 4,000 Btu/lb.

In order to obtain a functional relationship for this best fit curve, the data are presented in Figure 10 in the more conventional C_H/C_{H_0} versus $B'C_H/C_{H_0}$. It can be seen that the best fit curve for blowing reduction is given by the straight-line relationship

$$\frac{C_H}{C_{H_0}} = 1 - 0.59 B'C_H/C_{H_0} \quad (6)$$

The relation used in the CMA program is

$$\frac{C_H}{C_{H_0}} = \frac{\varphi}{e^\varphi - 1} \quad (7)$$

*The subject of heat-transfer coefficients with unequal diffusion coefficients is discussed in Section 4.

Figure 9. Heat-Transfer Coefficient for Quasisteady Stagnation Point Graphite Ablation Normalized by Nonablating Values as a Function of B' (Equal Diffusion, $R_{EFF} = 1$ inch, $\epsilon_w = 1.0$)

Figure 10. Heat-Transfer Coefficient for Quasisteady Stagnation Point Graphite Ablation
 Normalized by Nonablating Values as a Function of $B'C_H/C_{H_0}$ (Equal Diffusion,
 $R_{EFF} = 1$ inch, $\epsilon_w = 1.0$)

where

$$\varphi = 2\lambda B' C_H / C_{H_0} \quad (8)$$

Curves for $\lambda = 0.60, 0.65$ and 0.70 are also shown on Figure 10. It can be seen that the value of λ varies from 0.59 at the lowest values of $B' C_H / C_{H_0}$, consistent with Equation (6), to 0.66 at high values of $B' C_H / C_{H_0}$, due to the curvature of Equation (4).

It is very beneficial that the blowing correction from such a wide range of flight conditions reduces so well to a single curve, since one would anticipate success in applying this blowing correction curve to transient as well as quasisteady problems where the surface temperature and, hence, the B' , would in general be different for a given flight condition.

3.2.2.3 Skin-friction coefficient

The skin-friction coefficients for quasisteady graphite ablation with unequal diffusion coefficients normalized by the nonablating (air) values at the same wall temperature, C_f / C_{f_0} , are presented as a function of $B' C_f / C_{f_0}$ in Figure 11, the conventional blowing correction coordinates. It can be seen that the blowing correction on skin-friction coefficient does not collapse to a single curve as it did for the case of heat-transfer coefficient, but retains the edge conditions, H_T and P_{T_2} , as parameters. This correlation is not entirely satisfying since it is not a priori evident whether the functional relationship of Figure 11 would be retained under transient conditions when the surface temperature is modified. It is possible that the blowing correction is a function of H_T , P_{T_2} and T_w , of P_{T_2} and T_w , of H_T and T_w , or of some other combination of parameters. Additional solutions at reduced surface temperatures would help to resolve this matter. Lest the reader despair, however, it

Figure 11. Skin-Friction Coefficient For Quasisteady Stagnation Point Graphite Ablation Normalized by Nonablating Values as a Function of $B'C_f/C_{f_0}$ (Unequal Diffusion, $R_{EFF} = 1$ inch, $\epsilon_w = 1.0$)

should be mentioned that all of the results for skin-friction coefficients in the ablating cases are within 3 percent of a best-fit curve passing through the center of the data. Finally, it is significant that although Figure 11 is for unequal diffusion, it is for all practical purposes applicable to equal diffusion as well, the blowing correction for given flight conditions being in all cases within a fraction of a percent for equal and unequal diffusion coefficients.

3.2.2.4 Momentum and displacement thicknesses

The effect of ablation on momentum thickness and displacement thickness are summarized here only briefly. Curves could be generated by interpolating the air boundary-layer results of Table III for wall temperature and comparing to the quasisteady ablation results of Table XIII. As one would expect, both momentum thickness and displacement thickness are increased due to ablation, and the effect is largest when the blowing parameter, B' , is largest. The momentum thickness is increased about 3 to 10 percent for the lower B' , the effect being greater at the low pressure, high enthalpy cases than for the high pressure, low enthalpy cases for a given B' . For the highest B' considered, the increase in momentum thickness is about 20 percent. The effect of graphite ablation on displacement thickness is somewhat greater, especially at the lower total enthalpies, being as great as 50 percent at total enthalpies of 4,000 to 8,000 Btu/lb with a total pressure of 100 atmospheres.

3.3 Downstream Stations

Nonsimilar boundary-layer solutions for quasisteady graphite ablation were obtained for 8 total enthalpy, total pressure combinations, all for a one-inch nose radius, five-degree cone half angle, and with unequal diffusion coefficients. Five total pressures were considered for an enthalpy of 11,500 Btu/lb,

while four enthalpies were considered for a total pressure of 30 atmospheres. Time did not permit studies of the effect of nose radius, cone half angle, unequal-versus-equal diffusion coefficients, or a study of individual nonsimilar effects (pressure gradient, ablation rate distribution, surface temperature distribution). The quasisteady graphite ablation results are presented in Table XVI for parameters of primary interest. In particular, ablation rate, surface temperature, diffusive heat flux, wall shear, skin-friction coefficient, mass-transfer coefficient, blowing parameter, momentum thickness, and displacement thickness are presented in Table XVI(a) through (i), respectively. The results are discussed in the following subsections.

3.3.1 Ablation Rate

The ablation rates, presented in Table XVI(a), have an interesting and peculiar character in that they become negative* for s/R_N of the order of 1.5 to 2.0 for the more severe conditions (high enthalpy, high pressure). There is a logical explanation for this predicted behavior. As discussed previously, extensive sublimation occurs in the stagnation region under the more severe conditions. When this occurs, more carbon is injected into the boundary layer than the amount needed for stoichiometric combustion of carbon with the oxygen which diffuses across the boundary layer. At positions around the body, as the surface temperature drops off, the amount of carbon which can exist in the gas phase at the wall to maintain an equilibrium composition is decreased. Hence, the ablation rate is reduced below what it would be without this supply of excess carbon from upstream. Expressed differently, the oxygen which diffuses across the boundary layer reacts with excess carbon which is supplied from upstream, thereby decreasing the amount of oxygen which can react

* A negative mass transfer means in this case negative ablation rates, i.e., material deposition.

TABLE XVI
 NONSIMILAR BOUNDARY LAYER OVER A SPHERE CONE WITH
 QUASISTEADY ABLATION OF GRAPHITE
 Nose Radius = 1 Inch, Cone Half Angle = 5°,
 Surface Emissivity = 1, Unequal Diffusion Coefficients
 (a) Ablation Rate

Stream- wise Dimen- sion, ft.	S/R _N	Total Enthalpy, Btu/lb							
		11,500	11,500	11,500	11,500	11,500	10,000	8,000	4,000
		Total Pressure, atm							
		.01	.1	1	10	30	30	30	30
		Ablation Rate, $\dot{m}_w$ (lb/sec ft ²)							
0	0	2.581 - 3	8.829 - 3	4.873 - 2	2.199 - 1	4.170 - 1	3.751 - 1	3.145 - 1	1.801 - 1
0.01561	0.1873	2.504 - 3	8.483 - 3	4.648 - 2	2.115 - 1	4.020 - 1	3.615 - 1	3.029 - 1	1.713 - 1
0.03125	0.3750	2.331 - 3	7.733 - 3	4.146 - 2	1.932 - 1	3.691 - 1	3.314 - 1	2.771 - 1	1.587 - 1
0.04686	0.5623	1.966 - 3	6.296 - 3	3.077 - 2	1.538 - 1	2.989 - 1	2.675 - 1	2.226 - 1	1.291 - 1
0.05727	0.6872	1.722 - 3	5.491 - 3	2.437 - 2	1.280 - 1	2.528 - 1	2.256 - 1	1.868 - 1	1.109 - 1
0.07292	0.8750	1.320 - 3	4.201 - 3	1.470 - 2	8.641 - 2	1.778 - 1	1.575 - 1	1.290 - 1	7.902 - 2
0.08366	1.0039	1.045 - 3	3.346 - 3	9.114 - 3	5.900 - 2	1.279 - 1	1.125 - 1	9.120 - 2	5.773 - 2
0.10092	1.2110	6.593 - 4	2.120 - 3	4.186 - 3	2.269 - 2	6.020 - 2	5.170 - 2	4.103 - 2	3.312 - 2
0.12363	1.4836	3.124 - 4	9.940 - 4	2.353 - 3	2.203 - 3	5.686 - 3	4.550 - 3	5.491 - 3	1.536 - 2
0.13209	1.5851	2.675 - 4	8.435 - 4	2.010 - 3	3.182 - 3	2.294 - 3	2.435 - 3	1.141 - 3	1.174 - 2
0.14196	1.7035	2.439 - 4	7.726 - 4	1.919 - 3	1.081 - 3	2.429 - 3	1.210 - 3	3.571 - 3	1.119 - 2
0.16204	1.9445	2.168 - 4	6.894 - 4	1.796 - 3	7.580 - 4	1.366 - 3	8.174 - 4	4.733 - 3	1.001 - 2
0.20587	2.4704	1.789 - 4	5.716 - 4	1.563 - 3	2.159 - 3	5.256 - 4	2.591 - 3	5.299 - 3	8.342 - 3
0.25305	3.0366	1.520 - 4	4.868 - 4	1.367 - 3	2.489 - 3	1.618 - 3	3.361 - 3	5.319 - 3	7.322 - 3
0.33586	4.0303	1.208 - 4	3.875 - 4	1.122 - 3	2.496 - 3	2.763 - 3	3.848 - 3	5.038 - 3	6.156 - 3
0.46803	5.6164	9.215 - 5	2.960 - 4	8.807 - 4	2.237 - 3	3.143 - 3	3.704 - 3	4.413 - 3	4.974 - 3
0.60690	7.2828	7.499 - 5	2.411 - 4	7.319 - 4	1.976 - 3	2.994 - 3	3.360 - 3	3.860 - 3	4.256 - 3
1.0795	12.954	4.953 - 5	1.597 - 4	5.005 - 4	1.468 - 3	2.418 - 3	2.582 - 3	2.793 - 3	3.152 - 3
1.7136	20.563	3.734 - 5	1.207 - 4	3.836 - 4	1.176 - 3	1.992 - 3	2.090 - 3	2.186 - 3	2.534 - 3
2.7994	33.593	2.837 - 5	9.229 - 5	2.961 - 4	9.353 - 4	1.603 - 3	1.668 - 3	1.706 - 3	2.138 - 3

TABLE XVI (continued)
(b) Surface Temperature

Stream- wise Dimen- sion, ft.	S/R _N	Total Enthalpy, Btu/lb							
		11,500	11,500	11,500	11,500	11,500	10,000	8,000	4,000
		Total Pressure, atm							
		.01	.1	1	10	30	30	30	30
		Surface Temperature, T _w (°R)							
0	0	4,302	5,649	6,792	7,698	8,137	8,067	7,937	7,234
0.01561	0.1873	4,268	5,611	6,767	7,679	8,118	8,047	7,916	7,206
0.03125	0.3750	4,191	5,523	6,703	7,627	8,063	7,993	7,861	7,143
0.04686	0.5623	4,018	5,310	6,571	7,532	7,966	7,894	7,759	7,021
0.05727	0.6872	3,887	5,144	6,462	7,449	7,880	7,807	7,670	6,936
0.07292	0.8750	3,637	4,817	6,246	7,286	7,715	7,640	7,497	6,739
0.08366	1.0039	3,430	4,551	6,045	7,147	7,578	7,500	7,348	6,529
0.10092	1.2110	3,055	4,059	5,519	6,857	7,308	7,220	7,042	6,056
0.12363	1.4836	2,520	3,358	4,559	6,261	6,842	6,720	6,453	5,252
0.13209	1.5851	2,426	3,232	4,380	6,072	6,755	6,594	6,230	5,017
0.14196	1.7035	2,374	3,161	4,278	5,926	6,662	6,466	6,080	4,893
0.16204	1.9445	2,306	3,071	4,145	5,723	6,496	6,280	5,880	4,738
0.20587	2.4704	2,199	2,928	3,940	5,389	6,212	5,970	5,536	4,506
0.25305	3.0366	2,111	2,811	3,775	5,132	5,957	5,692	5,281	4,334
0.33586	4.0303	1,992	2,653	3,556	4,808	5,574	5,321	4,960	4,118
0.46803	5.6164	1,860	2,477	3,315	4,460	5,151	4,931	4,618	3,881
0.60690	7.2828	1,765	2,351	3,142	4,218	4,860	4,661	4,375	3,719
1.0795	12.954	1,589	2,116	2,823	3,772	4,334	4,167	3,992	3,424
1.7136	20.563	1,480	1,971	2,626	3,501	4,013	3,863	3,641	3,232
2.7994	33.593	1,385	1,843	2,452	3,262	3,735	3,598	3,392	3,091

TABLE XVI (continued)
(c) Diffusive Heat Flux

Stream- wise Dimen- sion, ft.	S/R_N	Total Enthalpy, Btu/lb							
		11,500	11,500	11,500	11,500	11,500	10,000	8,000	4,000
		Total Pressure, atm							
		.01	.1	1	10	30	30	30	30
		Diffusive Heat Flux, q_a (Btu/sec ft ²)							
0	0	1.658 + 2	4.995 + 2	1.185 + 3	2.824 + 3	4.465 + 3	3.943 + 3	3.226 + 3	1.660 + 3
0.01561	0.1873	1.607 + 2	4.858 + 2	1.158 + 3	2.755 + 3	4.346 + 3	3.842 + 3	3.148 + 3	1.662 + 3
0.03125	0.3750	1.495 + 2	4.548 + 2	1.096 + 3	2.600 + 3	4.081 + 3	3.612 + 3	2.968 + 3	1.539 + 3
0.04686	0.5623	1.261 + 2	3.879 + 2	9.765 + 2	2.291 + 3	3.551 + 3	3.155 + 3	2.616 + 3	1.393 + 3
0.05727	0.6872	1.103 + 2	3.405 + 2	8.952 + 2	2.078 + 3	3.187 + 3	2.845 + 3	2.371 + 3	1.297 + 3
0.07292	0.8750	8.441 + 1	2.617 + 2	7.594 + 2	1.727 + 3	2.594 + 3	2.332 + 3	1.969 + 3	1.111 + 3
0.08366	1.0039	6.684 + 1	2.080 + 2	6.559 + 2	1.448 + 3	2.191 + 3	1.986 + 3	1.697 + 3	9.501 + 2
0.10092	1.2110	4.197 + 1	1.315 + 2	4.510 + 2	1.135 + 3	1.622 + 3	1.489 + 3	1.292 + 3	6.818 + 2
0.12363	1.4836	1.938 + 1	6.135 + 1	2.090 + 2	7.356 + 2	1.071 + 3	9.922 + 2	8.434 + 2	3.772 + 2
0.13209	1.5851	1.663 + 1	5.258 + 1	1.778 + 2	6.493 + 2	9.951 + 2	9.039 + 2	7.270 + 2	3.122 + 2
0.14196	1.7035	1.521 + 1	4.813 + 1	1.619 + 2	5.916 + 2	9.417 + 2	8.393 + 2	6.619 + 2	2.835 + 2
0.16204	1.9445	1.355 + 1	4.284 + 1	1.427 + 2	5.171 + 2	8.544 + 2	7.489 + 2	5.790 + 2	2.473 + 2
0.20587	2.4704	1.121 + 1	3.538 + 1	1.164 + 2	4.073 + 2	7.171 + 2	6.144 + 2	4.560 + 2	2.016 + 2
0.25305	3.0366	9.525	3.003 + 1	9.808 + 1	3.355 + 2	6.079 + 2	5.078 + 2	3.779 + 2	1.728 + 2
0.33586	4.0303	7.553	2.380 + 1	7.716 + 1	2.584 + 2	4.665 + 2	3.884 + 2	2.943 + 2	1.407 + 2
0.46803	5.6164	5.734	1.807 + 1	5.819 + 1	1.915 + 2	3.407 + 2	2.867 + 2	2.209 + 2	1.107 + 2
0.60690	7.2828	4.649	1.465 + 1	4.696 + 1	1.531 + 2	2.702 + 2	2.289 + 2	1.779 + 2	9.311 + 1
1.0795	12.954	3.050	9.608	3.054 + 1	9.782 + 1	1.707 + 2	1.460 + 2	1.146 + 2	6.672 + 1
1.7136	20.563	2.297	7.228	2.285 + 1	7.245 + 1	1.254 + 2	1.077 + 2	8.492 + 1	5.288 + 1
2.7994	33.593	1.757	5.519	1.732 + 1	5.456 + 1	9.403 + 1	8.093 + 1	6.387 + 1	4.404 + 1

TABLE XVI(continued)

(d) Wall Shear

Stream- wise Dimen- sion, ft.	S/R _N	Total Enthalpy, Btu/lb							
		11,500	11,500	11,500	11,500	11,500	10,000	8,000	4,000
		Total Pressure, atm							
		.01	.1	1	10	30	30	30	30
		Wall Shear, τ_w (lb/ft ²)							
0	0	0	0	0	0	0	0	0	0
0.01561	0.1873	8.080 - 1	2.698	8.393	2.613 + 1	4.556 + 1	4.551 + 1	4.532 + 1	4.075 + 1
0.03125	0.3750	1.518	5.076	1.589 + 1	4.936 + 1	8.599 + 1	8.591 + 1	8.554 + 1	7.667 + 1
0.04686	0.5623	1.995	6.693	2.137 + 1	6.606 + 1	1.148 + 2	1.147 + 2	1.143 + 2	1.019 + 2
0.05727	0.6872	2.183	7.329	2.378 + 1	7.326 + 1	1.271 + 2	1.270 + 2	1.265 + 2	1.122 + 2
0.07292	0.8750	2.220	7.457	2.492 + 1	7.651 + 1	1.322 + 2	1.321 + 2	1.315 + 2	1.148 + 2
0.08366	1.0039	2.096	7.040	2.407 + 1	7.400 + 1	1.275 + 2	1.274 + 2	1.267 + 2	1.094 + 2
0.10092	1.2110	1.724	5.795	2.029 + 1	6.411 + 1	1.098 + 2	1.096 + 2	1.088 + 2	9.186 + 1
0.12363	1.4836	1.117	3.764	1.318 + 1	4.537 + 1	7.754 + 1	7.710 + 1	7.569 + 1	6.119 + 1
0.13209	1.5851	7.610 - 1	2.532	8.755	3.056 + 1	5.216 + 1	5.150 + 1	4.972 + 1	3.801 + 1
0.14196	1.7035	6.410 - 1	2.134	7.372	2.597 + 1	4.464 + 1	4.387 + 1	4.199 + 1	3.168 + 1
0.16204	1.9445	5.357 - 1	1.784	6.154	2.184 + 1	3.802 + 1	3.711 + 1	3.518 + 1	2.622 + 1
0.20587	2.4704	4.289 - 1	1.427	4.900	1.737 + 1	3.093 + 1	2.984 + 1	2.783 + 1	2.073 + 1
0.25305	3.0366	3.641 - 1	1.211	4.140	1.459 + 1	2.632 + 1	2.517 + 1	2.327 + 1	1.749 + 1
0.33586	4.0303	2.919 - 1	9.694 - 1	3.298	1.152 + 1	2.090 + 1	1.988 + 1	1.828 + 1	1.400 + 1
0.46803	5.6164	2.239 - 1	7.427 - 1	2.514	8.692	1.575 + 1	1.495 + 1	1.372 + 1	1.067 + 1
0.60690	7.2828	1.820 - 1	6.030 - 1	2.033	6.984	1.262 + 1	1.197 + 1	1.097 + 1	8.684
1.0795	12.954	1.177 - 1	3.892 - 1	1.302	4.419	7.931	7.522	6.874	5.851
1.7136	20.563	8.577 - 2	2.833 - 1	9.433 - 1	3.172	5.668	5.369	4.890	4.397
2.7994	33.593	6.278 - 2	2.070 - 1	6.863 - 1	2.289	4.071	3.850	3.494	3.304

TABLE XVI (continued)
(e) Skin Friction Coefficient

Stream- wise Dimen- sion, ft.	S/R N	Total Enthalpy, Btu/lb							
		11,500	11,500	11,500	11,500	11,500	10,000	8,000	4,000
		Total Pressure, atm							
		.01	.1	1	10	30	30	30	30
		Skin Friction Coefficient, $\rho_e u_e C_f / 2$ (lb/sec ft ²)							
0	0	1.672 - 2	5.338 - 2	1.579 - 1	4.682 - 1	7.973 - 1	8.375 - 1	9.000 - 1	1.062
0.01561	0.1873	1.632 - 2	5.213 - 2	1.547 - 1	4.584 - 1	7.803 - 1	8.196 - 1	8.809 - 1	1.038
0.03125	0.3750	1.531 - 2	4.898 - 2	1.464 - 1	4.329 - 1	7.364 - 1	7.735 - 1	8.312 - 1	9.780 - 1
0.04686	0.5623	1.338 - 2	4.297 - 2	1.310 - 1	3.860 - 1	6.555 - 1	6.885 - 1	7.399 - 1	8.689 - 1
0.05727	0.6872	1.197 - 2	3.847 - 2	1.193 - 1	3.505 - 1	5.942 - 1	6.240 - 1	6.706 - 1	7.851 - 1
0.07292	0.8750	9.557 - 3	3.078 - 2	9.839 - 2	2.884 - 1	4.875 - 1	5.119 - 1	5.498 - 1	6.362 - 1
0.08366	1.0039	7.884 - 3	2.541 - 2	8.319 - 2	2.444 - 1	4.120 - 1	4.324 - 1	4.643 - 1	5.335 - 1
0.10092	1.2110	5.409 - 3	1.747 - 2	5.866 - 2	1.775 - 1	2.977 - 1	3.122 - 1	3.345 - 1	3.775 - 1
0.12363	1.4836	2.908 - 3	9.434 - 3	3.177 - 2	1.050 - 1	1.760 - 1	1.838 - 1	1.952 - 1	2.116 - 1
0.13209	1.5851	1.974 - 3	6.322 - 3	2.102 - 2	7.048 - 2	1.180 - 1	1.224 - 1	1.278 - 1	1.311 - 1
0.14196	1.7035	1.654 - 3	5.303 - 3	1.762 - 2	5.962 - 2	1.005 - 1	1.038 - 1	1.074 - 1	1.088 - 1
0.16204	1.9445	1.370 - 3	4.393 - 3	1.457 - 2	4.970 - 2	8.488 - 2	8.700 - 2	8.923 - 2	8.939 - 2
0.20587	2.4704	1.075 - 3	3.446 - 3	1.138 - 2	3.878 - 2	6.774 - 2	6.866 - 2	6.930 - 2	6.949 - 2
0.25305	3.0366	8.955 - 4	2.869 - 3	9.442 - 3	3.199 - 2	5.665 - 2	5.691 - 2	5.694 - 2	5.773 - 2
0.33586	4.0303	6.986 - 4	2.237 - 3	7.328 - 3	2.462 - 2	4.387 - 2	4.381 - 2	4.365 - 2	4.516 - 2
0.46803	5.6164	5.192 - 4	1.661 - 3	5.416 - 3	1.803 - 2	3.209 - 2	3.200 - 2	3.184 - 2	3.356 - 2
0.60690	7.2828	4.123 - 4	1.318 - 3	4.283 - 3	1.418 - 2	2.516 - 2	2.509 - 2	2.495 - 2	2.686 - 2
1.0795	12.954	2.550 - 4	8.145 - 4	2.630 - 3	8.609 - 3	1.519 - 2	1.514 - 2	1.505 - 2	1.759 - 2
1.7136	20.563	1.809 - 4	5.775 - 4	1.857 - 3	6.028 - 3	1.059 - 2	1.055 - 2	1.047 - 2	1.300 - 2
2.7994	33.593	1.296 - 4	4.130 - 4	1.323 - 3	4.264 - 3	7.462 - 3	7.421 - 3	7.352 - 3	9.706 - 3

TABLE XVI (continued)
(f) Mass Transfer Coefficient *

Stream- wise Dimen- sion, ft.	S/R _N	Total Enthalpy, Btu/lb							
		11,500	11,500	11,500	11,500	11,500	10000	8,000	4,000
		Total Pressure, atm							
		.01	.1	1	10	30	30	30	30
		Mass Transfer Coefficient, $\rho_e U C_M$ (lb/sec ft ²)							
0	0	1.426 - 2	4.404 - 2	1.215 - 1	3.366 - 1	5.672 - 1	5.961 - 1	6.382 - 1	7.525 - 1
.01561	.1873	1.384 - 2	4.283 - 2	1.194 - 1	3.274 - 1	5.518 - 1	5.795 - 1	6.205 - 1	7.307 - 1
.03125	.3750	1.288 - 2	3.995 - 2	1.130 - 1	3.051 - 1	5.143 - 1	5.404 - 1	5.783 - 1	6.812 - 1
.04686	.5623	1.085 - 2	3.364 - 2	9.656 - 2	2.559 - 1	4.326 - 1	4.544 - 1	4.856 - 1	5.688 - 1
.05727	.6872	9.492 - 3	2.976 - 2	8.411 - 2	2.223 - 1	3.776 - 1	3.962 - 1	4.230 - 1	4.937 - 1
.07292	.8750	7.250 - 3	2.296 - 2	5.932 - 2	1.650 - 1	2.843 - 1	2.976 - 1	3.166 - 1	3.656 - 1
.08366	1.0039	5.731 - 3	1.830 - 2	4.100 - 2	1.236 - 1	2.185 - 1	2.279 - 1	2.411 - 1	2.807 - 1
.10092	1.2110	3.604 - 3	1.159 - 2	2.179 - 2	5.981 - 2	1.200 - 1	1.235 - 1	1.290 - 1	1.731 - 1
.12363	1.4836	1.707 - 3	5.387 - 3	1.270 - 2	9.390 - 3	1.605 - 2	1.540 - 2	2.372 - 2	8.458 - 2
.13209	1.5851	1.462 - 3	4.553 - 3	1.087 - 2	1.533 - 2	7.322 - 3	9.448 - 3	5.421 - 3	6.440 - 2
.14196	1.7035	1.330 - 3	4.196 - 3	1.037 - 2	5.484 - 3	8.735 - 3	5.314 - 3	1.781 - 2	6.146 - 2
.16204	1.9445	1.183 - 3	3.757 - 3	9.706 - 3	3.974 - 3	5.858 - 3	3.972 - 3	2.440 - 2	5.567 - 2
.20587	2.4704	9.755 - 4	3.116 - 3	8.432 - 3	1.164 - 2	2.563 - 3	1.332 - 2	2.786 - 2	4.635 - 2
.25305	3.0366	8.281 - 4	2.650 - 3	7.365 - 3	1.349 - 2	8.327 - 3	1.781 - 2	2.802 - 2	4.049 - 2
.33586	4.0303	6.569 - 4	2.106 - 3	6.027 - 3	1.350 - 2	1.475 - 2	2.054 - 2	2.636 - 2	3.399 - 2
.46803	5.6164	4.996 - 4	1.603 - 3	4.710 - 3	1.202 - 2	1.684 - 2	1.964 - 2	2.279 - 2	2.745 - 2
.60690	7.2828	4.059 - 4	1.303 - 3	3.916 - 3	1.055 - 2	1.599 - 2	1.769 - 2	1.972 - 2	2.365 - 2
1.0795	12.954	2.667 - 4	8.579 - 4	2.686 - 3	7.709 - 3	1.276 - 2	1.341 - 2	1.394 - 2	1.751 - 2
1.7136	20.563	2.006 - 4	6.458 - 4	2.044 - 3	6.173 - 3	1.039 - 2	1.069 - 2	1.086 - 2	1.422 - 2
2.7994	33.593	1.526 - 4	4.924 - 4	1.571 - 3	4.880 - 3	8.336 - 3	8.441 - 3	8.475 - 3	1.196 - 2

* Defined as $\rho_e U C_M = (-j_{k_w}) / (\tilde{z}_{k_e}^* - \tilde{z}_{k_w}^*)$

TABLE XVI (continued)
(g)* Blowing Parameter*

Stream- wise Dimen- sion, ft.	S/R _N	Total Enthalpy, Btu/lb							
		11,500	11,500	11,500	11,500	11,500	10,000	8,000	4,000
		Total Pressure, atm							
		.01	.1	1	10	30	30	30	30
		Blowing Parameter, B'							
0	0	.1810	.2005	.4011	.6533	.7352	.6293	.4928	.2393
.01561	.1873	.1809	.1981	.3893	.6460	.7285	.6238	.4882	.2344
.03125	.3750	.1810	.1936	.3669	.6332	.7177	.6132	.4792	.2330
.04686	.5623	.1812	.1872	.3187	.6010	.6909	.5887	.4584	.2270
.05727	.6872	.1814	.1845	.2897	.5758	.6695	.5694	.4416	.2246
.07292	.8750	.1820	.1830	.2478	.5237	.6254	.5292	.4074	.2161
.08366	1.0039	.1823	.1828	.2223	.4773	.5854	.4936	.3783	.2057
.10092	1.2110	.1829	.1829	.1921	.3794	.5017	.4186	.3181	.1913
.12363	1.4836	.1830	.1845	.1853	.2346	.3543	.2955	.2315	.1816
.13209	1.5851	.1830	.1853	.1849	.2076	.3133	.2577	.2105	.1823
.14196	1.7035	.1834	.1841	.1850	.1971	.2780	.2277	.2005	.1821
.16204	1.9445	.1833	.1835	.1850	.1907	.2332	.2058	.1940	.1798
.20587	2.4704	.1833	.1834	.1853	.1855	.2051	.1945	.1902	.1800
.25305	3.0366	.1835	.1837	.1856	.1845	.1943	.1887	.1898	.1808
.33586	4.0303	.1838	.1840	.1861	.1849	.1873	.1873	.1911	.1811
.46803	5.6164	.1844	.1847	.1870	.1861	.1866	.1886	.1936	.1812
.60690	7.2828	.1847	.1850	.1869	.1873	.1872	.1899	.1957	.1800
1.0795	12.954	.1857	.1862	.1863	.1904	.1895	.1925	.2004	.1800
1.7136	20.563	.1861	.1869	.1877	.1905	.1917	.1955	.2013	.1782
2.7994	33.593	.1859	.1874	.1885	.1917	.1922	.1976	.2013	.1788

* Defined as $B' = \dot{m}_w / \rho_e u_e C_M$ with $\rho_e u_e C_M$ defined by Eq. (4), see Table XVI(f).

TABLE XVI(continued)
(h) Momentum Thickness

Stream- wise Dimen- sion, ft.	S/R _N	Total Enthalpy, Btu/lb							
		11,500	11,500	11,500	11,500	11,500	10,000	8,000	4,000
		Total Pressure, atm							
		.01	.1	1	10	30	30	30	30
		Momentum Thickness, θ (ft)							
0	0	2.398 - 3	7.882 - 4	2.678 - 4	9.319 - 5	5.627 - 5	5.128 - 5	4.454 - 5	2.880 - 5
0.01561	0.1873	2.443 - 3	8.091 - 4	2.743 - 4	9.544 - 5	5.768 - 5	5.252 - 5	4.559 - 5	2.943 - 5
0.03125	0.3750	2.597 - 3	8.630 - 4	2.898 - 4	1.007 - 4	6.075 - 5	5.535 - 5	4.806 - 5	3.099 - 5
0.04686	0.5623	3.034 - 3	1.002 - 3	3.339 - 4	1.158 - 4	6.987 - 5	6.355 - 5	5.506 - 5	3.515 - 5
0.05727	0.6872	3.414 - 3	1.128 - 3	3.718 - 4	1.286 - 4	7.747 - 5	7.048 - 5	6.094 - 5	3.874 - 5
0.07292	0.8750	4.354 - 3	1.428 - 3	4.641 - 4	1.593 - 4	9.581 - 5	8.703 - 5	7.503 - 5	4.741 - 5
0.08366	1.0039	5.358 - 3	1.762 - 3	5.660 - 4	1.925 - 4	1.156 - 4	1.049 - 4	9.020 - 5	5.679 - 5
0.10092	1.2110	8.207 - 3	2.682 - 3	8.488 - 4	2.818 - 4	1.689 - 4	1.527 - 4	1.303 - 4	8.187 - 5
0.12363	1.4836	1.700 - 2	5.515 - 3	1.725 - 3	5.453 - 4	3.238 - 4	2.914 - 4	2.437 - 4	1.544 - 4
0.13209	1.5851	1.816 - 2	5.899 - 3	1.848 - 3	5.810 - 4	3.444 - 4	3.111 - 4	2.625 - 4	1.689 - 4
0.14196	1.7035	1.940 - 2	6.299 - 3	1.977 - 3	6.197 - 4	3.664 - 4	3.325 - 4	2.826 - 4	1.835 - 4
0.16204	1.9445	2.168 - 2	7.041 - 3	2.216 - 3	6.921 - 4	4.072 - 4	3.720 - 4	3.199 - 4	2.101 - 4
0.20587	2.4704	2.644 - 2	8.587 - 3	2.713 - 3	8.459 - 4	4.926 - 4	4.542 - 4	3.957 - 4	2.627 - 4
0.25305	3.0366	3.127 - 2	1.016 - 2	3.216 - 3	1.002 - 3	5.796 - 4	5.387 - 4	4.712 - 4	3.131 - 4
0.33586	4.0303	3.947 - 2	1.281 - 2	4.066 - 3	1.268 - 3	7.300 - 4	6.807 - 4	5.953 - 4	3.945 - 4
0.46803	5.6164	5.157 - 2	1.672 - 2	5.317 - 3	1.661 - 3	9.533 - 4	8.905 - 4	7.749 - 4	5.098 - 4
0.60690	7.2828	6.292 - 2	2.038 - 2	6.490 - 3	2.030 - 3	1.163 - 3	1.088 - 3	9.399 - 4	6.077 - 4
1.0795	12.954	9.341 - 2	3.020 - 2	9.633 - 3	3.017 - 3	1.735 - 3	1.621 - 3	1.368 - 3	8.420 - 4
1.7136	20.563	1.218 - 1	3.933 - 2	1.257 - 2	3.962 - 3	2.271 - 3	2.118 - 3	1.769 - 3	1.059 - 3
2.7994	33.593	1.574 - 1	5.078 - 2	1.635 - 2	5.139 - 3	2.952 - 3	2.743 - 3	2.269 - 3	1.272 - 3

TABLE XVI (concluded)
(i) Displacement Thickness

Stream- wise Dimen- sion, ft.	S/R _N	Total Enthalpy, Btu/lb							
		11,500	11,500	11,500	11,500	11,500	10,000	8,000	4,000
		Total Pressure, atm							
		.01	.1	1	10	30	30	30	30
		Displacement Thickness, δ^* (ft)							
0	0	1.106 - 3	4.050 - 4	1.624 - 4	5.830 - 5	3.470 - 5	3.946 - 5	4.579 - 5	5.458 - 5
0.01561	0.1873	1.129 - 3	4.123 - 4	1.663 - 4	5.976 - 5	3.562 - 5	4.049 - 5	4.696 - 5	5.577 - 5
0.03125	0.3750	1.215 - 3	4.414 - 4	1.800 - 4	6.480 - 5	3.875 - 5	4.384 - 5	5.064 - 5	5.951 - 5
0.04686	0.5623	1.370 - 3	4.949 - 4	2.071 - 4	7.471 - 5	4.493 - 5	5.079 - 5	5.861 - 5	6.768 - 5
0.05727	0.6872	1.524 - 3	5.504 - 4	2.332 - 4	8.456 - 5	5.108 - 5	5.756 - 5	6.624 - 5	7.494 - 5
0.07292	0.8750	1.882 - 3	6.818 - 4	2.927 - 4	1.082 - 4	6.597 - 5	7.393 - 5	8.468 - 5	9.108 - 5
0.08366	1.0039	2.236 - 3	8.147 - 4	3.512 - 4	1.332 - 4	8.181 - 5	9.140 - 5	1.045 - 4	1.070 - 4
0.10092	1.2110	3.009 - 3	1.114 - 3	4.814 - 4	1.968 - 4	1.224 - 4	1.366 - 4	1.563 - 4	1.480 - 4
0.12363	1.4836	4.106 - 3	1.605 - 3	7.634 - 4	3.589 - 4	2.268 - 4	2.547 - 4	2.938 - 4	2.598 - 4
0.13209	1.5851	6.725 - 3	2.487 - 3	1.060 - 3	4.765 - 4	2.970 - 4	3.272 - 4	3.679 - 4	3.174 - 4
0.14196	1.7035	8.936 - 3	3.223 - 3	1.306 - 3	5.705 - 4	3.580 - 4	3.899 - 4	4.272 - 4	3.659 - 4
0.16204	1.9445	1.224 - 2	4.329 - 3	1.677 - 3	7.039 - 4	4.529 - 4	4.834 - 4	5.067 - 4	4.420 - 4
0.20587	2.4704	1.782 - 2	6.212 - 3	2.320 - 3	9.270 - 4	6.110 - 4	6.354 - 4	6.430 - 4	5.778 - 4
0.25305	3.0366	2.288 - 2	7.926 - 3	2.912 - 3	1.135 - 3	7.480 - 4	7.655 - 4	7.731 - 4	7.030 - 4
0.33586	4.0303	3.111 - 2	1.072 - 2	3.880 - 3	1.480 - 3	9.632 - 4	9.795 - 4	9.911 - 4	9.060 - 4
0.46803	5.6164	4.367 - 2	1.498 - 2	5.350 - 3	2.003 - 3	1.285 - 3	1.302 - 3	1.326 - 3	1.206 - 3
0.60690	7.2828	5.625 - 2	1.924 - 2	6.810 - 3	2.520 - 3	1.602 - 3	1.620 - 3	1.659 - 3	1.475 - 3
1.0795	12.954	9.472 - 2	3.220 - 2	1.122 - 2	4.053 - 3	2.535 - 3	2.558 - 3	2.659 - 3	2.164 - 3
1.7136	20.563	1.372 - 1	4.642 - 2	1.606 - 2	5.706 - 3	3.532 - 3	3.571 - 3	3.741 - 3	2.844 - 3
2.7994	33.593	1.966 - 1	6.652 - 2	2.284 - 2	8.023 - 3	4.930 - 3	5.010 - 3	5.258 - 3	3.587 - 3

with the wall. If the amount of carbon supplied from upstream is by itself in excess of that needed to achieve equilibrium in the gas phase at the wall at the local station, carbon in the gas phase must condense. The BLIMP program currently does not allow the presence of a condensed phase within the boundary layer. Hence, carbon deposition is predicted to occur. (Actually, the condensate probably does form when predicted, but it may be swept away as small particulate matter, thus producing further effect on downstream stations.)

3.3.2 Mass-Transfer Coefficient and Blowing Parameter

There are two alternative procedures which one could choose to correlate these nonsimilar ablation results in terms of mass-transfer coefficients. One approach absorbs the nonsimilar effect into the mass-transfer coefficient, whereas the second approach absorbs the effect into the driving potential for mass transfer. Both of these approaches are considered here.

Following the first approach, one can define the mass-transfer coefficient in terms of the local $\tilde{J}_{k_w}$, $\tilde{Z}_{k_e}^*$ and $\tilde{Z}_{k_w}^*$ as in Equation (4). When this is done one gets the results for $\rho_e u_e C_M$ presented in Table XVI(f). It can be seen that these have the same general behavior as the ablation rates, that is, they become negative under the same conditions. This is shown very clearly by the presentation in Table XVI(g) of blowing parameters which are the ratio of ablation rates to mass-transfer coefficients (see Eq. (5)). It can be seen that the blowing parameter not only remains positive, but that it varies very smoothly from the stagnation-point values, passing through the region of negative ablation rates, to values representative of diffusion-controlled surface ablation.

This result is a direct consequence of the definitions of $\rho_e u_e C_M$ and B' . With these definitions and considering elemental mass balances at the wall, it can be shown that for

diffusion-controlled ablation the B' depends only upon the thermodynamic properties of the main stream, of the fluid in contact with the surface, and of the ablative substance and is, therefore, a function only of the boundary-layer-edge gas, the surface material, pressure, and surface temperature. The particular functional relationship for B' for the present problem has been shown in Figure 7 for equal and unequal diffusion coefficients. Although Figure 7 was prepared exclusively from stagnation-point results, it should apply to all stations around the body, even in the presence of the strong nonsimilar effects encountered in the present solutions. To be sure that the B' do satisfy this requirement and thus to obtain a further consistency check on the $\rho_e u_e C_M$, a number of B' for downstream stations were superimposed upon Figure 7 for the local pressures and surface temperatures and were seen to agree very favorably with the functional relationship for stagnation-point B' .

Following the second approach where the primary nonsimilar effect is absorbed into the driving potential for mass transfer, one can demand that the $\rho_e u_e C_M$ be given by some convenient correlation expression such as*

$$\rho_e u_e C_M = \rho_e u_e C_H (Le)^{2/3} \quad (9)$$

where Le is the Lewis number. In the present case the Lewis number is near unity. If one further takes $\rho_e u_e C_H$ as given by Equation (1) and defines B' in the conventional way, he obtains the streamwise distributions of B' presented in Figure 12 for the various flight conditions considered. It can be seen for the severe cases where the $\rho_e u_e C_M$ were seen to go negative when the approach discussed previously was used, the B' now become negative. Computationally, in order to utilize this approach, it is necessary to consider the composition of

*This relation is currently employed in the CMA program.

Figure 12. Blowing Parameter Around a Sphere Cone for Quasisteady Graphite Ablation:
 $R_N = 1$ inch, $\theta_c = 5^\circ$, $\epsilon_w = 1.0$

the edge gas to vary such as to yield the proper values of B' and, hence, the assumed values for $\rho_e u_e C_M$. That is, one must add carbon to the edge gas to simulate the fact that carbon is being fed in from upstream.

These two approaches are equally valid for correlating non-similar boundary-layer results. The first is more pleasing in the sense that no new definitions are required and one does not have to artificially alter the edge-gas composition. However, the second approach is more consistent with the spirit of the transfer-coefficient approach which is really based on similarity concepts. Furthermore, whereas it may be difficult to develop correlation relations for negative mass-transfer coefficients which can be extended with much degree of generality, it might be possible to develop fairly general relationships for predicting the amount of carbon to be considered in the edge gas to simulate upstream conditions. However, this is just conjecture at this point. It should be understood, however, that both approaches require the correlation of nonsimilar boundary-layer solutions.

3.3.3 Surface Temperature

Quasisteady surface temperature distributions are presented in Table XVI(b) and are shown graphically, normalized by stagnation-point temperature, in Figure 13. The solutions for a total enthalpy of 11,500 Btu/lb with various stagnation pressures are shown in Figure 13(a), whereas the solutions for a total pressure of 30 atmospheres for various total enthalpies are presented in Figure 13(b). Temperatures tend to drop off more rapidly going around the body as the severity of the flight condition lessens (reduction of total pressure or total enthalpy). No attempt has been made to correlate this behavior.

Figure 13. Wall Temperatures Normalized by Stagnation Values for Quasisteady Graphite Ablation
Around a Sphere Cone: $R_N = 1$ inch, $\theta_c = 5^\circ$, $\epsilon_w = 1.0$

a. Total Enthalpy = 11,500 BTU/lb

Figure 13 (concluded)
b. Total Pressure = 30 ATM

3.3.4 Additional Boundary-Layer Parameters

The other quantities presented in Table XVI, diffusive heat flux, wall shear, skin-friction coefficient, momentum thickness, and displacement thickness are presented for the record and will not be discussed in any detail here. Further solutions are required for ablating and nonablating boundary layers at various wall temperatures in order to develop any meaningful correlations.

3.4 Applicability of Quasisteady Ablation Approximation

The purpose of this section is to investigate the regions of applicability of the quasisteady conduction approximation which was employed in the graphite ablation calculations. In those regions where the approximation is found to be adequate, one can apply the quasisteady results of Sections 3.2 and 3.3 directly to obtain preliminary estimates of ablation rates and surface temperatures.

The heat conducted into the graphite stagnation point during quasisteady ablation, $\dot{m}_w(h_{cw} - h_\infty)$, is presented in Figure 14 for the complete matrix of total enthalpy and total pressure. For the purpose of exploring the applicability of the quasisteady ablation approximation, the following discussion is offered. Because of the small dimensions in the vicinity of the nose tip, the graphite body approaches that of a "lumped capacitance," that is, a body at a uniform temperature, T_c , for which

$$q_{\text{cond}} = \rho_c c_c \Delta l \frac{dT_c}{dt} \quad (10)$$

where ρ_c and c_c are the density and heat capacity of the surface material, respectively, and Δl is the effective thickness of the uniform slab. Prior to peak heating, dT_c/dt will probably often fall between 200 to 1,000°R/sec. Estimating the

Figure 14. Heat Conducted into Graphite Stagnation Point During Quasisteady Ablation:
 $R_{\text{EFF}} = 1$ inch, $\epsilon_w = 1.0$

body half-thickness in the vicinity of the nose tip as 0.5 inch, Equation (10) was used to estimate the transient q_{cond} and the results are superimposed on Figure 14. Regions are shown where the quasisteady assumption may be conservative, reasonably good, slightly conservative, and ultra-conservative just prior to and during peak heating. Three-dimensional geometry and heat conduction effects have been neglected here so the applicability bands should be considered as very approximate. With this word of caution, it is suggested that the quasisteady assumption is probably pretty good in the vicinity of the stagnation point during the region of peak heating (although slightly nonconservative in some cases), but is ultra-conservative early in the trajectory and at positions back on the cone.

In order to investigate further the applicability of the quasisteady approximation, transient and quasisteady solutions have been generated for stagnation-point graphite ablation for a typical, severe trajectory. The $p_e u_e C_{H_2O}$, H_T and P_{T_2} considered are presented in Figure 15; the results for surface recession and surface temperature are presented in Figure 16. It can be seen that quasisteady surface temperatures are achieved during the more severe portion of the trajectory, accounting for more than 90 percent of the total ablation, but that the quasisteady approximation is ultra-conservative at early trajectory times. The ablation rate, on the other hand, is seen to be underpredicted by about four percent at peak heating (the quasisteady approximation being nonconservative) and overpredicted at early trajectory times.

In summary, the quasisteady assumption is seen to produce the following effects for the trajectory considered: the recession rate is always predicted within 0.003 inches/second (to be compared with a peak recession rate of 0.079 inches/second) the total surface recession is very accurately predicted as a result of compensating errors early and late in the trajectory, the surface temperatures are accurately predicted over the peak

Figure 15. Typical Ballistic Reentry Trajectory

Figure 16. Comparison of Graphite Stagnation-Point Quasisteady and Transient Ablation Rates and Surface Temperatures for Typical Ballistic Reentry Trajectory

heating portion of the trajectory (where P_{T_2} exceeds 8 atmospheres or so), but the surface temperature is highly overpredicted early in the trajectory. These conclusions all tend to concur with the conclusions set forth on Figure 14.

4. CORRELATION OF BOUNDARY-LAYER SOLUTIONS WITH CONVECTIVE TRANSFER-COEFFICIENT APPROACH

The purpose of this section is 1) to review the convective transfer-coefficient model for considering unequal diffusion coefficients (employed in the CMA and ACE programs) which was proposed in Reference 7, 2) to subject this model to the test of correlating the quasisteady graphite boundary-layer solutions which were generated by the BLIMP program and are presented in Section 3, and 3) to establish compatibility between the BLIMP and CMA/ACE computer programs. These subjects are treated in Sections 4.2 through 4.4, respectively, following some introductory remarks in Section 4.1.

4.1 General Remarks

The basic problem under consideration here is the prediction of the thermal response of graphitic (or carbonaceous) material to a hot gaseous (air) environment flowing past the surface of the material. Such a situation involves a study of the boundary layer adjacent to the surface where the significant phenomena such as heat and mass transfer to the surface are controlled. In the theoretical treatment of such problems, two broad approaches have historically been taken - the one, a detailed study of the boundary layer which evokes the numerical solution of sets of nonlinear differential equations, and the other, a study of the algebraic relations obtained from a simplified treatment of the boundary layer in which details of the boundary layer are ignored, the so-called film transport or transfer coefficient analog. It has been shown in the past that for simple physical situations the simple algebraic relations in the film transport analog can be derived directly from the governing differential equations when certain simplifying assumptions are made which are also consistent with the physical situation being considered.

In this study the film transport relations are extended to situations where such a direct derivation cannot be made and the

relations must depend upon analogy with terms in the differential equations. For such situations, it is to be expected that the simplified treatment must lack the generality and/or the accuracy which is desired or which can be obtained from the detailed solutions. When the computational ease provided by a film transport approach greatly outweighs the resultant inaccuracies (as is found to be the case for the conditions of this study) then the consideration of such extensions is worthwhile.

4.2 Review of Transfer-Coefficient Formulation

The film transport analog is basically a simple physical concept in which the transport of the phenomena being considered, be it energy, matter, or momentum, can be described by a suitable driving potential in conjunction with an appropriate coefficient. This work is concerned with mass and energy coefficients along with the appropriate driving potential representation. A brief historical description of the use of the film concept and its development with further background references is found in Reference 6.

4.2.1 Mass Transport

Early investigators found that the heat flux to a surface could be correlated with the temperature difference and a single empirical coefficient. Subsequently, boundary-layer investigators found that solutions to the laminar boundary-layer species conservation differential equation with equal diffusion coefficients were frequently correlated when the mass fraction of a given species was used as the driving potential. This analogous relationship between the two phenomena has led to the acceptance of the following form for the mass-transport film-analog correlation expression:

$$-\tilde{j}_{k_w} = \rho_e u_e C_M (\tilde{K}_{k_e} - \tilde{K}_{k_w}) \quad (11)$$

where $\rho_e u_e C_M$, termed a mass-transfer coefficient, is the same for all K elements. Here the tilda ($\sim$) denotes the k^{th} element, independent of the molecular configurations involving that element. In this expression $\tilde{j}_{k_w}$ is the diffusional mass flux of the k^{th} configuration-independent element from the surface and $\tilde{K}_{k_e}$ and $\tilde{K}_{k_w}$ are mass fractions of the same element evaluated at the boundary-layer edge and at the wall, respectively.

When the analogy is extended to the unequal diffusion situation, the form of the film-transport driving potential becomes more tenuous. Since a considerable amount of effort was expended in this study on the unequal diffusion form of the equation, some detail on its development bears repeating (as found in References 6 and 7).

In the development of the differential equation describing the conservation of species under unequal-diffusion-coefficient conditions, it was found useful to define

$$\tilde{Z}_k \equiv \sum_{i=1}^N \alpha_{ki} Z_i \quad (12)$$

where N is the number of species in the chemical system of interest, α_{ki} is the mass fraction of element k in species i , and Z_i is defined as

$$Z_i \equiv \frac{\mathcal{M}_i x_i}{F_i} \bigg/ \sum_{j=1}^N \frac{\mathcal{M}_j x_j}{F_j} \quad (13)$$

with $\mathcal{M}_i$ the molecular weight, x_i the mole fraction, and F_i the diffusion factor for species i obtained from the bifurcation approximation for binary diffusion coefficients (Ref. 2) as

$$D_{ij} = \bar{D} / F_i F_j \quad (14)$$

where $\bar{D}$ is a reference diffusion coefficient. The F_i for a system of chemical species are found by a least-squares fit of the $N(N-1)/2 \mathcal{A}_{ij}$ for the system of interest.

It was shown in Reference 7 that this term dominates the diffusion portion of the species equation and, when the less important terms are dropped, the species equation for unequal diffusion is analogous to the equal-diffusion form of the equation with the exception that the mass-fraction-derivative term due to diffusion becomes:

$$\frac{\partial \tilde{K}_k}{\partial y} \rightarrow \frac{\partial \tilde{Z}_k}{\partial y} \quad (15)$$

and an overall effective diffusion coefficient must be defined. The point to be made is that Equation (15) has significant implication to the form of the driving potential for the film analog equation - that is, there seems to be justification for replacing $\tilde{K}_k$ in Equation (11) by $\tilde{Z}_k$:

$$-\tilde{j}_{k_w} = \rho_e u_e C_{M_k} (\tilde{Z}_{k_e} - \tilde{Z}_{k_w}) \quad (16)$$

The hope here is that the unequal diffusion effects are transferred from the mass-transfer coefficient to the driving potential so that the $\rho_e u_e C_{M_k}$ are the same, or nearly the same, for all elements K in the system. However, the subscript k in the $\rho_e u_e C_{M_k}$ is retained to illustrate that they are not necessarily equal. This form of the equation attains its greatest rigor when convective mass transfer is insignificant, as is the case with Couette flow, for example. For normal boundary-layer flow however, the convective effects are important and it can be argued that the unequal diffusive effects embodied in the $\tilde{Z}_k$ driving potential will be masked to some degree by convective effects. Therefore, an arbitrary definition for the

driving potential was devised based on a weighted average between the two extremes, equal and unequal diffusion coefficients:

$$\tilde{z}_k^* \equiv \sum_{i=1}^N \alpha_{ki} z_i^* \quad (17)$$

where

$$z_i^* \equiv \frac{z_i^\gamma K_i^{1-\gamma}}{\sum_{j=1}^N z_j^\gamma K_j^{1-\gamma}} \quad (18)$$

The defining relation for $\rho_e u_e C_{M_k}$ thus becomes

$$-\tilde{j}_{k_w} = \rho_e u_e C_{M_k} (\tilde{z}_{k_e}^* - \tilde{z}_{k_w}^*) \quad (19)$$

where γ is an arbitrary empirical factor the significance of which has yet to be completely evaluated. It is seen on examination of Equation (17) that $\gamma = 0$ produces a driving potential which is meaningful for equal diffusion problems. A $\gamma = 1.0$ is meaningful when unequal diffusion effects are not masked by convective effects (e.g., Couette flow). A value of $\gamma = 2/3$ was initially suggested in Reference 7. The results of a study of the effect of the γ parameter in correlating the quasisteady graphite ablation data are presented in Section 4.3.

4.2.2 Energy Transport

Attention is now turned to a parallel development for the energy transfer across the boundary layer. This development is unfortunately more obtuse than that for the mass-transfer case and depends quite heavily on intuition.

The form of the film-analog energy-transfer relation used here is an extension of a relation discussed by Lees (Ref. 9) for nonunity Lewis number to include unequal diffusion effects. In this form the energy flux to the wall is described by two terms, the first to account for sensible energy transported to the wall, and the second to account for the chemical energy released within the boundary layer due to chemical reactions occurring there. In the present terminology, the equal diffusion form can be expressed as

$$q_a = \rho_e u_e C_H (H_{s_r} - h_{s_w}) + \rho_e u_e C_M \sum_{i=1}^N (K_{i_e} - K_{i_w}) h_i^o \quad (20)$$

where H_{s_r} is the recovery enthalpy, h_{s_w} is the sensible enthalpy of the wall gas evaluated at wall temperature, h_i^o is the heat of formation of species i , and K_i is the mass fraction of species i .

For unequal diffusion coefficients, it was intuitively felt that meaningful and at least consistent correlations could be obtained by replacing the mass fraction driving potentials with the more general Z driving potential for the reasons similar to those stated in the preceding paragraphs. Such reasoning produced (Ref. 7):

$$q_a = \rho_e u_e C_H (H_{s_r} - h_{s_w}) + \rho_e u_e C_M \sum_{i=1}^N (Z_{i_e}^* - Z_{i_w}^*) h_i^o \quad (21)$$

Note that a single $\rho_e u_e C_M$ appears here whereas, in general, one might expect K values of $\rho_e u_e C_{M_k}$. Thus, the $\rho_e u_e C_{M_k}$ must be nearly equal for the correlation expression to be applicable.

Some question arises in establishing a basis for dividing the energy between the chemical and sensible terms. It was argued in Reference 6 that because the terms in the energy differential equation could be meaningfully grouped when the basis was chosen at a temperature equal to the wall temperature, that such a basis be used for Equation (21). Accordingly, Equation (21) becomes (as shown in Ref. 6)

$$q_a = \rho_e u_e C_H (h_{s_r} - h_{s_w})_{f_e} + \rho_e u_e C_M \sum_{i=1}^N (Z_{i_e}^* - Z_{i_w}^*) h_{i_w} \quad (22)$$

where the subscript f_e designates that both enthalpy terms are evaluated for the edge gas composition and h_{i_w} is the enthalpy of species i evaluated at the wall temperature. Equations (19) and (22) are employed in the CMA/ACE programs and are used in the subsequent sections to correlate the BLIMP data.

4.3 Application of Transfer-Coefficient Formulation to Graphite Boundary-Layer Solutions

The BLIMP and CMA/ACE programs employ the same energy and mass balances at the wall except that the CMA/ACE programs represent the diffusive heat and mass fluxes by transfer coefficients through Equations (19) and (22). In addition, the BLIMP and CMA/ACE programs utilize the same thermodynamic, transport, and material property data. Therefore, in order to establish an exact correspondance between BLIMP and CMA/ACE, it is necessary only to express the diffusive heat and mass fluxes computed by BLIMP in terms of $\rho_e u_e C_M$ and $\rho_e u_e C_H$ calculated by use of Equations (19) and (22). The success of this approach is not a priori evident, however, since it requires the K values of $\rho_e u_e C_{M_k}$ computed using Equation (19) to be constant, or nearly constant, for a given flight condition.

Examination of Equations (17) through (19) reveals that there is a parameter, γ , which can be freely selected to improve

the correlation of the $\rho_e u_e C_{M_k}$. Therefore, the first step in evaluating the success of the transfer-coefficient formulation was to evaluate $\rho_e u_e C_{M_k}$ from Equation (19), which can be expressed as

$$\rho_e u_e C_{M_k} = \frac{(-\tilde{j}_{k_w})}{\tilde{Z}_{k_e}^* - \tilde{Z}_{k_w}^*} \quad (23)$$

for values of γ from zero to unity. The $-\tilde{j}_{k_w}$ are given directly by BLIMP and the $\tilde{Z}_{k_e}^*$ and $\tilde{Z}_{k_w}^*$ are computed using Equations (17), (18) and (13) from the mole fractions at the boundary-layer edge and wall, respectively, also given by BLIMP. The results are presented in Figures A-1 through A-31 of Appendix A for all of the flight conditions listed in Table I.

It can be seen that in nearly all cases there is a value of γ between zero and unity for which the $\rho_e u_e C_{M_k}$ are not only nearly equal but are identically equal.* Furthermore, for the other cases, it appears that there would be a value of γ above unity or below zero where the three $\rho_e u_e C_{M_k}$ would be equal. The preciseness of this result must be more than coincidental and may not persist for systems with more than three elements. There may be an algebraic requirement for 3-element problems which states that if two curves for $\rho_e u_e C_{M_k}$ as a function of γ intersect, that the third $\rho_e u_e C_{M_k}$ must intersect at the same value of γ . (That this must be the case is apparent from the fact that Equation (19) is intended to correlate only first-order unequal diffusion effects, whereas the present BLIMP results contain numerical errors in the boundary-layer solutions for q_a ,

* Although these results were obtained for quasisteady ablation, the intersection points may not depend strongly on surface temperature, unequal diffusion effects being more dependent upon edge conditions, and thus may be applicable in transient problems.

$\tilde{j}_{k_w}$, and wall and edge concentrations, consider second-order unequal diffusion effects (see Ref. (7)), and consider thermal diffusion.) If this is indeed the case, then an exact intersection would not be anticipated when there are four or more elements in the system.

Values of $\rho_{e_e}^u \bar{C}_M$, defined arbitrarily as the arithmetic mean of the $\rho_{e_e}^u C_{M_k}$

$$\rho_{e_e}^u \bar{C}_M = \frac{1}{K} \sum_{k=1}^K \rho_{e_e}^u C_{M_k} \quad (24)$$

are also presented in Figures A-1 through A-31 as a function of γ . It can be seen, in general, that the $\rho_{e_e}^u \bar{C}_M$ is not strongly dependent upon γ .

Given the value of $\rho_{e_e}^u C_M$ at the intersection (or, alternatively, the value of $\rho_{e_e}^u \bar{C}_M$ for some other value of γ), the next step was to compute the $\rho_{e_e}^u C_H$ from Equation (22)

$$\rho_{e_e}^u C_H = \frac{q_a - \rho_{e_e}^u \bar{C}_M \sum_{i=1}^N (Z_{i_e}^* - Z_{i_w}^*) h_{i_w}}{(H_{s_r} - h_{s_w}) f_e} \quad (25)$$

where q_a is given directly by the BLIMP program and the other quantities can be calculated from wall and boundary-layer edge concentrations and species thermodynamic data. The results are presented as a function of γ in Figures B-1 through B-31 of Appendix B. The values for $\rho_{e_e}^u \bar{C}_M$ are also shown in these figures for reference. It is seen that the $\rho_{e_e}^u C_H$ is typically much more strongly dependent upon γ than is the $\rho_{e_e}^u \bar{C}_M$. Part of the reason for this may be due to the arbitrary definition of $\rho_{e_e}^u \bar{C}_M$ (Eq. (24)) which was imposed. If, for example, the system $\rho_{e_e}^u C_M$ were taken to be equal to the $\rho_{e_e}^u C_{M_k}$ for $k = \text{carbon}$,

for example, the effect of γ on $\rho_e u_e C_H$ may be appreciably reduced.

It should not be inferred from these results that the variation of γ for the various flight conditions is entirely representative of the extent to which unequal diffusion effects come into the mass fraction potentials. To the contrary, as mentioned previously, it may be the result of a mathematical identity.

The present results in themselves are not sufficient to determine whether $\gamma = 2/3$ is a good value for representing unequal diffusion,* whether some other value would be better, or whether γ should be considered to vary with flight conditions. In order to resolve this question, one could consider a problem where unequal diffusion effects are much more severe (e.g., hydrogen injection). Alternatively, one could start from nonablating $\rho_e u_e C_{H_0}$ and $\rho_e u_e C_{M_0}$, apply blowing corrections and wall temperature corrections, and look for a logical trend for $\rho_e u_e C_H$ (and thus for γ) to unfold. This latter approach was attempted in the present effort but was abandoned when it was found to be impossible to obtain no-blowing values for $\rho_e u_e C_{M_{k_0}}$ (and hence, through the same approach, $\rho_e u_e \bar{C}_{M_0}$ and $\rho_e u_e C_{H_0}$) with BLIMP. Of course, $\rho_e u_e C_{M_{k_0}}$ is undefined for zero blowing; hence, BLIMP runs were attempted with small blowing rates. As a result of problems apparently associated with small differences of large numbers or some minor error in the formulation of the BLIMP equations, as yet not located, it was not possible to get meaningful values for $\rho_e u_e C_{M_k}$ even for mass injection rates of carbon of as much as 10 percent of the quasisteady ablation values.**

*The bulk of the data do seem to favor a γ between 1/2 and 2/3 or so.

**The quasisteady values are believed to be accurate, however, since the $\rho_e u_e C_{M_k}$ for equal diffusion coefficients were all equal within two or three digits in the fourth significant figure. This problem with BLIMP has been eliminated since the completion of the effort reported herein.

Since exact values of $\rho_e u_e C_M$ can be obtained for most of the flight conditions by considering the intersection γ 's, and since an appropriate value for a single γ to represent unequal diffusion effects for all conditions was not determinable from the present results, $\rho_e u_e C_M$ at the intersection values of γ were selected for presentation in Section 3 (Tables XII and XVI(f)). The reader could choose a constant value of γ (say 1/2 or 2/3) and prepare revised tables of $\rho_e u_e C_M$ and B' .

The intersection values of $\rho_e u_e C_H$, obtained from Figures B-1 through B-31, are tabulated in Table XVII. In order to get some feeling for the nonablating $\rho_e u_e C_{H_0}$ and $\rho_e u_e C_{M_0}$, the blowing correction for equal diffusion coefficients, given by Equation (16), was applied to these $\rho_e u_e C_H$ and $\rho_e u_e C_M$ with the following result: the unequal-diffusion $\rho_e u_e C_{M_0}$ are typically several percent lower (especially at high total enthalpies) than their equal-diffusion counterparts, whereas the unequal-diffusion $\rho_e u_e C_{H_0}$ are considerably higher, again especially at high total enthalpies. Of course, these results, especially the $\rho_e u_e C_H$ and $\rho_e u_e C_{H_0}$ results, are very tenuous. In the first place, the $\rho_e u_e C_H$ are highly dependent upon the choice of γ and, possibly, upon the definition of $\rho_e u_e \bar{C}_M$. Then, there is no real substantiation for applying the equal diffusion blowing correction curve. Clearly, it would be highly desirable to resolve the numerical difficulties encountered when calculating mass-transfer coefficients with BLIMP for small mass injection rates, as this would enable the straightforward calculation of $\rho_e u_e C_{M_{k_0}}$ and $\rho_e u_e C_{H_0}$ as a function of γ for the flight conditions of interest.

4.4 Compatibility Between BLIMP and CMA/ACE Programs

It was recently reported in a previous study (Ref. 10) that quasisteady ablation results predicted by the BLIMP computer program were inconsistent with those predicted by the CMA/ACE computer program when the film transport coefficients determined by the BLIMP program were input into the latter program. The magnitude of the inconsistencies were reported in Reference 10 to vary between 5 percent and almost 30 percent on $\dot{m}_w$, for example. One

TABLE XVII

HEAT AND MASS TRANSFER COEFFICIENTS FOR QUASISTEADY
STAGNATION POINT GRAPHITE ABLATION WITH UNEQUAL
DIFFUSION CONSISTENT WITH AEROTHERM TRANSFER-COEFFICIENT MODEL

Total Enthalpy, H_T (Btu/lb)	Total Pressure, P_{T2} (atm)	γ_{int} (Value of γ to yield single $\rho_e u_e C_M$)	*Mass-Transfer Coefficient, (lb/sec-ft ²)	*Heat-Transfer Coefficient, (lb/sec ft ²)
13,000	0.01	0.736	0.0143	0.0148
13,000	0.1	$\gamma_{int} > 1.0$	0.0444	0.0448
13,000	1	0.562	0.1131	0.209
13,000	10	0.165	0.3217	0.591
13,000	30	0.022	0.540	0.987
13,000	100	$\gamma_{int} < 0$	0.962	1.710
11,500	0.01	0.514	0.0143	0.1537
11,500	0.1	0.822	0.0441	0.0487
11,500	1	0.401	0.1216	0.189
11,500	10	0.242	0.3367	0.593
11,500	30	0.152	0.566	0.982
11,500	100	0.003	1.007	1.750
10,000	0.01	0.510	0.01425	0.0149
10,000	0.1	0.578	0.0446	0.0491
10,000	1	0.316	0.1336	0.1304
10,000	10	0.300	0.353	0.590
10,000	30	0.228	0.595	0.980
10,000	100	0.131	1.059	1.732
8,000	1	0.368	0.1427	0.1343
8,000	10	0.371	0.379	0.574
8,000	30	0.310	0.638	0.969
8,000	100	0.239	1.135	1.726
6,000	10	0.440	0.419	0.502
6,000	30	0.402	0.690	0.931
6,000	100	0.353	1.222	1.697
4,000	10	0.647	0.430	0.470
4,000	30	0.471	0.753	0.703
4,000	100	0.528	1.319	1.590
2,000	10	$\gamma_{int} > 1.0$	0.428	0.499
2,000	30	$\gamma_{int} > 1.0$	0.743	1.025
2,000	100	$\gamma_{int} > 1.0$	1.350	3.90

*When $\gamma_{int} > 1.0$, $\gamma = 1.0$ values were used
When $\gamma_{int} < 0$, $\gamma = 0.0$ values were used

of the main aspects of this study was to resolve this inconsistency. In the present section, the discrepancy is resolved and the reason for the inconsistencies in the previous study is discussed.

The basis for the difficulty was that the $\rho_e^u C_M$ and $\rho_e^u C_H$ computed in BLIMP are different than the definitions utilized by CMA/ACE and these differences were not properly taken into account. In BLIMP, the mass-transfer coefficients are determined a posteriori from:

$$\rho_e^u C_{M_k} = \frac{-\tilde{j}_{k_w}}{\tilde{K}_{k_e} - \tilde{K}_{k_w}} \quad (26)$$

where the elemental mass-fraction potentials are used instead of the $\tilde{Z}_k^*$ potentials. Now, of course, Equation (26) is accurate for equal diffusion problem solutions. Note in Equation (26) the subscript k on C_M which connotes the fact that in general a different coefficient value is found for each element.

For the energy coefficient, the following equation is employed:

$$\rho_e^u C_H = \frac{q_a}{H_{T_e} - H_{T_w}} \quad (27)$$

Again, looking at Equation (25) it can be seen that this equation is precise for a solution where both the diffusion coefficients are equal and, more importantly, where $C_M = C_H$. For unequal diffusion solutions, relations (26) and (27) are obviously inaccurate compared to the $\tilde{Z}_k^*$ potential form of these equations (Eqs. (19) and (22)) which, and this must be emphasized, is the form used in the CMA/ACE programs. Moreover, for the conditions presented in Reference 10, C_M does not equal C_H . For these reasons then, it is understandable that agreement was not obtained between the two programs.

It was stated in Section 4.3 that the BLIMP and CMA/ACE programs should be completely compatible if Equations (19) and (22) are employed and if the $\rho_e u_e C_{M_k}$ can be reduced to a single $\rho_e u_e C_M$ through the selection of an appropriate γ in Equation (19). It was further shown that it is possible to select a γ for a given flight condition which leads to a single $\rho_e u_e C_M$, although the appropriate values of γ vary for the different flight conditions considered. Thus, one would expect complete compatibility between the BLIMP and CMA/ACE programs.

The values for the average mass-transfer coefficient, $\rho_e u_e \bar{C}_M$, and the corresponding values for the heat-transfer coefficient were input to the CMA/ACE program for two quasisteady graphite stagnation-point solutions. The $\rho_e u_e \bar{C}_M$ and $\rho_e u_e C_H$ were applied directly, that is, with no blowing corrections. The ablation response predicted by the CMA/ACE programs agreed very well with the BLIMP predictions for the intersection γ 's as shown in Table XVIII. This satisfying result eliminates the possibility that either theoretical or programming errors exist in either program (unless the same error exists in both codes). In conclusion, the two programs can be made self-consistent by modifying the coefficient determination technique in the BLIMP program. For example, one could present as BLIMP output, the $\rho_e u_e C_{M_k}$, $\rho_e u_e \bar{C}_M$, and $\rho_e u_e C_H$ as obtained by Equations (23) through (25) for several values of γ .

TABLE XVIII

COMPARISON OF BLIMP AND CMA/ACE PREDICTIONS
FOR QUASISTEADY GRAPHITE STAGNATION-POINT ABLATION

$H_T = 6,000 \text{ Btu/lb}, P_{T_2} = 30 \text{ atm}$						
γ	$\rho_{e^u e} \bar{C}_M$	$\rho_{e^u e} C_H$	Surface Temperature T_w ($^{\circ}\text{R}$)		Ablation Rate, $\dot{m}_w$ (lb/sec-ft ³)	
			CMA/ACE*	BLIMP	CMA/ACE*	BLIMP
0	0.731	0.999	7,708	7,721	0.254	0.248
0.33333	0.694	0.941	7,719		0.248	
0.66667	0.675	0.868	7,726		0.248	
1.0	0.670	0.781	7,739		0.256	
* γ at crossover = 0.40						

$H_T = 11,500 \text{ Btu/lb}, P_{T_2} = 100 \text{ atm}$						
γ	$\rho_e u_e \bar{C}_M$	$\rho_e u_e C_H$	Surface Temperature, T_w ($^{\circ}\text{R}$)		Ablation Rate, $\dot{m}_w$ (lb/sec-ft ²)	
			CMA/ACE*	BLIMP	CMA/ACE*	BLIMP
0	1.01	1.75	8,647	8,647	0.815	0.815
0.33333	1.01	1.62	8,650		0.815	
0.66667	1.02	1.49	8,656		0.820	
1.0	1.02	1.36	8,676		0.855	

* γ at crossover = 0

5. PROCEDURE FOR APPLYING QUASISTEADY GRAPHITE ABLATION RESULTS TO TRANSIENT PROBLEMS

Although the quasisteady graphite ablation results presented in Section 3 are useful for obtaining preliminary estimates of ablation rates and surface temperatures during ballistic reentry trajectories, an ultimate objective of the present effort was to develop procedures for using convective transfer coefficients with the CMA/ACE programs while maintaining the accurate representation of the boundary layer provided by the BLIMP program.

With regard to predicting the transient response of graphite at the stagnation point, the results presented in Sections 3.2 and 4, when considered in total, lead to the conclusion that it is better to consider equal diffusion coefficients. Corrections for unequal diffusion coefficients can then be applied to the results if desired. The recommended procedure is as follows.

There are two preliminary steps required for best results, in particular, options should be built into CMA which would enable 1) table lookup in terms of surface temperature, total pressure, and total enthalpy of the equal diffusion $\rho_e u_e C_{H_2O} / (P_{T2}/R_{eff})^{1/2}$ presented in Figure 2, and 2) use of the blowing correction given by Equation (6). With these options available, transient calculations could be performed by the following steps:

1. Calculate ACE tables for the range of reentry conditions of interest considering equal diffusion coefficients ($\gamma = 0$).
2. Perform CMA transient solution with equal diffusion, $\rho_e u_e C_M$ equal to $\rho_e u_e C_H$ and using the new options for calculating $\rho_e u_e C_{H_2O}$ at the ablation temperature and applying the blowing correction of Equation (6).
3. Correct these results for ablation rates and surface temperatures for unequal diffusion by applying corrections presented in Figure 4 and in Table XII, respectively.

There is a high degree of confidence in the accuracy of this approach, primarily since the ratio of $\rho_e u_e C_H$ to $\rho_e u_e C_{H_0}$ at the ablation temperature collapsed so well to a single curve for all flight conditions considered, thus suggesting that surface temperature is not a significant parameter in the blowing correction. The correction to include the effects of unequal diffusion coefficients is somewhat more tenuous, however, since it is not known whether the effects are primarily a function of flight conditions (as is implied in their direct application as a correction factor) or whether they would be significantly influenced by surface temperature.

One could also perform transient calculations for unequal diffusion coefficients directly using the BLIMP data presented in this report, but this is not recommended since the appropriate blowing correction is not known, let alone its dependence upon flight conditions and/or wall temperature; or, looking at the problem from another point of view, the nonablating $\rho_e u_e C_{H_0}$ and $\rho_e u_e C_{M_0}$ are not known. Thus, under transient conditions, where the surface temperature will in general differ from the quasisteady values, there is no accurate way for correcting the ablating values of $\rho_e u_e C_M$ and $\rho_e u_e C_H$ for surface temperature (and, hence, for B').

The scope of the effort did not permit the completion of general correlation procedures for nonsimilar solutions for positions back on the body. However, sufficient information is presented in Sections 3.3 and 4 to enable the reader to at least anticipate the types of problems which arise when applying convective transfer-coefficient methods to nonsimilar ablation problems. In particular, when substantial sublimation occurs in the stagnation region, he must obtain either $\rho_e u_e C_M$ or B' from a correlation of the data that is presented rather than by conventional methods, and that this parameter may even go negative shortly downstream of the sphere-cone juncture (implying carbon deposition).

6. SUMMARY OF CONCLUSIONS AND RECOMMENDATIONS

A matrix of solutions have been generated with the Boundary Layer Integral Matrix Procedure (BLIMP) program and are presented for nonsimilar air boundary layers and for nonsimilar boundary layers over ablating graphite for a range of flight conditions corresponding to moderate-to-severe ballistic reentry. As a point of reference, the graphite solutions have been obtained considering quasisteady conduction into the solid, the primary interest being application to the prediction of graphite ablation under transient conditions. However, it was seen that the quasisteady assumption is quite good during regions of peak heating and the results are directly applicable for estimating graphite ablation rates and surface temperatures under those conditions.

Correlation procedures for performing transient reentry predictions from these results using the Aerotherm Charring Material Ablation (CMA) and Aerotherm Chemical Equilibrium (ACE) programs are presented for the stagnation point. The approach which is presented in detail considers equal diffusion coefficients and then corrects the results for unequal diffusion effects, these effects actually being relatively small for graphite ablation. The application of this approach to transient problems should be quite successful, due primarily to the fact that a very simple relation was derived for a blowing reduction correction which was based on results from a wide range of total enthalpies and total pressures but did not depend upon these parameters and, hence, can be presumed not to depend on surface temperature.

A second approach is considered which utilizes BLIMP quasisteady unequal-diffusion-coefficient results directly. However, its use is not recommended since there are more uncertainties associated with the approach and, therefore, more chances for error when applied to transient problems.

In the development of the correlation procedure for unequal diffusion coefficients, it was found possible to express

elemental mass fluxes in terms of a single mass-transfer coefficient. In so doing, this represents, to some extent, a validity check on the transfer-coefficient correlation approach for unequal diffusion coefficients proposed in previous publications and used in the CMA/ACE programs (but never substantiated by comparison to boundary-layer solutions). Also, by expressing the diffusive heat and mass fluxes resulting from the boundary-layer (BLIMP) calculations in precisely the same formulation utilized in CMA/ACE (this being possible only because a single mass-transfer coefficient can be defined), an exact duplication of quasisteady ablation rates and surface temperatures were made with the BLIMP and CMA/ACE approaches.

Although a number of nonsimilar boundary-layer solutions were generated for stations around the body, the level of effort was not sufficient to permit the development of correlation procedures to enable one to use the CMA/ACE approach to duplicate BLIMP results. With regard to the nonsimilar boundary-layer solutions, a significant nonsimilar effect was predicted to occur. In particular, under reentry conditions sufficiently severe such that sublimation occurs in the stagnation region, ablation rates downstream of the sphere-cone juncture are reduced well below that which would be predicted on the basis of a similar boundary-layer procedure, even becoming negative (i.e., deposition) in the most severe cases.

Recommendations for further effort fall into three areas: 1) application of the present results and/or computational procedures, 2) completion of tasks which were only partially completed or which were suggested by the present effort, and 3) extension of the computational procedures to improved physical models and/or numerical solutions. Briefly, it is recommended that:

1. The changes be made in CMA which are recommended in Section 4 which will permit calculation of transient graphite ablation at the stagnation point considering equal diffusion

coefficients consistent with the BLIMP results. This computational approach should then be exercised and the results compared to BLIMP solutions with assigned values of heat conduction as obtained by CMA/ACE.

2. The problem encountered with BLIMP which makes it impossible to compute meaningful values of $\rho_e u_e C_{H_2O}$ and $\rho_e u_e C_{M_O}$ for unequal diffusion coefficients should be eliminated. With this accomplished, blowing corrections for $\rho_e u_e C_H$ for unequal diffusion could be better defined and the procedure for predicting transient graphite ablation rates at the stagnation point with unequal diffusion coefficients could be completed.

3. An enlarged matrix of nonsimilar boundary-layer solutions around graphite sphere cones should be conducted to study separately the effects of parameters such as dP/ds , dT_w/ds and $d\dot{m}_w/ds$ and cone angle on boundary-layer parameters of interest. It would also be desirable to include entropy layer and surface nonequilibrium effects into these calculations. The results could then be correlated into engineering design procedures for graphite sphere-cones.

4. In order to obtain a better understanding of the validity of the transfer-coefficient model utilized in this study, it would be desirable to perform studies for a chemical system with more than three elements to see if all $\rho_e u_e C_{M_k}$ curves still intersect at the same point for a given flight condition, and to study a problem where unequal diffusion effects are more important than they are for carbon so that physically meaningful values of γ could be determined.

5. The BLIMP code could be modified to include functionally dependent fail temperatures to enable the study of experimental carbon or carbon-phenolic ablation data to study thermomechanical ablation mechanisms.

6. The BLIMP program could be utilized to correlate transition data in terms of laminar boundary-layer parameters for actual flight vehicles.

7. There are two additional major program extensions which should be performed for application to the present problem: the BLIMP program should be extended to turbulent flow and a procedure should be developed which considers multidimensional conduction into the nose tip. Effort has been initiated in both of these areas under separate contracts.

APPENDIX A

ELEMENTAL MASS TRANSFER COEFFICIENTS VERSUS γ FOR QUASISTEADY GRAPHITE STAGNATION-POINT ABLATION

Symbols

$$\times \quad \rho_e^u C_{M_k}, \quad k = \text{carbon}$$

$$\nabla \quad \rho_e^u C_{M_k}, \quad k = \text{oxygen}$$

$$\triangle \quad \rho_e^u C_{M_k}, \quad k = \text{nitrogen}$$

$$\triangleleft \quad \rho_e^u \bar{C}_M \equiv \frac{1}{K} \sum_{k=1}^K \rho_e^u C_{M_k}$$

Flight Conditions

<u>Total Enthalpy, Btu/lb</u>	<u>Total Pressure, atm</u>	<u>Total Enthalpy, Btu/lb</u>	<u>Total Pressure, atm</u>
13,000	0.01	8,000	1
13,000	0.1	8,000	10
13,000	1	8,000	30
13,000	10	8,000	100
13,000	30	6,000	10
13,000	100	6,000	30
11,500	0.01	6,000	100
11,500	0.1	4,000	10
11,500	1	4,000	30
11,500	10	4,000	100
11,500	30	2,000	10
11,500	100	2,000	30
10,000	0.01	2,000	100
10,000	0.1		
10,000	1		
10,000	10		
10,000	30		
10,000	100

Figure A-1. Enthalpy 13000 Btu/lb, Pres .01 ATM

Figure A-2. Enthalpy 13000 BTU/lb, Pres 0.1 ATM

Figure A-3. Enthalpy 13000 BTU/lb, Pres 1 ATM

Figure A-4. Enthalpy 13000 BTU/lb, Pres 10 ATM

Figure A-5. Enthalpy 13000 BTU/lb, Pres 30 ATM

Figure A-6. Enthalpy 13000 BTU/lb, Pres 100 ATM

Figure A-7. Enthalpy 11500 BTU/lb, Pres .01 ATM

Figure A-8. Enthalpy 11500 BTU/lb, Pres 0.1 ATM

Figure A-9. Enthalpy 11500 BTU/lb, Pres 1 ATM

Figure A-10. Enthalpy 11500 BTU/lb, Pres 10 ATM

Figure A-11. Enthalpy 11500 BTU/lb, Pres 30 ATM

Figure A-12. Enthalpy 11500 BTU/lb, Pres 100 ATM

Figure A-13. Enthalpy 10000 BTU/lb, Pres .01 ATM

Figure A-14. Enthalpy 10000 BTU/lb, Pres 0.1 ATM.

Figure A-15. Enthalpy 10000 BTU/lb, Pres 1 ATM

Figure A-16. Enthalpy 10000 BTU/lb, Pres 10 ATM

Figure A-17. Enthalpy 10000 BTU/lb, Pres 30 ATM

Figure A-18. Enthalpy 10000 BTU/lb, Pres 100 ATM

Figure A-19. Enthalpy 8000 BTU/lb, Pres 1 ATM

Figure A-20. Enthalpy 8000 BTU/lb, Pres 10 ATM

Figure A-21. Enthalpy 8000 BTU/lb, Pres 30 ATM

Figure A-22. Enthalpy 8000 BTU/lb, Pres 100 ATM

Figure A-23. Enthalpy 6000 BTU/lb, Pres 10 ATM

Figure A-24. Enthalpy 6000 BTU/lb, Pres 30 ATM

Figure A-25. Enthalpy 6000 BTU/lb, Pres 100 ATM

Figure A-26. Enthalpy 4000 BTU/lb, Pres 10 ATM

Figure A-27. Enthalpy 4000 BTU/lb, Pres 30 ATM

Figure A-28. Enthalpy 4000 BTU/lb, Pres 100 ATM

Figure A-29. Enthalpy 2000 BTU/lb, Pres 10 ATM

Figure A-30. Enthalpy 2000 BTU/lb, Pres 30 ATM

Figure A-31. Enthalpy 2000 BTU/lb, Pres 100 ATM

APPENDIX B

HEAT AND MASS TRANSFER COEFFICIENTS VERSUS γ FOR QUASISTEADY GRAPHITE STAGNATION-POINT ABLATION

Symbols

$$\times \rho_e u_e \bar{C}_M$$

$$\Delta \rho_e u_e C_H$$

Flight Conditions

<u>Total Enthalpy, Btu/lb</u>	<u>Total Pressure, atm</u>	<u>Total Enthalpy, Btu/lb</u>	<u>Total Pressure, atm</u>
13,000	0.01	8,000	1
13,000	0.1	8,000	10
13,000	1	8,000	30
13,000	10	8,000	100
13,000	30	6,000	10
13,000	100	6,000	30
11,500	0.01	6,000	100
11,500	0.1	4,000	10
11,500	1	4,000	30
11,500	10	4,000	100
11,500	30	2,000	10
11,500	100	2,000	30
10,000	0.01	2,000	100
10,000	0.1		
10,000	1		
10,000	10		
10,000	30		
10,000	100

Figure B-1. Enthalpy 13000 BTU/lb, Pres .01 ATM

Figure B-2. Enthalpy 13000 BTU/lb, Pres 0.1 ATM

Figure B-3. Enthalpy 13000 BTU/lb, Pres 1 ATM

Figure B-4. Enthalpy 13000 BTU/lb, Pres 10 ATM

Figure B-5. Enthalpy 13000 BTU/lb, Pres 30 ATM

Figure B-6. Enthalpy 13000 BTU/lb, Pres 100 ATM

Figure B-7. Enthalpy 11500 BTU/lb, Pres .01 ATM

Figure B-8. Enthalpy 11500 BTU/lb, Pres 0.1 ATM

Figure B-9. Enthalpy 11500 BTU/lb, Pres 1 ATM

Figure B-10. Enthalpy 11500 BTU/lb, Pres. 10 ATM

Figure B-11. Enthalpy 11500 BTU/lb, Pres 30 ATM

Figure B-12. Enthalpy 11500 BTU/lb, Pres 100 ATM

Figure B-13. Enthalpy 10000 BTU/lb, Pres .01 ATM

Figure B-14. Enthalpy 10000 BTU/lb, Pres 0.1 ATM

Figure B-15. Enthalpy 10000 BTU/lb, Pres 1 ATM

Figure B-16. . Enthalpy 10000 BTU/lb, Pres 10 ATM

Figure B-17. Enthalpy 10000 BTU/lb, Pres 30 ATM

Figure B-18. Enthalpy 10000 BTU/lb, Pres 100 ATM

Figure B-19. Enthalpy 8000 BTU/lb, Pres 1 ATM

Figure B-20. Enthalpy 8000 BTU/lb, Pres 10 ATM

Figure B-21. Enthalpy 8000 BTU/lb, Pres 30 ATM

Figure B-22. Enthalpy 8000 BTU/lb, Pres 100 ATM

Figure B-23. Enthalpy 6000 BTU/lb, Pres 10 ATM

Figure B-24. Enthalpy 6000 BTU/lb, Pres 30 ATM

Figure B-25. Enthalpy 6000 BTU/lb, Pres 100 ATM

Figure B-26. Enthalpy 4000 BTU/lb, Pres 10 ATM

Figure B-27. Enthalpy 4000 BTU/lb, Pres 30 ATM

Figure B-28. Enthalpy 4000 BTU/lb, Pres. 100 ATM

Figure B-29. Enthalpy 2000 BTU/lb, Pres 10 ATM

Figure B-30. Enthalpy 2000 BTU/lb, Pres 30 ATM

Figure B-31. Enthalpy 2000 BTU/lb, Pres 100 ATM

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