Evolution of Pre-Collective Nuclei: Structural Signatures Near the Drip Lines

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Abstract

Recent studies have shown that the phenomenology of single-magic and near-magic nuclei has universal characteristics analogous to those of collective nuclei and that, moreover, this phenomenology attaches smoothly to that describing collective nuclei. This has led to a number of new signatures of structure as well as to a new, tripartite, classification of nuclear structure that embraces the gamut of structures from magic, through pre-collective, to fully collective and rotational nuclei. Aside from the natural appeal of simple global correlations of collective observables, these results have particular significance for soon-to-be accessible exotic nuclei near the drip lines since they rely on only the simplest-to-obtain data, in particular, the energies of just the first two excited states, $E(4_1^+)$ and $E(2_1^+)$, of even-even nuclei, and the $B(E2:2_1^+ \rightarrow 0_1^+)$ value. Indeed, without the need for more extensive level schemes, these basic data alone can reveal information about the goodness of seniority, about the validity of pair-addition mode relationships of adjacent even-even nuclei, about underlying shell structure (validity of magic numbers) and even about the shell model potential itself (e.g., the strengths of the $l \cdot s$ and l^2 terms).

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I. Introduction

The advent of a variety of radioactive nuclear beam (RNB) facilities worldwide in the next decade will open up entirely new horizons for nuclear structure. It will be interesting to study new examples of the kinds of structure familiar from known nuclei. However, the real excitement of exotic nuclei far from stability centers on the possibility that radically new nuclear properties will be encountered, unlike any known to date. This is especially likely near the neutron drip line.

These speculations are based on a number of factors and are backed by important recent calculations.¹ Consider the schematic single-particle levels in fig. 1. Near the neutron drip it is likely that the residual p-n interaction will be quite different than it is nearer stability because the large neutron excess implies that protons and neutrons will be filling rather different orbits. Since it is this interaction which is primarily responsible for configuration mixing, deformation and the development of collectivity in nuclei, it is reasonable to expect that such facets of structure might also be quite different.

Likewise, the pairing interaction--which is the main competitor to the p-n interaction in determining structure--will almost certainly have quite different manifestations. The pairing interaction normally scatters pairs of particles amongst the single-particle levels near the Fermi surface. When some of these levels are unbound, as occurs near the drip line, significant changes are expected.

Occupation of unbound or near-unbound levels could lead to a type of neutron skin or low density neutron gas surrounding a less neutron-rich core. There have already been speculations of possible core-skin collective modes such as mini-giant resonance oscillations of one against the other.² There are analogies (but also differences) between this skin region and the neutron halos that have recently been so extensively studied in light neutron-rich nuclei.

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Near the drip line the quantum mechanics necessarily will involve the interaction of bound and resonance or continuum states, and this should lead to single-particle and collective phenomena qualitatively different than those near stability. Finally, as single-particle levels merge into the continuum, the traditional symmetries of shell structure--SU(3) in light nuclei, pseudo-SU(3) in heavy nuclei--may lose relevance. There are two reasons for this. First, the sequencing and spacing of orbits will be different and, secondly, some orbits will be lost to the continuum, leading to the loss of a key characteristic of the shell structure of normal parity orbits near stability, namely the presence of all j values from $1/2$ to some maximum value.

Some of these ideas have taken concrete form in some important recent calculations^{1,3} which have presented some enticing results of relativistic mean-field calculations near the neutron drip line. These calculations suggest, for example, that even the shell-model central potential itself may not be the same. Specifically, they suggest that the l^2 term may be absent or significantly smaller than near stability. While this specific prediction may or may not be substantiated, it indicates the kind of effects that might come into play. The consequences of such effects are hard to know in detail at this time--ref. 3 itself warns of the dangers of using conventional calculational approaches in unconventional territory. But, it is equally hard to overestimate the importance of refs. 1,3. For example, the absence of an l^2 term, even viewed in a conventional shell model framework, will certainly alter the magic numbers. More than this, it will alter the familiar sequencing of shell model orbits that proceeds more or less smoothly from high j values early in a shell to low j at the top. Rather, the low j orbits will be concentrated near mid-shell. In addition, in medium and heavy mass nuclei, there would be no unique parity orbit and the myriad effects of this orbit at both low and high spin would therefore vanish.

These provocative efforts^{1,3} to probe the structure of medium and heavy drip line nuclei have set the stage for high expectations of exotic phenomena. At the same time, the results they embody will hardly be the last word. Other calculations⁴ already have reached somewhat different conclusions. The key point is that all the novel ingredients are at hand near the neutron drip line to foster new structural phenomena. Hopefully, these early forays into unknown territory will engender a greatly expanded theoretical effort to provide realistic predictions for the new nuclear regions that will soon be accessible, both to test our ideas of structure and to serve as preliminary guidelines for the choices of key experiments.

It is the purpose of this paper to take off from these kinds of speculations and to look at some of the possible experimental signatures of radically new structure. While the present results will be based as much as possible on the behavior of known nuclei and on the predictions of standard models, everything said above urges caution in blindly accepting the conclusions. Rather, we hope that the present results may encourage better and more sophisticated studies of the *empirical* consequences of the type of study Nazarewicz and colleagues have initiated. We will illustrate some of these ideas with predictions based on the specific suggestion of ref. 1, but the qualitative results we discuss are not wedded to any specific prediction. The focus will be on singly magic nuclei or those near closed shells, but occasional comments on collective and deformed nuclei will be added.

II. Near-Magic Nuclei

A. Basic Concepts

The standard approach to nuclei near closed shells is the shell model with short range and attractive residual interactions. Due to the relative weakness of the nuclear force, deeply bound nucleons are generally not excited to open shells in nucleon collisions/interactions. Therefore, the principal determinants of structure are the valence nucleons and the interactions amongst them. The residual valence

$T=0$ p-n interaction is the most important factor in the onset and development of collectivity and deformation but plays a lesser role near closed shells and, to first order, none at all in magic nuclei. For simplicity in most of what follows, we will deal with singly magic nuclei, and hence we will focus on the like nucleon interaction. In $J=0^+$ states this exhibits the pairing property to good approximation, but other aspects are extremely important for $J \neq 0^+$ states.

To start, we summarize two key properties of like-nucleon systems interacting with short-range attractive forces, namely the spectrum of a 2-particle configuration in a single j -orbit, and the consequences of the seniority scheme. We approximate the residual interaction with a surface δ force. Results with realistic interactions will differ in detail, but the key physics is almost always discernable with a δ interaction--at least in nuclei familiar to us; whether this persists near the drip lines is an interesting question. Consider the configuration of a pair of nucleons in an orbit j , $|j^2J\rangle$, where $J=0, 2, \dots, 2j-1$. Regardless of the value of j , the same type of level spectrum results. The 0^+ state in which the orbits of the two particles are co-planar lies lowest, by far, due to the strong attractive interaction when the two particles are, on average, closest to each other. The angles between the angular momentum vectors of the particles in the higher j states increase, hence their average internucleon distance increases, and the effect of the δ interaction decreases. [The interaction is weakest for the $J_{\max}$ state even though the orbits are again nearly co-planar because of the effects of total anti-symmetrization arising from the Pauli Principle.] The resultant $|j^2J\rangle$ spectrum is as shown in fig. 2. It is nearly independent of j . The key quantity

$$R_{4/2} \equiv E(4_1^+)/E(2_1^+) \quad (1)$$

is in the range 1.15-1.20 for all j values encountered.

Two extensions of these ideas to more than two particles and more than one j value allow the treatment of many realistic nuclei near closed shells. The first extension centers around the seniority concept, the second around configuration mixing. [Note that the latter is tantamount to, and essential for, deformation since it inherently involves an uneven distribution of m -substates.] The seniority scheme and its ramifications are fundamental to much of nuclear structure. Its essentials can even be discerned in the highly broken seniority environment of deformed nuclei. Basically, it provides a simple method to deduce, predict, and understand the properties of n -body systems from those of fewer-body systems (in practice, usually in terms of 2-body systems).

There are two particular results of the seniority scheme that are critical to our discussion. It is simple to show^{5,6}, in a j^n configuration with good seniority, v , that the excitation energies of all states are constant, independent of n . That is, the excitation energies of states $|j_v^n J\rangle$ are identical to those of $|j_v^v J\rangle$. In particular, for seniority $v=2$

$$E(j_{v=2}^n J) = E(j_{v=2}^2 J) \quad (2)$$

This is illustrated in fig. 3. The classic empirical example of this, made justly famous by Talmi, is the Sn isotopes, shown in fig. 4.

Another key observable that acts as a signature of structure is the $B(E2)$ value to the first 2^+ state, $B(E2:2_1^+ \rightarrow 0_1^+)$. [Since we will nearly always focus on this particular $B(E2)$ below, we will hereafter usually abbreviate it to just $B(E2)$.] The seniority scheme predicts a very specific n -dependence of $B(E2)$ values in j^n configurations. Taking the 2_1^+ state as having $v=2$, we have

$$B(E2:2_1^+ \rightarrow 0_1^+)_n = \frac{n(2j+1-n)}{2(2j-1)} B(E2:2_1^+ \rightarrow 0_1^+)_2 \quad (3)$$

Therefore, a good approximation to the n -dependence of the $B(E2)$ values is⁶

$$B(E2:2_1^+ \rightarrow 0_1^+)_n \propto f(1-f) \quad (4)$$

where $f = \frac{n}{2j+1}$ is the fractional occupation of the shell. This bell-shaped dependence peaking at mid-shell is illustrated in fig. 5. The qualitatively different behavior of energies and $B(E2)$ values ultimately originates in the properties of odd and even tensor operators in 2-particle systems.

B. Comparison with Data: Global Phenomenology

The comparison of eqs. 2 and 4 with the data is interesting and provides clues to what may happen far from stability. Although the constancy of 2_1^+ and 4_1^+ energies in Sn is well known, it is not the general case. This is illustrated in figs. 6a,b which shows $E(2_1^+)$, $E(4_1^+)$ and their energy difference

$$\Delta E_{4-2} \equiv E(4_1^+) - E(2_1^+) \quad (5)$$

for singly magic nuclei. These data reveal an interesting behavior. While, in general, neither $E(2_1^+)$ nor $E(4_1^+)$ is constant, they vary together so that their *energy difference* ΔE_{4-2} is *nearly constant*. An alternate form of eq. 5 with constant ΔE_{4-2} is

$$E(4_1^+) = E(2_1^+) + \text{constant}. \quad (6)$$

That is, $E(4_1^+)$ should be linear *against* $E(2_1^+)$ with a slope of unity. This is in fact the case as fig. 7 shows for several mass regions. The data are consistent with slopes very close to unity, except in the actinides. This phenomenology is incompatible with the seniority scheme. Note that an explanation of this does not simply

involve extension to a multi- j environment. For example, several degenerate j orbits interacting with an odd tensor interaction (e.g., a δ force) will have good generalized seniority in which the usual seniority predictions are retained.

To attempt to understand the empirical results of fig. 6a,b we have carried out simple shell model calculations with a surface δ interaction. The results⁷ for some of the neutron magic numbers are shown in fig. 6c and at the bottom of fig. 7. The calculations employed a variety of single particle energies, interaction strengths and orbits. Yet all lead to the same result—a slope of ~ 1.0 in eq. 6 corresponding to nearly constant ΔE_{4-2} . The same result emerges from realistic shell model calculations in medium and heavy nuclei.⁸

Before we interpret these results, we consider the $B(E2)$ values. Figure 8a shows $B(E2)$ values against A for singly magic nuclei. No obvious pattern emerges. However, fig. 8b shows the same data plotted against the fractional shell filling $n/(2j+1)$ where in this case $2j+1$ is taken as the full major shell degeneracy. Superposed on the data is the prediction of eq. 4. The agreement is excellent.

This pair of empirical results, figs. 6a,b and fig. 8b, seem contradictory: the energies point to a breakdown of seniority, but the $B(E2)$ values increase with shell filling in a manner expected in the seniority scheme.

It is possible to understand this seemingly contradictory situation by a schematic two- j orbit calculation that isolates the essential physics behind the calculations shown in figs. 6 and 7. We use an $h_{9/2}-f_{7/2}$ system and a surface δ interaction. As in figs. 6 and 7, the yrast energies vary, but ΔE_{4-2} is quite constant. Table 1 shows the major components of the calculated wave functions (in the column labeled "exact"). Analysis of these wave functions shows that seniority and generalized seniority are indeed broken but that a simple pair addition behavior retains validity. In this picture a state of spin J in a singly magic nucleus with n valence nucleons is constructed by adding a pair of nucleons coupled to 0^+ to the

state of the same spin in the nucleus with $n-2$ particles. Moreover, this 2-particle 0^+ state has the same correlated wave function as the 0^+ ground state of the two-valence nucleon nucleus. Given the wave functions for different J values in the $n=2$ nucleus, those for all others are obtained merely by the successive addition of 0^+ -coupled pairs. These pair wave functions are easily constructed from the $n=2$ wave functions in the table and are given in the "pair" column. The agreement with the exact wave functions is excellent. The energies $E(4_1^+)$ and $E(2_1^+)$ can vary with n in such a scheme but their energy difference remains constant (i.e., they vary in energy together) since both are constructed by addition of the same 0^+ -coupled 2-particle wave function. [Their excitation energies vary since the ground state binding energy is affected by the indistinguishability of its 0^+ -coupled pair constituents.]

In contrast to the energies, a calculation of the $B(E2)$ values in the pair addition scheme does *not* lead to results qualitatively different from the seniority scheme of fig. 8b. The $B(E2)$ values increase with n in the pair addition scheme as well. Indeed, it is difficult to concoct a model in which the $B(E2)$ values do not increase with valence nucleon number. The model differences show up in different functional forms for that increase. For example, simple calculation with eq. 3 for $j=13/2$ gives $B(E2)_{n=4} = 1.67 B(E2)_{n=2}$ while the pair addition result is a factor of 2, which is almost the same.

Thus the energies of the 2_1^+ and 4_1^+ states, perhaps combined with $B(E2)$ values, are a sensitive indicator of structure. Constant energies suggest a good seniority description. Varying energies with constant energy difference seem to point to a pair addition scheme of configuration-mixed states. A breakdown of this scheme should lead to $E(2_1^+)$, $E(4_1^+)$, and ΔE_{4-2} values that vary with n .

For singly magic nuclei, it may be possible to glean some ideas of the number of valence nucleons of the active type from the $B(E2)$ values by comparison with the

fractional filling plot of fig. 8. Early on in a shell, the $B(E2)$ values should increase roughly linearly with active valence nucleon number, while near mid-shell, they should be approximately flat.

In fig. 7, although the slope of $E(4_1^+)$ against $E(2_1^+)$ is constant, the ratio $R_{4/2}$ is continually changing, as is clear by rewriting eq. 6 as

$$R_{4/2} = E(4_1^+)/E(2_1^+) = 1.0 + \frac{\text{constant}}{E(2_1^+)} \quad (7)$$

An approximate $R_{4/2}$ scale is shown along the top in fig. 7. As $E(2_1^+)$ decreases in fig. 7, $R_{4/2}$ approaches the regime of collective values of 2 or more. An interesting aspect of fig. 7 is related to these $R_{4/2}$ values. Even though the slopes for different mass regions (except the actinides) are the same (unity), and even though the intercepts are nearly the same (~ 600 keV), $R_{4/2}$ values for nuclei in the different regions are not identical, especially for magic nuclei. This occurs because $R_{4/2}$ is correlated with $E(2_1^+)$ --compare top and bottom scales--and magic nuclei in different regions have 2_1^+ energies that differ. Thus it is interesting to separately study $R_{4/2}$ values. We will see that they have a deep connection to the underlying shell structure and can act as signatures of new types of shell structure.

To pursue this we first recall that 2-identical particles in a single-j shell have $R_{4/2} \sim 1.2$. Similar $R_{4/2}$ values result for multi-j configurations of the form $(j_1^{n_1} j_1 j_2^{n_2} j_2 \dots j_k^{n_k} j_k J)$ with n_i even. Thus it is hardly expected that some of the best "shell model" nuclei do not have low $R_{4/2}$ values. Indeed, some of these $R_{4/2}$ approach collective values. For example, for ^{18}O and ^{42}Ca , $R_{4/2} \sim 1.8$. In contrast $R_{4/2}$ is indeed ~ 1.2 in Sn, and for ^{134}Te , ^{210}Pb and ^{210}Po . Some of the $R_{4/2}$ data for magic nuclei is plotted in fig. 9 as a function of A.

The results are intriguing and heretofore unrecognized⁹. There is an extremely systematic behavior, with $R_{4/2}$ steadily decreasing toward " $|j^2J\rangle$ "--values in heavy

nuclei. The effects at work here are simple and useful to understand. Consider fig. 10 which shows angular momentum vector diagrams for 2^+ states constructed from 2 particles, one each occupying orbits j_1 and j_2 . The case on the right is typical of most cases in which the 2^+ level is constructed by coupling the two angular momenta at some angle between their orbital planes. The 4^+ state corresponds to a somewhat larger angle. With a short range (e.g., δ) force between the particles, the energy difference in the interaction for these two angles is small. The orbital planes are already far apart enough in the 2^+ state that there is little interaction. Hence the spectrum here is similar to that shown in fig. 2.

The left side of fig. 10, though, shows a special case. Suppose $j_1 = j_2 \pm 2$. Then, for this case *only*, the 2^+ state is constructed with *co-planar orbits*. Just as pairwise co-planar occupation of the same orbit leads to a low-lying 0^+ state in the pairing case, here occupation of *two different* orbits, with $\Delta j = \pm 2$, leads to a large attractive interaction in the 2^+ state. This state therefore drops in energy, and $R_{4/2}$ consequently rises. This is shown by the simple δ function calculations at the bottom of the figure. Generalizing this, it turns out⁹ that $R_{4/2}$ is *very closely* correlated in multi- j shell-model calculations with the summed probability for the occupation of $\Delta j = \pm 2$ orbit pairs by *odd* numbers of nucleons. We call this probability $\Sigma a_{\text{odd-n}, \Delta j=2}^2$ or Σa^2 , for short. The calculated $R_{4/2}$ - Σa^2 correlation is illustrated for the 50-82 shell in fig. 11.

It is now immediately clear why the apparently strange phenomenology of $R_{4/2}$ in fig. 9 occurs. In light nuclei, each shell (e.g., 8-20 or 20-40) has a low-lying $\Delta j=2$ pair ($d_{5/2}$ - $s_{1/2}$ and $f_{7/2}$ - $p_{3/2}$, respectively). [Note, incidentally, that pairs such as $d_{3/2}$ - $s_{1/2}$ or $p_{3/2}$ - $p_{1/2}$ are also $\Delta j=2$.] Hence, the 2^+ state is lowered and $R_{4/2}$ is large. The values of ~ 1.7 found experimentally are easy to reproduce in shell model calculations (see fig. 11). In medium and heavy nuclei, however, the l -s and l^2 terms in the shell model Hamiltonian force the unique parity orbit down into the next

lower shell, leaving *no* low-lying $\Delta j=2$ pair. For example, in the 82-126 shell the lowest orbits are $h_{9/2}$ and $f_{7/2}$. In these cases, there is no anomalous lowering of $E(2_1^+)$, the wave function remains dominated by configurations with even- n occupation of each j orbit, and $R_{4/2}$ is close to shell model values for $|j^2J\rangle$ configurations. Simple calculations⁹, one with constant interaction strength, V_0 , and one with decreasing strength, are shown in fig. 12. The concordance with the trend in fig. 9 is immediately obvious. Finally, it is worth noting an interesting feature of this discussion of odd n - $\Delta j=2$ configurations in pre-collective nuclei: such "particle-hole" type excitations link naturally onto the characteristic "RPA-wave functions" of collective nuclei. Pursuit of this idea may help understand the transition from pre-collective to collective structures at $R_{4/2}\sim 2.0$.

III. New Closed Shell Regions:

A. New Signatures of Structure

With RNB facilities it is likely that new closed shell or near-closed shell regions will be encountered. More generally, new nuclei with "pre-collective" structure ($R_{4/2} < 2.0$) are likely to be discovered. New magic numbers may be identified and traditional ones may lose their validity. The latter, in fact, has already occurred in the $N=20$ (^{32}Mg) and $Z=40$ (^{80}Zr and ^{100}Zr) regions. Shell structure, even close to the valley of stability, is hardly immutable: witness the appearance and disappearance of the $Z = 38$ or 40 and $Z = 64$ (sub)-shell gaps or even the relative change in the $1g_{7/2}$ orbit between ^{91}Zr and ^{131}Sn . The question arises then whether some of the above ideas and phenomenology may provide new signatures of perhaps exotic structures.

How would one identify potential magic nuclei in new regions, in particular, near the neutron drip line, where radical departures from traditional structure are anticipated? The traditional requirement that the first excited states are high lying is perhaps too vague, as an absolute standard of comparison may not be well established. $R_{4/2}$ alone is clearly not a good indicator of nuclei near magic numbers

since ^{18}O , ^{42}Ca and other light magic nuclei offer abundant examples of relatively high $R_{4/2}$ values--even values near the collective regime of 2.0. (We will see later that the role of $R_{4/2}$ is nevertheless quite important, albeit for different kinds of information.) The yrast energies discussed above, however, may provide clues. Constant values suggest regions of good seniority since even small breaking of seniority can induce large effects.

It is important to note that the "seniority breaking" need not involve wave functions with components of different v (e.g., $v=2, 4, \dots$). Variations in $E(2_1^+)$ and $E(4_1^+)$ can be produced with pure seniority $v=2$ wave functions as long as the amplitudes are spread over several non-degenerate j orbits. This is illustrated in fig. 13, which shows the energy difference of the 2_1^+ state for $n=6$ and $n=2$ particles, with $h_{9/2}$ and $f_{7/2}$ orbits interacting under a δ function interaction, as a function of the single particle energy (s.p.e.) difference $\Delta\epsilon \equiv \epsilon_{7/2} - \epsilon_{9/2}$. In these calculations we *fixed* the seniority as $v=0$ for the ground state and $v=2$ for the 2^+ level. Hence, any violation of the energy constancy required by good seniority cannot be due to seniority *mixing* but rather only to configuration mixing of non-degenerate orbits. For $\Delta\epsilon=0$, the orbit degeneracy satisfies the conditions of generalized seniority and hence $E(2_1^+)$ is constant. For large $\Delta\epsilon$, the interaction is not strong enough to excite particles to the $f_{7/2}$ orbit so that one has, effectively, a single j -good seniority situation. For intermediate $\Delta\epsilon$ values the conditions for generalized seniority break down and $E(2_1^+)$ changes, although (see above), ΔE_{4-2} may remain constant..

It is also possible to test the validity of magic numbers in new mass regions with $B(E2)$ values. To do this, it is necessary to develop some global understanding of the variations of $B(E2)$ values across the nuclear chart. Consider figs. 14 and 15, which show¹⁰ $B(E2)$ values for all even-even nuclei as a function of Z (fig. 14) and versus the valence nucleon product $N_p N_n$ in fig. 15. In fig. 14, while there are some evident trends, the data are still highly complex. They easily spread over an order-

of-magnitude for a given Z (and three orders-of-magnitude total--hence the need for a log scale). In contrast, in fig. 15a, the $B(E2)$ values, which are expressed in W.u. and, additionally are normalized by another factor of A , lie on a simple compact trajectory which is linear well into each shell. [A few specific nuclei that do *not* fit this tight correlation are omitted from fig. 15a. They are included in fig. 15b and will be discussed later.] Of course, a prerequisite for fig. 15 is an accurate and known accounting of $N_p N_n$. If the shell structure is altered, from that defined by the traditional magic numbers, or is unknown, then the use of incorrect N_p and N_n values can generate large deviations from the empirical correlation. Two examples illustrate this. Figure 16 shows three plots of $B(E2)$ values against $N_p N_n$ for the $A \sim 100$ region. On the left, the standard magic numbers are used, namely $Z=38,50$ for $50 < N < 60$, $Z=28,50$ for $N \geq 60$. A compact correlation is achieved. If the well known subshell at $Z=38$ for $N < 60$ were *not* assumed, however, the correlation becomes messy. This is shown in the middle panel of fig. 16 which assumes proton magic numbers of $Z=28$ and 50 throughout.

We can use this approach to test other magic numbers. Suppose, for example, that $Z=50$ is not taken as a magic number in constructing fig. 16. Then the plot on the right of fig. 16 is obtained. Clearly, the compact correlation for $B(E2)$ values against $N_p N_n$ that is found in all other mass regions is destroyed. Even in advance of specific level scheme data on ^{100}Sn , this certainly suggests that the $Z=50$ proton shell closure is intact, at least for nuclei within a few proton or neutron numbers of 50. Of course, it does not prove that ^{100}Sn itself is doubly magic since the entire precept we are discussing is that magic numbers can change, and it could still happen that the proton s.p.e.'s are different when $N=50$.

The second example concerns ^{32}Mg . Although it has $N=20$, it is a quasi-deformed nucleus. This suggests that $N=20$ is not magic. We can test this notion using the $B(E2)$ value. If $N=20$ were magic, then N_n would be zero as would $N_p N_n$,

giving a vanishing $B(E2)$ value. If $N=20$ is not magic, then the nearest closed neutron shell is $N=28$, and $N_p N_n = 4 \times 8 = 32$. One merely has to read the $B(E2)$ value from fig. 15a. Correcting to $e^2 b^2$, one predicts $B(E2: 2_1^+ \rightarrow 0_1^+) = 0.04 e^2 b^2$. This prediction is in agreement with the recently measured¹¹ value of $0.046(9) e^2 b^2$.

Although the following comment concerns collective nuclei rather than those near closed shells, a short digression is worthwhile since it highlights how global correlations can unveil otherwise inaccessible information and how simple data--energies and $B(E2)$ values--can be much more informative than one might think.

The phenomenon of unit slope in plots (e.g., fig. 7) of $E(4_1^+)$ vs. $E(2_1^+)$ is actually part of a remarkable and nearly universal correlation. Extension of $E(4_1^+)-E(2_1^+)$ plots to all types of nuclei, including collective species with $R_{4/2} > 2.0$, reveals a new, tripartite, classification of nuclear structure from magic nuclei to deformed rotors. This is illustrated in fig. 17 for the $Z=50-82$ region. The data fall along a series of three straight line segments, with slopes 3.33, 2.00 and 1.0. The first of these is the rotor region: the last we have just discussed. The middle region is a surprising recent discovery¹² that is not yet understood. All these collective, *non*-rotational nuclei ($2.05 \leq R_{4/2} \leq 3.15$) have a slope of 2.00 and constant intercept. That is

$$E(4_1^+) = 2E(2_1^+) + \epsilon_4 \quad (8)$$

This is exactly the equation of an *anharmonic vibrator* (AHV) with *constant anharmonicity* ϵ_4 . Note the widely varying $R_{4/2}$ values along the top scale. How such a wide variety of nuclei, with such different intrinsic or mean field structure, can exhibit almost constant anharmonicity is a central microscopic question raised by these results, as is the extremely rapid phase transition between rotor and AHV regions.

Another critical question is whether approximately "global" (insofar as known nuclei are concerned) correlations such as the tripartite classification of structural evolution in fig. 17 are characteristic only of the nuclei already accessible near stability, and, if so, what is special about these nuclei, or, are they completely general and, if so, what generic features of the shell model and nucleonic interactions are involved. The answers to these questions go right to the heart of our basic understanding of nuclear structure.

The lower part of fig. 15 shows an unexpected use of the compact $B(E2)-N_p N_n$ correlation. In contrast to fig. 15a, this plot includes a few points that deviate from the trendline. These deviations, of 20-30%, would never have been visible in fig. 14 but clearly stand out in the global $N_p N_n$ plot. They do not correspond to random nuclei but to nuclei with large positive (points above the main trend) or negative (points below) hexadecapole deformations. An expansion of the quadrupole moment [$\propto B(E2)$] in multipole deformation components contains a term linear in β_4 . Hence the $B(E)$ value, without the need for other, more difficult-to-obtain data, can even give clues to higher order deformations. It is worth stressing that the prerequisite for this is a compact, universal correlation.

Having looked at energies and $B(E2)$ values separately, it is also informative to combine them. As we shall now see, a plot of $B(E2)$ values against $R_{4/2}$ is even sensitive to the γ degree of freedom^{10,13}. Consider fig. 18 which plots two collective observables, $B(E2)$ and $R_{4/2}$, against each other. Individually, each can be a useful structural signature. Together they reveal even more information. For $R_{4/2} \gtrsim 2.5$, the plot shows a 2-pronged trajectory. The structure of the nuclei along these two tracks is known. The upper track is primarily composed of nuclei in a spherical vibrator to rotor transitional region while the lower track consists of γ -soft $\rightarrow$ rotor nuclei. [The presence of some $A \sim 130$ region Ba nuclei on the upper track is a bit

puzzling and deserves further study of the relationship of O(6) structure to structures enroute from U(5) to SU(3).]

Finally, there is another kind of signature, based on binding energies of neighboring even-even ground states, that can also signal either closed shell or phase transitional character. By forming double differences of binding energies, it is possible to isolate particular interactions amongst particular nucleons. There has been much study of the p-n interaction, δV_{pn} , of the last proton and last neutron.¹⁴

More useful here is the *non-pairing* interaction of the last two neutrons (or protons, but we focus on neutrons to avoid the complication of the Coulomb interaction). Called δV_{nn} , it is defined¹⁵ in fig. 19: it is designed to remove the strong attractive pairing contribution. As is well known, what remains is actually a repulsive interaction^{5,6,15}, as seen from empirical δV_{nn} values in fig. 20. The key points of the figure for our purposes are the spikes. Careful inspection of the defining formula for δV_{nn} shows that it is *invalid* if the core for the (N-1)th, Nth, and (N+1)th neutrons is not the same. Hence δV_{nn} is undefined if N is magic or if there is a phase transition. Nevertheless, numerically, δV_{nn} values can still be obtained for such cases and are very informative. These "invalid" points are, in fact, the spikes in fig. 20, positive at magic numbers and negative at phase transitions. Thus, binding energies far off stability, which in some cases are easier and in other cases more difficult to obtain than excitation energies or B(E2) values, provide an alternate signature for shell structure or spherical-deformed transition regions.

B. A Specific Example

The work of Nazarewicz and colleagues^{1,3} that we cited earlier on shell structure near the neutron drip lines is particularly intriguing. While more study is necessary, it is interesting to take their suggestion that the shell model potential itself is altered and that it lacks an l^2 term. Clearly, this will change the magic numbers in medium and heavy nuclei and also the traditional ordering of orbits

within a shell. We illustrate this in fig. 21a, and we ask if there are simple empirical signatures of such effects. Figure 21a shows the shell model orbits corresponding to 20-50 nucleons with a normal potential (the s.p.e.'s are taken from the Nilsson diagram at $\beta=0$) and, with the same potential with no l^2 term. With no l^2 term, the $f_{7/2}$ orbit rises in energy and so does the $g_{9/2}$ orbit. These "unique" parity orbits now are joined to others with the same parity. Hence $Z=50$ and 82 are no longer magic, being replaced by 40 and 70 . This effect could be disclosed by $B(E2)-N_p N_n$ plots of very neutron-rich $Z=50$ nuclei. But there is an even more dramatic and immediate consequence. Recall the plot of $R_{4/2}$ vs. A in fig. 9 and the discussion (fig. 11) of the correlation of $R_{4/2}$ values with Σa^2 . The principal point was that if $\Delta j=2$ orbital pairs are present, the 2^+ state is energetically favored, and hence $R_{4/2}$ increases. This is not the only effect at work (the binding energy of the 0_1^+ state, and hence the 2_1^+ and 4_1^+ excitation energies, also depends on the distribution of pairwise occupation of different j orbits), but we focus on it here. In normal heavy nuclei, the unique parity orbit is lowered into the next lower shell, eliminating the principal $\Delta j=2$ pair. Hence $R_{4/2} \sim 1.2-1.3$ for singly-magic medium and heavy-mass nuclei. Now, suppose the s.p.e.'s on the right in fig. 21a apply. Now a favored $g_{9/2}-d_{5/2}$ pair occurs low in the new shell. Hence one may expect $R_{4/2}$ to be anomalously large. This effect is illustrated in fig. 21b which shows calculations as a function of the l^2 strength. As anticipated, $R_{4/2}$ does indeed increase substantially (for a realistic δ function strength V_0). The lower panel of fig. 21 confirms that, as expected, the increase in $R_{4/2}$ as the l^2 term decreases is correlated with a rise in the occupation of $\Delta j=2$ orbits by odd numbers of nucleons. The effect on $R_{4/2}$ is shown in fig. 22 which repeats fig. 12 but with a point corresponding to an " $l^2=0$ " calculation for 44 protons, that is, for 4 protons in the shell 40-70 that appears when the l^2 term vanishes. Observation of such anomalous R values may therefore signal major changes in the shell model

potential without the need for extensive transfer reaction data to establish s.p.e.'s and spectroscopic strengths in new regions.

This signature needs further study, especially with realistic shell-model calculations, to establish whether other effects could mask or swamp it. Such an effort, however, is certainly worthwhile: the idea that subtle details of the shell model potential can be revealed simply from the $E(4_1^+)/E(2_1^+)$ ratio in a single magic nucleus is too appealing not to pursue.

IV. Conclusion

We have discussed a number of signatures of structure that are remarkably simple, that are global, or nearly so, in character, that are based on simple ideas of the shell model and residual interactions, and, most importantly, which may be the easiest-to-obtain signatures of structure near closed shells in nuclei far from stability. Near the neutron drip line it seems plausible,¹ perhaps even likely, that there may be radical departures from shell structure and nuclear structure compared to anything we have seen near stability. Hints of this already exist in the N=20 region (³²Mg), in the N=Z nucleus ⁸⁰Zr, and in the evanescence of shell structure in the A=100 (Z=38 or 40 shell gap) and A=150 (Z=64 gap) regions. The merging of bound levels into the continuum, the changes expected in the pairing interaction near zero binding energy, and the changes in the residual p-n interaction when neutrons are filling much higher orbits than protons, will all contribute to engender new shell structure, even new shell model potentials, and new forms of collectivity.

With RNB's, we will need new and highly efficient signatures of structure. This paper has discussed a number of these, with emphasis on magic and near-magic nuclei. Three of the most important are the energies, and energy differences, of the 2_1^+ and 4_1^+ states, their ratio $R_{4/2}$, and the $B(E2:2_1^+ \rightarrow 0_1^+)$ value.

The constancy of yrast energies can signal states of good seniority which, in turn, likely signals either an effectively single-j environment or a nearly-degenerate

multi-j one. Non-constancy of these energies but constancy of their difference, can point to a pair addition mode where the $v \neq 0$ part of the wave functions of yrast states is nearly independent of valence nucleon number. The behavior of the $B(E2)$ value for such nuclei can give evidence concerning the fractional filling of a shell if the nucleus is singly magic or, via an $N_p N_n$ analysis, concerning the validity of proposed magic numbers.

When near-closed shell, especially magic, nuclei are identified, the $R_{4/2}$ ratio can point to the underlying shell structure and even to the content of the shell model Hamiltonian itself (e.g., the presence or absence of an l^2 term). $B(E2:2_1^+ \rightarrow 0_1^+)$ values for collective nuclei can also provide evidence for the γ degree of freedom and for higher-order deformation multipoles (e.g., β_4). Finally, appropriate double differences of binding energies can also point both to magic nuclei (positive spikes in δV_{nn}) or to shape/phase transitions (negative spikes in δV_{nn}).

Nuclei far from stability will also allow us to test the remarkable nearly-universal correlations of structure that have recently been discussed, such as the tripartite classification of structure into rotor, AHV and pre-collective regimes via $E(4_1^+) - E(2_1^+)$ plots, the smooth decrease of $R_{4/2}$ values for singly magic nuclei with A , the global $B(E2:2_1^+ \rightarrow 0_1^+)$ vs. $N_p N_n$ correlation, and the $B(E2:2_1^+ \rightarrow 0_1^+) - R_{4/2}$ two-tracked trajectory.

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References

1. J. Dobaczewski et al., Phys. Rev. Lett. **72** (1994) 981.
2. P. Van Isacker, M. A. Nagarajan and D. D. Warner, Phys. Rev. **C45** (1992) R13.
3. W. Nazarewicz, T. R. Werner and J. Dobaczewski, to be published.
4. M. M. Sharma et al., Phys. Rev. Lett. **72** (1994) 1431.
5. A. de Shalit and I. Talmi, Nuclear Shell Theory (Academic Press, New York, 1963).
6. R. F. Casten, Nuclear Structure from a Simple Perspective (Oxford University Press, New York, 1990).
7. N. V. Zamfir, R. F. Casten and D. S. Brenner, Phys. Rev. Lett. **72** (1994) 3480.
8. Y. K. Gambhir, S. Hag and J. K. Suri, Ann. Phys. (NY) **133** (1981) 154.
9. R. F. Casten, N. V. Zamfir and D. S. Brenner, Phys. Lett. **B324** (1994) 267.
10. R. F. Casten and N. V. Zamfir, Phys. Rev. Lett. **70** (1993) 402.
11. T. Motobayashi et al., to be published.
12. R. F. Casten, N. V. Zamfir and D. S. Brenner, Phys. Rev. Lett. **71** (1993) 227.
13. N. V. Zamfir and R. F. Casten, Phys. Lett. **B305** (1993) 317.
14. J.-Y. Zhang, R. F. Casten and D. S. Brenner, Phys. Lett. **B227** (1989) 1.
15. N. V. Zamfir and R. F. Casten, Phys. Rev. **C43** (1991) 2879.

Table 1. Main components of the exact and pair addition mode ($h_{9/2}f_{7/2}$) wave function probabilities (squared amplitudes) (in %). From ref. 7.

J^π	number of particles (n)							
	2		4			6		
	w.fa)	exact ^{b)}	w.fa)	exact ^{b)}	pair ^{c)}	w.fa)	exact ^{b)}	pair ^{c)}
0 ⁺	(20,00)	74	(40,00)	51	55	(40,20)	51	45
	(00,20)	26	(20,20)	43	38	(60,00)	32	38
			(00,40)	6	7	(20,40)	16	16
2 ⁺	(22,00)	92	(42,00)	62	68	(42,20)	41	36
	(00,22)	6	(22,20)	27	24	(62,00)	34	46
	(11,11)	2	(20,22)	7	4	(40,22)	9	5
4 ⁺	(22,00)	94	(42,00)	64	70	(42,20)	41	36
	(11,11)	4	(22,20)	27	24	(62,00)	35	47
	(00,22)	2	(31,11)	5	3	(22,40)	7	7

- a) This column lists the principal wave function components of the 2-orbit system, with the notation $[(nv)_{h_{9/2}} (nv)_{f_{7/2}}]$.
- b) The "exact" wave function probabilities are those obtained by diagonalizing a surface δ interaction in the 2-orbit space with an interaction strength $V_0 = 1.0$ MeV.
- c) For $n = 4, 6$, the "pair" column gives the "pair addition mode" wave function probabilities for n -particles in a state of spin J calculated by forming the product of the 2-particle 0⁺ state (upper left box) with the $(n-2)$ -particle wave function probabilities of spin J .

Figure Captions

Fig. 1. Schematic depiction of proton and neutron levels near the neutron drip line.

Fig. 2. Levels of the configuration $|j^2J\rangle$ with $j=7/2$ and a δ function interaction.

Fig. 3. Yrast energy levels of $v=2$ states in a j^n configuration. From ref. 6.

Fig. 4. Levels of the Sn isotopes.

Fig. 5. $B(E2:2_1^+ \rightarrow 0_1^+)$ values for a j^n configuration with good seniority. The function plotted is $f(1-f)$. See eq. 4.

Fig. 6. $E(2_1^+)$, $E(4_1^+)$ and ΔE_{4-2} values. a) Magic proton nuclei, b) Magic neutron nuclei, c) Calculations (see text). Partly based on ref. 7.

Fig. 7. Plot of empirical $E(4_1^+)$ against $E(2_1^+)$ values for nuclei with $R_{4/2} < 2.0$. The bottom panel shows calculations described in the text. From ref. 7.

Fig. 8. $B(E2)$ values a) against mass number A and b) against the fractional filling factor f for singly magic nuclei with $Z, N \geq 50$. The curve on the right is the function $f(1-f)$ and is normalized to the data.

Fig. 9. Empirical $R_{4/2}$ values in singly magic nuclei.

Fig. 10. Schematic, semi-classical angular momentum diagram for the construction of a 2^+ state from angular momenta j_1 and j_2 . Left: the co-planar case of $|j_1 \pm j_2| = 2$. Right: illustration of all other cases-- a $\Delta j = 1$ example. At the bottom, level schemes for these two cases are shown, for a surface δ function interaction with strength, $V_0 = 1.0$ MeV, and degenerate orbits. From ref. 9.

Fig. 11. Plots of $R_{4/2}$ vs. Σa^2 , the summed percentage of wave function components corresponding to the occupation of each of two orbits, with $\Delta j = 2$, by an odd number of nucleons, for sets of shell model calculations with a surface δ interaction. Calculations for the 50-82 shell. The following calculations are included: s.p.e.'s as follows, $g_{7/2}$ (0.0 MeV), $d_{5/2}$ (0.8 MeV), $h_{11/2}$ (1.9 MeV), $d_{3/2}$ (3.5 MeV), $s_{1/2}$ (3.9 MeV), $V_0 = 1.0$ MeV, seniority $v \leq 4$, up to 2 particles in the $h_{11/2}$ orbit, for 2, 4, 6, and 8 active particles (star symbols); same except for 4 particles and interaction

strength, V_0 , varying from 0.2 to 1.2 MeV (circles); 4 particles, $V_0 = 1.0$ MeV and several choices for single particle energies (crosses).

Fig. 12. Shell model calculations of $R_{4/2}$ values for singly magic nuclei with a surface δ interaction. The calculations are done for 4 particles in each of several major shells. The solid line gives the results for a calculation with *constant* interaction strength, $V_0 = 0.8$ MeV. The dashed lines give slightly more realistic results with an interaction strength that decreases with A . $V_0 = 1.2$ MeV for the 8-20 and 20-40 shells, 0.6 MeV for the 50-82 shell and 0.3 MeV for the $Z > 82$ shell.

Fig. 13. The energy difference $E(2_1^+) - E(2_2^+)$ for a system with $h_{9/2}$ and $f_{7/2}$ orbits as a function of the single particle energy difference $\Delta\epsilon$ between the two orbits, for a δ function strength of $V_0 = 1.0$ MeV.

Fig. 14. $B(E2)$ values for $Z \geq 40$. Based on ref. 10.

Fig. 15. $B(E2)$ values in W.u., divided by mass number A , plotted against $N_p N_n$. a) Excludes a few deviant points. This plot includes a bi-linear fit of the form $B(E2) = a + b N_p N_n$ where $a = 0.07$, $b = 0.0065$ W.u./ A for $N_p N_n < 175$, and $a = 1.14$, $b = 0.0004$ W.u./ A for $N_p N_n > 175$. b) Data points for nuclei deviating from the main sequence are now included (filled circles). Based on ref. 10.

Fig. 16. $B(E2)$ values for the $A=100$ region (Zr-Sn) against $N_p N_n$ assuming normal shell and subshell closures (left), ignoring the $Z=38$ subshell closure (middle), and ignoring in addition the $Z=50$ closure (right). Partly based on ref. 10.

Fig. 17. A plot, for the $Z = 50-82$, $N = 82-126$ region, of $E(4_1^+)$ against $E(2_1^+)$ showing the division of the structural evolution into a tripartite classification of "seniority", anharmonic vibrator (AHV) and rotor (R) regions, corresponding to slopes of 1.0, 2.0, and 3.33. These regions are labelled at the bottom and demarcated by vertical dotted lines separated by narrow zones in which the structure varies rapidly in a way that resembles phase transitional behavior seen in other physical systems. From ref. 7.

Fig. 18. Empirical $B(E2)$ values against $R_{4/2}$ values for nuclei with Z values from 50-82. Similar to ref. 13.

Fig. 19. Definition of δV_{nn} in terms of a double difference of binding energies. From ref. 15.

Fig. 20. Empirical δV_{nn} for $N > 30$. The solid lines connect isotopes. Based on ref. 15.

Fig. 21. a) Evolution of s.p.e.'s for nucleon numbers 20-50 as a function of the strength of the l^2 term. b) Calculated $R_{4/2}$ values as a function of the strength of the l^2 term. c) Corresponding Σa^2 values.

Fig. 22. $R_{4/2}$ against Z . The dashed line is from fig 12. The square symbol gives the result of a δ -function calculation (with $V_0 = 0.6$ MeV) for 4 particles in the 40-70 shell that results when there is no l^2 term in the shell model Hamiltonian.

200

0

B.F.

$$\begin{array}{c} 6^+ \\ 4^+ \\ 2^+ \end{array}$$

$$0^+$$

$$(f_{7/2})^2$$

$$\Delta j = 2$$

$$g_{7/2} \quad d_{3/2}$$

$$4^+ \text{ ————— } 4.88$$

$$E \text{ (MeV)} \quad 2^+ \text{ ————— } 2.62$$

$$0^+ \text{ ————— } 0$$

$$R_{4/2} = 1.86$$

$$\Delta j = 1$$

$$g_{7/2} \quad d_{5/2}$$

$$4^+ \text{ ————— } 5.72$$

$$2^+ \text{ ————— } 5.12$$

$$0^+ \text{ ————— } 0$$

$$R_{4/2} = 1.12$$

$$= \frac{1}{2} \{ [B(N+1,Z) - B(N-1,Z)] - [B(N,Z) - B(N-2,Z)] \}$$

$$\delta V_{nn}(N,N+1) = \frac{1}{2} \left\{ \left[\begin{array}{c} \bullet \bullet \\ \bullet \\ \boxed{N-2} \\ \boxed{Z} \end{array} \right] - \left[\begin{array}{c} \bullet \\ \bullet \bullet \\ \boxed{N} \end{array} \right] - \left[\begin{array}{c} \bullet \bullet \\ \bullet \\ \boxed{N} \end{array} \right] - \left[\begin{array}{c} \bullet \bullet \\ \bullet \\ \boxed{N-2} \end{array} \right] \right\}$$