Comment on "Flavor-changing processes and CP violation in the $S_3 \times Z_3$ model"

L. Lavoura

Department of Physics, Carnegie-Mellon University,
Pittsburgh, Pennsylvania 15213, U.S.A.

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Abstract

Deshpande, Gupta and Pal neglected the presence of a complex phase in the mass matrix and in the Yukawa coupling matrices of the down-type quarks. I argue that that phase should not be neglected, because it can explain the observed CP violation. The mechanism of CP violation considered by those authors may certainly be present, but for simplicity one may want to eliminate it, by restricting the soft-breaking sector of the Higgs potential in such a way that complex Higgs-bosons propagators do not occur.

In the paper on which I comment [1], Deshpande, Gupta and Pal (DGP) studied the scalar flavor-changing neutral currents present in Ma's $S_3 \times Z_3$ model [2], and in particular the CP violation induced by them. They neglected the presence of complex phases in some Yukawa couplings (and sometimes neglected those couplings altogether), with the argument that those couplings are proportional to $m_u/m_c \sim 10^{-3}$. As a consequence, in order to explain the observed CP violation, they had to introduce complex Higgs-bosons propagators, or equivalently scalar-pseudoscalar mixing, which occur if CP is spontaneously broken. I show here that one cannot really neglect the complex terms in the Yukawa couplings, which are not always suppressed by factors m_u/m_c , and which can easily explain the observed CP violation. (Even if all the complex Yukawa couplings were suppressed by m_u/m_c , one should not neglect them, because the ratio of the observed CP-violating to the CP-conserving amplitudes is precisely of the order of magnitude of m_u/m_c .)

I first show that there is no need for scalar-pseudoscalar mixing to be present in the $S_3 \times Z_3$ model (though it may certainly be present), because the Higgs potential may be chosen so constrained as to avoid it. The most general Higgs potential consistent with the $S_3 \times Z_3$ symmetry includes only terms which are insensitive to the phases of the Higgs doublets, and which have real coefficients, because of Hermiticity. The only exception is a term $f[(\eta_2^\dagger \phi_1)(\eta_1^\dagger \phi_2) + (\eta_1 \leftrightarrow \eta_2)] + h.c.$, in which f is a complex coupling constant. As a consequence of these facts, the Higgs potential has, besides the desired $S_3 \times Z_3$ and gauge symmetries, two additional U(1) symmetries: under the first

$U(1)$, ϕ_1 and ϕ_2 get rephased by opposite phases, and under the second $U(1)$, the same thing happens (independently) to η_1 and η_2 . These symmetries give rise to unwanted Goldstone bosons upon spontaneous symmetry breaking. In order to eliminate these Goldstone bosons, the $U(1)$ symmetries must be eliminated, by introducing terms which break them, and which of course also break $S_3 \times Z_3$, softly. Another motivation for introducing soft-breaking terms is allowing for the vacuum expectation values (vevs) of η_1^0 and η_2^0 to be different. A potential symmetric under $S_3 \times Z_3$ is also symmetric under the interchange $\eta_1 \leftrightarrow \eta_2$, and leads to $\langle \eta_1^0 \rangle = \langle \eta_2^0 \rangle$. But we know that in the $S_3 \times Z_3$ model $\langle \eta_1^0 \rangle / \langle \eta_2^0 \rangle = m_u / m_c$. A minimal set of soft-breaking terms is

$$\mu(\eta_1^\dagger \eta_1 - \eta_2^\dagger \eta_2) + (\rho \phi_1^\dagger \phi_2 + \sigma \eta_1^\dagger \eta_2 + h.c.), \quad (1)$$

where μ is real but ρ and σ are complex. It is consistent to use only these soft-breaking terms, because then the complete Lagrangian enjoys from a discrete symmetry, under which $\eta_1, \eta_2, q_{3L}, b_R, t_R, c_R$ and u_R change sign; this discrete symmetry avoids the appearance, via radiative corrections, of other soft-breaking terms in the Higgs potential, of the type $\phi_j^\dagger \eta_k$ and $\eta_k^\dagger \phi_j$. Now, with the simple soft-breaking sector of Eq. 1, there are only three complex parameters in the Higgs potential, f, ρ and σ . As there are also three relative phases among the vevs of the four Higgs doublets, it is clear that the phases of the vevs will just be such as to offset the phases of f, ρ and σ , and to render both these parameters and the vevs effectively real. Then, no scalar-pseudoscalar mixing among the neutral Higgs fields will be induced. If those neutral Higgs fields are expanded as $\phi_j^0 = (v_j + \tau_j + i\varpi_j) \exp(i\alpha_j) / \sqrt{2}$, where from now on I denote η_1 by ϕ_3 and η_2 by ϕ_4 , and $v_j \exp(i\alpha_j) / \sqrt{2} = \langle \phi_j^0 \rangle$, then the four τ_j and the four ϖ_j will have separate mass matrices and will not mix with each other.

If one cares not to neglect $\langle \phi_3^0 \rangle$, Eq. 14 of the paper commented upon reads

$$\begin{pmatrix} \tau_1 + i\varpi_1 \\ \tau_2 + i\varpi_2 \\ \tau_3 + i\varpi_3 \\ \tau_4 + i\varpi_4 \end{pmatrix} = \begin{pmatrix} \frac{v_1}{v} & \frac{v_2}{v_{12}} & \frac{v_1 v_4}{v_{12} v_{124}} & -\frac{v_1 v_3}{v_{124} v} \\ \frac{v_2}{v} & -\frac{v_1}{v_{12}} & \frac{v_2 v_4}{v_{12} v_{124}} & -\frac{v_2 v_3}{v_{124} v} \\ \frac{v_3}{v} & 0 & 0 & \frac{v_{124}}{v} \\ \frac{v_4}{v} & 0 & -\frac{v_{12}}{v_{124}} & -\frac{v_3 v_4}{v_{124} v} \end{pmatrix} \begin{pmatrix} h + ig \\ r_1 + ij_1 \\ r_2 + ij_2 \\ r_3 + ij_3 \end{pmatrix}, \quad (2)$$

where $v_{12} := \sqrt{v_1^2 + v_2^2}$, $v_{124} := \sqrt{v_{12}^2 + v_4^2}$ and $v := \sqrt{v_{124}^2 + v_3^2} \approx 246$ GeV. g is the longitudinal component of the Z^0 . h is the analogous of the sole Higgs boson of the standard model, but it mixes with the three r_k . Similarly, the three fields j_k mix. The mass terms of the down-type quarks, and their Yukawa couplings to the neutral scalar fields, are given by

$$- \begin{pmatrix} \bar{d}_L & \bar{s}_L & \bar{b}_L \end{pmatrix} V_L^\dagger M U_R \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix} + h.c., \quad (3)$$

where

$$M := \left(1 + \frac{h + ig}{v}\right) \begin{pmatrix} 0 & a & \xi b \\ a & 0 & b \\ \xi c & c & d \end{pmatrix} + \frac{r_1 + ij_1}{v_{12}} \begin{pmatrix} 0 & -\frac{v_1}{v_2} a & 0 \\ -\frac{v_1}{v_2} a & 0 & 0 \\ 0 & 0 & \frac{v_2}{v_1} d \end{pmatrix}$$

$$\begin{aligned}
& + \frac{r_2 + ij_2}{v_{124}} \begin{pmatrix} 0 & \frac{v_4}{v_{12}} a & 0 \\ \frac{v_4}{v_{12}} a & 0 & -\frac{v_{12}}{v_4} b \\ 0 & -\frac{v_{12}}{v_4} c & \frac{v_4}{v_{12}} d \end{pmatrix} \\
& + \frac{r_3 + ij_3}{v} \begin{pmatrix} 0 & -\frac{v_3}{v_{124}} a & \frac{v_{124}}{v_3} \xi b \\ -\frac{v_3}{v_{124}} a & 0 & -\frac{v_3}{v_{124}} b \\ \frac{v_{124}}{v_3} \xi c & -\frac{v_3}{v_{124}} c & -\frac{v_3}{v_{124}} d \end{pmatrix}. \tag{4}
\end{aligned}$$

This equation corrects the Eq. 16 of the paper commented upon, to the case in which neither v_3 nor the complex phase are neglected. V_L and U_R are the unitary matrices which bi-diagonalize the mass matrix, which is also the matrix of the Yukawa couplings to h and g . All the matrix elements in Eq. 4 are real, except for $\xi = (m_u/m_c) \exp(i\chi)$. DGP have set $\chi = 0$, in which case all the matrices become real. Their only mechanism of CP violation is then the mixing between the r_k and the j_k , which they provoke by using a soft-symmetry-breaking sector richer than the one in Eq. 1. But χ is in general not zero, even in the case considered by DGP. Therefore, it is more economical to consider that all the CP violation in Ma's model originates in χ , and that there is no scalar-pseudoscalar mixing.

Now let us keep track of the factors $|\xi| = m_u/m_c$. As all the complex matrix elements of the mass matrix are proportional to $|\xi|$, all the imaginary parts in V_L and U_R will be of that order. As all the complexities in the Yukawa couplings of r_1, j_1, r_2 and j_2 stem from the complexities in V_L and U_R , they will also be of order $|\xi|$. But the same does not occur in the Yukawa couplings of r_3 and j_3 , which include terms $(v_{124}/v_3)\xi$ (see Eq. 4). As $v_{124}/v_3 > v_4/v_3 = 1/|\xi|$, the complexities in these Yukawa couplings are not suppressed. Therefore, neglecting χ with the argument that its effects are suppressed by m_u/m_c is untenable. Of course, the exchange of r_3 and of j_3 will be the main source of CP violation, if there is no scalar-pseudoscalar mixing, and if the masses of all the neutral Higgs bosons (except maybe h) are not too different.

I have worked out numerically the following example. I have set $a^2 = 0.001702 \text{ GeV}^2$, $b^2 = 0.063426 \text{ GeV}^2$, $c^2 = 9.067551 \text{ GeV}^2$ and $d^2 = 20.048207 \text{ GeV}^2$. I used $v_1 = v_2 = 44 \text{ GeV}$, $v_3 = 0.9 \text{ GeV}$ and $v_4 = 238 \text{ GeV}$. Of course then, $|\xi| = 9/2380$. With χ varying between 0° and 180° , one then obtains $m_b = 5.4 \text{ GeV}$, $m_s = 0.15 \text{ GeV}$, m_s/m_d varying between 16.4 and 15.5 [3], $|V_{cb}| = 0.0387$, $|V_{ub}/V_{cb}|$ varying between 0.114 and 0.106, and $|V_{us}|$ varying between 0.2172 and 0.2228. We do not know how the neutral Higgs bosons (h, r_1, r_2, r_3) and (j_1, j_2, j_3) mix. There are various ways of simulating a solution to this problem. I have proceeded in the following way. I considered three real Higgs fields, one of them — H_1 — with the Yukawa couplings of $(r_1 + ij_1)$, the second one — H_2 — with the couplings of $(r_2 + ij_2)$, and the third one — H_3 — with the couplings of $(r_3 + ij_3)$. I required that the tree-level exchange of each of these three fictitious Higgs bosons, separately, does not give contributions to any of the three quantities Δm_K , Δm_B and ϵ_K , larger than the experimental values. From this requirement, I have obtained that:

- 1) The mass of H_1 should be larger than about 3.1 TeV. This bound comes from

the contribution of H_1 to Δm_B . Its contribution to Δm_K is then at least about two orders of magnitude smaller than the experimental value. Its contribution to ϵ_K is at least one order of magnitude smaller than the experimental value.

2) The mass of H_2 should be larger than 3.2 TeV, in order for the contribution of its exchange to Δm_B not to be too large. If $20^\circ < |\chi| < 160^\circ$, the contribution of H_2 to ϵ_K is also important, and yields a stronger bound on the mass of that scalar, going up to 6 TeV, for $|\chi| = 90^\circ$. The contribution of H_2 to Δm_K is never larger than 75% of the experimental value.

3) H_3 gives an important contribution to ϵ_K . This contribution is at its largest when $|\chi|$ is either 45° or 135° . The lower bound on the mass of H_3 varies between 3.5 TeV, for $|\chi| = 0^\circ$ (in that case, the bound comes from Δm_B), and 14 TeV, for an angle $|\chi|$ of 45° or 135° . The contribution of H_3 to Δm_K is at least one order of magnitude smaller than the experimental value.

The fact that the lower bounds on the mass of H_3 that I obtain are lower than the estimate given by DGP, 12 to 40 TeV, demonstrates that the CP-violating effects of the phase χ , neglected by those authors, are at least as important as the CP-violating effects of the scalar-pseudoscalar mixing.

As a final remark on the paper of DGP, I want to point out the following. DGP assert that Ma's model is a realization of the superweak theory of CP violation. This is not rigorously true. The charged Higgs bosons also mediate CP violation. If their masses happen not to be very high, then a measurable ϵ'/ϵ could be generated. The three physical charged Higgs bosons may, as in Weinberg's model of CP violation [4], mix via a unitary matrix with one complex phase. Moreover, the Yukawa couplings of those three Higgs bosons will be complex (and for one of them the complexities will not be suppressed by factors m_u/m_c). As the masses of the charged Higgs bosons are in principle independent of the ones of the neutral Higgs bosons, it is not obvious that ϵ'/ϵ should be much smaller than 10^{-3} in this model.

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