2

E. I. DU PONT DE NEMOURS AND COMPANY

EXPLOSIVES DEPARTMENT

WILMINGTON 98, DELAWARE

DPW-56-233

SR/H--816

**DOES NOT CONTAIN
UNCLASSIFIED CONTROLLED
NUCLEAR INFORMATION**

Reviewing
Official

C. J. Banick

C. J. Banick, AED Class Officer

Date

1/28/87

cc: Hood Worthington
M. H. Wahl-C. W. J. Wende
Dale F. Babcock
L. Bernath-R. L. Menegus
-H. F. Ring
W. File (2)
ST File

May 9, 1956

MEMORANDUM

TO: L. FALK

FROM: L. BERNATH-R. L. MENEGUS-H. F. RING

MIXING IN A RIVER

Many rivers are burdened with tributary streams of warm water and/or liquid wastes containing dissolved or suspended matter. The warm water and waste matter mix thoroughly with the river water some distance downstream from the point of entry of the tributary, but near the point of entry there may be high local temperatures or concentrations of waste.

It is often necessary to know the local temperatures or concentrations of waste. We have used a formula for computing the turbulent mixing that takes place in such a situation; this formula fits quite well in the case of one Southern river.

O. G. Sutton⁽¹⁾ has studied turbulent mixing in the atmosphere using the theory of the "drunkard's walk", which was a subject of study by Taylor, Pearson, Kluyver and Rayleigh. We have adapted Sutton's theory to fit the turbulent mixing of the main river flow, and the formula derived describes the local temperatures or concentrations as a function of position in the river.

Consider a river and tributary in the following plan form:

MASTER
DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, make any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

$$u = \frac{T - T_r}{T_c - T_r} = \frac{\sqrt{\frac{2}{\pi}} c v}{\operatorname{erf} v} e^{-\frac{1}{2}\left(\frac{xy}{X}\right)^2} \quad (1)$$

$$v = \frac{\sqrt{2} X}{ay^m + c\sqrt{\frac{4}{\pi}} X} ; \quad \operatorname{erf} v = \frac{1}{\sqrt{2\pi}} \int_{-v}^v e^{-\frac{t^2}{2}} dt \quad (\text{Tabulated})^*$$

$$c = \frac{\text{Tributary Flow}}{\text{Tributary Flow} + \text{River Flow}} \ll 0.3$$

X = Width of the river in miles, strictly equal to the wetted perimeter of the river minus twice the average depth.

x = Distance in miles measured across the river from the point of entry.

y = Distance in miles measured downstream from the point of entry.

T = Temperature or concentration of any point in the river downstream of the point of entry.

T_r = Temperature or concentration in the river upstream of the point of entry.

T_c = Temperature or concentration in the tributary.

a = Mixing constant.

m = Mixing exponent varying between 0.5 to 1, depending on the turbulence in the river.

* The error function has a value of unity for $v > 4$.

Note that Equation (1) has the following characteristics:

When: $\underline{x}$	and $\underline{y}$	then: $\underline{u}$
0	0	1
0	∞	c
X	0	0
X	∞	c

The constants a and m should depend mainly on the Reynolds number of the river, which may be written approximately as:

$$R = \frac{\text{River flow}}{\text{River width}} \frac{\text{ft}^3/\text{sec}}{\text{mile}}$$

Hence, it is expected that all rivers having a regular channel and the same Reynolds number should have the same mixing constants, a and m. A river having numerous cataracts or narrow gorges will perhaps not fit the above equation at all.

In one case, in which $R = 10^5 \text{ ft}^3/\text{mile-sec}$, a was found to be 0.3075, and m, 0.964. These constants were fitted by means of the principle of maximum likelihood for 22 local temperatures in the river downstream from a tributary of warm water.

The attached graph allows one to determine the value of u/c at any point in the river (calculate x/X and v at the point in question).

(1) Sutton, O. G., A New Theory of Atmospheric Diffusion, March 12, 1930, PORTON-790 (SECRET)

$$u = \frac{T - T_r}{T_c - T_r} = \frac{\sqrt{\pi} c v}{\text{erf } v} e^{-\frac{1}{2} \left(\frac{v}{X} \right)^2} \quad (1)$$

$$v = \frac{\sqrt{2} X}{\sigma y^m + c \sqrt{\frac{4}{\pi}} X}$$

$$\text{erf } v = \frac{1}{\sqrt{\pi}} \int_0^v e^{-\frac{t^2}{2}} dt \quad (\text{Tabulated})^*$$

$$C = \frac{\text{TRIBUTARY FLOW}}{\text{TRIBUTARY FLOW} + \text{RIVER FLOW}} \leq 0.5$$

X = WIDTH OF THE RIVER IN MILES, STRICTLY EQUAL TO THE WETTED PERIMETER OF THE RIVER MINUS TWICE THE AVERAGE DEPTH.

X = DISTANCE IN MILES MEASURED ACROSS THE RIVER FROM THE POINT OF ENTRY.

y = DISTANCE IN MILES MEASURED DOWNSTREAM FROM THE POINT OF ENTRY.

T = TEMPERATURE OR CONCENTRATION OF ANY POINT IN THE RIVER DOWNSTREAM OF THE POINT OF ENTRY.

T_r = TEMPERATURE OR CONCENTRATION IN THE RIVER UPSTREAM OF THE POINT OF ENTRY.

T_c = TEMPERATURE OR CONCENTRATION IN THE TRIBUTARY.

σ = MIXING CONSTANT.

m = MIXING EXPONENT VARYING BETWEEN 0.5 TO 1, DEPENDING ON THE TURBULENCE IN THE RIVER.

* THE ERROR FUNCTION HAS A VALUE OF UNITY FOR $v > 4$.

MIXING IN A RIVER

Note: v is the distance parameter, varying inversely with distance y .
 At $y = 0$, $v = \frac{1}{\sigma} \sqrt{\frac{\pi}{2}}$.
 By eq. 1 at the source $y = 0$, $\frac{X}{y} = 0$, $u = 1$ since $\text{erf } v \cong 1$ for large values of v .
 Hence the mix intercept on this plot corresponding to $y = 0$ @ $\frac{X}{y} = 0$ is equal to $\frac{1}{\sigma}$.