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A PROPOSED MODIFICATION OF SCALING LAWS
FOR EXPLOSIVE PHENOMENA

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ABSTRACT

A semi-acoustic theory of blast energy propagation, incorporating Sachs' scaling modified for altitude of observation, gives excellent agreement with experiments providing Hilliar's Similarity Principle is restricted to similar ambient (weather) conditions.

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A PROPOSED MODIFICATION OF SCALING LAWS
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Hilliar's Principle of Similarity,^{1/} "Pressures and the other properties of a shock wave will be unchanged if the scales of length and time are changed by the same factor as the dimensions of the charge," is not without earlier recognized limitations^{2/} when the transmitting medium is the atmosphere. However, it is surprising that, with but small modification, this Principle, originally announced to explain phenomena of underwater shock overpressures in the range of tenth-tons per square inch, appears to account well for air-transmitted blast where overpressure levels are but microbars.

An equivalent and familiar expression of the Principle, applicable to spherical explosives, is: "Distances to equal overpressures, and time values at radii of equal overpressures, are proportional to the cube roots of charge weights." From this unmodified statement would come the corollary that blast overpressures at fixed distances from identical explosions should be independent of direction and ambient conditions. Figure 1 demonstrates the fallacy of such generality.

1. H. W. Hilliar, Experiments on the Pressure Wave Thrown Out by Submarine Explosions, D.S.R. Report R.E. 142, (1919); R. H. Cole, Underwater Explosions, Princeton Univ. Press (1943), Chapter 4.
2. R. G. Sachs, The Dependence of Blast on Ambient Pressure and Temperature, Ballistic Research Laboratory (Unclassified) Report 466, (1944);

A semi-acoustic approach^{3/} to the problem of predicting blast overpressures from explosions, accompanied by a modified form of Hilliar's Principle, seems to fit experimental evidence.

From an explosion releasing energy E at altitude B above sea level, the increment of energy emitted through an elemental solid angle at azimuthal angle ϕ_B , and starting elevation angle $\theta_{\phi,B}$, will be

$$dE = E \cdot \cos \theta_{\phi,B} \cdot d\theta_{\phi,B} \cdot d\phi_B / 4\pi \quad (1)^*$$

Much of this energy will be lost from the shock wave through non-adiabatic processes. Let the fraction remaining as blast energy after the shock has traveled a distance D be represented as ϵ_D . By refraction, this reduced increment of energy may pass a point of interest at altitude h above sea level and horizontal distance x_ϕ measured from the shot point. At that point it will present a wave-front area

$$dA_1 = x_\phi \cdot d\phi \cdot dx_{\phi,h} \cdot \sin \theta_{\phi,h} \quad (2)$$

where $\theta_{\phi,h}$ is the sound-ray elevation angle at that point. The blast-wave energy per unit area normal to the direction of propagation, similar except for time dependence to the acoustical term intensity, will be

3. This approach starts like that given by B. Gutenberg, J. Acous. Soc. Amer. 14, 151 (1942).

* The denominator of this expression ideally becomes 2π if the explosion occurs on the ground. However, since digging a crater takes some energy, the denominator may take on a value up to twice that size, depending on soil conditions.

$$\xi_{\text{direct}} = \frac{\epsilon_D \cdot dE}{d\Lambda_1} = \frac{\epsilon_D E \cos \theta_{\phi,B} \cdot d\theta_{\phi,B} \cdot d\phi_B}{4\pi x_{\phi} \sin \theta_{\phi,h} \cdot dx_{\phi,h} \cdot d\phi_{\phi}} \quad (3)$$

Meteorological gradients capable of refracting blast or sound waves are almost always much stronger in vertical than horizontal directions. Horizontal expansion of the wave front will be essentially cylindrical; so neglecting refraction in azimuthal directions, $d\phi_{\phi}$ will equal $d\phi_B$. At a point where the horizontal distance x_{ϕ} is large compared with vertical distance $|h-B|$ from the shot point, the quotient $\cos \theta_{\phi,B} \cdot d\theta_{\phi,B} / \sin \theta_{\phi,h} \cdot dx_{\phi,h}$ is determinable from the vector form^{4/} of Snell's law,

$$V_{\phi,h} \cos \theta_{\phi,B} = V_{\phi,B} \cos \theta_{\phi,h} \quad (4)$$

Terms $V_{\phi,h}$ and $V_{\phi,B}$ represent, at altitudes h and B respectively, the sums of sound speeds in still air (c), and the horizontal ϕ components of wind velocities. Only for no wind and no ambient temperature gradient would the direct blast energy concentration about an explosion be spherically symmetrical. For no wind and a vertical temperature gradient, direct blast energy concentration would be cylindrically symmetrical about a vertical line through the explosion point. In the general case blast energy concentration at any point depends on distance from the shot point, vertical temperature gradients even beyond the span between B and h , and relative speeds and angles of winds with respect to the selected azimuth, ϕ , of the point of interest.

4. E. F. Cox, H. J. Plagge, and J. W. Reed, Bull. Amer. Meteorol. Soc. 35, 95 (1954).

Allowable energy transmission paths between explosions and points of interest are generally more numerous than those considered "direct." When either the explosion or the point of interest is near the ground, allowable paths involve reflections as well as refractions. Figure 2, two examples plotted automatically by an analogue computer named the RAYPAC, shows computed ray paths for two explosions at the Atomic Energy Commission's Nevada Test Site, one at "high altitude," one near the ground. At each strike against ground, some of the blast energy is lost; the remaining fraction, f , is reflected. If the point of measurement is at ground level, reflection raises energy concentration at the instrument.

Few weather situations produce sound-velocity-versus-altitude patterns susceptible to simple algebraic computation of blast concentration. Because of this fact, Sandia Laboratory constructed the RAYPAC for use in Nevada. But to illustrate ideas which will be applied later in more complex situations, a simple case, that of no wind and no temperature gradient (i.e., a quiet, isothermal atmosphere with pressure and density gradients), will be treated first.

At distance D in any direction from an explosion, Eq 3 reduces in this instance to

$$\xi_{\text{direct}} = \frac{\epsilon_D E}{4\pi D^2} = \frac{\epsilon_D b a^3}{3D^2}, \quad (5)$$

where $\underline{b}$ is the energy released by a unit cube of explosive, and $\underline{a}$ is the radius of the spherical charge. If the point of observation is sufficiently far from a reflector, such as the earth, direct energy will be the total energy at the receiver. The more usual experimental arrangement has receivers on or near the ground, in which case the total energy observed would be $(1+f)$ times Eq 5.

The author is unaware of any instrument satisfactory for measuring total energy per unit area of the wave front from an explosion. Furthermore, damage which sometimes occurs to nearby property appears to be more nearly related to peak overpressure, or to impulse, than to energy density, and good instruments are available for recording perturbation pressure as a function of time. A relation between ξ and root mean square perturbation pressure, $\overline{\Delta p_h}$, is

$$\xi_{\text{total}} = \frac{\overline{\Delta p_h}^2 \tau_h}{\rho_h v_h} \quad , \quad (6)$$

where ρ_h is air density; and τ_h is the effective time duration of the perturbation, all measured at altitude $\underline{h}$. Assuming the pressure-time wave form intermediate between sinusoidal (which would give a factor of 2) and N-shaped^{5/} (which would give a factor of 3), one may write a measurable quantity

$$(\Delta p_h)_{\text{peak}} \approx (2.5 \overline{\Delta p_h}^2)^{1/2} \quad , \quad (7)$$

5. J. W. M. DuMond, E. R. Cohen, W. K. H. Panofsky, and E. Deeds, J. Acoust. Soc. Amer. 18, 97 (1946).

and by combining equations, arrive at

$$(\Delta p_h)^2_{\text{peak}} = \frac{2.5 \rho_h V_h b}{3} \cdot \frac{\epsilon_D a^3}{r_h D^2} \quad (8)$$

Curtis^{6/} has experimentally measured values for ϵ_D , which at distances exceeding forty spherical-charge radii he reports as represented by $(4.7 + 1.1 \times 10^3 a/D)$ per cent. Spread of data, taken at distances between 40 and 300 charge radii, is so large that it can be as well represented as $8 \exp(60 a/D)$ per cent. His data cannot be used to decide whether 4.7 or 8 per cent of the originally released energy remains acoustical energy at infinite distance, so the exponential form will be used here.

With success of Sachs' unmodified and modified scaling^{2/} as primary support, let $a_{s,h} = (P_s/P_h)^{1/3} a$, where P_s and P_h are ambient air pressures at sea level and altitude of measurement, and assume*

$$\epsilon_D = 8 \times 10^{-2} \exp(60 a_{s,h}/D) \quad (9)$$

Curtis's measurements, all conducted at sea level, neither confirm nor refute this assumption. The term $a_{s,h}$ may be construed as the apparent charge radius when viewed from altitude h .

6. W. Curtis, Ballistic Research Laboratory Memorandum Report 544 (1951).

* Results of other assumptions are discussed at the end of this article.

Next needed for an evaluation of Eq 8 is an expression for τ_h , the effective time duration of pressure variation at the point of measurement. In an attempt to find such an expression, assume* both Hilliar's and modified Sachs' scaling valid. Both will be satisfied by Eq 8 if

$$\tau_h = k a_{s,h} \beta D^{1-\beta} / v_h, \quad \beta \neq 1 \quad (10)$$

Experiments can determine whether the resultant relation is true, and simultaneously evaluate β and the proportionality constant k .

From experiments of the types reported by Curtis^{6/} and Shreve^{7/} (see Fig. 3), β appears to be about one-half, and the constant k appears to be about 1.3. For TNT, which has a specific gravity 1.6 and releases 4.5×10^3 joules/gram, the value of b is 7.2×10^3 joules/cm³. These numbers, plus the Laplace equation for sound speed, entered into Eq 8 give

$$(\Delta p_h)_{\text{peak}} / p_h = 71 (a_{s,h} / D)^{5/4} \exp(30 a_{s,h} / D) \quad (11)^{\dagger}$$

Lord Rayleigh derives^{8/} an expression for energy reflection coefficient, which coupled with values for topsoil and air densities and sound-transmission speeds would show $r = 99.7$ per cent. Peak relative perturbation pressures observed at ground level would therefore be $2^{1/2}$ times those predicted by Eq 11.

* Results of other assumptions are discussed at the end of this article.

7. J. D. Shreve, Unpublished data from explosives measurements conducted by Sandia Corporation.

† For observations taking place at sea level, $(D/a_{s,h})$ for TNT becomes 7.5λ , where λ , the "scaled distance" often used by British and Americans for explosives measurements, is expressed in ft lb^{-1/3}. Eq 11 becomes $5.6 \lambda^{-5/4} \exp(4/\lambda)$; effective time duration at 0°C becomes $\tau_h = 0.45 W_{lb}^{1/3} \lambda^{1/2}$ milliseconds.

8. Lord Rayleigh, Theory of Sound, Dover Publication (1945), Sec. 270.

Figure 3 compares both the free-air and ground-level values of Eq 11 with experimental data. It is encouraging to note that although values of both β and the τ -proportionality constant were selected to match Curtis' and Shreve's relative peak pressure measurements, the τ_h values computed therefrom are within a factor of 1.5 of the positive-overpressure durations reported by Curtis, and in the correct direction for these short distances in view of the approximation made in Eq 7.

Because of the small physical size of a , peak pressure in a non-moving isothermal atmosphere fails to be interesting at very great distances. For a one-pound TNT charge, $a = 1.6$ inches; for one metric ton, $a = 53$ cm; and for even a nominal 20-kiloton-equivalent atomic bomb, a is only 46 feet! At ground level, peak overpressure from a one-metric-ton explosion would be only one millibar at 5.3 kilometers (3.3 miles). Interesting situations arise only when meteorological gradients destroy spherical divergence of energy and concentrate the energy in some particular direction.

One interesting and relatively frequently occurring situation is easily computed (Fig. 4): that related to an explosion on the ground, points of interest on the ground, and sound velocity in the azimuth of interest increasing linearly with altitude from ground up to the determined height of inversion, h_i . Values of functions at ground level are indicated by zero subscript.

Blast rays emanating from the source at angles less steep than arc $\cos (V_o/V_i)$ will be refracted back to earth at varying distances^{4/}

$$x = 2 (h_i - h_o) V_o \tan \theta_o / (V_i - V_o) , \quad (12)$$

depending on the initial elevation angle θ_o . The maximum direct striking distance will be

$$R = 2 (h_i - h_o) \left[(V_i + V_o) / (V_i - V_o) \right]^{1/2} , \quad (13)$$

and the minimum will be zero distance. Each point on the ground along this azimuth thus has an infinite number of rays striking it.

Combination of Eqs 3 and 12 gives the direct energy per unit area of wave front at any distance between zero and R :

$$\xi_{\text{direct}} = \frac{\epsilon_D E \cos^2 \theta_o}{2\pi x^2} . \quad (14)$$

In the real atmosphere arc $\cos (V_o/V_i)$ is never large, so $\cos^2 \theta_o$ may be taken as unity, ϵ_D as ϵ_x . The total incident energy per unit area at a point like P_1 (Fig. 4) will be that coming directly from the source, plus that received after one reflection from an area one-fourth as large at one-half the distance, plus that received after two reflections from an area one-ninth as large at one-third the distance, etc. At a point like P_2 all the incident energy has undergone reflection. Thus the total incident and reflected energy per unit area of wave front at x will be

$$\xi_{\text{total}} = \frac{\epsilon_x E (1+f)}{2\pi x^2} \sum_{n=N}^{n=\infty} f^n = \frac{\epsilon_x E}{2\pi x^2} \left(\frac{1+f}{1-f} \right) f^N, \quad (15)$$

where N is the whole number of maximum-length paths between the source and the point of interest.

For this inversion situation we again assume the reasonableness of Eqs 6, 7, and 9; and again the numerator term $(1+f)$, which results from only the final reflection, is replaced by two. The counterpart of Eq 8 thus becomes

$$(\Delta p_o)^2_{\text{peak}} = \frac{0.80 \rho_o V_o b a^3 f^N \exp(60a_{s,h}/x)}{3(1-f) \gamma_2 x^2}. \quad (16)$$

We come next to an evaluation of the effective signal duration under an inversion. It would appear logical that a strong inversion, hence a relatively small R from Eq 13, would condense duration; but duration should increase with distance when R is fixed. Also a highly reflecting surface might be expected to cause longer durations than would a poor reflector. Lastly, except for a factor of root two (see footnote on Eq 1), pressures for the inversion case should transform into those in quiet, isothermal air when R becomes infinite. These conditions are met if

$$\gamma_2 = k a_{s,h} (x/a_{s,h})^{1/2} / V_o (1-f)(1+x/R)^\sigma, \quad \left(\frac{1}{2} \geq \sigma > 0 \right), \quad (17)$$

although other functions could satisfy these criteria.

In the summation performed in Eq 15, which led to the term $f^N/(1-f)$, no account was taken of the phase differences associated with the infinite number of allowable paths, which must become important when the result is to be applied to peak overpressure rather than energy measurements. Even if the land were perfectly smooth compared with the wavelengths involved, transit times along paths illustrated by Fig. 4 depend on heights of ray apexes; and general roughness of terrain augments phase differences. As a result, the practical value of f over rather rough land appears to be about seventy per cent, resembling in this respect an "effective value."

Meteorological functions from which values of R are calculated, and which thus affect experimental determination of σ , are naturally varying;^{9/} but data collected at the Nevada Test Site indicate a σ of about one-half. Numerically, Eq 16 then becomes

$$(\Delta P_o)_{\text{peak}}/P_o = 1.4 \times 10^2 (0.7)^{N/2} (a_{s,h}/x)^{5/4} (1+x/R)^{1/4} \exp(30a_{s,h}/x) \quad (18)$$

Measured peak overpressures are compared with this equation in Fig. 5. Typical microbarographic recordings under inversion conditions are shown in Fig. 6. Effective signal time durations computed from Eq 17 and graphed in Fig. 5 are not unreasonable when compared with Fig. 6.

In contrast to the small physical dimension a , in nature R is seldom shorter than kilometers or miles. At a distance $x \gtrsim R$, peak overpressure

9. H. K. Schilling, et al, Amer. J. Phys. 14, 343 (1946).

would be some seventy per cent higher for a surface burst under an inversion than it would be at the same distance from a low air-burst in a quiet isothermal atmosphere. In other words, observed from such a distance, an explosion would appear 3.5 times as energetic under an inversion.

It is now noted that both Eqs 17 and 18 contain "weather" terms $\underline{N}$ and $\underline{R}$. But $\underline{N}$ is the characteristic of the ratio x/R , so the only appearance of "weather" terms comes in x/R ratio form. From Eq 13 it is seen that $\underline{R}$ is directly proportional to height of the linear inversion. If this height, too, is imagined to change by the same factor as radius of the charge, Hilliar's Principle is found to hold for the inversion case. However, for it to hold, components of winds between ground and the top of the inversion, as resolved along the line connecting shot point and measuring point, must be unchanged in view of the vector nature of Eqs 3, 12, and 13.

A simple semi-acoustical treatment has thus arrived at fair approximations for peak relative perturbation pressures (1) in free air and (2) at ground level in an isothermal still atmosphere, and (3) under a linear inversion, where an infinite number of "rays" strike the receiver. For these three cases, by assumption of an "effective signal duration," Hilliar's Principle and Sachs' modified scaling have been found valid providing allowance is made for the vector nature of winds, and providing all distances, meteorological ones included, are scaled.

In the treatment, Eqs 3, 4, 6, 7, and 9 are general, and to good approximation should be considered applicable to all types of weather and firing conditions. When conditions are complex like those shown in Fig. 2, computations

of direct energy concentration, indicated by Eq 3, and of total energy concentration received at a point are tedious; tabular computational aids, such as Tables II and III of Ref. 4, or an analogue machine such as the RAYPAC are usually employed.

But even after energy concentration has been computed, peak overpressure cannot be predicted until a τ value has been found. It is the author's hypothesis, based on the three preceding examples and the success of modified Sachs' scaling, that effective time duration can be well represented by a function $\tau_h(a_{s,h}/V_h, x/a_{s,h}, f, x/h_j)$, where h_j represents one or more meteorological heights. By allowing any meteorological situation to degenerate into the non-moving isothermal atmosphere case, the τ -function would need to change into Eq 10; thus it is further hypothesized that

$$\tau_h = k(a_{s,h}/V_h)(x/a_{s,h})^{1/2} F(f, x/h_j) \quad , \quad (19)$$

where the function F becomes unity for the degenerate situation.

If this hypothesis is true, then Hilliar's Principle, modified to read "Under similar ambient conditions, pressures and other properties of a shock wave will be unchanged if all scales of length and time are changed by the same factor as the dimensions of the charge," will be true. When the transmitting medium is air, the italicized phrase refers to relative vector winds as well as ambient temperatures and pressures.

Modification of Hilliar's Principle as here proposed does not settle the problem of "scaling" in an inhomogeneous atmosphere. Assumptions in Eqs 9 and 10 were based on modified Sachs' scaling. Had it been assumed that

$$\epsilon_D = 8 \times 10^{-2} \exp(60a_{B,h}/D), \text{ and } \tau_h = k a_{B,h}^{1/2} D^{1/2} P_s/P_{B,h} V_h, \quad (20)$$

where $a_{B,h} = (P_B/P_h)^{1/3} a$ is the apparent charge radius when exploded at altitude B and viewed from altitude h , then an equation identical to Eq 11, with $a_{B,h}$ replacing $a_{s,h}$, would have produced a different "scaling" system. Use of Eq 20 with P_h in place of P_s in the τ_h expression would have given

$$(\Delta P_h)_{\text{peak}} / (\rho_h V_h)^{1/2} = (0.2bV_h/3k)^{1/2} (a_{B,h}/D)^{5/4} \exp(30a_{B,h}/D), \quad (21)$$

which resembles an "acoustic scaling method" considered by Haskell.

Any number of other assumptions could have been made; presumably one pair would result in a close approximation to Haskell's "inverse acoustic scaling," another to the Ledsham-Pike pressure relation, still another pair to the Theilheimer-Rudlin relation. Because of its simplicity and the good agreement between experimental data and modified Sachs' scaling as shown by Broyles, the author now prefers that scaling.

Fig. 1
MEASURED
PEAK-TO-PEAK PRESSURES
(MILLIBARS)
FROM IDENTICAL 1.2-TON
T.N.T. EXPLOSIONS
UNDER DIFFERENT
WEATHER CONDITIONS

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TWO EXAMPLES OF RAYPAC WORK

The left curved line depicts input data: ratios of sound velocities V toward a selected azimuth (bearing), at altitudes, to burst-height sound velocity V_B . Shot-time air temperatures, and winds resolved toward the east (i.e., bearing 100°), for the High Altitude Shot, introduced on the RAYPAC, produced the upper representation of sound rays. No blast struck earth more than 20 miles east from the shot, but all rays emitted in the vertical angle between plus and minus 12° from the horizontal were channeled between 11,000 and 36,000 feet above terrain. The lower example utilized predicted yield and forecast temperatures and winds resolved toward the southeast for proposed shot of an atomic device called "Apple 2." Strong blast focuses are predicted at 32, 64 and 96 miles, and also at 41 and 83 miles. Had the shot been fired under the predicted conditions, both Las Vegas and Boulder City would have received glass-breaking shock overpressures, 4.5 and 2.6 millibars respectively.

Fig. 3. Equation 11 is compared with free-air blast pressure measurement taken at sea level, and with ground-level data taken at Albuquerque, New Mexico, 6000 feet above sea level. Curtis used one- and five-pound charges; Shreve used 256-pound charges. Close to an explosion, ground-level pressure values change with burst height (see Sec. 3.20-3.72, The Effects of Atomic Weapons, McGraw-Hill Book Co., New York, 1950).

Fig. 4.

Fig. 5. Peak pressure fluctuations under inversions, measured at the AEC Nevada Test Site, 4000 feet above sea level, are compared with Eq. 18. Wave forms are shown in Fig. 6. Effective duration values computed from Eq. 17 are plotted at top of figure.

WAVE FORMS FROM 1.2-TON TNT EXPLOSION
NEVADA PROVING GROUND - 0300 PST, 18 APRIL 1953
MAXIMUM DIRECT STRIKING RANGE = 3.19 MILES

WAVE FORMS FROM 1.2-TON TNT EXPLOSION
NEVADA PROVING GROUND - 0300 PST, 25 APRIL 1953
MAXIMUM DIRECT STRIKING RANGE = 4.15 MILES

Fig. 6. Wave forms recorded by microbarographs at various distances from explosions. Meteorological soundings on both occasions showed a linear inversion like Fig. 4.

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