CROSSOVER FROM BCS TO BOSE SUPERCONDUCTIVITY: A FUNCTIONAL INTEGRAL APPROACH*

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CROSSOVER FROM BCS TO BOSE SUPERCONDUCTIVITY: A FUNCTIONAL INTEGRAL APPROACH

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We use a functional integral formulation to study the crossover from cooperative Cooper pairing to the formation and condensation of tightly bound pairs in a 3D continuum model of fermions with attractive interactions. The inadequacy of a saddle point approximation with increasing coupling is pointed out, and the importance of temporal (quantum) fluctuations for normal state properties at intermediate and strong coupling is emphasized. In addition to recovering the Nozières-Schmitt-Rink interpolation scheme for T_c , and the Leggett variational results for $T = 0$, we also present results for evolution of the time-dependent Ginzburg-Landau equation and collective mode spectrum as a function of the coupling.

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The problem of the crossover [1,2] from BCS theory with cooperative Cooper pairing to the formation and condensation of composite bosons has attracted considerable attention [3-5] after the discovery of the high T_c superconductors which have pair sizes comparable to the average carrier separation. In this paper we summarize some of the results of an extensive analysis [6] of this problem in a 3D continuum model of fermions (mass m , Fermi energy ϵ_F) with attraction characterized by an s -wave scattering length a_s .

We use a standard imaginary-time functional integral formulation at temperature β^{-1} and decouple the fermion interaction with a pair field Δ . The effective action, written in terms of the Nambu propagator G , is then given by

$$S_{\text{eff}} = \frac{1}{g} \int_0^\beta d\tau \int dx |\Delta(x)|^2 - \text{Tr} \ln G^{-1}[\Delta(x)].$$

The bare attraction g is replaced [3] by the scattering length a_s .

First we consider a saddle-point estimate of the transition temperature T_0 , defined by the solution of $\delta S_{\text{eff}}/\delta \Delta[\Delta = 0] = 0$, and $N = -\partial \Omega/\partial \mu$. The saddle point approximation for the thermodynamic potential, $\Omega_0 = S_{\text{eff}}[\Delta = 0]/\beta$, then allows one to solve for T_0 and the chemical potential μ as a function of the coupling. In the weak coupling $g \rightarrow 0$ limit, $(k_F a_s)^{-1} \rightarrow -\infty$

and we find the BCS results $\mu \simeq \epsilon_F$ and $T_0 \simeq 8e^{-2}\gamma\pi^{-1}\epsilon_F \exp(-\pi/2k_F|a_s|)$.

However, the saddle-point approximation becomes progressively worse with increasing coupling. In the strong coupling $g \rightarrow \infty$ limit $(k_F a_s)^{-1} \rightarrow +\infty$ and one finds tightly bound pairs with binding energy $E_b = 1/ma_s^2$. The nondegenerate Fermi system has $\mu \simeq -E_b/2$ and $T_0 \simeq (E_b/2)/\ln(E_b/\epsilon_F)^{3/2}$. This result is an artifact of the approximation and the strong coupling T_0 is related to the pair dissociation scale rather than the T_c ($\ll T_0$) at which coherence is established.

We next include the effects of gaussian fluctuations using $S_{\text{gauss}} = S_{\text{eff}}[\Delta = 0] + \sum_{\mathbf{q}, i\mathbf{q}_m} \Gamma^{-1}(\mathbf{q}, i\mathbf{q}_m) |\Delta(\mathbf{q}, i\mathbf{q}_m)|^2$, with $\Gamma^{-1} = 1/g - \chi_{\text{pair}}^0$, to obtain the Ω used in the number equation $N = -\partial \Omega/\partial \mu$. While the weak coupling results are unaffected, the strong coupling results are completely changed by the appearance of a bound state pole in Γ . Within a pole approximation the $g \rightarrow \infty$ partition function reduces to that of a free Bose gas (mass $2m$ and density $n/2$) with the Bose condensation $T_c = [n/2\zeta(3/2)]^{2/3} \pi/m$.

We see that the bosonic degrees of freedom, which become important with increasing g , can only be adequately described by retaining the full frequency dependent gaussian fluctuations in the normal state. Numerical calculations show that

these quantum fluctuations affect T_c significantly even in the intermediate coupling regime. The gaussian theory for T_c is identical to the Nozières-Schmitt-Rink interpolation [2]. Going beyond this approximation we find a Δ^4 term, which for $g \rightarrow \infty$ represents a two-body repulsion between the composite bosons as shown below.

We next turn to the evolution of the time-dependent Ginzburg-Landau (TDGL) equation near T_c from the BCS to the Bose regime. We first expand S_{eff} to order Δ^4 and then in powers of $|q|$ and ω (after analytically continuing $iq_m \rightarrow \omega + i0^+$). On either side of the point $\mu(T_c) = 0$ (see below) one has a well defined TDGL equation, for $\varepsilon = (T - T_c)/T_c \ll 1$, provided $\Delta \ll |\omega| \ll \min(T_c, \mu(T_c))$.

In the BCS regime we obtain the usual TDGL with characteristic length scale $\xi(T) = \xi_0 \varepsilon^{-1/2}$ and time scale $\tau(T) = \tau_0 \varepsilon^{-1}$ with $\xi_0 \simeq v_F/T_c$ ($\sim \xi_{\text{pair}}$, the pair size), and $\tau_0 \simeq 1/T_c$. The width of the Ginzburg region is $(T_c/\varepsilon_F)^4$. The dynamics of the order parameter is overdamped reflecting the continuum of fermionic excitations into which a pair can decay. There is in addition a small $\mathcal{O}(T_c/\varepsilon_F)$ propagating part since the model is not particle-hole symmetric.

With increasing coupling the propagating piece grows while the damped part diminishes. A singular point is obtained at an intermediate coupling where $\mu(T_c) = 0$. The vanishing energy scale does not allow a low frequency expansion and even a linearized TDGL description fails.

Beyond this singular point an essentially propagating mode is obtained. In the extreme strong coupling Bose limit the TDGL reduces to the Gross-Pitaevskii equation for a dilute Bose gas. The repulsion between bosons is characterized by a (boson) scattering length $a_b = 2a_s > 0$ with $n_b a_b^3 \ll 1$, where $n_b = n/2 \sim k_F^3$ (in terms of the k_F of the constituent fermions). The prefactors of the divergent length and time scales at the transitions are given by $\xi_0 \simeq k_F^{-1}/\sqrt{k_F a_s} \gg \xi_{\text{pair}} \simeq a_s$ and $\tau_0 \simeq T_c^{-1}/(k_F a_s)$ respectively. These are both much longer than the microscopic scales because of the diluteness condition $k_F a_s \ll 1$.

We obtain the following results for the broken symmetry state. The saddle point gap and num-

ber equations at $T = 0$ are identical to Leggett's variational equations [1]. The non-trivial saddle point of S_{eff} includes the non-perturbative effects of bound state formation and condensation and we find that, at least for $T \ll T_c$, the gaussian fluctuations make only a small correction even for strong coupling (in marked contrast to the normal state discussed above).

The collective excitation spectrum at $T = 0$ is obtained from the gaussian fluctuations. For the charge neutral system considered here we find a sound mode $\omega = cq$ for long wavelengths where explicit formulae have been obtained for c , and the eigenvector corresponding to this mode, as function of the coupling. In the two extreme limits we obtain the analytical results. In the BCS limit $c = v_F/\sqrt{3}$ corresponding to the Anderson-Bogoliubov result, which evolves smoothly into the Bogoliubov mode for the dilute Bose gas with $c = (\pi n a_s/m^2)^{1/2}$. Note that the repulsion between bosons discussed above is crucial for the collective mode spectrum. Also, unlike the regime close to T_c with its singular point when $\mu(T_c) = 0$, the low frequency expansion for $T \ll T_c$ is well defined for all couplings, since a gap in the spectrum precludes a vanishing energy scale, and that the sound mode evolves smoothly from one limit to another.

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