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Heat Loading Limits for Solid Transuranic Wastes Storage (U)

by

Thomas L. Spatz

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1.0 Summary

Heat loading limits have been established for four storage configurations of TRU wastes. The calculations were performed assuming the worst case scenario whereby all the heat generated within a drum was generated within one "cut" and that this cut was located in the very center of the drum. Poly-boxes containing one HEPA filter were assumed to have a uniform heat generation throughout the filter. The maximum allowable temperatures were based on the materials in the containers. The calculated heat load limits per container (i.e. 55 gallon drum, poly-box, or black box) are summarized in the table below.

	Without Anion Resin.	With Anion Resin.
55 Gallon drums in culvert on an open TRU pad.	30 Btu/hr (8.793 Watts)	2.5 Btu/hr (0.73 Watts)
55 Gallon drums on an enclosed TRU pad.	30 Btu/hr (8.793 Watts)	3 Btu/hr (0.879 Watts)
Poly-boxes in a culvert on an open TRU pad.	60 Btu/hr (17.58 Watts)	14 Btu/hr (4.1 Watts)
Black boxes on an open TRU pad.	22 Btu/hr (6.448 Watts)	0.5 Btu/hr (0.146 Watts)

A comparison between the drum center temperature for a uniform heat load distribution and for the center temperature when the heat load is confined to one cut in the center of the drum is also illustrated. This comparison showed that the heat load of a particular drum can be more than doubled by distributing the sources of heat uniformly throughout the container.

2.0 Introduction

Job-control TRU waste generated at the Savannah River Site (SRS) is packed in multiple barrier layers of plastic and stored in 55 gallon drums, carbon-steel black boxes, polyethylene boxes, and casks. The waste generated before 1986 was stored on concrete slabs, referred to as TRU pads, under 2 to 4 feet of earthen cover in the Solid Waste Disposal Facility (SWDF). Safe heat loading limits have previously been calculated for this earthen-covered storage configuration [1]. The waste generated after 1986 has been stored above ground on TRU pads. The above ground storage configurations were investigated in

this report under the task plan 'Heat Loading Limit Calculation for Storage of Solid Transuranium Wastes' (Task number 93-013-1).

There are four storage configurations for the above-ground storage of solid transuranic wastes that are presented.

- 55 gallon drums in concrete culvert on an open TRU pad.
Job-control waste is bagged into a PVC bag and placed into a 5 gallon pail which is lined with a polyethylene bag. The bag is removed from the pail, after assay, and referred to as a "cut". Seven of these cuts are placed into a 55 gallon drum. The 55 gallon drum is lined with both a polyethylene bag and then a polyethylene liner. Each concrete culvert contains a total of 14 drums in two layers of 7 drums per layer. The concrete culvert is stored on an open TRU pad. Details of the "cut", drum, polyethylene liner, and culvert can be found in Ref. [2,3,4,5, and 6].
- 55 gallon drums on enclosed TRU pad.
These are the same 55 gallon drums and contents as mentioned above. These drums are not placed inside of the concrete culvert but are simply stacked, 2 high, on a TRU pad. One TRU pad may contain 4200 drums. The TRU pad is covered with a free-span portable enclosure that can be built around an existing TRU pad. Details of the enclosure can be found in Ref. [7].
- Poly-boxes in concrete culverts on an open TRU pad.
Poly-boxes are constructed of 1/8" thick cross-linked polyethylene. They are rectangular in shape with an approximate volume of 65 gallons. There is a total of 8 poly-boxes placed in a concrete culvert in 2 layers of 4 boxes per layer. Each poly-box contains one HEPA filter [8]. The HEPA filter was assumed to have a uniform heat generation throughout the filter.
- Black boxes on an open TRU pad.
Black boxes are large carbon steel boxes (12'x18'x7') that are placed on an open TRU pad. The contents of the black boxes are larger items than the drums can hold; such as cabinets, tanks, glove boxes, desks, etc.. Details of the black boxes can be found in Ref. [9].

Safe heat loading limits are defined to prevent the ignition of anion exchange resin, the ignition of paper, the melting of low-density polyethylene, and the melting of high-density polyethylene. A table of limiting temperatures is given below.

SUBSTANCE [Ref.]	DEGRADATION TEMP.	MELT/FAILURE TEMP.
High-density polyethylene [10]	109±24.7°C (228.2±44.46°F)	123.5±23.3°C (254.3±41.94°F)
Low-density polyethylene [10]	82.5±31.8°C (180.5±57.24°F)	92.8±17.7°C (199.04±31.86°F)
Anion resin decomposition [1]	87.78°C (190°F)	
Paper ignition [1]		232.2°C (450°F)
ALARA [10]	37°C (98.6°F)	79°C (174.2°F)
PVC [10]	66°C (150.8°F)	89.3±17.9°C (192.74±32.22°F)

The melting temperature of high-density polyethylene corresponds to the melting of the drum liner and the melting temperature of the poly-boxes. The ignition temperature of paper corresponds to a maximum temperature within a cut, as long as the cut does not contain anion resin. The anion resin decomposition temperature corresponds to the maximum temperature within a cut that does contain anion resin. The failure of the polyethylene bag containing one cut corresponds to the melting temperature of low-density polyethylene. The failure temperature of PVC corresponds to the melting of the HEPA filter liner bag. ALARA is packing material used in the black boxes and was not considered in establishing of the heat load limits of the black boxes. ALARA was not used in any of the other storage configurations.

3.0 Calculation Method

In this section a description of the assumptions made and the approach used will be discussed. It is important to note that all assumptions made were conservative. Each configuration will be discussed separately. Details of the actual numbers used to yield the results are given in Section 9, "Attachments; Calculation Sheets".

To keep the scope of this task within the guidelines of a "Small Task" (see Ref. [11]) and due to the time-frame constraints the following assumption was made. The heat transfer within the drums and poly-boxes was simplified from the 2-D and 3-D conduction with heat generation problems to a one dimensional conduction with heat generation problem by using a spherical coordinate system. The

drum, for example, was assumed to be encompassed by a sphere. The sphere's surface temperature was assumed to be the same temperature as the drum surface would be. This imaginary sphere would result in a higher center temperature than the actual drums center temperature, which makes this simplification very conservative. The hottest any drum can get is when all the heat being generated is evolved by one cut at the very center of this imaginary sphere. This "one cut generation" scenario will be used to establish the heat load limits. The "cut" will also be characterized by a sphere of radius 0.54 ft and located at the center of the drum (or poly-boxes), also modeled by an imaginary sphere [12,13]. The total heat generated within one culvert will be assumed to be the product of the heat generated by the hottest drum and the number of drums inside the culvert.

The thermal conductivity of the job-control waste is non-uniform and unknown. Previous calculations have used 0.05 Btu/hr/ft/°F as the assumed thermal conductivity of the bagged waste [1]. To be thoroughly conservative the calculations will be made for the thermal conductivity of air (0.02 Btu/hr/ft/°F) and for the same thermal conductivity that was used in previous calculations (0.05 Btu/hr/ft/°F). Conclusions drawn from the results will be made upon the calculations with the thermal conductivity of air since they are the more conservative of the two calculations.

Convective heat transfer is present in every storage configuration under investigation. For example there is convection from the drums to the air under the enclosure or there is convection from the poly-boxes to the air inside the culvert and from the air inside the culvert to the culvert walls. Previous calculations have used the following two equations for the vertical and horizontal heat transfer coefficients [1].

$$h_v = 0.23 * (\Delta T)^{\frac{1}{3}}$$

$$h_H = 0.25 * (\Delta T)^{\frac{1}{3}}$$

These equations are valid for heat transfer from a vertical or horizontal surface into a semi-infinite space. For the packing matrix of either the drums or the poly-boxes inside of the culvert these equations are not totally valid. The center drum in a culvert, for example, with six drums surrounding it offers more resistance to air flow along its vertical surfaces than the same drum would

without the other six drums surrounding it. The scope of this task did not allow for any detailed analysis of the heat transfer coefficients; therefore, another conservative simplifying assumption had to be made. The heat transfer coefficient was divided by two for any surface that appeared to be in close proximity with another surface.

In every storage configuration under investigation there is an outer surface that is exposed to the atmosphere. This is a significantly different scenario than the under-ground storage of solid transuranium wastes. These outer surfaces are subject to incident solar radiant heat transfer, emittance of thermal radiation heat transfer, convective heat transfer to the atmosphere, and the generated heat from the waste. The incident solar radiation, just outside the atmosphere, is reported to be 442.8 Btu/hr/ft² [14,15]. An additional 20-40% is absorbed by the earth's atmosphere, but for this study it was assumed that only 10% is absorbed by the atmosphere and that 90% of the 442.8 Btu/hr/ft² strikes the surface in question [16].

The amount of thermal radiation and convection leaving the outer surface is dependent upon the air temperature. Once again, in an attempt to be conservative the air temperature was assumed to be 110°F. This was a reasonable assumption for the worst case air temperature.

The incident solar radiation on a surface and the emitted thermal radiation from a surface are dependent on the surface absorbtivity and emissivity, respectively, of the surface. These properties are not known with any degree of certainty for the three types of exposed surfaces under investigation. This was the primary source of uncertainty for these calculations. A literature search was performed in an attempt to establish these surface properties within a 95% confidence interval. The discussion of these surface properties will be given in the "Uncertainty Analysis", Sec. 5.0.

The modes of heat transfer for each of the 4 storage configurations will be discussed next. Details of the analysis can be seen in the calculation sheets in the "Attachments", Sec. 9.0.

3.1 55 gallon drums in concrete culvert on an open TRU pad.

- Conduction with generation through the cut at the center of the drum.

- Conduction through the rest of the drum.
- Convection from the drum walls to the air inside the culvert.
- Convection from the air inside the culvert to the culvert walls and lid.
- Conduction through the culvert walls and lid.
- Incident solar radiation, emitted thermal radiation, convection to atmosphere, and heat generated from waste at the culvert outer surface.

3.2 55 gallon drums on enclosed TRU pad.

- Conduction with generation through the cut at the center of the drum.
- Conduction through the rest of the drum.
- Convection from the drum walls to the air inside the enclosure.
- Convection from the air inside the enclosure to the enclosure surface.
- Incident solar radiation, emitted thermal radiation, convection to atmosphere, and heat generated from waste at the enclosure surface.

3.3 Poly-boxes in concrete culvert on an open TRU pad.

- Conduction with generation within the HEPA filter.
- Conduction through the rest of the box.
- Convection from the box walls to the air inside the culvert.
- Convection from the air inside the culvert to the culvert walls and lid.
- Conduction through the culvert walls and lid.
- Incident solar radiation, emitted thermal radiation, convection to atmosphere, and heat generated from waste at the culvert outer surface.

3.4 Black boxes on an open TRU pad.

- Conduction with generation through the cut at the center of the box.
- Conduction through the rest of the box.
- Incident solar radiation, emitted thermal radiation, convection to atmosphere, and heat generated from waste at the black box outer surface.

4.0 Results

The results will be presented for each configuration separately.
Graphs will be presented where each surface of interest is plotted

versus the heat generated per container. Also plotted will be error bars for the surface temperatures. The heat loading limits will be based on the most conservative assumed thermal conductivity of the waste of 0.02 Btu/hr/ft/°F.

4.1 55 gallon drums in concrete culvert on an open TRU pad.

The results from the calculations for the 55 gallon drums inside a concrete culvert on an open TRU pad are shown in Fig. 1. The shaded areas represent the constraints of the melting temperature of high-density polyethylene, from 212.4°F to 296°F, and for the ignition temperature of paper, above 450°F. The uncertainty of all the surface temperature measurements is $\pm 7.5^\circ\text{F}$. The surface temperature uncertainty will be discussed in detail in the "Uncertainty Analysis" Sec. 5.0. The results in Fig. 1 represent the worst case scenario where all the heat generated within the drum is generated within one "cut" at the center of the drum. It will be shown later by comparison with a uniform heat generation scenario that the "one cut generation" assumption yields the hottest internal drum temperature.

The three temperature curves of interest, in Fig. 1, are the drum surface temperature, the temperature at the cut boundary, and the center of the cut temperature (or center of drum temperature).

- The drum surface temperature is constrained by the lowest melt/failure temperature of the high-density polyethylene liner. Figure 1 illustrates that even at 100 Btu/hr (29.3 Watts) the drum surface will not get hot enough to melt the liner.
- The center of cut temperature is constrained by the anion resin decomposition temperature in the presence of anion resin, and the ignition temperature of paper when anion resin is not present. As a reminder, the anion resin decomposition temperature is 190°F. In the presence of anion resin the heat load per drum in the culvert is only about 2.5 Btu/hr (0.73 Watts). When anion resin is not present the heat load jumps to about 30 Btu/hr (8.793 Watts).
- The temperature at the cut boundary was not considered a constraint in the previous calculations but is mentioned here for completeness. The bag containing the cut (cut boundary) will melt at a heat load of about 10 Btu/hr (2.93 Watts). This bag melting may not be of any concern as long as the drum liner does not melt and there is no ignition within the drum.

The constraint of primary concern in this configuration is the ignition temperature at the center of the cut (or drum). The heat

load limit for this configuration is 30 Btu/hr (8.793 Watts) per drum when anion resin is not present and 2.5 Btu/hr (0.73 Watts) when anion resin is present.

4.2 55 gallon drums stacked on an enclosed TRU pad.

The results from the calculations for the 55 gallon drums stacked on an enclosed TRU pad are shown in Fig. 2. The shaded areas represent the constraints of the melting temperature of high-density polyethylene, from 212.4°F to 296°F, and for the ignition temperature of paper, above 450°F. The uncertainty of all the surface temperature measurements is $\pm 20^\circ\text{F}$. The surface temperature uncertainty will be discussed in detail in the "Uncertainty Analysis" Sec. 5.0. The results in Fig. 2 represent the worst case scenario where all the heat generated within the drum is generated within one "cut" at the center of the drum.

The three temperature curves of interest, in Fig. 2, are the drum surface temperature, the temperature at the cut boundary, and the center of the cut temperature (or center of drum temperature).

- The drum surface temperature is constrained by the lowest melt/failure temperature of the high-density polyethylene liner. Figure 2 illustrates that even at 80 Btu/hr (23.45 Watts) the drum surface will not get hot enough to melt the liner.
- The center of cut temperature is constrained by the anion resin decomposition temperature, in the presence of anion resin, and the ignition temperature of paper, when anion resin is not present. As a reminder, the anion resin decomposition temperature is 190°F. In the presence of anion resin the heat load per drum in the culvert is only about 3 Btu/hr (0.879 Watts). When anion resin is not present the heat load jumps to about 30 Btu/hr (8.793 Watts).
- The temperature at the cut boundary was not considered a constraint in the previous calculations but is mentioned here for completeness. The bag containing the cut (cut boundary) will melt at a heat load of about 6 Btu/hr (1.758 Watts). This bag melting may not be of any concern as long as the drum liner does not melt and there is no ignition within the drum.

The constraint of primary concern in this configuration is the ignition temperature at the center of the cut (or drum). The heat load limit for this configuration is 30 Btu/hr (8.793 Watts) per drum when anion resin is not present and 3 Btu/hr (0.879 Watts) when anion resin is present.

4.3 Poly-boxes in concrete culverts on an open TRU pad.

The results from the calculations for the poly-boxes inside a concrete culvert on an open TRU pad are shown in Fig. 3. The shaded areas represent the constraints of the melting temperature of high-density polyethylene from 212.4°F to 296°F, the melting temperature of PVC from 160°F to 225°F, and for the ignition temperature of paper above 450°F. The uncertainty of all the surface temperature measurements is $\pm 7.5^\circ\text{F}$. The surface temperature uncertainty will be discussed in detail in the "Uncertainty Analysis" Sec. 5.0. The results in Fig. 4 represent the worst case scenario where all the heat generated within the poly-box is uniformly generated within the HEPA filter. Each poly-box contains one HEPA filter surrounded by two PVC bags [8].

The three temperature curves of interest, in Fig. 3, are the poly-box surface temperature, the temperature at the filter boundary, and the center of the filter temperature (or center of poly-box temperature).

- The box surface temperature is constrained by the lowest melt/failure temperature of the high-density poly-box. Figure 3 illustrates that even at 100 Btu/hr (29.3 Watts) the box surface will not get hot enough to melt the poly-box itself.
- The center of filter temperature is constrained by the anion resin decomposition temperature, in the presence of anion resin, and the ignition temperature of paper, when anion resin is not present. As a reminder, the anion resin decomposition temperature is 190°F. In the presence of anion resin the heat load per poly-box in the culvert is only about 14 Btu/hr (4.1 Watts). When anion resin is not present the heat load jumps to about 130 Btu/hr (38.09 Watts).
- The temperature at the filter bag was not considered a constraint in the previous calculations but is mentioned here for completeness. The bag containing the HEPA filter will melt at a heat load of about 6 Btu/hr (1.758 Watts). This bag melting may not be of any concern as long as the poly-box does not melt and there is no ignition within the poly-box.

The constraint of primary concern in this configuration is the failure temperature of the high-density polyethylene poly-box in conjunction with the center of the filter temperature in the presence of anion resin. The heat load limit for this configuration is 60 Btu/hr (17.58 Watts) per box when anion resin is not present. This 60 Btu/hr limit assumes that there is a possibility of the inner liner bag failing but that the final barrier, the poly-box itself, will

not fail. The heat load limit reduces to 14 Btu/hr (4.1 Watts) when anion resin is present.

4.4 Black boxes on an open TRU pad.

The contents of the black boxes is varied. A typical black box contents may be a limestone drum cabinet surrounded by bags of miscellaneous wastes [17]. The results presented here assume that all the heat generated within the black box is generated within one of the bags of miscellaneous wastes and that this bag is located in the middle of the black box.

The results from the calculations for the black boxes on an open TRU pad are shown in Fig. 4. The shaded areas represent the constraints of the melting temperature of high-density polyethylene, from 212.4°F to 296°F, and for the ignition temperature of paper, above 450°F. The uncertainty of all the surface temperature measurements is $\pm 15^\circ\text{F}$. The surface temperature uncertainty will be discussed in detail in the "Uncertainty Analysis" Sec. 5.0. The results in Fig. 4 represent the worst case scenario where all the heat generated within the black box is generated within one "cut" at the center of the box.

The two temperature curves of interest, in Fig. 4, are the temperature at the cut boundary and the center of the cut temperature (or center of box temperature).

- The center of cut temperature is constrained by the anion resin decomposition temperature, in the presence of anion resin, and the ignition temperature of paper, when anion resin is not present. As a reminder, the anion resin decomposition temperature is 190°F. In the presence of anion resin the heat load for the black box is 0.5 Btu/hr (0.146 Watts). When anion resin is not present the heat load jumps to about 22 Btu/hr (6.448 Watts).
- The temperature at the cut boundary was not considered a constraint in the previous calculations but is mentioned here for completeness. The bag containing the cut (cut boundary) will melt at a heat load of about 2 Btu/hr (0.586 Watts). This bag melting may not be of any concern as long as there is no ignition within the box.

The constraint of primary concern in this configuration is the ignition temperature at the center of the cut (or box). The heat load limit for this configuration is 22 Btu/hr (6.448 Watts) per black box

when anion resin is not present and 0.5 Btu/hr (0.146 Watts) when anion resin is present.

4.5 Uniform heat distribution.

All the graphs presented in Figs. 1 to 4 present data with the assumption that all the heat generated has been generated in one cut at the center of the drum (or poly-box). It was mentioned that this "one cut generation" assumption was the worst case scenario for the maximum temperature within a drum (or poly-box). Figure 5 illustrates the difference in the temperature at the center of the drum for a uniform heat load distribution and for the "one cut generation". This illustrates that a higher heat load per drum can be attained by dispersing the heat source over all the cuts within the drum. The constraining criteria then changes from the ignition of paper at the drum center to the melting of the drum liner at a heat load in excess of 100 Btu/hr (29.3 Watts) per drum.

5.0 Uncertainty analysis.

In all four storage configurations under investigation the worst case scenario was assumed. The temperature increases within the waste container (drum or poly-box) was based on the thermal conductivity of air to yield a conservatively high temperature at the center of the waste container. The uncertainty in the calculations comes from the lack of knowledge about the thermal radiation surface properties of the three external surfaces under investigation. These three external surfaces are the concrete culverts, the enclosure, and the black boxes (carbon black steel).

A literature search was done to obtain values of the emissivity and solar absorptivity of the three external surfaces. In the literature, something resembling the surface in question had to be assumed to have similar surface properties; for example, the black carbon steel was approximated as flat black paint. The results of the literature search are given in the table below.

		ϵ	α_s
Black carbon steel	Carbon flame spray [18]		.68
	Black paint [15]	.96-.98	
	ACI-HW (60Ni,12Cr)[19]	.89-.82	

	Flat black lacquer [19]	.92-.96	
	Black silicon paint(flat)[22]	.81	.89
	Carbon black in acrylic binder[22]	.83	.94
	Black gloss paint [23]	.9	
	Black lacquer paint [23]	.8-.93	
	graphite [23]		.85
	Flat black epoxy paint [14]	.888	.951
	Mean $\pm 2\sigma$	.89067 $\pm$.12576	.8622 $\pm$.21922
Concrete	Concrete [15]	.94	
	Concrete [20]	.95	
	Concrete roof tiles [21]	.88-.93	
	Concrete [21]	.88	.6
	Concrete [23]	.94	
	Mean $\pm 2\sigma$	.92 $\pm$.065	.6 $\pm$.06(assumed)
Enclosure	White paint [14]	.882	.248
	White paper [15]	.95	.28
	White paper [19]	.92-.96	
	Cloth [20]	.95	
	White silicon paint (glossy) [22]	.75	.26
	Papers [23]	.91-.95	
	White paints [23]	.89-.96	
	Mean $\pm 2\sigma$	.9122 $\pm$.1276	.2245 $\pm$.1549

The mean values for the emissivity and absorptivity were obtained and the 95% confidence interval of 2σ was also obtained. The surface temperatures were evaluated at the two extreme cases of $\epsilon+2\sigma$, $\alpha-2\sigma$ and $\epsilon-2\sigma$, $\alpha+2\sigma$. The difference between these two extreme

surface temperatures were used to establish the uncertainty of the surface temperatures that is shown on the graphs of Figs. 1-4. The uncertainty of the surface temperatures is independent of the heat load applied from inside the container, but is related solely to the uncertainty of the thermal radiation surface properties. The uncertainty of the external surface temperature propagates to every surface and boundary temperature inside that surface. To summarize, for the four external surfaces under investigation the uncertainty of any temperature related to that storage configuration is given in the table below.

Storage configuration	Temp. uncertainties
55 gallon drums in concrete culvert on an open TRU pad	$\pm 7.5^{\circ}\text{F}$
55 gallon drums stacked on an enclosed TRU pad	$\pm 20^{\circ}\text{F}$
Poly-boxes in concrete culverts on an open TRU pad	$\pm 7.5^{\circ}\text{F}$
Black boxes on an open TRU pad	$\pm 15^{\circ}\text{F}$

6.0 Conclusions

Heat loading limits have been established for four storage configurations under the Small Task "Heat Loading Limits Calculation for Storage of Solid Transuranium Wastes", task number 93-013-1. The calculations were performed assuming the worst case scenario where all the heat was generated within one "cut" and that this cut was located in the very center of the drum, or poly-box. The largest uncertainty for the temperature calculations was $\pm 20^{\circ}\text{F}$ in addition to all the conservative assumptions made. The heat load limits per container (i.e. 55 gallon drum, poly-box, or black box) are summarized in the table given in the "Summary", Sec. 1.0.

The concrete culvert storage configurations yielded the highest heat loads of all the configurations (30 Btu/hr (8.793 Watts) per drum). The thermal radiation properties of the concrete maintains a low external surface temperature. A transient analysis would yield an even greater heat load capacity due to the large thermal mass of the concrete culvert. A transient analysis was beyond the scope of this task but could be performed in the future if increased heat loads are anticipated.

A comparison between the drum center temperature for a uniform heat load distribution and for the center temperature when the heat load is confined to one cut in the center of the drum is also illustrated. This comparison showed that the heat load of a particular container can be more than doubled by distributing the sources of heat uniformly throughout the container.

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8.0 Figures

Figure 1

DRUMS UNDER ENCLOSURE ($k=0.02$ Btu/h/ft/°F)

DRUMS UNDER ENCLOSURE ($k=0.05$ Btu/h/ft/°F)

Figure 2

POLY-BOXES INSIDE CULVERT ($k=0.02$ Btu/h/ft/°F)

POLY-BOXES INSIDE CULVERT ($k=0.05$ Btu/h/ft/°F)

Figure 3

Figure 4

Figure 5

9.0 Attachments; Calculation Sheets

Heat load limit calculations are performed for the following set of conditions:

- I) Poly-boxes inside concrete culvert on an open TRU pad.
- II) 55 Gallon drums inside concrete culvert on an open TRU pad.
- III) 55 Gallon drums (stacked 2 tall) on an enclosed TRU pad.
- IV) Black boxes on an open TRU pad.

Microsoft Excel™ spreadsheets were used to obtain surface temperatures over a range of heat generation. The results were plotted using Cricket Graph™ software.

Note: None of the thermal radiation surface properties are given on these calculation sheets.

Evaluation of Surface Areas

55 Gallon Drums:

$$A_{\text{Horizontal}} = \pi r^2 = \pi(1 \text{ ft})^2 = 3.1416 \text{ ft}^2$$

$$A_{\text{Vertical}} = \pi DH = \pi(2 \text{ ft})(3 \text{ ft}) = 18.85 \text{ ft}^2$$

Poly-Boxes:

$$A_{\text{Horizontal}} = (16.75 \text{ in})(29.25 \text{ in}) = 489.9375 \text{ in}^2 = 3.40234 \text{ ft}^2$$

$$A_{\text{Vertical}} = 2(16.75 \text{ in})(30.75) + 2(29.25)(30.75) = 2829 \text{ in}^2 = 19.646 \text{ ft}^2$$

Concrete Culvert:

Inside:

$$A_{\text{Horizontal}} = \pi r^2 = \pi(3.0833 \text{ ft})^2 = 29.87 \text{ ft}^2$$

$$A_{\text{Vertical}} = \pi DH = 2\pi(3.0833 \text{ ft})(6.5 \text{ ft}) = 125.92 \text{ ft}^2$$

Outside:

$$A_{\text{Horizontal}} = \pi r^2 = \pi (3.66667 \text{ ft})^2 = 42.24 \text{ ft}^2$$

$$A_{\text{Vertical}} = \pi DH = 2\pi (3.66667 \text{ ft})(7.66667 \text{ ft}) = 176.63 \text{ ft}^2$$

$$\text{Culvert thickness} = 7 \text{ in} = 0.58333 \text{ ft}$$

$$\text{Culvert height} = 6.5 \text{ ft}$$

$$\begin{aligned} \text{Culvert:} \quad r_i &= 3.08333 \text{ ft.} \\ r_o &= 3.66666 \text{ ft.} \end{aligned}$$

Note: The horizontal area is only for the top of the particular enclosure. It is assumed that the amount of the generated heat that is conducted out the bottom of the enclosure is negligible. This is a conservative assumption.

Projected Areas

The projected areas need to be evaluated for the incident solar radiation.

Culvert:

The total projected area is made up of the rectangle and two half ellipses, or the rectangle and an ellipse.

$$A_{\text{Total}} = A_{\text{Rectangle}} + A_{\text{Ellipse}} = (2r)(H \sin \theta) + \pi(r^2)(\cos \theta)$$

The maximum area $\Rightarrow dA/d\theta = 0$

or,

$$(2r)(H \cos \theta) - \pi(r^2)(\sin \theta) = 0$$

$$\tan \theta = 2H/\pi r$$

Culvert:

$$H = 7.66667$$

then,

and,

$$r = 3.66667$$

$$\theta = 53.084 \text{ deg.}$$

$$A_{\text{Maximum}} = 70.32 \text{ ft}^2.$$

Black Boxes:

Maximum area with respect to θ :

$$A = (bc)(\cos \theta) + (ab)(\sin \theta)$$

$$dA/d\theta = 0 \Rightarrow \tan \theta = a/c \quad \text{or } \theta = 21 \text{ deg.}$$

Next, rotate about an angle perpendicular to the ray.

$$A = (bc \cos \theta + ab \sin \theta) \cos \phi + ac \sin \phi$$

$$\frac{dA}{d\phi} = 0 = ac \cos \phi - (bc \cos \theta + ab \sin \theta) \sin \phi$$

$$\tan\phi = \frac{ac + ab \cos\theta - ab \sin\theta}{bc \cos\theta + ab \sin\theta} \Rightarrow \phi = 28.7 \text{ deg.}$$

Then, $A_{\text{Maximum}} = \underline{263.8 \text{ ft}^2}$

Convective Heat Transfer Coefficients

The information given in this report about the heat transfer coefficient and the thermal conductivity of the waste material is from the following article.

Nuclear Safety and Heat Load Limits for Above-Grade Storage of Solid Transuranium Wastes, by B.G. Clontz, [DPSPU 77-11-24].

The above-mentioned article determined the heat load limits for the storage of solid transuranium wastes in 55 gallon drums within concrete culverts that are buried underground. I used these heat transfer correlations and properties to be consistent with the previously established limits.

Convective Heat Transfer Coefficients:

$$h_{\text{Horizontal}} = 0.25(\Delta T)^{1/3} \left(\frac{\text{BTU}}{\text{Hr} \cdot ^\circ\text{F} \cdot \text{ft}^2} \right)$$

$$h_{\text{Vertical}} = 0.23(\Delta T)^{1/3} \left(\frac{\text{BTU}}{\text{Hr} \cdot ^\circ\text{F} \cdot \text{ft}^2} \right)$$

Where flow is obstructed:

$$h = \frac{0.23}{2}(\Delta T)^{1/3} = \frac{h_v}{2}$$

Thermal Conductivities:

$$(k_{\text{waste}})_{\text{conservative}} = k_{\text{air}} = 0.02 \frac{\text{BTU}}{\text{Hr} \cdot ^\circ\text{F} \cdot \text{ft}}$$

I. Poly-boxes inside concrete culvert

Definition of some surface temperatures:

$T_{\infty} = 110^{\circ}\text{F}$

T_1 = Box surface temperature
 T_2 = Air inside culvert temp.
 T_3 = Inside culvert wall temp.
 T_4 = Outside culvert wall temp.
 T_{∞} = Worst case outdoor temp.

- A. Convection from the box surface to the air inside the culvert.
 (q = heat generated in one polybox)

Bottom boxes at these locations are the worst case (restricted convection over half the vertical area, and no convection out the top)

$$h_v = \frac{0.23}{2}(\Delta T)^{1/3}$$

$$h_v = 0.23(\Delta T)^{1/3}$$

$$q = \left[\left(\frac{0.23}{2} \right) \left(\frac{19.646}{2} \right) + 0.23 \left(\frac{19.646}{2} \right) \right] (T_1 - T_2)^{4/3}$$

$$q = 3.388935(T_1 - T_2)^{4/3}$$

or

$$T_1 = T_2 + \left[\frac{q}{3.388935} \right]^{3/4}$$

- B. Convection to culvert inner wall surface:

$$8q = h_V A_V (T_2 - T_3) + h_H A_H (T_2 - T_3)$$

$$= (.23)(125.92)(T_2 - T_3)^{4/3} + (.25)(29.87)(T_2 - T_3)^{4/3}$$

$$q = 455364(T_2 - T_3)^{4/3}$$

or,

$$T_2 = T_3 + \left(\frac{q}{4.55364} \right)^{3/4}$$

C. Conduction through the concrete culvert walls and lid:

$$8q = k \frac{2\pi H}{\ln\left(\frac{r_o}{r_i}\right)} (T_3 - T_4) + k \frac{\pi r^2}{\tau} (T_3 - T_4)$$

$$8q = \left[\frac{(0.4)2\pi(6.5)}{\ln\left(\frac{3.6666}{3.08333}\right)} + \frac{(0.4)\pi(3.08333)^2}{0.58333} \right] (T_3 - T_4)$$

$$q = 14.346(T_3 - T_4) \quad \text{or,} \quad T_3 = T_4 + \frac{q}{14.346}$$

D. Convection and radiation from outer wall to atmosphere:

Generation + Incident = Emitted + Convection
Solar Radiation

$$8q + \alpha_s A_i q_s = \epsilon \sigma A_T (T_4^4 - T_\infty^4) + h_V A_V (T_4 - T_\infty) + h_H A_H (T_4 - T_\infty)$$

$$8q + \alpha_s(70.32)(0.9442) = \epsilon(0.1714 \times 10^{-8})(42.24 + 176.63)((T_4 + 459.67)^4 - 569.67^4) \\ + (0.23)(176.63)(T_4 - 110)^{4/3} \\ + (0.25)(42.24)(T_4 - 110)^{4/3}$$

Summarizing:

$$T_1 = T_2 + \left(\frac{q}{3.388925} \right)^{3/4}$$

$$T_2 = T_3 + \left(\frac{q}{4.55364} \right)^{3/4}$$

$$T_3 = T_4 + \left(\frac{q}{14.346} \right)$$

$$8q + \alpha_s(70.32)(0.9442) = \epsilon(0.1714 \times 10^{-8})(218.87)((T_4 + 459.67)^4 - 569.67^4) \\ + 51.1849(T_4 - 110)^{4/3}$$

II. Drums Inside Concrete Culverts:

There are 14 drums inside a concrete culvert in two layers of 7 drums per layer. The concrete culvert is outdoors on an open tru pad.

Temperature
Definitions:

T_1 = Drum Surface Temperature
 T_2 = Air Inside Culvert Temp.
 T_3 = Inner Culvert Wall Temp.
 T_4 = Outer Culvert Wall Temp.
 T_{∞} = Ambient Air (= 110°F) Worst Case

A. Convection From the Drum Surface to the Inside Culvert.

Worst Case: Center bottom drum has restricted convection and only convects from vertical surface.

$$A_H = 0 \quad A_V = 18.85 \quad h_V = \frac{0.23}{2}(\Delta T)^{1/3}$$

Then,

$$q = \frac{0.23}{2}(18.85)(T_1 - T_2)^{4/3}$$

or,

$$T_1 = T_2 + \left(\frac{q}{2.16775} \right)^{3/4}$$

- B. Convection From Air to Inner Culvert Wall: (Similar to Poly-box to Culvert Analysis)

$$14q = 36.4291 (T_2 - T_3)^{4/3} \quad \text{or, } T_2 = T_3 + (q/2.60208)^{3/4}$$

- C. Conduction Through Culvert: (Similar to Ploy-box to Culvert Analysis)

$$14q = 114.768(T_3 - T_4) \quad \text{or, } T_3 = T_4 + (q/8.1973)$$

- D. Convection and Radiation from Outer Wall to Atmosphere:

$$14 \cdot q + \alpha_s A_i q_s = \epsilon \sigma A (T_4^4 - T_\infty^4) + h_V A_V (T_4 - T_\infty) + h_H A_H (T_4 - T_\infty)$$

$$\begin{aligned} 14 \cdot q + \alpha_s (70.32)(0.9)(442) &= \epsilon (0.1714 \times 10^{-8})(42.24 + 176.63) \\ &\quad \times ((T_4 + 459.67)^4 - 569.67^4) \\ &\quad + 0.23(176.63)(T_4 - T_\infty)^{4/3} \\ &\quad + 0.25(42.24)(T_4 - T_\infty)^{4/3} \end{aligned}$$

$$\begin{aligned} 14 \cdot q + \alpha_s (70.32)(0.9)(442) &= \epsilon (0.1714 \times 10^{-8})(42.24 + 176.63) \\ &\quad \times ((T_4 + 459.67)^4 - 569.67^4) \\ &\quad + 51.1849(T_4 - T_\infty)^{4/3} \end{aligned}$$

Summarizing:

$$T_1 = T_2 + \left(\frac{q}{2.16775} \right)^{3/4}$$

$$T_2 = T_3 + \left(\frac{q}{2.60208} \right)^{3/4}$$

$$T_3 = T_4 + \left(\frac{q}{8.1973} \right)$$

$$14 \cdot q + \alpha_s(70.32)(0.9)(442) = \epsilon(0.1714 \times 10^{-8})(42.24 + 176.63) \\ \times ((T_4 + 459.67)^4 - 569.67^4) \\ + 51.1849(T_4 - T_\infty)^{4/3}$$

III. Drums on Enclosed Tru Pads.

In this scenario, drums are stacked 2 high on a tru pad that is enclosed with a PVC-coated polyester cover (see OPS-WMT-900495 for enclosure details).

A. The Total Surface Area of the Enclosure Outer Cover is Given Next

$$A_{\text{Total}} = 2(A_{\text{Doors}}) + A_{\text{Sides \& Top}}$$

$$A_T = 2[(60)(25.95) - 2(1/2)((12.95)(2.28) + (11.39)(6.58) + (7.25)(5.66))] \\ + (160)(2)(13.149 + 13.154 + 9.1977 + 9.84) \\ A_T = 2(1557 - 145.5072) + 320(45.3407)$$

$$A_{\text{Total}} = 17332 \text{ ft}^2$$

(See figure on next page for details)

B. The Area for Incident Solar Radiation is Given As:

$$A_s = 1 \text{ door} + (60' \times 160')$$

Conservative: Larger Than Actual Incident Area

$$A_s = 1411.4928 + 9600 = 11011.5 \text{ ft}^2$$

RUBB - 60' SPAN HV RANGE

60' x 160'

C. For convective heat transfer from the enclosures cover, the vertical heat transfer coefficient will be used with the total area. This is a conservative simplification for the complex shape of the enclosure.

D. Convection from the Drums: (m = # of stacks)($2m$ = total drums)

Worst Case: Drum at bottom-inside row gets inhibited convection along vertical surface and no convection along horizontal surface.

$$q = \left(\frac{.23}{2} \right) (18.85) (T_1 - T_2)^{4/3}$$

$$\text{or, } T_1 = T_2 + (q/2.16775)^{3/4}$$

E. From Air to Cover:

$$2qm = (.23)(17332.01)(T_2 - T_3)^{4/3}$$

$$q = \frac{1993.18}{m} (T_2 - T_3)^{4/3}$$

$$\text{or, } T_2 = T_3 + \left(\frac{qm}{1993.18} \right)^{3/4}$$

The maximum number of drums on any tru pad is given as 4200. Therefore, $m = 2100$ number of stacks. Then,

$$T_2 = T_3 + \left(\frac{q}{0.94913} \right)^{\frac{3}{4}}$$

F. The Cover

$$2qm + \alpha_s A_s q_s = \epsilon \sigma A (T_3^4 - T_\infty^4) + h_v A_T (T_3 - T_\infty)$$

$$\begin{aligned} 2q(2100) + \alpha_s(11011.5)(0.9)(442) = \\ \epsilon(0.1714 \times 10^{-8})(17332)((T_3 + 459.67)^4 - 569.67^4) \\ + 0.23(17332)(T_3 - 110)^{4/3} \end{aligned}$$

Summarizing:

$$T_1 = T_2 + \left(\frac{q}{2.16775} \right)^{\frac{3}{4}}$$

$$T_2 = T_3 + \left(\frac{q}{0.94913} \right)^{\frac{3}{4}}$$

$$\begin{aligned} 4200q + \alpha_s(11011.5)(0.9)(442) = \\ \epsilon(0.1714 \times 10^{-8})(17332)((T_3 + 459.67)^4 - 569.67^4) \\ + 3986.36(T_3 - 110)^{4/3} \end{aligned}$$

IV. Black Boxes on Open Tru Pad:

$$q + \alpha_s A_i q_s = \epsilon \sigma A_T (T_s^4 - T_\infty^4) + h_H A_H (T_s - T_\infty) + h_V A_V (T_s - T_\infty)$$

$$q + \alpha_s (265)(0.9)(442) = \epsilon (0.1714 \times 10^{-8})(636)((T_s + 459.67)^4 - 569.67^4) \\ + 0.23(420)(T_s - 110)^{4/3} \\ + 0.25(216)(T_s - 110)^{4/3}$$

Simplifying:

$$q + \alpha_s (265)(0.9)(442) = \epsilon (0.1714 \times 10^{-8})(636)((T_s + 459.67)^4 - 569.67^4) \\ + 150.6(T_s - 110)^{4/3}$$

Temperature Profile Within The Individual Container

The surface temperatures of a particular drum or box configuration as a function of the heat generated in that container is given in the previous analysis. The customer is interested in knowing what the heat load limits are for a particular container, if all the heat is generated in one 'cut'. A cut is a bag of waste that can be idealized by a sphere of radius 0.54 ft. To simplify this analysis, I will assume spherical conduction and conduction with generation.

$$R_1 = 1.803 \text{ ft}$$

$$R_* = 0.54 \text{ ft}$$

$$R_1 = 1.90106 \text{ ft}$$

HEPA Filter

$$2r = (12^2 + 24^2 + 24^2)^{1/2}$$

$$2r = 36$$

$$r = 18 \text{ in.} = 1.5 \text{ ft} = r_*$$

For pure conduction from R_* to R_1 :

$$T_* = T_1 + \frac{q}{4\pi k} \left(\frac{1}{r_*} - \frac{1}{r_1} \right)$$

For generation from 0 to R_* :

$$T(0) = T_{\infty} + \frac{q}{8\pi k} \left(\frac{1}{r_{\infty}} \right)$$

The derivation of these temperature equations follows:

The governing equation for the region surrounding the cut is a spherical, pure conduction, equation. This applies from the outside of the cut, at the center of the drum (or box), to the surface of the imaginary sphere that encompasses the drum (or box).

$$\frac{d}{dr} \left(r^2 \frac{dT}{dr} \right) = 0$$

Integrating twice yields:

$$T(r) = \frac{C_1}{r} + C_2$$

Applying boundary condition:

$$T(r = r_1) = T_1 \quad \text{Drum Surface Temp.}$$

$$T(r = r_{\infty}) = T_{\infty} \quad \text{Cut Surface Temp.}$$

$$C_1 = \frac{T_1 - T_{\infty}}{\left(\frac{1}{r_1} - \frac{1}{r_{\infty}} \right)}$$

$$C_2 = T_1 - \frac{T_1 - T_{\infty}}{\left(1 - \frac{r_1}{r_{\infty}} \right)}$$

After simplifying, the temperature profile is:

$$T(r) = \left[\frac{T_1 - T_{\infty}}{\left(1 - \frac{r_1}{r_{\infty}} \right)} \right] \left(\frac{r_1}{r} \right) + T_1 - \left[\frac{T_1 - T_{\infty}}{\left(1 - \frac{r_1}{r_{\infty}} \right)} \right]$$

or,

$$T(r) = T_1 + \left[\frac{T_1 - T_{\infty}}{\left(1 - \frac{r_1}{r_{\infty}} \right)} \right] \left(\frac{r_1}{r} - 1 \right)$$

$$\frac{dT}{dr} = \left[\frac{T_1 - T_{\infty}}{\left(1 - \frac{r_1}{r_{\infty}} \right)} \right] \left(-\frac{r_1}{r^2} \right)$$

At $r = r_*$: (the flux leaving the cut) = (flux entering rest of drum)

Determination of temperature profile within a cut and within the non-generating surface around the cut.

The governing equation for the sphere (cut) with heat generation is as follows:

$$k \frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{dT}{dr} \right) + q''' = 0 \quad \text{where:} \quad q''' = \frac{q}{V}$$

Integrating twice yields:

$$T(r) = -\frac{q'''}{6K} r^2 + \frac{C_1}{r} + C_2$$

Boundary conditions: $T(r \rightarrow 0) = r_0 \Rightarrow C_1 = 0 \quad C_2 = T_0$

$$T(r) = T_0 - \frac{q'''}{6K} r^2$$

and $\frac{dT}{dr} = -\frac{q'''}{3K} r$

If I know that T at $r = r_*$ (cut boundary), then I can find T_0 from the boundary condition $T(r = r_*) = T_*$

$$T_* = T_0 - \frac{q'''}{6K} r_*^2 \quad \text{or,} \quad T_0 = T_* + \frac{q'''}{6K} r_*^2$$

$$V = \frac{4}{3} \pi r_*^3$$

substituting

$$T_0 = T_* + \frac{q}{8\pi k} \left(\frac{1}{r_*} \right)$$

Applying the condition of the heat flux leaving the generating region is the same as the heat flux entering the non-generating region yields the temperature at the interface between the two regions.

$$\left(-k \frac{dT_*}{dr} \Big|_{r_*} \right)_{\text{generating region}} = \left(-k \frac{dT}{dr} \Big|_{r_*} \right)_{\text{non-generating region}}$$

The conductivity is the same for both regions

$$-\left(\frac{-q'''}{3k} r_* \right) = -\left(\frac{T_1 - T_*}{1 - \frac{r_1}{r_*}} \right) \left(-\frac{r_1}{r_*^2} \right)$$

substituting;

$$q''' = \frac{q}{\frac{4}{3}\pi r_*^3}$$

$$\frac{-qr_*}{\left(\frac{4}{3}\pi r_*^3 \right) (3k)} = \frac{-(T_1 - T_*)}{\left(1 - \frac{r_1}{r_*} \right)} \left(\frac{r_1}{r_*^2} \right)$$

simplifying some yields:

$$\frac{-q}{4\pi k} = \frac{T_* - T_1}{\left(\frac{1}{r_1} - \frac{1}{r_*} \right)}$$

or,

$$T_* = T_1 + \frac{q}{4\pi k} \left(\frac{1}{r_*} - \frac{1}{r_1} \right)$$

Then the following two equations give the temperature profile within the generating and non-generating regions of the drum (or box)

At $r = 0$;

$$T_0 = T_* + \frac{q}{8\pi k} \left(\frac{1}{r_*} \right)$$

At $r = r_*$;

$$T_* = T_1 + \frac{q}{4\pi k} \left(\frac{1}{r_*} - \frac{1}{r_1} \right)$$

END

**DATE
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