DEPARTMENT OF NUCLEAR ENGINEERING
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Cambridge, Massachusetts 02139

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AN APPROXIMATE METHOD FOR PREDICTING
TEMPERATURE DISTRIBUTION IN WIRE WRAPPED
FUEL ASSEMBLIES OF A LMFBR

MASTER

by

B. Mikic
E.C. Khan
N.E. Todreas

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An Approximate Method for Predicting Temperature Distribution

in Wire Wrapped Fuel Assemblies of a LMFBR

by

B. Mikic**, E.U. Khan* and N.E. Todreas**

1.0 Introduction:

In Liquid Metal Fast Breeder Reactors (LMFBR) the fuel and blanket assemblies are subject to radial gradients in the rate of heat generations and have a wide range of flow conditions. Core designers have evolved a number of numerical methods for predicting temperature and pressure distributions within these assemblies in order to determine cladding hot spots, assembly housing bowing and deformation, and the points of incipient boiling and voiding.

Here we present an approximate method for predicting temperature distributions in wire wrapped fuel assemblies based on a porous body model which was previously formulated by Khan et al.¹ Numerical techniques were used for the solution of the equation set in Ref. 1. However it was indicated that an analytical solution is possible for the case where the bundle is operating in forced convection only (negligible free convection). Here we present an analytical method which, when combined with a very simple graphical construction, provides an engineering estimate of the temperature distribution in wire wrapped fuel rod assemblies.

The results of the analytical method compare well with those obtained in Ref. 1 using the numerical solution. (The numerical solution has already been shown¹ to match well with experimental data). The present approximate method is useful as a design tool for the following reasons. It provides a better insight into the physics of the problem compared to the numerical solution since the results permit explicit interpretation of the influence of

*Argonne National Lab., Argonne, Illinois

**Mass. Inst. of Tech., Cambridge, MA 02139

various parameters on the temperature distribution. Most important of all computational costs are considerably reduced compared to those required for the numerical solution.

2.0 The Porous Body Model

The present method retains the significant features of the porous body model (developed in Ref. 1) for predicting temperature distribution in wire wrapped assemblies. The porous body model is briefly described below. The significant difference between the present method and that in Ref. 1 is that it is assumed that the hexagonal fuel rod bundle can be approximately represented by an equivalent circular assembly (Fig. 1). This assumption has been justified based on comparison of temperature distributions between the hexagonal and the equivalent circular bundle.

The porous body model described in Ref. 1 is similar in principle to the one which has been used to study heat and mass transfer in porous media. The packed bed is analogous to the bundle of rods in the present case. The model assumes the existence of a large number of closely spaced rods and treats the rod assembly as a porous medium, approximating it as a continuum. The difference in the flow pattern in the outer and inner regions of a wire wrapped LMFB assembly suggests that the bundle be divided into two regions. An inner core surrounded by an outer region close to the housing wall. The energy generation by the rods is modelled by a continuous volumetric heat source distribution $W(x,y,z)$ which simulates the heat generation in the rods. The energy transfer in the transverse direction is modelled by an effective eddy diffusivity which includes the fluid oscillations due to the presence of wire wraps and also all other means of energy transport such as natural turbulence.

In what follows the inner region will be called region I and it is assumed to have a uniform superficial axial velocity U_1 (i.e. uniform volu-

metric flow rate per unit total area). The outer region which also has a peripheral velocity in addition to an axial velocity component is called region II. Both of these velocity components are assumed spatially uniform. The velocity vector U_g in region II is the vectorial sum of these two velocities. The distance from the housing wall at which the circumferential (swirl) velocity becomes negligibly small must determine the thickness of the outer region. In the present study, as also in Ref. 1, the thickness of region II is taken equal to the perpendicular distance between the wall and the line joining the centers of the peripheral rods.

3.0 The Present Method

The radial distribution of volumetric heat generation rate in the core of a LMFBR can reasonably be assumed to be linear across a fuel assembly. Fig. 2 shows the variation of the volumetric heat generation rate within the assembly which is assumed to be

$$W_1 = W_0 + \Delta W \frac{r}{r_1} \cos \theta \quad \text{in region I}$$

$$\text{and} \quad W'_1 = W'_0 + \Delta W' \frac{r}{r_0} \cos \theta \quad \text{in region II}$$

where the positions of maximum heat generation in regions I and II correspond to $\theta = 0$ and $r = r_1$ and $r = r_0$ respectively.

It is necessary to divide the bundle into two regions because region II has a circumferential component of velocity and because the average axial velocities in regions I and II could be different. The temperature distribution in region I is determined first assuming an insulated boundary condition at the interface of regions I and II ($r=r_1$). Next the temperature distribution in region II is determined with a boundary condition of zero heat flux at $r=r_0$ and the previously calculated temperature distribution at $r=r_1$ (note the temperature distribution $T(r_1, \theta)$ is only approximate because it was assumed that at $r=r_1, \partial T / \partial r = 0$, although in reality it is not).

A simple graphical construction then provides the corrected temperature distribution within the entire bundle.

The energy equation for region I may be written as follows:

$$\rho C_p U_1 \frac{\partial T}{\partial z} = (\rho C_p \epsilon + k) \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} \right] + W_1(r, \theta, z) \quad (1)$$

where $T = T(r, \theta, z)$ is the temperature distribution at any point within the region.

ϵ = enhanced effective eddy diffusivity which includes the mixing effect due to wire wraps; empirically determined in Ref. 1.

U_1 = superficial (based on total area of the region) axial velocity, known from data¹.

W_1 = volumetric heat generation rate

$$= W_0 + \Delta W \frac{r}{r_0} \cos \theta$$

W_0 = average volumetric heat generation rate in region I.

r_0 = equivalent radius of bundle.

ΔW = Difference in vol. heat gen. rate at $r = r_1$ and $r = 0$.

Since we are only considering region I, at first, the boundary conditions are:

$$z = 0; T = T_i; \text{ uniform inlet temperature} \quad (2a)$$

$$r = r_1; \frac{\partial T}{\partial r} = 0, \text{ assumed insulated boundary at } r = r_1 \quad (2b)$$

$$r = 0; T = \text{finite} \quad (2c)$$

$$\theta = 0, 2\pi; \frac{\partial T}{\partial \theta} = 0 \quad (2d)$$

The solution to equation (1) with boundary conditions (2) is given by

(for the region $0 \leq r \leq r_1$)

$$T = T_i + \left(\frac{W_o}{\rho C_p} \right) \frac{z}{U_I} + \frac{\Delta W}{2K_e} r_i^2 \cos \theta \left\{ \frac{1}{4} \left\{ 3 \left(\frac{r}{r_i} \right) - \left(\frac{r}{r_i} \right)^3 \right\} \right. \\ \left. - \sum_{n=1}^{\infty} e^{-a_t \alpha_n^2 \frac{z}{U_I}} \frac{(\alpha_n r_i) J_2(\alpha_n r_i) + J_3(\alpha_n r_i)}{[(\alpha_n r_i)^2 - 1] J_1^2(\alpha_n r_i)} J_1(\alpha_n r) \right\} \quad (3)$$

where K_e = effective thermal conductivity = $(\rho C_p + k)$

k = thermal conductivity of coolant

$a_t = (\epsilon + \alpha)$

α = thermal diffusivity

and the eigenvalues are the root of the equation

$$(\alpha_n r_i) J_0(\alpha_n r_i) - J_1(\alpha_n r_i) = 0$$

The following was done in order to validate the assumption of modelling the hexagonal geometry of the bundle by an equivalent circular region. A comparison was made between the temperature distribution obtained from equation (3) for a circular region I and the temperature distribution obtained numerically for a hexagonal region I under the same operating condition. Fig. 3 shows that the two temperature distributions match closely.

In region II both axial, U_{II} , and circumferential, U_θ , velocities exist. Therefore, in order to obtain the temperature distribution it is more convenient to calculate the temperature of a volume of fluid as it moves along a streamline with a velocity U_s where $\vec{U}_s = \vec{U}_{II} + \vec{U}_\theta$. At $z = 0$ the location of the fluid material volume in region II is given by $\theta = \theta_o$. It is fairly simple to show the following two relationships:

$$s = z \sqrt{1 + C^2}$$

and

$$\theta = C \frac{z}{r} + \theta_o = \frac{Cs}{r \sqrt{1 + C^2}} + \theta_o$$

where z = axial (vertical) distance from bundle inlet

$C = \frac{U_\theta}{U_{II}}$ = ratio of circumferential to axial velocities in region II.

r = average radius of region II ($\bar{r} = \frac{r_1 + r_0}{2}$).

The energy equation for this material volume may be written as

$$\rho C_p U_s \frac{dT}{ds} = \frac{(\rho C_p \delta + k) [T_W(s) - T]}{\eta_e} + W' \quad (4)$$

where δ = thickness of region II, determined in Ref. 1.

η_e = effective distance for heat transfer between region I and II.

In this study it was found that $\eta_e = 2\delta$

gave good results over a wide range of power skew.

T = Temperature distribution in region II

s = distance along the streamline of the material volume, measured from inlet of the bundle

$$\vec{U}_s = \vec{U}_\theta + \vec{U}_{II}$$

$T_W(s)$ = temperature distribution at the interface of regions I and II

$W' = W'(s)$, known distribution of volumetric heat generation rate in region II

$$W' = W'_0 + \Delta W' \cos \theta$$

W'_0 = average volumetric heat generation rate in region II

$\Delta W'$ = difference between heat generation rate at $r = r_0$ and average volumetric heat generate rate (Fig. 2)

Equation (4) may be written as

$$\frac{dT}{ds} + A_1 T = A_1 T_W(s) + \frac{1}{\rho C_p U_s} [W'_0 + \Delta W' \cos \theta] \quad (5)$$

where

$$A_1 = \frac{1}{\eta_e \delta} \frac{(\rho C_p e + k)}{\rho C_p U_s}$$

It is important to note that the present method lumps the transverse temperature distribution in region II. It calculates some bulk temperature across the region and the boundary condition at the inner ($T = T_w(s)$, equation 6b below) and at the outer ($\partial T / \partial r = 0$) boundaries are incorporated in the energy equation. The initial condition is given by:

$$s \text{ or } z = 0 \quad T = T_{IN} \quad (\text{known inlet temperature}) \quad (6a)$$

$$\text{and} \quad T_w(s) = T_1 + \frac{W_o s}{\rho C_p U_I \sqrt{1+C^2}} + \frac{\Delta W}{2K_e} r_i^2 \cos \left\{ \frac{1}{2} - \sum_{n=1}^{\infty} e^{-\left(\frac{s \alpha_n^2}{U_I \sqrt{1+C^2}} \right)} JSUM(\alpha_n r_i) \right\} \quad (6b)$$

where

$$JSUM(\alpha_n r_i) = \frac{(\alpha_n r_i) J_2(\alpha_n r_i) + J_3(\alpha_n r_i)}{[(\alpha_n r_i)^2 - 1] J_1(\alpha_n r_i)} \quad (7)$$

The solution of equation (5) for the region $r_i \leq r \leq r_o$ is given by:

$$\begin{aligned} T = T_1 + \frac{W_o}{\rho C_p U_I \sqrt{1+C^2}} & \left[\left(s - \frac{1}{A_1} \right) + \frac{e^{-A_1 s}}{A_1} \right] + \frac{W_o}{\rho C_p U_s} \left(\frac{1-e^{-A_1 s}}{A_1} \right) \\ & + \frac{R}{(b^2 + A_1^2)} \left\{ \left[b \sin (bs + \theta_o) + A_1 \cos (bs + \theta_o) \right] e^{-A_1 s} b \sin \theta_o - e^{-A_1 s} A_1 \cos \theta_o \right. \\ & - \sum_{n=1}^{\infty} \left\{ \frac{\left(\frac{A_1 \Delta W r_o^2}{2K_e} \right)}{\left[b^2 + \left(A_1 - \frac{s \alpha_n^2}{U_I \sqrt{1+C^2}} \right)^2 \right]} JSUM(\alpha_n r_o) \left[e^{-\frac{s \alpha_n^2}{U_I \sqrt{1+C^2}}} \left\{ b \sin \theta \right. \right. \right. \\ & \left. \left. \left. + \left(A_1 - \frac{s \alpha_n^2}{U_I \sqrt{1+C^2}} \right) \cos \theta - e^{-A_1 s} b \sin \theta_o - e^{-A_1 s} \left(A_1 - \frac{s \alpha_n^2}{U_I \sqrt{1+C^2}} \right) \cos \theta_o \right\} \right] \right\} \quad (8) \end{aligned}$$

$$\text{where } R = \frac{A_1 \Delta W r_o^2}{4K_s} + \frac{\Delta W}{\rho C U_s} \text{ and } b = \frac{Cr}{\sqrt{1+C^2}} \quad (9)$$

It was found that for the problem under investigation the infinite series converges very rapidly.

4.0 Example Computation of the Temperature Distribution Across a Bundle.

Of importance in LMFBF design are the magnitudes and location of the peak and minimum temperature distribution in the LMFBF bundle. Equation (8) can be used to determine the temperature distribution in the wall region. However, to determine the transverse temperature distribution across the bundle it is necessary to graphically construct the temperature profile. This is because in our method we neglected the boundary condition of continuity of temperature gradient (or heat flux) at the interface of regions I and II.

The graphical construction is shown in Figure 4 for a typical 217 pin wire wrapped LMFBF bundle with the following base parameters:

- d = 0.23 in.
- p/d = 1.24
- h/d = 52.
- Peak to avg. transverse power skew = 1.2
- Peak to avg. axial power skew = 1.23 (chopped cosine)
- Average temp. rise across the active bundle = 300°F

In Figure 4 the dotted curve is the temperature distribution obtained from Reference 1 where a finite difference technique was used to solve the temperature field in the entire bundle. In order to construct the exit temperature distribution from the present method we proceed as follows. In Figure 4 the line A-A represents the exit transverse temperature profile in region I assuming an insulated boundary on the periphery of region I.

- (a) The point B represents the region II temperature calculated by Eq. (8).

Determine the temperature at the point C which lies on the interface

between the two regions by taking an arithmetic mean between the temperatures at A and B $\left[T_C = \frac{T_A + T_B}{2} \right]$.

- (b) If L is the axial length of the bundle, point D on line A-A is at a distance $\left(\pi a_c \frac{L}{U_I} \right)^{\frac{1}{2}}$ from the interface of regions I and II. This is approximately the penetration distance for interface effects to propagate into the interior of region I. Connect C with the temperature in region I located $\left(\pi a_c \frac{L}{U_I} \right)^{\frac{1}{2}}$ from the interface (i.e. connect C to D, so that BCD-DCB is a smooth continuous curve).

The rationale for the above construction is as follows. Since A is not the correct temperature of the interface and since there must be a continuity of temperature between the two regions of the bundle, an average temperature between A and B is chosen for the interface temperature. In addition, not only is A inexact but the entire region which lies approximately at a distance $\left(\pi a_c \frac{L}{U_I} \right)^{\frac{1}{2}}$ is affected by the heat interaction at the interface between the two zones. However the temperature at the point $\left(\pi a_c \frac{L}{U_I} \right)^{\frac{1}{2}}$ from the interface approaches the correct temperature. Hence one can approximate the edge zone temperature by a line C-D.

One finds that the temperature profiles match reasonably well. Comparison of the temperature profiles for the same bundle but with a uniform radial power distribution was also found to be good. Generally one is interested in the peak temperature within the bundle; the present approximate method will provide it and the results will be on the conservative side from a designer's point of view.

5.0 Limitations of the Present Method

The present method has been tested for linear or flat power skews across the bundle with a peak to average power ratio from 1.0 to 1.2. The bundle is assumed to operate in forced convection only (negligible natural convection).

The range of applicability of the model covers the entire steady state range of operation of fuel assemblies in a Typical Liquid Metal Fast Breeder Reactor.

6.0 Conclusions

The method is an approximate method for rapidly estimating the temperature distribution in an assembly and thus is suited well to the designer's needs. It gives conservative results for the peak temperature in the bundle.

The method is attractive since it reduces computation costs compared to the numerical solution. The temperature distribution predicted by the method can be improved iteratively but this is not recommended as the iterations would require considerable computational costs, thereby defeating the purpose for which the method was originally intended. The method does provide a better insight into the physics of the problem than the numerical solution since the results permit explicit interpretation of the influence of various parameters on the temperature distribution.

Nomenclature

- a_e - effective thermal diffusivity ($= \epsilon + \alpha$)
- b - defined by equation (9)
- C - ratio of U_θ/U_{II}
- C_p - specific heat
- k - thermal conductivity
- K_e - effective thermal conductivity ($= \rho C_p \epsilon + k$)
- L - bundle length
- r - radial coordinate
- $\bar{r}$ - mean radius of region II
- s - stream line coordinate in region II
- T - Temperature ($T(r, \theta, z)$)
- T_i - inlet temperature
- U - axial velocity (superficial)
- U_I - axial velocity in region I
- U_{II} - axial velocity in region II
- U_θ - circumferential velocity in region II
- U_s - $\vec{U}_s = \vec{U}_\theta + \vec{U}_{II}$
- W_i - volumetric heat generation rate in region I
- W_o - average heat generation rate in region I
- ΔW - difference in volumetric heat generation rate at $r = r_i$ and $r = 0$
- W' - volumetric heat generation rate in region II
- W'_o - average volumetric heat generation rate in region II
- $\Delta W'$ - (Maximum minus average) volumetric heat generation rate (see Fig. 2)
- z - axial coordinate

Greek Symbols

- ρ - density
- ϵ - eddy diffusivity due to presence of wire wraps
- α_n - eigenvalues

- η_e - effective distance for heat transfer between regions I and II
taken to be equal to 2δ
- δ - thickness of region II
- θ - circumferential coordinate
- θ_0 - location of fluid material volume at $z = 0$ with respect to $\theta = 0$

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Fig. 1

Fig. 2

Fig. 3

SAMPLE CALCULATION

REGION I:

$$r_o = 2.0915'' = 0.1743'$$

n	$\alpha_n r_o$	$\alpha_n (\text{ft}^{-1})$	$J_o(\alpha_n r_o)$	$J_1(\alpha_n r_o)$	$J_3(\alpha_n r_o)$
1	1.841120046	10.563441	0.5818649368	0.5818649368	0.106649371
2	5.331441546	30.589192	-0.3461258543	-0.3461258543	0.297029376
3	8.536314955	48.977182	0.273294563	0.273294563	-0.257471658
4	11.706003854	67.163296	-0.233300241	-0.233300241	0.226007495
5	14.863587864	85.279632	0.0146703925	0.2070113257	-0.203062364
6	18.1	103.84891	≈ 0	-0.2	0.2

$$T_i = 600^\circ\text{F} \quad Z = 3 \text{ ft.}$$

$$\bar{U}_1 = 29747.9 \text{ ft/hr}$$

$$t = \frac{Z}{\bar{U}_1} = 1.0084745 \times 10^{-4} \text{ hr.}$$

$$W_o = 5.399 \times 10^7 \text{ Btu/hr-ft}^3$$

$$\frac{W_o}{\rho C_p} t = \frac{5.399 \times 10^7 \times 1.0084745 \times 10^{-4}}{53 \times 0.305} = 3.368236 \times 10^2 \text{ }^\circ\text{F}$$

$$\frac{\Delta W}{2Ke} r_o^2 = \frac{1.0799589 \times 10^7 \times (2.0915 \times \frac{1}{12})^2}{2 \times 107.926} = 1.5199 \times 10^3 \text{ }^\circ\text{F}$$

$$T = T_i + \left(\frac{W_o}{\rho C_p} \right) t + \frac{\Delta W}{2Ke} r_o^2 \cos \theta \left[4 \left(\frac{r}{r_o} \right) - \left(\frac{r}{r_o} \right)^3 \right]$$

$$- \sum_{n=1}^{\infty} e^{-\alpha_n^2 t} \frac{J_1(\alpha_n r_o) + J_3(\alpha_n r_o)}{[(\alpha_n r_o)^2 - 1] J_1^2(\alpha_n r_o)} J_1(\alpha_n r)$$

Eqn. 3

$$\text{where } t \equiv Z/\bar{U}_1$$

r in	r/r ₀	$\frac{1}{4}[3(\frac{r}{r_0}) - (\frac{r}{r_0})^3]$
0.16598	0.0793595	0.0593944
1.3279	0.6349031	0.4521943
1.6599	0.7936409	0.4702586
1.82592	0.8730193	0.4884184

Let $I_n^{(1)} = e^{-a_n^2 t}$ $I_n^{(2)} = \frac{J_1(\alpha_n r_0) + J_3(\alpha_n r_0)}{[(\alpha_n r_0)^2 - 1]J_1^2(\alpha_n r_0)}$

n	$I_n^{(1)}$	$I_n^{(2)}$	$I_n^{(1)} \times I_n^{(2)}$
1	0.927627	0.8470751	0.7857697
2	0.53261	-0.0149433	-0.0079589
3	0.198894	0.0029477	0.0005862
4	0.23117	-0.0009849	-0.0002276

	r = 0.16598		r = 1.3279		r = 1.6599		r = 1.82592	
n	$\alpha_n r$	$J_1(\alpha_n r)$	$\alpha_n r$	$J_1(\alpha_n r)$	$\alpha_n r$	$J_1(\alpha_n r)$	$\alpha_n r$	$J_1(\alpha_n r)$
1	0.14611	0.07280	1.16893	0.49018	1.46119	0.55217	1.60733	0.57058
2	0.42310	0.20680	3.38495	0.18547	4.23125	-0.14919	4.65445	-0.026998
3	0.67744	0.31947	5.41973	-0.34485	6.77477	-0.07277	7.45237	0.12314

r	$\sum I_n^{(1)} x I_n^{(2)} J_1(\alpha_n r)$	$k[3(\frac{r}{r_o}) - (\frac{r}{r_o})^3] - \sum I_n^{(1)} I_n^{(2)} J_1(\alpha_n r)$
0.16598	0.0557453	0.0036491
0.13279	0.3834903	0.0287040
1.6599	0.4350231	0.0352355
1.82592	0.4505652	0.03827664

$$A = \frac{\Delta W}{2Ke} r_o^2 \cos\theta \{ k[3(\frac{r}{r_o}) - (\frac{r}{r_o})^3] - \sum e^{-\alpha_n z} \frac{J_1(\alpha_n r_o) + J_3(\alpha_n r_o)}{[(\alpha_n r_o)^2 - 1] J_1^2(\alpha_n r_o)} J_1(\alpha_n r) \}$$

θ	r	A	T at $z = 3$ ft.
0	0.16598	5.546267	942.37
0	1.3279	43.627209	980.45
0	1.6599	53.664436	990.38
0	1.82592	125.79665	991.2
180	1.82592	-125.79665	886.1
120	1.3279	-21.813604	915.01

REGION II

$$\begin{aligned}
 T = T_i &+ \frac{W_o}{\sqrt{1+C^2} \rho C_p U_I} \left[\left(S_o \frac{1}{A_1} \right) + \frac{e^{-A_1 S}}{A_1} \right] + \frac{W_o'}{\rho C_p U_S} \left(\frac{1-e^{-A_1 S}}{A_1} \right) + \\
 &+ \frac{R}{b^2 + A_1^2} \left[\left(b \sin(bs+\theta_o) + A_1 \cos(bs+\theta_o) \right) - e^{-A_1 S} A_1 \cos\theta_o \right] \quad \text{term IV} \\
 &- \sum_{n=1}^{\infty} \frac{\left(\frac{A_1 \Delta W r_o^2}{2Ke} \right) J_{\text{SUM}}(\alpha_n r_o)}{\left(b^2 + \left(A_1 - \frac{a_t \alpha_n^2}{U_S'} \right)^2 \right)} \left[e^{-\frac{a_t \alpha_n^2 S}{U_S'}} \left(b \sin\theta + \left(A_1 - \frac{a_t \alpha_n^2}{U_S'} \right) \cos\theta \right) - \right. \\
 &\quad \left. - e^{-A_1 S} b \sin\theta_o - e^{-A_1 S} \left(A_1 - \frac{a_t \alpha_n^2}{U_S'} \right) \cos\theta_o \right] \quad \text{Eqn. 8}
 \end{aligned}$$

$$\text{where } U_S' = U_I \sqrt{1+C^2}$$

$$\begin{aligned}
 T_i &= 600 \text{ } ^\circ\text{F} \\
 r_o &= 2.0915 \text{ in.} = 0.1742915 \text{ ft.} \\
 \rho &= 53 \text{ lbm/ft}^3 \\
 C_p &= 0.305 \text{ Btu/lbm } ^\circ\text{F} \\
 W_o &= 5.399 \times 10^7 \text{ Btu/hrft}^3 \\
 U_I &= 29747.9 \text{ ft/hr} \\
 Ke &= 107.925 \text{ Btu/hrft } ^\circ\text{F} \\
 \Delta W &= 1.0799589 \times 10^7 \\
 a_t &= 6.676 \text{ ft}^2/\text{hr} \\
 C &= 0.07 \\
 S &= 3 \sqrt{1+C^2} = 3.007341 \\
 A_1 &= 0.362/\text{ft} \\
 W_o' &= 3.8866 \times 10^7 \text{ Btu/hr } \text{ft}^3 \\
 Ke &= 114 \text{ Btu/hr-ft } ^\circ\text{F}
 \end{aligned}$$

$$\begin{aligned}
 R &= \frac{A_1 \Delta W r_o^2}{4Ke} + \frac{\Delta W'}{\rho C_p U_S} \\
 &= 260.43767 + 11.10218 = 271.53985 \\
 U_S' &= U_I \sqrt{1+C^2} = 2.9820693 \times 10^4 \text{ ft/hr} \\
 b &= \frac{c/\bar{r}}{\sqrt{1+C^2}} = 0.3840018 \text{ ft}^{-1} \\
 \bar{r} &= \frac{1}{2}(r_o + r_{ex}) = 0.1818458 \text{ ft} \\
 r_{ex} &= 2.2728 \text{ in.} = 0.1894 \text{ ft.} \\
 \theta_o &= \theta - bS \\
 \Delta W' &= 7.7732 \times 10^6 \\
 U_S &= 43207.02 \sqrt{1+C^2} = 4.3312747 \times 10^4 \text{ ft/hr} \\
 e^{-A_1 S} &= 0.336668
 \end{aligned}$$

n	α_n	$\alpha_n r_o$	$J_o(\alpha_n r_o)$	$J_3(\alpha_n r_o)$
1	10.563441	1.841120046	0.316716582	0.106649371
2	30.589192	5.331441546	-0.0654210578	0.297029376
3	48.977182	8.536314955	0.0337423961	-0.257471658

$$JSUM(\alpha_n r_o) = \frac{J_1(\alpha_n r_o) + J_3(\alpha_n r_o)}{[(\alpha_n r_o)^2 - 1] J_1^2(\alpha_n r_o)} J_1(\alpha_n r_o)$$

$$\begin{aligned} JSUM(\alpha_1 r_o) &= 0.4951573 & \gamma &= 0.3840018 & e^{-A_1 S} &= 0.336668 \\ JSUM(\alpha_2 r_o) &= 0.0051722 & A_1 &= 0.362/\text{ft} & A_t &= 6.676 \text{ ft}^2/\text{hr} \\ JSUM(\alpha_3 r_o) &= 0.0008055 & U'_S &= 2.9820695 \times 10^4 \text{ ft/hr} & S &= 3.007341 \end{aligned}$$

n	$A_1 - \frac{a_t \alpha_n^2}{U'_S}$	$e^{-\frac{a_t \alpha_n^2 S}{U'_S}}$
1	0.3370191	0.927627
2	0.1525239	0.53261
3	-0.1750146	0.198894

$$I_1 = e^{-\frac{a_t \alpha_n^2 S}{U'_S}} \{b \sin \theta + (A_1 - \frac{a_t \alpha_n^2}{U'_S}) \cos \theta\}, \quad I_2 = e^{-\frac{a_t \alpha_n^2 S}{U'_S}} \{b \sin \theta_o + (A_1 - \frac{a_t \alpha_n^2}{U'_S}) \cos \theta_o\}$$

$$\begin{aligned} \theta &= 0 \\ \theta_o &= 1.1548243 \end{aligned}$$

n	I_1	I_2	$I_1 + I_2$
1	0.312628	0.0724081	0.3850361
2	0.0812357	0.097507	0.1787427
3	-0.0348093	0.1420656	0.1072563

$$\theta = \pi$$

$$\theta_0 = 1.9867683$$

n	I_1	I_2	$I_1 + I_2$
1	-0.312628	-0.0724082	-0.3850361
2	-0.0812357	-0.097507	-0.1787427
3	0.0348093	-0.1420655	-0.1072563

$$III_n = \frac{\left(\frac{A_1 \Delta W r_o^2}{2K\epsilon} \right) JSUM(\alpha r_o)}{2} \left[e^{-\frac{a_t \alpha_n^2 S}{U'_S}} \left(b \sin \theta + \left(A_1 - \frac{a_t \alpha_n^2}{U'_S} \right) \cos \theta \right) \right. \\ \left. - e^{-\frac{A_1 S}{U'_S}} b \sin \theta_0 - e^{-\frac{A_1 S}{U'_S}} \left(A_1 - \frac{a_t \alpha_n^2}{U'_S} \right) \cos \theta_0 \right]$$

III _n		
n	$\theta = 0$	$\theta = \pi$
1	380.42826	-380.42826
2	2.8203774	-2.8203774
3	0.2524125	-0.2524125
Term III	383.50104	-383.50104

$$\text{term I} = T_1 = 600^\circ \text{F}$$

$$\text{term II} = 131.59288$$

$$\text{term III} = 101.7186$$

$$\text{term IV} = \begin{cases} 420.23612 & \theta = 0 \\ -420.23612 & \theta = \pi \end{cases}$$

$$\text{term V} = \begin{cases} 383.50104 & \theta = 0 \\ -383.50104 & \theta = \pi \end{cases}$$

$$T = I + II + III + IV + V$$

$$= \begin{cases} 870.0466^\circ \text{F} & \theta = 0 \\ 796.57620^\circ \text{F} & \theta = \pi \end{cases}$$