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INSTRUMENT UNCERTAINTY PREDICTIONS (U)

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JULY, 1991

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INSTRUMENT UNCERTAINTY PREDICTIONS (U)

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1.0 SUMMARY

The accuracy of measurements and correlations should normally be provided for most experimental activities. The uncertainty is a measure of the accuracy of a stated value or equation. The uncertainty term reflects a combination of instrument errors, modeling limitations, and phenomena understanding deficiencies. This report provides several methodologies to estimate an instrument's uncertainty when used in experimental work. Methods will be shown to predict both the pretest and post-test uncertainty.

2.0 INTRODUCTION

For most engineering work requiring measurement it is necessary to judge if an instrument is accurate enough to provide usable data. In most instances the required measurement accuracy is not explicitly stated and it is left to the judgement of the user. Linear measurements are an example of this. If a design drawing specifies a plate thickness of $1/2 \pm 1/32$ " it is common practice to measure the plate using a mechanic's ruler. If the tolerance is ± 0.010 " then a micrometer or similar device would normally be used. No specific thought is given to the appropriateness of the method since past practice has proven when each method is appropriate.

The selection of instruments for many measurement tasks is often not as straightforward. As a general rule of thumb the accuracy of the measurement instrument should be at least 25% of the required measurement accuracy if the instrument accuracy is to have a negligible effect. This assumes that the required measurement accuracy and available instrument accuracy are explicitly known. This is often not the case.

The required measurement accuracy as requested by the customer is often: "As accurate as possible." This coupled with the other two significant requirements: "As fast as possible" and "as cheap as possible" results in a managerial conflict which must be resolved and points out the four principle limitations on all engineering efforts: manpower, schedule, funding, and quality. It is normally not possible to change one without affecting the others. Before it is possible to select the measurement uncertainty estimate method, it is necessary to know (even if it is only in general terms) what the answer to the four limits is. This report provides some discussion of the four limits and how each affects the measurement accuracy effort.

3.0 RESOURCES AND REQUIREMENTS

3.1 Accuracy of Task Output

Experimental accuracy can be considered a measure of how representative the reported information is of the actual physical phenomena which is being evaluated. Uncertainty is a measure of the accuracy for an experimental task or program. As an example assume the goal of the task is to develop a

correlation to predict the onset of a thermal excursion. Based on knowledge of the field the assumed relationship is:

$$T_{\text{critical}} = \psi(T_{\text{inlet}}, T_{\text{sat}}, L_H, D_H, G, c_{\text{pt}})$$

Tests would be conducted to evaluate the effect of each parameter. After all of the data collection, data reduction and analysis the final relation takes the form¹:

$$R = \frac{1}{1 + \frac{\eta}{\left(\frac{L_H}{D_H}\right)}} = \frac{T_{\text{critical}} - T_{\text{inlet}}}{T_{\text{sat}} - T_{\text{inlet}}}$$

When discussing the accuracy of this correlation appropriate statements might be:

1. The uncertainty of this correlation to predict the actual value of T_{critical} is $\pm 5^\circ\text{C}$ (95% coverage). (It should be understood that this stated uncertainty contains both the correlation uncertainties and the instrument uncertainties.)
2. The uncertainty of this correlation to predict the actual value of T_{critical} is:

$$\omega = 0.05 \sqrt{T_{\text{critical}} + \frac{T_{\text{critical}}}{T_{\text{sat}}}} \quad (95\% \text{ coverage})$$
 (If the uncertainty is not constant.)
3. This correlation predicted the experimental value of T_{critical} with an uncertainty of $\pm 4^\circ\text{C}$ (95% coverage). (This stated uncertainty does not necessarily include the instrument uncertainties; an explanation is warranted.)
4. The uncertainty of this correlation to predict the actual value of T_{critical} is $\pm 5^\circ\text{C}$ (95% coverage) where: $B = 3.0^\circ\text{C}$, $S = 1.4^\circ\text{C}$, $v = 4$, $t = 2.776$. (This stated uncertainty includes both the correlation and instrument uncertainties. It indicates that there is a possible bias in the equation of 3°C and a precision error of 1.4°C . The degrees of freedom is a measure of the number of samples used to evaluate the correlation. Four is a very small number. The Student-t value is a function of the confidence interval and the degrees of freedom. It accounts for the possible error produced by using small sample sets. As the degrees of freedom approach 30 the Student-t value will approach 2.0.)

The fourth method is the preferred method (given unlimited resources) for stating uncertainties. It also is the most resource demanding method. For most experimental work the first method is the best way of stating correlation uncertainty because of its simplicity and low resource demand.

¹ This is the Whittle and Forgan correlation to predict the temperature at the minimum pressure drop during subcooled boiling in a single channel.

If the purpose of the task was to evaluate the coefficient η then the uncertainty should be stated differently.

$$\eta = 25 \pm 0.3 \text{ (95\% coverage)}$$

For a task which is evaluating η it may be appropriate to report the uncertainty of η and the uncertainty of the correlation when the reported value of η is used.

3.2 Manpower, Schedule, Funding and Quality

The pressure transducer example used in this report is relatively straightforward and simple. Its primary purpose is demonstrate how instrument uncertainty can be estimated. The appropriateness of each method requires an understanding of how accurate the results must be. Without an answer to this question it is not possible to establish a justifiable balance between manpower, schedule, cost, and quality². These constraints always exist although they may not be recognized. Often the quality of the output is taken as fixed until manpower, funding or schedule runs out; when these resources run out, the program stops, therefore the quality is redefined. (i.e. All of the original program goals are not met.)

For many experimental tasks the manpower, schedule, and funding are fixed or have limited flexibility. In this instance the quality of the outputs will be limited. Examples of how the quality of the output can be lowered are:

1. Reducing the number of test samples or runs made,
2. Minimizing the depth of the analytical evaluation of the test data, and
3. Scaling the test apparatus to a smaller size.

Reducing the number of samples will increase the potential that the measurements are not representative of the situation being evaluated. Minimizing the analytical evaluation results in the possibility that inconsistencies in the measurements, techniques and evaluations may be overlooked. Down-sizing the test apparatus may result in scaling errors which would not be present in a prototypic model. Each of these effects produces an impact on the quality of the program primarily by reducing the veracity (i.e. increasing the uncertainty) of the program outputs (e.g. the finished product, measurement, equation, etc).

²

The term quality for this report is defined as: The degree to which the requested task outputs are met. Quality Assurance (QA) should not be confused with the measure of quality. QA is only one of many aspects which must be measured in establishing the quality of the output.

4.0 STAGES OF UNCERTAINTY ANALYSIS

4.1 Pretest Uncertainty

For most instruments there are two data banks available which can be used to estimate the uncertainty of the data which is obtained with the instrument. These are the vendor's data sheet and the individual M&TE calibration history file. A third source is a collective history file for a given type, brand or model of instrument. Often this third source is the best for pretest uncertainty estimates, unfortunately it often does not exist. A sample of the data provided on a vendor's data sheet is listed in Table 1. A sample calibration history file is presented in Table 2. Neither is in a form where the uncertainty answer is explicitly provided.

4.2 Post-test Uncertainty

In addition to the information available for estimating the pretest uncertainty, the post-test calibration and test operating data can be used to estimate the post-test uncertainty. These coupled with the information sources listed in section 4.1 can and should be used to estimate the uncertainty of the instrument while it was being used and the measurement uncertainty for the data taken with the instrument.

The test operating data may consist of daily zero checks, replicated test runs (i.e. Where all test variables are held constant.), loop function checks, and comparison with repetitive instrumentation. This information provides a basis to estimate field effect uncertainties.

5.0 UNCERTAINTY ESTIMATE METHODS

5.1 Vendor Data Method

5.1.1 Combining Uncertainty Terms

Table 1 lists the operating data and uncertainty information for a sample pressure transducer. The uncertainty of the transducer can be estimated (neglecting most of the field effects³) using the Root Sum Square (RSS) of the stated values. At an operating temperature of 25°C the uncertainty of a 7.5 psig reading is:

Linearity	0.009 psi
Hysteresis	0.006
Sensitivity	0.008
Thermal sensitivity	0.009
Thermal zero drift	0.011

³ The operating temperature is often treated as a field effect.

The total combined uncertainty based on these values is 0.02 psi. This can be considered the expected uncertainty of data taken with this transducer model with a confidence value of 95%. (For most vendor data a confidence value is not provided. It is a common practice to assume 95%.) This value does not contain any estimate of field effects except for the operating temperature. The recorded data for this measurement would take the form:

$$P = 7.50 \text{ psig} \pm 0.02 \text{ psi (95\% coverage)}$$

The intent of this statement is that the technician recording the data has a 95% confidence that pressure is actually somewhere between 7.48 and 7.52 psig.

5.1.2 Curve Fit Estimate

The curve fit for the sample transducer can be estimated from the vendor data. At 0 psig the instrument should output 0 volts, at 15 psig the instrument should output 5 volts. Since the instrument is linear:

$$\begin{aligned} 0 \text{ volts} &= a + b (0 \text{ psig}) \\ 5 \text{ volts} &= a + b (15 \text{ psig}) \end{aligned}$$

Solving these simultaneous equations:

$$\begin{aligned} a &= 0 \text{ volts} \\ b &= 0.333 \text{ volts/psi} \end{aligned}$$

$$V = 0.333 P$$

5.1.3 Variation of Uncertainty

Seldom is a single measurement made where the ambient conditions are well controlled. Figures 1 and 2 show the effects of varying the temperature and reading. Figure 1 demonstrates how the uncertainty of measurements made using the sample transducer varies with the reading. When the operating temperature is 30°C (86°F) the uncertainty can range from 0.025 to 0.047 psi. Selecting the value to be used for the uncertainty must consider the range which is actually used. If the range of readings is very limited; say 6.0 to 7.0 psig, then the range of uncertainty would be 0.030 to 0.031. Clearly there is no advantage in using multiple uncertainties over this limited range.

If the instrument is being used over its full range then the uncertainty selection is not as straightforward. The simplest uncertainty estimate could be to use the largest value. In this case it would be at 15 psig: $\omega = 0.047$ psig. If the mean uncertainty value (0.03 psig) were to be used, it is possible that the uncertainty would be understated for a significant number of readings (all data between 6 and 15 psig).

If the uncertainty estimate based on the largest possible uncertainty does not provide the requested measurement uncertainty there are a number of

alternatives to reduce the uncertainty: Replace the instrument with a more accurate unit, redesign the test apparatus, or report the uncertainty as a function of the reading. The first two options are outside of the scope of this report, the third will require a significant expenditure of effort (manpower, funding and schedule resources) during data reduction. If the reduced uncertainty is justifiable then the additional resources needed during data reduction is one method which may be used to reduce uncertainties.

5.1.4 Field Effect Uncertainty Estimates

A field effect is an uncertainty which is introduced after the calibration considerations. Examples of field effects are emergent stem error for thermometers and dynamic lag in pressure transducers. The thermal sensitivity and thermal zero drift uncertainties listed in Table 1 can be considered field effect uncertainties. If it were possible to maintain the temperature of the transducers at 20°C (the calibration temperature) there would be no uncertainty related to temperature effects. It is seldom practical to hold the temperature constant.

The sample transducer is significantly affected by operating temperature. Figure 2 shows that for a reading of 7.5 psig the temperature effect uncertainties are the largest uncertainties when the operating temperature varies from the calibration temperature by more than 4°C. The ambient air temperature in the HTL commonly drifts from 10 to 30°C. This can have a considerable impact on the uncertainty. In addition, for certain rigs the instrument operating temperature can be higher than the room temperature by 5 to 10°C. This means that the actual temperature which should be used in the uncertainty estimate is not known unless measurements are recorded. Lacking measurements the uncertainty can be estimated based on the highest expected temperature: 30°C for an ambient rig and 40°C for a heated rig. Based on the 40°C operating temperature the uncertainty of a 7.5 psig reading is ± 0.06 psi.

5.1.5 Uncertainty Calculation

If it is necessary to estimate the overall uncertainty for this instrument using the least resource demanding method some assumptions are required. (1) The instrument will operate over its full range of 0 to 15 psig. (2) The maximum expected operating temperature is 40°C. (3) The most demanding uncertainty conditions will occur simultaneously. (4) The vendor data is accurate and represents a 95% confidence interval. Using these assumptions the instrument uncertainty would be:

Linearity	0.009 psi
Hysteresis	0.006
Sensitivity	0.015
Thermal sensitivity	0.075
Thermal zero drift	0.045

$$\omega_T = [(0.009)^2 + (0.006)^2 + (0.015)^2 + (0.075)^2 + (0.045)^2]^{1/2}$$

$$\omega_T = 0.089 \text{ psi}$$

This should be rounded to 0.09 psi.

5.2 Pretest Calibration Data Uncertainty Estimate

Table 2 provides information which is typically found in an M&TE history file. for purposes of this example assume than only the first two calibrations are available. (The later calibrations will be discussed in a post-test context.)

5.2.1 Curve Fit Estimate

The 5/12/91 calibration data is plotted in Figure 3. From this graph it is possible to prepare an alternate to the vendor curve fit. The y intercept is directly read as 0.00. Since the instrument is linear:

$$\begin{aligned} a &= 0.00 \text{ volts} \\ 4.99 \text{ volts} &= a + b (15 \text{ psig}) \end{aligned}$$

Solving these equations:

$$\begin{aligned} a &= 0 \text{ volts} \\ b &= 0.33 \text{ volts/psi} \end{aligned}$$

The uncertainty of each term may be estimated. The uncertainty of a is considered to be 0.05 volts based on the graph subdivisions. The uncertainty of the coefficient b is estimated using the RSS method.

$$\omega_b = \left[\left(\frac{\partial b}{\partial V} \omega_V \right)^2 + \left(\frac{\partial b}{\partial P} \omega_P \right)^2 + \left(\frac{\partial b}{\partial a} \omega_a \right)^2 \right]^{1/2}$$

$$\omega_b = \left[\left(\frac{1}{P} \omega_V \right)^2 + \left(\frac{-1}{P^2} \omega_P \right)^2 + \left(\frac{-1}{P} \omega_a \right)^2 \right]^{1/2}$$

$$\omega_b = \left[\left(\frac{1}{P} 0.05 \text{ volts} \right)^2 + \left(\frac{a - V}{P^2} 0.1 \text{ psi} \right)^2 + \left(\frac{-1}{P} 0.05 \text{ volts} \right)^2 \right]^{1/2} = \frac{0.078 \text{ volts}}{P}$$

The analysis indicates that the uncertainty of b is very dependant on P. As the range approaches zero the uncertainty of the slope will become infinite. This is to be expected since as P decreases to zero, the line will become a point with an undefined slope. Using the opposite logic the slope should be calculated using pressure data with as much separation as feasible to ensure the lowest uncertainty. If the maximum range for the transducer is used, ω_b becomes 0.005 volts/psi. The uncertainty of predicting the voltage for a given pressure input would be:

$$\omega_V = [(\omega_a)^2 + (b \omega_P)^2 + (P \omega_b)^2]^{1/2}$$

$$\omega_V = [(0.05)^2 + (0.33 \cdot 0.1)^2 + (15 \cdot 0.005)^2]^{1/2} = 0.096 \text{ volts}$$

$$\omega_P = \frac{0.096 \text{ volts}}{0.33 \text{ volts/psi}} = 0.29 \text{ psi}$$

This large uncertainty (in comparison to the vendor's stated uncertainties) indicates that a graphical solution to the calibration problem is not appropriate. It must be kept in mind that the above uncertainty reflects only the uncertainty of the calculations. Any uncertainty which would result from calibration data scatter and calibration standard uncertainties is not considered.

5.2.2 HTL Simplified Calibration Method⁴

The HTL has developed a simplified uncertainty analysis for the calibration of linear instruments. Based on this analysis, if the calibration standard uncertainty is negligible:

$a = 0.000432 \text{ volts}$	The first parameter for the calibration curve.
$b = 0.333 \text{ volts/psi}$	The second parameter for the calibration curve.
$\omega_a = 0.000496 \text{ volts}$	The uncertainty associated with a.
$\omega_b = 0.0000558 \text{ volts/psi}$	The uncertainty associated with b.
$\omega_E = 0.00402 \text{ psi}$	The uncertainty associated with the curve fit which is a result of data scatter.
$\omega_F = 0.0119 \text{ psi}$	The uncertainty introduced if the vendor's curve is used to reduce data.
$\omega_T = 0.013 \text{ psi}$	The total uncertainty for the instrument.

There are many instances when the total uncertainty as calculated from the calibration data can be used as the uncertainty of the instrument during use. This is certainly not appropriate in this case. The sample transducer displays a significant temperature effect and unless the transducer is used at the calibration temperature the uncertainty will be significantly underestimated.

For most calibrations the vendor's stated uncertainty and the calibration uncertainty should be compared. From Figure 1 the estimated uncertainty when the sample transducer is operating at 30°C is 0.047 psi. This is significantly higher than the calibration data uncertainty. Note that at the calibration temperature the uncertainty terms are:

⁴ Reference WSRC-TR-91-106, Appendix C.

Linearity	0.009 psi
Hysteresis	0.006
Sensitivity	0.015
Thermal sensitivity	0.000
Thermal zero drift	0.000

The combined uncertainty would be 0.018 psi which is in close agreement with the calibration uncertainty of 0.013 psi. To estimate the expected uncertainty during usage the greater of these two terms should be combined with the field effect uncertainties.

5.3 Post-test Calibration Data Estimate

Using the pretest and post-test calibration data to estimate uncertainties can often be confusing. The most difficult part of the effort is determining what is applicable and what is superfluous. Assume that the tests being evaluated were conducted between May 15 and November 1, 1988. The applicable calibration data sets for this evaluation are listed in Table 2. The two which should be used are the May 12, 1988 and November 9, 1988 as found. The November 1988 as left data would not be applicable since it represents the transducer after it was adjusted. The same would be true for the 1991 data. It would be appropriate to use the August 6, 1987 data as part of the evaluation. This would possibly provide an indication of drift in the instrument. It would also take additional effort. For most instances only the calibration data just preceding and following testing need be considered. A comparison of the curve fits calculated from each set of data are listed in Table 3.

Any of the curve fits listed in Table 3 may be used to reduce data. Normally the simplest method should be selected. This often is the vendor's curve. The advantage to this curve is that it will be consistent for multiple traducers being used on a given task. This reduces the number of equations which must be used in the reduction of data and simplifies the calculations. The other methods all have advantages. Use of the pretest calibration curve allows reduction of data prior to the post-test calibration. If the pretest or vendor's curve are used for reduction of data to allow real-time evaluation of the testing it is often advantageous to not repeat the data reduction process after the post-test calibration. Whatever curve is selected it should be representative of the instrument's behavior during the test. Its selection is a matter of technical judgement. Table 4 lists the uncertainties calculated using a variety of curves.

5.3.1 Pretest Calibration Curve

Figure 4 shows the difference between the applied pressure and the calculated pressure based on the pretest calibration curve. The difference between these two values is referred to as error. Note that the linear curve fit for the pretest error has a zero slope and zero y-intercept. This is expected since the pretest calibration curve was estimated to minimize the difference between the calculated pressure (based on a voltage reading) and the applied pressure.

5.3.2 Post-test Calibration Curve

Figure 5 shows the difference between the applied pressure and the calculated pressure based on the post-test calibration curve. The slope and y-intercept are zero for the post-test data.

5.3.3 Pooled Calibration Curve

The pooled data curve is based on all of the pretest and post-test calibration data being used to estimate a single linear curve fit. This method works well if only a pretest and post-test calibration are available. The error for this curve is shown in Figure 6. If multiple calibration data are available then the averaged coefficient method might be more appropriate. Note that the data for the pretest and post-test can be used to provide error curves. These curves can be used to estimate the possible bias error.

5.3.4 Averaged Coefficient Calibration Curve

The averaged coefficient calibration curve is based on an average of the linear coefficients for all available calibration curve fits. This method works well if multiple calibrations are available. The error for this curve is shown in Figure 7. This is perhaps the best method of estimating the correct curve fit. The uncertainty of each coefficient can be estimated using the same methodology used in section 5.4.

5.3.5 Vendor's Calibration Curve

The least resource demanding method to estimate the correct curve fit is to use the vendor's data. The drawback to this method is that it normally results in the largest uncertainties. The errors for this curve fit are shown in Figure 8. Note that the vendor curve has the highest errors of the methods proposed. If the bias error must be estimated the maximum difference between the two curves shown should be used. For this case at 10 psig the bias would be 0.015 psi.

5.4 Bias and Precision Errors

One method which can be used to reduce the uncertainty of measurements made with a given instrument is to average a set of readings at a given state. When this is done it is important to recognize which uncertainties can be reduced by the method. In general bias errors can not be reduced by averaging a set of readings. Precision errors can be reduced. Normally the precision errors are considered to occur in a gaussian distribution. For the sample transducer assume that the test readings (in volts) at a given state are as listed in Table 5. The pressure values were calculated using the pretest calibration curve. If field effects (including temperature) are considered negligible; then from Table 4 the expected uncertainties would be:

$$\omega_E = 0.0040 \text{ or } 0.0063 \text{ psi (use largest)}$$

$$\omega_F = 0.0092 \text{ psi}$$

$$\omega_T = 0.011 \text{ psi}$$

The fixed curve error should be considered a bias since it is not possible to select the "correct" curve. The curve fit uncertainty can be treated as a precision error if the calibration data sets contain a significant number of samples.⁵ (A significant number for most applications would be 20 well spaced samples.) If a set of readings at a given state is averaged the precision of the average should approach zero if all data scatter is due to the instrument. The precision uncertainty of the mean can be estimated based on the standard deviation of the readings. For the ten sample case in Table 5:

$$\bar{p} = \frac{\sum_{i=1}^n p_i}{n} = 12.61 \text{ psi}$$

$$S = \left[\frac{\sum_{i=1}^n (p_i - \bar{p})^2}{n - 1} \right]^{1/2} = 0.00887 \text{ psi}$$

$$S_m = \frac{S}{\sqrt{n}} = 0.00284 \text{ psi}$$

$$v = n - 1$$

$$T(v, 95\%) = 2.262$$

$$\omega_S = t S_m = 2.262 (0.00284) = 0.00633 \text{ psi}$$

$$B = 0.0092 \text{ psi}$$

$$\omega_T = [B^2 + (t S_m)^2]^{1/2} = 0.0112 \text{ psi}$$

For this case the precision uncertainty based on the working data (0.00633) is equivalent to the precision uncertainty based on the calibration data (0.0063). There is no significant advantage in estimating the uncertainty by either method. Table 6 demonstrates how the uncertainties can vary with sample size. Note that as the sample size increases the total uncertainty will approach the bias uncertainty.

$$\omega_T = \lim_{\substack{n \rightarrow \infty \\ t \rightarrow 2}} \left[B^2 + \left(\frac{t S}{\sqrt{n}} \right)^2 \right]^{1/2} = B$$

6.0 CONCLUSION

- 6.1 The method used to estimate uncertainties is a matter of technical judgement. The method used should minimize the resource demands on manpower, funding, and schedule but ensure that the quality goals are met.
- 6.2 Uncertainties should be reported such that both the interval and the confidence level are documented.

7.0 REFERENCES

Coutts, D. Allan, *Uncertainty and Calibration Analysis*, WSRC-TR-91-106, March 1991.

Table 1	
Vendor's Instrument Data	
Input Range, psig	0 to 15 psig
Excitation	15 VDC
Output Range	0 to 5 volts
Linearity error	0.06% full scale
Hysteresis error	0.04% full scale
Sensitivity error	0.1% of reading
Thermal sensitivity error	0.025%/°C of reading
Thermal zero drift	0.015%/°C full scale
Temperature range	0 to 50

Table 2									
Calibration History									
8/6/87		5/12/88		11/9/88		11/9/88		2/12/91	
psig	volts	psig	volts	psig	volts	psig	volts	psig	volts
0.00	0.000	0.00	0.000	0.00	0.000	0.00	0.000	0.00	0.000
3.75	1.251	1.50	0.500	1.50	0.500	1.50	0.500	1.50	0.500
7.50	2.500	3.00	1.000	3.00	0.999	3.00	1.000	3.00	1.001
11.25	3.749	4.50	1.500	4.50	1.499	4.50	1.500	4.50	1.502
15.00	4.999	6.00	2.000	6.00	1.999	6.00	2.000	6.00	2.002
		7.50	2.499	7.50	2.498	7.50	2.499	7.50	2.502
		9.00	2.998	9.00	2.996	9.00	2.998	9.00	3.001
		10.50	3.497	10.50	3.495	10.50	3.497	10.50	3.501
		12.00	3.997	12.00	3.994	12.00	3.997	12.00	4.002
		13.50	4.496	13.50	4.493	13.50	4.497	13.50	4.502
		15.00	4.996	15.00	4.992	15.00	4.996	15.00	5.002
		15.00	4.996	15.00	4.992	15.00	4.996	13.50	4.502
		13.50	4.496	13.50	4.493	13.50	4.497	12.00	4.001
		12.00	3.997	12.00	3.993	12.00	3.997	10.50	3.501
		10.50	3.497	10.50	3.495	10.50	3.497	9.00	3.001
		9.00	2.997	9.00	2.995	9.00	2.997	7.50	2.502
		7.50	2.499	7.50	2.498	7.50	2.500	6.00	2.001
		6.00	1.999	6.00	1.998	6.00	1.999	4.50	1.500
		4.50	1.499	4.50	1.498	4.50	1.498	3.00	1.000
		3.00	0.999	3.00	0.998	3.00	1.000	1.50	0.499
		1.50	0.499	1.50	0.498	1.50	0.499	0.00	-0.001
		0.00	0.000	0.00	-0.001	0.00	0.000		

Table 3		
Calibration Curve Coefficients		
	a	b
Pretest Coefficients	0.000432	0.3330
Post-test Coefficients	0.000386	0.3328
Pooled-data Coefficients	0.000409	0.3333
Averaged Coefficients	0.000409	0.3329
Vendor's Curve	0.000000	0.3333

Table 4			
Calibration Uncertainties			
Uncertainty Basis	Pretest Data, psi	Post-test Data, psi	Pooled Data, psi
Curve Fit Uncertainty	0.0040	0.0063	0.0082
Fixed Curve Uncertainty			
Pretest Curve	0.0000	0.0092	NA
Post-test Curve	NA	0.0000	NA
Pooled Curve	NA	NA	0.0000
Vendor's Curve	0.0119	0.0229	0.0183
Combined Uncertainty			
Pretest Curve	0.004	0.011	NA
Post-test Curve	NA	0.009	NA
Pooled Curve	NA	NA	0.008
Vendor's Curve	0.013	0.024	0.020

Table 5					
Field Measurements					
5 samples		10 samples		20 samples	
measured voltage	calculated pressure, psi	measured voltage	calculated pressure, psi	measured voltage	calculated pressure, psi
4.202	12.617	4.200	12.611	4.203	12.620
4.203	12.620	4.198	12.605	4.196	12.599
4.196	12.599	4.199	12.608	4.202	12.617
4.203	12.620	4.202	12.617	4.200	12.611
4.201	12.614	4.203	12.620	4.197	12.602
		4.196	12.599	4.203	12.620
		4.205	12.626	4.195	12.596
		4.196	12.599	4.199	12.608
		4.198	12.605	4.202	12.617
		4.198	12.605	4.199	12.608
				4.199	12.608
				4.197	12.602
				4.197	12.602
				4.202	12.617
				4.202	12.617
				4.196	12.599
				4.196	12.599
				4.195	12.596
				4.201	12.614
				4.199	12.608

Table 6			
Uncertainty Estimate Variation with Sample Set Size			
Number of samples	5	10	20
Student-t value	2.776	2.262	2.093
Mean pressure	12.609 psi	12.614 psi	12.608 psi
Standard Deviation of the Mean	0.00391 psi	0.00284 psi	0.00261 psi
Bias	0.0092 psi	0.0092 psi	0.0092 psi
Precision ($t S_m$)	0.01083 psi	0.00633 psi	0.00588 psi
Total Uncertainty	0.014 psi	0.011 psi	0.011 psi

Figure 1, Uncertainty Estimate Based on Vendor's Data at 30°C

Figure 2, Uncertainty Estimate Based on Vendor's Data at 7.5 psig

Figure 3, Pretest Calibration Data

Figure 4, Errors based on Pretest Calibration Curve

Figure 5, Errors based on Post-test Calibration Curve

Figure 6, Errors based on Pooled Calibration Data Curve

Figure 7, Errors based on Averaged Coefficient Curve

Figure 8, Errors based on Vendor's Calibration Curve

END

**DATE
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11/22/93