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A VIBRATION MODEL FOR CENTRIFUGAL CONTACTORS

by

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MASTER

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ABSTRACT

Using the transfer matrix method, we created the Excel worksheet "Beam" for analyzing vibrations in centrifugal contactors. With this worksheet, a user can calculate the first natural frequency of the motor/rotor system for a centrifugal contactor. We determined a typical value for the bearing stiffness (k_b) of a motor after measuring the k_b value for three different motors. The k_b value is an important parameter in this model, but it is not normally available for motors. The assumptions that we made in creating the Beam worksheet were verified by comparing the calculated results with those from a VAX computer program, BEAM IV. The Beam worksheet was applied to several contactor designs for which we have experimental data and found to work well.

I. INTRODUCTION

Our objective was to develop a vibration model for the rotating motor/contactor-rotor, or motor/rotor, assemblies of centrifugal contactors that was simple to use yet powerful in its analytical capabilities. Using this model, new designs could be analyzed for vibrational problems prior to construction. Thus, an appropriate contactor could be designed, built, and tested in a shorter time and with lower development costs.

Since its development in the late 1960s, the Argonne centrifugal contactor has proven to be effective for carrying out solvent extraction processes [BERNSTEIN, LEONARD-1980, -1983, -1985, -1988]. As various contactor designs have been built over the years, we have experienced a variety of vibrational problems including an out-of-balance rotor operation near the first natural frequency of the support structure, and, more recently, as we used inexpensive off-the-shelf motors, operation near the first natural frequency of the motor/rotor assembly. An out-of-balance rotor is fixed by balancing the rotor. Operation near the first natural frequency of the support structure is solved by making the support structure stiffer and thus increasing its first natural frequency [LEONARD-1988]. Similarly, operation near the first natural frequency of the motor/rotor assembly is solved by making the assembly stiffer. The American Petroleum Institute (API) standard states that the steady-state running speed of rotating equipment should not be within $\pm 20\%$ of any natural frequency [MITCHELL-1984]. Since our model determines the first natural frequency of rotating systems to within $\pm 20\%$ of its actual value, a conservative design criterion is a calculated first natural frequency that is 40% above its operating speed.

Vibration calculations for the motor/rotor assembly were previously made using BEAM IV, a VAX mainframe computer program to analyze rotating beams [MITCHELL-1984]. We developed a simplified version of the BEAM IV program for the Macintosh personal computer, an Excel worksheet called "Beam". The Beam worksheet is not only easy to use, but it has been enhanced to compute needed beam masses, area moments of inertia, and weight moments of inertia from the dimensions and material specifications. In addition, an Excel macro called "Beam_driver" locates the first natural frequency using the motor/rotor data in the Beam worksheet. Finally, an Excel chart called "Beam_chart" shows if more than one natural frequency exists within a specified frequency domain.

This report has four parts. First, the underlying theory based on the method of transfer matrices is presented. Second, experimental measurements needed to determine bearing stiffness are reported for several motors. Third, the Beam worksheet, which uses this bearing stiffness data, is developed and its calculated results are compared with those of the BEAM IV program and with experimental data on contactor vibrations of motor/rotor assemblies. Finally, various factors important for applying the Beam worksheet to other systems are discussed. In addition, there are a series of appendices: (1) a manual for using the Beam worksheet, (2) complete documentation of the Beam worksheet, and (3) details of the various Beam and BEAM IV models discussed in the report.

II. THEORY

We used the same method of transfer matrices that was used in the BEAM IV program [PESTEL, PILKEY] to create the Beam worksheet. In this method, a continuous, complicated beam is divided into simple component parts (sections) with simple elastic and dynamic properties that can be expressed in matrix form. These section matrices, when combined by matrix multiplication and evaluated with the proper boundary conditions, give the vibrational response for the entire beam.

As applied to the centrifugal contactor, the motor and attached contactor rotor are divided into 11 beam sections. (See Fig. 1.) Each point i , which is at the right end of the beam section i , has a state vector Z_i given as

$$Z_i = \begin{bmatrix} w \\ \psi \\ M \\ V \end{bmatrix}_i \quad (1)$$

where w_i is the vertical deflection, ψ_i is the slope, M_i is the internal moment, and V_i is the internal shear. The beam section i , which lies between the points $i-1$ and i , is considered to be a massless beam with a concentrated mass m_i at point i . The appropriate field transfer matrix F_i for a massless beam section i between points $i-1$ and i is given by

$$F_i = \begin{bmatrix} 1 & L & \frac{L^2}{2EI} & \frac{L^3}{6EI} \\ 0 & 1 & \frac{L}{EI} & \frac{L^2}{2EI} \\ 0 & 0 & 1 & L \\ 0 & 0 & 0 & 1 \end{bmatrix}_i \quad (2)$$

where L is the section length, E is the modulus of elasticity of the section, and I is the area moment of inertia of the section.

Fig. 1.

Schematic Showing the Sections of the Motor/Rotor System Used in the Beam Worksheet. The number on the right side of each section is the section number (e.g., Section 1 lies between 0 and 1).

Between each field transfer matrix is a point transfer matrix. The appropriate point transfer matrix P_i for point i with (1) a concentrated mass m_i that is vibrating at a frequency ω is

$$P_{m,i} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ m_i \omega^2 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

(2) an elastic support or bearing stiffness k_i (sections 2 and 5 in Fig. 1) is

$$P_{k,i} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -k_i & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

and (3) a weight moment of inertia J_i for the concentrated mass is

$$P_{J,i} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & hJ_i \omega^2 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (5)$$

where

$$h = \begin{cases} +1 & \text{for a rotating shaft (equal angular direction of whirl and rotation)} \\ -1 & \text{for a static beam (bending vibration)} \\ -3 & \text{for a rotating shaft (opposite angular direction of whirl and rotation)} \end{cases} \quad (6)$$

To obtain a more general point transfer matrix for point i that contains the concentrated mass, the bearing stiffness, and the weight movement of inertia, the three point matrices ($P_{m,i}$, $P_{k,i}$, and $P_{J,i}$ given by Eqs. 3, 4, and 5) are multiplied together to give

$$P_i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & hJ_i \omega^2 & 1 & 0 \\ m_i \omega^2 - k_i & 0 & 0 & 1 \end{bmatrix} \quad (7)$$

Now, for the model shown in Fig. 1, the state vector at the right end, z_{11} , is related to the state vector at the left end, z_0 , by the transfer matrix equation

$$z_{11} = P_{11}F_{11}P_{10}F_{10}P_9F_9 \dots P_2F_2P_1F_1z_0 \quad (8)$$

If the intermediate matrices are multiplied together to give the overall transfer matrix U , Eq. 8 becomes

$$z_{11} = Uz_0 \quad (9)$$

where U is a 4 x 4 matrix of the form

$$U = \begin{bmatrix} u_{11} & u_{12} & u_{13} & u_{14} \\ u_{21} & u_{22} & u_{23} & u_{24} \\ u_{31} & u_{32} & u_{33} & u_{34} \\ u_{41} & u_{42} & u_{43} & u_{44} \end{bmatrix} \quad (10)$$

where u_{ij} is the matrix element in row i and column j . Now, because M_0 , V_0 , M_{11} , and V_{11} are zero in the end vectors z_0 and z_{11} (i.e., both ends of the motor/rotor assembly are free), the following Δ determinant

$$\Delta = \begin{vmatrix} u_{31} & u_{32} \\ u_{41} & u_{42} \end{vmatrix} \quad (11)$$

will be zero for those values of ω that are critical rotational speeds for the system shown in Fig. 1. Thus, by evaluating Eq. 11 for various frequencies and finding those at which Δ becomes zero, we can calculate the various natural frequencies of the motor/rotor system in the contactor, including the first natural frequency (first critical speed).

By setting up these matrix equations in the Excel Beam worksheet, we created a simple way to use the power of the transfer matrix method for modeling the rotating motor/rotor system in a centrifugal contactor.

III. EXPERIMENTAL SECTION

We had to determine a typical value for the bearing stiffness in motors. To do this, we first needed measurements of natural frequencies of motors. These data would be used in BEAM IV to calculate bearing stiffness. We measured the first three natural frequencies for three different types of motors using a Zonic Real Time Spectrum Analyzer (RTSA) (Zonic Corp., Milford, OH), which can measure up to about 40 kHz. The motors tested are identified in Table 1.

Table 2 shows the natural frequencies obtained, along with the various techniques that were used. These techniques included (1) using other spectrum analyzers besides the Zonic RTSA, (2) isolating the motor from its surroundings by various means, and (3) mounting the accelerometer to the motor shaft by two methods. While there are some variations in the measurement of natural frequency due to different techniques, they are usually not large. Suspending the motor with bungee cords and attaching the accelerometer to the motor shaft with quick-drying cement produced displays that were more easily read on the Zonic screen. For one system where the rotor was attached to the motor, we found that it was better to clamp the motor to a solid surface than to lay it on a foam-damped surface. After we switched to a clamped motor, the first natural frequency went from a very broad band with two peaks to a single sharp peak. Thus, for any given system, several methods should be tested and the one that works best should be used.

Four of the motors listed in Table 1 were disassembled, and the dimensions of each motor shaft were measured. These measurements were used to get an average dimensionless diameter and length for each section. Using these data, default values can be generated for any motor if the user knows its overall length, output shaft length, and output shaft diameter. The measurements and the derived dimensionless parameters are listed in Table 3. They are used in the Beam worksheet to generate default values for motor dimensions if the user does not have the actual values.

Results from three of the motors listed in Table 1 (motors 1, 3, and 4) were used to get the values for bearing stiffness described in the next section (Sect. IV.A). For these calculations, we used both the natural frequencies in Table 2 and the actual motor dimensions in Table 3.

Table 1. Motor Identification

Identification Number	Motor Mfr.	Motor Model	Power, W(hp)	No. of Phases	Voltage	Nominal Speed, Hz (rpm)	Motor Serial Number
1	Bodine ^a	710	0.8(1/930)	1	115	60 (3,600)	NH4154
2	Bodine	710	0.8(1/930)	1	115	60 (3,600)	NH4185
3	Bodine	special ^b	12.4(1/60)	1	115	57.5 (3,450)	M0052027
4	Bodine	254	124(1/6)	1	115	28.3 (1,700)	MG9026
5	Bodine	284	249(1/3)	1	115/230	28.3 (1,700)	KF5050
6	Baldor ^c	VL3501	249(1/3)	1	115/208-230	28.8 (1,725)	W0787
7	Baldor	VL3510T	746(1)	1	115/208-230	28.8 (1,725)	F686
8	Baldor	VM3546T	746(1)	3	208-230/460	28.8 (1,725)	F587

*Bodine Electric Company, Chicago, IL.

^bThis special-order motor, available only in quantities of 25 or more, does not have a model number. It is a combination of two Bodine-type N-1 motors, model 219, which has a speed of 3450 rpm, and model 625, which has a totally-enclosed face-mounted design.

^cBaldor, Fort Smith, AR.

Table 2. Natural Frequencies For Several Motors

Motor Identification Number ^a	Measured Natural Frequency, ^b Hz			Support Method	Accelerometer Mounting Method ^c	Notes
	1st	2nd	3rd			
1	2,430	3,888	--	d	e	
2	--	--	5,200	d	f	g
	--	--	5,260; 5,380	d	f	
	2,300	3,472	5,264; 5,360	d	f	h
3	869	2,310	3,195	i	e	
	863	2,295	3,075	i	e	j
	855	2,295	3,030	k	e	l
	868	2,325	3,300	k	f	l
4	1,065	2,490	--	k	e	
	1,095	2,790	16,200	k	f	

^aSee Table 1.

^bUnless specified otherwise in notes, natural frequencies were measured using the Zonic 6088 RTSA. A dash indicates that although a value was expected based on other tests with the same motor, no peak was found for the natural frequency in this range. All Zonic measurements were made using an impulse force hammer from PCB Piezotronics (Depew, NY), Model 086B01, with aluminum extender and red rubber tip installed. This unit has a sensitivity of 10.2 mV/N and a mass of about 90 g. The accelerometer attached to the motors is PCB model 303A, which has a sensitivity of 10.31 mV/G, where $G = 9.81 \text{ m/s}^2$, and a mass of about 2 g. The test setup included two power units set to a gain of x1. Most of the measurements were made by averaging three to five hammer hits; some measurements were made with only one hit.

^cAccelerometer is mounted on flat of motor shaft unless specified otherwise in notes.

^dMotor in on benchtop. If padded surface was used, this information was not recorded.

^e"Petro-wax" or beeswax.

^fQuick-drying cement.

^gThese data were measured using an oscilloscope.

^hThese data were measured by T. M. Mulcahy (ANL Materials and Components Technology Div.) using a Hewlett-Packard Multichannel Analyzer.

ⁱPadded surface.

^jAccelerometer mounted 180° from flat on motor shaft.

^kSuspended using bungee cords.

^lData obtained after the motor was disassembled and re-assembled.

Table 3. Motor Dimensions					
Motor Identification Number	Section Number ^a	Diameter, mm (in.)	Length, mm (in.)	Dimensionless Diameter ^b	Dimensionless Length ^c
1	1 & 2 ^d	5.99 (0.236)	2.37 (0.094)	1.261	0.047
	3	6.59 (0.259)	6.39 (0.252)	1.385	0.125
	4	28.65 (1.128)	29.18 (1.149)	6.026	0.572
	5	6.63 (0.261)	6.20 (0.244)	1.395	0.121
	6 ^e	4.98 (0.196)	6.88 (0.271)	1.048	0.135
	Shaft ^f	4.75 (0.187)	--		
3	1 & 2 ^d	7.99 (0.315)	3.38 (0.133)	1.008	0.029
	3	9.07 (0.357)	29.95 (1.179)	1.144	0.254
	4	40.91 (1.610)	51.51 (2.028)	5.162	0.437
	5	9.20 (0.362)	27.47 (1.081)	1.161	0.233
	6 ^e	7.97 (0.314)	5.57 (0.219)	1.006	0.047
	Shaft ^f	7.92 (0.312)	--		
4	1 & 2 ^d	9.29 (0.366)	31.75 (1.250)	0.976	0.189
	3	13.60 (0.535)	31.50 (1.240)	1.429	0.188
	4	54.17 (2.133)	69.47 (2.735)	5.690	0.414
	5	13.48 (0.531)	26.44 (1.041)	1.416	0.158
	6 ^e	9.76 (0.384)	8.68 (0.342)	1.025	0.052
	Shaft ^f	9.52 (0.375)	--		
8	1 & 2 ^d	17.03 (0.670)	40.40 (1.591)	0.766	0.161
	3	28.41 (1.118)	50.46 (1.987)	1.278	0.201
	4	99.37 (3.912)	66.01 (2.599)	4.471	0.263
	5	30.45 (1.199)	70.95 (2.794)	1.370	0.283
	6 ^e	30.56 (1.203)	23.33 (0.919)	1.375	0.093
	Shaft ^f	22.23 (0.875)	--		
Averages ^g	1 & 2 ^d	--	--	1.003±0.203	0.106±0.080
	3	--	--	1.309±0.127	0.192±0.053
	4	--	--	5.34±0.68	0.421±0.127
	5	--	--	1.335±0.118	0.199±0.073
	6 ^e	--	--	1.113±0.175 ^h	0.082±0.041
Footnotes on next page.					

Footnotes To Table 3

^aSee Fig. 1 for identification of each section.

^bDiameter for each section divided by diameter of motor shaft as it extends from face of the motor.

^cLength for each section divided by total length of sections 1 through 6.

^dDiameter and length for sections 1 and 2 are lumped together because section 1 is very short, only 0.025 mm (0.001 in.) long.

^eDoes not include that portion of motor shaft extending from the face of the motor.

^fThe motor shaft diameter that extends from the face of the motor, i.e., from section 6. Shaft length is omitted as it is incased in the motor/rotor coupling.

^gWith the standard deviation.

^hDefault motor model uses a dimensionless diameter of 1.0 for section 6 because it also includes that portion of the motor shaft extending from the face of the motor.

IV. BEAM MODEL

The Beam model calculates the first natural frequency for the motor/rotor assembly of a typical contactor design when it is operating. First, the bearing stiffness is determined using BEAM IV for a typical contactor motor. Second, this bearing stiffness and the other quantities and calculations included in the Beam worksheet are discussed. Third, the model used in the Beam worksheet is compared with three other models. Finally, the Beam worksheet is tested using vibrational data from 2-, 4-, and 10-cm contactors. Appendix A is a user manual for the Beam worksheet. A detailed layout of the Beam worksheet and documentation for the Beam_driver macro are presented in Appendix B. The various Beam worksheet and BEAM IV models discussed here are documented in Appendix C.

A. Bearing Stiffness

The k_b values of the various off-the-shelf motors used for centrifugal contactors are generally not known. However, our rotating beam model requires a value for bearing stiffness. To get k_b , the first three natural frequencies (f_n with $n = 1, 2$, and 3) were measured for three different motor types. (See Table 2.) Then, a model of the motor/rotor was created in BEAM IV using the dimensions given in Table 3. Since k_b is not known, f_n was calculated for various k_b values for the first three natural frequencies. The k_b value that best fit the model to the experimentally measured f_n values became the k_b value for the motor bearings.

As an example of this procedure, the natural frequency was calculated as a function of bearing stiffness for the Bodine 710 motor. (See Fig. 2.) The three dashed horizontal lines on this figure give the first three natural frequencies that were measured experimentally. Their intersections with their respective model (calculated) curves give values for k_b . The average value for k_b for the Bodine 710 motor, $2.4 \times 10^5 \pm 1.0 \times 10^5$ lb_f/in. ($4.2 \times 10^7 \pm 1.8 \times 10^7$ N/m) is given in Table 4 along with the average k_b values for the other two motor types tested. Based on these BEAM IV results, a bearing stiffness of 2.4×10^5 lb_f/in. (4.2×10^7 N/m) was chosen for the contactor motors in the Beam worksheet. Note that the two k_b values are clearly marked in the Beam worksheet described in Appendix A so that they can be changed if necessary.

B. Worksheet Design

The design for the Beam worksheet used the transfer matrix method already discussed. The motor/rotor assembly is partitioned into eleven sections, as shown in Fig. 1. The user then enters the specified motor and contactor-rotor dimensions, the density of the liquid in the rotor, the appropriate value for h (see Eq. 6), the fraction of copper in the motor armature, the search range for the natural frequency (to guide the Beam_driver macro), and the number of significant digits needed for f_n . The Beam worksheet treats the rest of the motor rotor and the contactor rotor as steel and uses appropriate values for density and modulus of elasticity. These values can be changed by the user. The user can also include additional mass and additional weight moment of inertia at each section. The worksheet calculates the concentrated masses and the various moments of inertia based on these input parameters. The transfer matrix for each section i ($P_i F_i$) is formed by multiplying the matrices P_i (Eq. 7) and F_i (Eq. 2) together to get

$$P_i F_i = \begin{bmatrix} 1 & L & \frac{L^2}{2EI} & \frac{L^3}{6EI} \\ 0 & 1 & \frac{L}{EI} & \frac{L^2}{2EI} \\ 0 & hJ\omega^2 & 1 + \frac{hJ\omega^2 L}{EI} & L + \frac{hJ\omega^2 L^2}{2EI} \\ m\omega^2 - k & (m\omega^2 - k)L & \frac{(m\omega^2 - k)L^2}{2EI} & 1 + \frac{(m\omega^2 - k)L^3}{6EI} \end{bmatrix} \quad (12)$$

Fig. 2. Comparison of BEAM IV Model Calculated Natural Frequencies with Actual Natural Frequencies for a Nonrotating Bodine 710 Motor. The intersections of the calculated and experimental curves are the values for k_B . Model name in Appendix C is BODINE_710.001.

Table 4. Average Bearing Stiffness Values For Several Motors		
Motor Identification Number ^a	Bearing Stiffness, $lb_f / in.$ (N/m)	Beam IV Model
1, 2	$2.4 \times 10^5 \pm 1.0 \times 10^5$ ($4.2 \times 10^7 \pm 1.8 \times 10^7$)	BODINE_710.001
3	$9.8 \times 10^4 \pm 1.6 \times 10^4$ ($1.7 \times 10^7 \pm 0.3 \times 10^7$)	BODINE_SPC.001
4	$4.4 \times 10^5 \pm 1.5 \times 10^5$ ($7.7 \times 10^7 \pm 4.4 \times 10^7$) ^c	BODINE_254.001

^aSee Table 1.

^bBased on natural frequency measurements analyzed using the BEAM IV program for vibrational analysis.

^cThis average bearing stiffness includes measurements made by Prof. Larry D. Mitchell and K. J. Young [MITCHELL-1987].

Using the input data and the calculated intermediate quantities, the transfer matrix for each section is formed; these matrices are combined to give the overall transfer matrix (U). The Δ determinant function for U is then calculated to see if it is zero: that is, if the frequency used in the calculation is a natural frequency.

Since it can be tedious to supply the different frequencies necessary to converge on the frequency that makes the Δ determinant zero, a Beam_driver macro was set up to do this job automatically. To speed up this calculation further, the calculated values for the length, modulus of elasticity, mass, area moment of inertia, weight moment of inertia, and bearing stiffness are copied into a new section of the worksheet, where they are available for the matrix calculations. The macro copy procedure speeds up the calculation by 25 to 30%. On a Macintosh II running at 16 MHz, the macro calculation takes 55 s; on a Macintosh IIcx running at 25 MHz, it takes 35 s. A second Beam_driver macro uses the Excel table function in the Beam worksheet to generate values for a chart (Beam_chart) that shows the effect of frequency on the determinant function in the user-specified interval. This chart shows if there is more than one natural frequency in the user-specified frequency interval. If there is, then the determinant function will cross the zero axis more than once. Details on using these Beam_driver macros and the Beam_chart are given in Appendix A.

C. Comparison with Other Models

The Beam worksheet was compared with three other models that also calculate natural frequencies. All four models are based on the method of transfer matrices. To compare the models, the first natural frequency (f_1) for the motor/rotor assembly of a 2-cm contactor with a rotor 12.7 cm (5 in.) long was calculated using all four models. The results, given in Table 5, agree well. The massless BEAM IV model should be identical to the Beam worksheet in technique, although it is executed in a very different way (i.e., as a BASIC program running on a VAX computer.) The results, which are essentially the same, provide a check for these two models. There is an extended Beam worksheet that resembles the Beam worksheet, only there are 21 sections, rather than 11, for more accuracy. Using it increased the calculated first natural frequency by only 0.5%, indicating that eleven sections are sufficient. For higher natural frequencies, the 21-section model would be better. Mitchell et al. [MITCHELL-1984] recommend using at least four sections for each natural frequency that you want to calculate. The continuum BEAM IV model has 11 sections, like the Beam worksheet; however, the mass of the shaft is distributed uniformly along the section rather than represented as lumped masses at the ends of the section. The calculated first natural frequency was increased by only 0.6%, again indicating that the massless beam approximation is sufficient. Thus, the choices made in setting up the Beam worksheet are reasonable approximations when using transfer matrices to calculate the first natural frequency for the motor/rotor assembly of centrifugal contactors.

Table 5. Comparison of the Beam Worksheet with Three Other Models

Model ^a	Type of Beam Sections ^b	Number of Sections	Calculated First Natural Frequency, (Hz)	Notes
Beam Worksheet	Massless	11	38.745	c
Massless BEAM IV	Massless	11	38.728	d
Extended Beam Worksheet	Massless	21	38.937	e
Continuum BEAM IV	Continuum	11	38.961	f

^aIn each case, the system modeled is the motor/rotor assembly for a 2-cm contactor with a Bodine 710 motor and a solid rotor that is 12.7 cm (5 in.) long and 2.5 cm (1 in.) in diameter.

^bEither massless or continuum beam sections can be used in the BEAM IV program. The Beam worksheet only supports the massless model, in which the section mass is included as a mass lumped at the end of each section.

^cThis model uses the Beam worksheets as presented in this report. The Excel worksheet used is file "Beam11/2cm(solid)" described in Appendix C.

^dThis massless BEAM IV model is identical to the Beam worksheet; however, it is a BASIC program running on a VAX mainframe computer. The VAX file used, "2CM_SOL.001," is described in Appendix C.

^eThis model uses a special version of the Beam worksheet that has 21 sections instead of the 11 sections used for the Beam worksheet described here. The Excel worksheet used is file "Beam21/2cm(solid)" described in Appendix C.

^fThis continuum BEAM IV model is identical to the Beam worksheet except that the mass of the shaft in each section is distributed continuously along the beam rather than being included as a mass lumped at the end of each section. The VAX file used, "2CM_CON.001," is described in Appendix C.

As a final check of these models, the calculated first natural frequency (38.8 ± 0.1 Hz) was compared with the measured value for this system (37.5 ± 2.5 Hz, see Rotor Code 5.0A2 in Table 6) and found to agree. Other comparisons of the Beam worksheet results to actual contactor data are given in the next section.

D. Application to Contactors

To evaluate the ability of the Beam worksheet to calculate the first natural frequency (f_1) of various motor/rotor systems, worksheet values were compared with experimental data on vibrations for 2-, 4-, and 10-cm contactors. The results of these comparisons are discussed. A description of the Beam worksheet models for the various motor/rotor assemblies is given in Appendix C.

One of the key features of the Beam worksheet is its ability to create a default motor model. Essentially, the dimensions for sections 1 through 6 in Fig. 1 are estimated from (1) the diameter of the motor shaft and (2) the overall motor length minus the length of the motor shaft that protrudes from the motor face. With these two motor dimensions, the average dimensionless diameters and lengths for sections 1 through 6 in Table 3 are used to generate

Table 6. Comparison of Experimental f_1 Values for the 2-cm Motor/Rotor Assemblies with Results from the Beam Worksheet

Test No.	Actual or Default Motor ^a	Type of Rotor ^b	Rotor Code ^c	Rotor Dia., mm (in.)	Rotor Length, mm (in.)	Motor/ Rotor Gap, ^d mm (in.)	Beam Worksheet ^e	First Natural Frequency, ^f Hz	
								Meas.	Calc.
1	A	W ^a	N/A	22.2 (0.875)	64.0 (2.518)	0.8 (1/32)	Beam11/2cm	>72 ^b	130
	D	W ^a	N/A	22.2 (0.875)	64.0 (2.518)	0.8 (1/32)	Beam11/2cm	>72 ^b	144
2	A	W ⁱ	N/A	22.2 (0.875)	64.0 (2.518)	0.8 (1/32)	Beam11/2cm	>72 ^b	127
	D	W ⁱ	N/A	22.2 (0.875)	64.0 (2.518)	0.8 (1.32)	Beam11/2cm	>72 ^b	141
3	A	W ^j	N/A	22.2 (0.875)	64.0 (2.518)	0.8 (1/32)	Beam11/2cm	>72 ^b	125
	D	W ^j	N/A	22.2 (0.875)	64.0 (2.518)	0.8 (1/32)	Beam11/2cm	>72 ^b	138
4	A	S	2.5A1	25.4 (1.00)	63.5 (2.50)	1.6 (1/16)	Beam11/2cm (solid)	>72 ^b	78.0
	D	S	2.5A1	25.4 (1.00)	63.5 (2.50)	1.6 (1/16)	Beam11/2cm (solid)	>72 ^b	85.4
5	A	H	2.5A2	25.4 (1.00)	63.5 (2.50)	1.6 (1/16)	Beam11/2cm (solid)	>72 ^b	86.2
	D	H	2.5A2	25.4 (1.00)	63.5 (2.50)	1.6 (1/16)	Beam11/2cm (solid)	>72 ^b	94.5
6	A	S	5.0A2	25.4 (1.00)	127 (5.00)	1.6 (1/16)	Beam11/2cm (solid)	37.5±2.5	38.7
	D	S	5.0A2	25.4 (1.00)	127 (5.00)	1.6 (1/16)	Beam11/2cm (solid)	37.5±2.5	42.2
7	A	S	7.5	25.4 (1.00)	191 (7.50)	1.6 (1/16)	Beam11/2cm (solid)	25±2.5	24.2
	D	S	7.5	25.4 (1.00)	191 (7.50)	1.6 (1/16)	Beam11/2cm (solid)	25±2.5	26.3
8	A	S	10.0	25.4 (1.00)	254 (10.0)	1.6 (1/16)	Beam11/2cm (solid)	17.5±2.5	16.9
	D	S	10.0	25.4 (1.00)	254 (10.0)	1.6 (1/16)	Beam11/2cm (solid)	17.5±2.5	18.3

Footnotes on next page.

Footnotes to Table 6

^aThe dimensionless diameters and lengths in Table 3 were used to calculate the values for the default motor (D). For the actual motor (A) models, the motor dimensions were measured. The measured values are those given in Table 3 for motor 1.

^b"W" refers to a rotor designed with a separating zone, weirs, and underflow openings so that it will work with two liquid phases as a solvent extraction stage. "S" refers to a solid steel rod that is a stand-in for an actual rotor. "H" refers to a solid rotor with a 1.1-cm- (7/16-in-) dia. hole drilled 5.7 cm (2.25-in.) into its bottom end.

^cActual measurements of the first natural frequency for some rotors were first reported in [STEINDLER-1990]. The rotor codes used there are shown in this column.

^dGap between the top end of the rotor shaft (coupling portion that slips over the motor shaft) and the face of the motor.

^eThese worksheets are given in Appendix C.

^fThese values are for the motor/rotor assemblies as they are spinning. The measured frequencies were obtained as follows. The motor driving the contactor rotor was turned off while they were operating at 60 Hz. Using a strobe light, the lowest rotational speed before excessive vibration occurred and the highest rotational speed after excessive vibration ceased were determined. The average of these two rotational speeds is the measured first natural frequency reported. The actual speeds are used to get the range for the first natural frequency. Excessive vibration is defined as the rotor banging on the metal housing surrounding the rotor.

^gThe rotor is empty in this case.

^hThis value was determined on the basis that, since the motor/rotor assembly did not vibrate at or below 60Hz, the natural frequency is at least 20% higher than 60 Hz.

ⁱIn this case, the rotor contains water, which has a density of 1000 g/L.

^jIn this case, the rotor contains tetrachloroethylene (TCE), which has a density of 1631 g/L.

the default motor model. In each of the tests reported here, the motor was later disassembled and the actual motor dimensions were also determined. The first natural frequency was calculated for both motor models, and the results are included in the tables along with the measured value for this frequency.

When the actual shaft diameter varies within a section of the model, an average shaft diameter is used. The area moment of inertia, which is proportional to the spring stiffness, is the most important parameter affected by this diameter. Since this is a fourth-order effect, the average diameter $\bar{D}$ for these different diameters which are treated as springs in series is given by

$$\bar{D} = \left(\sum_i \frac{f_i}{D_i^4} \right)^{-1/4} \quad (13)$$

where f_i is the fraction of the section length that has the diameter D_i [SHIGLEY]. For a conservative estimate, which will lower the calculated first natural frequency slightly, the smallest shaft diameter could be used for the entire section.

1. 2-cm Contactor

Eight different motor/rotor assemblies were tested for the 2-cm contactor using the Bodine 710 motor. The results, listed in Table 6, show that the calculated f_1 values for all models using actual motor dimensions are within the measured range for f_1 . The same is true for the f_1 values calculated using the default motor dimensions, except for one case (test 6) where the calculated frequency was 5% above the measured range.

The first three tests in Table 6 are for a working 2-cm contactor having a rotor with a diameter of 2.2 cm (0.87 in.) and a length of 6.5 cm (2.5 in.). The system was operated with the rotor spinning (1) empty, (2) filled with water, and (3) filled with tetrachloroethylene (TCE) at 60 Hz (3600 rpm). In each case, the calculated natural frequency was well above 72 Hz, the minimum f_1 value that is possible based on the contactor operating well during all three tests. Because the calculated values for f_1 are so much higher than the lower limit for f_1 established by these experimental tests, this was not a good test of the Beam worksheet.

A better test of the Beam worksheet with the Bodine 710 motor was obtained by using a series of solid rotors. (See tests 4-8 in Table 6.) These tests are better either because they are closer to, but above, the first natural frequency (tests 4 and 5) or because the rotor can pass through the first natural frequency so that this frequency can be bracketed, as explained in Table 6, and an actual value measured (tests 6, 7, and 8). The rotor in test 5 was hollow: that is, a 1.1-cm-diameter hole had been drilled from the bottom surface most of the way through the solid rotor to reduce its mass. The (completely) solid rotors were made progressively longer to increase their mass and therefore decrease the f_1 . As can be seen in Table 6, the calculated value for f_1 in test 4 was just slightly above the detection level of the test procedure. This was reflected in the experimental tests. When test 4 was repeated using a different Bodine 710 motor, a motor that was later found to have a slightly bent shaft, the rotor banged against the housing wall, indicating that f_1 was now less than 72 Hz. In tests 6 through 8, the measured natural frequency agreed with the calculated natural frequency in all three cases when actual motor dimensions were used.

2. 4-cm Contactor

For the 4-cm contactor, two motor/rotor assemblies were tested using a special Bodine motor, number 3 in Table 1. The results, listed in Table 7, show that the calculated f_1 values for both models (using both the default and actual motor dimensions) are within the range expected based on measured f_1 values.

The first three tests were with the "clamp nut" coupling. These three tests provide another good evaluation of the Beam worksheet, as the rotating motor/rotor system was so close to f_1 . The operation went from good (no rotors banging) to bad (all rotors banging) as the mass of liquid in the rotor was increased: that is, as we went from an empty rotor to one filled with water to one filled with TCE. In the clamp-nut coupling design used for these tests, [LEONARD-1988], there is a small shaft diameter for the motor/rotor coupling above the rotor so that the clamping nut can be taken off when the motor and rotor are

Table 7. Comparison of Experimental f_1 Values for the 4-cm Motor/Rotor Assemblies with Results from the Beam Worksheet

Test Number	Actual or Default Motor ^a	Type of Coupling Nut	Rotor Contents	Beam Worksheet ^b	First Natural Frequency, ^c (Hz)	
					Measured ^d	Calculated
1	Actual	Clamp	Empty	Beam11/4cm (clamp nut)	>72	83.8
	Default	Clamp	Empty	Beam11/4cm (clamp nut)	>72	82.9
2	Actual	Clamp	Water	Beam11/4cm (clamp nut)	<72 ^e	73.8
	Default	Clamp	Water	Beam11/4cm (clamp nut)	<72 ^e	73.0
3	Actual	Clamp	TCE	Beam11/4cm (clamp nut)	<72	69.1
	Default	Clamp	TCE	Beam11/4cm (clamp nut)	<72	68.3
4	Actual	Coupling	Empty	Beam11/4cm (coupling)	>72	107
	Default	Coupling	Empty	Beam11/4cm (coupling)	>72	107
5	Actual	Coupling	Water	Beam11/4cm (coupling)	>72	94.9
	Default	Coupling	Water	Beam11/4cm (coupling)	>72	95.0
6	Actual	Coupling	TCE	Beam11/4cm (coupling)	>72	89.1
	Default	Coupling	TCE	Beam11/4cm (coupling)	>72	89.3

^aThe dimensionless diameters and lengths in Table 3 were used to calculate the values for the default motor model. For the actual motor models, the motor dimensions were measured.

^bThese worksheets are given in Appendix C.

^cThese values are for the motor/rotor assemblies as they are spinning ($h = +1$).

^dThese values were determined on the basis of whether or not the motor/rotor assembly banged on the stationary housing when operated at 60 Hz (3600 rpm). If banging occurred, f_1 was taken to be less than 72 Hz, that is, no greater than 20% above the operating speed.

^eFor this test, only three out of the eight rotors banged. Thus, this measurement might more appropriately be written as 72 ± 12 Hz.

disassembled. Thus, for this case, the section assignments for the motor/rotor model given in Appendix A were modified as shown in Fig. 3. Note that the shaft into the rotor (section 9 in Fig. 3) has a larger diameter than the motor/rotor coupling (section 7). In these calculations, Eq. 13 was used to get the average diameter for section 9, which had some diameter changes along its length.

Fig. 3. Modified Section Model for the Simple Clamp-Nut Design

The second set of three tests in Table 7 were with the "coupling nut" coupling. In these three tests, operation remained good as the mass of liquid in the rotor was increased. In the coupling-nut design, the coupling and the captive nut were merged into a single part [STEINDLER-1991]. Thus, the shaft into the rotor (section 9 in Fig. 3) has a smaller diameter than that of the motor/rotor coupling (section 7), which became the coupling nut. Since the diameter of the shaft into the rotor is unchanged, the overall coupling is stiffer than the clamp-nut design and allows better operation, as shown in Table 7. The measured results, which agree with the f_1 values calculated using the Beam worksheet, again demonstrate the usefulness of the Beam worksheet in designing the rotating motor/rotor system.

3. 10-cm Contactor

Four motor/rotor assemblies, each having a different motor, were tested on the 10-cm contactor with the rotor (1) empty, (2) filled with water, and (3) filled with TCE. Since each motor/rotor coupling was of the captive clamp-nut type, the way that sections are allocated in the Beam worksheet was modified as shown in Fig. 4. The results, listed in Table 8, show that the calculated f_1 values for all models using actual motor dimensions are within 10% of the measured range. All f_1 values calculated from the default motor dimensions are within 20% of the measured range. For one case, motor 4 (Bodine 254 motor), the default motor dimensions give f_1 values that are only 67% of those for the actual motor dimensions. Since the default motor dimensions give f_1 values that are lower than those for the measured motor dimensions, the default motor model would seem to be conservative.

Proceeding from motor 4 to motor 7 in Table 8, both the absolute size of the motor and its size relative to that of the rotor increase. At the same time, the differences in the calculated f_1 values using the default and actual motor dimensions decrease. When the rotor

length was increased for the solid 2-cm rotor using the same motor, we noted little change in the relative difference between the calculated f_1 values using the default and actual motor dimensions (see Table 6). Thus, the default motor model appears to work better for larger motors. In particular, it worked well for three motors (motors 5, 6, and 7) that were not used to generate the dimensionless lengths and diameters for the default motor model.

Fig. 4. Modified Section Model for the Captive Clamp-Nut Design

Table 8. Comparison of Experimental f_1 Values for the 10-cm Motor/Rotor Assemblies with Results from the Beam Worksheet

Test Number	Motor ID ^a	Motor Shaft Dia., mm (in.)	Motor/Coupling Gap, ^b mm (in.)	Rotor Contents	Beam Worksheet ^c	Actual or Default Motor ^d	First Natural Frequency, ^e Hz	
							Meas. ^f	Calc.
1	4	9.52(0.375)	4.76(0.1875)	Empty	Beam11/10cm /254	Actual Default	<36	34.8 23.2
2				Water		Actual Default	<36	27.0 18.0
3				TCE		Actual Default	NM ^g	24.2 16.2
4	5	15.9(0.625)	3.18(0.125)	Empty	Beam11/10cm /284	Actual Default	>36	51.1 55.3
5				Water		Actual Default	<36	39.7 43.0
6				TCE		Actual Default	NM ^g	35.7 38.6
7	6	15.9(0.625)	7.94(0.3125)	Empty	Beam11/10cm /3501	Actual Default	>36	50.5 48.4
8				Water		Actual Default	>36	39.3 37.7
9				TCE		Actual Default	<36	35.4 33.9
10	7	22.2(0.875)	-2.38 ^h (-0.0938)	Empty	Beam11/10cm /3510	Actual Default	>36	68.7 68.6
11				Water		Actual Default	>36	53.5 53.4
12				TCE		Actual Default	>36	48.2 48.0

^aSee Table 1.

^bGap between the top surface of the rotor coupling on the motor shaft and the face of the motor.

^cThese worksheets are given in Appendix C.

^dThe dimensionless diameters and lengths in Table 3 were used to calculate f_1 for the default motor models. For the actual motor models, the actual motor dimensions were used.

^eThese values are for the motor/rotor assemblies as they are spinning.

^fThese values were determined on the basis of whether or not the motor/rotor assembly banged on the stationary housing when operated at its normal speed, essentially 30 Hz (1800 rpm). If banging occurred, f_1 was taken to be less than 36 Hz, that is, no greater than 20% above the operating speed.

^gNot measured. This value must be <36 Hz as TCE in the rotor adds even more mass than does water.

^hA recessed area in the center of the motor face allows the end of the coupling to be below the face of the motor.

V. DISCUSSION

From the application of the Beam worksheet to various 2-, 4-, and 10-cm contactors, it appears that the first natural frequency can be calculated to within $\pm 10\%$ if actual motor dimensions are used and to within $\pm 20\%$ if the default motor dimensions are used. These results show that the transfer matrix method works well for calculating the f_1 value of the rotating motor/rotor mass in the centrifugal contactor. Since the model has only eleven sections including a very short one on the left end (see Fig. 1), it should only be used for calculating the first two natural frequencies.

In the future, the Beam worksheet could be extended to high-temperature contactors, to the support frame for a bank of contactors, and to the legs for the support frame. For a high-temperature contactor, the rotor would be hot while the motor would remain cold. For this case, the coupling between the motor and the rotor (section 7 in Fig. 1) would need to be much longer and would be broken up into several sections, each with its own temperature. The modulus of elasticity is then entered in the worksheet as a function of temperature and used to model the effect of temperature on the stiffness of the long coupling between the motor and the rotor. For the support frame and the legs, there is no bearing, so bearing stiffness is zero. Since there is no rotation, h would always be -1. Finally, the boundary conditions at the end of these two beams are not free-free, as is the case for the rotating motor/rotor system, so the Δ determinant given by Eq. 11 no longer applies. If the boundary condition is pinned-pinned (e.g., one bolt holds each end of the support frame), the appropriate Δ determinant is

$$\Delta \text{ (pinned-pinned)} = \begin{vmatrix} u_{12} & u_{14} \\ u_{32} & u_{34} \end{vmatrix} \quad (14)$$

Note that the boundary conditions for a pinned point are that w_i and M_i are zero. If the boundary condition is fixed-fixed (e.g., four bolts hold each end of the support frame or it is welded to the legs), the appropriate Δ determinant is

$$\Delta \text{ (fixed-fixed)} = \begin{vmatrix} u_{13} & u_{14} \\ u_{23} & u_{24} \end{vmatrix} \quad (15)$$

Note that the boundary conditions for a fixed point are that w_i and ψ_i are zero. If the boundary condition is fixed-pinned (e.g., four bolts hold one end of the support frame and one bolt holds the other end), the appropriate Δ determinant is

$$\Delta \text{ (fixed-pinned)} = \begin{vmatrix} u_{13} & u_{14} \\ u_{33} & u_{34} \end{vmatrix} \quad (16)$$

If the boundary condition is fixed-free where the left end (section 1 in Fig. 1) is the fixed end and the right end is the free end, the appropriate Δ determinant is

$$\Delta \text{ (fixed-free)} = \begin{vmatrix} u_{33} & u_{34} \\ u_{43} & u_{44} \end{vmatrix} \quad (17)$$

Note that the boundary conditions for a free point are that M_i and V_i are zero. Equation 17 is good for modeling the contactor leg. The bottom of the leg is the fixed end, while the top of the leg is the free end with the mass of the support frame and contactors acting on it. Note that the top of the leg for the contactor support frame is not actually free; it is constrained in some way that depends on how the leg is attached to the support frame. Since Eq. 17 represents the worse case, its use should result in legs with the required stiffness and, perhaps, with some built-in safety factor.

When the motor is considered alone, the effect of the bearing stiffness on the first natural frequency is significant, as can be seen in Fig. 2 for the Bodine 710 motor. However, when this same motor is considered with a solid rotor that is 6.4 cm (2.5 in.) long and 2.5 cm (1 in.) in diameter, f_1 is almost independent of k_b , as shown in Fig. 5. Since the important criterion for centrifugal-contactor design is to determine f_1 , the estimate for the bearing stiffness of a motor derived from Table 4, $2.4 \times 10^5 \text{ lb}_f/\text{in.}$ ($4.2 \times 10^7 \text{ N/m}$), should be sufficient for calculating the first natural frequency with the Beam worksheet.

The mode shape plots for deflection produced by BEAM IV for the 2-cm solid rotor that is 6.4 cm (2.5 in.) long is shown in Fig. 6. Note that the BEAM IV sketch, Fig. 6a, has the same orientation as the deflection curves but is about 40% longer. The stiffness of the motor bearings at the left side of the sketch are indicated by a resistance, or spring, marked with the spring constant k . The solid rotor is the thick striped area on the right side of the sketch. Concentrated weight at the end of a section is indicated by a circle. Thus, the first natural frequency is seen to correspond to the vibrating (ringing) of the contactor rotor on the motor shaft.

Fig. 5. Effect of Bearing Stiffness on Natural Frequency from BEAM IV for a Bodine 710 Motor with a Nonrotating 2.5-in.-Long Solid Rotor. Model name in Appendix C is MINI_710.001.

For the Bodine 710 motor itself, the BEAM IV model indicates that this ringing of the shaft that extends from the face of the motor occurs at the second natural frequency f_2 . This ringing can be seen in Fig. 7, which gives the mode shape plots for deflection for the Bodine 710 motor alone. Note that Fig. 7 shows that the third natural frequency f_3 for the Bodine 710 motor involves a ringing of the extended motor shaft, along with some movement of the motor shaft between the motor bearings. As indicated in Table 2, this peak had two fine peaks at the top of the main f_3 peak. These two fine peaks may be the result of the area moment of inertia of the motor shaft extending from the motor face. This shaft extension has a different area moment of inertia for the x- and y-axes because of the flat on this shaft extension. However, using the two area moments of inertia associated with the shaft extension does not give as much separation of the two fine peaks as was actually measured.

The mode shapes for deflection for f_1 and f_3 of the Bodine 710 motor given in Table 2 and shown in Fig. 7 correspond to f_3 and f_2 , respectively, in Fig. 6. Thus, the addition of the solid rotor to the Bodine 710 motor has almost no effect on the value of f_1 (now f_3) and reduces f_3 (now f_2) by a factor of 4. The biggest change is in the value of f_2 (now f_1), the ringing peak, which goes from 3528 Hz to 72 Hz (50 times lower) when the solid rotor is added. This again shows how f_1 for the motor/rotor assembly, that is, the ringing of the rotor, is more dependent on the length and diameter of the motor/rotor coupling and on the mass of the rotor than on bearing stiffness.

a) BEAM IV Sketch of the Model

b) First Natural Frequency

c) Second Natural Frequency

d) Third Natural Frequency

Fig. 6. Mode Shape Plots for Deflection from BEAM IV Model of Bodine 710 Motor with 2.5-in.-Long Solid Rotor. Model name in Appendix C is MINI_710.001. Static (nonrotating) case is shown.

Fig. 7. Mode Shape Plots for Deflection from BEAM IV Model of Bodine 710 Motor Alone. Model name in Appendix C is BODINE_710.001. Static (nonrotating) case is shown.

VI. SUMMARY

A new method, the Beam worksheet, has been developed for calculating the first natural frequency of a rotating motor/rotor assembly in a centrifugal contactor. The model was tested over a wide range of contactor sizes and found to be accurate to within $\pm 20\%$. As such, the Beam worksheet becomes a powerful tool for evaluating the designs of future centrifugal contactors prior to construction.

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APPENDIX A

MANUAL FOR THE BEAM WORKSHEET

This manual tells how to use the Beam worksheet to calculate the first natural frequency of the motor/rotor system in an Argonne centrifugal contactor.

A. Introduction

The Beam worksheet, set up using Microsoft Excel 2.2, calculates the natural frequencies of a vibrating beam system in both stationary and rotating modes. Most importantly, the worksheet finds the first natural frequency of a rotating system consisting of the rotor in the centrifugal contactor and the rotor in the motor. The Beam worksheet allows a preliminary evaluation of the motor/rotor system to be made easily while the contactor is still being designed. As more specific information becomes available to the designer, the system parameters can be added to the worksheet and so improve upon the preliminary calculations.

Worksheet calculations are based on the transfer matrix method for analyzing beams. This method breaks complicated beams up into short sections that can be analyzed easily using a transfer matrix. The transfer matrices are then multiplied together to get an overall transfer matrix that reflects the mechanical characteristics of the overall beam and from which the natural frequencies of the overall beam can be calculated. The Beam worksheet was checked using the BEAM IV program, which is also based on the method of transfer matrices, licensed from Virginia Polytechnic Institute and State University, Blacksburg, Virginia.

The Beam worksheet and the Beam_driver macro sheet must be run on a Macintosh personal computer with Microsoft Excel 2.2 or higher. Then, following the procedure given, the beam parameters are entered into the worksheet and a macro is run to determine the natural frequency for the beam in the range of interest.

B. Using the Beam Worksheet

The user must enter the required beam parameters into the worksheet and run one of the macros on the Beam_driver macro sheet to get the natural frequency within a specified range. These operations are described here along with a discussion of possible errors.

1. Data Input

Data needed for the Beam worksheet include dimensions from the motor, the motor/rotor coupling, the contactor rotor, and some material properties. Worksheet cells that allow or require user input are underlined. Note that default values are shown for many cells. They should be replaced with more appropriate values, if available.

In order to enter data, first open the Beam worksheet. For information on how to turn on the Macintosh personal computer and to open the Excel worksheet, refer to the user

manuals supplied with the Macintosh and with Microsoft Excel. It should also be possible to use the Beam worksheet and Beam_driver macro on an IBM personal computer or clone that runs Microsoft Excel for the IBM PC with few or no changes.

Note that the original Beam worksheet should never be used. Instead, select the unopened worksheet and choose "Duplicate" from the Macintosh File menu. Work only with this copy of the Beam worksheet. Thus, the original Beam worksheet with appropriate default values and formulas in the data input cells will be available for future use.

In the motor/rotor model in the Beam worksheet, the rotating beam was divided into eleven sections with section end points as shown in Fig. A-1. Data must be input in four areas of the worksheet. These input areas, shown in Figs. A-2 through -5, provide (1) general information, (2) motor and rotor details, (3) physical constants and conversion factors, and (4) parameters for each beam section. The data required for each input area is now described in some detail.

Fig. A-1.

Schematic of the 11 Sections of the Motor/Rotor System Used in the Beam Worksheet

	A	B	C	D	E	F	G
1	BEAM VIBRATION ANALYSIS						
2	Date of last revision: April 1, 1991						
3	General Input Area						
4	Name of Value	Value	Units	Description			
5	DMS	0.625	in.	Diameter of Output Shaft of Motor			
6	LOAM	5	in.	Overall Length of Motor (less shaft beyond motor face)			
7	GAP	0.03125	in.	Gap Between Motor Face and End of Motor/Rotor Coupling			
8	rho_LIQ	997	g/L	Density of Liquid in Rotor			
9	h	+1	-	Whirl Direction (Static = -1, Forward = +1, Backward = -3)			
10	Frequency Range to Search						
11	F_LOW	0	Hz	Lower Frequency Limit			
12	F_HIGH	500	Hz	Upper Frequency Limit			
13	ACCURACY	4	-	Number of Significant Digits Needed			
14							
15	Output Area						
16							
17	FREQ Hz	1.277E+02	Hz	Natural Frequency			
18							

Fig. A-2. General Input and Output Areas for the Beam Worksheet

	H	I	J	K	L	M
1	Motor and Rotor Details					
2	Name of Value	Value	Units	Description		
3	Motor details					
4	Cu_Pct	35%	-	Fraction of Cu in Motor Armature (section 4)		
5	Rotor details					
6	D_upper	0.250	in.	Upper Weir Diameter (nominal)		
7	Up_weir_width	0.200	in.	Upper Weir Width		
8	Riser_width	0.020	in.	Width of Riser to Upper Weir		
9	D_lower	0.250	in.	Lower Weir Diameter (nominal)		
10	D_in	0.250	in.	Rotor Inlet Diameter		
11	OD_rotor	1.575	in.	Rotor Outer Diameter		
12	ID_rotor	1.500	in.	Rotor Inner Diameter		
13	Cavity_width	0.250	in.	Width of Lower Weir Cavity		
14	No_Vanes	4	-	Number of Vanes in Rotor Body		
15	T_Vanes	0.050	in.	Thickness of Vanes		

Fig. A-3. Input Area for Motor and Rotor Details

	N	O	P	Q	R
1	Physical Constants and Conversion Factors				
2	Name of Value	Value	Units	Description	
3	CF_Gc	386.4	lbm•in/(lbf•s ²)	Gravitational Conversion Factor	
4	CF_gpL_pci	3.613E-05	(lbm/in ³)/(g/L)	Converts from g/L to lbm/in ³	
5	rho_steel	0.28	lb/in ³	Density of Steel	
6	rho_copper	0.322	lb/in ³	Density of Copper	
7	E_steel	2.9E+07	lbf/in ²	Elastic Modulus of Steel	
8	E_copper	1.6E+07	lbf/in ²	Elastic Modulus of Copper	

Fig. A-4. Input Area for Physical Constants and Conversion Factors

	A	B	C	D	E	F	G
2 2	Parameters for Each Beam Section						
2 3	In the table below, user may put data in all underlined cells below the title line.						
2 4	Note that some input values are already calculated from data entered elsewhere.						
2 5				Motor Data			
2 6	Description	Units	SECTION	1	2	3	4
2 7	Elastic Modulus	lb/in ²	Mod_e	2.9E+07	2.9E+07	2.9E+07	2.4E+07
2 8	Density of Material	lb/in ³	Density	2.800E-01	2.800E-01	2.800E-01	2.947E-01
2 9	Outer Diameter	in.	OD	6.269E-01	6.269E-01	8.181E-01	3.338E+00
3 0	Inner Diameter	in.	ID	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3 1	Length	in.	LEN	1.000E-03	5.290E-01	9.600E-01	2.105E+00
3 2	Additional Mass	lb	Add_mass	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3 3	Additional Weight Moment of Inertia						
3 4		lb·in ²	Add_IW	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3 5	Bearing Stiffness	lbf/in	BS	2.4E+5			

Fig. A-5a. Row Captions and First Four Sections

	H	I	J	K	L	M	N
2 2	Parameters for Each Beam Section (cont.)						
2 3	In the table below, user may put data in all underlined cells below the title line.						
2 4	Note that some input values are already calculated from data entered elsewhere.						
2 5			Coupling Data	Rotor Data			
2 6	5	6	7	8	9	10	11
2 7	2.9E+07	2.9E+07	2.9E+07	2.9E+07	2.9E+07	2.9E+07	2.9E+07
2 8	2.800E-01	2.800E-01	2.800E-01	2.800E-01	2.800E-01	2.800E-01	2.800E-01
2 9	8.344E-01	6.250E-01	3.750E-01	2.500E-01	1.575E+00	1.575E+00	1.575E+00
3 0	0.000E+00	0.000E+00	0.000E+00	0.000E+00	2.500E-01	1.500E+00	2.500E-01
3 1	9.950E-01	4.413E-01	5.000E-01	5.000E-01	3.000E-01	3.500E+00	1.061E-01
3 2	0.000E+00	0.000E+00	0.000E+00	1.238E-03	0.000E+00	2.008E-01	0.000E+00
3 3							
3 4	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3 5	2.4E+5						

Fig. A-5b. Last Seven Sections

Fig. A-5. Input Area Giving Parameters for Each Beam Section

a. General Input and Output Areas

General Beam data are entered in the input area shown in Fig. A-2. Enter the diameter of the output shaft of the motor (DMS) and the overall length of the motor less the shaft beyond the motor face (LOAM), and allow the worksheet to generate typical dimensions for the internal parts of the motor, sections 1 through 6 in Fig. A-1. (Note that each section is referred to by the number at the right end of the section.) This overall motor information is usually available from vendor catalogs. If internal motor dimensions are available, they should be entered directly in the input area for motor data shown in Fig. A-5. The gap between the motor face and the top of the motor/rotor coupling (GAP) completes the information needed to specify the length of section 6.

Part of the rotor mass comes from the liquid in the rotor. The Beam worksheet calculates this liquid volume in each section of the rotor and gives it the density (rho_LIQ) specified here. If the density is zero, the rotor is empty. In general, the density of the more dense phase should be used to get the worst-case (lowest) value for the first natural frequency, f_1 .

The whirl direction (h) allows the user to calculate the natural frequency, f_n , for the beam shown in Fig. A-1 under three different conditions. If the beam is stationary, set h to -1. If the beam is rotating in a normal manner so that forward whirl is expected, set h to +1. If the beam is rotating under special conditions so that backward whirl is possible, set h to -3. In general, for rotating beams use the forward whirl condition with an h of +1.

Enter the upper and lower limits of the range in which the natural frequency is to be calculated (F_HIGH and F_LOW). Choose this range carefully so that there is only one natural frequency within the range. Further discussion on selecting the frequency range and catching possible errors can be found in sections 2 and 3 below. For the first natural frequency, a range from ten to a few hundred hertz is usually appropriate. Another parameter related to these calculations is the ACCURACY, the number of significant digits needed for the natural frequency being calculated. Since the worksheet converges on the natural frequency by successive macro-driven iterations, calculation time can be minimized by not asking for more significant figures than needed. When four significant figures are requested, the iterative calculation required to get the natural frequency takes from 40 to 55 seconds on a Mac II and from 80 to 135 seconds on a Mac SE or Plus, depending on which of the macros described here are used. Note that Excel can handle up to 14 significant figures.

The natural frequency in hertz is placed in the output area in Fig. A-2, shown between the two shaded rows, by a Beam_driver macro. The use of the macros is discussed in section 2. Note that the value of the natural frequency used in the worksheet calculations has units of $\text{lb}_f^{0.5}/(\text{lb}_m \cdot \text{in.})^{0.5}$.

b. Motor and Rotor Details

The input area containing the motor and rotor details is shown in Fig. A-3. The fraction of copper contained in the armature (section 4 in Fig. A-1) is specified at Cu_Pct. If this parameter is zero, the calculations will be based on a steel armature.

Next, the rotor details are entered. Figure A-6 is a schematic of the rotor and the motor/rotor coupling, with most of the detailed dimensions named in the worksheet identified. Also shown in Fig. A-6 are details on how the rotor is divided to calculate sections 7 through 11 in Fig. A-1. Figure A-3 lists the names and descriptions of the data needed here. Since D_upper and D_lower are normally not known in the early design stages, the use of D_in is recommended as a conservative initial value. Two other rotor details are needed for the input area shown in Fig. A-3: (1) the number of vanes in the rotor body (No_vanes) and (2) the thickness of these vanes (T_vanes).

Fig. A-6. Schematic Showing Rotor Details for the Beam Worksheet

c. Physical Constants and Conversion Factors

The input area for physical constants and conversion factors is shown in Fig. A-4. Here, two conversion factors, the gravitational conversion factor (CF_Gc) and the density conversion factor (CF_gpL_pci), are given. The square root of CF_Gc is used to convert the frequency in $[lb_f / (lb_m \cdot in.)]^{0.5}$ to rad/s. This number is then divided by 2π to get the frequency in Hz. CF_gpL_pci is used to convert g/L (or kg/m³) to lb_m/in³. In addition, the density (rho_xxx) and modulus of elasticity (E_xxx) are given for both steel and copper. Any of these values can be changed by the user.

d. Parameters for Each Beam Section

The remaining rotor and motor/rotor coupling dimensions are entered in the Parameters for Each Beam Section area shown in Fig. A-5. The outer and inner diameters (OD and ID) of section 7 (the rotor coupling) are entered here. When a section has several OD values, use an average OD value. A conservative choice is to use the smallest value for OD that should give the lowest value for f_1 . The lengths (LEN) of sections 7 through 11 are also entered here.

In this area the user can enter individual values for the modulus of elasticity (Mod_e) and the density of the material (Density) for each section of the model. The default values are for a system that uses steel throughout, except for section 4, which contains the armature. This section is assumed to be made of steel mixed with the percentage of copper specified by the value of Cu_Pct.

The stiffness of the motor bearings (BS) is also specified in this area. The worksheet positions the bearings at the right ends of sections 2 and 5 by having default values for BS in the cells shown in Fig. A-5. The user may change the value of BS, but the value provided, which is based on a detailed evaluation of three motors, is recommended. The position of the bearings is changed by moving the BS values to different sections (cells) in the BS row shown in Fig. A-5.

Finally, two other parameters can be specified by the user. These quantities, additional mass and additional weight moment of inertia, can be added to any of the sections at the user's discretion. Lines have been provided for these entries as Add_mass and Add_IW, respectively. Note that a value specified for Add_mass will not affect the weight moment of inertia of a section. Also, please note that a formula has already been entered for Add_mass in sections 8 and 10. This calculation represents the mass of the liquid inside the rotor in these sections. To add further mass to these sections, append the formula already present with a plus (+) sign followed by a value or formula for the additional mass. Finally, please note that Add_mass and Add_IW entries to section i are treated in subsequent calculations as if they are uniformly distributed in section i.

2. Calculation of Natural Frequency

To calculate the natural frequency after the input data have been entered, go to the general input area shown in Fig. A-2 and specify the range (F_LOW, F_HIGH) in which to search for the natural frequency. Some trial and error may be required before the appropriate range is found. If the first natural frequency is desired, a good starting point is to set F_LOW at 10 Hz and F_HIGH at 300 Hz. The ACCURACY, the number of significant digits of accuracy desired, is specified beneath the frequency range. Three or four significant digits is typical.

Now open the Beam_driver macro sheet, which has three macros. (What these three macros (b, c, and B) do and how to use them are discussed in the next three paragraphs.) Also, open the Excel chart Beam_chart, which is used by two of the three macros on the macro sheet. Finally, make the Beam worksheet the active window on the

computer screen. When this is done, the six horizontal lines of its title bar will be visible. Now, the macro "b" can be used to calculate the natural frequency by pressing Command-Option-b. It can also be run using the "Macro Run..." item on the Excel menu bar. The calculations take about 40 seconds on a Mac II and about 80 seconds on a Mac SE or Mac Plus. The frequency can be seen changing in the output area box (see Fig. A-2) as it converges to the natural frequency. When the calculation is finished, an alert box will appear stating the natural frequency. This frequency will also be found in the Freq_Hz box in the output area. When the alert box is acknowledged and the macro stops, the active cell will be the same one that was active when the macro was started. The frequency is displayed in scientific notation, with the number of digits specified in ACCURACY. All the digits shown are accurate.

The chart, when used with the macro "c", becomes a plot of the determinant function over the frequency range specified in the general input area shown in Fig. A-2 via the F_LOW and F_HIGH boxes. At each point where the curve of the determinant function becomes zero (crosses the horizontal axis), a natural frequency exists. Thus, the chart can be used to determine the number of natural frequencies in a specified range and to give their approximate values. One such chart, (see Fig A-7) shows that three natural frequencies occur in the range from 50 to 3000 Hz and estimates the value of each. The user then chooses a smaller range containing only one of the frequencies to calculate the exact value of the frequency using the b macro. If the frequency range specified is too broad, the chart may not be able to show all the natural frequencies in the range, since it uses only eleven equally spaced points. As the frequency range is narrowed, any other natural frequencies could appear.

Fig. A-7. Use of Determinant Function to Locate Natural Frequencies

To use the chart, invoke the c macro using Command-Option-c with the Beam worksheet active. As with the b macro, this macro can be run by selecting the appropriate line from the "Macro Run..." item on the Excel menu bar. The calculation will be based on

the frequency range in the Beam worksheet. If desired, the calculation of the exact natural frequency in a range can be performed along with the calculation of the chart for the same range. This combination constitutes the B macro, which essentially combines the b and c macros into one. The user presses Command-Option-Shift-b (also written as Command-Option-B) with the Beam worksheet active. This macro can also be selected using the appropriate line from the "Macro Run..." item on the Excel menu bar.

Once the desired natural frequency of a motor/rotor system has been found, it is a simple matter to change one or more parameters and see their effect on the natural frequency. In this way, a variety of design options can be explored easily and rapidly.

3. Errors

The transfer matrix model used in the Beam worksheet is based on a design where each beam section is massless with a lumped mass at its right end. The more beam sections in the model, the better the model is. We recommend using four beam sections for each natural frequency to be calculated. Since this model has 11 sections, it should only be used to calculate the first and second natural frequencies.

When a frequency range is specified, it should contain only one natural frequency. If this is not the case, the natural frequency will not be calculated and an alert box, "No natural frequency or more than one natural frequency between limits" is displayed. However, if the B macro was used to calculate the natural frequency, the chart will be updated to show what roots, if any, are in the specified interval.

If there are an odd number of natural frequencies (e.g., three, five, or seven), this multiple root problem will not be detected and no alert box will appear. Instead, the macro will find one of the natural frequencies. This error can be detected if the frequency chart is calculated at the same time that the natural frequency is determined.

To prevent computational errors, each beam section should have some length, even if it is only 10^{-9} inches. In addition, the bearing (support) stiffness cannot be infinite. However, it can be very large, up to 10^{10} lb/in. which is essentially a rigid support.

Beams containing rotor supports (e.g., the motor bearings) or tuned absorbers become branched systems and, as such, will have infinity wraps; that is, the system becomes infinitely stiff and so poses no vibration problem. These infinity wraps appear as sharp changes in the determinant plot given in the chart. As the determinant function is calculated for discrete points, the curve appears to go up slightly, then it comes down sharply, crossing the horizontal axis at a natural frequency. However, this is really a false zero, not a natural frequency. Instead, the curve is going off to plus infinity and coming back from minus infinity, or visa versa, on either side of the false natural frequency. The user can check for an infinity wrap by making the frequency range closer and closer to the apparent natural frequency. As this is done, the value of the determinant function increases.

APPENDIX B

DOCUMENTATION FOR BEAM WORKSHEET AND MACRO SHEET

The Beam worksheet used in this report is documented here in Figs. B-1 through B-8. The various areas of the worksheet are printed out in "formula" view to show the formulas used. In some cases, the formulas were too long, so the complete formulas are given below the figure as text. In formula view, text cells are sometimes cut short because text cannot be displayed outside the cell in this view. These truncated text lines, if not given, can be read from the figures in Appendix A. Note that some of the matrices are omitted in Fig. B-6. Those matrices not shown can be reconstructed from the forms shown. Each of the 4 x 4 matrices where matrix multiplication takes place, MATRIX.1 through FINAL_MATRIX, must be designated as an array, as specified in the Excel manual. The names used in the Beam worksheet and the cells they refer to are listed in Fig. B-8.

There are two other factors involved in creating the Beam worksheet. (1) The matrix input values in Fig. B-5 do not need to be added, as the appropriate values are put here by the Beam_driver macro. (2) The table function shown in Fig. B-7 is put in place by selecting the whole table and following the procedure given in the Excel manual.

As the Beam worksheet is set up, the additional mass (Add_mass) and the additional weight moment inertia (Add_IW) are treated as if they were uniformly distributed across the section. If the user wants to have the additional mass and the additional weight moment of inertia apply to the right-hand side of each section, two rows of the worksheet have to be changed. In row 47 (see Fig B-4), all "Add_mass/2" terms should be changed to "Add_mass" and all "Add_mass" terms within the INDEX function should be deleted. Similarly, in row 49, all "Add_IW/2" terms should be changed to "Add_IW" and all "Add_IW" terms within the INDEX function should be deleted.

The macro sheet for the Beam driver macro is documented here in Figs. B-9, 10, and 11. In the macro sheet format used, the actual macros are in column B. The words in column A are names for specific macro cells if they are formatted as plain text. Column C is used to document the macro commands in column B. The names used and the cell or cells they refer to are listed in Fig. B-11. Please see an Excel manual for further details on the use of macros.

	A	B	C	D
1	BEAM			
2				
3	General Input			
4	Name of Value	Value	Units	Description
5	DMS	= 5/8	in.	Diameter of Output Shaft of Motor
6	LOAM	5	in.	Overall Length of Motor (less shaft beyond motor face)
7	GAP	= 1/32	in.	Gap Between Motor Face and End of Motor/Rotor Coupling
8	rho_LIQ	997	g/L	Density of Liquid in Rotor
9	h	1	-	Whirl Direction (Static = -1, Forward = +1, Backward = -3)
10	Frequency Range			
11	F_LOW	0	Hz	Lower Frequency Limit
12	F_HIGH	500	Hz	Upper Frequency Limit
13	ACCURACY	4	-	Number of Significant Digits Needed
14				
15	Output Area			
16				
17	FREQ_Hz	=freq*SQRT(CF_Gc)/(2*PI()), Hz		Natural Frequency
18				
19	freq	40.8202835678118	$\frac{(\text{lbf})^{0.5}}{(\text{lbm} \cdot \text{in})^{0.5}}$	Natural frequency shown here is the value used in the calculations below. The odd units are equivalent to
20				
21	="(&CF Gc&")^{0.5} rad/sec"			

Fig. B-1. General Input and Output Areas of the Beam Worksheet

	N	O	P	Q
1	Physical Cons			
2	Name of Value	Value	Units	Description
3	CF_Gc	= 32.2*12	lbm·in/(lbf·s^2)	Gravitational Conversion Factor
4	CF_gpL_pci	= 0.036127/1000	(lbm/in^3)/(g/L)	Converts from g/L to lbm/in^3
5	rho_steel	0.28	lb/in^3	Density of Steel
6	rho_copper	0.322	lb/in^3	Density of Copper
7	E_steel	29000000	lbf/in^2	Elastic Modulus of Steel
8	E_copper	16000000	lbf/in^2	Elastic Modulus of Copper

Fig. B-2. Physical Constants and Conversion Factors Area of the Beam Worksheet

	A	B	C	D	E	F	G
2 2	Parameters for Ea						
2 3	In the table below, u						
2 4	Note that some input						
2 5							
2 6	Description	Units	SECTION	1	2	3	4
2 7	Elastic Modulus	lb/in^2	Mod_e	=E_steel	=E_steel	=E_steel	=(1-Cu_Pct)*E_steel+Copper_Pct*E_copper
2 8	Density of Material	lb/in^3	Density	=rho_steel	=rho_steel	=rho_steel	=(1-Cu_Pct)*rho_steel+Copper_Pct*rho_copper
2 9	Outer Diameter	in	OD	=DMS*1.0026	=DMS*1.0026	=DMS*1.30909	=DMS*5.3372
3 0	Inner Diameter	in	ID	0	0	0	0
3 1	Length	in	LEN	0.001	=LOAM*0.10631-0.001	=LOAM*0.191951	=LOAM*0.42142
3 2	Additional Mass	lb	Add_mass	0	0	0	0
3 3	Additional Weight Moment						
3 4		lb-in^2	Add_IW	0	0	0	0
3 5	Bearing Stiffness	lb/in	BS		240000		
3 6							
3 7	END OF USER INPUT						

Fig. B-3a. Row Headings and Input for First Four Beam Sections

	H	I	J	K	L	M	N
2 2	Parameters for Ea						
2 3	In the table below, u						
2 4	Note that some input						
2 5							
2 6	5	6	Coupling Data	7	Rotor Data	9	10
2 7	=E_steel	=E_steel	=E_steel	=E_steel	=E_steel	=E_steel	=E_steel
2 8	=rho_steel	=rho_steel	=rho_steel	=rho_steel	=rho_steel	=rho_steel	=rho_steel
2 9	=DMS*1.33528	=DMS	0.375	0.25	=OD_rotor	=OD_rotor	=OD_rotor
3 0	0	0	0	0	=D_lower	=ID_rotor	=D_in
3 1	=LOAM*0.198639	=LOAM*0.081685+GAP	0.5	0.5	0.3	3.5	0.106135
3 2	0	0	0	(a)	0	(b)	0
3 3							
3 4	0	0	0	0	0	0	0
3 5	240000						
3 6							
3 7	END OF USER INPUT A1						

Fig. B-3b. Input for Last Seven Beam Sections

- (a) $=\text{rho_LIQ} * \text{CF_gpL_pci} * \text{PI}() / 4 * (\text{ID_rotor}^2 - \text{D_upper}^2) * \text{Riser_width}$
- (b) $=\text{rho_LIQ} * \text{CF_gpL_pci} * (\text{PI}() / 4 * (\text{ID}^2 - \text{D_lower}^2) - \text{No_Vanes} * (\text{ID} - \text{D_lower}) / 2 * \text{T_Vanes}) * \text{LEN}$

Fig. B-3c. Listing of Formulas from Line 32 (too long to be displayed in worksheet cells)

Fig. B-3. Sectional Input Area of the Beam Worksheet

	A	B	C	D	E	F	G
3 8							
3 9	Intermediate Calculations						
4 0	Mass Calculations	lbm	Mass	0	(a)	(b)	(b)
4 1	Area Moment Calc's	in^4	IA	(c)	(c)	(c)	(c)
4 2	Weight Moment Calc's	lb•in^2	IW	(d)	(d)	(d)	(d)
4 3							
4 4	Final Input Calculations						
4 5	Length	in	L_Final	=LEN	=LEN	=LEN	=LEN
4 6	Modulus of Elasticity	psi	E_Final	=Mod_e	=Mod_e	=Mod_e	=Mod_e
4 7	Mass	lbm	M_Final	(e)	(f)	(f)	(f)
4 8	Area Moment of Inertia	in^4	IA_Final	=IA	=IA	=IA	=IA
4 9	Wt. Moment of Inertia	lb•in^2	IW_Final	(g)	(h)	(h)	(h)
5 0	Bearing Stiffness	lbf/in	K_Final	=BS	=BS	=BS	=BS

Fig. B-4a. Row Headings and Calculations for First Four Beam Sections

	H	I	J	K	L	M	N
3 8							
3 9							
4 0	(b)	(b)	(b)	(i)	(j)	(k)	(l)
4 1	(c)	(c)	(c)	(c)	(c)	(c)	(c)
4 2	(d)	(d)	(d)	(d)	(d)	(d)	(d)
4 3							
4 4							
4 5	=LEN	=LEN	=LEN	=LEN	=LEN	=LEN	=LEN
4 6	=Mod_e	=Mod_e	=Mod_e	=Mod_e	=Mod_e	=Mod_e	=Mod_e
4 7	(f)	(f)	(f)	(f)	(f)	(f)	=Mass/2+Add_mass/2
4 8	=IA	=IA	=IA	=IA	=IA	=IA	=IA
4 9	(h)	(h)	(h)	(h)	(h)	(h)	=IW/2+Add_IW/2
5 0	=BS	=BS	=BS	=BS	=BS	=BS	=BS

Fig. B-4b. Calculations for Last Seven Beam Sections

- (a) =Density*PI()/4*(OD^2-ID^2)*(LEN+0.001)
 (b) =Density*PE()/4*(OD^2-ID^2)*LEN
 (c) =PI()*(OD^4-ID^4)/64
 (d) =Mass/48*(3*OD^2+3*ID^2+4*LEN^2)
 (e) =Mass+Add_mass/2+INDEX(Mass+Add_mass,SECTION+1)/2
 (f) =Mass/2+Add_mass/2+INDEX(Mass+Add_mass,SECTION+1)/2
 (g) =IW+Add_IW/2+INDEX(IW+Add_IW,SECTION+1)/2
 (h) =IW/2+Add_IW/2+INDEX(IW+Add_IW,SECTION+1)/2
 (i) =Density*PI()/4*(OD^2*LEN+(OD_rotor^2-D_upper^2)*Up_weir_width+OD_rotor^2-ID_rotor^2)*Riser_width)
 (j) =Density*PI()/4*(OD^2-ID^2)*LEN+PI()*(D_lower^2/4*(LEN-Cavity_width)
 (k) =Density*(PI()/4*(OD^2-ID^2)+No_Vanes*ID/2*T_Vanes)*LEN
 (l) =Density*PI()/4*(OD^2-ID^2)*LEN

Fig. B-4c. Listing of Formulas from Lines 40 to 49 (too long to be displayed in worksheet cells)

Fig. B-4. Intermediate and Final Input Calculation Areas of the Beam Worksheet

	A	B	C	D	E	F	G
5 1							
5 2	Matrix Input Values						
5 3	Use Beam_driver macro to Input these values or						
5 4			Section 1	2	3	4	
5 5	Length	ln	L_I	0.001	0.529	0.96	2.105
5 6	Modulus of Elasticity	psi	E_I	29000000	29000000	29000000	24450000
5 7	Mass	lbm	M_I	0.0229010	0.0935537	2.7841829	2.7896966
5 8	Area Moment of Inertia	ln^4	IA_I	0.0075804	0.0075804	0.0219911	6.0905287
5 9	Wt. Moment of Inertia	lb*ln^2	IW_I	0.0010965	0.0094782	2.8994683	2.9006846
6 0	Bearing Stiffness	lb/ln	K_I	240000			

Fig. B-5a. Row Headings and Matrix Input for First Four Beam Sections

	H	I	J	K	L	M	N
5 1							
5 2							
5 3							
5 4	5	6	7	8	9	10	11
5 5	0.995	0.44125	0.5	0.5	0.3	3.5	0.1061347
5 6	29000000	29000000	29000000	29000000	29000000	29000000	29000000
5 7	0.0951187	0.0266835	0.0654557	0.1386972	0.3434036	0.2906434	0.0282125
5 8	0.0237911	0.0074901	0.0009707	0.0001917	0.3017165	0.0534033	0.3017165
5 9	0.0103682	0.0009992	0.0016417	0.0148871	0.2267621	0.2177973	0.0045096
6 0	240000						

Fig. B-5b. Matrix Input for the Last Seven Beam Sections

Fig. B-5. Matrix Input Area of the Beam Worksheet

	A	B	C
62	MATRIX_ONE	1	=L_1
63		0	1
64		0	=H*IW_1*freq^2
65		=M_1*freq^2	=M_1*freq^2*L_1
66			
112	MATRIX_ELEVEN	1	=L_11
113		0	1
114		0	=H*IW_11*freq^2
115		=M_11*freq^2	=M_11*freq^2*L_11
116			
117	MATRIX.1	=MMULT(MATRIX_ELEVEN,MATRIX_TEN)	=MMULT(MATRIX_ELEVEN,MATRIX_TEN)
118		=MMULT(MATRIX_ELEVEN,MATRIX_TEN)	=MMULT(MATRIX_ELEVEN,MATRIX_TEN)
119		=MMULT(MATRIX_ELEVEN,MATRIX_TEN)	=MMULT(MATRIX_ELEVEN,MATRIX_TEN)
120		=MMULT(MATRIX_ELEVEN,MATRIX_TEN)	=MMULT(MATRIX_ELEVEN,MATRIX_TEN)
121			
122	MATRIX.2	=MMULT(MATRIX.1,MATRIX_NINE)	=MMULT(MATRIX.1,MATRIX_NINE)
123		=MMULT(MATRIX.1,MATRIX_NINE)	=MMULT(MATRIX.1,MATRIX_NINE)
124		=MMULT(MATRIX.1,MATRIX_NINE)	=MMULT(MATRIX.1,MATRIX_NINE)
125		=MMULT(MATRIX.1,MATRIX_NINE)	=MMULT(MATRIX.1,MATRIX_NINE)
126			
157	MATRIX.9	=MMULT(MATRIX.8,MATRIX_TWO)	=MMULT(MATRIX.8,MATRIX_TWO)
158		=MMULT(MATRIX.8,MATRIX_TWO)	=MMULT(MATRIX.8,MATRIX_TWO)
159		=MMULT(MATRIX.8,MATRIX_TWO)	=MMULT(MATRIX.8,MATRIX_TWO)
160		=MMULT(MATRIX.8,MATRIX_TWO)	=MMULT(MATRIX.8,MATRIX_TWO)
161			
162	FINAL_MATRIX	=MMULT(MATRIX.9,MATRIX_ONE)	=MMULT(MATRIX.9,MATRIX_ONE)
163		=MMULT(MATRIX.9,MATRIX_ONE)	=MMULT(MATRIX.9,MATRIX_ONE)
164		=MMULT(MATRIX.9,MATRIX_ONE)	=MMULT(MATRIX.9,MATRIX_ONE)
165		=MMULT(MATRIX.9,MATRIX_ONE)	=MMULT(MATRIX.9,MATRIX_ONE)
166			
167	DETERMINANT (D)	=MDETERM(B164:C165)	
168	LOGDETERMINANT	= (LOG10(1+ABS(D))) * IF (D<0,-1,1)	

Fig. B-6a. Matrix Names Plus First Two Columns of Each Matrix

	D	E
62	=L_1^2/(2*E_1*IA_1)	=L_1^3/(6*E_1*IA_1)
63	=L_1/(E_1*IA_1)	=L_1^2/(2*E_1*IA_1)
64	=1+H*IW_1*freq^2*L_1/E_1/IA_1	=L_1+H*IW_1*freq^2*L_1^2/2/E_1/IA_1
65	=M_1*freq^2*L_1^2/(2*E_1*IA_1)	=1+(M_1*(freq^2)*(L_1^3)/(6*E_1*IA_1))
66		
112	=L_11^2/(2*E_11*IA_11)	=L_11^3/(6*E_11*IA_11)
113	=L_11/(E_11*IA_11)	=L_11^2/(2*E_11*IA_11)
114	=1+H*IW_11*freq^2*L_11/E_11/IA_11	=L_11+H*IW_11*freq^2*L_11^2/2/E_11/IA_11
115	=M_11*freq^2*L_11^2/(2*E_11*IA_11)	=1+(M_11*freq^2*L_11^3/(6*E_11*IA_11))
116		
117	=MMULT(MATRIX_ELEVEN,MATRIX_TEN)	=MMULT(MATRIX_ELEVEN,MATRIX_TEN)
118	=MMULT(MATRIX_ELEVEN,MATRIX_TEN)	=MMULT(MATRIX_ELEVEN,MATRIX_TEN)
119	=MMULT(MATRIX_ELEVEN,MATRIX_TEN)	=MMULT(MATRIX_ELEVEN,MATRIX_TEN)
120	=MMULT(MATRIX_ELEVEN,MATRIX_TEN)	=MMULT(MATRIX_ELEVEN,MATRIX_TEN)
121		
122	=MMULT(MATRIX.1,MATRIX_NINE)	=MMULT(MATRIX.1,MATRIX_NINE)
123	=MMULT(MATRIX.1,MATRIX_NINE)	=MMULT(MATRIX.1,MATRIX_NINE)
124	=MMULT(MATRIX.1,MATRIX_NINE)	=MMULT(MATRIX.1,MATRIX_NINE)
125	=MMULT(MATRIX.1,MATRIX_NINE)	=MMULT(MATRIX.1,MATRIX_NINE)
126		
157	=MMULT(MATRIX.8,MATRIX_TWO)	=MMULT(MATRIX.8,MATRIX_TWO)
158	=MMULT(MATRIX.8,MATRIX_TWO)	=MMULT(MATRIX.8,MATRIX_TWO)
159	=MMULT(MATRIX.8,MATRIX_TWO)	=MMULT(MATRIX.8,MATRIX_TWO)
160	=MMULT(MATRIX.8,MATRIX_TWO)	=MMULT(MATRIX.8,MATRIX_TWO)
161		
162	=MMULT(MATRIX.9,MATRIX_ONE)	=MMULT(MATRIX.9,MATRIX_ONE)
163	=MMULT(MATRIX.9,MATRIX_ONE)	=MMULT(MATRIX.9,MATRIX_ONE)
164	=MMULT(MATRIX.9,MATRIX_ONE)	=MMULT(MATRIX.9,MATRIX_ONE)
165	=MMULT(MATRIX.9,MATRIX_ONE)	=MMULT(MATRIX.9,MATRIX_ONE)

Fig. B-6b. Last Two Columns of Each Matrix

Fig. B-6. Matrix Calculation Area of the Beam Worksheet

	P	Q	R
10	Function(Determinant) vs. Frequency		
11	Table of values over the frequency range		
12	Frequency, rad/s	f(Determinant)	Frequency, Hz
13		=LOGDETERMINANT	=freq*SQRT(CF_Gc)/(2*PI())
14	=F_LOW*2*PI()/SQRT(CF_Gc)	=TABLE(.B19)	=TABLE(.B19)
15	=(F_LOW+0.1*(F_HIGH-F_LOW))*2*PI()/SQRT(CF_Gc)	=TABLE(.B19)	=TABLE(.B19)
16	=(F_LOW+0.2*(F_HIGH-F_LOW))*2*PI()/SQRT(CF_Gc)	=TABLE(.B19)	=TABLE(.B19)
17	=(F_LOW+0.3*(F_HIGH-F_LOW))*2*PI()/SQRT(CF_Gc)	=TABLE(.B19)	=TABLE(.B19)
18	=(F_LOW+0.4*(F_HIGH-F_LOW))*2*PI()/SQRT(CF_Gc)	=TABLE(.B19)	=TABLE(.B19)
19	=(F_LOW+0.5*(F_HIGH-F_LOW))*2*PI()/SQRT(CF_Gc)	=TABLE(.B19)	=TABLE(.B19)
20	=(F_LOW+0.6*(F_HIGH-F_LOW))*2*PI()/SQRT(CF_Gc)	=TABLE(.B19)	=TABLE(.B19)
21	=(F_LOW+0.7*(F_HIGH-F_LOW))*2*PI()/SQRT(CF_Gc)	=TABLE(.B19)	=TABLE(.B19)
22	=(F_LOW+0.8*(F_HIGH-F_LOW))*2*PI()/SQRT(CF_Gc)	=TABLE(.B19)	=TABLE(.B19)
23	=(F_LOW+0.9*(F_HIGH-F_LOW))*2*PI()/SQRT(CF_Gc)	=TABLE(.B19)	=TABLE(.B19)
24	=F_HIGH*2*PI()/SQRT(CF_Gc)	=TABLE(.B19)	=TABLE(.B19)
25	Chart values over the frequency range		
26	Frequency, Hz	f(Determinant)	
27	Chart_freq	Chart_fdet	
28	=R14	=Q14	
29	=R15	=Q15	
30	=R16	=Q16	
31	=R17	=Q17	
32	=R18	=Q18	
33	=R19	=Q19	
34	=R20	=Q20	
35	=R21	=Q21	
36	=R22	=Q22	
37	=R23	=Q23	
38	=R24	=Q24	

Fig. B-7. Chart Data Area of the Beam Worksheet

ACCURACY	=\$B\$13	K 3	=\$F\$60
Add IW	=\$D\$34:\$N\$34	K 4	=\$G\$60
Add mass	=\$D\$32:\$N\$32	K 5	=\$H\$60
BS	=\$D\$35:\$N\$35	K 6	=\$I\$60
Cavity width	=\$I\$13	K 7	=\$J\$60
CF Gc	=\$O\$3	K 8	=\$K\$60
CF gpL pct	=\$O\$4	K 9	=\$L\$60
Chart fdet	=\$Q\$28:\$Q\$38	K Final	=\$D\$50:\$N\$50
Chart freq	=\$P\$28:\$P\$38	LEN	=\$D\$31:\$N\$31
Cu Pct	=\$I\$4	LOAM	=\$B\$6
D	=\$B\$167	LOGDETERMINANT	=\$B\$168
Density	=\$D\$28:\$N\$28	L 1	=\$D\$55
DMS	=\$B\$5	L 10	=\$M\$55
D in	=\$I\$10	L 11	=\$N\$55
D low	=\$I\$9	L 2	=\$E\$55
D upper	=\$I\$6	L 3	=\$F\$55
E 1	=\$D\$56	L 4	=\$G\$55
E 10	=\$M\$56	L 5	=\$H\$55
E 11	=\$N\$56	L 6	=\$I\$55
E 2	=\$E\$56	L 7	=\$J\$55
E 3	=\$F\$56	L 8	=\$K\$55
E 4	=\$G\$56	L 9	=\$L\$55
E 5	=\$H\$56	L Final	=\$D\$45:\$N\$45
E 6	=\$I\$56	Mass	=\$D\$40:\$N\$40
E 7	=\$J\$56	MATRIX.1	=\$B\$117:\$E\$120
E 8	=\$K\$56	MATRIX.2	=\$B\$122:\$E\$125
E 9	=\$L\$56	MATRIX.3	=\$B\$127:\$E\$130
E copper	=\$O\$8	MATRIX.4	=\$B\$132:\$E\$135
E Final	=\$D\$46:\$N\$46	MATRIX.5	=\$B\$137:\$E\$140
E steel	=\$O\$7	MATRIX.6	=\$B\$142:\$E\$145
Final input	=\$D\$45:\$N\$50	MATRIX.7	=\$B\$147:\$E\$150
FINAL MATRIX	=\$B\$162:\$E\$165	MATRIX.8	=\$B\$152:\$E\$155
freq	=\$B\$19	MATRIX.9	=\$B\$157:\$E\$160
FREQ Hz	=\$B\$17	MATRIX EIGHT	=\$B\$97:\$E\$100
F HIGH	=\$B\$12	MATRIX ELEVEN	=\$B\$112:\$E\$115
F LOW	=\$B\$11	MATRIX FIVE	=\$B\$82:\$E\$85
GAP	=\$B\$7	MATRIX FOUR	=\$B\$77:\$E\$80
h	=\$B\$9	Matrix input	=\$D\$55
IA	=\$D\$41:\$N\$41	MATRIX NINE	=\$B\$102:\$E\$105
IA 1	=\$D\$58	MATRIX ONE	=\$B\$62:\$E\$65
IA 10	=\$M\$58	MATRIX SEVEN	=\$B\$92:\$E\$95
IA 11	=\$N\$58	MATRIX SIX	=\$B\$87:\$E\$90
IA 2	=\$E\$58	MATRIX TEN	=\$B\$107:\$E\$110
IA 3	=\$F\$58	MATRIX THREE	=\$B\$72:\$E\$75
IA 4	=\$G\$58	MATRIX TWO	=\$B\$67:\$E\$70
IA 5	=\$H\$58	Mod e	=\$D\$27:\$N\$27
IA 6	=\$I\$58	M 1	=\$D\$57
IA 7	=\$J\$58	M 10	=\$M\$57
IA 8	=\$K\$58	M 11	=\$N\$57
IA 9	=\$L\$58	M 2	=\$E\$57
IA Final	=\$D\$48:\$N\$48	M 3	=\$F\$57
ID	=\$D\$30:\$N\$30	M 4	=\$G\$57
ID rotor	=\$I\$12	M 5	=\$H\$57
IW	=\$D\$42:\$N\$42	M 6	=\$I\$57
IW 1	=\$D\$59	M 7	=\$J\$57
IW 10	=\$M\$59	M 8	=\$K\$57
IW 11	=\$N\$59	M 9	=\$L\$57
IW 2	=\$E\$59	M Final	=\$D\$47:\$N\$47
IW 3	=\$F\$59	No Vanes	=\$I\$14
IW 4	=\$G\$59	OD	=\$D\$29:\$N\$29
IW 5	=\$H\$59	OD rotor	=\$I\$11
IW 6	=\$I\$59	Print Area	=\$A\$1:\$G\$21
IW 7	=\$J\$59	rho copper	=\$O\$6
IW 8	=\$K\$59	rho LIQ	=\$B\$8
IW 9	=\$L\$59	rho steel	=\$O\$5
IW Final	=\$D\$49:\$N\$49	Riser width	=\$I\$8
K 1	=\$D\$60	SECTION	=\$D\$26:\$N\$26
K 10	=\$M\$60	T Vanes	=\$I\$15
K 11	=\$N\$60	Up weir width	=\$I\$7
K 2	=\$E\$60		

Fig. B-8. Names Used in the Beam Worksheet. Names are listed in the first and third columns. Cell or cells associated with these names are listed in the second and fourth columns.

	A	B	C
1	Command	Beam	Locates natural frequency within specified range.
2		Option-Command-b	
3			Last revision--10/20/89.
4		=Save_starting_point()	And hide copy and paste.
5		=Find_natl_freq()	Show updated values of f(n).
6		=Return_to_starting_point()	And show current worksheet.
7		=RETURN()	
8			
9	Command	Chart	Updates values for chart.
10		Option-Command-c	
11			Last revision--10/20/89.
12		=Save_starting_point()	And hide copy and paste.
13		=CALCULATE.NOW()	Recalculate table for chart.
14		=Return_to_starting_point()	And show current worksheet.
15		=BEEP()	Indicate calculation is done.
16		=RETURN()	
17			
18	Command	Beam_and_chart	Locates natural frequency and updates chart.
19		Option-Command-B	
20			Last revision--11/29/89.
21		=Save_starting_point()	And hide copy and paste.
22		=CALCULATE.NOW()	Recalculate table for chart.
23		=Find_natl_freq()	Show updated values of f(n).
24		=Return_to_starting_point()	And show current worksheet.
25		=RETURN()	
26			
27	Command	Save_starting_point	Remember worksheet in its original condition.
28			Last revision--12/14/89.
29			If worksheet, type is 1.
30	doc 2	=GET.DOCUMENT(3)	Stop macro if active sheet is a macro sheet or a chart.
31		=IF(doc_2=1,GOTO(doc_1))	
32		=ALERT("Active window must be on a worksheet, not a macro sheet or a chart.",3)	
33		=HALT()	
34	doc 1	=GET.DOCUMENT(14)	Type of calculation currently on active worksheet.
35			Automatic calculation without tables.
36		=CALCULATION(2)	Set marker at starting point.
37			
38		=DEFINE.NAME("junk")	Selects data values and copies.
39		=ECHO(FALSE)	Selects upper left cell and pastes the Final input values and formats w/o their equations. This separates input calculations from the matrix calculations and speeds up the worksheet.
40		=SELECT(IFinal input)	
41		=COPY()	
42		=SELECT(IMatrix input)	
43		=PASTE.SPECIAL(3,1)	
44		=PASTE.SPECIAL(4,1)	
45			
46			
47			
48			
49		=RETURN()	
50			
51	Command	Return_to_starting_point	Return worksheet to its original condition.
52			Last revision--12/14/89.
53			Set worksheet to calculational mode in use before macro was invoked.
54		=CALCULATION(doc_1)	Return to starting point.
55			Delete marker.
56			Show current screen.
57		=FORMULA.GOTO(!junk)	
58		=DELETE.NAME("junk")	
59		=ECHO(TRUE)	
60		=RETURN()	
61			

Fig. B-9. First Part of Beam_driver Macro

	A	B	C
62	Command	Find_natl_freq	Iterate f(n) until reach value that makes det zero.
63			
64			Last revision--10/20/89.
65		=SELECT(!freq_hz)	Selects frequency (Hz) box.
66		=ECHO(TRUE)	
67	convert	=!cf_gc	Enters g(c) conv. factor.
68	acc	=!accuracy	Sets accuracy.
69		=SET.NAME("form","0.")	This loop formats the value of the frequency to show only the desired number of significant digits.
70		=FOR("count",2,acc)	
71		=SET.NAME("form",form&"0")	
72		=NEXT()	
73		=SET.NAME("form",form&"E+00")	
74		=FORMAT.NUMBER(iorm)	
75	freql	=!f_low*2*PI()/SQRT(convert)	Enters lower bound and converts to SI.
76			
77	freqh	=!f_high*2*PI()/SQRT(convert)	Enters upper bound and converts to SI.
78			
79		=FORMULA(freql,freqh)	Puts lower frequency into frequency box and recalcs.
80			Enters determinant value.
81	deth	=!logdeterminant	Puts upper frequency into frequency box and recalcs.
82		=FORMULA(freqh,freqh)	Enters determinant value.
83			Tests for only one root.
84	deth	=!logdeterminant	
85		=IF(SIGN(deth)<>SIGN(deth),GOTO(f_Hz_1))	
86	message_1	=ALERT("No natural frequency or more than one natural frequency between limits.",2)	
87		=GOTO(end)	
88	f_Hz_1	=TEXT(!freq_hz,form)	Enter last frequency in Hz.
89	slope	=(deth-deth)/(!freqh-freql)	These lines calculate the new frequency.
90	freqm	=-deth/slope+freql	Puts new frequency into frequency box and recalcs.
91		=FORMULA(freqm,freqh)	Enter new frequency in Hz.
92			Compares first "acc" digits of current and previous frequencies.
93	f_Hz_2	=TEXT(!freq_hz,form)	Enters determinant value for new frequency.
94		=IF(LEFT(f_Hz_1,acc+1)=LEFT(f_Hz_2,acc+1),GOTO(end1))	Change frequency limits for next iteration.
95			Change determinant limits for next iteration.
96			Makes new frequency the last frequency.
97	detm	=!logdeterminant	
98			
99		=IF(SIGN(deth)=SIGN(detm),SET.VALUE(freql,freqm),SET.VALUE(freqh,freqm))	
100			
101		=IF(SIGN(deth)=SIGN(detm),SET.VALUE(deth,detm),SET.VALUE(deth,detm))	
102			
103		=SET.VALUE(f_Hz_1,f_Hz_2)	
104			
105		=GOTO(slope)	
106	end1	=ALERT("The natural frequency is "&f_Hz_2&" Hz.",2)	Show result.
107	end		
108		=ECHO(FALSE)	Do not update screen.
109		=RETURN()	

Fig. B-10. Rest of Beam_driver Macro

acc	=\$B\$68	0	
Beam	=\$B\$1	2	b
Beam_and_chart	=\$B\$18	2	B
beep_1	=1	0	
Chart	=\$B\$9	2	c
convert	=\$B\$67	0	
count	=5	0	
deth	=\$B\$84	0	
detl	=\$B\$81	0	
detm	=\$B\$97	0	
doc_1	=\$B\$34	0	
doc_2	=\$B\$30	0	
end	=\$B\$107	0	
end1	=\$B\$106	0	
Find_natl_freq	=\$B\$62	2	
form	= "0.000E+00"	0	
freqh	=\$B\$77	0	
freql	=\$B\$75	0	
freqm	=\$B\$90	0	
f_Hz_1	=\$B\$88	0	
f_Hz_2	=\$B\$93	0	
junk	=\$34:\$34	0	
message_1	=\$B\$86	0	
Return_to_starting_point	=\$B\$51	2	
Save_starting_point	=\$B\$27	2	
slope	=\$B\$89	0	

Fig. B-11. Names Used in the Beam_driver Macro. Names are in column 1. Cell locations or definitions are in column 2. Column 3 indicates if the name is a command macro, "2," or not, "0". Column 4 shows any shortcut keys for the command macros.

APPENDIX C

INPUT DATA FOR MODELS USED IN THE BEAM WORKSHEET AND IN BEAM IV

The various models based on the Beam worksheet are identified in Table C-1. In cases where worksheet values given in Appendix A were changed, the changed portion of the worksheet is shown in Figs. C-1 through C-15. The appropriate figures for the various models are identified in Table C-1.

The six BEAM IV models discussed in this report are listed here as Figs. C-16 through 21. The sketch at the top of each model shows the relative lengths and diameters and the bearing locations (k). Beams that are filled with equally-spaced diagonal lines are continuum, rather than massless, beams. Circles at the right end of a section represent a concentrated mass. Note that two continuum models shown in Figs. C-16 and 17, have a concentrated weight near their right end. This concentrated mass is the accelerometer used in making static ($h = -1$) tests. A user who wants to run the models using BEAM IV on a VAX computer can obtain a license for the BEAM IV program [MITCHELL-1984].

Table C-1. Input Values for Beam Models				
Worksheet Name	Values for General Input Area*			Figures Showing Input Values
	DMS	LOAM	GAP	
Beam11/2cm(solid)	0.1875	2.01	0.0625	C-2, C-3
Beam21/1cm(solid)	0.1875	2.01	0.0625	C-2, C-4
Beam11/2cm	0.187	2.01	0.0312	C-5, C-6
Beam11/4cm(clamp nut)	0.312	4.94	0.218	C-7, C-8
Beam11/4cm(coupling)	0.312	4.74	0	C-9, C-10
Beam11/10cm/254	0.375	6.57	0.1875	C-11, C-12
Beam11/10cm/284	0.625	7.78	0.1250	C-11, C-13
Beam11/10cm/3501	0.625	9.31	0.312	C-11, C-14
Beam11/10cm/3510	0.875	11.25	-0.0938	C-11, C-15
*Other parameters are given in Fig. C-1. If the rotor is empty or solid, rho _LIQ is zero, as shown in Fig. C-1. The value rho _LIQ is 1000 g/L if the rotor is filled with water and 1631 g/L if it is filled with TCE. If the rotor is turning forward, h is +1, as shown in Fig. C-1. If it is not turning (static), h is -1.				

	A	B	C	D	E	F	G
1	BEAM VIBRATION ANALYSIS						
2	Date of last revision: April 1, 1991						
3	General Input Area						
4	Name of Value	Value	Units	Description			
5	DMS	(From Table C-1)	in.	Diameter of Output Shaft of Motor			
6	LOAM	(From Table C-1)	in.	Overall Length of Motor (less shaft beyond motor face)			
7	GAP	(From Table C-1)	in.	Gap Between Motor Face and End of Motor/Rotor Coupling			
8	rho_LIQ	0	g/L	Density of Liquid in Rotor			
9	h	+1	-	Whirl Direction (Static = -1, Forward = +1, Backward = -3)			
10	Frequency Range to Search						
11	F_LOW	50	Hz	Lower Frequency Limit			
12	F_HIGH	500	Hz	Upper Frequency Limit			
13	ACCURACY	4	-	Number of Significant Digits Needed			

Fig. C-1. General Input Area of the Beam Worksheet

	H	I	J
1	Motor and Rotor Details		
2	Name of Value	Value	Units
3	Motor details		
4	Cu_Pct	59%	-
5	Rotor details		
6	D_upper	0.000	in.
7	Up_weir_width	0.000	in.
8	Riser_width	0.000	in.
9	D_lower	0.000	in.
10	D_in	0.000	in.
11	OD_rotor	1.000	in.
12	ID_rotor	0.000	in.
13	Cavity_width	0.000	in.
14	No_Vanes	0	-
15	T_Vanes	0.000	in.

Fig. C-2. Motor and Rotor Input Values for
"Beam11/2cm(solid)" and
"Beam21/2cm(solid)" Models

	A	B	C	D	E	F	G
2 5				Motor Data			
2 6	Description	Units	SECTION	1	2	3	4
2 7	Elastic Modulus	lb/in ²	Mod_e	2.9E+07	2.9E+07	2.9E+07	2.1E+07
2 8	Density of Material	lb/in ³	Density	2.900E-01	2.900E-01	2.900E-01	3.089E-01
2 9	Outer Diameter	in.	OD	2.360E-01	2.360E-01	2.593E-01	1.128E+00
3 0	Inner Diameter	in.	ID	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3 1	Length	in.	LEN	1.000E-03	9.250E-02	2.515E-01	1.149E+00
3 2	Additional Mass	lb	Add_mass	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3 3	Additional Weight Moment of Inertia						
3 4		lb•in ²	Add_IW	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3 5	Bearing Stiffness	lbf/in	BS	2.4E+5			

Fig. C-3a. Row Headings and First Four Columns of Input Values

	H	I	J	K	L	M	N
2 5			Coupling Data	Rotor Data			
2 6	5	6	7	8	9	10	11
2 7	2.9E+07	2.9E+07	2.9E+07	2.9E+07	2.9E+07	2.9E+07	2.9E+07
2 8	2.900E-01	2.900E-01	2.900E-01	2.900E-01	2.900E-01	2.900E-01	2.900E-01
2 9	2.611E-01	1.875E-01	4.375E-01	4.375E-01	1.000E+00	1.000E+00	1.000E+00
3 0	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3 1	2.440E-01	3.335E-01	1.563E+00	3.125E-01	1.667E+00	1.667E+00	1.667E+00
3 2	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3 3							
3 4	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3 5	2.4E+5						

Fig. C-3b. Last Seven Columns of Input Values

Fig. C-3. Sectional Input Values for "Beam11/2cm(solid)" Model

	A	B	C	D	E	F	G	H
2 5	Motor Data							
2 6	Description	Units	SECTION	1	2	3	4	5
2 7	Elastic Modulus	lb/in ²	Mod_e	2.9E+07	2.9E+07	2.9E+07	2.1E+07	2.9E+07
2 8	Density of Material	lb/in ³	Density	2.900E-01	2.900E-01	2.900E-01	3.089E-01	2.900E-01
2 9	Outer Diameter	in.	OD	2.360E-01	2.360E-01	2.593E-01	1.128E+00	2.611E-01
3 0	Inner Diameter	in.	ID	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3 1	Length	in.	LEN	1.000E-03	9.250E-02	2.515E-01	1.149E+00	2.440E-01
3 2	Additional Mass	lb	Add_mass	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3 3	Additional Weight Moment of Inertia							
3 4		lb·in ²	Add_IW	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3 5	Bearing Stiffness	lbf/in	BS		2.4E+5			2.4E+5

Fig. C-4a. Row Headings and First Five Columns of Input Values

	I	J	K	L	M	N	O	P
2 5	Motor/Rotor Coupling Data							
2 6	6	7	8	9	10	11	12	13
2 7	2.9E+07	2.9E+07	2.9E+07	2.9E+07	2.9E+07	2.9E+07	2.9E+07	2.9E+07
2 8	2.900E-01	2.900E-01	2.900E-01	2.900E-01	2.900E-01	2.900E-01	2.900E-01	2.900E-01
2 9	1.875E-01	4.375E-01	4.375E-01	4.375E-01	4.375E-01	4.375E-01	4.375E-01	1.000E+00
3 0	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3 1	3.335E-01	5.208E-01	5.208E-01	5.208E-01	1.042E-01	1.042E-01	1.042E-01	5.556E-01
3 2	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3 3								
3 4	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3 5								

Fig. C-4b. Middle Eight Columns of Input Values

	Q	R	S	T	U	V	W	X
2 5	Rotor Data							
2 6	14	15	16	17	18	19	20	21
2 7	2.9E+07	2.9E+07	2.9E+07	2.9E+07	2.9E+07	2.9E+07	2.9E+07	2.9E+07
2 8	2.900E-01	2.900E-01	2.900E-01	2.900E-01	2.900E-01	2.900E-01	2.900E-01	2.900E-01
2 9	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00
3 0	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3 1	5.556E-01	5.556E-01	5.556E-01	5.556E-01	5.556E-01	5.556E-01	5.556E-01	5.556E-01
3 2	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3 3								
3 4	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3 5								

Fig. C-4c. Last Eight Columns of Input Values

Fig. C-4. Sectional Input Values for "Beam21/2cm(solid)" Model

	H	I	J
1	Motor and Rotor Details		
2	Name of Value	Value	Units
3	Motor details		
4	Cu_Pct	59%	-
5	Rotor details		
6	D_upper	0.400	in.
7	Up_weir_width	0.063	in.
8	Riser_width	0.093	in.
9	D_lower	0.400	in.
10	D_in	0.400	in.
11	OD_rotor	0.875	in.
12	ID_rotor	0.639	in.
13	Cavity_width	0.156	in.
14	No_Vanes	4	-
15	T_Vanes	0.060	in.

Fig. C-5. Motor and Rotor Input Values for
"Beam11/2cm" Model

	A	B	C	D	E	F	G
2 2	Parameters for Each Beam Section						
2 3	In the table below, user may put data in all underlined cells below the title line.						
2 4	Note that some input values are already calculated from data entered elsewhere.						
2 5				Motor Data			
2 6	Description	Units	SECTION	1	2	3	4
2 7	Elastic Modulus	lb/in ²	Mod_e	2.9E+07	2.9E+07	2.9E+07	2.1E+07
2 8	Density of Material	lb/in ³	Density	2.900E-01	2.900E-01	2.900E-01	3.089E-01
2 9	Outer Diameter	in.	OD	2.360E-01	2.360E-01	2.593E-01	1.128E+00
3 0	Inner Diameter	in.	ID	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3 1	Length	in.	LEN	1.000E-03	9.250E-02	2.515E-01	1.149E+00
3 2	Additional Mass	lb	Add_mass	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3 3	Additional Weight Moment of Inertia						
3 4		lb·in ²	Add_IW	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3 5	Bearing Stiffness	lbf/in	BS		2.4E+5		

Fig. C-6a. Row Headings and First Four Columns of Input Values

	H	I	J	K	L	M	N
2 5			Coupling Data	Rotor Data			
2 6	5	6	7	8	9	10	11
2 7	2.9E+07	2.9E+07	2.9E+07	2.9E+07	2.9E+07	2.9E+07	2.9E+07
2 8	2.900E-01	2.900E-01	2.900E-01	2.900E-01	2.900E-01	2.900E-01	2.900E-01
2 9	2.611E-01	1.870E-01	4.690E-01	3.125E-01	8.750E-01	8.750E-01	8.750E-01
3 0	0.000E+00	0.000E+00	0.000E+00	0.000E+00	4.000E-01	6.390E-01	4.000E-01
3 1	2.440E-01	3.023E-01	1.219E+00	2.500E-01	3.438E-01	2.089E+00	8.515E-02
3 2	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3 3							
3 4	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3 5	2.4E+5						

Fig. C-6b. Last Seven Columns of Input Values

Fig. C-6. Sectional Input Values for "Beam11/2cm" Model

	H	I	J
1	Motor and Rotor Details		
2	Name of Value	Value	Units
3	Motor details		
4	Cu_Pct	35%	-
5	Rotor details		
6	D_upper	0.550	in.
7	Up_weir_width	0.250	in.
8	Riser_width	0.188	in.
9	D_lower	0.550	in.
10	D_in	0.550	in.
11	OD_rotor	1.625	in.
12	ID_rotor	1.500	in.
13	Cavity_width	0.313	in.
14	No_Vanes	4	-
15	T_Vanes	0.029	in.

Fig. C-7. Motor and Rotor Input Values for
"Beam 1 1/4cm (clamp nut)" Model

	A	B	C	D	E	F	G
25	Motor Data						
26	Description	Units	SECTION	1	2	3	4
27	Elastic Modulus	lb/in ²	Mod_e	2.9E+07	2.9E+07	2.9E+07	2.4E+07
28	Density of Material	lb/in ³	Density	2.800E-01	2.800E-01	2.800E-01	2.947E-01
29	Outer Diameter	in.	OD	4.600E-01	4.600E-01	3.762E-01	1.556E+00
30	Inner Diameter	in.	ID	0.000E+00	0.000E+00	0.000E+00	7.650E-01
31	Length	in.	LEN	1.000E-03	1.405E-01	1.188E+00	2.030E+00
32	Additional Mass	lb	Add_mass	0.000E+00	5.905E-02	0.000E+00	2.370E-01
33	Additional Weight Moment of Inertia						
34		lb·in ²	Add_IW	0.000E+00	0.000E+00	0.000E+00	9.518E-02
35	Bearing Stiffness	lbf/in	BS		2.4E+5		

Fig. C-8a. Row Headings and First Four Columns of Input Values

	H	I	J	K	L	M	N
25	Coupling Data				Rotor Data		
26	5	6	7	8	9	10	11
27	2.9E+07	2.9E+07	2.9E+07	2.9E+07	2.9E+07	2.9E+07	2.9E+07
28	2.800E-01	2.800E-01	2.800E-01	2.800E-01	2.800E-01	2.800E-01	2.800E-01
29	3.854E-01	3.120E-01	4.370E-01	7.500E-01	5.543E-01	1.625E+00	1.625E+00
30	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	1.500E+00	5.500E-01
31	1.123E+00	4.805E-01	1.750E+00	5.000E-01	1.125E+00	2.920E+00	1.614E-01
32	0.000E+00	0.000E+00	0.000E+00	0.000E+00	3.053E-01	0.000E+00	0.000E+00
33							
34	0.000E+00	0.000E+00	0.000E+00	0.000E+00	2.345E-01	0.000E+00	0.000E+00
35	2.4E+5						

Fig. C-8b. Last Seven Columns of Input Values

Fig. C-8. Sectional Input Values for "Beam11/4cm(clamp nut)" Model

	H	I	J
1	Motor and Rotor Details		
2	Name of Value	Value	Units
3	Motor details		
4	Cu_Pct	35%	-
5	Rotor details		
6	D_upper	0.550	in.
7	Up_weir_width	0.250	in.
8	Riser_width	0.188	in.
9	D_lower	0.550	in.
10	D_in	0.550	in.
11	OD_rotor	1.625	in.
12	ID_rotor	1.500	in.
13	Cavity_width	0.313	in.
14	No_Vanes	4	-
15	T_Vanes	0.029	in.

Fig. C-9. Motor and Rotor Input Values for
"Beam11/4cm(coupling)" Model

	A	B	C	D	E	F	G
2 5				Motor Data			
2 6	Description	Units	SECTION	1	2	3	4
2 7	Elastic Modulus	lb/in ²	Mod_e	2.9E+07	2.9E+07	2.9E+07	2.4E+07
2 8	Density of Material	lb/in ³	Density	2.800E-01	2.800E-01	2.800E-01	2.947E-01
2 9	Outer Diameter	in.	OD	4.600E-01	4.600E-01	3.762E-01	1.556E+00
3 0	Inner Diameter	in.	ID	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3 1	Length	in.	LEN	1.000E-03	1.405E-01	1.188E+00	2.030E+00
3 2	Additional Mass	lb	Add_mass	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3 3	Additional Weight Moment of Inertia						
3 4		lb·in ²	Add_IW	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3 5	Bearing Stiffness	lbf/in	BS	2.4E+5			

Fig. C-10a. Row Headings and First Four Columns of Input Values

	H	I	J	K	L	M	N
2 5			Coupling Data	Rotor Data			
2 6	5	6	7	8	9	10	11
2 7	2.9E+07	2.9E+07	2.9E+07	2.9E+07	2.9E+07	2.9E+07	2.9E+07
2 8	2.800E-01	2.800E-01	2.800E-01	2.800E-01	2.800E-01	2.800E-01	2.800E-01
2 9	3.854E-01	3.120E-01	8.326E-01	4.978E-01	1.625E+00	1.625E+00	1.625E+00
3 0	0.000E+00	0.000E+00	0.000E+00	0.000E+00	5.500E-01	1.500E+00	5.500E-01
3 1	1.123E+00	2.620E-01	2.250E+00	6.875E-01	4.375E-01	2.920E+00	1.614E-01
3 2	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3 3							
3 4	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3 5	2.4E+5						

Fig. C-10b. Last Seven Columns of Input Values

Fig. C-10. Sectional Input Values for "Beam11/4cm(coupling)" Model

	H	I	J
1	Motor and Rotor Details		
2	Name of Value	Value	Units
3	Motor details		
4	C _u _Pct	35%	-
5	Rotor details		
6	D_upper	1.600	in.
7	Up_weir_width	0.250	in.
8	Riser_width	0.250	in.
9	D_lower	1.600	in.
10	D_in	0.875	in.
11	OD_rotor	4.000	in.
12	ID_rotor	3.875	in.
13	Cavity_width	0.625	in.
14	No_Vanes	4	-
15	T_Vanes	0.060	in.

Fig. C-11. Motor and Rotor Input Values for All "Beam11/10cm" Models

	A	B	C	D	E	F	G
2 5				Motor Data			
2 6	Description	Units	SECTION	1	2	3	4
2 7	Elastic Modulus	lb/in ²	Mod_e	2.9E+07	2.9E+07	2.9E+07	2.4E+07
2 8	Density of Material	lb/in ³	Density	2.900E-01	2.900E-01	2.900E-01	3.012E-01
2 9	Outer Diameter	in.	OD	3.748E-01	3.748E-01	5.242E-01	2.133E+00
3 0	Inner Diameter	in.	ID	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3 1	Length	in.	LEN	1.000E-03	1.249E+00	1.420E+00	2.375E+00
3 2	Additional Mass	lb	Add_mass	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3 3	Additional Weight Moment of Inertia						
3 4		lb•in ²	Add_IW	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3 5	Bearing Stiffness	lb/in	BS	2.4E+5			

Fig. C-12a. Row Headings and First Four Columns of Input Values

	H	I	J	K	L	M	N
2 5				Coupling Data		Rotor Data	
2 6	5	6	7	8	9	10	11
2 7	2.9E+07	2.9E+07	2.9E+07	2.9E+07	2.9E+07	2.9E+07	2.9E+07
2 8	2.900E-01	2.900E-01	2.900E-01	2.900E-01	2.900E-01	2.900E-01	2.900E-01
2 9	6.250E-01	3.748E-01	8.120E-01	8.120E-01	1.375E+00	4.000E+00	4.000E+00
3 0	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	3.875E+00	8.750E-01
3 1	1.221E+00	3.075E-01	1.156E+00	1.156E+00	2.438E+00	8.205E+00	2.161E-01
3 2	0.000E+00	0.000E+00	0.000E+00	0.000E+00	2.823E+00	2.819E+00	0.000E+00
3 3							
3 4	0.000E+00	0.000E+00	0.000E+00	0.000E+00	1.293E+01	0.000E+00	0.000E+00
3 5	2.4E+5						

Fig. C-12b. Last Seven Columns of Input Values

Fig. C-12. Sectional Input Values for "Beam11/10cm/254" Model

	A	B	C	D	E	F	G
2 5				Motor Data			
2 6	Description	Units	SECTION	1	2	3	4
2 7	Elastic Modulus	lb/in ²	Mod_e	2.9E+07	2.9E+07	2.9E+07	2.4E+07
2 8	Density of Material	lb/in ³	Density	2.900E-01	2.900E-01	2.900E-01	3.012E-01
2 9	Outer Diameter	in.	OD	5.452E-01	5.452E-01	6.063E-01	2.721E+00
3 0	Inner Diameter	in.	ID	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3 1	Length	in.	LEN	1.000E-03	1.535E+00	1.710E+00	2.812E+00
3 2	Additional Mass	lb	Add_mass	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3 3	Additional Weight Moment of Inertia						
3 4		lb•in ²	Add_IW	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3 5	Bearing Stiffness	lb/in	BS	2.4E+5			

Fig. C-13a. Row Headings and First Four Columns of Input Values

	H	I	J	K	L	M	N
2 5				Coupling Data		Rotor Data	
2 6	5	6	7	8	9	10	11
2 7	2.9E+07	2.9E+07	2.9E+07	2.9E+07	2.9E+07	2.9E+07	2.9E+07
2 8	2.900E-01	2.900E-01	2.900E-01	2.900E-01	2.900E-01	2.900E-01	2.900E-01
2 9	6.483E-01	6.248E-01	1.728E+00	8.120E-01	1.375E+00	4.000E+00	4.000E+00
3 0	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	3.875E+00	8.750E-01
3 1	1.265E+00	4.888E-01	1.719E+00	8.438E-01	2.438E+00	8.205E+00	2.161E-01
3 2	0.000E+00	0.000E+00	0.000E+00	0.000E+00	2.823E+00	2.819E+00	0.000E+00
3 3							
3 4	0.000E+00	0.000E+00	0.000E+00	0.000E+00	1.293E+01	0.000E+00	0.000E+00
3 5	2.4E+5						

Fig. C-13b. Last Seven Columns of Input Values

Fig. C-13. Sectional Input Values for "Beam11/10cm/284" Model

	A	B	C	D	E	F	G
2 5				Motor Data			
2 6	Description	Units	SECTION	1	2	3	4
2 7	Elastic Modulus	lb/in ²	Mod_e	2.9E+07	2.9E+07	2.9E+07	2.4E+07
2 8	Density of Material	lb/in ³	Density	2.900E-01	2.900E-01	2.900E-01	2.226E-01
2 9	Outer Diameter	in.	OD	6.858E-01	6.858E-01	8.340E-01	3.050E+00
3 0	Inner Diameter	in.	ID	0.000E+00	0.000E+00	0.000E+00	2.250E+00
3 1	Length	in.	LEN	1.000E-03	1.205E+00	2.213E+00	2.445E+00
3 2	Additional Mass	lb	Add_mass	0.000E+00	0.000E+00	1.958E-01	1.753E+00
3 3	Additional Weight Moment of Inertia						
3 4		lb•in ²	Add_IW	0.000E+00	0.000E+00	0.000E+00	3.092E+00
3 5	Bearing Stiffness	lb/in	BS	2.4E+5			

Fig. C-14a. Row Headings and First Four Columns of Input Values

	H	I	J	K	L	M	N
2 5			Coupling Data		Rotor Data		
2 6	5	6	7	8	9	10	11
2 7	2.9E+07	2.9E+07	2.9E+07	2.9E+07	2.9E+07	2.9E+07	2.9E+07
2 8	2.900E-01	2.900E-01	2.900E-01	2.900E-01	2.900E-01	2.900E-01	2.900E-01
2 9	9.391E-01	6.250E-01	1.728E+00	8.120E-01	1.375E+00	4.000E+00	4.000E+00
3 0	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	3.875E+00	8.750E-01
3 1	1.863E+00	1.087E+00	1.719E+00	8.438E-01	2.438E+00	8.205E+00	2.161E-01
3 2	0.000E+00	0.000E+00	0.000E+00	0.000E+00	2.735E+00	0.000E+00	0.000E+00
3 3							
3 4	0.000E+00	0.000E+00	0.000E+00	0.000E+00	1.293E+01	0.000E+00	0.000E+00
3 5	2.4E+5						

Fig. C-14b. Last Seven Columns of Input Values

Fig. C-14. Sectional Input Values for "Beam11/10cm/3501" Model

	A	B	C	D	E	F	G
2 5				Motor Data			
2 6	Description	Units	SECTION	1	2	3	4
2 7	Elastic Modulus	lb/in ²	Mod_e	2.9E+07	2.9E+07	2.9E+07	2.4E+07
2 8	Density of Material	lb/in ³	Density	2.900E-01	2.900E-01	2.900E-01	2.226E-01
2 9	Outer Diameter	in.	OD	7.095E-01	7.095E-01	8.587E-01	3.800E+00
3 0	Inner Diameter	in.	ID	0.000E+00	0.000E+00	0.000E+00	2.750E+00
3 1	Length	in.	LEN	1.000E-03	1.549E+00	2.050E+00	3.500E+00
3 2	Additional Mass	lb	Add_mass	0.000E+00	0.000E+00	2.006E-01	3.897E+00
3 3	Additional Weight Moment of Inertia						
3 4		lb•in ²	Add_IW	0.000E+00	0.000E+00	0.000E+00	1.135E+01
3 5	Bearing Stiffness	lbf/in	BS	2.4E+5			

Fig. C-15a. Row Headings and First Four Columns of Input Values

	H	I	J	K	L	M	N
2 5			Coupling Data		Rotor Data		
2 6	5	6	7	8	9	10	11
2 7	2.9E+07	2.9E+07	2.9E+07	2.9E+07	2.9E+07	2.9E+07	2.9E+07
2 8	2.900E-01	2.900E-01	2.900E-01	2.900E-01	2.900E-01	2.900E-01	2.900E-01
2 9	1.188E+00	8.750E-01	1.625E+00	8.120E-01	1.375E+00	4.000E+00	4.000E+00
3 0	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	3.875E+00	8.750E-01
3 1	2.938E+00	7.188E-01	2.500E+00	1.500E+00	2.438E+00	8.205E+00	2.161E-01
3 2	4.008E-01	0.000E+00	0.000E+00	0.000E+00	2.876E+00	4.510E+00	0.000E+00
3 3							
3 4	0.000E+00	0.000E+00	0.000E+00	0.000E+00	1.293E+01	0.000E+00	0.000E+00
3 5	2.4E+5						

Fig. C-15b. Last Seven Columns of Input Values

Fig. C-15. Sectional Input Values of "Beam11/10cm/3510" Model

PROBLEM DATA:

BODINE_710.001

For section # 1 :

Length of this section.....=9.3500E-02 in
 Modulus of Elasticity.....=2.9000E+07 psi
 Area Moment of Inertia.....=1.5200E-04 in⁴
 Linear Density.....=1.2200E-02 lb/in
 Stiffness of Support.....=2.4000E+05 lb/in
 Magnitude of Concentrated Weight.....=7.0000E-03 lb
 Weight Moment of Inertia about a diameter
 of disc or element (W*Rad.Gyr.^2).....=1.0000E-04 lb-in²

For section # 2 :

Length of this section.....=9.3500E-02 in
 Modulus of Elasticity.....=2.9000E+07 psi
 Area Moment of Inertia.....=1.5200E-04 in⁴
 Linear Density.....=1.2200E-02 lb/in

For section # 3 :

Length of this section.....=6.0000E-02 in
 Modulus of Elasticity.....=2.9000E+07 psi
 Area Moment of Inertia.....=1.2200E-04 in⁴
 Linear Density.....=1.1000E-02 lb/in

For section # 4 :

Length of this section.....=1.6600E-01 in
 Modulus of Elasticity.....=2.9000E+07 psi
 Area Moment of Inertia.....=4.6500E-04 in⁴
 Linear Density.....=2.1400E-02 lb/in
 Magnitude of Concentrated Weight.....=2.1800E-02 lb
 Weight Moment of Inertia about a diameter
 of disc or element (W*Rad.Gyr.^2).....=2.2000E-03 lb-in²

For section # 5 :

Length of this section.....=5.1150E-01 in
 Modulus of Elasticity.....=2.9000E+07 psi
 Area Moment of Inertia.....=9.4500E-02 in⁴
 Linear Density.....=3.2320E-01 lb/in
 Weight Moment of Inertia about a diameter
 of disc or element (W*Rad.Gyr.^2).....=5.5900E-02 lb-in²

Fig. C-16. Input Data for BEAM IV Model "BODINE_710.001"

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For section # 6 :
Length of this section.....=5.1150E-01 in
Modulus of Elasticity.....=2.9000E+07 psi
Area Moment of Inertia.....=9.4500E-02 in^4
Linear Density.....=3.2320E-01 lb/in
Magnitude of Concentrated Weight.....=2.1800E-02 lb
Weight Moment of Inertia about a diameter
of disc or element (W*Rad.Gyr.^2).....=2.4000E-03 lb-in^2

For section # 7 :
Length of this section.....=1.6600E-01 in
Modulus of Elasticity.....=2.9000E+07 psi
Area Moment of Inertia.....=4.6500E-04 in^4
Linear Density.....=2.1400E-02 lb/in

For section # 8 :
Length of this section.....=6.0000E-02 in
Modulus of Elasticity.....=2.9000E+07 psi
Area Moment of Inertia.....=1.2200E-04 in^4
Linear Density.....=1.1000E-02 lb/in

For section # 9 :
Length of this section.....=7.6000E-02 in
Modulus of Elasticity.....=2.9000E+07 psi
Area Moment of Inertia.....=1.5200E-04 in^4
Linear Density.....=1.2200E-02 lb/in
Stiffness of Support.....=2.4000E+05 lb/in
Magnitude of Concentrated Weight.....=7.0000E-03 lb
Weight Moment of Inertia about a diameter
of disc or element (W*Rad.Gyr.^2).....=1.0000E-04 lb-in^2

For section # 10 :
Length of this section.....=7.6000E-02 in
Modulus of Elasticity.....=2.9000E+07 psi
Area Moment of Inertia.....=1.5200E-04 in^4
Linear Density.....=1.2200E-02 lb/in

For section # 11 :
Length of this section.....=2.9400E-01 in
Modulus of Elasticity.....=2.9000E+07 psi
Area Moment of Inertia.....=6.0000E-05 in^4
Linear Density.....=7.7000E-03 lb/in

For section # 12 :
Length of this section.....=2.5000E-01 in
Modulus of Elasticity.....=2.9000E+07 psi
Area Moment of Inertia.....=5.2000E-05 in^4
Linear Density.....=7.4000E-03 lb/in
Magnitude of Concentrated Weight.....=7.9000E-03 lb

For section # 13 :
Length of this section.....=2.5000E-01 in
Modulus of Elasticity.....=2.9000E+07 psi
Area Moment of Inertia.....=5.2000E-05 in^4
Linear Density.....=7.4000E-03 lb/in

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Fig. C-16. Input Data for BEAM IV Model "BODINE_710.001" (contd)

For section # 14 :
Length of this section.....=1.0000E-03 in
Modulus of Elasticity.....=2.9000E+07 psi
Area Moment of Inertia.....=6.0000E-05 in⁴
Linear Density.....=7.7000E-03 lb/in

For section # 15 :
Length of this section.....=1.0000E-03 in
Modulus of Elasticity.....=2.9000E+07 psi
Area Moment of Inertia.....=6.0000E-05 in⁴
Linear Density.....=7.7000E-03 lb/in

Fig. C-16. Input Data for BEAM IV Model "BODINE_710.001" (contd)

PROBLEM DATA:

BODINE_SPC.001

For section # 1 :

Length of this section.....=1.4100E-01 in
 Modulus of Elasticity.....=2.9000E+07 psi
 Area Moment of Inertia.....=4.7300E-04 in⁴
 Linear Density.....=1.9700E-02 lb/in
 Stiffness of Support.....=9.8000E+04 lb/in
 Magnitude of Concentrated Weight.....=6.8000E-03 lb
 Weight Moment of Inertia about a diameter
 of disc or element (W*Rad.Gyr.^2).....=2.0000E-04 lb-in²

For section # 2 :

Length of this section.....=3.1000E-02 in
 Modulus of Elasticity.....=2.9000E+07 psi
 Area Moment of Inertia.....=4.7300E-04 in⁴
 Linear Density.....=1.9700E-02 lb/in

For section # 3 :

Length of this section.....=1.0900E-01 in
 Modulus of Elasticity.....=2.9000E+07 psi
 Area Moment of Inertia.....=4.7700E-04 in⁴
 Linear Density.....=2.1700E-02 lb/in

For section # 4 :

Length of this section.....=3.7500E-01 in
 Modulus of Elasticity.....=2.9000E+07 psi
 Area Moment of Inertia.....=8.7100E-04 in⁴
 Linear Density.....=2.9300E-02 lb/in

For section # 5 :

Length of this section.....=2.1200E-01 in
 Modulus of Elasticity.....=2.2000E+07 psi
 Area Moment of Inertia.....=9.6570E-03 in⁴
 Linear Density.....=1.0410E-01 lb/in

For section # 6 :

Length of this section.....=3.2000E-02 in
 Modulus of Elasticity.....=2.9000E+07 psi
 Area Moment of Inertia.....=7.4900E-03 in⁴
 Linear Density.....=8.5900E-02 lb/in

For section # 7 :

Length of this section.....=1.5600E-01 in
 Modulus of Elasticity.....=2.9000E+07 psi
 Area Moment of Inertia.....=1.2680E-03 in⁴
 Linear Density.....=3.4700E-02 lb/in

Fig. C-17. Input Data for BEAM IV Model "BODINE_SPC.001"

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For section # 8 :
Length of this section.....=3.2000E-02 in
Modulus of Elasticity.....=2.9000E+07 psi
Area Moment of Inertia.....=3.0014E-02 in^4
Linear Density.....=1.4510E-01 lb/in

For section # 9 :
Length of this section.....=1.5600E-01 in
Modulus of Elasticity.....=2.9000E+07 psi
Area Moment of Inertia.....=1.3340E-03 in^4
Linear Density.....=3.6200E-02 lb/in

For section # 10 :
Length of this section.....=9.4000E-02 in
Modulus of Elasticity.....=2.9000E+07 psi
Area Moment of Inertia.....=8.7100E-04 in^4
Linear Density.....=2.9300E-02 lb/in

For section # 11 :
Length of this section.....=1.4100E-01 in
Modulus of Elasticity.....=2.9000E+07 psi
Area Moment of Inertia.....=2.7093E-01 in^4
Linear Density.....=1.4060E-01 lb/in

For section # 12 :
Length of this section.....=1.7500E+00 in
Modulus of Elasticity.....=2.9000E+07 psi
Area Moment of Inertia.....=3.3809E-01 in^4
Linear Density.....=5.3890E-01 lb/in

For section # 13 :
Length of this section.....=1.4100E-01 in
Modulus of Elasticity.....=2.9000E+07 psi
Area Moment of Inertia.....=2.7093E-01 in^4
Linear Density.....=1.4060E-01 lb/in

For section # 14 :
Length of this section.....=9.2200E-01 in
Modulus of Elasticity.....=2.9000E+07 psi
Area Moment of Inertia.....=9.7100E-04 in^4
Linear Density.....=3.0900E-02 lb/in

For section # 15 :
Length of this section.....=1.3800E-01 in
Modulus of Elasticity.....=2.9000E+07 psi
Area Moment of Inertia.....=4.7700E-04 in^4
Linear Density.....=2.1700E-02 lb/in
Stiffness of Support.....=9.8000E+04 lb/in
Magnitude of Concentrated Weight.....=6.8000E-03 lb
Weight Moment of Inertia about a diameter
of disc or element (W*Rad.Gyr.^2).....=2.0000E-04 lb-in^2

For section # 16 :
Length of this section.....=1.3800E-01 in
Modulus of Elasticity.....=2.9000E+07 psi
Area Moment of Inertia.....=4.7700E-04 in^4
Linear Density.....=2.1700E-02 lb/in

```

Fig. C-17. Input Data for BEAM IV Model "BODINE_SPC.001" (contd)

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For section # 17 :
Length of this section.....=3.1200E-01 in
Modulus of Elasticity.....=2.9000E+07 psi
Area Moment of Inertia.....=4.6500E-04 in^4
Linear Density.....=2.1400E-02 lb/in

For section # 18 :
Length of this section.....=3.7500E-01 in
Modulus of Elasticity.....=2.9000E+07 psi
Area Moment of Inertia.....=4.1700E-04 in^4
Linear Density.....=2.1400E-02 lb/in
Magnitude of Concentrated Weight.....=7.9000E-03 lb

For section # 19 :
Length of this section.....=2.0310E-01 in
Modulus of Elasticity.....=2.9000E+07 psi
Area Moment of Inertia.....=4.1700E-04 in^4
Linear Density.....=2.1400E-02 lb/in

For section # 20 :
Length of this section.....=1.7190E-01 in
Modulus of Elasticity.....=2.9000E+07 psi
Area Moment of Inertia.....=4.1300E-04 in^4
Linear Density.....=1.9500E-02 lb/in

For section # 21 :
Length of this section.....=1.0000E-06 in
Modulus of Elasticity.....=2.9000E+07 psi
Area Moment of Inertia.....=4.1300E-04 in^4
Linear Density.....=1.9500E-02 lb/in

For section # 22 :
Length of this section.....=1.0000E-06 in
Modulus of Elasticity.....=2.9000E+07 psi
Area Moment of Inertia.....=4.1300E-04 in^4
Linear Density.....=1.9500E-02 lb/in

For section # 23 :
Length of this section.....=1.0000E-06 in
Modulus of Elasticity.....=2.9000E+07 psi
Area Moment of Inertia.....=4.1300E-04 in^4
Linear Density.....=1.9500E-02 lb/in

For section # 24 :
Length of this section.....=1.0000E-06 in
Modulus of Elasticity.....=2.9000E+07 psi
Area Moment of Inertia.....=4.1300E-04 in^4
Linear Density.....=1.9500E-02 lb/in

For section # 25 :
Length of this section.....=1.0000E-06 in
Modulus of Elasticity.....=2.9000E+07 psi
Area Moment of Inertia.....=4.1300E-04 in^4
Linear Density.....=1.9500E-02 lb/in

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Fig. C-17. Input Data for BEAM IV Model "BODINE_SPC.001" (contd)

PROBLEM DATA:

BODINE_254.001

For section # 1 :

Length of this section.....=6.9000E-01 in
 Modulus of Elasticity.....=3.0000E+07 psi
 Area Moment of Inertia.....=9.6556E-04 in⁴
 Linear Density.....=3.1063E-02 lb/in
 Magnitude of Concentrated Weight.....=5.6100E-02 lb
 Weight Moment of Inertia about a diameter
 of disc or element (W*Rad.Gyr.^2).....=6.2210E-02 lb-in²

For section # 2 :

Length of this section.....=5.6000E-01 in
 Modulus of Elasticity.....=3.0000E+07 psi
 Area Moment of Inertia.....=9.7072E-04 in⁴
 Linear Density.....=3.1146E-02 lb/in
 Stiffness of Support.....=4.4000E+05 lb/in

For section # 3 :

Length of this section.....=1.8000E-01 in
 Modulus of Elasticity.....=3.0000E+07 psi
 Area Moment of Inertia.....=1.1781E-03 in⁴
 Linear Density.....=3.4312E-02 lb/in

For section # 4 :

Length of this section.....=3.8000E-01 in
 Modulus of Elasticity.....=3.0000E+07 psi
 Area Moment of Inertia.....=6.6200E-03 in⁴
 Linear Density.....=8.1336E-02 lb/in
 Magnitude of Concentrated Weight.....=6.9533E-02 lb
 Weight Moment of Inertia about a diameter
 of disc or element (W*Rad.Gyr.^2).....=8.8260E-02 lb-in²

For section # 5 :

Length of this section.....=8.6000E-01 in
 Modulus of Elasticity.....=3.0000E+07 psi
 Area Moment of Inertia.....=7.4901E-03 in⁴
 Linear Density.....=8.3498E-02 lb/in
 Magnitude of Concentrated Weight.....=1.3020E-01 lb
 Weight Moment of Inertia about a diameter
 of disc or element (W*Rad.Gyr.^2).....=2.1383E-01 lb-in²

For section # 6 :

Length of this section.....=1.1875E+00 in
 Modulus of Elasticity.....=3.0000E+07 psi
 Area Moment of Inertia.....=1.0142E+00 in⁴
 Linear Density.....=8.1163E-01 lb/in
 Weight Moment of Inertia about a diameter
 of disc or element (W*Rad.Gyr.^2).....=3.4424E-01 lb-in²

Fig. C-18. Input Data for BEAM IV Model "BODINE_254.001"

For section # 7 :

Length of this section.....	=1.1875E+00	in
Modulus of Elasticity.....	=3.0000E+07	psi
Area Moment of Inertia.....	=1.0142E+00	in ⁴
Linear Density.....	=8.1163E-01	lb/in
Magnitude of Concentrated Weight.....	=1.3020E-01	lb
Weight Moment of Inertia about a diameter of disc or element (W*Rad.Gyr.^2).....	=2.1383E-01	lb-in ²

For section # 8 :

Length of this section.....	=1.0410E+00	in
Modulus of Elasticity.....	=3.0000E+07	psi
Area Moment of Inertia.....	=7.4901E-03	in ⁴
Linear Density.....	=8.6517E-02	lb/in

For section # 9 :

Length of this section.....	=1.8000E-01	in
Modulus of Elasticity.....	=3.0000E+07	psi
Area Moment of Inertia.....	=1.1781E-03	in ⁴
Linear Density.....	=3.4312E-02	lb/in
Stiffness of Support.....	=4.4000E+05	lb/in

For section # 10 :

Length of this section.....	=1.6700E+00	in
Modulus of Elasticity.....	=3.0000E+07	psi
Area Moment of Inertia.....	=9.6041E-04	in ⁴
Linear Density.....	=3.0980E-02	lb/in

Fig. C-18. Input Data for BEAM IV Model "BODINE_254.001" (contd)

PROBLEM DATA:

2CM_SOL.001

For section # 1 :

Length of this section.....=1.0000E-03 in
 Modulus of Elasticity.....=2.9000E+07 psi
 Area Moment of Inertia.....=1.5227E-04 in⁴
 Magnitude of Concentrated Weight.....=5.9305E-04 lb

For section # 2 :

Length of this section.....=9.2500E-02 in
 Modulus of Elasticity.....=2.9000E+07 psi
 Area Moment of Inertia.....=1.5227E-04 in⁴
 Stiffness of Support.....=2.4000E+05 lb/in
 Magnitude of Concentrated Weight.....=2.5190E-03 lb
 Weight Moment of Inertia about a diameter
 of disc or element ($W \cdot \text{Rad.Gyr.}^2$).....=5.1208E-05 lb-in²

For section # 3 :

Length of this section.....=2.5150E-01 in
 Modulus of Elasticity.....=2.9000E+07 psi
 Area Moment of Inertia.....=2.2195E-04 in⁴
 Magnitude of Concentrated Weight.....=1.7725E-01 lb
 Weight Moment of Inertia about a diameter
 of disc or element ($W \cdot \text{Rad.Gyr.}^2$).....=7.4668E-02 lb-in²

For section # 4 :

Length of this section.....=1.1490E+00 in
 Modulus of Elasticity.....=2.1300E+07 psi
 Area Moment of Inertia.....=7.9471E-02 in⁴
 Magnitude of Concentrated Weight.....=1.7722E-01 lb
 Weight Moment of Inertia about a diameter
 of disc or element ($W \cdot \text{Rad.Gyr.}^2$).....=7.4667E-02 lb-in²

For section # 5 :

Length of this section.....=2.4400E-01 in
 Modulus of Elasticity.....=2.9000E+07 psi
 Area Moment of Inertia.....=2.2802E-04 in⁴
 Stiffness of Support.....=2.4000E+05 lb/in
 Magnitude of Concentrated Weight.....=3.2291E-03 lb
 Weight Moment of Inertia about a diameter
 of disc or element ($W \cdot \text{Rad.Gyr.}^2$).....=6.5777E-05 lb-in²

For section # 6 :

Length of this section.....=3.3350E-01 in
 Modulus of Elasticity.....=2.9000E+07 psi
 Area Moment of Inertia.....=6.0670E-05 in⁴
 Magnitude of Concentrated Weight.....=3.5394E-02 lb
 Weight Moment of Inertia about a diameter
 of disc or element ($W \cdot \text{Rad.Gyr.}^2$).....=8.5833E-03 lb-in²

Fig. C-19. Input Data for BEAM IV Model "2CM_SOL.001"

```

For section # 7 :
Length of this section.....=1.5625E+00 in
Modulus of Elasticity.....=2.9000E+07 psi
Area Moment of Inertia.....=1.7984E-03 in^4
Magnitude of Concentrated Weight.....=4.0871E-02 lb
Weight Moment of Inertia about a diameter
of disc or element (W*Rad.Gyr.^2).....=8.9405E-03 lb-in^2

For section # 8 :
Length of this section.....=3.1250E-01 in
Modulus of Elasticity.....=2.9000E+07 psi
Area Moment of Inertia.....=1.7984E-03 in^4
Magnitude of Concentrated Weight.....=1.9700E-01 lb
Weight Moment of Inertia about a diameter
of disc or element (W*Rad.Gyr.^2).....=9.1800E-02 lb-in^2

For section # 9 :
Length of this section.....=1.6667E+00 in
Modulus of Elasticity.....=2.9000E+07 psi
Area Moment of Inertia.....=4.9087E-02 in^4
Magnitude of Concentrated Weight.....=3.7960E-01 lb
Weight Moment of Inertia about a diameter
of disc or element (W*Rad.Gyr.^2).....=1.8277E-01 lb-in^2

For section # 10 :
Length of this section.....=1.6667E+00 in
Modulus of Elasticity.....=2.9000E+07 psi
Area Moment of Inertia.....=4.9087E-02 in^4
Magnitude of Concentrated Weight.....=3.7960E-01 lb
Weight Moment of Inertia about a diameter
of disc or element (W*Rad.Gyr.^2).....=1.8277E-01 lb-in^2

For section # 11 :
Length of this section.....=1.6667E+00 in
Modulus of Elasticity.....=2.9000E+07 psi
Area Moment of Inertia.....=4.9087E-02 in^4
Magnitude of Concentrated Weight.....=1.8980E-01 lb
Weight Moment of Inertia about a diameter
of disc or element (W*Rad.Gyr.^2).....=9.1387E-02 lb-in^2

```

Fig. C-19. Input Data for BEAM IV Model "2CM_SOL.001" (contd)

PROBLEM DATA:

2CM_CON.001

For section # 1 :

Length of this section.....=1.0000E-03 in
 Modulus of Elasticity.....=2.9000E+07 psi
 Area Moment of Inertia.....=1.5227E-04 in⁴
 Linear Density.....=1.2690E-02 lb/in

For section # 2 :

Length of this section.....=9.2500E-02 in
 Modulus of Elasticity.....=2.9000E+07 psi
 Area Moment of Inertia.....=1.5227E-04 in⁴
 Linear Density.....=1.2690E-02 lb/in
 Stiffness of Support.....=2.4000E+05 lb/in
 Weight Moment of Inertia about a diameter
 of disc or element ($W \cdot \text{Rad.Gyr.}^2$).....=1.7360E-05 lb-in²

For section # 3 :

Length of this section.....=2.5150E-01 in
 Modulus of Elasticity.....=2.9000E+07 psi
 Area Moment of Inertia.....=2.2195E-04 in⁴
 Linear Density.....=1.5320E-02 lb/in
 Weight Moment of Inertia about a diameter
 of disc or element ($W \cdot \text{Rad.Gyr.}^2$).....=8.5060E-05 lb-in²

For section # 4 :

Length of this section.....=1.1490E+00 in
 Modulus of Elasticity.....=2.1300E+07 psi
 Area Moment of Inertia.....=7.9471E-02 in⁴
 Linear Density.....=3.0870E-01 lb/in
 Weight Moment of Inertia about a diameter
 of disc or element ($W \cdot \text{Rad.Gyr.}^2$).....=1.4930E-01 lb-in²

For section # 5 :

Length of this section.....=2.4400E-01 in
 Modulus of Elasticity.....=2.9000E+07 psi
 Area Moment of Inertia.....=2.2802E-04 in⁴
 Linear Density.....=1.5520E-02 lb/in
 Stiffness of Support.....=2.4000E+05 lb/in
 Weight Moment of Inertia about a diameter
 of disc or element ($W \cdot \text{Rad.Gyr.}^2$).....=8.3330E-05 lb-in²

For section # 6 :

Length of this section.....=3.3350E-01 in
 Modulus of Elasticity.....=2.9000E+07 psi
 Area Moment of Inertia.....=6.0670E-05 in⁴
 Linear Density.....=8.0070E-03 lb/in
 Weight Moment of Inertia about a diameter
 of disc or element ($W \cdot \text{Rad.Gyr.}^2$).....=4.8220E-05 lb-in²

Fig. C-20. Input Data for BEAM IV Model "2CM_CON.001"

```

For section # 7 :
Length of this section.....=1.5625E+00 in
Modulus of Elasticity.....=2.9000E+07 psi
Area Moment of Inertia.....=1.7984E-03 in^4
Linear Density.....=4.3600E-02 lb/in
Weight Moment of Inertia about a diameter
of disc or element (W*Rad.Gyr.^2).....=1.7120E-02 lb-in^2

For section # 8 :
Length of this section.....=3.1250E-01 in
Modulus of Elasticity.....=2.9000E+07 psi
Area Moment of Inertia.....=1.7984E-03 in^4
Linear Density.....=4.3600E-02 lb/in
Weight Moment of Inertia about a diameter
of disc or element (W*Rad.Gyr.^2).....=7.6280E-04 lb-in^2

For section # 9 :
Length of this section.....=2.5000E-01 in
Modulus of Elasticity.....=2.9000E+07 psi
Area Moment of Inertia.....=4.9087E-02 in^4
Linear Density.....=2.2780E-01 lb/in
Weight Moment of Inertia about a diameter
of disc or element (W*Rad.Gyr.^2).....=1.4530E-02 lb-in^2

For section # 10 :
Length of this section.....=4.5000E+00 in
Modulus of Elasticity.....=2.9000E+07 psi
Area Moment of Inertia.....=4.9087E-02 in^4
Linear Density.....=2.2780E-01 lb/in
Weight Moment of Inertia about a diameter
of disc or element (W*Rad.Gyr.^2).....=2.6570E-01 lb-in^2

For section # 11 :
Length of this section.....=2.5000E-01 in
Modulus of Elasticity.....=2.9000E+07 psi
Area Moment of Inertia.....=4.9087E-02 in^4
Linear Density.....=2.2780E-01 lb/in
Weight Moment of Inertia about a diameter
of disc or element (W*Rad.Gyr.^2).....=1.4530E-02 lb-in^2

```

Fig. C-20. Input Data for BEAM IV Model "2CM_CON.001" (contd)

PROBLEM DATA: MINI_710.001

For section # 1 :

Length of this section.....=9.3500E-02 in
 Modulus of Elasticity.....=2.9000E+07 psi
 Area Moment of Inertia.....=1.5200E-04 in⁴
 Linear Density.....=1.2200E-02 lb/in
 Stiffness of Support.....=2.4000E+05 lb/in
 Magnitude of Concentrated Weight.....=7.0000E-03 lb
 Weight Moment of Inertia about a diameter
 of disc or element ($W \cdot \text{Rad.Gyr.}^2$).....=1.0000E-04 lb-in²

For section # 2 :

Length of this section.....=9.3500E-02 in
 Modulus of Elasticity.....=2.9000E+07 psi
 Area Moment of Inertia.....=1.5200E-04 in⁴
 Linear Density.....=1.2200E-02 lb/in

For section # 3 :

Length of this section.....=6.0000E-02 in
 Modulus of Elasticity.....=2.9000E+07 psi
 Area Moment of Inertia.....=1.2200E-04 in⁴
 Linear Density.....=1.1000E-02 lb/in

For section # 4 :

Length of this section.....=1.6600E-01 in
 Modulus of Elasticity.....=2.9000E+07 psi
 Area Moment of Inertia.....=4.6500E-04 in⁴
 Linear Density.....=2.1400E-02 lb/in
 Magnitude of Concentrated Weight.....=2.1800E-02 lb
 Weight Moment of Inertia about a diameter
 of disc or element ($W \cdot \text{Rad.Gyr.}^2$).....=2.2000E-03 lb-in²

For section # 5 :

Length of this section.....=5.1150E-01 in
 Modulus of Elasticity.....=2.9000E+07 psi
 Area Moment of Inertia.....=9.4500E-02 in⁴
 Linear Density.....=3.2320E-01 lb/in
 Weight Moment of Inertia about a diameter
 of disc or element ($W \cdot \text{Rad.Gyr.}^2$).....=5.5900E-02 lb-in²

For section # 6 :

Length of this section.....=5.1150E-01 in
 Modulus of Elasticity.....=2.9000E+07 psi
 Area Moment of Inertia.....=9.4500E-02 in⁴
 Linear Density.....=3.2320E-01 lb/in
 Magnitude of Concentrated Weight.....=2.1800E-02 lb
 Weight Moment of Inertia about a diameter
 of disc or element ($W \cdot \text{Rad.Gyr.}^2$).....=2.4000E-03 lb-in²

Fig. C-21. Input Data for BEAM IV Model "MINI_710.001"

```

For section # 7 :
Length of this section.....=1.6600E-01 in
Modulus of Elasticity.....=2.9000E+07 psi
Area Moment of Inertia.....=4.6500E-04 in^4
Linear Density.....=2.1400E-02 lb/in

For section # 8 :
Length of this section.....=6.0000E-02 in
Modulus of Elasticity.....=2.9000E+07 psi
Area Moment of Inertia.....=1.2200E-04 in^4
Linear Density.....=1.1000E-02 lb/in

For section # 9 :
Length of this section.....=7.6000E-02 in
Modulus of Elasticity.....=2.9000E+07 psi
Area Moment of Inertia.....=1.5200E-04 in^4
Linear Density.....=1.2200E-02 lb/in
Stiffness of Support.....=2.4000E+05 lb/in
Magnitude of Concentrated Weight.....=7.0000E-03 lb
Weight Moment of Inertia about a diameter
of disc or element (W*Rad.Gyr.^2).....=1.0000E-04 lb-in^2

For section # 10 :
Length of this section.....=7.6000E-02 in
Modulus of Elasticity.....=2.9000E+07 psi
Area Moment of Inertia.....=1.5200E-04 in^4
Linear Density.....=1.2200E-02 lb/in

For section # 11 :
Length of this section.....=2.9400E-01 in
Modulus of Elasticity.....=2.9000E+07 psi
Area Moment of Inertia.....=6.0000E-05 in^4
Linear Density.....=7.7000E-03 lb/in

For section # 12 :
Length of this section.....=3.1250E-02 in
Modulus of Elasticity.....=2.9000E+07 psi
Area Moment of Inertia.....=6.0000E-05 in^4
Linear Density.....=7.4000E-03 lb/in

For section # 13 :
Length of this section.....=1.5690E+00 in
Modulus of Elasticity.....=2.9000E+07 psi
Area Moment of Inertia.....=1.8070E-03 in^4
Linear Density.....=4.2200E-02 lb/in
Weight Moment of Inertia about a diameter
of disc or element (W*Rad.Gyr.^2).....=1.4373E-02 lb-in^2

For section # 14 :
Length of this section.....=3.1250E-01 in
Modulus of Elasticity.....=2.9000E+07 psi
Area Moment of Inertia.....=9.3500E-04 in^4
Linear Density.....=3.4900E-02 lb/in

```

Fig. C-21. Input Data for BEAM IV Model "MINI_710.001" (contd)

For section # 15 :
 Length of this section.....=2.5000E+00 in
 Modulus of Elasticity.....=2.9000E+07 psi
 Area Moment of Inertia.....=4.9087E-02 in⁴
 Linear Density.....=2.1990E-01 lb/in
 Magnitude of Concentrated Weight.....=7.9000E-03 lb
 Weight Moment of Inertia about a diameter
 of disc or element (W*Rad.Gyr.^2).....=3.2070E-01 lb-in²

Fig. C-21. Input Data for BEAM IV Model "MINI_710.001" (contd)

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