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SEPTEMBER, 1977

MASTER

STRUCTURAL ANALYSIS METHODS
AND
DESIGN SOLUTIONS FOR OHMIC HEATING SOLENOIDS

**fusion power
systems department**

Westinghouse Electric Corporation
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DESIGN SOLUTIONS FOR OHMIC HEATING SOLENOIDS

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FOREWORD

The Division of Magnetic Fusion Energy within the U.S. Energy Research and Development Administration has initiated within the fusion development program for tokamak power reactors a series of systems studies aimed at the definition of subsequent generations of tokamak devices leading to a commercial prototype reactor. Since April, 1976, a design team composed of representatives from the ORNL Fusion Energy Division and the Westinghouse Fusion Power Systems Department has been engaged in scoping studies associated with the definition of The Next Step (TNS) in the tokamak program after the TFTR. Provisional goals established for TNS include:

- achievement of ignition
- demonstration of burning dynamics
- evaluation of design requirements and solutions for long pulse operation
- features which extrapolate to a viable power reactor
- availability in the mid-to-late 1980's

It is in this context that the work reported herein was performed.

ABSTRACT

Design equations with solutions are derived for the ohmic heating coil solenoid unique to a tokamak fusion device. The solutions are provided to perform a detailed structural analysis of every conducting turn in the radial build of the solenoid. The derivations follow from the theory of elasticity for a body possessing cylindrical anisotropy where the material properties are different in the radial and tangential directions. Distinction is made to provide separate models for the region of the solenoid: (a) where turn-to-turn radial contact is maintained with radial compression or with a bond tensile strength in regions of radial tension, and (b) where there is turn-to-turn radial separation due to the absence or the loss of bonding in locations of would-be radial tension.

The results are intended for the engineer who must design a solenoid to generate a required flux swing while simultaneously meeting the limitations imposed by cooling, insulation, current density, power supply, stress and the outer geometric boundary. The solutions are made practical by presenting: (a) the equations and solutions that relate current density, conductor packing fraction and solenoid geometry to the required flux swing, (b) the derivations for computing equivalent material properties in the radial and the tangential directions for a composite structure composed of conductor, coolant holes and insulation for the different regions of the solenoid distinguished by turn-to-turn contact or separation and (c) the methods for reducing stress and for relocating the relative position for potential turn-to-turn radial delamination by permitting an arbitrary traction at the outer radial surface of the solenoid in the form of pressure or displacement such as may be applied by a containment or a shrink fit structural cylinder.

ACKNOWLEDGEMENT

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NOMENCLATURE

CF	=	Coolant Fraction, the ratio of coolant hole area to that area plus the copper area.
SF	=	Stacking Fraction, the ratio of copper plus coolant hole area to that area plus the insulation area.
PF	=	Packing Factor SF (1-CF), the ratio of copper area to the copper plus coolant hole plus insulation area.
J	=	Current Density, the Amps/m ² in the copper portion of the conductor.
J _a	=	Apparent Current Density, an averaging of current density for the purpose of field or force calculation created in a composite structure.
A	=	Area (m ²)
E	=	Young's Modulus (Pa)
R _o	=	Solenoid outer radius (m)
R _i	=	Solenoid inner radius (m)
h	=	Solenoid height (m)
B	=	Solenoid magnetic field (T)
B _i	=	Field at solenoid inner radius, (T)
B _o	=	Field at solenoid outer radius, (T)
φ	=	Solenoid half-swing flux, (Webers)
a	=	Inner radius of a particular region of the solenoid that is being analyzed with the boundary condition of zero radial stress. (m)
b	=	Outer radius of a particular region of the solenoid that is being analyzed with the boundary condition of specified displacement or specified radial pressure or zero radial pressure.
ε	=	strain in the composite structure.
σ	=	stress in the composite structure, (Pa)

- u = radial direction displacement in the composite structure, (m)
 S = stress in a particular material within the composite structure, (Pa)
 C_r = Material Constant = $E_r / (1 - \nu_{r\theta} \nu_{\theta r})$, (Pa)
 C_θ = Material Constant = $E_\theta / (1 - \nu_{r\theta} \nu_{\theta r})$, (Pa)
 F = Body force per unit volume, (N m^{-3})
 α = $\sqrt{(E_\theta / E_r)}$
 D_1 = Unit body force property constant (m^{-2})
 D_2 = Unit body force property constant (m^{-1})
 I_2 = Integration constant ($\text{m}^{1+\alpha}$)
 I_1 = Integration constant ($\text{m}^{1-\alpha}$)
 P = Radial stress at the solenoid outer radius, (Pa)
 D = Displacement at the solenoid outer radius, (m)

Subscripts

- i = insulation (B_i is exception: see nomenclature)
 c = copper
 r = radial coordinate direction
 θ = tangential (hoop) coordinate direction
 1 = coolant hole (only for area, $A_1 = A_{\text{coolant hole}}$)
 2 = copper (only for area, $A_2 = A_c$)
 3 = insulation (only for area, $A_3 = A_i$)
 s = series of equivalent stiffness for copper, coolant hole and insulation

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1.0 INTRODUCTION

The work provided in this study presents methods for achieving and recommendations for improving the engineering design solutions for ohmic heating solenoids. The solutions relate the requirements imposed by structural integrity to the need for producing sufficient flux swing to initiate a plasma current in a tokamak fusion device. A similar study provided in Reference (1) developed a theoretical approach and presented sample results for the stress analysis of transversely isotropic, homogeneous, axisymmetric solenoids loaded by linearly varying radial electromagnetic forces. Additionally, an application of the solution to multi-layered, non-homogeneous coils was presented. The Reference (1) work is significant in that it was directed at some of the structural design problems unique to superconducting solenoids. It describes a method that was employed to include the effects of winding preload and of thermal cooldown to superconducting temperatures. By comparison, the work presented here is developed for direct applications to solenoids constructed from copper conductors with emphasis on the relationship between conductor stress and the solenoid flux requirement which can be computed with great accuracy by using the detailed magnetic field data generated for solenoids in the Reference (2) work. In Reference (2) a parametric study was presented which allows a rapid evaluation of pertinent solenoid field values with two given geometric parameters which are the ratio of solenoid height to inner radius and the ratio of solenoid outer radius to inner radius. The work presented here utilizes the detailed information provided by the Reference (2) study so that comparisons are made between realistic solenoid geometries and the ideal approximations based upon the assumption of a tall thin-walled solenoid.

Computer programs are presented in the appendices of this report which have been developed for the structural analysis of both the ideal approximation and the actual more realistic relationship between conductor stress and solenoid flux. These programs are based upon the derivations presented in Sections 2.0, 3.0 and 4.0 of this report.

Section 2.0 provides a general definition of the conductor cross-section with respect to coolant channel, insulation and conductor fractions. Here the material property equations

are derived to define the solenoid anisotropy for two different conditions. These included (1) the properties in the region of the solenoid where there is radial compaction between turns, and (2) the properties in the region of the solenoid where there is radial separation between conductor turns.

Section 3.0 presents the magnetic field computation methods from a simplified approximation and from the detailed evaluations from the Reference (2) report. This section also includes the derivation of equations for flux and the Lorentz force which is later used in the solution to the equations of elasticity.

Section 4.0 provides the derivations of equations for the stress and displacement of solenoids from the equations of elasticity and equilibrium for solenoids with a region of radial compaction between turns and for solenoids with a region of radial separation between turns. An additional aspect to the solutions derived here that was not treated in the Reference (1) and (2) study reports includes the provision for evaluating the effect of a shrink fit or containment structural cylinder on the solenoid.

Section 5.0 presents some numerical results for the analysis of tall solenoids with radial compacting turns and for solenoids with radial separating turns. Section 6.0 of this report makes the conclusions and recommendations from the analysis for improved solenoid design configurations and methods of evaluation.

2.0 CONDUCTOR DEFINITION

The conductor is defined in cross-section as square shaped with a single circular coolant channel and with insulation on its outer surfaces, schematically described as the following:

A_1	=	Coolant Hole Area
A_2	=	Copper Area
$A_1 + A_2$	=	Conductor Area
A_3	=	Insulation Area

With these areas, three different terms are defined.

$$CF = A_1 / (A_1 + A_2) = \text{Coolant Fraction}$$

The coolant fraction is the ratio of coolant hole area to conductor area.

$$SF = (A_1 + A_2) / (A_1 + A_2 + A_3) = \text{Stacking Fraction}$$

The stacking fraction is the ratio of conductor area to conductor plus insulation area.

$$PF = A_2 / (A_1 + A_2 + A_3) = SF (1 - CF) = \text{Packing Factor}$$

The packing factor is the ratio of the copper area to the total area.

2.1 CURRENT DENSITY

The current density (J) is the amps (I) per square meter (A_2) in the copper portion of the conductor defined by the above schematic, or

$$J = I / A_2. \quad (2-1)$$

When the magnetic field is being calculated at any location within the solenoid and where it involves the product of current density with other terms such as geometry and the permeability of air, the current density must be modified to be an apparent current density to average the current over the total area.

$$J_a = J \cdot PF = J \cdot SF(1 - CF) \quad (2-2)$$

Similar modifications are required when the Lorentz force is being computed as the current density times an area crossed on the magnetic field vector. When the force is being computed on a single delaminated turn, the current density is modified to be an apparent current density over the conductor area.

$$J_a = J \cdot (1 - CF) \quad (2-3)$$

When the force is being computed in the homogenized outer set of turns where radial contact is retained, then the current density is modified as shown by equation (2-2). Further detail is provided on the Lorentz force in Section 3.0 of this work.

2.2 EQUIVALENT MATERIAL PROPERTIES

The material properties that need to be derived to modify the structural stiffness of the solenoid to account for coolant holes and insulation are the Young's modulus in the radial and hoop directions for the two conditions of radial separation between conductor turns and radial contact between conductor turns. Thus, four different material properties must be derived.

2.2.1 Radial Contact Between Conductor Turns

Radial contact between conductor turns can be maintained by either of two mechanisms. The first is provided by natural radial compression produced by an outer turn having a lower radial expansion than its neighboring inner turn which then tends to pile into the outer turn as the higher expansion tendency of the inner turn is restricted by the lower expansion tendency of the outer turn. As one moves, radially inward, there is a location where this trend can be reversed, whereby the next inner turn has a lower free radial expansion than its neighboring outer turn. Thus separation between turns will occur unless there is a bond at the winding interfaces which can carry a tensile load. Hence the second mechanism by which radial contact can be maintained is with a bond tensile strength between turns that would permit radial tensile stress in that portion of the solenoid.

The region of the solenoid where radial contact is maintained is modelled for the purpose of strain and deflection calculations as a composite structure composed of copper, insulation and coolant holes. In the hoop direction, the members in the composite act in parallel such that:

$$E_{\theta} (A_1 + A_2 + A_3) = E_i (A_3) + E_c (A_2)$$

$$E_{\theta} = E_i \left(\frac{A_1 + A_2 + A_3 - A_1 - A_2}{A_1 + A_2 + A_3} \right) + E_c \left(\frac{A_2}{A_1 + A_2 + A_3} \right)$$

$$E_{\theta} = E_i (1 - SF) + E_c (PF)$$

$$E_{\theta} = E_i (1 - SF) + E_c \cdot SF (1 - CF) \quad (2-4)$$

The radial direction modulus is derived from a combination of parallel and series equivalent springs. The parallel combination is the insulation with a series of insulation, copper and coolant hole designated here as E_s .

$$E_r \sqrt{A_1 + A_2 + A_3} = E_i \left(\sqrt{A_1 + A_2 + A_3} - \sqrt{A_1 + A_2} \right) + E_s \sqrt{A_1 + A_2}$$

This is rewritten as

$$E_r = E_i (1 - \sqrt{SF}) + E_s \sqrt{SF}$$

The series of insulation and copper as modified by the coolant channel is written as the sum of the inverse spring constants as

$$\frac{1}{E_s \frac{1 \cdot \sqrt{A_1 + A_2}}{\sqrt{A_1 + A_2 + A_3}}} = \frac{1}{E_i \frac{1 \cdot \sqrt{A_1 + A_2}}{\sqrt{A_1 + A_2 + A_3} - \sqrt{A_1 + A_2}}} + \frac{1}{E_c \frac{1 \cdot \sqrt{A_1 + A_2}}{\sqrt{A_1 + A_2} - \sqrt{4A_1/\pi}}} + \frac{1}{E_c \frac{1 (\sqrt{A_1 + A_2} - \sqrt{4A_1/\pi})}{\sqrt{4A_1/\pi}}}$$

The first term represents the radial thickness of the insulation not included in the parallel equation written above. The second term represents the copper of full area whose length is decreased by the channel diameter, and the third term represents the copper whose area is decreased by the channel diameter and whose length is only that of a channel diameter. The above expression when inverted is written as

$$E_s \sqrt{SF} = \frac{E_c \sqrt{SF}}{\frac{E_c}{E_i} (1 - \sqrt{SF}) + \sqrt{SF} + \frac{CF \sqrt{SF}}{\left(\frac{\pi}{4}\right) - \sqrt{\frac{\pi}{4}} CF}}$$

Substitution of this series expression into the above parallel expression provides the equation for the equivalent Youngs modulus in the radial direction.

$$E_r = E_i (1 - \sqrt{SF}) + \frac{E_c \sqrt{SF}}{\frac{E_c}{E_i} (1 - \sqrt{SF}) + \sqrt{SF} + \frac{CF \sqrt{SF}}{\left(\frac{\pi}{4}\right) - \sqrt{\left(\frac{\pi}{4}\right)} CF}} \quad (2-5)$$

2.2.2 Radial Separation Between Conductor Turns

The analysis of individual turns will be performed under the assumption that the insulation material is not present. There are three basic reasons for the assumption. The first is that a slightly conservative answer will be obtained by not including some of the stiffness, because the hoop strain and consequently the radial displacement will be slightly larger than if this stiffness were included. The second reason is that the insulation no longer provides a soft modulus in the radial direction because there is no turn-to-turn contact, and therefore the radial stress distribution in the copper turn itself will be more meaningful in terms of the Poisson ratio effect that it has upon the more important hoop stress in the copper turn. The third reason is that since there is no current in the insulation and therefore no Lorentz force, by throwing out the insulator the current density will not be averaged over that entire area, but it will be averaged where it actually exists consistent with the averaged

material properties to provide a larger and more accurate Lorentz force on the copper itself. Finally the true radial dimensions of the copper turn will be that which is used in the analysis.

The region of the turns where radial contact is not maintained is modelled for the purpose of strain and deflection calculations as a composite structure composed of copper with a coolant channel. The hoop direction modulus is modelled in parallel as:

$$E_{\theta} (A_1 + A_2) = E_c (A_2) = E_c (A_2 + A_1 - A_1)$$

$$E_{\theta} = E_c (1 - CF) \quad (2-6)$$

The radial direction modulus is derived from a combination of series springs represented by a length of copper reduced by the coolant channel diameter and an area of copper reduced by the coolant hole diameter whose length is the channel diameter.

$$\frac{1}{\frac{E_r}{\sqrt{A_1 + A_2}} \cdot 1} = \frac{1}{\frac{E_c}{\sqrt{A_1 + A_2}} \cdot 1} + \frac{1}{\frac{E_c (\sqrt{A_1 + A_2} - \sqrt{4A_1/\pi})}{\sqrt{4A_1\pi}} \cdot 1}$$

or

$$E_r = \frac{E_c \left(1 - \sqrt{\left(\frac{4}{\pi}\right)} CF \right)}{\left(1 - \sqrt{\left(\frac{4}{\pi}\right)} CF \right) + \left(\frac{4}{\pi}\right) CF} \quad (2-7)$$

3.0 MAGNETIC FIELDS

The magnetic field that is created by the solenoid is needed to compute the flux swing that can be produced by the ohmic heating coils, and it is also required to compute the Lorentz force acting upon the coils.

A linear field approximation is assumed to exist with the larger value present at the inner radial surface. It has been shown⁽²⁾ that this assumption will yield quite accurate results when comparing the total generated force between that computed by the linear approximation to that computed from the true field shape provided the solenoid height is at least the size of its inner radius. It is further shown⁽²⁾ that if the height becomes small compared to the inner radius then very large errors in the force computation can result when the outer radius gets two or more times larger than the inner radius.

The assumed linear function is:

$$B = \left[B_i + R_i \left(\frac{B_i - B_o}{R_o - R_i} \right) \right] - \left[\frac{B_i - B_o}{R_o - R_i} \right] r \quad (3-1)$$

Another approximation to the magnetic field that can be used instead of a magnetic field computer code that can be used to compute B_i and B_o is (where with MKS units, $\mu = 4 \pi \times 10^{-7}$):

$$B = \mu J (R_o - r) SF (1 - CF) \quad (3-2)$$

$$B_o = 0 \quad (3-3)$$

$$B_i = \mu J (R_o - R_i) SF (1 - CF) \quad (3-4)$$

The validity of this particular equation for accuracy is very dependent upon the solenoid geometry. For example; if $R_i/R_o = .33$, the height (h) of the solenoid must be about 8.0 times the inner radius for B_i to be reasonably accurate with equation (3-4). If $R_i/R_o = .50$, the height of the solenoid must be about 5.75 times the inner radius, and if $R_i/R_o = .67$ the height must be about 4.5 times the inner radius.

In relation to the solenoid outer radius, the equations (3-3) and (3-4) are reasonably accurate at $R_i/R_o = .33$ if $h \geq 2.66 R_o$, at $R_i/R_o = .5$ if $h \geq 2.87 R_o$ and at $R_i/R_o = .67$ if the height is about or greater than 3 times the outer radius.

These observations or comparisons of validity have been made with the use of the detailed magnetic field calculations performed at ORNL for the reference (2) report. This magnetic field information generated at ORNL by W. H. Gray and J. K. Ballou is copied here and presented as figures 3-1 and 3-2. From these figures the fields are computer (with MKS units) for particular geometric ratios as:

$$B_o = B_o^* R_i J (1-CF) SF \quad (3-3a)$$

$$B_i = B_i^* R_i J (1-CF) SF \quad (3-4a)$$

It should be noted that the asymptotic value of B_i with equation (3-4a) for large (h/R_i) ratios is approaching the value of B_i as computed from equation (3-3). Furthermore the small negative value of B_o with equation (3-3a) approaches the zero value indicated by equation (3-3) as the (h/R_i) ratio becomes large. Additional curves copied from reference (2) and presented here as figures 3-3 and 3-4 extend the range of the geometric ratio values for the determination of B_i^* and B_o^* .

3.1 LORENTZ FORCE

The Lorentz force is computed from the vector cross product of current and magnetic field. The derivations presented here will be for the increment of force on the unit of volume that will be used in a later section for the derivation of the equations of equilibrium. For the unit volume illustrated below the increment of force is in the radial direction, the current is in the tangential direction and the field is in the axial direction.

Figure 3-1. Contour plots of the normalized field factor values at the inside solenoid radius for different geometric ratio values. (Copied from ORNL/TM-5528 originals by W. H. Gray and J. K. Ballou)

Figure 3-2. Contour plots of the normalized field factor values at the outside solenoid radius for different geometric ratio values. (Copied from ORNL/TM-5528 originals by W. H. Gray and J. K. Ballou).

Figure 3-3. Contour plots of normalized field factor values at the inside solenoid radius for extended geometric ratio values. (Copied from ORNL/TM-5528 originals by W. H. Gray and J. K. Ballou).

Figure 3-4. Contour plots of the normalized field factor values at the outside solenoid radius for extended geometric ratio values. (Copied from ORNL/TM-5528 originals by W. H. Gray and J. K. Ballou).

$$d\mathbf{F} = I d\mathbf{\ell} \times \mathbf{B}$$

$$d\mathbf{\ell} = r d\mathbf{\theta}$$

For the region of the solenoid where radial contact is maintained between turns the current is averaged over the region where the material properties were averaged to give

$I = J \, dr \, dz \, (1 - CF) \, SF$, and the incremental body force is

$$dF = [J (1-CF) \, SF \, B] \, r \, dr \, d\theta \, dz = F \, r \, dr \, d\theta \, dz \quad (3-5)$$

For the region of the solenoid where there is turn-to-turn radial separation the current is averaged only over the copper and its coolant hole consistent with the regions over which material properties were averaged. The current for the analysis of a single free turn is

$I = J \, dr \, dz \, (1-CF)$, and the increment of body force is

$$dF = [J (1-CF) \, B] \, r \, dr \, d\theta \, dz = F \, r \, dr \, d\theta \, dz \quad (3-6)$$

In the above equations F is the body force per unit volume.

3.2 FLUX SWING

The purpose of the ohmic heating solenoid is to produce sufficient flux swing (from a positive to a negative value by a current reversal) to initiate current flow in the plasma. Therefore the flux swing is a requirement to be met by the OH solenoid design. The following is a derivation for the flux half-swing (ϕ). The full flux swing would be twice the half-swing value. The magnetic field is the flux density, and schematically the field distributions is shown below, and it is assumed to have no contribution beyond the

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solenoid outer radius.

$$\phi = \pi R_i^2 B_i + 2\pi \int_{R_i}^{R_o} B r dr$$

(3-7)

Substitution of equation (3-1) into equation (3-7) gives:

$$\phi = \pi R_i^2 B_i + 2\pi \int_{R_i}^{R_o} \left\{ \left[B_i + a \left(\frac{B_i - B_o}{R_o - R_i} \right) r \right] - \left[\frac{B_i - B_o}{R_o - R_i} \right] r^2 \right\} dr$$

$$\phi = \left(\pi/3 \right) (B_i - B_o) (R_i^2 + R_i R_o + R_o^2) + \pi B_o R_o^2$$

(3-8)

Substitution of equations (3-3a) and (3-4a) into equation (3-8) gives

$$\phi = R_i J (1-CF) SF \left\{ \left(\pi/3 \right) (B_i^* - B_o^*) (R_i^2 + R_i R_o + R_o^2) + \pi B_o^* R_o^2 \right\}$$

(3-9)

Substitution of the approximation to the magnetic field given by equation (3-2) into equation (3-7) gives the following expression for the flux half-swing for which the accuracy will depend upon the solenoid geometry.

$$\phi = \mu J (1-CF) SF \left\{ \left(\pi/3 \right) (R_o^3 - R_i^3) \right\}$$

(3-10)

A rearrangement of this equation in a form that is normalized to some likely values of coolant fraction, stacking fraction and current density is provided below for estimating the outer radius of the solenoid once the flux half-swing requirement is established.

$$R_o \sqrt[3]{\frac{J}{15 \times 10^6}} \sqrt[3]{\frac{(1. - .1) (.9)}{(1. - CF) (SF)}} = .3969 \sqrt[3]{\frac{\phi}{1 - (R_i/R_o)^3}} \quad (3-11)$$

This equation is plotted on figure (3-5), and in order to be accurate the solenoid must be tall in comparison to its outer radius. If $R_i/R_o = .3$, then $h \geq 2.6 R_o$. If $R_i/R_o = .5$, then $h \geq 2.87 R_o$, and if $R_i/R_o = .70$, then $h \geq 3 R_o$. Otherwise equation (3-9) must be used to solve for the relationship between flux and the solenoid radial dimensions.

$$\left\{ R_o \cdot \sqrt[3]{\frac{J}{15 \times 10^6}} \right\} \sqrt[3]{\frac{(1. - .1) (.9)}{(1. - CF) (SF)}} , (m)$$

Figure 3-5. Outer radius versus the OH solenoid half-swing flux for different inner to outer radius ratios for the approximating equation (3-10).

4.0 ELASTICITY

The equations of elasticity provide the relationship between the components of stress and the components of strain in an elastic body. In what follows, it is assumed that the strain components are linear functions of the stress components. There is a continuous body which satisfies Hooke's law, and the body possesses cylindrical anisotropy with the different elastic constants in the r and θ directions that were derived in section 2.2 of this work. In the region of the solenoid where turn-to-turn radial contact is maintained, that entire region is the continuous body. In the region of the solenoid where the turns separate, each turn is the continuous body with a different set of equations, elastic constants and boundary conditions as compared to the region where contact is maintained. The general form of the equations common to both regions include stress-strain, strain-displacement and equilibrium. Any axial loading due to a radial field component near the ends of the solenoid is neglected. Should a solenoid be designed such that (small height to radius ratio) this assumption produces significant error a separate solution could be performed for the axial load contributions to stress, and then a superposition could be made with the solutions presented here. The strain-stress relationships for an axisymmetrically loaded body with cylindrical anisotropy are:

$$\epsilon_r = \frac{1}{E_r} (\sigma_r - \nu_{\theta r} \sigma_\theta) \quad (4-1)$$

$$\epsilon_\theta = \frac{1}{E_\theta} (\sigma_\theta - \nu_{r\theta} \sigma_r) \quad (4-2)$$

$$E_r \nu_{r\theta} = E_\theta \nu_{\theta r} \quad (4-3)$$

The double subscript, for the Poisson ratio is interpreted as the second being the direction of a decreased value for the first being the direction of an applied tension.

The strain displacement relationships for an axisymmetrically loaded body in the radial and hoop directions are:

$$\epsilon_{\theta} = \frac{u}{r} \quad (4-4)$$

$$\epsilon_r = \frac{du}{dr} \quad (4-5)$$

The combination of the above five equations provides the following relationships between stress and displacement:

$$\sigma_r = C_r \left[\frac{du}{dr} + \nu_{r\theta} \frac{u}{r} \right] \quad (4-6)$$

$$\sigma_{\theta} = C_{\theta} \left[\frac{u}{r} + \nu_{\theta r} \frac{du}{dr} \right] \quad (4-7)$$

$$C_r = \frac{E_r}{(1 - \nu_{r\theta} \nu_{\theta r})} \text{ and } C_{\theta} E_r = C_r E_{\theta} \quad (4-8)$$

The equation of equilibrium is derived from the following free body diagram by summation of forces in the radial direction:

$$\left(\sigma_r + \frac{d\sigma_r}{dr} dr \right) (r + dr) d\theta dz - \sigma_r r d\theta dz - 2\sigma_\theta dr dz \sin\left(\frac{d\theta}{2}\right) + r F d\theta dz dr = 0$$

Rearrangement gives:

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} + F = 0 \quad (4-9)$$

In the above equation F is the body force per unit volume defined by either equation (3-5) or (3-6) depending on the region of analysis. Substitution of equations (4-6), (4-7) and (4-8) into equation (4-9) gives the following differential equation for displacement:

$$\frac{d^2 u}{dr^2} + \frac{1}{r} \frac{du}{dr} - \frac{u \alpha^2}{r^2} + \frac{F}{C_r} = 0 \quad (4-10)$$

where

$$\alpha = \sqrt{(E_\theta / E_r)} \quad (4-11)$$

Equation (4-10) can be rewritten as

$$\frac{1}{r^{1-\alpha}} \frac{d}{dr} \left[r^{1-2\alpha} \frac{d}{dr} (r^\alpha u) \right] = D_1 r - D_2 \quad (4-12)$$

where

$$D_1 r - D_2 = -F/C_r \quad (4-13)$$

Since the value of F differs with the combinations of equation (3-1) with (3-5) or (3-6) in the two regions where solutions are being sought, the general solution will first be presented and then the separate solutions in the two regions will be solved with appropriate and different boundary conditions. The integration of equation (4-12) two times gives the following

equation for displacement in terms of the unit body force property constants D_1 and D_2 and the two integration constants I_1 and I_2 :

$$u = \frac{D_1 r^3}{9 - \alpha^2} - \frac{D_2 r^2}{4 - \alpha^2} + \frac{I_1 r^\alpha}{2\alpha} + \frac{I_2}{r^\alpha} \quad (4-14)$$

$$\epsilon_\theta = \frac{u}{r} = \frac{D_1 r^2}{9 - \alpha^2} - \frac{D_2 r}{4 - \alpha^2} + \frac{I_1 r^{\alpha-1}}{2\alpha} + \frac{I_2}{r^{\alpha+1}} \quad (4-15)$$

$$\epsilon_r = \frac{du}{dr} = \frac{3D_1 r^2}{9 - \alpha^2} - \frac{2D_2 r}{4 - \alpha^2} + \frac{I_1 r^{\alpha-1}}{2} - \frac{\alpha I_2}{r^{\alpha+1}} \quad (4-16)$$

The substitution of equation (4-15), (4-16) and (4-8) into equation (4-7) gives the tangential stress in the composite structure as:

$$\begin{aligned} \sigma_\theta = & D_1 C_r^2 \left[\left(\frac{3 \nu_{r\theta} + \alpha^2}{9 - \alpha^2} \right) - \frac{D_2}{D_1 r} \left(\frac{2 \nu_{r\theta} + \alpha^2}{4 - \alpha^2} \right) \right] \\ & + I_1 C_r \left[\frac{(\alpha + \nu_{r\theta}) r^{\alpha-1}}{2} \right] + I_2 C_r \left[\frac{\alpha (\alpha - \nu_{r\theta})}{r^{\alpha+1}} \right] \end{aligned} \quad (4-17)$$

The substitution of equations (4-15), (4-16) and (4-8) into equation (4-6) into (4-6)

give the radial stress expression in the composite structure:

$$\sigma_r = D_1 C_r^2 \left[\left(\frac{3 + \nu}{9 - \alpha} \frac{r\theta}{2} \right) - \frac{D_2}{D_1 r} \left(\frac{2 + \nu}{4 - \alpha} \frac{r\theta}{2} \right) \right] + I_1 C_r \left[\frac{r^{\alpha-1}}{2\alpha} (\alpha + \nu r\theta) \right] + I_2 C_r \left[r^{-\alpha-1} (\nu r\theta - \alpha) \right] \quad (4-18)$$

- The above equations are the general expressions that can be applied to the two different regions of the solenoid.

4.1 RADIAL CONTACT BETWEEN TURNS

The combination of equations (3-1), (3-5) and (4-13) provides the expressions for D_1 and D_2 in the region of the solenoid where radial contact is maintained between turns either by radial compressive stress or by bonding in the presence of some radial tension.

$$-F/C_r = J (1-CF) SF \left\{ \left[\frac{B_i - B_o}{R_o - R_i} \right] r - \left[B_i + R_i \left(\frac{B_i - B_o}{R_o - R_i} \right) \right] \right\} / C_r$$

$$D_1 = J (1 - CF) SF \left[\frac{B_i - B_o}{R_o - R_i} \right] / C_r \quad (4-19)$$

$$D_2 = J (1-CF) SF \left[B_i + R_i \left(\frac{B_i - B_o}{R_o - R_i} \right) \right] / C_r \quad (4-20)$$

For geometric configurations where equation (3-2) is an accurate expression for the magnetic field,

D_1 and D_2 further simplify to

$$D_1 = \mu \left[J (1-CF) SF \right]^2 / Cr \quad (4-19, a)$$

$$D_2 = R_o D_1 = b D_1 \quad (4-20, a)$$

For this condition figures (3-1) through (3-4) are not required for the determination of B_i and B_o . The integration constants I_1 and I_2 are evaluated for two different boundary conditions that can be used to simulate the restraint of a containment or shrink fit cylinder. These will include a specified displacement at the outer radius or a specified pressure at the outer radius where for both cases the radial stress at the inner surface is zero.

4.1.1 Specified Radial Pressure at the Outer Radius

The integration constants are evaluated by setting $\sigma_r = 0$ at $r = a$ and $\sigma_r = P$ at $r = b = R_o$. From equation (4-18) the two equations with two unknowns are:

$$\begin{aligned} 0 = & D_1 C_r a^2 \left[\left(\frac{3 + \nu_{r\theta}}{9 - \alpha^2} \right) - \frac{D_2}{D_1 a} \left(\frac{2 + \nu_{r\theta}}{4 - \alpha^2} \right) \right] \\ & + I_1 C_r \left[\frac{a^{\alpha-1}}{2\alpha} (\alpha + \nu_{r\theta}) \right] + I_2 C_r \left[a^{-\alpha-1} (\nu_{r\theta} - \alpha) \right] \end{aligned} \quad (4-21)$$

$$P = D_1 C_r b^2 \left[\left(\frac{3 + \nu_{r\theta}}{9 - \alpha^2} \right) - \frac{D_2}{D_1 b} \left(\frac{2 + \nu_{r\theta}}{4 - \alpha^2} \right) \right] + I_1 C_r \left[\frac{b^{\alpha-1}}{2} (\alpha + \nu_{r\theta}) \right] + I_2 C_r \left[b^{-\alpha-1} (\nu_{r\theta} - \alpha) \right] \quad (4-22)$$

The simultaneous solution of these equations provides the expressions for the integration constants I_1 and I_2

$$I_1 = \frac{P + D_1 C_r b^2 \left\{ \left(\frac{3 + \nu_{r\theta}}{9 - \alpha^2} \right) \left[\left(\frac{a}{b} \right)^{\alpha+3} - 1 \right] + \left(\frac{2 + \nu_{r\theta}}{4 - \alpha^2} \right) \left(\frac{D_2}{D_1 b} \right) \left[1 - \left(\frac{a}{b} \right)^{\alpha+2} \right] \right\}}{\frac{b^{\alpha-1} C_r (\alpha + \nu_{r\theta})}{2\alpha} \left[1 - \left(\frac{a}{b} \right)^{2\alpha} \right]} \quad (4-23)$$

$$I_2 = \frac{\left(\frac{a}{b} \right)^{2\alpha} \left\{ P + D_1 C_r b^2 \left\{ \left(\frac{3 + \nu_{r\theta}}{9 - \alpha^2} \right) \left[\left(\frac{a}{b} \right)^{3-\alpha} - 1 \right] + \left(\frac{2 + \nu_{r\theta}}{4 - \alpha^2} \right) \left(\frac{D_2}{D_1 b} \right) \left[1 - \left(\frac{a}{b} \right)^{2-\alpha} \right] \right\} \right\}}{b^{-\alpha-1} C_r (\alpha - \nu_{r\theta}) \left[1 - \left(\frac{a}{b} \right)^{2\alpha} \right]} \quad (4-24)$$

4.1.1.1 Displacement Solution with Specified Pressure

The displacement equation (4-14) can be rewritten in non-dimensional form

as:

$$\frac{u}{D_1 b^3} = \left(\frac{1}{9 - \alpha^2} \right) \left(\frac{r}{b} \right)^3 - \left(\frac{1}{4 - \alpha^2} \right) \left(\frac{D_2}{D_1 b} \right) \left(\frac{r}{b} \right)^2 + \frac{I_1 r^\alpha}{2\alpha D_1 b^3} + \frac{I_2}{r^\alpha D_1 b^3} \quad (4-14, a)$$

The substitution of equations (4-23) and (4-24) into the above equation provides after some rearrangement the following relationship for computing displacement in the region of the solenoid where turn-to-turn radial contact is maintained and where an arbitrary value of pressure may be specified at the solenoid outer radial surface:

$$\begin{aligned} \frac{u}{D_1 b^3} = & \left(\frac{1}{9 - \alpha^2} \right) \left(\frac{r}{b} \right)^3 - \frac{D_2}{D_1 b} \left(\frac{r}{b} \right)^2 \left(\frac{1}{(4 - \alpha^2)} \right) \\ & + \frac{\left(\frac{a}{b} \right)^{\alpha+2} \left[\left(\frac{3 + \nu_{r\theta}}{9 - \alpha^2} \right) \left(\frac{a}{b} \right) - \left(\frac{D_2}{D_1 b} \right) \left(\frac{2 + \nu_{r\theta}}{4 - \alpha^2} \right) \right]}{\left(\frac{r}{b} \right)^\alpha (\alpha - \nu_{r\theta})} \\ & + \frac{\left(\frac{r}{b} \right)^\alpha \left[1 + \left(\frac{\alpha + \nu_{r\theta}}{\alpha - \nu_{r\theta}} \right) \left(\frac{a}{b} \right)^{2\alpha} \left(\frac{b}{r} \right)^{2\alpha} \right]}{(\alpha + \nu_{r\theta}) \left[1 - \left(\frac{a}{b} \right)^{2\alpha} \right]} \left\{ \frac{P}{C_r D_1 b^2} \right. \\ & \left. - \left(\frac{3 + \nu_{r\theta}}{9 - \alpha^2} \right) \left[1 - \left(\frac{a}{b} \right)^{\alpha+3} \right] + \left(\frac{2 + \nu_{r\theta}}{4 - \alpha^2} \right) \left(\frac{D_2}{D_1 b} \right) \left[1 - \left(\frac{a}{b} \right)^{\alpha+2} \right] \right\} \end{aligned} \quad (4-25)$$

4.1.1.2 Tangential Stress Solution with Specified Pressure

The tangential stress equation (4-17) is rewritten in the following non-dimensional form:

$$\frac{\sigma_\theta}{D_1 C_r b^2} = \left(\frac{r}{b} \right)^2 \left[\left(\frac{3 + \nu_{r\theta} + \alpha^2}{9 - \alpha^2} \right) - \frac{D_2}{D_1 r} \left(\frac{2 + \nu_{r\theta} + \alpha^2}{4 - \alpha^2} \right) \right] +$$

$$+ \frac{l_1}{D_1 b^2} \left[\frac{(\alpha + \nu_{r\theta}) r^{\alpha-1}}{2} \right] + \frac{l_2}{D_1 b^2} \left[\frac{\alpha (\alpha - \nu_{r\theta})}{r^{\alpha+1}} \right] \quad (4-17, a)$$

The substitution of equations (4-23) and (4-24) into this equation provides after some rearrangement the following relationship for computing tangential stress in the composite structure where turn-to-turn radial contact is maintained and where an arbitrary value of pressure may be specified at the solenoid outer radial surface:

$$\begin{aligned} \frac{\sigma_{\theta}}{D_1 C_r b^2} = & \left(\frac{r}{b}\right)^2 \left[\left(\frac{3 \nu_{r\theta} + \alpha^2}{9 - \alpha^2} \right) - \frac{D_2}{D_1 b} \left(\frac{2 \nu_{r\theta} + \alpha^2}{4 - \alpha^2} \right) \left(\frac{b}{r}\right) \right] \\ & + \alpha \left(\frac{b}{r}\right)^{\alpha+1} \left(\frac{a}{b}\right)^{\alpha+2} \left[\left(\frac{3 + \nu_{r\theta}}{9 - \alpha^2} \right) \left(\frac{a}{b}\right) - \left(\frac{D_2}{D_1 b}\right) \left(\frac{2 + \nu_{r\theta}}{4 - \alpha^2} \right) \right] \\ & + \alpha \left(\frac{r}{b}\right)^{\alpha-1} \frac{\left[1 + \left(\frac{b}{r}\right)^{2\alpha} \left(\frac{a}{b}\right)^{2\alpha} \right]}{\left[1 - \left(\frac{a}{b}\right)^{2\alpha} \right]} \left\{ \frac{P}{C_r D_1 b^2} \right. \\ & \left. - \left(\frac{3 + \nu_{r\theta}}{9 - \alpha^2} \right) \left[1 - \left(\frac{a}{b}\right)^{\alpha+3} \right] + \left(\frac{2 + \nu_{r\theta}}{4 - \alpha^2} \right) \left(\frac{D_2}{D_1 b}\right) \left[1 - \left(\frac{a}{b}\right)^{\alpha+2} \right] \right\} \end{aligned} \quad (4-26)$$

The stress in the different materials of the composite structure are evaluated in the copper and epoxy from the above evaluation by:

$$S_{\theta c} = \sigma_{\theta} E_c / E_{\theta} \quad (4-27)$$

$$S_{\theta e} = \sigma_{\theta} E_e / E_{\theta} \quad (4-28)$$

4.1.1.3 Radial Stress Solution with Specified Pressure

The radial stress equation (4-18) is rewritten in the following non-dimensional form:

$$\begin{aligned} \frac{\sigma_r}{D_1 C_r b^2} = & \left(\frac{r}{b}\right)^2 \left[\left(\frac{3 + \nu_{r\theta}}{9 - \alpha^2} \right) - \frac{D_2}{D_1 r} \left(\frac{2 + \nu_{r\theta}}{4 - \alpha^2} \right) \right] \\ & + \frac{I_1}{D_1 b^2} \left[\frac{r^{\alpha-1}}{2\alpha} (\alpha + \nu_{r\theta}) \right] + \frac{I_2}{D_1 b^2} \left[r^{-\alpha-1} (\nu_{r\theta} - \alpha) \right] \end{aligned} \quad (4-18, a)$$

Substitution of equations (4-23) and (4-24) into this equations with same rearrangement provides the following relationship for computing the radial stress in the composite structure where turn-to-turn radial contact is maintained and where an arbitrary value of pressure may be specified at the solenoid outer radial surface:

$$\begin{aligned} \frac{\sigma_r}{D_1 C_r b^2} = & \left(\frac{r}{b}\right)^2 \left\{ \left[1 - \left(\frac{b}{r}\right)^{\alpha+3} \left(\frac{a}{b}\right)^{\alpha+3} \right] \left(\frac{3 + \nu_{r\theta}}{9 - \alpha^2} \right) - \right. \\ & \left. \left(\frac{b}{r}\right) \left(\frac{2 + \nu_{r\theta}}{4 - \alpha^2} \right) \left(\frac{D_2}{D_1 b} \right) \left[1 - \left(\frac{b}{r}\right)^{\alpha+2} \left(\frac{a}{b}\right)^{\alpha+2} \right] \right\} + \left(\frac{r}{b}\right)^{\alpha-1} \left\{ \frac{\left[1 - \left(\frac{b}{r}\right)^{2\alpha} \left(\frac{a}{b}\right)^{2\alpha} \right]}{\left[1 - \left(\frac{a}{b}\right)^{2\alpha} \right]} \right\} \frac{P}{D_1 C_r b^2} \\ & - \left(\frac{3 + \nu_{r\theta}}{9 - \alpha^2} \right) \left[1 - \left(\frac{a}{b}\right)^{\alpha+3} \right] + \left(\frac{2 + \nu_{r\theta}}{4 - \alpha^2} \right) \left(\frac{D_2}{D_1 b} \right) \left[1 - \left(\frac{a}{b}\right)^{\alpha+2} \right] \end{aligned} \quad (4-29)$$

In the regions of the composite material where the insulation and the copper are acting in series such as a compressive interface the stress is the same in each material and is of the value as provided by equation (4-29).

In the regions of the composite material where the insulation the coolant hole and the copper are locally acting in parallel the stress in the copper and epoxy are evaluated as:

$$S_{r_c} = \sigma_r E_c / E_r \quad (4-30)$$

$$S_{r_e} = \sigma_r E_e / E_r \quad (4-31)$$

The stress in the copper may even be carried a step further by considering a stress concentration of about a factor of three on the inside surface of the coolant channel due to the presence of the hole.

4.1.2 Specified Displacement at the Outer Radius

The integration constants for this condition are evaluated by setting $\sigma_r = 0$ at $r = a$ and $u = D$ at $r = b = R_o$. From equations (4-18) and (4-14) the two equations to be solved for the two unknowns I_1 and I_2 are:

$$0 = D_1 C_r a^2 \left[\left(\frac{3 + \nu_{r\theta}}{9 - \alpha^2} \right) - \frac{D_2}{D_1 a} \left(\frac{2 + \nu_{r\theta}}{4 - \alpha^2} \right) \right] + I_1 C_r \left[\frac{a^{\alpha-1}}{2\alpha} (\alpha + \nu_{r\theta}) \right] + I_2 C_r \left[a^{-\alpha-1} (\nu_{r\theta} - \alpha) \right] \quad (4-32)$$

$$D = \frac{D_1 b^3}{9 - \alpha^2} - \frac{D_2 b^2}{4 - \alpha^2} + \frac{I_1 b^\alpha}{2\alpha} + \frac{I_2}{b^\alpha} \quad (4-33)$$

The simultaneous solution of these equations provides the expressions for the integration constants I_1 and I_2 .

$$I_1 = \frac{\frac{D}{D_1 b^3} - \frac{1}{9 - \alpha^2} + \frac{D_2}{D_1 b (4 - \alpha^2)} + \left(\frac{1}{\alpha - \nu_{r\theta}}\right) \left(\frac{a}{b}\right)^{\alpha+2} \frac{D_2}{D_1 b} \left[\left(\frac{2 + \nu_{r\theta}}{4 - \alpha^2}\right) - \left(\frac{3 + \nu_{r\theta}}{9 - \alpha^2}\right) \left(\frac{a}{b}\right) \right]}{\left(\frac{1}{D_1 b^3}\right) \frac{b^\alpha}{2\alpha} \left[1 + \left(\frac{\alpha + \nu_{r\theta}}{\alpha - \nu_{r\theta}}\right) \left(\frac{a}{b}\right)^{2\alpha} \right]} \quad (4-34)$$

$$I_2 = \frac{\left(\frac{a}{b}\right)^\alpha \left(\frac{a + \nu_{r\theta}}{\alpha - \nu_{r\theta}}\right) \left[\frac{D}{D_1 b^3} - \frac{1}{9 - \alpha^2} + \frac{D_2}{D_1 b (4 - \alpha^2)} \right]}{\left(\frac{1}{a}\right)^\alpha \frac{1}{D_1 b^3} \left[1 + \left(\frac{\alpha + \nu_{r\theta}}{\alpha - \nu_{r\theta}}\right) \left(\frac{a}{b}\right)^{2\alpha} \right]} - \frac{\left(\frac{1}{\alpha - \nu_{r\theta}}\right) \left(\frac{a}{b}\right)^2 \left[\frac{D_2}{D_1 b} \left(\frac{2 + \nu_{r\theta}}{4 - \alpha^2}\right) - \left(\frac{3 + \nu_{r\theta}}{9 - \alpha^2}\right) \left(\frac{a}{b}\right) \right]}{\left(\frac{1}{a}\right)^\alpha \frac{1}{D_1 b^3} \left[1 + \left(\frac{\alpha + \nu_{r\theta}}{\alpha - \nu_{r\theta}}\right) \left(\frac{a}{b}\right)^{2\alpha} \right]} \quad (4-35)$$

4.1.2.1 Displacement Solution with Specified Displacement

The displacement equation (4-14) is rewritten in non-dimensional form as:

$$\frac{u}{D_1 b^3} = \left(\frac{1}{9 - \alpha^2} \right) \left(\frac{r}{b} \right)^3 - \left(\frac{1}{4 - \alpha^2} \right) \left(\frac{D_2}{D_1 b} \right) \left(\frac{r}{b} \right)^2 + \frac{l_1 r^\alpha}{2\alpha D_1 b^3} + \frac{l_2}{r^\alpha D_1 b^3} \quad (4-14, a)$$

The substitution of equations (4-34) and (4-35) into the above equation provides after some rearrangement the following relationship for computing displacement in the region of the solenoid where turn-to-turn radial contact is maintained and where an arbitrary value of displacement (D) may be specified at the solenoid outer radial surface:

$$\begin{aligned} \frac{u}{D_1 b^3} = & \left(\frac{1}{9 - \alpha^2} \right) \left(\frac{r}{b} \right)^3 - \frac{D_2}{D_1 b} \left(\frac{1}{4 - \alpha^2} \right) \left(\frac{r}{b} \right)^2 \\ & + \left(\frac{b}{r} \right)^\alpha \left(\frac{1}{\alpha - \nu_{r\theta}} \right) \left(\frac{a}{b} \right)^{\alpha+2} \left[\left(\frac{3 + \nu_{r\theta}}{9 - \alpha^2} \right) \left(\frac{a}{b} \right) - \frac{D_2}{D_1 b} \left(\frac{2 + \nu_{r\theta}}{4 - \alpha^2} \right) \right] \\ & + \frac{\left(\frac{r}{b} \right)^\alpha \left[1 - \left(\frac{b}{r} \right)^{2\alpha} \left(\frac{a}{b} \right)^{2\alpha} \left(\frac{\alpha + \nu_{r\theta}}{\alpha - \nu_{r\theta}} \right) \right]}{\left[1 + \left(\frac{\alpha - \nu_{r\theta}}{\alpha + \nu_{r\theta}} \right) \left(\frac{a}{b} \right)^{2\alpha} \right]} \left\{ \frac{D}{D_1 b^3} - \frac{1}{9 - \alpha^2} \right. \\ & \left. + \frac{D_2}{D_1 b (4 - \alpha^2)} - \left(\frac{1}{\alpha - \nu_{r\theta}} \right) \left(\frac{a}{b} \right)^{\alpha+2} \left[\left(\frac{3 + \nu_{r\theta}}{9 - \alpha^2} \right) \left(\frac{a}{b} \right) - \frac{D_2}{D_1 b} \left(\frac{2 + \nu_{r\theta}}{4 - \alpha^2} \right) \right] \right\} \end{aligned} \quad (4-36)$$

4.1.2.2 Tangential Stress Solution with Specified Displacement

The tangential stress equation (4-17) is rewritten in the following non-dimensional form:

$$\begin{aligned} \frac{\sigma_{\theta}}{D_1 C_r b^2} = & \left(\frac{r}{b} \right)^2 \left[\left(\frac{3 \nu_{r\theta} + \alpha^2}{9 - \alpha^2} \right) - \frac{D_2}{D_1 r} \left(\frac{2 \nu_{r\theta} + \alpha^2}{4 - \alpha^2} \right) \right] \\ & + \frac{I_1}{D_1 b^2} \left[\frac{(\alpha + \nu_{r\theta}) r^{\alpha-1}}{2} \right] + \frac{I_2}{D_1 b^2} \left[\frac{\alpha (\alpha - \nu_{r\theta})}{r^{\alpha+1}} \right] \end{aligned} \quad (4-17, a)$$

The substitution of equations (4-34) and (4-35) into this equation provides after some rearrangement the following relationship for computing the hoop stress in the composite structure where turn-to-turn radial contact is maintained and where an arbitrary value of displacement may be specified at the outer solenoid outer radial surface:

$$\begin{aligned} \frac{\sigma_{\theta}}{D_1 C_r b^2} = & \left(\frac{r}{b} \right)^2 \left[\left(\frac{3 \nu_{r\theta} + \alpha^2}{9 - \alpha^2} \right) - \frac{D_2}{D_1 b} \left(\frac{2 \nu_{r\theta} + \alpha^2}{4 - \alpha^2} \right) \left(\frac{b}{r} \right) \right] \quad (4-37) \\ & + \alpha (\alpha + \nu_{r\theta}) \left(\frac{r}{b} \right)^{\alpha-1} \frac{\left[1 + \left(\frac{b}{r} \right)^{2\alpha} \left(\frac{a}{b} \right)^{2\alpha} \right]}{\left[1 + \left(\frac{\alpha + \nu_{r\theta}}{\alpha - \nu_{r\theta}} \right) \left(\frac{a}{b} \right)^{2\alpha} \right]} \left\{ \frac{D}{D_1 b^3} - \frac{1}{9 - \alpha^2} + \right. \end{aligned}$$

$$\begin{aligned}
& + \frac{D_2}{D_1 b (4 - \alpha^2)} + \left(\frac{1}{\alpha - \nu_{r\theta}} \right) \left(\frac{a}{b} \right)^{\alpha+2} \left[\frac{D_2}{D_1 b} \left(\frac{2 + \nu_{r\theta}}{4 - \alpha^2} \right) - \left(\frac{a}{b} \right) \left(\frac{3 + \nu_{r\theta}}{9 - \alpha^2} \right) \right] \Bigg\} \\
& + \alpha \left(\frac{b}{r} \right)^{\alpha+1} \left(\frac{a}{b} \right)^{\alpha+2} \left[\left(\frac{a}{b} \right) \left(\frac{3 + \nu_{r\theta}}{9 - \alpha^2} \right) - \frac{D_2}{D_1 b} \left(\frac{2 + \nu_{r\theta}}{4 - \alpha^2} \right) \right]
\end{aligned}$$

The stress solutions in the different materials of the composite structure are evaluated in the copper and epoxy from the above evaluation by:

$$S_{\theta c} = \sigma_{\theta} E_c / E_{\theta} \quad (4-27)$$

$$S_{\theta e} = \sigma_{\theta} E_e / E_{\theta} \quad (4-28)$$

4.1.2.3 Radial Stress Solution with Specified Displacement

The radial stress equation (4-18) is re-written in the following non-dimensional form:

$$\begin{aligned}
\frac{\sigma_r}{D_1 C_r b^2} &= \left(\frac{r}{b} \right)^2 \left[\left(\frac{3 + \nu_{r\theta}}{9 - \alpha^2} \right) - \frac{D_2}{D_1 r} \left(\frac{2 + \nu_{r\theta}}{4 - \alpha^2} \right) \right] \\
&+ \frac{I_1}{D_1 b^2} \left[\frac{r^{\alpha-1}}{2\alpha} (\alpha + \nu_{r\theta}) \right] + \frac{I_2}{D_1 b^2} \left[r^{-\alpha-1} (\nu_{r\theta} - \alpha) \right]
\end{aligned} \quad (4-18, a)$$

Substitution of equations (4-34) and (4-35) into this equation with same re-arrangement provides the following relationship for computing the radial stress in the composite structure where turn-to-turn radial contact is maintained and where an arbitrary value of displacement may be specified at the solenoid outer

radial surface:

$$\begin{aligned}
 \frac{\sigma_r}{D_1 C_r b^2} = & \left(\frac{r}{b} \right)^2 \left\{ \left(\frac{3 + \nu_{r\theta}}{9 - \alpha^2} \right) \left[1 - \left(\frac{r}{b} \right)^{-\alpha - 3} \left(\frac{a}{b} \right)^{\alpha + 3} \right] \right. \\
 & \left. \left(\frac{b}{r} \right) \left(\frac{2 + \nu_{r\theta}}{4 - \alpha^2} \right) \left(\frac{D_2}{D_1 b} \right) \left[\left(\frac{r}{b} \right)^{-\alpha - 2} \left(\frac{a}{b} \right)^{\alpha + 2} - 1 \right] \right\} \\
 & + (\alpha + \nu_{r\theta}) \left(\frac{r}{b} \right)^{\alpha - 1} \frac{\left[1 - \left(\frac{r}{b} \right)^{-2\alpha} \left(\frac{a}{b} \right)^{2\alpha} \right]}{\left[1 + \left(\frac{\alpha + \nu_{r\theta}}{\alpha - \nu_{r\theta}} \right) \left(\frac{a}{b} \right)^{2\alpha} \right]} \left\{ \frac{D}{D_1 b^3} - \frac{1}{9 - \alpha^2} \right. \\
 & \left. + \frac{D_2}{D_1 b (4 - \alpha^2)} + \left(\frac{1}{\alpha - \nu_{r\theta}} \right) \left(\frac{a}{b} \right)^{\alpha + 2} \left[\frac{D_2}{D_1 b} \left(\frac{2 + \nu_{r\theta}}{4 - \alpha^2} \right) - \left(\frac{a}{b} \right) \left(\frac{3 + \nu_{r\theta}}{9 - \alpha^2} \right) \right] \right\}
 \end{aligned} \tag{4-38}$$

This equation provides the value of radial stress in the regions of the composite material where the insulation and the copper are acting in series such as at the radial interfaces where the stress is the same in the different materials. The radial stress in the copper and epoxy are different in the region of the composite where the insulation, the coolant hole and the copper are locally acting in parallel. These values are:

$$S_{rc} = \sigma_r \frac{E_c}{E_r} \tag{4-30}$$

$$S_{re} = \sigma_r \frac{E_e}{E_r} \tag{4-31}$$

In this region the stress in the copper can also experience a local concentration by about a factor of three on the inside surface of the coolant channel due to the presence of the hole.

4.2 RADIAL SEPARATION BETWEEN TURNS

The combination of equations (3-1), (3-6) and (4-13) provides the expressions for D_1 and D_2 in the region of the solenoid where there is radial separation between turns. This is the region where each turn is individually analyzed for displacement and stress.

$$- F/C_r = J (1 - CF) \left\{ \left[\frac{B_i - B_o}{R_o - R_i} \right] r - \left[B_i + R_i \left(\frac{B_i - B_o}{R_o - R_i} \right) \right] \right\} / C_r$$

$$D_1 = J (1 - CF) \left[\frac{B_i - B_o}{R_o - R_i} \right] / C_r \quad (4-39)$$

$$D_2 = J (1 - CF) \left[B_i + R_i \left(\frac{B_i - B_o}{R_o - R_i} \right) \right] / C_r \quad (4-40)$$

For geometric configurations where the magnetic field equation (3-2) is accurate, the constants D_1 and D_2 can be further simplified as follows eliminating the need to use figures (3-1) through (3-4) for the determination of B_i and B_o :

$$D_1 = \mu \left[J (1 - CF) \right]^2 / C_r \quad (4-39, a)$$

$$D_2 = R_o D_1 \quad (4-40, a)$$

Since each turn is analyzed with a radial separation from its neighbor, the integration constants I_1 and I_2 are evaluated for the boundary conditions of zero radial surface traction. The two equations with two unknowns are obtained from equation (4-18) by setting $\sigma_r = 0$ at $r = a$ and $\sigma_r = 0$ at $r = b$.

$$0 = D_1 C_r a^2 \left[\left(\frac{3 + \nu_{r\theta}}{9 - \alpha^2} \right) - \frac{D_2}{D_1 a} \left(\frac{2 + \nu_{r\theta}}{4 - \alpha^2} \right) \right] + I_1 C_r \left[\frac{a^{\alpha-1}}{2\alpha} (\alpha + \nu_{r\theta}) \right] + I_2 C_r \left[a^{-\alpha-1} (\nu_{r\theta} - \alpha) \right] \quad (4-41)$$

$$0 = D_1 C_r b^2 \left[\left(\frac{3 + \nu_{r\theta}}{9 - \alpha^2} \right) - \frac{D_2}{D_1 b} \left(\frac{2 + \nu_{r\theta}}{4 - \alpha^2} \right) \right] + I_1 C_r \left[\frac{b^{\alpha-1}}{2\alpha} (\alpha + \nu_{r\theta}) \right] + I_2 C_r \left[b^{-\alpha-1} (\nu_{r\theta} - \alpha) \right] \quad (4-42)$$

The simultaneous solution of these equations provides the expressions for I_1 and I_2 .

$$I_1 = \frac{D_1 b^2 \left\{ \left(\frac{3 + \nu_{r\theta}}{9 - \alpha^2} \right) \left[\left(\frac{a}{b} \right)^{\alpha+3} - 1 \right] + \left(\frac{2 + \nu_{r\theta}}{4 - \alpha^2} \right) \left(\frac{D_2}{D_1 b} \right) \left[1 - \left(\frac{a}{b} \right)^{\alpha+2} \right] \right\}}{\frac{b^{\alpha-1} (\alpha + \nu_{r\theta})}{2\alpha} \left[1 - \left(\frac{a}{b} \right)^{2\alpha} \right]} \quad (4-43)$$

$$I_2 = \frac{\left(\frac{a}{b} \right)^{2\alpha} D_1 b^2 \left\{ \left(\frac{3 + \nu_{r\theta}}{9 - \alpha^2} \right) \left[\left(\frac{a}{b} \right)^{3-\alpha} - 1 \right] + \left(\frac{2 + \nu_{r\theta}}{4 - \alpha^2} \right) \left(\frac{D_2}{D_1 b} \right) \left[1 - \left(\frac{a}{b} \right)^{2-\alpha} \right] \right\}}{b^{-\alpha-1} (\alpha - \nu_{r\theta}) \left[1 - \left(\frac{a}{b} \right)^{2\alpha} \right]} \quad (4-44)$$

These expressions could have been obtained directly from equations (4-23) and (4-24) by setting $P = 0$.

4.2.1 Individual Turn Displacement Solution

The displacement solution for the individual turns can be obtained directly from equation (4-25) by setting $P = 0$.

$$\begin{aligned} \frac{u}{D_1 b^3} = & \left(\frac{1}{9 - \alpha^2} \right) \left(\frac{r}{b} \right)^3 - \frac{D_2}{D_1 b} \left(\frac{r}{b} \right)^2 \left(\frac{1}{4 - \alpha^2} \right) \\ & + \left(\frac{3 + \nu_{r\theta}}{9 - \alpha^2} \right) \left(\frac{1}{\alpha + \nu_{r\theta}} \right) \left\{ \left(\frac{r}{b} \right)^\alpha \left[\left(\frac{a}{b} \right)^{\alpha+3} - 1 \right] \right. \\ & + \left. \left(\frac{\alpha + \nu_{r\theta}}{\alpha - \nu_{r\theta}} \right) \left(\frac{b}{r} \right)^\alpha \left[\left(\frac{a}{b} \right)^{\alpha+3} - \left(\frac{a}{b} \right)^{2\alpha} \right] \right\} / \left[1 - \left(\frac{a}{b} \right)^{2\alpha} \right] \\ & - \left(\frac{2 + \nu_{r\theta}}{4 - \alpha^2} \right) \left(\frac{1}{\alpha + \nu_{r\theta}} \right) \left(\frac{D_2}{D_1 b} \right) \left\{ \left(\frac{r}{b} \right)^\alpha \left[\left(\frac{a}{b} \right)^{\alpha+2} - 1 \right] \right. \\ & + \left. \left(\frac{\alpha + \nu_{r\theta}}{\alpha - \nu_{r\theta}} \right) \left(\frac{b}{r} \right)^\alpha \left[\left(\frac{a}{b} \right)^{\alpha+2} - \left(\frac{a}{b} \right)^{2\alpha} \right] \right\} / \left[1 - \left(\frac{a}{b} \right)^{2\alpha} \right] \end{aligned} \quad (4-45)$$

4.2.2 Individual Turn Tangential Stress Solution

The tangential stress solution for the individual turns can be obtained directly from equation (4-26) by setting $P = 0$.

$$\begin{aligned} \frac{\sigma_\theta}{D_1 C_r b^2} = & \left(\frac{r}{b} \right)^2 \left(\frac{3 \nu_{r\theta} + \alpha^2}{9 - \alpha^2} \right) - \left(\frac{D_2}{D_1 b} \right) \left(\frac{r}{b} \right) \left(\frac{2 \nu_{r\theta} + \alpha^2}{4 - \alpha^2} \right) \\ & + \alpha \left(\frac{3 + \nu_{r\theta}}{9 - \alpha^2} \right) \left\{ \left(\frac{r}{b} \right)^{\alpha-1} \left[\left(\frac{a}{b} \right)^{\alpha+3} - 1 \right] \right. \end{aligned} \quad (4-46)$$

$$\begin{aligned}
& + \left(\frac{b}{r}\right)^{\alpha+1} \left[\left(\frac{a}{b}\right)^{\alpha+3} - \left(\frac{a}{b}\right)^{2\alpha} \right] \Bigg/ \left[1 - \left(\frac{a}{b}\right)^{2\alpha} \right] \\
& - \alpha \left(\frac{2 + \nu_{r\theta}}{9 - \alpha^2} \right) \left(\frac{D_2}{D_1 b} \right) \left\{ \left(\frac{r}{b}\right)^{\alpha-1} \left[\left(\frac{a}{b}\right)^{\alpha+2} - 1 \right] \right. \\
& \left. + \left(\frac{b}{r}\right)^{\alpha+1} \left[\left(\frac{a}{b}\right)^{\alpha+2} - \left(\frac{a}{b}\right)^{2\alpha} \right] \right\} \Bigg/ \left[1 - \left(\frac{a}{b}\right)^{2\alpha} \right]
\end{aligned}$$

The above equation provides the average stress in the composite structure. The stress in the copper and epoxy are evaluated with equations (4-27) and (4-28).

4.2.3 Individual Turn Radial Stress Solution

The radial stress solution for the composite turn can be obtained directly from equation (4-29) by setting $P = 0$.

$$\begin{aligned}
\frac{\sigma_r}{D_1 C_r b^2} &= \left(\frac{3 + \nu_{r\theta}}{9 - \alpha^2} \right) \left(\frac{r}{b} \right)^2 - \left(\frac{2 + \nu_{r\theta}}{9 - \alpha^2} \right) \left(\frac{D_2}{D_1 b} \right) \left(\frac{r}{b} \right) \quad (4-47) \\
&+ \left(\frac{3 + \nu_{r\theta}}{9 - \alpha^2} \right) \left\{ \left(\frac{r}{b} \right)^{\alpha-1} \left[\left(\frac{a}{b}\right)^{\alpha+3} - 1 \right] \right. \\
&+ \left(\frac{b}{r}\right)^{\alpha+1} \left[\left(\frac{a}{b}\right)^{2\alpha} - \left(\frac{a}{b}\right)^{\alpha+3} \right] \Bigg/ \left[1 - \left(\frac{a}{b}\right)^{2\alpha} \right] \\
&+ \left(\frac{2 + \nu_{r\theta}}{4 - \alpha^2} \right) \left(\frac{D_2}{D_1 b} \right) \left\{ \left(\frac{r}{b} \right)^{\alpha-1} \left[1 - \left(\frac{a}{b}\right)^{\alpha+2} \right] \right. \\
&\left. + \left(\frac{b}{r}\right)^{\alpha+1} \left[\left(\frac{a}{b}\right)^{\alpha+2} - \left(\frac{a}{b}\right)^{2\alpha} \right] \right\} \Bigg/ \left[1 - \left(\frac{a}{b}\right)^{2\alpha} \right]
\end{aligned}$$

5.0 NUMERICAL ANALYSIS

Numerical solutions to the equations of elasticity are provided for two solenoid design classifications. The first is for the design which is considered tall relative to the inner radius such that the field equation (3-2), the flux equation (3-10) and the relationships between flux and geometry provided by Figure 3-5 are all considered to be accurate. The second problem class for which numerical solutions are provided include geometries different from the above idealization whereby the magnetic field is approximated by the linear function provided by equation (3-1) as supplemented with equations (3-3a) and (3-4a) thereby making use of the detailed magnetic field calculations performed at ORNL in the reference (2) report.

The distinction between the two problem classifications defined above is that the former can be displayed numerically in non-dimensional form such that solutions can be achieved without selecting a specific geometry. The latter class of problems is solved only after a height, an inner radius and an outer radius of the solenoid are specified. The latter solution method is the more accurate. The former idealized solution for "tall" solenoids tends to predict stresses greater than the actual, which is conservative; but at the same time it predicts a flux swing greater than the actual which is non-conservative.

5.1 ANALYSIS RESULTS AND COMPARISONS FOR "TALL" SOLENOIDS WITH RADIAL CONTACTING TURNS

The general problem for which analysis results are presented here includes only "tall" solenoids and solenoids where radial contact is maintained between turns either by compression or by bonding where there is a radial tensile stress. The individual turns are not treated here, but they will be analyzed in the next section of this report for the condition where turn-to-turn radial contact is not maintained. For the problem being treated here the inner radius is $a = R_i$ and the outer radius is $b = R_o$. The entire solenoid is treated as a homogeneous, anisotropic structure with the equivalent material properties defined by equations (2-4) and (2-5). The magnetic field is defined by equation (3-2), and the flux swing is given by equation (3-10). The solutions are for the equations of elasticity found in Section 4.1 with the constants D_1 and D_2 defined by

equations (4-19, a) and (4-20, a). With all of the above stated assumptions, the non-dimensional form of solutions that are provided by Figures 5-1 through 5-8 are readily obtained. Since these solution results are restricted to the "tall" solenoid, their value lies in showing the locations in a solenoid where radial delamination can be expected or by showing the bond strength values that would be required to preclude delamination. If there is delamination in a solenoid the solutions presented here are no longer correct, and it then becomes necessary to refer to the solutions and methods provided in the next section of this report. The additional value of the solutions present here is to show the relative effects that can result from an externally applied pressure or displacement at the outer radius of the solenoid such as may be applied by a shrink fit or a structural containment cylinder. Other comparisons provided here show the sensitivity of solutions to the coolant channel fraction, the insulation fraction and the assumed value of Poisson's ratio. All of the numerical solutions were carried out with the assumption of a copper conductor with a Young's modulus of 117×10^9 Pa (17×10^6 psi) and with an epoxy insulation with a Young's modulus of 3.78×10^9 Pa ($.55 \times 10^6$ psi). A listing of the computer program that was used to generate the numerical solutions is provided in Appendix A of this report.

Figure 5-1 shows a non-dimensional plot of displacement versus the radial location along the coil for the condition of no constraint at the coil outer radius. It should be noted that the average flux density in the non-dimensional displacement function is normalized with respect to 7.0 Tesla. The same has been done to the non-dimensional stress functions plotted on figures (5-2) through (5-8).

Figure (5-2) shows the non-dimensional radial stress distribution in the composite structure for two different external boundary conditions, namely, no external pressure and a pressure of -30 M Pa at the outer radial surface. A tensile radial stress can exist in the composite solenoid only if there is turn to turn bonding. Otherwise there will be turn to turn separation for which case the solutions presented in this section of the report would no longer be valid. It shows for the zero pressure condition that without bonding radial separation begins to occur when the ratio of the inner to outer radial dimension becomes smaller than about 0.53, and

Figure 5-1. Tall Solenoid Displacement Distribution with No Outer Surface Constraint for Different (a/b) Ratios

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Figure 5-2. Tall Solenoid Radial Stress Distribution with Different Outer Surface Tractions (P) for Different (a/b) Ratios

Figure 5-3. Tall Solenoid Copper Hoop Stress Distributions

Figure 5-4. Tall Solenoid Tangential Stress at the Inner Radial Surface for Different Surface Traction (P) versus the (a/b) Ratio

Figure 5-5. Tall Solenoid Tangential Stress at the Inner Radial Surface for Different Outer Radial Surface Displacements (D) versus the (a/b) Ratio

Westinghouse

Figure 5-6. Sensitivity of Tall Solenoid Maximum Tangential Stress to the Stacking and Cooling Fractions for Different Surface Traction (P) versus the (a/b) Ratio

Figure 5-7. Sensitivity of Tall Solenoid Maximum Tangential Stress to the Poisson Ratio Value for Different Surface Traction (P) versus the (a/b) Ratio

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Figure 5-8. Sensitivity of Tall Solenoid Maximum Tangential Stress to the Poisson Ratio Value for Different Outer Radial Surface Displacements (D) versus the (a/b) Ratio

with an externally applied pressure of $-30. \text{ M Pa}$ the turns would begin separating at an (a/b) ratio of approximately 0.35.

Figure 5-3 shows how the tangential stress is a maximum at the solenoid inner radial surface, and it demonstrates the advantage of stress reduction to be gained by the application of an external constraint.

Figure 5-4 plots the maximum tangential stress which occurs at the solenoid inner radial surface for different values of externally applied pressure. Figure 5-5 also plots the maximum tangential stress. Here, strain control at the outer surface is the variable. Some cross-plots of interest are the zero strain condition shown on Figure 5-4 and the zero radial stress condition shown on Figure 5-5.

The sensitivity of maximum tangential stress relative to the coolant fraction and the stacking fraction is provided by Figure 5-6. The trend for solenoids of most probable (a/b) ratios is increasing stress with increased packing fraction. At small (a/b) ratios this trend appears to reverse, but this is not the likely case because without bonding between turns to withstand the large radial stress depicted by figure 5-2 at the small (a/b) ratios these solutions are no longer valid and correct solutions would be obtained from the methods presented in the next section of this report where each turn is treated on an individual basis. With radial separation between turns it is shown that each turn loads itself, and its stresses are therefore not changed by the loads in neighboring turns.

The sensitivity of maximum hoop stress to the assumed value for the Poisson ratio is shown on figures 5-7 and 5-8 for the two different type of externally applied boundary conditions. These figures would indicate that a conservative estimate would be a large value for the Poisson ratio in the absence of measured data for the anisotropic structure.

The set of figures displayed in this section of the report can be considered valid solutions only if: a) the solenoid is very tall in comparison to its radial dimension and b) there is

no radial separation from turn to turn. Therefore these should not be considered design curves, and they should only be used as a guide to gain an understanding of consequences, trends and solution sensitivity.

5.2 ANALYSIS RESULTS AND COMPARISONS FOR SOLENOIDS WITH RADIAL SEPARATING TURNS

The general problem for which analysis results are presented here includes solenoids which are not "tall" and where radial separation between turns is not prevented with bonding. The entire solenoid is treated as consisting of two regions. An outer region and an inner region. The outer region is the region of radial contact between turns as defined by the existence of a compressive radial stress, and this region is analyzed the same way as it was in the previous section with the exception of the magnetic field equations. The field equations used here are (3-1), (3-3a) and (3-4a) instead of the idealized (for "tall" solenoids) equations (3-2), (3-3) and (3-4). These field equations are dependent upon geometry in such a way that the solutions in this section of the report cannot be presented in non-dimensional form. Solutions are dependent upon the selection of a solenoid height, inner radius, outer radius and a current density. The flux half-swing is computed with equation (3-9) instead of with the idealized (for tall solenoids) equation (3-10). The inner region of the solenoid is the region where there is turn to turn separation. In this region the turn thickness must be defined, and every turn is then analyzed with the equations of elasticity found in section 4.2. Each turn is treated as a homogenous anisotropic structure with the equivalent material properties defined by equations (2-6) and (2-7). In this work all of the numerical solutions were carried out with the assumptions of a copper conductor with a Young's modulus of 117×10^9 Pa. (17×10^6 psi) and with an epoxy insulation with a Young's modulus of 3.78×10^9 Pa ($.55 \times 10^6$ psi). A listing of the computer program that was used to generate the numerical solutions is provided in Appendix B of this report. This program is used to calculate the stresses and deflections in both of the previously described solenoid regions, and separate runs are required for each region. For both runs the true inner and outer radii are specified and B_i^* and B_o^* values are specified consistent with the solenoid height to inner radius ratio. With this information the magnetic field boundaries can be computed for any turn in the solenoid from equation (3-1). The outer

region of the solenoid is specified as a single turn, and it is analyzed as a homogeneous isotropic structure. Some trial and error is usually required to achieve the correct guess for the inner radius of this outer region. The correct inner radius is the smallest possible value which still results in a compressive radial stress distribution. If the solution shows some tensile radial stress near the inner radial location of this outer region, then the selected inner radius is too small. A fairly accurate initial guess can be achieved by using Figure 5-2 as an aid for the (a/b) ratio which satisfies the compressive radial stress criterion. The computer run for the separate turns of the inner region of the coil utilizes a solution where every turn of the solenoid is evaluated with a single run. The solutions for the individual turns in the outer region of the solenoid are disregarded as incorrect as evidenced by their displacement values. In the inner region of the solenoid the displacements of the individual turns show separation between turns. These solutions are retained for every turn up to the turn which displaces the same amount as the inner surface of the outer region which was previously analyzed as one very thick turn with the radial compression. Solutions for turns beyond that radial location are disregarded as incorrect because they don't account for the compacting into one another which actually occurs in this outer region.

Some sample solutions were generated with the Appendix B computer program in order to:

a) compare solutions with different turn thicknesses, b) with different height to inner radius ratios, c) with different external boundary conditions and d) with different outer radial dimensions.

Figures 5-9 and 5-10 show the tangential stress and the radial displacement solutions for a solenoid which has been assumed to be very tall in comparison to its inner radius. In the region of separation between turns it compares turns that are either 2 or 4 cm thick. These curves were plotted from the output in Appendix B from data sets one and two. The output for the 4 cm thick turns is not in Appendix B. These solutions for the "tall" solenoid assumption are provided to show that the stress solution is conservative, but the flux swing solution is non-conservative when compared to a solution which uses the field calculations which account for the height to inner radius ratio. Figures 5-11 and 5-12 show the

5-14

Figure 5-9. Tall Solenoid Tangential Stress with Indicated Flux Half Swing and with no Tensile Bond Strength Between Turns

Figure 5-10. Tall Solenoid Deflection with No Bond Between Turns

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Figure 5-11. Tangential Stress for Solenoid with $h = 6 R_i$, with Indicated Flux Half Swings and with no Tensile Bond Strength Between Turns

Figure 5-12. Radial Deflection for Solenoid with $h = 6 R_i$ and with no Tensile Bond Strength Between Turns

tangential stress and displacement solutions for the assumption that the solenoid ($h/R_i = 6$) height is six times the inner radius. With $(R_o/R_i) = 5$, $B_o^* = - .33 \times 10^{-6}$ is found on Figure 3-4, and $B_i^* = 4.5 \times 10^{-6}$ is found on Figure 3-3. The first comparison to be made between Figure 5-9 and Figure 5-11 is the maximum tangential stress. With the tall solenoid its predicted value is about 234. M Pa, and with the solenoid where $h = 6 R_i$ the predicted value is about 196. M Pa with no external pressure. The second comparison to be made between Figures 5-9 and 5-11 pertains to the value of the flux half swing. With the tall solenoid the flux is (109 Wb from the compacted region with a gain of an additional 18 Wb from the separating region) 127 webers. The flux half swing is only 102 webers from the solenoid where $h = 6 R_i$. Figures 5-13 and 5-14 compare the tangential stress and the radial deflections for tall solenoids with no external constraints where the outer dimension is either 1.0, 1.5 or 2.0 meters. Also superimposed on Figure 5-13 is the flux half swings achieved by each solenoid which are 127, 53.5 and 15.9 Wb for the three different outer dimensions. This same comparison for the solenoid with $h = 6 R_i$ is made on Figures 5-15 and 5-16. Here the flux half swings achieved by the three different solenoids are 102, 42.9 and 12.7 Wb for the three different outer dimensions.

Figure 5-13. Tall Solenoid Tangential Stresses with Indicated Flux Half Swings and with no Tensile Bond Strength Between Turns

Figure 5-14. Tall Solenoid Radial Deflections with no Tensile Bond Strength Between Turns

Figure 5-15. Tangential Stress for Solenoids with $h = 6 R_i$, with Indicated Flux Half Swings and with no Tensile Bond Strength Between Turns

Figure 5-16. Radial Deflection for Solenoids with $h = 6 R_1$ and with no Tensile Bond Strength Between Turns

6.0 CONCLUSIONS/RECOMMENDATIONS

The major conclusions that can be made from the results of this study relate to the desirability and consequences of a) bonding between turns, b) providing external constraint with a containment or shrink fit structural cylinder and c) the axial height to inner radius ratio where performance is compared to the fabrication and operating cost. Other conclusions relate to the effects of the overall packing factor and certain material property values.

Bonding between turns can be provided on both the radial and axial planes of the turns. Bonding on the axial planes of the turns is of little consequence, because there will always exist some axial compression of the solenoid induced within itself by the horizontal component of the field which tends to increase toward the axial ends of the solenoid at increasing distances from the solenoid horizontal midplane. This component of load has not been included in this work. It is a separate effect which can be superimposed upon the stresses evaluated in this study. The net effect of axial compression would be to add to the hoop and the radial stress values some additional tension. This effect should be small in comparison to the stress values computed from the vertical component of the magnetic field, and the actual value can only be computed after a detailed field evaluation is made. A detailed field evaluation method is not presented as a part of this report. It is however recommended that it should eventually be incorporated as part of the Appendix B computer code to calculate the actual field values at the inner and outer surfaces of the solenoid. The computer program subroutine FIFIL documented in reference (3) could be used for this purpose. This subroutine will provide a detailed field evaluation for an arbitrary inner radius to height ratio solenoid. Returning to the topic of bonding, it appears that bonding on the radial surfaces of turns to prevent turn to turn separation can only result in larger tangential stresses for solenoids with practical inner to outer radius ratios. Solenoids with an inner to outer radius ratio of value greater than about 0.53 will tend to radially compact the turns so that bonding between turns is of no consequence. The value of this ratio can be smaller than 0.53 in the presence of an external constraint as indicated by Figure 5-2, and bonding between turns is still of no consequence for (R_i/R_o) ratios greater than this smaller value. The other approach that one could apply to these totally compacted solenoids where

bonding is of no consequence would be the opposite to bonding. This is fabrication with an initial radial gap between turns which would permit each turn to freely expand without compacting into its neighbor. With this fabrication method, the turn at the inner surface would have the same tangential stress at that location as would exist on the compacted solenoids. Moving radially outward the turns would tend to have a tangential stress greater than the compacted solenoid. Then there would be a reversal where turns near the outer radial location would have a tangential stress less than the compacted solenoid. The net effect is no real stress reduction at the inner most turn where the tangential stress is a maximum.

When there is no external constraint on the solenoid, the turns on a solenoid of (R_i/R_o) ratio of less than about 0.53 will tend to radially separate without bonding between turns. The example solutions for tangential stress and radial displacement provided by Figures 5-13 through 5-16 were all for solenoids with an $(R_i/R_o) = 0.20$ ratio. If the turns on these solenoids are bonded with sufficient strength to withstand the radial tensile stress predicted by Figure 5-2, then the tangential stress with bonding would be about the worst possible value as indicated by the $(a/b) = 0.20$ ratio curve on Figure 5-3. On Figures 5-13 and 5-15 the tangential stress would follow the trend of the dotted curve shown for $R_o = 200$ cm if there were bonding between turns of sufficient strength to support the radial tension. The conclusion to be made here with respect to bonding between turns is that there is no advantage to be gained from bonding to reduce stresses. In fact, it is shown that stress values can be significantly lower in solenoids where there is separation between turns with the absence of bonding.

The second major conclusion to be drawn from this study pertains to the application of external constraints. A solenoid with an a/b ratio of about 0.35 or greater will radially compact with an external constraint equivalent to about -30 MPa at the outer surface, and a solenoid with an (a/b) ratio greater than about 0.53 will radially compact without an external constraint. Under this range of conditions the solutions for a tall solenoid presented on Figure 5-4 are valid without bonding between turns. Following this line of reasoning,

Figure 5-4 shows that for solenoids with an (a/b) ratio between about 0.30 and 0.80 that a condition of no radial deflection at the outer surface of the solenoid is achieved with an external pressure somewhere between -25 and -30 MPa. This represents the ideal upper limit for a containment cylinder without a pretension shrinkfit, because the radial deflection must approach zero at the inner radial surface of the containment cylinder to achieve the zero hoop isostrain condition shown on Figure 5-4. The major conclusion therefore which can be made on the basis of this study is that in order to achieve by external constraint better stress reductions than are shown on Figure 5-4 for the maximum tangential stress that the outer surface condition on the solenoid will necessarily have to be applied by a preload from a shrinkfit or possibly from the centering forces of the toroidal field coils when the solenoid is part of a tokamak. It is not meant to be implied here that all ohmic heating solenoids in a tokamak should or will require an external support. Many combined factors such as the required flux swing, the outer radius of the solenoid, the current density in the turns, the fatigue strength of the conductor and insulator and the need for conductor connectors will influence the need for solenoid containment support. Figures 5-13 and 5-15 can be used to establish a likely range of tangential stress values that can be expected for the condition of no external constraint. These figures show what can be expected for the flux half-swing as a function of increasing outer radius. With fixed geometry the flux swing can be increased directly with the current density J , but stress will increase proportionately with the current density squared. Combining the flux swing requirements with geometric constraints and with the allowable fatigue strength of the conductor for the solenoid duty cycles one can then use Figures 5-13 and 5-15 or a newly generated one for a different (h/R_i) ratio to establish the need for solenoid external constraint.

The third major conclusion of this study pertains to the ohmic heating solenoid performance. Since the primary purpose of the ohmic heating solenoid is to produce enough flux swing to initiate a current in the plasma, flux swing is the parameter of interest. An immediate observation that can be made from this study from the flux half swing numbers provided on Figures 5-13 and 5-15 is that for a realistic geometry $(h/R_i = 6)$ the flux half swing is only 80% of that which would be predicted by the idealized "tall" solenoid assumption

provided by equation (3-10) when $(R_o/R_i) = 5.0$. As (R_o/R_i) decreases, then smaller values of (h/R_i) approximate the "tall" solenoid assumption. For example the flux from a solenoid with $(R_o/R_i) = 2.0$ and $(h/R_i) = 6$ would very closely approximate the flux computed by equation (3-10) for the "tall" solenoid approximation. Another observation from both Figures 5-13 and 5-15 is that the flux contribution by all of the turns with radial separation is only about 12 and 14% of the total for the two respective (h/R_i) conditions. All of the above suggests that there is an optimum (h/R_i) and (R_o/R_i) that will maximize the flux half swing with respect to minimizing the amount of copper and the amount of power supply which is needed. The final recommendation that follows from this work is that a study be performed to evaluate the maximum achievable flux with respect to required power supply. The basis for this study will result in comparisons between the short solenoid with a relatively thick radial build as compared to the tall solenoid with a relatively thin radial build. If the optimum combination is found to result in an (R_o/R_i) which is greater than about 2.0 there would be no impact on the solenoid stress reduction. But if the optimum combination is found to result in an (R_o/R_i) which is less than 2.0, the consequence with respect to stress reductions in the solenoid can be favorable. In any event the result of this optimization study would be beneficial from the flux versus required power supply viewpoint.

Other conclusions which relate the conductor packing fraction and certain material property values to stress levels are self-explanatory on Figures 5-6, 5-7 and 5-8.

7.0 REFERENCES

- (1) N. E. Johnson, W. H. Gray and R. A. Weed, "Stress Analysis of Non-Homogeneous, Transversely Isotropic Superconducting Solenoids," Proceedings of the Sixth Symposium on Engineering Problems of Fusion Research, November, 1975, pp. 243-247.
- (2) W. H. Gray and J. K. Ballou, "Electro-Mechanical Stress Analysis of Transversely Isotropic Solenoids," ORNL/TM-5528, March, 1977.
- (3) Robert A. Smith, "Field Load and Displacement Boundary Condition Computer Program Used for the Finite Element Analysis and Design of Toroidal Field Coils in a Tokamak," WFPS TME-002, June, 1975.

APPENDIX A

COMPUTER PROGRAM FOR "TALL" HOMOGENEOUS SOLENOID APPROXIMATION

C *** STRESS AND DEFLECTIONS FOR THE OH SOLENOID ***
 C FOR TALL, ANISOTROPIC, HOMOGENEOUS SOLENOID APPROXIMATION-R.A.SMITH
 C NON-DIMENSIONAL SOLUTIONS NORMALIZED TO FLUX DENSITY HALF-SWING
 C DIMENSION SZ(10), CZ(10), AZ(11), PZ(10), UZ(10)
 C SF=STACKING FACTOR, RATIO OF CONDUCTOR OUTER DIMENSIONAL AREA TO THAT
 C AREA PLUS THE INSULATION AREA=SZ
 C CF=COOLANT FRACTION, RATIO OF COOLANT HOLE AREA TO THE CONDUCTOR OUTER
 C DIMENSIONAL AREA=CZ
 C AB=RATIO OF OUTER TO INNER RADIUS OF SOLENOID =AZ
 C DO NOT INPUT AN AB VALUE OF 0.0 OR 1.0
 C READ(5,1)L,M,N,(SZ(I),I=1,L),(CZ(J),J=1,M),(AZ(K),K=1,N)
 C 1 FORMAT(3I3,1F4.2,1F4.2,11F4.2)
 C EI=MODULUS OF INSULATING MATERIAL,(PA)
 C EC=MODULUS OF CONDUCTOR MATERIAL,(PA)
 C VR=POISSON RATIO WITH TENSION IN THE R-DIRECTION
 C VT=POISSON RATIO WITH TENSION IN THE THETA DIRECTION
 C BOH=THE HALF-SWING FLUX DENSITY,(TESLA)
 C READ(5,2)EI,EC,VR,BOH
 C 2 FORMAT(2E12.5,F6.3,F6.2)
 C PI=3.1415926
 C U=PI*4.0E-7
 C READ(5,3)LL,DEL,(PZ(I),I=1,LL)
 C THE THREE (3) FORMAT CARD NEEDS LLE12.5
 C 3 FORMAT(I6,F6.3,1E12.5)
 C DEL=THE DESIRED INCREMENT ON THE X PARAMETER BETWEEN AB AND 1.0
 C PZ(I)=THE EXTERNAL PRESSURE AT R=B,(PA)
 C READ(5,4)KPRES,KDISP,MM,(UZ(J),J=1,MM)
 C THE FOUR (4) FORMAT CARD NEEDS MME12.5
 C 4 FORMAT(3I3,6F10.6)
 C KPRES AND KDISP ARE KEYS FOR PRESSURE OR DISPLACEMENT SOLUTION AT R=B
 C IF KPRES=1,P AT R=B IS DESIRED SOLUTION, OTHERWISE LET KPRES=0
 C IF KDISP=1,U AT R=B IS DESIRED SOLUTION, OTHERWISE LET KDISP=0
 C UZ(J)=THE HOOP STRAIN AT R=B,(NON-DIM.DISP.AT R=B)
 C DO400 I=1,L
 C DO300 J=1,M
 C SF=SZ(I)
 C CF=CZ(J)
 C ET=EI*(1.-SF)+EC*SF*(1.-CF)
 C SSF=SQRT(SF)
 C SCF=SQRT(.25*PI*CF)

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      ER=E1*(1.-SSF)+EC*SSF/((EC/E1)*(1.-SSF)+SSF+SSF*CF/(.25*PI-SCF))
C ET=THE HOMOGENEOUS HOOP MODULUS
C ER=THE HOMOGENEOUS RADIAL MODULUS
      A=SQRT(ET/ER)
      VT=VR*ER/ET
      C1=(3.+VR)/(9.-A*A)
      C3=(2.+VR)/(4.-A*A)
      C7=(3.*VR+A*A)/(9.-A*A)
      C8=(2.*VR+A*A)/(4.-A*A)
      C9=1./(9.-A*A)
      C10=1./(4.-A*A)
      C11=A-VR
      C12=A+VR
      C13=C12/C11
      IF(KPRES.EQ.1)LLL=LL
      IF(KDISP.EQ.1)LLL=MM
      DO200 K=1,LLL
      IF(KPRES.EQ.1)DZ=PZ(K)
      IF(KDISP.EQ.1)DZ=UZ(K)
      WRITE(7,25)
25  FORMAT(1H1)
      WRITE(7,10)E1,EC,VR,VT,ER,ET,A,DZ,SF,CF
10  FORMAT(1H ,2E12.5,2F6.3,2E12.5,F6.3,F19.6,2F6.3)
      DO100 KK=1,N
      WRITE(7,11)KK
11  FORMAT(1H ,I4)
      AB=AZ(KK)
      BJSF=3.*BOH/(U*(1.-AB)*(1.+AB+AB*AB))
      C2=AB** (A+3.)
      C4=AB** (A+2.)
      C5=AB** (2.*A)
      C6=1.-C5
      IF(KPRES.EQ.1)P=DZ/(U*BJSF*BJSF)
      IF(KPRES.EQ.1)CI=P-C1*(1.-C2)+C3*(1.-C4)
      IF(KDISP.EQ.1)Q=ER*DZ/(U*BJSF*BJSF*(1.-VR*VT))
      IF(KDISP.EQ.1)CJ=Q-C9+C10-C1*C2/C11+C3*C4/C11
      X=AB
      GOT016

```

15 IF(DEL.NE.0.0)X=X+DEL
IF(DEL.EQ.0.0)X=1.0

16 D=1./X
D1=X**(A-1.)
D2=D**A
D3=D**(A+1.)
D4=D2*D2
D5=D*D3
D6=D*D5
D7=X*D1

C *** START OF NON-DIMENSIONAL RADIAL STRESS,SRND ***
IF(KPRES,EQ.1)SRND=X*X*(C1*(1.-C2*D6)-C3*D*(1.-C4*D5))+
1 D1*(1./C6)*(1.-D4*C5)*C1
IF(KDISP,EQ.1)SRND=X*X*(C1*(1.-C2*D6)-C3*D*(1.-C4*D5))+
2 D1*C12*CJ*(1.-D4*C5)/(1.+C13*C5)
SR=SRND*U*BJSF*BJSF

C *** END OF RADIAL STRESS ***

C *** START OF NON DIMENSIONAL HOOP STRESS,STND ***
IF(KPRES,EQ.1)STND=X*X*(C7-D*C8)+A*D3*C2*(C1-C3/AB)+
3 A*D1*(1./C6)*(1.+D4*C5)*C1
IF(KDISP,EQ.1)STND=X*X*(C7-D*C8)+A*D3*C2*(C1-C3/AB)+
4 A*C12*D1*CJ*(1.+D4*C5)/(1.+C13*C5)
ST=STND*U*BJSF*BJSF
STC=ST*EC/ET

C *** END OF HOOP STRESS ***

C *** START OF NON DIMENSIONAL RADIAL DISPLACEMENT,UND ***
IF(KPRES,EQ.1)UND=X**3*(C9-D*C10)+D2*C2*(C1-C3/AB)/C11+
5 D7*(1.+C13*D4*C5)*C1/(C12*C6)
IF(KDISP,EQ.1)UND=X**3*(C9-D*C10)+D2*C2*(C1-C3/AB)/C11+
6 D7*CJ*(1.+C13*D4*C5)/(1.+C13*C5)
UB=UND*U*BJSF*BJSF*(1.-VR*VT)/ER

C *** END OF RADIAL DISPLACEMENT ***
WRITE(7,20)AB,X,SRND,STND,UND,SR,ST,STC,UB

20 FORMAT(1H ,2F6.3,3E12.5,5X,4E12.5)
IF(X.LT.0.999)GOTO15

100 CONTINUE
200 CONTINUE
300 CONTINUE
400 CONTINUE
STOP
END

C SAMPLE INPUT DATA FOR P=0, WHERE THE A/B RATIO IS VARIED FROM .05 TO .95

1 1 11 .90 .10 .05 .10 .20 .30 .40 .50 .60 .70 .80 .90 .95

0.00378E+12 0.11700E+12 .300 7.00

1 .050 0.0E+6

1 0 6 0.000000 0.000200 0.000400 0.000600 0.000800 0.001000

PROGRAM OUTPUT WITH ABOVE INPUT DATA

.37800E 10	.11700E 12	.300	.129	.40929E 11	.95148E 11	1.525	.000000	.900	.100
1									
.050	.050	.63898E-11	.36749E 00	.82223E-02	.22430E-02	.12900E 09	.15863E 09	.67789E-04	
.050	.100	.11014E 00	.26093E 00	.10197E-01	.38661E 08	.91592E 08	.11263E 09	.84073E-04	
.050	.150	.12271E 00	.25502E 00	.14647E-01	.43073E 08	.89519E 08	.11008E 09	.12075E-03	
.050	.200	.11933E 00	.25120E 00	.19277E-01	.41887E 08	.88177E 08	.10843E 09	.15893E-03	
.050	.250	.11021E 00	.24401E 00	.23599E-01	.38685E 08	.85654E 08	.10533E 09	.19456E-03	
.050	.300	.98501E-01	.23387E 00	.27429E-01	.34576E 08	.82094E 08	.10095E 09	.22614E-03	
.050	.350	.85678E-01	.22167E 00	.30692E-01	.30075E 08	.77811E 08	.95681E 08	.25304E-03	
.050	.400	.72573E-01	.20817E 00	.33364E-01	.25475E 08	.73073E 08	.89855E 08	.27507E-03	
.050	.450	.59725E-01	.19398E 00	.35453E-01	.20965E 08	.68091E 08	.83729E 08	.29229E-03	
.050	.500	.47513E-01	.17956E 00	.36985E-01	.16678E 08	.63028E 08	.77504E 08	.30492E-03	
.050	.550	.36214E-01	.16527E 00	.38001E-01	.12712E 08	.58012E 08	.71336E 08	.31329E-03	
.050	.600	.26039E-01	.15140E 00	.38552E-01	.91404E 07	.53145E 08	.65351E 08	.31784E-03	
.050	.650	.17157E-01	.13819E 00	.38698E-01	.60224E 07	.48509E 08	.59649E 08	.31904E-03	
.050	.700	.97004E-02	.12583E 00	.38504E-01	.34051E 07	.44171E 08	.54315E 08	.31745E-03	
.050	.750	.37806E-02	.11449E 00	.38042E-01	.13271E 07	.40187E 08	.49417E 08	.31364E-03	
.050	.800	-.51089E-03	.10428E 00	.37386E-01	-.17934E 06	.36606E 08	.45013E 08	.30823E-03	
.050	.850	-.30965E-02	.95337E-01	.36616E-01	-.10869E 07	.33466E 08	.41152E 08	.30188E-03	
.050	.900	-.39100E-02	.87750E-01	.35812E-01	-.13725E 07	.30802E 08	.37876E 08	.29525E-03	
.050	.950	-.28944E-02	.81604E-01	.35059E-01	-.10160E 07	.28645E 08	.35224E 08	.28905E-03	
.050	1.000	.72760E-11	.76975E-01	.34445E-01	.25540E-02	.27020E 08	.33225E 08	.28398E-03	

2

.100	.100	-.67457E-11	.42915E 00	.19204E-01	-.23720E-02	.15091E 09	.18557E 09	.15860E-03
.100	.150	.83224E-01	.31559E 00	.19507E-01	.29265E 08	.11098E 09	.13646E 09	.16111E-03
.100	.200	.10031E 00	.28062E 00	.22421E-01	.35274E 08	.98678E 08	.12134E 09	.18518E-03
.100	.250	.99459E-01	.26088E 00	.25847E-01	.34974E 08	.91736E 08	.11280E 09	.21347E-03
.100	.300	.91794E-01	.24463E 00	.29143E-01	.32278E 08	.86022E 08	.10578E 09	.24069E-03
.100	.350	.81205E-01	.22907E 00	.32061E-01	.28555E 08	.80549E 08	.99049E 08	.26479E-03
.100	.400	.69447E-01	.21356E 00	.34496E-01	.24420E 08	.75095E 08	.92342E 08	.28490E-03
.100	.450	.57469E-01	.19808E 00	.36415E-01	.20208E 08	.69652E 08	.85649E 08	.30075E-03
.100	.500	.45847E-01	.18279E 00	.37821E-01	.16122E 08	.64278E 08	.79040E 08	.31236E-03
.100	.550	.34965E-01	.16790E 00	.38742E-01	.12295E 08	.59042E 08	.72602E 08	.31997E-03
.100	.600	.25095E-01	.15361E 00	.39221E-01	.88244E 07	.54015E 08	.66420E 08	.32392E-03
.100	.650	.16442E-01	.14008E 00	.39310E-01	.57817E 07	.49259E 08	.60571E 08	.32466E-03
.100	.700	.91631E-02	.12749E 00	.39072E-01	.32221E 07	.44829E 08	.55125E 08	.32269E-03
.100	.750	.33829E-02	.11596E 00	.38575E-01	.11896E 07	.40775E 08	.50139E 08	.31859E-03
.100	.800	-.79644E-03	.10561E 00	.37893E-01	-.28006E 06	.37137E 08	.45666E 08	.31295E-03
.100	.850	-.32905E-02	.96555E-01	.37101E-01	-.11571E 07	.33953E 08	.41750E 08	.30642E-03
.100	.900	-.40281E-02	.88880E-01	.36281E-01	-.14164E 07	.31254E 08	.38432E 08	.29965E-03
.100	.950	-.29487E-02	.82664E-01	.35517E-01	-.10369E 07	.29068E 08	.35744E 08	.29333E-03
.100	1.000	.72760E-11	.77979E-01	.34894E-01	.25585E-02	.27420E 08	.33718E 08	.28819E-03

3

.200	.200	-.19060E-10	.43584E 00	.39006E-01	-.67973E-02	.15543E 09	.19113E 09	.32671E-03
.200	.250	.42767E-01	.34988E 00	.37706E-01	.15252E 08	.12477E 09	.15343E 09	.31582E-03
.200	.300	.56410E-01	.30140E 00	.38189E-01	.20117E 08	.10748E 09	.13217E 09	.31987E-03
.200	.350	.57604E-01	.26811E 00	.39284E-01	.20543E 08	.95612E 08	.11757E 09	.32904E-03
.200	.400	.52960E-01	.24197E 00	.40467E-01	.18887E 08	.86292E 08	.10611E 09	.33895E-03
.200	.450	.45568E-01	.21971E 00	.41489E-01	.16250E 08	.78353E 08	.96348E 08	.34751E-03
.200	.500	.37057E-01	.19988E 00	.42234E-01	.13215E 08	.71281E 08	.87651E 08	.35374E-03
.200	.550	.28375E-01	.18182E 00	.42654E-01	.10119E 08	.64842E 08	.79734E 08	.35727E-03
.200	.600	.20115E-01	.16525E 00	.42748E-01	.71733E 07	.58933E 08	.72467E 08	.35806E-03
.200	.650	.12673E-01	.15006E 00	.42540E-01	.45195E 07	.53513E 08	.65803E 08	.35631E-03
.200	.700	.63286E-02	.13620E 00	.42069E-01	.22569E 07	.48572E 08	.59727E 08	.35236E-03
.200	.750	.12850E-02	.12371E 00	.41389E-01	.45826E 06	.44117E 08	.54250E 08	.34667E-03
.200	.800	-.23028E-02	.11262E 00	.40564E-01	-.82123E 06	.40163E 08	.49386E 08	.33976E-03
.200	.850	-.43138E-02	.10298E 00	.39662E-01	-.15384E 07	.36725E 08	.45159E 08	.33220E-03
.200	.900	-.46509E-02	.94842E-01	.38758E-01	-.16586E 07	.33823E 08	.41591E 08	.32463E-03
.200	.950	-.32350E-02	.88258E-01	.37932E-01	-.11537E 07	.31475E 08	.38703E 08	.31771E-03
.200	1.000	.00000E 00	.83277E-01	.37265E-01	.00000E 00	.29698E 08	.36519E 08	.31213E-03

4

.300	.300	.13782E-11	.39190E 00	.52610E-01	.51086E-03	.14527E 09	.17863E 09	.45803E-03
.300	.350	.19979E-01	.33034E 00	.50799E-01	.74057E 07	.12245E 09	.15058E 09	.44227E-03
.300	.400	.26675E-01	.28727E 00	.49987E-01	.98881E 07	.10649E 09	.13094E 09	.43519E-03
.300	.450	.26594E-01	.25419E 00	.49580E-01	.98579E 07	.94226E 08	.11587E 09	.43165E-03
.300	.500	.23044E-01	.22712E 00	.49268E-01	.85419E 07	.84188E 08	.10352E 09	.42894E-03
.300	.550	.17869E-01	.20401E 00	.48891E-01	.66237E 07	.75624E 08	.92993E 08	.42566E-03
.300	.600	.12175E-01	.18382E 00	.48373E-01	.45129E 07	.68139E 08	.83788E 08	.42115E-03
.300	.650	.66644E-02	.16595E 00	.47688E-01	.24704E 07	.61516E 08	.75644E 08	.41518E-03
.300	.700	.18098E-02	.15010E 00	.46845E-01	.67087E 06	.55638E 08	.68416E 08	.40785E-03
.300	.750	-.20596E-02	.13607E 00	.45875E-01	-.76345E 06	.50440E 08	.62024E 08	.39939E-03
.300	.800	-.47044E-02	.12379E 00	.44822E-01	-.17438E 07	.45889E 08	.56428E 08	.39023E-03
.300	.850	-.59452E-02	.11322E 00	.43744E-01	-.22038E 07	.41970E 08	.51609E 08	.38084E-03
.300	.900	-.56440E-02	.10435E 00	.42706E-01	-.20921E 07	.38680E 08	.47564E 08	.37181E-03
.300	.950	-.36915E-02	.97176E-01	.41781E-01	-.13684E 07	.36021E 08	.44294E 08	.36375E-03
.300	1.000	.00000E 00	.91723E-01	.41044E-01	.00000E 00	.34000E 08	.41809E 08	.35734E-03

5

.400	.400	.16937E-11	.33324E 00	.59648E-01	.67843E-03	.13349E 09	.16414E 09	.56117E-03
.400	.450	.73380E-02	.28919E 00	.57790E-01	.29394E 07	.11584E 09	.14245E 09	.54370E-03
.400	.500	.88222E-02	.25476E 00	.56407E-01	.35339E 07	.10205E 09	.12548E 09	.53069E-03
.400	.550	.72070E-02	.22653E 00	.55221E-01	.28869E 07	.90742E 08	.11158E 09	.51953E-03
.400	.600	.41165E-02	.20266E 00	.54081E-01	.16489E 07	.81180E 08	.99824E 08	.50880E-03
.400	.650	.56639E-03	.18209E 00	.52913E-01	.22688E 06	.72939E 08	.89691E 08	.49782E-03
.400	.700	-.27761E-02	.16420E 00	.51693E-01	-.11120E 07	.65772E 08	.80877E 08	.48634E-03
.400	.750	-.54538E-02	.14862E 00	.50427E-01	-.21846E 07	.59532E 08	.73204E 08	.47442E-03
.400	.800	-.71416E-02	.13514E 00	.49143E-01	-.28607E 07	.54131E 08	.66563E 08	.46235E-03
.400	.850	-.76009E-02	.12362E 00	.47887E-01	-.30447E 07	.49518E 08	.60890E 08	.45052E-03
.400	.900	-.66517E-02	.11400E 00	.46713E-01	-.26645E 07	.45663E 08	.56150E 08	.43948E-03
.400	.950	-.41548E-02	.10623E 00	.45687E-01	-.16643E 07	.42551E 08	.52325E 08	.42983E-03
.400	1.000	.29104E-10	.10029E 00	.44880E-01	.11658E-01	.40175E 08	.49401E 08	.42223E-03

6

.500	.500	-.17985E-11	.27190E 00	.60836E-01	-.82436E-03	.12463E 09	.15326E 09	.65494E-03
.500	.550	.59293E-03	.24050E 00	.59148E-01	.27178E 06	.11024E 09	.13556E 09	.63676E-03
.500	.600	-.88227E-03	.21435E 00	.57622E-01	-.40441E 06	.98251E 08	.12082E 09	.62034E-03
.500	.650	-.32165E-02	.19210E 00	.56155E-01	-.14743E 07	.88051E 08	.10827E 09	.60454E-03
.500	.700	-.56210E-02	.17294E 00	.54700E-01	-.25765E 07	.79272E 08	.97477E 08	.58888E-03
.500	.750	-.75594E-02	.15640E 00	.53251E-01	-.34650E 07	.71689E 08	.88153E 08	.57328E-03
.500	.800	-.86535E-02	.14217E 00	.51824E-01	-.39665E 07	.65166E 08	.80132E 08	.55792E-03
.500	.850	-.86280E-02	.13007E 00	.50457E-01	-.39548E 07	.59619E 08	.73311E 08	.54320E-03
.500	.900	-.72769E-02	.11998E 00	.49199E-01	-.33355E 07	.54995E 08	.67625E 08	.52966E-03
.500	.950	-.44422E-02	.11184E 00	.48110E-01	-.20361E 07	.51264E 08	.63037E 08	.51794E-03
.500	1.000	.14552E-10	.10561E 00	.47259E-01	.66701E-02	.48409E 08	.59526E 08	.50877E-03

A7

7									
.600	.600	.93104E-11	.21229E 00	.56997E-01	.53157E-02	.12121E 09	.14904E 09	.76432E-03	
.600	.650	-.25488E-02	.19033E 00	.55583E-01	-.14552E 07	.10867E 09	.13363E 09	.74535E-03	
.600	.700	-.51189E-02	.17140E 00	.54170E-01	-.29226E 07	.97860E 08	.12034E 09	.72641E-03	
.600	.750	-.71878E-02	.15503E 00	.52752E-01	-.41039E 07	.88512E 08	.10884E 09	.70740E-03	
.600	.800	-.83867E-02	.14093E 00	.51351E-01	-.47884E 07	.80463E 08	.98942E 08	.68861E-03	
.600	.850	-.84467E-02	.12893E 00	.50003E-01	-.48226E 07	.73612E 08	.90518E 08	.67053E-03	
.600	.900	-.71665E-02	.11892E 00	.48760E-01	-.40917E 07	.67899E 08	.83493E 08	.65387E-03	
.600	.950	-.43914E-02	.11085E 00	.47683E-01	-.25073E 07	.63289E 08	.77824E 08	.63942E-03	
.600	1.000	.10914E-10	.10467E 00	.46839E-01	.62313E-02	.59763E 08	.73488E 08	.62810E-03	
8									
.700	.700	.20978E-10	.15566E 00	.48758E-01	.17055E-01	.12655E 09	.15562E 09	.93105E-03	
.700	.750	-.33991E-02	.14102E 00	.47671E-01	-.27635E 07	.11465E 09	.14099E 09	.91029E-03	
.700	.800	-.56662E-02	.12827E 00	.46527E-01	-.46067E 07	.10429E 09	.12824E 09	.88845E-03	
.700	.850	-.65986E-02	.11733E 00	.45379E-01	-.53648E 07	.95388E 08	.11729E 09	.86652E-03	
.700	.900	-.60417E-02	.10816E 00	.44288E-01	-.49120E 07	.87932E 08	.10813E 09	.84568E-03	
.700	.950	-.38743E-02	.10075E 00	.43322E-01	-.31499E 07	.81909E 08	.10072E 09	.82725E-03	
.700	1.000	.18190E-10	.95105E-01	.42558E-01	.14789E-01	.77322E 08	.95080E 08	.81265E-03	
9									
.800	.800	.20442E-10	.10191E 00	.36480E-01	.30124E-01	.15017E 09	.18466E 09	.12626E-02	
.800	.850	-.27494E-02	.93158E-01	.35747E-01	-.40516E 07	.13728E 09	.16881E 09	.12373E-02	
.800	.900	-.36988E-02	.85727E-01	.34972E-01	-.54506E 07	.12633E 09	.15534E 09	.12104E-02	
.800	.950	-.27973E-02	.79707E-01	.34241E-01	-.41222E 07	.11746E 09	.14443E 09	.11851E-02	
.800	1.000	-.10914E-10	.75178E-01	.33641E-01	-.16083E-01	.11078E 09	.13623E 09	.11643E-02	
10									
.900	.900	.14964E-10	.50320E-01	.20266E-01	.71506E-01	.24045E 09	.29568E 09	.22744E-02	
.900	.950	-.10969E-02	.46490E-01	.19903E-01	-.52417E 07	.22215E 09	.27317E 09	.22338E-02	
.900	1.000	.36380E-11	.43718E-01	.19563E-01	.17384E-01	.20891E 09	.25688E 09	.21956E-02	
11									
.950	.950	.26665E-11	.25061E-01	.10653E-01	.46002E-01	.43235E 09	.53164E 09	.43167E-02	
.950	1.000	.50022E-11	.23422E-01	.10481E-01	.86298E-01	.40408E 09	.49688E 09	.42469E-02	

APPENDIX B

COMPUTER PROGRAM FOR INDIVIDUAL TURN
ANALYSIS FOR EVERY TURN IN SOLENOIDS
OF ARBITRARY HEIGHT TO INNER RADIUS RATIO

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C *** STRESS, DEFLECTION AND FLUX FOR OH SOLENOIDS ****
C PROGRAM FOR EVALUATING THE OUTER COMPACTED REGION OR THE INDIVIDUAL
C SEPARATING TURNS WITH THE HEIGHT TO INNER RADIUS RATIO AS ARBITRARY
C
C R.A.SMITH
C
C LOAD CASE 1 IS TALL SOLENOID, OUTER REGION
C LOAD CASE 2 IS TALL SOLENOID, INNER REGION
C LOAD CASE 3 IS H=6RI SOLENOID, OUTER REGION
C LOAD CASE 4 IS H=6RI SOLENOID, INNER REGION
C LOAD CASE 5 IS H=6RI SOLENOID, OUTER REGION, P=-25MPA
C
C DIMENSION SZ(10), CZ(10), AZ(99), RO(99), RI(99), DC(10), DORP(10)
C READ(5,1) L,M,NT, (SZ(I), I=1,L), (CZ(J), J=1,M)
C NT=NUMBER OF TURNS
C SF=STACKING FACTOR, RATIO OF CONDUCTOR OUTER DIMENSIONAL AREA TO THAT
C AREA PLUS THE INSULATION AREA=SZ
C CF=COOLANT FRACTION, RATIO OF COOLANT HOLE AREA TO THE CONDUCTOR OUTER
C DIMENSIONAL AREA=CZ
C 1 FORMAT(3I3,1F4.2,1F4.2)
C READ(5,2) EI, EC, VR, BB, TT, TRUT
C EI=MODULUS OF INSULATING MATERIAL, (PA)
C EC=MODULUS OF CONDUCTOR MATERIAL, (PA)
C VR=POISSON RATIO WITH TENSION IN THE R-DIRECTION
C VT=POISSON RATIO WITH TENSION IN THE THETA DIRECTION
C BB=OUTER MOST RADIUS OF SOLENOID, (M)
C TT=TURN THICKNESS INCLUDING INSULATION, (M)
C IF NT=1, TT IS THE HOMOGENEOUS SOLENOID THICKNESS.
C TRUT=THE TRUE TURN THICKNESS. IT IS USED TO CALCULATE TH CORRECTLY WHEN
C WHEN TT IS THE SOLENOID THICKNESS.
C 2 FORMAT(2E12.5,1F6.3,3F12.5)
C PI=3.1415926
C U=PI*4.0E-7
C READ(5,4) LL, DIV, (DC(J), J=1,LL)
C DIV=DIVISIONS ON A TURN FOR STRESS CALC LOCATIONS
C CD=CURRENT DENSITY=DC(J), (AMP/SQ.M)
C 4 FORMAT(16,F6.3,1F12.0)
C READ(5,5) KSTR, BOSTR, BISTR, ATRU

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C KSTR=A KEY SELECTOR FOR USING(ORNL/TM-5528)FLUX VALUES AS INPUT, THE
 C INPUT IS IGNORED IF KSTR=0. IF KSTR=1, THEN THE VALUES OF BOSTR AND
 C BISTR ARE USED TO CALCULATE THE FIELD VALUES
 C BOSTR AND BISTR = NORMALIZED FIELD FACTORS, P.19, ORNL/TM-5528. THESE
 C INPUT VALUES SHOULD BE BASED UPON THE AB WHICH ALSO INCLUDES
 C THE SEPARATING TURNS.
 C ATRU=THE TRUE SOLENOID INNER RADIUS FOR THE PURPOSE OF COMPUTING
 C THE BISTR VALUE ON THAT PORTION OF THE SOLENOID TREATED AS A
 C HOMOGENEOUS NON-SEPARATING STRUCTURE. REASON-WHEN(NT.EQ.1), ATRU
 C CAN BE LESS THAN AA.
 5 FORMAT(16,2E12.5,1F12.4)
 READ(5,6)KDISP,KPRES,KD,(DORP(I),I=1,KD)
 C KPRES AND KDISP ARE KEYS FOR PRESSURE OR DISPLACEMENT SOLUTION AT R=B
 C LET KDISP=1 IF DISPLACEMENTS ARE INPUT, AND LET KPRES=0
 C LET KPRES=1 IF PRESSURES ARE INPUT, AND LET KDISP=0
 C DORP(I)=DISPLACEMENT OR PRESSURE B.C. AT R=B, (M) OR (PA)
 C THE SIX(6) FORMAT CARD NEEDS KDF10.6 FOR THE AMOUNT OF INPUT DATA
 6 FORMAT(3I2, F10.6).
 TN=NT
 C AA=SOLENOID INNER RADIUS, (M)
 DO500 I1=1, KD
 DZ=DORP(I1)
 DO400 I=1, L
 SF=SZ(I)
 IF(I.GT.1)GOTO32
 IF(I1.GT.1)GOTO32
 TH=.5*TRUT*(1.-SQRT(SF))
 C TH=INSULATION THICKNESS ON TURNS, (M)
 BB=BB-TH
 IF(KSTR.EQ.0)BOSTR=0.0
 IF(KSTR.EQ.0)BISTR=U*(BB/ATRU-1.)
 BTSTR=BISTR
 C BTSTR DOES NOT GET MODIFIED
 AA=BB-TN*TT+2.*TH
 XSTR=BISTR+ATRU*(BISTR-BOSTR)/(BB-ATRU)
 YSTR=(BISTR-BOSTR)/(BB-ATRU)
 IF(NT.EQ.1)BISTR=(XSTR-YSTR*AA)*ATRU/AA
 32 CONTINUE

```

R0(1)=BB
R1(1)=BB-TT+TH*2.
AZ(1)=R1(1)/R0(1)
IF(NT.EQ.1)GOTO 33
DO3 K=2,NT
LK=K-1
R0(K)=R0(LK)-TT
R1(K)=R1(LK)-TT
AZ(K)=R1(K)/R0(K)
3 CONTINUE
33 CONTINUE
DO300 J=1,M
CF=CZ(J)
PF=SF*(1,-CF)
ET=EI*(1,-SF)+EC*SF*(1.-CF)
C ET=THE HOMOGENEOUS HOOP MODULUS,(PA)
SSF=SQRT(SF)
SCF=SQRT(.25*PI*CF)
SFC=SQRT(4.*CF/PI)
ER=EI*(1,-SSF)+EC*SSF/((EC/EI)*(1.-SSF)+SSF+SSF*CF/((.25*PI)-SCF))
C ER=THE HOMOGENEOUS RADIAL MODULUS,(PA)
IF(NT.GT.1)EI=EC*(1,-CF)
IF(NT.GT.1)ER=EC*(1,-SFC)/((1.-SFC)+4.*CF/PI)
A=SQRT(ET/ER)
VT=VR*ER/ET
C1=(3.+VR)/(9,-A*A)
C3=(2.+VR)/(4,-A*A)
C7=(3.*VR+A*A)/(9,-A*A)
C8=(2.*VR+A*A)/(4,-A*A)
C9=1./(9,-A*A)
C10=1./(4,-A*A)
C11=A-VR
C12=A+VR
C13=C12/C11
DO200 K=1,LL
CD=DC(K)
WRITE(7,25)
25 FORMAT(1H1)
WRITE(7,7)KDISP,KPRES,11,KD,DZ
7 FORMAT(4I3,E12.6)

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AT=ATRU
BBB=BOSTR*AT*CD*PF
BAA=BISTR*AA*CD*PF
BAT=BTSTR*AT*CD*PF IF(NT.GT.1) BAA = BAT
XXX=BAA+AA*(BAA-BBB)/(BB-AA)
YYY=(BAA-BBB)/(BB-AA)
FLUX=(PI/3.)*(BAA-BBB)*(AA*AA+AA*BB+BB*BB)+PI*BBB*BB*BB
TLUX=(PI/3.)*(BAT-BBB)*(AT*AT+AT*BB+BB*BB)+PI*BBB*BB*BB
WRITE(7,8)KSTR,BOSTR,BISTR,BTSTR,BBB,BAA,BAT,FLUX,TLUX
C BBB=FIELD AT SOLENOID OUTER RADIUS,(T)
C BAA=FIELD AT SOLENOID INNER RADIUS,(T),FOR THE HOMOGENEOUS PORTION
C BAT=FIELD AT SOLENOID INNER RADIUS,(T)
C FLUX=THE HALF SWING OF OH FLUX,(WEBERS),BASED ON THE RADIUS=AA
C TLUX=TRUE FLUX BASED ON AT=ATRU=SOLENOID INNER RADIUS
  8 FORMAT(1H ,110,3E12,5,5F10,3)
  WRITE(7,10)EI,EC,VR,VT,ER,ET,A,CD
  WRITE(7,9)ATRU,AA,BB,TH,TT,SF,CF,PF
  9 FORMAT(1H ,3F6.3,F9.6,F9.6,3F6.3)
 10 FORMAT(1H ,2E12,5,2F6.3,2E12,5,F6.3,F12.0)
  DO100 KK=1,NT
  AB=AZ(KK)
  ROT=RO(KK)
  RIN=RI(KK)
  DEL=(1.-AB)/DIV
  WRITE(7,11)KK,AB,DEL,RIN,ROT
C AB=RATIO OF INNER TO OUTER RADIUS OF A TURN=AZ
C DEL=THE DESIRED INCREMENT ON THE X PARAMETER BETWEEN AB AND 1.0
C RO=OUTER RADIUS OF A TURN,(M)
C RI=INNER RADIUS OF A TURN,(M)
 11 FORMAT(1H ,14,2F7,4,2F7,3)
  C2=AB**(A+3.)
  C4=AB**(A+2.)
  C5=AB**(2,*A)
  C6=1.-C5
  C14=XXX/(YYY*ROT)
  IF(KPRES.EQ.1)P=DZ/(CD*PF*YYY*ROT*ROT)
  IF(KPRES.EQ.1)C1=P-C1*(1.-C2)+C3*C14*(1.-C4)
  IF(KDISP.EQ.1)Q=DZ/(CD*PF*YYY*ROT**3*(1.-VR*VT)/ER)
  IF(KDISP.EQ.1)CJ=Q-C9+C10*C14-(1./C11)*C4*(C1*AB-C14*C3)
  X=AB
  GOT016

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15 IF(DEL.NE.0.0)X=X+DEL
   IF(DEL.EQ.0.0)X=1.0
16 D=1./X
   D1=X**(A-1.)
   D2=D**A
   D3=D**(A+1.)
   D4=D2*D2
   D5=D*D3
   D6=D*D5
   D7=X*D1
C   *** START OF NON-DIMENSIONAL RADIAL STRESS,SRND ***
   IF(KPRES.EQ.1)SRND=X*X*((1.-D6*C2)*C1-D*C3*C14*(1.-D5*C4))+
1  D1*(1./C6)*(1.-D4*C5)*C1
   IF(KDISP.EQ.1)SRND=X*X*((1.-D6*C2)*C1-D*C3*C14*(1.-D5*C4))+
2  D1*C12*CJ*(1.-D4*C5)/(1.+C13*C5)
   SR=SRND*CD*PF*YYY*ROT*ROT
   IF(NT.GT.1)SR=SR*(1.-CF)/PF
C   *** END OF RADIAL STRESS ***
C   *** START OF NON DIMENSIONAL HOOP STRESS,STND ***
   IF(KPRES.EQ.1)STND=C7*X*X-X*C8*C14+A*D3*C4*(C1*AB-C14*C3)+
3  A*D1*(1./C6)*(1.+D4*C5)*C1
   IF(KDISP.EQ.1)STND=C7*X*X-X*C8*C14+A*D3*C4*(C1*AB-C14*C3)+
4  A*C12*D1*CJ*(1.+D4*C5)/(1.+C13*C5)
   ST=STND*CD*PF*YYY*ROT*ROT*EC/ET
   IF(NT.GT.1)ST=ST*(1.-CF)/PF
C   *** END OF HOOP STRESS ***
C   *** START OF NON DIMENSIONAL RADIAL DISPLACEMENT,UND ***
   IF(KPRES.EQ.1)UND=C9*X**3-C14*C10*X*X+D2*C4*(C1*AB-C14*C3)/C11+
5  D7*(1.+C13*D4*C5)*C1/(C12*C6)
   IF(KDISP.EQ.1)UND=C9*X**3-C14*C10*X*X+D2*C4*(C1*AB-C14*C3)/C11+
6  D7*CJ*(1.+C13*D4*C5)/(1.+C13*C5)
   UB=UND*CD*PF*YYY*ROT*ROT*ROT*(1.-VR*VT)/ER
   IF(NT.GT.1)UB=UB*(1.-CF)/PF
C   *** END OF RADIAL DISPLACEMENT ***
   WRITE(7,20)AB,X,SRND,STND,UND,SR,ST,UB
20 FORMAT(1H ,2F6.3,3E12.5,5X,3E12.5)
   IF(X.LT.0.998)GOTO15
100 CONTINUE
200 CONTINUE
300 CONTINUE

```

400 CONTINUE
500 CONTINUE
STOP
END

\$EOF

C INPUT DATA FOR LOAD CASE 1 FOR RO=200.CM

1 1 1 .90 .10

0.00378E+12 0.11700E+12 .3002.000513 .941026 .02

1 47. 15000000.

0 -.33000E-6 .45000E-5 .4

0 1 1.0

C INPUT DATA FOR LOAD CASE 2 FOR RO=200.CM

1 1 80 .90 .10

0.00378E+12 0.11700E+12 .300 2.000513 .02 .02

1 4. 15000000.

0 -.33000E-6 .45000E-5 .4

0 1 1.0

C INPUT DATA FOR LOAD CASE 3 FOR RO=200.CM

1 1 1 .90 .10

0.00378E+12 0.11700E+12 .3002.000513 1.041026 .02

1 52. 15000000.

1 -.33000E-6 .45000E-5 .4

0 1 1.0

C INPUT DATA FOR LOAD CASE 4 FOR RO=200.CM

1 1 80 .90 .10

0.00378E+12 0.11700E+12 .300 2.000513 .02 .02

1 4. 15000000.

1 -.33000E-6 .45000E-5 .4

0 1 1.0

C INPUT DATA FOR LOAD CASE 5 FOR RO=200.CM

1 1 1 .90 .10

0.00378E+12 0.11700E+12 .3002.000513 1.241026 .02

1 62. 15000000.

1 -.33000E-6 .45000E-5 .4

0 1 1-25000000.

PROGRAM OUTPUT WITH DATA INPUT FROM LOAD CASE 1

```

0 1 1 1 .000000E 00
      0 .00000E 00 .11144E-05 .50265E-05 .000 14,352 24,429 108,867 126.887
.37800E 10 .11700E 12 .300 .129 .40929E 11 .95148E 11 1,525 15000000.
.400 1.060 2.000 .000513 .941026 .900 .100 .810
1 .5300 .0100 1.060 2.000
.530 .530 -.69996E-11 .25374E 00 .60179E-01 -.51939E-02 .23153E 09 .20976E-02
.530 .540 .35478E-04 .24766E 00 .59842E-01 .26326E 05 .22598E 09 .20859E-02
.530 .550 -.24262E-04 .24181E 00 .59514E-01 -.18003E 05 .22064E 09 .20744E-02
.530 .560 -.16667E-03 .23616E 00 .59192E-01 -.12367E 06 .21549E 09 .20632E-02
.530 .570 -.38031E-03 .23071E 00 .58876E-01 -.28220E 06 .21052E 09 .20522E-02
.530 .580 -.65475E-03 .22545E 00 .58565E-01 -.48585E 06 .20571E 09 .20413E-02
.530 .590 -.98047E-03 .22036E 00 .58257E-01 -.72754E 06 .20107E 09 .20306E-02
.530 .600 -.13487E-02 .21544E 00 .57952E-01 -.10008E 07 .19658E 09 .20200E-02
.530 .610 -.17516E-02 .21068E 00 .57650E-01 -.12997E 07 .19223E 09 .20095E-02
.530 .620 -.21816E-02 .20606E 00 .57350E-01 -.16188E 07 .18802E 09 .19990E-02
.530 .630 -.26320E-02 .20158E 00 .57051E-01 -.19530E 07 .18393E 09 .19886E-02
.530 .640 -.30965E-02 .19724E 00 .56754E-01 -.22977E 07 .17997E 09 .19782E-02
.530 .650 -.35695E-02 .19303E 00 .56457E-01 -.26487E 07 .17613E 09 .19679E-02
.530 .660 -.40455E-02 .18895E 00 .56161E-01 -.30019E 07 .17240E 09 .19576E-02
.530 .670 -.45197E-02 .18498E 00 .55866E-01 -.33537E 07 .16879E 09 .19473E-02
.530 .680 -.49873E-02 .18113E 00 .55571E-01 -.37008E 07 .16527E 09 .19370E-02
.530 .690 -.54443E-02 .17739E 00 .55276E-01 -.40398E 07 .16186E 09 .19267E-02
.530 .700 -.58865E-02 .17376E 00 .54981E-01 -.43680E 07 .15855E 09 .19164E-02
.530 .710 -.63103E-02 .17023E 00 .54687E-01 -.46825E 07 .15533E 09 .19062E-02
.530 .720 -.67123E-02 .16681E 00 .54393E-01 -.49807E 07 .15221E 09 .18959E-02
.530 .730 -.70892E-02 .16349E 00 .54099E-01 -.52604E 07 .14917E 09 .18857E-02
.530 .740 -.74380E-02 .16026E 00 .53806E-01 -.55192E 07 .14623E 09 .18755E-02
.530 .750 -.77559E-02 .15713E 00 .53514E-01 -.57551E 07 .14337E 09 .18653E-02
.530 .760 -.80403E-02 .15409E 00 .53223E-01 -.59661E 07 .14060E 09 .18552E-02
.530 .770 -.82886E-02 .15114E 00 .52933E-01 -.61504E 07 .13791E 09 .18451E-02
.530 .780 -.84985E-02 .14828E 00 .52645E-01 -.63062E 07 .13530E 09 .18350E-02
.530 .790 -.86679E-02 .14551E 00 .52359E-01 -.64319E 07 .13277E 09 .18250E-02

```

PROGRAM OUTPUT WITH DATA INPUT FROM LOAD CASE 2

[illegible]

3	.9903	.0024	1.941	1.960				
.990	.990	.22410E-11	.25280E-01	.25335E-01	.17801E-02	.22313E 08	.37017E-03	
.990	.993	-.82761E-05	.25197E-01	.25317E-01	-.65743E 04	.22240E 08	.36990E-03	
.990	.995	-.11030E-04	.25117E-01	.25298E-01	-.87622E 04	.22169E 08	.36963E-03	
.990	.998	-.82694E-05	.25039E-01	.25280E-01	-.65690E 04	.22100E 08	.36936E-03	
.990	1.000	.28422E-13	.24962E-01	.25261E-01	.22577E-04	.22033E 08	.36909E-03	
4	.9902	.0024	1.921	1.940				
.990	.990	-.57682E-11	.35866E-01	.35940E-01	-.44890E-02	.31013E 08	.50921E-03	
.990	.993	-.82278E-05	.35748E-01	.35914E-01	-.64032E 04	.30912E 08	.50883E-03	
.990	.995	-.10967E-04	.35633E-01	.35887E-01	-.85348E 04	.30813E 08	.50846E-03	
.990	.998	-.82224E-05	.35521E-01	.35861E-01	-.63990E 04	.30715E 08	.50808E-03	
.990	1.000	.28422E-13	.35410E-01	.35834E-01	.22119E-04	.30620E 08	.50771E-03	

OUTPUT UP TO THIS POINT IS MEANINGLESS FOR THE
INDIVIDUAL TURNS IN THE OUTER REGION WHERE THE
TURNS ARE ACTUALLY COMPACTING.

45	.9831	.0042	1.101	1.120			
.983	.983	.31961E-10	.79623E 00	.79212E 00	.82903E-02	.22948E 09	.21595E-02
.983	.987	.22916E-04	.79180E 00	.79109E 00	.59441E 04	.22820E 09	.21567E-02
.983	.992	.30212E-04	.78741E 00	.79008E 00	.78367E 04	.22694E 09	.21540E-02
.983	.996	.22405E-04	.78306E 00	.78907E 00	.58116E 04	.22568E 09	.21512E-02
.983	1.000	-.11369E-11	.77874E 00	.78807E 00	-.29489E-03	.22444E 09	.21485E-02
46	.9828	.0043	1.081	1.100			
.983	.983	.56776E-10	.82898E 00	.82443E 00	.14206E-01	.23046E 09	.21293E-02
.983	.987	.25893E-04	.82428E 00	.82335E 00	.64786E 04	.22915E 09	.21266E-02
.983	.991	.34146E-04	.81962E 00	.82228E 00	.85435E 04	.22786E 09	.21238E-02
.983	.996	.25329E-04	.81501E 00	.82121E 00	.63375E 04	.22658E 09	.21210E-02
.983	1.000	.45475E-12	.81044E 00	.82014E 00	.11378E-03	.22531E 09	.21183E-02

START OF INDIVIDUAL TURN OUTPUT WHERE TURNS ARE SEPARATING

47	.9824	.0044	1.061	1.080				
.982	.982	.14605E-10	.86294E 00	.85793E 00	.35227E-02	.23126E 09	.20972E-02	
.982	.987	.29160E-04	.85796E 00	.85678E 00	.70332E 04	.22992E 09	.20944E-02	
.982	.991	.38463E-04	.85302E 00	.85564E 00	.92768E 04	.22860E 09	.20916E-02	
.982	.996	.28537E-04	.84813E 00	.85451E 00	.68829E 04	.22729E 09	.20888E-02	
.982	1.000	.00000E 00	.84328E 00	.85338E 00	.00000E 00	.22599E 09	.20861E-02	
48	.9821	.0045	1.041	1.060				
.982	.982	-.11285E-11	.89818E 00	.89267E 00	-.26219E-03	.23187E 09	.20631E-02	
.982	.987	.32749E-04	.89290E 00	.89145E 00	.76090E 04	.23051E 09	.20603E-02	
.982	.991	.43204E-04	.88767E 00	.89025E 00	.10038E 05	.22916E 09	.20575E-02	
.982	.996	.32061E-04	.88248E 00	.88904E 00	.74490E 04	.22782E 09	.20547E-02	
.982	1.000	.68212E-11	.87734E 00	.88785E 00	.15848E-02	.22649E 09	.20520E-02	
49	.9818	.0046	1.021	1.040				
.982	.982	.27846E-10	.93479E 00	.92873E 00	.62279E-02	.23230E 09	.20272E-02	
.982	.986	.36696E-04	.92919E 00	.92744E 00	.82072E 04	.23091E 09	.20244E-02	
.982	.991	.48417E-04	.92363E 00	.92615E 00	.10829E 05	.22953E 09	.20216E-02	
.982	.995	.35934E-04	.91813E 00	.92488E 00	.80369E 04	.22816E 09	.20188E-02	
.982	1.000	-.12278E-10	.91268E 00	.92361E 00	-.27461E-02	.22681E 09	.20161E-02	
50	.9814	.0047	1.001	1.020				
.981	.981	.12467E-10	.97284E 00	.96617E 00	.26822E-02	.23255E 09	.19896E-02	
.981	.986	.41041E-04	.96689E 00	.96481E 00	.88294E 04	.23112E 09	.19868E-02	
.981	.991	.54156E-04	.96100E 00	.96344E 00	.11651E 05	.22972E 09	.19840E-02	
.981	.995	.40198E-04	.95516E 00	.96209E 00	.86479E 04	.22832E 09	.19812E-02	
.981	1.000	-.90949E-12	.94938E 00	.96075E 00	-.19566E-03	.22694E 09	.19784E-02	
51	.9810	.0047	.981	1.000				
.981	.981	.24931E-11	.10124E 01	.10051E 01	.51552E-03	.23261E 09	.19504E-02	
.981	.986	.45830E-04	.10061E 01	.10036E 01	.94768E 04	.23116E 09	.19476E-02	
.981	.991	.60479E-04	.99984E 00	.10022E 01	.12506E 05	.22972E 09	.19448E-02	
.981	.995	.44895E-04	.99365E 00	.10008E 01	.92834E 04	.22830E 09	.19420E-02	
.981	1.000	-.22737E-12	.98751E 00	.99934E 00	-.47017E-04	.22689E 09	.19392E-02	
52	.9806	.0048	.961	.980				
.981	.981	.39283E-10	.10536E 01	.10456E 01	.78013E-02	.23249E 09	.19096E-02	
.981	.985	.51115E-04	.10469E 01	.10440E 01	.10151E 05	.23101E 09	.19068E-02	
.981	.990	.67457E-04	.10403E 01	.10425E 01	.13396E 05	.22954E 09	.19040E-02	
.981	.995	.50077E-04	.10337E 01	.10410E 01	.99450E 04	.22809E 09	.19012E-02	
.981	1.000	-.68212E-12	.10272E 01	.10395E 01	-.13546E-03	.22665E 09	.18985E-02	

53	.9802	.0049	.941	.960				
.980	.980	.48729E-10	.10965E 01	.10877E 01	.92863E-02	.23218E 09	.18674E-02	
.980	.985	.56955E-04	.10894E 01	.10861E 01	.10854E 05	.23067E 09	.18646E-02	
.980	.990	.75166E-04	.10823E 01	.10844E 01	.14324E 05	.22918E 09	.18618E-02	
.980	.995	.55802E-04	.10753E 01	.10828E 01	.10634E 05	.22770E 09	.18590E-02	
.980	1.000	-.45475E-12	.10684E 01	.10812E 01	-.86661E-04	.22623E 09	.18563E-02	
54	.9798	.0050	.921	.940				
.980	.980	.11426E-10	.11413E 01	.11316E 01	.20876E-02	.23169E 09	.18239E-02	
.980	.985	.63417E-04	.11337E 01	.11299E 01	.11587E 05	.23015E 09	.18211E-02	
.980	.990	.83696E-04	.11262E 01	.11281E 01	.15292E 05	.22863E 09	.18183E-02	
.980	.995	.62135E-04	.11188E 01	.11264E 01	.11353E 05	.22712E 09	.18155E-02	
.980	1.000	.68212E-11	.11114E 01	.11247E 01	.12463E-02	.22563E 09	.18128E-02	
55	.9794	.0052	.901	.920				
.979	.979	.33902E-10	.11880E 01	.11774E 01	.59335E-02	.23102E 09	.17791E-02	
.979	.985	.70579E-04	.11799E 01	.11755E 01	.12353E 05	.22945E 09	.17763E-02	
.979	.990	.93147E-04	.11719E 01	.11737E 01	.16303E 05	.22790E 09	.17735E-02	
.979	.995	.69151E-04	.11640E 01	.11719E 01	.12103E 05	.22636E 09	.17707E-02	
.979	1.000	-.27285E-11	.11562E 01	.11701E 01	-.47754E-03	.22484E 09	.17680E-02	
56	.9789	.0053	.881	.900				
.979	.979	.40583E-10	.12367E 01	.12252E 01	.67974E-02	.23016E 09	.17331E-02	
.979	.984	.78529E-04	.12282E 01	.12232E 01	.13153E 05	.22856E 09	.17304E-02	
.979	.989	.10364E-03	.12197E 01	.12212E 01	.17358E 05	.22698E 09	.17276E-02	
.979	.995	.76936E-04	.12113E 01	.12193E 01	.12886E 05	.22542E 09	.17248E-02	
.979	1.000	-.54570E-11	.12030E 01	.12174E 01	-.91400E-03	.22387E 09	.17221E-02	
57	.9784	.0054	.861	.880				
.978	.978	.26463E-10	.12877E 01	.12751E 01	.42375E-02	.22912E 09	.16861E-02	
.978	.984	.87368E-04	.12786E 01	.12730E 01	.13990E 05	.22750E 09	.16834E-02	
.978	.989	.11530E-03	.12696E 01	.12709E 01	.18463E 05	.22589E 09	.16806E-02	
.978	.995	.85589E-04	.12606E 01	.12688E 01	.13705E 05	.22429E 09	.16779E-02	
.978	1.000	.11369E-10	.12518E 01	.12668E 01	.18205E-02	.22272E 09	.16752E-02	
58	.9779	.0055	.841	.860				
.978	.978	.66080E-10	.13411E 01	.13273E 01	.10106E-01	.22790E 09	.16382E-02	
.978	.983	.97213E-04	.13314E 01	.13250E 01	.14867E 05	.22624E 09	.16354E-02	
.978	.989	.12828E-03	.13218E 01	.13228E 01	.19619E 05	.22460E 09	.16327E-02	
.978	.994	.95222E-04	.13122E 01	.13206E 01	.14563E 05	.22298E 09	.16300E-02	
.978	1.000	.11369E-10	.13028E 01	.13184E 01	.17387E-02	.22138E 09	.16273E-02	
59	.9774	.0056	.821	.840				
.977	.977	-.69078E-11	.13971E 01	.13819E 01	-.10079E-02	.22649E 09	.15893E-02	
.977	.983	.10820E-03	.13867E 01	.13795E 01	.15787E 05	.22480E 09	.15866E-02	
.977	.989	.14277E-03	.13764E 01	.13771E 01	.20831E 05	.22314E 09	.15839E-02	
.977	.994	.10597E-03	.13662E 01	.13748E 01	.15461E 05	.22149E 09	.15812E-02	
.977	1.000	.00000E 00	.13562E 01	.13724E 01	.00000E 00	.21986E 09	.15785E-02	

60	.9769	.0058	.801	.820				
.977	.977	-.70708E-11	.14557E 01	.14391E 01	-.98312E-03	.22490E 09	.15397E-02	
.977	.983	.12048E-03	.14447E 01	.14365E 01	.16752E 05	.22318E 09	.15370E-02	
.977	.988	.15896E-03	.14337E 01	.14340E 01	.22102E 05	.22149E 09	.15343E-02	
.977	.994	.11798E-03	.14228E 01	.14315E 01	.16404E 05	.21981E 09	.15316E-02	
.977	1.000	.00000E 00	.14121E 01	.14290E 01	.00000E 00	.21816E 09	.15290E-02	
61	.9763	.0059	.781	.800				
.976	.976	.16364E-10	.15174E 01	.14991E 01	.21656E-02	.22312E 09	.14894E-02	
.976	.982	.13425E-03	.15055E 01	.14964E 01	.17767E 05	.22138E 09	.14867E-02	
.976	.988	.17711E-03	.14938E 01	.14937E 01	.23438E 05	.21965E 09	.14840E-02	
.976	.994	.13143E-03	.14822E 01	.14910E 01	.17393E 05	.21795E 09	.14814E-02	
.976	1.000	.68212E-11	.14708E 01	.14884E 01	.90272E-03	.21627E 09	.14787E-02	
62	.9757	.0061	.761	.780				
.976	.976	.33818E-10	.15822E 01	.15622E 01	.42545E-02	.22116E 09	.14385E-02	
.976	.982	.14971E-03	.15695E 01	.15593E 01	.18834E 05	.21939E 09	.14359E-02	
.976	.988	.19747E-03	.15569E 01	.15564E 01	.24843E 05	.21764E 09	.14332E-02	
.976	.994	.14653E-03	.15446E 01	.15535E 01	.18434E 05	.21590E 09	.14306E-02	
.976	1.000	-.45475E-11	.15323E 01	.15507E 01	-.57210E-03	.21419E 09	.14280E-02	
63	.9750	.0062	.741	.760				
.975	.975	.34404E-10	.16504E 01	.16285E 01	.41091E-02	.21902E 09	.13872E-02	
.975	.981	.16711E-03	.16368E 01	.16253E 01	.19959E 05	.21722E 09	.13845E-02	
.975	.988	.22040E-03	.16234E 01	.16222E 01	.26324E 05	.21543E 09	.13819E-02	
.975	.994	.16351E-03	.16101E 01	.16192E 01	.19530E 05	.21368E 09	.13793E-02	
.975	1.000	-.40927E-11	.15970E 01	.16161E 01	-.48882E-03	.21194E 09	.13767E-02	
64	.9744	.0064	.721	.740				
.974	.974	.14556E-10	.17223E 01	.16982E 01	.16482E-02	.21669E 09	.13354E-02	
.974	.981	.18675E-03	.17077E 01	.16949E 01	.21146E 05	.21486E 09	.13328E-02	
.974	.987	.24626E-03	.16934E 01	.16916E 01	.27885E 05	.21305E 09	.13302E-02	
.974	.994	.18267E-03	.16792E 01	.16883E 01	.20685E 05	.21126E 09	.13276E-02	
.974	1.000	-.45475E-12	.16651E 01	.16851E 01	-.51493E-04	.20950E 09	.13250E-02	
65	.9736	.0066	.701	.720				
.974	.974	.24700E-10	.17982E 01	.17718E 01	.26477E-02	.21418E 09	.12833E-02	
.974	.980	.20897E-03	.17826E 01	.17682E 01	.22401E 05	.21232E 09	.12807E-02	
.974	.987	.27552E-03	.17672E 01	.17647E 01	.29534E 05	.21048E 09	.12781E-02	
.974	.993	.20434E-03	.17519E 01	.17612E 01	.21904E 05	.20867E 09	.12756E-02	
.974	1.000	-.18190E-11	.17369E 01	.17577E 01	-.19499E-03	.20687E 09	.12731E-02	

66	.9729	.0068	.681	.700				
.973	.973	.2020E-10	.18785E 01	.18495E 01	.20469E-02	.21149E 09	.12310E-02	
.973	.980	.23419E-03	.18617E 01	.18456E 01	.23729E 05	.20959E 09	.12284E-02	
.973	.986	.30870E-03	.18451E 01	.18418E 01	.31279E 05	.20773E 09	.12259E-02	
.973	.993	.22891E-03	.18288E 01	.18381E 01	.23194E 05	.20588E 09	.12234E-02	
.973	1.000	.00000E 00	.18126E 01	.18343E 01	.00000E 00	.20407E 09	.12209E-02	
67	.9721	.0070	.661	.680				
.972	.972	.50635E-10	.19636E 01	.19316E 01	.48415E-02	.20861E 09	.11786E-02	
.972	.979	.26289E-03	.19455E 01	.19275E 01	.25137E 05	.20669E 09	.11761E-02	
.972	.986	.34646E-03	.19276E 01	.19234E 01	.33127E 05	.20479E 09	.11736E-02	
.972	.993	.25686E-03	.19100E 01	.19193E 01	.24559E 05	.20292E 09	.11711E-02	
.972	1.000	-.90949E-12	.18926E 01	.19153E 01	-.86962E-04	.20107E 09	.11686E-02	
68	.9713	.0072	.641	.660				
.971	.971	.98768E-10	.20538E 01	.20186E 01	.88964E-02	.20555E 09	.11262E-02	
.971	.978	.29567E-03	.20343E 01	.20142E 01	.26632E 05	.20359E 09	.11237E-02	
.971	.986	.38957E-03	.20150E 01	.20097E 01	.35090E 05	.20167E 09	.11212E-02	
.971	.993	.28874E-03	.19961E 01	.20054E 01	.26008E 05	.19977E 09	.11188E-02	
.971	1.000	-.20918E-10	.19774E 01	.20010E 01	-.18842E-02	.19790E 09	.11163E-02	
69	.9704	.0074	.621	.640				
.970	.970	.54093E-10	.21497E 01	.21109E 01	.45816E-02	.20230E 09	.10738E-02	
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.967	.975	.48407E-03	.24504E 01	.24187E 01	.33673E 05	.18939E 09	.91576E-03	
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