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Page 1 of 1

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TANK 241-T-111

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CONTENTS

1.0	SUMMARY	1-1
2.0	INTRODUCTION	2-1
3.0	MEAN CONCENTRATION ESTIMATES	3-1
3.1	STATISTICAL METHODS	3-1
3.2	STATISTICAL RESULTS	3-1
4.0	COMPARISON OF THE VARIANCE COMPONENT ESTIMATES	4-1
4.1	STATISTICAL METHODS	4-1
4.2	STATISTICAL RESULTS	4-1
5.0	REFERENCES	5-1

LIST OF APPENDICES

A	CORE COMPOSITE DATA	A-1
B	CORE COMPOSITE DATA PLOTS	B-1
C	MEAN CONCENTRATION CALCULATION METHODS	C-1

LIST OF TABLES

2-1	T-111 Core Recoveries	2-1
2-2	Special Analyte List.	2-3
3-1	Concentration Estimate Statistics (Units $\mu\text{g/g}$ Except Radionuclides $\mu\text{Ci/g}$)	3-2
4-1	Variance Component Estimates	4-2
A-1	Core Composite Data (Units $\mu\text{g/g}$ except Radionuclides $\mu\text{Ci/g}$)	A-3

LIST OF FIGURES

2-1	Riser Location	2-1
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LIST OF TERMS

95% LL	lower limit of a 95% confidence interval on the mean
95% UL	upper limit of a 95% confidence interval on the mean
ANOVA	analysis of variance
T-111	tank 241-T-111
CI	confidence interval
df	degrees of freedom
.dir	direct
DL	detection limit
GEA	gamma energy analysis
IC	ion chromatography
IC.w.	ion chromatography of a water leached sample
ICP	inductively coupled plasma
ICP.a.	inductively coupled plasma analysis of an acid digested sample
ICP.f.	inductively coupled plasma analysis of a fusion digested sample
ICP.w.	inductively coupled plasma analysis of a water leached sample
KOH/Ni	Potassium Hydroxide/Nickel
Mean/DL	the mean of two sample results (sample and duplicate) divided by the detection limit
mL	milliliters
NA	not available
REML	Restricted Maximum Likelihood Estimation
TGA	thermal gravimetric analysis
TIC	total inorganic carbon
TOC	total organic carbon
$\bar{y}$	weighted mean of the concentration data
$\hat{\sigma}^2(\bar{y})$	estimated variance of $\bar{y}$
$\mu\text{g/g}$	micrograms per gram
$\mu\text{Ci/g}$	microcuries per gram

STATISTICAL CHARACTERIZATION REPORT
FOR
SINGLE-SHELL TANK 241-T-111

1.0 SUMMARY

This report contains the results of the statistical analysis of data from two core samples obtained from single-shell tank 241-T-111 (T-111).

Section 2.0 contains a description of the core samples and the chemical analyses performed on the core samples.

Section 3.0 contains mean concentration estimates and associated 95% confidence intervals (CIs) on the mean for each of the analytes found in the core samples from T-111.

Section 4.0 contains estimates of the spatial variability (variability between cores) and estimates of the analytical variability from the core composite data.

Two types of analytical variability were estimated from the core composite data: (1) sample composite variability (variability between composite samples within the same core) and (2) analytical measurement variability (variability between the primary and duplicate analyses within each core composite sample). Estimates of the analytical measurement variability were used as the reference value to test the significance of the spatial and sample composite variability. Spatial variability was significantly different from zero¹ for 39 out of 85 analytes. The sample composite variance was significantly different from zero for (a different) 39 out of the 85 analytes.

¹ Significantly different from zero at the $\alpha = 0.05$ level.

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2.0 INTRODUCTION

Two core samples (cores 31 and 33) were taken from T-111. Cores 31 and 33 were obtained from risers 6 and 3, respectively. The location of the risers are given in Figure 2-1. The segment recoveries for each core are given in Table 2-1. Core 31 had nearly 100% segment recovery for all segments except segments 1 and 6. There was nearly 100% segment recovery for all of the segments in Core 33.

Figure 2-1. Riser Location.

Table 2-1. T-111 Core Recoveries.

Core	Segment number								
	1	2	3	4	5	6	7	8	9
31	27%	80- 100%	95- 100%	80- 100%	100%	0%	90- 100%	100%	100%
33	100%	100%	87- 100%	75- 85%	88%	100%	100%	100%	100%

Due to the incomplete core sample recovery, the concentration estimates given in this report are biased. The magnitude of the bias is unknown.

Two core composite samples were made from each core sample from the homogenized solid segment waste. Primary and duplicate results were obtained from each core composite sample.

The chemical analysis of the primary and duplicate samples consisted of the following routine analytical methods and sample preparations (the notation for the method and preparation is also listed):

- inductively coupled plasma (ICP) analysis of an acid digested sample (ICP.a)
- ICP of a potassium hydroxide/nickel (KOH/Ni) fused sample (ICP.f)
- ICP analysis of a water leached sample (ICP.w)
- ion chromatography (IC) of a water leached sample (IC.w)
- radiochemistry
- gamma energy analysis (GEA).

Whenever appropriate, the data given here are identified using the notation in the above list. For example, the notation ICP.a.A1 refers to the A1 concentration from an ICP analysis of an acid digested sample. However, it needs to be noted that, occasionally, other analytical methods are used. The results from such methods are also given in the tables in Sections 3.0 and 4.0 and the table in Appendix A.

The core composite sample data are contained in Table A-1 (Appendix A). Ratios of the mean of each primary and duplicate result divided by the detection limit (DL) for that pair are also included. The ratios are provided to show how large the analyte concentrations are relative to the DL. The data packages for T-111 (Kocher 1993 and McKinney 1993) contain a complete report of the sample results along with the laboratory quality control data. The core composite data for each analyte are illustrated in the figures provided in Appendix B.

Analysis of variance (ANOVA) models were fit to the data for all analytes that did not have any "less than" values. All of the data used in the statistical analyses are reported in Table A-1. In addition, this table reports the data for the analytes with "less than" values.

Analytical Evaluation and Reporting personnel, within the Tank Waste Remediation System Information Management Systems, requested that ANOVA models be fit to the data for analytes whose concentrations were greater than ten times the DL. In addition, they identified a list of "special analytes," given in Table 2-2. For this list, they requested that the ANOVA models be fit to the data, provided the concentration values were greater than three times the DL.

Table 2-2. Special Analyte List.

Aluminum	Nitrate
Bismuth	Nitrite
Calcium	Phosphate
Chromium	Carbonate
Iron	Fluoride
Silicon	Chloride
Sodium	Total Organic Carbon
Zirconium	Cyanide

The analytes in Tables 3-1, 4-1, and A-1 are denoted by a "*" or a "★" whenever a concentration is less than three times or ten times the DL. The results for such analytes are to be used with caution. However, in Table A-1, some of the data are marked with a "■." For these analytes, the results are so close to the DL, that the concentration values are questionable. ANOVA models were not fit to the data marked with a "■."

A close examination of the data plots given in Appendix B revealed several potential anomalies in the core composite data. They are listed below.

- ICP.a.Co The primary result for core 31, composite 2 of 11.7 $\mu\text{g/g}$ is over three times as large as the duplicate result. The other results for ICP.a.Co all fall in the range 2.7 to 3.8 $\mu\text{g/g}$.
- ICP.a.Cu The duplicate result for core 31, composite 2 of 127 $\mu\text{g/g}$ is about four times as large as the primary result of 31.7 $\mu\text{g/g}$.
- Spec.w.NO2 Both the primary and duplicate results from core 31, composite 2 are about one half the results for core 31, composite 1. The average of the results for core 31, composite 1 is 952 $\mu\text{g/g}$, the average for composite two is 525 $\mu\text{g/g}$.

There is no direct evidence that the results noted above are due to analytical measurement variability. Consequently, the statistical analysis was performed on the data as reported in Table A-1.

WHC-SD-WM-TI-650, Rev. 0

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3.0 MEAN CONCENTRATION ESTIMATES

One of the tasks outlined in the Tank Waste Characterization Plan, Section 5.1.1.2 (Bell 1993), is to estimate the inventories of the analytes in the waste. The inventories are estimated by computing mean concentration and 95% CIs on the mean concentration for each analyte. The estimate of the inventory and CI on the inventory of an analyte in the tank are equal to the corresponding mean concentration estimates and CI multiplied by the volume of waste in the tank. The estimates of the tank inventory are not given in this document.

Table A-1 contains the core composite data used to compute the mean concentration estimates and the CIs.

3.1 STATISTICAL METHODS

The concentration estimates are given in the form of 95% CIs on the mean concentration. It is assumed that each primary sample and its duplicate are analyzed independently of one another. The two analytical results are used to estimate the analytical measurement variability. Due to the hierarchical structure of the data, the analytical measurement variability alone is not the appropriate variance term to use in computing the CIs. A linear combination of the analytical measurement variance and the spatial variance is the appropriate variance of the mean for the CIs (Snedecor and Cochran 1980). Appendix C contains a description of the statistical model and formulas used to calculate the estimate of the mean, the variance of the mean, and the CI on the mean.

3.2 STATISTICAL RESULTS

Table 3-1 contains the summary statistics, by analyte, for ICP acid digestion, ICP water leach, ICP KOH/Ni fusion dissolution, radiochemistry, IC, and the other analyses. The summary statistics are defined as follows:

$\bar{y}$	mean of the concentration data
$\hat{\sigma}^2(\bar{y})$	estimated variance of $\bar{y}$
df	degrees of freedom
95% LL	lower limit of the 95% CI on the mean
95% UL	upper limit of the 95% CI on the mean.

For some analytes, the lower confidence limit (95% LL) was negative. Since concentrations are greater than or equal to zero, any negative 95% LL values were set equal to zero.

The CIs in Table 3-1 are wide relative to the range of the data because only two cores were used to estimate the spatial variability. Two core samples are the minimum number needed to estimate a tank's spatial variability.

Table 3-1. Concentration Estimate Statistics (Units $\mu\text{g/g}$
Except Radionuclides $\mu\text{Ci/g}$). (sheet 1 of 3)

Analyte	$\bar{y}$	$\hat{\sigma}^2(\bar{y})$	df	95% LL	95% UL
ICP.a.Ag	1.26E+02	7.86E+03	1	0.00	1.25E+03
ICP.a.Al	5.41E+02	1.06E+04	1	0.00	1.85E+03
ICP.a.B	2.80E+01	7.56E+00	1	0.00	6.30E+01
ICP.a.Ba	6.90E+01	6.46E+01	1	0.00	1.71E+02
ICP.a.Bi	2.59E+04	6.38E+06	1	0.00	5.80E+04
ICP.a.Ca	1.88E+03	2.12E+05	1	0.00	7.72E+03
ICP.a.Cd*	5.80E+00	3.03E+00	1	0.00	2.79E+01
ICP.a.Ce*	3.37E+01	9.69E+00	1	0.00	7.33E+01
ICP.a.Co*	4.30E+00	1.63E+00	1	0.00	2.05E+01
ICP.a.Cr	1.98E+03	1.63E+04	1	3.57E+02	3.60E+03
ICP.a.Cu	3.35E+01	3.53E+02	1	0.00	2.72E+02
ICP.a.Fe	1.85E+04	1.21E+06	1	4.55E+03	3.25E+04
ICP.a.K	1.14E+03	2.24E+03	1	5.34E+02	1.74E+03
ICP.a.La	4.22E+03	3.00E+05	1	0.00	1.12E+04
ICP.a.Mg	3.77E+02	6.36E+03	1	0.00	1.39E+03
ICP.a.Mn	6.33E+03	2.68E+04	1	4.25E+03	8.41E+03
ICP.a.Na	3.69E+04	1.56E+06	1	2.10E+04	5.27E+04
ICP.a.Ni	1.32E+02	5.12E+02	1	0.00	4.19E+02
ICP.a.P	1.03E+04	1.21E+05	1	5.90E+03	1.47E+04
ICP.a.Pb	3.47E+02	2.64E+04	1	0.00	2.41E+03
ICP.a.S	1.21E+03	1.06E+03	1	8.00E+02	1.63E+03
ICP.a.Sb*	3.14E+01	1.65E+01	1	0.00	8.30E+01
ICP.a.Si	4.69E+02	9.17E+02	1	8.40E+01	8.54E+02
ICP.a.Sr	3.00E+02	3.75E+02	1	5.39E+01	5.46E+02
ICP.a.Ti	1.95E+01	1.39E+02	1	0.00	1.69E+02
ICP.a.V	1.45E+01	6.58E+00	1	0.00	4.71E+01
ICP.a.Zn	6.50E+01	6.46E+02	1	0.00	3.88E+02
ICP.f.Ag	1.28E+02	8.05E+03	1	0.00	1.27E+03
ICP.f.Al	5.70E+02	9.70E+03	1	0.00	1.82E+03
ICP.f.Ba	6.46E+01	2.45E+01	1	1.73E+00	1.28E+02
ICP.f.Bi	2.36E+04	9.08E+06	1	0.00	6.18E+04
ICP.f.Ca*	2.42E+03	8.27E+04	1	0.00	6.07E+03
ICP.f.Cd*	8.12E+00	1.76E+00	1	0.00	2.50E+01
ICP.f.Co*	1.15E+01	1.16E+00	1	0.00	2.52E+01
ICP.f.Cr	1.80E+03	1.56E+03	1	1.30E+03	2.30E+03

*: Analytes with a portion of the data below three times the DL.

*: Analytes with a portion of the data below ten times the DL.

Table 3-1. Concentration Estimate Statistics (Units $\mu\text{g/g}$
Except Radionuclides $\mu\text{Ci/g}$). (sheet 2 of 3)

Analyte	$\bar{y}$	$s^2(\bar{y})$	df	95% LL	95% UL
ICP.f.Cu*	2.39E+01	3.56E+01	1	0.00	1.05E+02
ICP.f.Fe	1.80E+04	4.05E+06	1	0.00	4.36E+04
ICP.f.La	4.11E+03	3.08E+05	1	0.00	1.12E+04
ICP.f.Mg	3.55E+02	7.31E+03	1	0.00	1.44E+03
ICP.f.Mn	6.28E+03	1.88E+04	1	4.54E+03	8.02E+03
ICP.f.Na	3.70E+04	6.00E+06	1	5.82E+03	6.81E+04
ICP.f.Ni	8.14E+03	6.41E+06	1	0.00	4.03E+04
ICP.f.P	1.04E+04	8.42E+05	1	0.00	2.21E+04
ICP.f.Pb*	3.65E+02	9.38E+03	1	0.00	1.60E+03
ICP.f.S	1.23E+03	1.13E+04	1	0.00	2.58E+03
ICP.f.Si	5.67E+03	5.41E+04	1	2.71E+03	8.62E+03
ICP.f.Sr	2.98E+02	6.24E+01	1	1.97E+02	3.98E+02
ICP.f.Ti	4.79E+01	6.09E+02	1	0.00	3.62E+02
ICP.f.V*	1.47E+01	8.32E-01	1	3.06E+00	2.62E+01
ICP.f.Zn*	1.06E+02	7.17E+00	1	7.22E+01	1.40E+02
ICP.w.Al*	1.09E+01	5.75E+00	1	0.00	4.14E+01
ICP.w.B*	4.07E+00	6.77E-01	1	0.00	1.45E+01
ICP.w.Bi	2.02E+02	2.40E+03	1	0.00	8.24E+02
ICP.w.Ca*	6.16E+01	3.32E+01	1	0.00	1.35E+02
ICP.w.Cr	2.18E+02	2.45E+01	1	1.55E+02	2.81E+02
ICP.w.Fe	1.28E+02	3.11E+02	1	0.00	3.52E+02
ICP.w.K	7.19E+02	1.54E+03	1	2.21E+02	1.22E+03
ICP.w.La*	1.10E+01	1.42E+01	1	0.00	5.89E+01
ICP.w.Mg*	3.64E+00	5.45E-02	1	6.75E-01	6.61E+00
ICP.w.Mn	2.47E+01	2.36E+01	1	0.00	8.65E+01
ICP.w.Na	3.30E+04	2.44E+06	1	1.31E+04	5.28E+04
ICP.w.P	5.68E+03	3.24E+04	1	3.39E+03	7.97E+03
ICP.w.S	1.15E+03	2.38E+03	1	5.29E+02	1.77E+03
ICP.w.Si	5.72E+02	5.35E+03	1	0.00	1.50E+03
ICP.w.Sr*	1.96E+00	8.65E-02	1	0.00	5.70E+00
IC.w.Cl	4.50E+02	1.11E+03	1	2.56E+01	8.74E+02
IC.w.F	2.30E+03	6.46E+05	1	0.00	1.25E+04
IC.w.NO3	4.12E+04	7.77E+06	1	5.82E+03	7.67E+04
IC.w.PO4	1.55E+04	1.53E+06	1	0.00	3.13E+04
IC.w.SO4	3.54E+03	2.85E+04	1	1.40E+03	5.69E+03

*: Analytes with a portion of the data below three times the DL.

*: Analytes with a portion of the data below ten times the DL.

Table 3-1. Concentration Estimate Statistics (Units $\mu\text{g/g}$
Except Radionuclides $\mu\text{Ci/g}$). (sheet 3 of 3)

Analyte	$\bar{y}$	$\sigma^2(\bar{y})$	df	95% LL	95% UL
GEA.Am-241	4.24E-02	2.61E-06	1	2.19E-02	6.29E-02
GEA.Cs-137	1.66E-01	3.35E-03	1	0.00	9.02E-01
Gross.alpha	3.73E-01	1.96E-04	1	1.95E-01	5.51E-01
Gross.beta	1.51E+01	3.48E+01	1	0.00	9.00E+01
TGA.%.H2O	7.65E+01	2.23E+01	1	1.64E+01	1.37E+02
Am-241*	4.26E-02	6.65E-25	1	4.26E-02	4.26E-02
Hg*	1.43E+00	1.53E-01	1	0.00	6.40E+00
Spec.w.N02	7.93E+02	8.76E+03	1	0.00	1.98E+03
Percent.H2O	7.60E+01	5.81E-01	1	6.63E+01	8.57E+01
Pu-239/40	1.39E-01	9.19E-06	1	1.00E-01	1.77E-01
Sr-90	5.41E+00	3.53E+00	1	0.00	2.93E+01
TOC*	3.12E+03	3.83E+05	1	0.00	1.10E+04
Tc-99*	7.92E-03	8.90E-06	1	0.00	4.58E-02
U*	2.79E+03	2.01E+05	1	0.00	8.50E+03
pH	9.98E+00	7.79E-03	1	8.86E+00	1.11E+01

*: Analytes with a portion of the data below three times the DL.

*: Analytes with a portion of the data below ten times the DL.

4.0 COMPARISON OF THE VARIANCE COMPONENT ESTIMATES

Using the hierarchical structure of the core composite data, estimates of the spatial variability, the sample composite variability, and the analytical measurement variability can be obtained. The spatial variance [$\sigma^2(S)$] is a measure of the variability between cores. The sample composite variance [$\sigma^2(C)$] measures the variability between composite samples within the same core. The analytical measurement variance [$\sigma^2(A)$] includes, among other things, the sample handling variability and the chemical analysis variability. This variance is a measure of the difference between the analytical results from the primary and duplicate samples.

The estimate of the variance of the mean is a linear function of the spatial, sample composite, and analytical measurement variances. To help evaluate the magnitude of these three variance components, estimates of each variance component are given in Table 4-1.

4.1 STATISTICAL METHODS

Estimates of the spatial variance [$\hat{\sigma}^2(S)$], sample composite variance [$\hat{\sigma}^2(C)$], and analytical measurement variance [$\hat{\sigma}^2(A)$] were obtained for each analyte using Restricted Maximum Likelihood Estimation (REML) methods. The REML methods are discussed in Harville (1977).

To test the significance of the variance components, an ANOVA was calculated using the hierarchical statistical model described in Appendix C. The mean square error terms in the ANOVA tables were used to perform F-tests on the spatial variability and F-tests on the composite variability. These tests determine whether or not $\sigma^2(S)$ and $\sigma^2(C)$ are significantly different than zero. The p-values given in Table 4-1 were derived from the results of these tests. The p-value is the attained level of significance of the statistical test.

4.2 STATISTICAL RESULTS

The REML estimates of each component of variability, along with the p-values from the F-tests, are given in Table 4-1. A p-value less than 0.05 indicates that $\sigma^2(S)$ or $\sigma^2(C)$ is significantly different from zero at the 0.05 level of significance.

The p-values from the tests on $\sigma^2(S)$ were less than 0.05 for 39 out of the 85 analytes. For these 39 cases, the two cores were, statistically, significantly different from each other.

The p-values from the tests on $\sigma^2(C)$ were less than 0.05 for (a different) 39 out of the 85 analytes. This implies, that for these 39 analytes, the two composite samples were, statistically, significantly different from each other. Conversely, for 46 out of 85 analytes, differences between composite samples were not, statistically, significantly different from each other.

Table 4-1. Variance Component Estimates. (sheet 1 of 3)

Analyte	$\sigma^2(s)$	Test: $\sigma^2(s)=0$ p-value	$\sigma^2(c)$	Test: $\sigma^2(c)=0$ p-value	$\sigma^2(A)$
ICP.a.Ag	1.56E+04	0.001	1.94E+02	0.000	3.16E+00
ICP.a.Al	1.89E+04	0.055	4.79E+03	0.000	1.06E+01
ICP.a.B	1.25E+01	0.093	9.28E-01	0.387	8.59E+00
ICP.a.Ba	6.91E+01	0.263	1.20E+02	0.000	3.54E-01
ICP.a.Bi	1.27E+07	0.000	6.25E+03	0.360	3.75E+04
ICP.a.Ca	4.10E+05	0.010	2.17E+04	0.074	9.95E+03
ICP.a.Cd*	5.96E+00	0.004	1.90E-01	0.039	5.34E-02
ICP.a.Ce*	1.69E+01	0.064	9.66E-01	0.380	7.87E+00
ICP.a.Co*	8.92E-01	0.376	8.08E-01	0.389	7.84E+00
ICP.a.Cr	3.17E+04	0.007	1.36E+03	0.081	6.75E+02
ICP.a.Cu	3.39E+02	0.290	1.67E+02	0.369	1.14E+03
ICP.a.Fe	2.33E+06	0.013	1.55E+05	0.063	6.25E+04
ICP.a.K	2.45E-24	0.911	8.86E+03	0.000	1.75E+02
ICP.a.La	5.91E+05	0.003	1.63E+04	0.019	2.88E+03
ICP.a.Mg	1.25E+04	0.005	5.18E+02	0.001	1.58E+01
ICP.a.Mn	2.97E+04	0.254	4.29E+04	0.030	1.00E+04
ICP.a.Na	2.76E+06	0.057	6.73E+05	0.016	1.05E+05
ICP.a.Ni	1.02E+03	0.000	6.13E+00	0.091	3.38E+00
ICP.a.P	2.37E-22	0.608	4.77E+05	0.000	1.25E+04
ICP.a.Pb	5.20E+04	0.003	1.41E+03	0.000	1.91E+01
ICP.a.S	1.11E+03	0.268	1.94E+03	0.003	1.25E+02
ICP.a.Sb*	1.11E-20	0.739	1.44E-14	0.578	1.32E+02
ICP.a.Si	2.57E-31	0.946	7.61E-14	0.418	7.34E+03
ICP.a.Sr	6.38E+02	0.079	2.18E+02	0.004	1.54E+01
ICP.a.Ti	2.75E+02	0.001	4.69E+00	0.000	7.83E-02
ICP.a.V	1.83E+00	0.423	2.25E+01	0.000	2.30E-01
ICP.a.Zn	1.22E+03	0.021	1.35E+02	0.008	1.42E+01
ICP.f.Ag	1.61E+04	0.000	6.23E+00	0.295	1.83E+01
ICP.f.Al	1.90E+04	0.005	6.78E+02	0.030	1.57E+02
ICP.f.Ba	3.99E+01	0.102	1.71E+01	0.012	2.19E+00
ICP.f.Bi	1.81E+07	0.001	2.88E+04	0.392	2.94E+05
ICP.f.Ca*	1.51E+05	0.008	2.07E-19	0.804	5.86E+04
ICP.f.Cd*	2.17E+00	0.126	1.21E-20	0.649	5.38E+00
ICP.f.Co*	1.44E+00	0.078	1.01E-29	0.755	3.48E+00
ICP.f.Cr	3.72E-15	0.994	5.63E+03	0.013	1.21E+03

*: Analytes with a portion of the data below three times the DL.

*: Analytes with a portion of the data below ten times the DL.

Table 4-1. Variance Component Estimates. (sheet 2 of 3)

Analyte	$\sigma^2(S)$	Test: $\sigma^2(S)=0$ p-value	$\sigma^2(C)$	Test: $\sigma^2(C)=0$ p-value	$\sigma^2(A)$
ICP.f.Cu*	6.97E+01	0.005	2.00E+00	0.119	1.43E+00
ICP.f.Fe	8.00E+06	0.003	1.68E+05	0.079	8.13E+04
ICP.f.La	5.96E+05	0.010	3.82E+04	0.012	4.98E+03
ICP.f.Mg	1.46E+04	0.000	5.15E-44	0.913	1.60E+02
ICP.f.Mn	1.66E-11	0.553	7.02E+04	0.011	9.70E+03
ICP.f.Na	1.17E+07	0.007	5.43E+05	0.044	1.65E+05
ICP.f.Ni	1.17E+07	0.033	4.18E-12	0.498	4.39E+06
ICP.f.P	1.57E+06	0.028	1.95E+05	0.049	6.45E+04
ICP.f.Pb*	1.85E+04	0.003	4.35E+02	0.030	1.02E+02
ICP.f.S	2.15E+04	0.017	2.00E+03	0.008	2.13E+02
ICP.f.Si	1.06E+05	0.005	1.00E+03	0.369	6.83E+03
ICP.f.Sr	6.12E-16	0.661	1.15E+02	0.222	2.69E+02
ICP.f.Ti	1.22E+03	0.000	2.66E-01	0.331	1.12E+00
ICP.f.V*	5.86E-19	0.905	2.17E+00	0.107	2.31E+00
ICP.f.Zn*	5.69E-23	0.440	1.10E-22	0.839	5.74E+01
ICP.w.Al*	7.35E+00	0.208	6.93E+00	0.061	2.72E+00
ICP.w.B*	1.14E+00	0.082	3.99E-01	0.008	3.99E-02
ICP.w.Bi	3.90E+03	0.102	1.30E+03	0.123	9.66E+02
ICP.w.Ca*	1.86E-11	0.144	1.69E-22	0.862	2.66E+02
ICP.w.Cr	5.66E-20	0.979	9.52E+01	0.001	5.62E+00
ICP.w.Fe	6.16E+01	0.436	9.05E+02	0.077	4.29E+02
ICP.w.K	2.28E+03	0.148	1.57E+03	0.001	4.73E+01
ICP.w.La*	2.71E+01	0.015	1.34E+00	0.226	2.23E+00
ICP.w.Mg*	3.04E-21	0.597	1.61E-01	0.098	1.14E-01
ICP.w.Mn	2.51E+01	0.266	3.19E+01	0.130	2.51E+01
ICP.w.Na	4.53E+06	0.030	6.84E+05	0.002	2.88E+04
ICP.w.P	3.98E+04	0.222	3.94E+04	0.089	2.13E+04
ICP.w.S	3.95E+03	0.091	1.50E+03	0.014	2.13E+02
ICP.w.Si	8.53E+03	0.113	1.98E+03	0.272	4.72E+03
ICP.w.Sr*	6.35E-02	0.340	1.83E-01	0.061	7.19E-02
IC.w.Cl	1.99E+03	0.043	1.15E-10	0.509	9.66E+02
IC.w.F	1.28E+06	0.001	1.37E+04	0.102	8.44E+03
IC.w.NO3	1.42E+07	0.038	2.46E+06	0.013	3.44E+05
IC.w.PO4	2.59E+06	0.082	8.54E+05	0.030	1.99E+05

*: Analytes with a portion of the data below three times the DL.

*: Analytes with a portion of the data below ten times the DL.

Table 4-1. Variance Component Estimates. (sheet 3 of 3)

Analyte	$\sigma^2(S)$	Test: $\sigma^2(S)=0$ p-value	$\sigma^2(C)$	Test: $\sigma^2(C)=0$ p-value	$\sigma^2(A)$
IC.w.S04	5.27E+04	0.032	7.04E+03	0.071	3.14E+03
GEA.Am-241	2.99E-32	0.786	8.13E-06	0.062	4.59E-06
GEA.Cs-137	6.61E-03	0.003	1.87E-04	0.000	1.88E-06
Gross.alpha	3.42E-04	0.065	7.74E-05	0.098	4.58E-05
Gross.beta	6.93E+01	0.000	3.14E-01	0.026	6.62E-02
TGA.%.H2O	3.17E+01	0.008	3.78E-33	0.935	5.18E+01
Am-241*	6.65E-25	0.976	7.41E-06	0.173	1.66E-05
Hg*	1.53E-01	0.022	1.36E-02	0.115	9.38E-03
Spec.w.NO2	1.92E-14	0.789	3.46E+04	0.000	9.50E+02
Percent.H2O	8.99E-01	0.128	2.55E-01	0.259	5.46E-01
Pu-239/40	2.49E-28	0.736	2.01E-05	0.171	3.33E-05
Sr-90	7.04E+00	0.000	1.79E-02	0.209	2.61E-02
Total organic carbon (TOC)*	6.10E+05	0.113	2.98E+05	0.006	2.72E+04
Tc-99*	1.76E-05	0.001	2.63E-07	0.027	5.75E-08
U*	6.24E-06	0.831	7.91E+05	0.001	2.99E+04
pH	7.11E-28	0.601	3.06E-02	0.001	1.19E-03

*: Analytes with a portion of the data below three times the DL.

*: Analytes with a portion of the data below ten times the DL.

5.0 REFERENCES

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APPENDIX A
CORE COMPOSITE DATA

WHC-SD-WM-TI-650, Rev. 0

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Table A-1. Core Composite Data (Units $\mu\text{g/g}$ except Radionuclides $\mu\text{Ci/g}$). (sheet 1 of 9)

Analyte	Core 31 Comp. 1	Mean/DL	Core 31 Comp. 2	Mean/DL	Core 33 Comp. 1	Mean/DL	Core 33 Comp. 2	Mean/DL
ICP.a.Ag	2.02E+02 2.03E+02	4.05E+02	2.25E+02 2.28E+02	4.53E+02	4.48E+01 4.39E+01	8.87E+01	2.81E+01 3.19E+01	6.00E+01
ICP.a.Al	5.80E+02 5.88E+02	2.43E+02	7.03E+02 7.07E+02	2.94E+02	4.71E+02 4.73E+02	1.97E+02	4.05E+02 4.04E+02	1.69E+02
ICP.a.As	■3.13E+00 3.17E+00	NA	■3.02E+00 ■2.81E+00	NA	3.39E+00 3.50E+00	1.15E+00	3.23E+00 ■2.90E+00	NA
ICP.a.B	2.34E+01 3.08E+01	4.52E+01	2.53E+01 2.16E+01	3.91E+01	2.92E+01 2.96E+01	4.90E+01	3.20E+01 3.23E+01	5.36E+01
ICP.a.Ba	5.66E+01 5.75E+01	1.90E+02	6.53E+01 6.46E+01	2.17E+02	6.67E+01 6.70E+01	2.23E+02	8.67E+01 8.79E+01	2.91E+02
ICP.a.Be	■1.04E-01 1.06E-01	NA	■1.01E-01 ■9.38E-02	NA	1.13E-01 1.20E-01	1.17E+00	1.08E-01 ■1.00E-01	NA
ICP.a.Bi	2.34E+04 2.37E+04	3.14E+03	2.33E+04 2.32E+04	3.10E+03	2.84E+04 2.86E+04	3.80E+03	2.86E+04 2.82E+04	3.79E+03
ICP.a.Ca	2.13E+03 2.26E+03	4.99E+02	2.36E+03 2.61E+03	5.65E+02	1.49E+03 1.50E+03	3.40E+02	1.35E+03 1.34E+03	3.06E+02
ICP.a.Cd*	7.19E+00 7.25E+00	1.81E+01	7.78E+00 7.94E+00	1.97E+01	4.09E+00 4.70E+00	1.10E+01	3.64E+00 3.80E+00	9.30E+00
ICP.a.Ce*	3.39E+01 3.13E+01	3.23E+00	2.90E+01 2.83E+01	2.84E+00	4.00E+01 3.57E+01	3.75E+00	3.28E+01 3.89E+01	3.55E+00
ICP.a.Co*	3.42E+00 3.38E+00	4.25E+00	3.79E+00 1.17E+01	9.68E+00	3.15E+00 3.10E+00	3.91E+00	3.13E+00 2.70E+00	3.64E+00
ICP.a.Cr	1.83E+03 1.89E+03	2.07E+03	1.84E+03 1.84E+03	2.04E+03	2.05E+03 2.08E+03	2.29E+03	2.16E+03 2.13E+03	2.38E+03
ICP.a.Cu	2.48E+01 2.57E+01	6.31E+01	1.27E+02 3.17E+01	1.98E+02	1.64E+01 1.65E+01	4.11E+01	1.29E+01 1.30E+01	3.24E+01
ICP.a.Fe	1.89E+04 1.95E+04	1.92E+04	2.01E+04 2.00E+04	2.01E+04	1.74E+04 1.76E+04	1.75E+04	1.75E+04 1.72E+04	1.74E+04
ICP.a.K	1.08E+03 1.11E+03	9.78E+01	1.22E+03 1.20E+03	1.08E+02	1.21E+03 1.22E+03	1.08E+02	1.02E+03 1.02E+03	9.11E+01

NA: Not available.

■ : Value less than DL or less than five percent above the DL.

• : Analytes with a portion of the data below three times the DL.

* : Analytes with a portion of the data below ten times the DL.

Table A-1. Core Composite Data (Units $\mu\text{g/g}$ except Radionuclides $\mu\text{Ci/g}$). (sheet 2 of 9)

Analyte	Core 31 Comp. 1	Mean/DL	Core 31 Comp. 2	Mean/DL	Core 33 Comp. 1	Mean/DL	Core 33 Comp. 2	Mean/DL
ICP.a.La	3.66E+03 3.78E+03	2.66E+03	3.64E+03 3.59E+03	2.58E+03	4.61E+03 4.67E+03	3.31E+03	4.91E+03 4.86E+03	3.49E+03
ICP.a.Mg	4.32E+02 4.38E+02	1.45E+03	4.75E+02 4.82E+02	1.60E+03	3.02E+02 3.07E+02	1.02E+03	2.88E+02 2.92E+02	9.67E+02
ICP.a.Mn	6.07E+03 6.31E+03	3.10E+04	6.14E+03 6.14E+03	3.07E+04	6.65E+03 6.77E+03	3.36E+04	6.32E+03 6.23E+03	3.14E+04
ICP.a.Na	3.71E+04 3.80E+04	1.21E+04	3.87E+04 3.86E+04	1.25E+04	3.49E+04 3.50E+04	1.13E+04	3.63E+04 3.62E+04	1.17E+04
ICP.a.Ni	1.49E+02 1.54E+02	8.91E+01	1.57E+02 1.57E+02	9.24E+01	1.09E+02 1.10E+02	6.44E+01	1.09E+02 1.08E+02	6.38E+01
ICP.a.P	1.02E+04 1.00E+04	1.74E+03	9.94E+03 9.98E+03	1.72E+03	9.97E+03 9.75E+03	1.70E+03	1.13E+04 1.14E+04	1.96E+03
ICP.a.Pb	4.69E+02 4.81E+02	7.66E+01	5.42E+02 5.44E+02	8.76E+01	2.00E+02 2.01E+02	3.23E+01	1.67E+02 1.69E+02	2.71E+01
ICP.a.S	1.21E+03 1.24E+03	4.54E+02	1.26E+03 1.27E+03	4.69E+02	1.14E+03 1.14E+03	4.22E+02	1.22E+03 1.22E+03	4.52E+02
ICP.a.Sb*	3.90E+01 2.19E+01	1.72E+00	3.61E+01 3.69E+01	2.06E+00	2.00E+01 5.18E+01	2.03E+00	1.91E+01 2.61E+01	1.28E+00
ICP.a.Se	7.92E+00 8.04E+00	NA	7.66E+00 7.13E+00	NA	8.60E+00 1.20E+01	1.36E+00	8.18E+00 7.40E+00	NA
ICP.a.Si	5.29E+02 4.36E+02	3.71E+02	4.18E+02 5.24E+02	3.62E+02	5.75E+02 4.80E+02	4.06E+02	4.90E+02 2.98E+02	3.03E+02
ICP.a.Sn	4.28E+00 4.13E+00	2.63E+00	1.74E+00 3.13E+00	1.52E+00	1.81E+00 1.80E+00	1.13E+00	1.72E+00 1.50E+00	NA
ICP.a.Sr	2.78E+02 2.85E+02	9.38E+02	2.80E+02 2.80E+02	9.33E+02	3.01E+02 3.08E+02	1.02E+03	3.37E+02 3.32E+02	1.12E+03
ICP.a.Ti	2.96E+01 2.93E+01	7.36E+01	3.34E+01 3.27E+01	8.26E+01	9.00E+00 8.80E+00	2.23E+01	6.42E+00 6.50E+00	1.62E+01
ICP.a.V	1.23E+01 1.31E+01	2.54E+01	2.13E+01 2.15E+01	4.28E+01	1.35E+01 1.42E+01	2.77E+01	9.58E+00 1.04E+01	2.00E+01

NA: Not available.

■ : Value less than DL or less than five percent above the DL.

• : Analytes with a portion of the data below three times the DL.

* : Analytes with a portion of the data below ten times the DL.

Table A-1. Core Composite Data (Units $\mu\text{g/g}$ except Radionuclides $\mu\text{Ci/g}$). (sheet 3 of 9)

Analyte	Core 31 Comp. 1	Mean/DL	Core 31 Comp. 2	Mean/DL	Core 33 Comp. 1	Mean/DL	Core 33 Comp. 2	Mean/DL
ICP.a.Zn	7.68E+01 8.20E+01	2.65E+02	9.68E+01 1.06E+02	3.38E+02	4.36E+01 4.47E+01	1.47E+02	3.45E+01 3.54E+01	1.17E+02
ICP.a.Zr	8.34E-01 8.47E-01	NA	8.07E-01 7.51E-01	NA	9.05E-01 9.20E-01	NA	8.61E-01 7.70E-01	NA
ICP.f.Ag	2.10E+02 2.18E+02	8.56E+01	2.17E+02 2.26E+02	8.86E+01	3.89E+01 4.00E+01	1.36E+01	3.69E+01 3.74E+01	1.28E+01
ICP.f.Al	6.32E+02 6.56E+02	5.37E+01	6.80E+02 7.06E+02	5.78E+01	4.83E+02 4.85E+02	4.03E+01	4.59E+02 4.59E+02	3.83E+01
ICP.f.As	■1.50E+01 ■1.50E+01	NA	■1.50E+01 ■1.50E+01	NA	■1.50E+01 ■1.50E+01	NA	■1.50E+02 ■1.50E+02	NA
ICP.f.B	■3.01E+00 ■3.00E+00	NA	■2.99E+00 ■2.99E+00	NA	■5.36E+00 ■4.32E+00	NA	■4.58E+00 ■5.10E+00	NA
ICP.f.Ba	5.72E+01 6.04E+01	3.92E+01	5.96E+01 6.15E+01	4.04E+01	6.60E+01 6.48E+01	4.36E+01	7.45E+01 7.30E+01	4.92E+01
ICP.f.Be	■5.01E-01 ■5.00E-01	NA	■4.99E-01 ■4.99E-01	NA	■4.99E-01 ■5.00E-01	NA	■5.00E-01 ■4.99E-01	NA
ICP.f.Bi	2.05E+04 2.14E+04	5.59E+02	2.02E+04 2.01E+04	5.37E+02	2.66E+04 2.63E+04	7.05E+02	2.73E+04 2.61E+04	7.12E+02
ICP.f.Ca*	2.58E+03 2.93E+03	1.25E+02	2.49E+03 2.83E+03	1.21E+02	2.50E+03 1.93E+03	9.23E+00	2.18E+03 1.92E+03	8.54E+00
ICP.f.Cd*	7.34E+00 9.16E+00	4.13E+00	7.18E+00 1.41E+01	5.32E+00	6.76E+00 6.08E+00	3.21E+00	6.86E+00 7.48E+00	3.59E+00
ICP.f.Ce	■5.06E+01 ■5.05E+01	NA	■5.04E+01 ■5.04E+01	NA	■5.04E+01 ■5.05E+01	NA	■5.05E+01 ■5.04E+01	NA
ICP.f.Co*	9.70E+00 1.05E+01	2.53E+00	1.11E+01 1.05E+01	2.70E+00	1.37E+01 1.30E+01	3.34E+00	8.90E+00 1.48E+01	2.96E+00
ICP.f.Cr	1.86E+03 1.92E+03	4.20E+02	1.67E+03 1.73E+03	3.78E+02	1.81E+03 1.76E+03	3.97E+02	1.82E+03 1.82E+03	4.04E+02
ICP.f.Cu*	3.68E+01 3.59E+01	1.82E+01	3.41E+01 3.42E+01	1.71E+01	2.16E+01 2.26E+01	9.82E+00	2.30E+01 2.61E+01	1.09E+01

NA: Not available.

■ : Value less than DL or less than five percent above the DL.

• : Analytes with a portion of the data below three times the DL.

* : Analytes with a portion of the data below ten times the DL.

Table A-1. Core Composite Data (Units $\mu\text{g/g}$ except Radionuclides $\mu\text{Ci/g}$). (sheet 4 of 9)

Analyte	Core 31 Comp. 1	Mean/DL	Core 31 Comp. 2	Mean/DL	Core 33 Comp. 1	Mean/DL	Core 33 Comp. 2	Mean/DL
ICP.f.Fe	2.02E+04 2.08E+04	4.10E+03	1.95E+04 1.97E+04	3.92E+03	1.62E+04 1.57E+04	6.16E+02	1.61E+04 1.61E+04	6.22E+02
ICP.f.K	■5.61E+01 *5.60E+01	NA	■5.59E+01 ■5.59E+01	NA	■5.59E+01 ■5.60E+01	NA	■5.60E+01 ■5.59E+01	NA
ICP.f.La	3.63E+03 3.75E+03	5.27E+02	3.38E+03 3.45E+03	4.88E+02	4.58E+03 4.45E+03	6.45E+02	4.84E+03 4.78E+03	6.87E+02
ICP.f.Mg	4.24E+02 4.52E+02	2.92E+02	4.31E+02 4.56E+02	2.96E+02	2.78E+02 2.58E+02	3.46E+01	2.74E+02 2.69E+02	3.50E+01
ICP.f.Mn	6.29E+03 6.47E+03	6.38E+03	5.86E+03 6.02E+03	5.94E+03	6.29E+03 6.15E+03	4.01E+02	6.59E+03 6.59E+03	4.25E+02
ICP.f.Na	3.96E+04 4.01E+04	2.57E+03	3.85E+04 3.94E+04	2.51E+03	3.41E+04 3.36E+04	5.34E+01	3.51E+04 3.52E+04	5.54E+01
ICP.f.Ni	5.94E+03 5.61E+03	6.79E+02	7.49E+03 3.40E+03	6.41E+02	9.49E+03 9.09E+03	8.68E+01	9.81E+03 1.43E+04	1.13E+02
ICP.f.P	1.12E+04 1.19E+04	3.98E+02	1.11E+04 1.11E+04	3.83E+02	8.99E+03 9.15E+03	1.04E+02	9.91E+03 9.91E+03	1.13E+02
ICP.f.Pb*	4.27E+02 4.53E+02	1.42E+01	4.82E+02 4.86E+02	1.56E+01	2.62E+02 2.72E+02	8.61E+00	2.72E+02 2.67E+02	8.69E+00
ICP.f.S	1.35E+03 1.36E+03	1.00E+02	1.29E+03 1.33E+03	9.70E+01	1.08E+03 1.08E+03	8.00E+01	1.16E+03 1.16E+03	8.59E+01
ICP.f.Sb	■8.87E+01 ■8.85E+01	NA	■8.83E+01 ■8.83E+01	NA	1.29E+02 ■8.85E+01	NA	■8.85E+01 ■8.83E+01	NA
ICP.f.Se	■3.81E+01 ■3.80E+01	NA	■3.79E+01 ■3.79E+01	NA	■3.79E+01 ■3.80E+01	NA	■3.80E+01 ■3.79E+01	NA
ICP.f.Si	5.88E+03 6.04E+03	9.17E+02	5.78E+03 5.89E+03	8.98E+02	5.52E+03 5.39E+03	7.43E+01	5.41E+03 5.41E+03	7.37E+01
ICP.f.Sn	■8.02E+00 ■8.00E+00	NA	■7.98E+00 ■7.98E+00	NA	■7.98E+00 ■8.00E+00	NA	■8.00E+00 ■7.98E+00	NA
ICP.f.Sr	3.08E+02 2.98E+02	2.02E+02	2.70E+02 2.91E+02	1.87E+02	2.97E+02 2.84E+02	6.65E+01	2.98E+02 3.36E+02	7.25E+01

NA: Not available.

■ : Value less than DL or less than five percent above the DL.

• : Analytes with a portion of the data below three times the DL.

* : Analytes with a portion of the data below ten times the DL.

Table A-1. Core Composite Data (Units $\mu\text{g/g}$ except Radionuclides $\mu\text{Ci/g}$). (sheet 5 of 9)

Analyte	Core 31 Comp. 1	Mean/DL	Core 31 Comp. 2	Mean/DL	Core 33 Comp. 1	Mean/DL	Core 33 Comp. 2	Mean/DL
ICP.f.Ti	7.32E+01 7.25E+01	3.64E+01	7.13E+01 7.34E+01	3.62E+01	2.25E+01 2.22E+01	1.12E+01	2.51E+01 2.31E+01	1.21E+01
ICP.f.V*	1.11E+01 1.32E+01	4.86E+00	1.55E+01 1.74E+01	6.58E+00	1.38E+01 1.69E+01	6.14E+00	1.42E+01 1.51E+01	5.86E+00
ICP.f.Zn*	9.71E+01 1.11E+02	6.94E+01	1.11E+02 1.00E+02	7.03E+01	1.10E+02 1.00E+02	7.66E+00	1.02E+02 1.19E+02	8.07E+00
ICP.f.Zr	4.01E+00 4.00E+00	NA	3.99E+00 3.99E+00	NA	3.99E+00 4.00E+00	NA	4.00E+00 3.99E+00	NA
ICP.w.Ag	7.80E-01 5.00E-01	NA	1.26E+00 1.07E+00	2.33E+00	4.99E-01 6.08E-01	NA	4.99E-01 5.00E-01	NA
ICP.w.Al*	6.99E+00 6.43E+00	2.80E+00	1.04E+01 1.02E+01	4.29E+00	1.34E+01 1.77E+01	6.48E+00	1.19E+01 1.02E+01	4.60E+00
ICP.w.As	3.00E+00 3.00E+00	NA	3.00E+00 3.00E+00	NA	2.99E+00 2.99E+00	NA	2.99E+00 3.00E+00	NA
ICP.w.B*	3.50E+00 3.11E+00	5.51E+00	3.27E+00 3.12E+00	5.33E+00	5.54E+00 5.54E+00	9.23E+00	4.44E+00 4.06E+00	7.08E+00
ICP.w.Ba	3.09E-01 3.00E-01	NA	3.00E-01 5.32E-01	NA	3.78E-01 7.15E-01	1.82E+00	5.32E-01 5.01E-01	1.72E+00
ICP.w.Be	1.00E-01 9.99E-02	NA	9.99E-02 9.99E-02	NA	9.98E-02 9.98E-02	NA	9.98E-02 9.99E-02	NA
ICP.w.Bi	8.34E+01 1.47E+02	1.54E+01	1.76E+02 2.05E+02	2.54E+01	2.05E+02 2.58E+02	3.09E+01	2.73E+02 2.67E+02	3.60E+01
ICP.w.Ca*	5.05E+01 5.12E+01	1.16E+01	5.43E+01 6.89E+01	1.40E+01	7.16E+01 6.13E+01	1.51E+01	9.35E+01 4.15E+01	1.53E+01
ICP.w.Cd	4.00E-01 4.00E-01	NA	4.00E-01 4.00E-01	NA	3.99E-01 3.99E-01	NA	3.99E-01 4.00E-01	NA
ICP.w.Ce	1.01E+01 1.01E+01	NA	1.01E+01 1.01E+01	NA	1.01E+01 1.01E+01	NA	1.01E+01 1.01E+01	NA
ICP.w.Co	8.00E-01 7.99E-01	NA	7.99E-01 8.51E-01	NA	7.98E-01 8.43E-01	NA	8.50E-01 7.99E-01	NA

NA: Not available.

■ : Value less than DL or less than five percent above the DL.

• : Analytes with a portion of the data below three times the DL.

* : Analytes with a portion of the data below ten times the DL.

Table A-1. Core Composite Data (Units $\mu\text{g/g}$ except Radionuclides $\mu\text{Ci/g}$). (sheet 6 of 9)

Analyte	Core 31 Comp. 1	Mean/DL	Core 31 Comp. 2	Mean/DL	Core 33 Comp. 1	Mean/DL	Core 33 Comp. 2	Mean/DL
ICP.w.Cr	2.11E+02 2.07E+02	2.32E+02	2.28E+02 2.30E+02	2.54E+02	2.22E+02 2.26E+02	2.49E+02	2.12E+02 2.09E+02	2.34E+02
ICP.w.Cu	■4.00E-01 ■4.00E-01	NA	■4.00E-01 ■4.00E-01	NA	■3.99E-01 ■3.99E-01	NA	■3.99E-01 ■4.00E-01	NA
ICP.w.Fe	5.70E+01 1.02E+02	7.95E+01	1.30E+02 1.51E+02	1.41E+02	1.16E+02 1.47E+02	1.32E+02	1.60E+02 1.58E+02	1.59E+02
ICP.w.K	7.40E+02 7.28E+02	6.55E+01	7.83E+02 7.83E+02	6.99E+01	7.04E+02 7.19E+02	6.35E+01	6.47E+02 6.50E+02	5.79E+01
ICP.w.La*	5.00E+00 7.05E+00	4.30E+00	7.69E+00 9.35E+00	6.09E+00	1.22E+01 1.55E+01	9.89E+00	1.57E+01 1.58E+01	1.13E+01
ICP.w.Mg*	2.67E+00 3.22E+00	9.82E+00	3.66E+00 4.23E+00	1.32E+01	3.60E+00 4.08E+00	1.28E+01	3.95E+00 3.72E+00	1.28E+01
ICP.w.Mn	1.01E+01 1.93E+01	7.35E+01	2.31E+01 2.70E+01	1.25E+02	2.04E+01 3.04E+01	1.27E+02	3.34E+01 3.42E+01	1.69E+02
ICP.w.Na	3.41E+04 3.39E+04	1.10E+04	3.51E+04 3.50E+04	1.13E+04	3.06E+04 3.09E+04	9.92E+03	3.19E+04 3.22E+04	1.03E+04
ICP.w.Ni	■1.70E+00 ■1.70E+00	NA	■1.70E+00 ■1.70E+00	NA	■1.70E+00 ■1.70E+00	NA	■1.70E+00 ■1.70E+00	NA
ICP.w.P	5.63E+03 5.89E+03	9.93E+02	5.81E+03 6.11E+03	1.03E+03	5.26E+03 5.34E+03	9.14E+02	5.66E+03 5.74E+03	9.83E+02
ICP.w.Pb	7.91E+00 ■6.19E+00	NA	6.93E+00 8.92E+00	1.28E+00	■6.19E+00 ■6.28E+00	NA	■6.19E+00 ■6.19E+00	NA
ICP.w.S	1.21E+03 1.18E+03	4.43E+02	1.19E+03 1.21E+03	4.44E+02	1.05E+03 1.07E+03	3.93E+02	1.14E+03 1.14E+03	4.22E+02
ICP.w.Sb	■1.77E+01 ■1.77E+01	NA	■1.77E+01 ■1.77E+01	NA	■1.77E+01 ■1.77E+01	NA	■1.77E+01 ■1.77E+01	NA
ICP.w.Se	■7.60E+00 ■7.59E+00	NA	■7.59E+00 8.44E+00	NA	■7.58E+00 ■7.58E+00	NA	■7.58E+00 ■7.59E+00	NA
ICP.w.Si	3.45E+02 5.30E+02	3.37E+02	5.30E+02 5.89E+02	4.30E+02	6.68E+02 6.71E+02	5.15E+02	6.22E+02 6.18E+02	4.77E+02

NA: Not available.

■ : Value less than DL or less than five percent above the DL.

• : Analytes with a portion of the data below three times the DL.

* : Analytes with a portion of the data below ten times the DL.

Table A-1. Core Composite Data (Units $\mu\text{g/g}$ except Radionuclides $\mu\text{Ci/g}$). (sheet 7 of 9)

Analyte	Core 31 Comp. 1	Mean/DL	Core 31 Comp. 2	Mean/DL	Core 33 Comp. 1	Mean/DL	Core 33 Comp. 2	Mean/DL
ICP.W.Sn	■1.60E+00 ■1.60E+00	NA	■1.60E+00 ■1.60E+00	NA	■1.60E+00 ■1.60E+00	NA	■1.60E+00 ■1.60E+00	NA
ICP.W.Sr*	9.37E-01 1.48E+00	4.03E+00	1.97E+00 2.29E+00	7.10E+00	1.97E+00 2.38E+00	7.25E+00	2.39E+00 2.29E+00	7.80E+00
ICP.W.Ti	■4.00E-01 ■4.00E-01	NA	■4.00E-01 ■4.00E-01	NA	■3.99E-01 ■3.99E-01	NA	■3.99E-01 ■4.00E-01	NA
ICP.W.V	■5.00E-01 ■5.00E-01	NA	5.94E-01 8.20E-01	1.41E+00	■4.99E-01 ■4.99E-01	NA	9.21E-01 6.76E-01	1.60E+00
ICP.W.Zn	■3.00E-01 ■3.00E-01	NA	■3.00E-01 ■3.00E-01	NA	■2.99E-01 ■2.99E-01	NA	■2.99E-01 ■3.00E-01	NA
ICP.W.Zr	8.00E-01 7.99E-01	NA	7.99E-01 7.99E-01	NA	7.98E-01 7.98E-01	NA	7.98E-01 7.99E-01	NA
IC.W.Cl	4.73E+02 4.66E+02	4.70E+01	5.18E+02 4.75E+02	4.97E+01	4.40E+02 3.62E+02	4.01E+01	4.40E+02 4.23E+02	4.32E+01
IC.W.F	3.14E+03 3.03E+03	3.09E+02	3.16E+03 3.09E+03	3.13E+02	1.47E+03 1.26E+03	1.37E+02	1.67E+03 1.59E+03	1.63E+02
IC.W.NO2	<1.10E+03 <1.10E+03	NA	8.71E+02 <1.10E+03	NA	8.42E+02 7.04E+02	7.73E+00	7.59E+02 7.04E+02	7.32E+00
IC.W.NO3	4.45E+04 4.41E+04	4.43E+02	4.36E+04 4.39E+04	4.38E+02	3.76E+04 3.61E+04	3.69E+02	3.98E+04 4.03E+04	4.01E+02
IC.W.PO4	1.56E+04 1.67E+04	1.62E+02	1.71E+04 1.77E+04	1.74E+02	1.36E+04 1.35E+04	1.36E+02	1.51E+04 1.50E+04	1.51E+02
IC.W.SO4	3.69E+03 3.69E+03	3.69E+01	3.72E+03 3.75E+03	3.74E+01	3.23E+03 3.34E+03	3.29E+01	3.41E+03 3.52E+03	3.47E+01
GEA.Am-241	4.37E-02 4.80E-02	4.02E+01	4.02E-02 4.16E-02	3.59E+01	3.95E-02 3.79E-02	3.39E+01	4.24E-02 4.61E-02	3.88E+02
GEA.Co-60	<4.21E-04 <3.39E-04	NA	<3.85E-04 <3.75E-04	NA	<3.45E-04 <3.70E-04	NA	<3.29E-04 <3.45E-04	NA
GEA.Cs-137	2.12E-01 2.11E-01	5.72E+02	2.36E-01 2.38E-01	6.41E+02	1.15E-01 1.12E-01	3.07E+02	1.03E-01 1.04E-01	2.80E+02

NA: Not available.

■ : Value less than DL or less than five percent above the DL.

* : Analytes with a portion of the data below three times the DL.

* : Analytes with a portion of the data below ten times the DL.

Table A-1. Core Composite Data (Units $\mu\text{g/g}$ except Radionuclides $\mu\text{Ci/g}$). (sheet 8 of 9)

Analyte	Core 31 Comp. 1	Mean/DL	Core 31 Comp. 2	Mean/DL	Core 33 Comp. 1	Mean/DL	Core 33 Comp. 2	Mean/DL
GEA.Eu-154	■1.09E-03 ■1.07E-03	NA	<1.06E-03 3.24E-03	NA	<1.11E-03 <1.01E-03	NA	<1.05E-03 <9.64E-04	NA
GEA.Eu-155	<2.08E-03 <2.09E-03	NA	<2.13E-03 <2.12E-03	NA	2.97E-03 3.16E-03	5.15E+00	<1.49E-03 <1.49E-03	NA
Gross.alpha	3.59E-01 3.58E-01	5.05E+01	3.50E-01 3.69E-01	5.06E+01	3.78E-01 3.76E-01	7.11E+02	3.97E-01 3.97E-01	7.49E+02
Gross.beta	2.04E+01 2.07E+01	2.20E+02	2.16E+01 2.13E+01	2.29E+02	9.86E+00 9.32E+00	1.56E+02	8.95E+00 8.71E+00	1.44E+02
TIC.w.CO3	<5.00E+02 6.50E+02	NA	6.49E+02 9.99E+02	1.65E+00	7.49E+02 8.98E+02	1.65E+00	7.99E+02 1.40E+03	2.20E+00
TGA.%.H2O	6.13E+01 8.53E+01	NA	6.92E+01 7.12E+01	NA	8.10E+01 8.22E+01	NA	7.86E+01 8.30E+01	NA
Am-241*	4.43E-02 3.85E-02	7.26E+00	4.66E-02 3.96E-02	7.56E+00	3.71E-02 3.94E-02	6.59E+00	4.44E-02 5.11E-02	7.96E+00
C-14	3.80E-04 <2.25E-04	NA	<2.23E-04 <2.25E-04	NA	<2.25E-04 <2.25E-04	NA	<2.25E-04 <2.25E-04	NA
CN	<4.50E+00 <4.00E+00	NA	<3.41E+00 <3.57E+00	NA	<4.90E+00 <4.81E+00	NA	<4.76E+00 <4.61E+00	NA
H-3	<3.15E-04 <3.15E-04	NA	<3.12E-04 <3.15E-04	NA	<3.15E-04 <3.15E-04	NA	<3.15E-04 <3.15E-04	NA
Hg*	1.70E+00 1.48E+00	1.27E+01	1.79E+00 1.88E+00	1.47E+01	1.18E+00 1.22E+00	9.60E+00	1.02E+00 1.15E+00	8.68E+00
I-129	<1.74E-02 <1.72E-02	NA	<2.40E-02 <1.92E-02	NA	<2.15E-02 <1.44E-02	NA	<1.75E-02 <2.17E-02	NA
NH3	<4.50E+03 <4.50E+03	NA	<4.50E+03 <4.50E+03	NA	<4.50E+03 <4.50E+03	NA	<4.50E+03 <4.50E+03	NA
Spec.w.NO2	9.49E+02 9.55E+02	1.90E+01	5.22E+02 5.27E+02	1.05E+01	8.72E+02 8.84E+02	1.76E+01	7.74E+02 8.60E+02	1.63E+01
Np-237	<3.25E-02 <3.24E-02	NA	<3.24E-02 <3.24E-02	NA	<3.24E-02 <3.24E-02	NA	<3.24E-02 <3.24E-02	NA

NA: Not available.

■ : Value less than DL or less than five percent above the DL.

• : Analytes with a portion of the data below three times the DL.

* : Analytes with a portion of the data below ten times the DL.

TIC: Total inorganic carbon.

Table A-1. Core Composite Data (Units $\mu\text{g/g}$ except Radionuclides $\mu\text{Ci/g}$). (sheet 9 of 9)

Analyte	Core 31 Comp. 1	Mean/DL	Core 31 Comp. 2	Mean/DL	Core 33 Comp. 1	Mean/DL	Core 33 Comp. 2	Mean/DL
Percent.H2O	7.48E+01 7.44E+01	NA	7.59E+01 7.59E+01	NA	7.72E+01 7.57E+01	NA	7.78E+01 7.64E+01	NA
Pu-238	<1.04E-02 <1.07E-02	NA	<1.03E-02 <1.04E-02	NA	<1.05E-02 <1.00E-02	NA	<1.13E-02 <1.04E-02	NA
Pu-239/40	1.35E-01 1.41E-01	3.83E+01	1.34E-01 1.37E-01	3.76E+01	1.39E-01 1.29E-01	3.83E+01	1.53E-01 1.42E-01	4.21E+01
Se	<6.05E-05 <6.11E-05	NA	6.17E-05 6.31E-05	1.11E+00	<1.23E-04 <1.26E-04	NA	<1.29E-04 <1.26E-04	NA
Sr-90	6.97E+00 7.34E+00	1.01E+03	7.55E+00 7.31E+00	1.62E+03	3.67E+00 3.62E+00	1.70E+03	3.37E+00 3.48E+00	1.59E+03
TOC*	3.30E+03 3.68E+03	6.98E+00	4.12E+03 3.85E+03	7.97E+00	2.00E+03 2.00E+03	4.00E+00	3.00E+03 3.00E+03	6.00E+00
Tc-99*	4.95E-03 5.33E-03	6.05E+00	4.90E-03 4.56E-03	5.56E+00	1.12E-02 1.16E-02	2.48E+00	1.03E-02 1.05E-02	2.26E+00
U*	2.14E+03 2.21E+03	8.70E+00	4.00E+03 3.75E+03	1.55E+01	3.01E+03 3.34E+03	9.34E+00	2.07E+03 1.82E+03	5.72E+00
pH	1.02E+01 1.02E+01	NA	9.91E+00 9.94E+00	NA	1.01E+01 1.00E+01	NA	9.72E+00 9.81E+00	NA

NA: Not available.

- : Value less than DL or less than five percent above the DL.
- : Analytes with a portion of the data below three times the DL.
- * : Analytes with a portion of the data below ten times the DL.

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APPENDIX B
CORE COMPOSITE DATA PLOTS

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Composite 1 = O, Composite 2 = X
Units ug/g, except for radionuclides uCi/g

Composite 1 = O, Composite 2 = X
Units ug/g, except for radionuclides uCi/g

Composite 1 = O, Composite 2 = X
Units ug/g, except for radionuclides uCi/g

Composite 1 = O, Composite 2 = X
Units ug/g, except for radionuclides uCi/g

Composite 1 = O, Composite 2 = X
Units ug/g, except for radionuclides uCi/g

Composite 1 = O, Composite 2 = X
Units ug/g, except for radionuclides uCi/g

Composite 1 = O, Composite 2 = X
Units ug/g, except for radionuclides uCi/g

Composite 1 = O, Composite 2 = X
Units ug/g, except for radionuclides uCi/g

Composite 1 = O, Composite 2 = X
Units ug/g, except for radionuclides uCi/g

Composite 1 = O, Composite 2 = X
Units ug/g, except for radionuclides uCi/g

Composite 1 = O, Composite 2 = X
Units ug/g, except for radionuclides uCi/g

Composite 1 = O, Composite 2 = X
Units ug/g, except for radionuclides uCi/g

Composite 1 = O, Composite 2 = X
Units ug/g, except for radionuclides uCi/g

Composite 1 = O, Composite 2 = X
Units ug/g, except for radionuclides uCi/g

ICP.W.Sr

IC.W.Cl

B-19

IC.W.F

IC.W.NO3

Composite 1 = O, Composite 2 = X
Units ug/g, except for radionuclides uCi/g

IC.W.SO4

GEA.Cs-137

IC.W.PO4

GEA.Am-241

Composite 1 = O, Composite 2 = X
Units ug/g, except for radionuclides uCi/g

Composite 1 = O, Composite 2 = X
Units ug/g, except for radionuclides uCi/g

Composite 1 = O, Composite 2 = X
Units ug/g, except for radionuclides uCi/g

Composite 1 = O, Composite 2 = X
Units ug/g, except for radionuclides uCi/g

Composite 1 = O , Composite 2 = X

APPENDIX C
MEAN CONCENTRATION CALCULATION METHODS

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This appendix includes a description of the statistical model that describes the structure of the data from core samples taken from tank 241-T-111. Equations are also presented for estimates of the mean concentration, the variance for the mean concentration, and the CI on the mean concentration.

The statistical model that describes the structure of the core composite data is

$$y_{ijk} = \mu + S_i + C_{ij} + A_{ijk}, \quad i=1, \dots, a, \quad j=1, \dots, b_i, \quad k=1, \dots, n_{ij}, \quad (1)$$

where

- y_{ijk} = laboratory results from the k^{th} duplicate of the j^{th} composite of the i^{th} core from the tank
- μ = the grand mean
- S_i = the effect of the i^{th} core (spatial effect)
- C_{ij} = the effect of the j^{th} composite sample from the i^{th} core
- A_{ijk} = the analytical error associated with the k^{th} duplicate in the j^{th} composite from the i^{th} core
- a = the number of cores
- b_i = the number of composite samples in the i^{th} core
- n_{ij} = the number of analytical results from the j^{th} composite sample in the i^{th} core.

There are two core composite samples (i.e., $b_i = 2$) for cores 31 and 33.

The variables S_i and C_{ij} are treated as random effects. It is assumed that S_i , C_{ij} , and A_{ijk} are each distributed normally with mean zero and variances $\sigma^2(S)$, $\sigma^2(C)$, and $\sigma^2(A)$, respectively. Estimates of $\sigma^2(S)$, $\sigma^2(C)$, and $\sigma^2(A)$ were obtained using Restricted Maximum Likelihood Estimation (REML). This method applied to variance component estimation is described in Harville (1977).

The mean concentration of each analyte of interest in the tank was calculated using the following equation:

$$\bar{y} = \frac{1}{a} \sum_{i=1}^a \bar{y}_{i++} = \frac{1}{a} \sum_{i=1}^a \frac{\sum_{j=1}^{b_i} \sum_{k=1}^{n_{ij}} y_{ijk}}{n_{i+}} = \frac{1}{a} \sum_{i=1}^a \frac{\sum_{j=1}^{b_i} \sum_{k=1}^{n_{ij}} (\mu + S_i + C_{ij} + A_{ijk})}{n_{i+}}, \quad (2)$$

where

$$\bar{y}_{i++} = \frac{\sum_{j=1}^{b_i} \sum_{k=1}^{n_{ij}} y_{ijk}}{n_{i+}} \quad \text{and} \quad n_{i+} = \sum_{j=1}^{b_i} n_{ij}. \quad (3)$$

This mean gives the results from each core the same weight regardless of the unbalance that may exist for a particular analyte.

The variance of $\bar{y}$ is

$$V(\bar{y}) = C_1 \sigma^2(S) + C_2 \sigma^2(C) + C_3 \sigma^2(A) \quad (4)$$

where

$$C_1 = \frac{1}{a}, \quad C_2 = \frac{1}{a^2} \sum_{i=1}^a \left(\frac{1}{n_{i+}} \right)^2 \left[\sum_{j=1}^{b_i} n_{ij}^2 \right], \quad C_3 = \frac{1}{a^2} \sum_{i=1}^a \left(\frac{1}{n_{i+}} \right). \quad (5)$$

Using $\hat{\sigma}^2(S)$, $\hat{\sigma}^2(C)$, and $\hat{\sigma}^2(A)$ (REML variance component estimates), an estimated variance of $\bar{y}$ is

$$\hat{\sigma}^2(\bar{y}) = C_1 \hat{\sigma}^2(S) + C_2 \hat{\sigma}^2(C) + C_3 \hat{\sigma}^2(A). \quad (6)$$

The approximate degrees of freedom used for $\hat{\sigma}^2(\bar{y})$ is the number of cores with data minus one.

The lower and upper 95% CI limits (95% LL and 95% UL, respectively) on the mean concentration are

$$95\% \text{ LL} = \bar{y} - t_{0.975} \sqrt{\hat{\sigma}^2(\bar{y})} \quad \text{and} \quad 95\% \text{ UL} = \bar{y} + t_{0.975} \sqrt{\hat{\sigma}^2(\bar{y})} \quad (7)$$

where $t_{0.975}$ is the 0.975 percentile from a Student's t-distribution with the approximate degrees of freedom associated with $\hat{\sigma}^2(\bar{y})$.