Theater SBI Cost-Effectiveness Ratios

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ABSTRACT

To address M missiles spaced at intervals longer than the constellation reconstitution time t , the defense needs at the absentee ratio N_a of SBIs to fill the belt plus the M SBIs needed for the intercepts; the resulting cost effectiveness scales as $M/(M + N_a)$. N_a is large and CER small for small ranges and numbers of missiles. For several-hundred missile threats, CERs are greater than unity for ranges of interest.

I. INTRODUCTION

In theaters, cost-effectiveness ratios are less important than simple constellation sizes because the threats are rarely at the margin. Still, it is useful to have some rough feel for their magnitudes. This note derives the cost-effectiveness ratios for space-based interceptors (SBIs) for a range of threat sizes and ranges.

II. MIDCOURSE INTERCEPT ANALYSIS

Midcourse intercept kinematics for short-range missile trajectories are reviewed elsewhere.^{1,2} The main results are reviewed here. Ignoring drag and powered flight, for ranges of interest it is adequate to note that a missile launched on a minimum-energy trajectory of range R requires a velocity $v = \sqrt{gR}$, reaches an altitude $h = R/4$, and takes a total flight time $T = \sqrt{2R/g}$, where $g = 0.01 \text{ km/s}^2$ is the value of gravity at the Earth's surface. Minimum-energy trajectories maximize range and

minimize error for given missiles;³ they are probably those of most concern for theater launches. For $R = 1,000$ km, $v = 3.2$ km/s; $h \approx 250$ km; $T \approx 450$ s; and the time to apogee is ≈ 250 s.⁴

If each SBI is capable of reaching missiles with apogees within a radius r , it can defend an area of $\approx \pi r^2$,⁵ so the number needed for contiguous coverage is $N \approx 4\pi R_e^2 / z\pi r^2$, where R_e is the Earth's radius and z is a numerical factor that accounts for the concentration of the constellation possible when only regional coverage is required.⁶ For coverage concentrated over a band of 20° to 30° latitude, $z \approx 3.6$, so $N \approx (R_e/r)^2$. The SBI must be able to reach the missile while it is still accessible, which means that the SBI constellation must be dense enough to allow the SBI to reach the missile or weapon before it falls back into the atmosphere.

It is assumed that SBIs are capable of intercepting missiles at any time up to apogee, which gives a maximum fly-out time of $T/2 = \sqrt{R/2g}$. Thus, SBIs from ranges less than $r \approx V \cdot T/2 = V\sqrt{R/2g}$ with average velocity V with sensors sufficiently sensitive to detect cold bodies rather than hot missile plumes could engage missiles on minimum energy trajectories by apogee. The number of SBIs required to have at least one SBI in range of apogee is

$$N_a \approx (R_e/r)^2 \approx (2R_e/VT)^2 \approx 2gR_e^2/V^2R, \quad (1)$$

which gives the apogee "absentee ratio," i.e., the number of SBIs needed on orbit to intercept one missile at its apogee, which is shown as a function of range in Fig. 1. For a 1,000 km range and a typical divert velocity of $V = 6$ km/s, $N_a \approx 23$. N_a scales as R^{-1} , so for $R = 2,500$ km, $N_a \approx 9$, although ranges that long lead to $\approx 10\%$ differences between planar approximations and spherical-earth trajectories.⁷

It can be argued that for the launch of M missiles, the number of SBIs should be increased by a factor of M , but unless the missiles are launched within a time $t \approx 2r/V_0$ of each other, the movement of the SBIs in their constellation brings a new interceptor into position in time for intercept. For

$R = 1,000 \text{ km}$, $r \approx V\sqrt{(R/2g)} \approx 1,350 \text{ km}$ and $t \approx 2 \cdot 1,350 \text{ km} / 8 \text{ km/s} \approx 330 \text{ s} \approx 6 \text{ minutes}$, where $V_0 \approx 8 \text{ km/s}$ is the SBIs' orbital velocity.

III. COST EFFECTIVENESS

To address M missiles spaced at intervals longer than the reconstitution time t , the defense needs at N_a SBIs to fill the belt plus the M SBIs needed for the intercepts. If short-range missiles have unit variable costs m_0 , their masses and costs grow as $e^{V/c}$, where $c \approx 2.5 \text{ km/s}$ is the SBIs' effective exhaust velocity, and the SBIs have unit variable costs s , the SBIs' cost effectiveness ratio (CER) is

$$\text{CER} \approx mM/s(M + N_a) \approx m_0 e^{\sqrt{(gR)}/c} M/s(M + 2gR_e^2/V^2R), \quad (2)$$

which is shown in Fig. 2 for threats as functions of range for $M = 10, 30$, and 100 missiles for $m_0/s \approx 1$, i.e., SBIs that cost about as much as short-range missiles. For M small, the CERs are less than unit for very short ranges. They reach unity for ranges of about $1,000, 500$, and 300 km for threats of $10, 30$, and 100 missiles, respectively. For the several-hundred missile threats generally discussed,⁸ CERs are greater than unity for all ranges of interest. They fall, of course, as m_0/s for lower missile or higher SBI costs. They increase with SBI velocity V , if it does not greatly increase cost. If more than one missile is assumed to be in the air at once, that number multiplies N_a in the denominator of CER, which effectively rescales R .

IV. SUMMARY AND CONCLUSIONS

This note has derived the cost-effectiveness ratios for space-based interceptors (SBIs) for a range of threat sizes and ranges. Absentee ratios are calculated to estimate the number of SBIs needed to fill the defensive constellations. To address M missiles spaced at intervals longer than the reconstitution time t , the defense needs at N_a SBIs to fill the belt plus the M SBIs needed for the intercepts; the resulting cost effectiveness scales as $mM/s(M + N_a)$. N_a is large and CER small for small ranges and numbers of missiles. For the several-hundred missile

threats generally discussed, CERS are greater than unity for all ranges of interest.

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Fig. 1 SBI absentee ratio versus range

Fig. 2 Cost effectiveness versus range

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