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New 2BPM-1K Scheme and Resistive Wall Instability

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Abstract

A new transverse damping correction scheme is developed, using two monitors and one kicker, which allows full cancellation of the coherent beam motion within two turns, when no other perturbation exists. This scheme is capable of much better control of the resistive wall instability in the SSC Collider than the conventional scheme.

1.0 INTRODUCTION

One of the main goals of a transverse damping feedback system for the Superconducting Super Collider (SSC) is to control the transverse multibunch instabilities. These instabilities arise due to the interaction of the bunches with the wake field left by previous bunches traveling in the ring. The wake field is the electromagnetic field resulting from the interaction of charged particles in the bunch with the surrounding conductors elements (beam pipe, scrapers, kickers, rf-cavities, *etc.*), and it can also be characterized by the associated impedance (Fourier transformation of the wake field).¹ The instability that arises due to the resistive beam pipe is called resistive wall instability. Since the SSC Collider machine the injection time is higher than one hour² and the storage time at high energy (20 TeV) must be at least 20 hours, the control of the resistive wall instability is of fundamental importance for the Collider to be operational. The most dangerous mode of the beam oscillation is the dipole one which has an estimated growth time³ between 56 msec and 150 msec. Reference 3 shows that the traditional feedback scheme* was not good enough to control (with low gain) the Dipole Mode Multibunch Instability (DMMI) due to the resistive wall impedance. Consequently, a novel two BPM-two kickers transverse feedback system was proposed to control the DMMI much better (the gain could be one order of magnitude lower). This novel system was optimized later on,⁴⁻⁵ and it is still under study. Since this system has two kicker, one would like to reduce the cost of the system by looking for the possibility of using one single kicker with other correction scheme.⁶ In this paper, a new scheme is shown, using two BPM's and a single kicker, which allows excellent control of the DMMI due to resistive wall impedance and allows a complete cancellation of the coherent oscillations within two turns, when the gain is equal to one and there is no other perturbations. Although this system was mentioned elsewhere,⁴ it was not fully analyzed, and its value was underestimated.

2.0 ANALYSIS

A simplified diagram of a two BPM's and one kicker feedback system is shown in Figure 1 where a fully decoupled motion is assumed, and the motion is restricted to a single plane, say horizontal. B's and K represent the location of the two BPM's and the kicker, ϕ and ψ represent the relative phase advances between these elements, (α 's, β 's) are the Courant-Snyder⁷ parameters associated with the location of each element, μ is the tune of the machine, and X 's and X' 's represent the amplitude of the oscillation and the angle made by the bunch. Using the Courant-Snyder map, the coordinates of the bunch at the point B2 are given in terms of those at the point B1 as

* Using a Beam Position Monitor (BPM) and a single kicker where the correction kick is proportional to the measured displacement at the BPM.

$$\begin{pmatrix} X_2 \\ X'_2 \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{\beta_2}{\beta_1}}(\cos \phi + \alpha_1 \sin \phi) & \sqrt{\beta_2 \beta_1} \sin \phi \\ -\frac{1 + \alpha_1 \alpha_2}{\sqrt{\beta_2 \beta_1}} \sin \phi + \frac{\alpha_1 - \alpha_2}{\sqrt{\beta_2 \beta_1}} \cos \phi & \sqrt{\beta_1 \beta_2}(\cos \phi - \alpha_2 \sin \phi) \end{pmatrix} \begin{pmatrix} X_1 \\ X'_1 \end{pmatrix}. \quad (1)$$

Figure 1. Model for two-turns cancellation of coherent motion.

Since the variables X_1 and X_2 are measured, using Eq. (1), X'_1 and X'_2 are given by

$$X'_1 = -\frac{X_1}{\beta_1} \left(\frac{\cos \phi}{\sin \phi} + \alpha_1 \right) + \frac{X_2}{\sqrt{\beta_1 \beta_2} \sin \phi} \quad (2)$$

and

$$X'_2 = -\frac{X_1}{\sqrt{\beta_1 \beta_2} \sin \phi} + \frac{X_2}{\beta_2} \left(\frac{\cos \phi}{\sin \phi} - \alpha_2 \right). \quad (3)$$

The coordinates at the kicker location are connected with those at the B2 as

$$\begin{pmatrix} X_3 \\ X'_3 \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{\beta_3}{\beta_2}}(\cos \psi + \alpha_2 \sin \psi) & \sqrt{\beta_2 \beta_3} \sin \psi \\ -\frac{1 + \alpha_2 \alpha_3}{\sqrt{\beta_2 \beta_3}} \sin \psi + \frac{\alpha_2 - \alpha_3}{\sqrt{\beta_2 \beta_3}} \cos \psi & \sqrt{\frac{\beta_2}{\beta_3}}(\cos \psi - \alpha_3 \sin \psi) \end{pmatrix} \begin{pmatrix} X_2 \\ X'_2 \end{pmatrix} + \begin{pmatrix} 0 \\ U_1 \end{pmatrix} \quad (4)$$

where the kick, U_1 , is the correction made by the kicker which at this moment is unknown. Using Eqs. (2) and (3), and making some rearrangements, it follows

$$X_3 = -X_1 \sqrt{\frac{\beta_3}{\beta_1}} \frac{\sin \psi}{\sin \phi} + X_2 \sqrt{\frac{\beta_3}{\beta_2}} \frac{\sin(\phi + \psi)}{\sin \phi} \quad (5a)$$

and

$$X'_3 = -\frac{X_1}{\sqrt{\beta_3\beta_1}} \frac{\cos \psi - \alpha_3 \sin \psi}{\sin \phi} + \frac{X_2}{\sqrt{\beta_3\beta_2}} \frac{\cos(\phi + \psi) - \alpha_3 \sin(\phi + \psi)}{\sin \phi} + U_1. \quad (5b)$$

Then, a one-turn map is made in order to effectuate the second turn of the bunch. Its coordinates at the kicker location, (Y_3, Y'_3) , are derived from the transformation

$$\begin{pmatrix} Y_3 \\ Y'_3 \end{pmatrix} = \begin{pmatrix} \cos \mu + \alpha_3 \sin \mu & \beta_3 \sin \mu \\ -\frac{1 + \alpha_3^2}{\beta_3} \sin \mu & \cos \mu - \alpha_3 \sin \mu \end{pmatrix} \begin{pmatrix} X_3 \\ X'_3 \end{pmatrix} + \begin{pmatrix} 0 \\ U_2 \end{pmatrix}, \quad (6)$$

where U_2 represents the new kick given to the bunch. Using Eqs. (5a) and (5b) and in Eq. (6), the new coordinates of the bunch at the kicker location can be written as

$$Y_3 = -X_1 \sqrt{\frac{\beta_3}{\beta_1}} \frac{\sin(\mu + \psi)}{\sin \phi} + X_2 \sqrt{\frac{\beta_3}{\beta_2}} \frac{\sin(\mu + \phi + \psi)}{\sin \phi} + \beta_3 \sin \mu U_1 \quad (7a)$$

and

$$\begin{aligned} Y'_3 = & \frac{-X_1}{\sqrt{\beta_3\beta_1}} \frac{\cos(\mu + \psi) - \alpha_3 \sin(\mu + \psi)}{\sin \phi} + \frac{X_2}{\sqrt{\beta_3\beta_2}} \frac{\cos(\mu + \phi + \psi) - \alpha_3 \sin(\mu + \phi + \psi)}{\sin \phi} \\ & + (\cos \mu - \alpha_3 \sin \mu) U_1 + U_2. \end{aligned} \quad (7b)$$

The complete cancellation of the coherent oscillation at this second turn is satisfied if kicks U_1 and U_2 are chosen such that $Y_3 = 0$ and $Y'_3 = 0$. Doing this in Eqs. (7a) and (7b), the following expressions for the kicks are brought about

$$U_1 = \frac{X_1}{\sqrt{\beta_3\beta_1}} \frac{\sin(\mu + \psi)}{\sin \phi} - \frac{X_2}{\sqrt{\beta_3\beta_2}} \frac{\sin(\mu + \phi + \psi)}{\sin \phi} \quad (8)$$

and

$$\begin{aligned} U_2 = & (-\cos \mu + \alpha_3 \sin \mu) U_1 + \frac{X_1}{\sqrt{\beta_3\beta_1}} \frac{\cos(\mu + \psi) - \alpha_3 \sin(\mu + \psi)}{\sin \phi} \\ & - \frac{X_2}{\sqrt{\beta_3\beta_2}} \frac{\cos(\mu + \phi + \psi) - \alpha_3 \sin(\mu + \phi + \psi)}{\sin \phi}. \end{aligned} \quad (9)$$

These last two expressions represent the new scheme for damping the DMMI, and they will be used in the following sections for controlling the resistive wall instability.

3.0 RESISTIVE WALL IMPEDANCE APPROXIMATION

The bunch is assumed to be a single point macroparticle which effectuates rigid motion, *i.e.*, one restricts oneself to analyze the Dipole Mode Multibunch Instability (DMMI). Now, the real part of the transverse resistive wall impedance, for a uniform beam pipe, has a peak at low frequencies (the lowest frequency of interest is the angular revolution frequency, ω_o , times the fractional part of the tune, f_μ). Hence, it can be approximated as a resonator impedance (see Figure 2), as it was initially studied in Reference 3. The negative frequencies correspond to unstable oscillation modes of the beam. These modes occur at the frequencies

$$\omega_{(k)} = (k + \Delta_\beta)\omega_o ,$$

where Δ_β is the fractional part of the tune such that $-1/2 < \Delta_\beta < 1/2$, and k can range from 0 or -1 to $M/2$, where M the total number of bunches (17 424). The most dangerous mode corresponds to ω_{-1} if $\Delta_\beta > 0$, or ω_0 if $\Delta_\beta < 0$. The growth time of this mode for a stainless steel beam pipe (1 mm of thickness) coated with copper (0.1 mm of thickness, Residual Resistivity Ratio (RRR) = 30) at injection (2 TeV) is between 56 msec to about 100 msec, depending on the beam pipe radius (1.622 cm to 2.077 cm) and the fractional part of the tune. Assuming a fractional part of the tune of 0.77, it follows that $\Delta_\beta = -0.23$.* So, the angular frequency of the most dangerous mode is $\omega_0 = |\Delta_\beta|\omega_o$ which is about 4.9 kHz, and the resistive wall impedance, $\text{Re } Z_\perp(\omega_0)$, is about 2.4 G Ω /m. The approximation consists on taking ω_0 as the resonant frequency and $\text{Re } Z_\perp(\omega_0)$ as the peak resonant impedance.

Figure 2. Resistive wall impedance (uniform pipe).

* Actually, the fractional part of the Collider tune at injection is about 0.28. However, the above value has been assumed here in order to have a wider safety margin and possible tune variation. In addition, this allows to apply the results at 20 TeV where the fractional part of the tune may be about 0.77.

4.0 CONTROLLING THE RESISTIVE WALL INSTABILITY, FIRST MODES

The model used for the ring is shown in Figure 3. The relative separation among BPM1, BPM2, and kicker is taken as $\pi/2$ in phase advance. The resonator is located at an arbitrary phase advance $\tilde{\phi}$. The Courant-Snyder map is used to transport each of the 17 424 bunches from element to element. The wake field felt by the bunch is the one generated at the resonator (**R**). The correction kick at turn “ n ” is given, according to (8) and (9) with $\phi = \psi = \pi/2$, by

$$U_n = (-\cos \mu + \alpha_3 \sin \mu)U_{n-1} - \frac{gX_1^{(n-1)}}{\sqrt{\beta_3\beta_2}}(\sin \mu + \alpha_3 \cos \mu) + \frac{gX_2^{(n-1)}}{\sqrt{\beta_3\beta_2}}(\cos \mu - \alpha_3 \sin \mu) \quad (10)$$

and

$$U_1 = \frac{gX_1}{\sqrt{\beta_3\beta_1}} \frac{\cos \mu}{\sin \mu} + \frac{gX_2}{\sqrt{\beta_3\beta_2}}, \quad (11)$$

where g is gain of the feedback system. All of this was implemented in the code TADIMMI (see Reference 3) that simulates the Dipole Mode Multibunch Instabilities (DMMI) for a resonator impedance. Figure 4 shows the average displacement of the bunches, as seen by BPM2, as a function of the number of turns. The behavior of some particular bunches is shown in Figures 5 and 6. The kick given to the bunches as a function of the number of turns is shown in Figure 7. The gain used in this simulation was 0.03. Therefore, this feedback system scheme can control much better the DMMI than the traditional one (see Reference 3) which requires very large gain to be able to do this. In addition, the requirement of having an initial kick given by Eq. (11) is not really needed since one can start with $U_1 = 0$ without major difference in the result. Even more, it is possible to make $X_2^{(n-1)} = 0$ and still maintain a good damping effect. That is, this damping scheme allows a great deal of flexibility and is highly stable.

Figure 8 shows the average displacement at BPM2 for a feedback working with a gain equal to one. Figure 9 shows one particular bunch behavior, and Figure 10 shows the response of the kicker. Thus, full control of the resistive wall instability is obtained in a couple of turns for a gain equal to one. These figures are explicitly showing that the above scheme does not correspond to a normal damper process (the amplitude of the oscillations decrease exponentially with the number of turns), but the above scheme represents a killer of the coherent oscillations (the same behavior as two BPM-two kicker system, see References 2, 5). This is the reason why this scheme works much better than a normal damper.

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Figure 3. Model for computer simulations.

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Figure 4. Average value of the oscillations.

Figure 5. First bunch damping behavior.

Figure 6. Bunch 2000 damping behavior.

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Figure 7. Kick function Eq. (10) behavior.

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Figure 8. Average behavior for gain equal to one.

Figure 9. Bunch 1000 behavior for gain equal to one.

Figure 10. Kick function Eq. (10) for gain equal to one.

5.0 CONTROLLING THE RESISTIVE WALL INSTABILITY, HIGHER MODES

The preceding feedback system can control few (< 10) of the most dangerous modes. However, there are thousands of modes at higher frequencies that are also unstable, and they must be controlled (even the last one, $\omega_{M/2}$, that has a growth time of about 10 s). A wide band feedback system used to damp bunch by bunch oscillations (frequency bandwidth ≥ 30 MHz) can control modes at very high frequency ($\sim$ MHz). There remains something to be done for the modes with frequencies in between ($\sim kHz < \omega_k < \sim$ MHz). However, one has to take into account the decoherence time of the bunch. That is, the effect on the emittance growth caused by the motion of the particles in the phase space which try to fill out a bigger region when the bunch effectuate coherent oscillations. The time (τ_ν) the particles take to accomplish this is called decoherent time, and it depends on the tune spread, σ_ν , as

$$\tau_\nu = T_o / \sigma_\nu , \quad (12)$$

where T_o is the revolution time in the Collider (about 0.29 msec). If the growth time of the instability is higher than the decoherent time, this instability is Landau damped, and then, it is not important. Frequencies higher than ω_ν at which the instability growth time is equal to the decoherent time, are not really important since the instabilities associated are Landau damped (see Figure 2 dashed area). Since the resistive wall instability growth time is proportional to the square root of the frequency,⁸ all the modes k such that $|k| > |k_1|$ will be Landau damped if the tune spread satisfies

$$\sigma_\nu \geq \frac{T_o}{\tau_o} \sqrt{\frac{\Delta_\beta}{|k_1 + \Delta_\beta|}} , \quad (13)$$

where τ_o is the growth time of the fundamental mode ($k = 0$). Using the previous data, $\Delta_\beta = -0.23$ and $\tau_o = 100$ msec, and assuming $k_1 = 5$, the required tune spread is

$$\sigma_\nu \geq 10^{-3} . \quad (14)$$

If the tune spread within the bunch is like Eq. (14), all the resistive wall instability modes can be controlled by the new feedback system scheme Eq. (10) and by Landau damping.

6.0 SELECTION OF THE GAIN AND BPM RESOLUTION

Due to the same decoherent phenomenon mentioned above, it is required to damp the coherent oscillations faster than the decoherent time,⁹ otherwise, the increment of the

emittance could cause a real problem in achieving or maintaining the desired luminosity in the Collider. If τ_d is the damping time

$$\tau_d = \frac{2T_o}{g}, \quad (15)$$

the condition is $\tau_d \ll \tau_\nu/4\pi$ (see Reference 9), *i.e.*,

$$g \gg 8\pi\sigma_\nu. \quad (16)$$

Using the value of Eq. (14), the gain must satisfy the relation $g \gg 0.025$. Therefore, the gain of the feedback system can be chosen as $g \geq 0.25$.

On the other hand, the BPM resolution (noise), σ_x , has the bad effect of increasing the emittance of the beam. If one restricts oneself to a 24 hour doubling time (the time in which the emittance growth doubles its initial value) and a tune spread is given by Eq. (14), the BPM resolution (see Reference 9) must satisfy $\sigma_x \leq 0.25 \mu\text{m}$ which is within the theoretical limits of the BPM resolution.¹³ However, this last result may require computer simulation verification.

7.0 GETTING THE TUNE SPREAD

As it was seen in Section 5.0, the tune spread Eq. (14) within the bunch is required to fully control the resistive wall instabilities. At injection (2 TeV), the natural tune spread due mainly to octupoles (or octupole component of the CDM dipole) is about 10^{-4} , if the chromaticity, ξ , is zero.¹⁰ This tune spread is not enough to control the resistive wall instability. However, within the Collider, there are no single bunch instabilities (the number of particles in the bunch is not large), therefore, it is possible to work with a chromaticity that is not zero, $15 < \xi < 35$. Note that even if there were single bunch instability (head-tail), its most dangerous mode,¹¹ would be damped with this positive chromaticity.

At 20 TeV (Collider operation), the resistive wall instability growth times are essentially one order of magnitude higher than at injection (preceding sections). As a consequence, the tune spread required to Landau damp higher modes may be one order of magnitude less than Eq. (14). The beam-beam iteration, mainly the long range, provides enough tune spread within the bunch to achieve this value. The crossing angle, α , can not be too small since the tune spread may be too large, causing losses of particles due to resonant instabilities. It can not be too large either, since betatron-synchrotron resonants can be induced into the beam. Therefore, the possible range of variation would be $120 \mu\text{rad} \leq \alpha < 175 \mu\text{rad}$. With this angle, the tune spread required is achieved.

8.0 COMMENTS

One might think that using a wide band bunch by bunch feedback system (bandwidth ≥ 30 MHz), all the possible instabilities of the beam could be controlled. In this case, the kicker would require one turn delay to make the correction (processing signal from the BPM and sending to the kicker is slower than the particle speed). This condition enter in a natural way in the preceding scheme Eq. (10). However, it is found that this wide band feedback system can not damp the fundamental mode of the resistive wall instability (independently of the gain selected).^{*} It is though that this is due to fact that the resistive wall impedance is too large ($\sim G\Omega/m$) at low frequencies, which is three orders of magnitude larger than any rf-cavity impedance. Therefore, to control the resistive wall instability, it requires a low frequency bandwidth feedback system (for the first few more dangerous modes) and the tune spread to Landau damp higher frequencies modes.

^{*} This was also found using the traditional feedback scheme.

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