BASIC Program for Power Spectrum Estimation*

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Abstract

A standard data set is presented to be used in testing routines that are developed for evaluating the one-dimensional periodogram spectral estimate of a series of uniformly spaced data points representing measurements of a surface profile. These data were generated using the GWBASIC program "FFTPSD" which is included in the Appendix of this report. The original purpose of this data set was to allow us to test and debug our own periodogram estimation routines, written in various programming languages, against a standard data set. At the request of numerous individuals, we are making it available to others who may wish to do the same. We have also included a summary explanation of periodogram estimators and the various considerations that go into generating computer algorithms for power spectrum estimation.

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Nomenclature

The Power Spectral Density, $S(f_x)$, is the most important statistical property of the profiles of an optical surface. Statistical quantities, however, are defined in terms of averages taken over an infinite set of equivalent surfaces. In statistical terminology the set is an ensemble, the ensemble averaging is denoted by the symbol $\langle \dots \rangle$, and a single member of the ensemble is called a realization.

This presents us with a practical problem: in the real world we have only a finite number of profile data sets (realizations) -- perhaps only one --, the data refer to only a finite profile length, and consist of values taken at discrete points on the profile. How do we estimate the profile spectrum from one or more finite, sampled data sets?

This is done by defining an algebraic function of the profile data which approximates the profile spectrum, and evaluating it for the particular data sets on hand. In statistical terms this procedure is called statistical estimation and the function is called an estimator. The spectral estimator and its values are denoted by $\hat{S}(f_x)$, where the hat distinguishes it from the true spectrum, $S(f_x)$.

Periodogram Estimator

The particular estimator of the profile power spectral density considered in the present paper is the windowed periodogram^{1,2}:

$$\hat{S}(f_m) = \frac{2D}{N} |\text{FFT}|^2 K(j) \quad (1)$$

where

$$\text{FFT} = \sum_{n=1}^N e^{i2\pi(n-1)(m-1)/N} W(n) ZD(n) \quad (2)$$

is the Discrete Fourier Transform of the product of a data window, W , and the detrended surface profile ZD . The various quantities appearing in Eqs. (1,2) are defined in detail below.

We have written the discrete Fourier transform as "FFT" since it is almost always evaluated using a Fast Fourier Transform algorithm. The program provided in this report uses a particular routine written in BASIC, which requires that the total number of data points, N , be an integer power of two.

In practice it is more convenient to implement Eqs. (1,2) using a canned FFT routine written in a faster, higher-level language. However, one must be aware of the fact that although the normalization of the FFT in Eqs. (1,2) is the most common one, it is not universal. Higher level and more modern languages usually have built-in numerical routines for FFT computation. One must be aware that normalizations used in these FFT routines may differ from that used in this program. The FFT routine used here is translated into BASIC from the Fortran version given in Numerical Recipes³.

The BASIC program in the Appendix is intended to be used as a model to test other power spectrum calculation programs against the standard set of data presented in this report. Since it is written in a generic version of BASIC, it can also be used as a model for translation into other programming languages.

Definition of Symbols

$\hat{S}$ = the periodogram estimate of the profile power spectral density. This is the "one-sided" spectrum; that is, it involves positive frequencies only. Note that it has the dimensions of length-cubed.

$f_m = (m - 1) / ND$ is the spatial frequency along the x-axis -- i.e. discrete values of f_x .

$m = 1$ to $1 + N / 2$ is the frequency index. The lowest value, $m = 1$, corresponds to $f_x = 0$, the "DC" component of the profile. The highest value, $m = 1 + N / 2$, corresponds to $f_x = 1 / 2D$, the Nyquist frequency of the sampled measurement.

In practice these extreme values are generally omitted in the analysis of real data. However, it is advised to keep them in the analysis since, as explained later, their inclusion provides a convenient check on the written program

(The BASIC program uses j in place of m as the spectral index to avoid internal notational conflicts.)

- D** = the sampling interval of the profile measurements, i.e. the distance between successive data points.
- n** = 1 to N is the profile sampling index. The sampling positions are then $x = (n - 1)D$, and the sampled profile length is $(N - 1)D$.
- N** = the total number of points in the sampled profile. In the case of the BASIC program provided, this must be an integer power of two. A method of avoiding this restriction by zero-padding the data is discussed later.
- W(n)** = the data window function. Its purpose is to smooth the ends of the finite-length data set to eliminate spurious effects in the spectral estimate that would otherwise appear.

We recommend the Blackman window¹ for surface profile data:

$$W(n) = \sqrt{\frac{2}{1523}} \left[21 - 25 \cos(2\pi(n-1)/N) + 4 \cos(4\pi(n-1)/N) \right] \quad (3)$$

This begins on line 3080 of the program. (Other window functions begin on lines 2840, 2900, and 2990.)

- ZD(n)** = the detrended profile data^{1,4}; that is, the measured profile after various instrumental and surface artifacts have been removed.

This is usually done by subtracting a polynomial, i.e.

$$ZD(n) = Z(n) - [AA + BB(n-1) + CC(n-1)^2] \quad (4)$$

where $Z(n)$ is the original profile and the coefficients AA, BB, and CC of the detrending polynomial in the square brackets are determined by ordinary-least-squares fitting of the polynomial to the original data set; that is, by minimizing $\sum ZD^2$.

We recommend detrending with the full quadratic expression indicated. The resulting expressions for the coefficients in terms of $Z(n)$ are lengthy and are not repeated here¹. They are given in the subroutine on line 3480 in the program. (Subroutines for detrending lower-order polynomials begin on lines 3170, 3230 and 3340.)

- K(j)** = 1 except for $K(1) = K(1+N/2) = 1/2$. This is an end-effect or bookkeeping factor that only affects the spectral estimate at the DC and Nyquist frequencies. It is essential in order to have the estimate satisfy Parseval's Theorem:

$$\frac{1}{N} \sum_{n=1}^N Z^2(n) = \frac{1}{ND} \sum_{m=1}^{1+N/2} \hat{S}(j) \quad (6)$$

This is a mathematical identity which is valid for any profile data set using a rectangular data window, $W(n) = 1$.

Window Functions

If the data window is not rectangular, Parseval's Theorem (Eq. 6) is still valid *on the average*, providing the window function has the normalization

$$\frac{1}{N} \sum_{n=1}^N W(n)^2 = 1 \quad (7)$$

The Blackman window in Eq. (3) and the other windows given in the program have this normalization^{1,2}.

There are a number of other window functions discussed in the literature, but they frequently are normalized differently, and must be renormalized to satisfy Eq. (7) before being used in the present periodogram estimator. If the normalization is incorrect, then incorrect RMS roughness values will be calculated from the power spectral density estimator.

The effects of the windowing function on the RMS profile roughness deduced from the periodogram can be seen in the first column in Table IV below. The value for the rectangular window, 26.13517, is identical with that derived from the profile data. However, the values derived from the spectrum using other data windows differ from that value. Which one is correct? The answer depends on what is wanted. On the one hand, the rectangular window leads to RMS values that are exact for each realization of the surface profile. On the other hand, the end-smoothing data windows lead to better estimates of the average value for an ensemble of such surfaces.

Zero padding

The most common FFT algorithms require the number of data points, N , to be a power of two. If the number of measured data points, M , is not a power of two, these algorithms can still be used, by either discarding data and to fit the next lowest power of two, or adding $(N - M)$ zeros to either end of the measured data set to fit the next higher power of two. This last procedure is called zero padding.

To account for this zero padding, the periodogram of the padded data calculated from Eq.(1) must be renormalized by multiplying it by the factor N/M . The result satisfies Parseval's theorem. Also zero-padding the data set adds spurious information into the lowest frequency intervals, specifically into those frequencies that have index values $J \leq N/M$. The lowest usable spatial frequency in the estimate is $1/MD$ rather than $1/ND$. Since zero-padding is usually done to bring the number of points in the data set up to the nearest integral power of 2, $J=1$ is the usually the only frequency affected. To avoid adding spurious information into the periodogram, ignore or set equal to zero the periodogram values for which $J \leq N/M$. The Nyquist frequency remains $1/2D$, regardless of the extent of the zero-padding.

Connection with the two-dimensional spectrum

The power spectral density $S(f_x)$ discussed above is the profile or one-dimensional spectrum, as distinct from the area or two-dimensional spectrum, $S(f_x, f_y)$. The profile spectrum is the natural statistic of profile measurements while the area spectrum appears in the discussion of surface scattering and other areadependent properties of surfaces^{5,6}.

The two are related by

$$S(f_x) = 4 \int_0^{\infty} S(f_x, f_y) df_y \quad (8)$$

Although related, the two spectral densities are quite different quantities -- note that they even have different dimensions: $S(f_x)$ is length cubed while $S(f_x, f_y)$ is length to the fourth power. Sometimes the subscripts "1" and "2" are added to the two different S's to further distinguish them.

It is also important to note that the profile spectrum is not simply a special case of the area spectrum; that is, $S(f_x) \neq S(f_x, 0)$!

In general, Eq.(8) cannot be inverted to give the area spectrum in terms of the profile spectrum without further information about the form of the area spectrum. There are two special cases of interest: a corrugated (sinusoidal) surface and a statistically-isotropic (random) surface.

If the profile is taken across the corrugations, Eq.(8) gives

$$S(f_x, f_y) = S(f_x) \cdot \delta(f_y) \quad (8.5)$$

where $S(f_x)$ is the profile spectrum whose estimate is given by Eq.(1).

In the case of an isotropically-rough surface

$$S(f_x, f_y) = S(f) ; \text{ where } f = (f_x^2 + f_y^2)^{1/2} \quad (9)$$

and Eq.(8) can be rewritten:

$$S(f_x) = 4 \int_{f_x}^{\infty} \frac{f}{\sqrt{f^2 - f_x^2}} S(f) df \quad (10)$$

This integral equation can be solved to give the area spectrum in terms of the profile spectrum:

$$S(f) = -\frac{1}{2\pi} \int_f^{\infty} \frac{df_x}{\sqrt{f_x^2 - f^2}} \cdot \frac{d}{df_x} S(f_x) \quad (11)$$

It is important to note that Eqs(10,11) are relationships between spectra and not between spectral estimates. How then do we derive estimates of the area spectrum from estimates of the profile spectrum?

Strictly speaking, this can't be done because of the band-width-limited character of all real profile measurements. Since profile data contain no information for frequencies less than $1/ND$, Eq.(11) shows that this is also the lower limit of information that can be derived for the area spectrum. And more importantly, since the profile data contain no information for frequencies greater than the Nyquist frequency, $1/2D$, one can't carry out the integration in Eq.(11) since its upper limit goes to infinity. In order to estimate the area spectrum from profile measurements we must add additional a-priori information to compensate for these deficiencies.

The recommended procedure is to estimate the profile spectrum, to fit it to an analytic model, and to evaluate the area spectrum by substituting that expression into Eq.(11). This modelling procedure does three things simultaneously: it smoothes the statistical fluctuations in the profile spectral estimate, it condenses the information contained in the profile measurements into a few finish parameters, and adds -- through the physical content of the model -- the a-priori information required for the realistic extrapolation of the profile data outside its bandwidth.

A model that we have found to be particularly useful for this purpose is the ABC model, where the fitting parameters A, B and C are the finish parameters of the model^{1,7}:

$$S(f_x) = \frac{A}{[1 + (Bf_x)^2]^{C/2}} \quad (12)$$

which corresponds to the 2-dimensional spectrum:

$$S(f) = \frac{A'}{[1 + (Bf)^2]^{(C+1)/2}} \quad (13)$$

where

$$A' = \frac{1}{2\sqrt{\pi}} \cdot \frac{\Gamma\left(\frac{C+1}{2}\right)}{\Gamma\left(\frac{C}{2}\right)} \cdot AB \quad (14)$$

In the limit of large B (large-correlation-length limit) Eqs.(12-14) reduce to the inverse-power-law or fractal-like forms:

$$S(f_x) = \frac{K_n}{f_x^n} \quad (15)$$

and

$$S(f) = \frac{1}{2\sqrt{\pi}} \cdot \frac{\Gamma\left(\frac{(n+1)}{2}\right)}{\Gamma\left(\frac{n}{2}\right)} \cdot \frac{K_n}{f^{n+1}} \quad (16)$$

where $1 < n < 3$ is a number. K and n are the two finish parameters that appear in this limit.

In practice, measured profile spectra may consist of a sum of terms of the form indicated by Eqs(12 & 15), in which case the area spectrum for an isotropic surface is the corresponding sum of terms in Eq.(13,16). For each term in the sum, the two parameters, K_n and $1 < n < 3$, are the finish parameters that appear in this limit.

Numerical Test Case For Periodogram Spectral Estimation Routines

The data in the following Tables are provided to test routines for evaluating the one-dimensional periodogram spectral estimate of a series of uniformly spaced data points. These data were generated using the GWBASIC program "FFTPSD" appearing in the Appendix of this report.

Table I lists the standard input data set used in all subsequent computations:

Table I - Input data
A total of $N = 32$ data points:

n	Z(n)	n	Z(n)
1	-38	17	3
2	15	18	6
3	36	19	17
4	22	20	20
5	29	21	24
6	-43	22	16
7	-1	23	-5
8	-5	24	-17
9	-40	25	-35
10	45	26	23
11	20	27	4
12	3	28	-8
13	47	29	-45
14	-18	30	26
15	45	31	1
16	43	32	6

Mean value of these data:

$$\text{Mean} = (1/N) \sum_{n=1}^N Z(n) = 6.125.$$

RMS variation about that mean:

$$(\text{RMS})^2 = (1/N) \sum_{n=1}^N [Z(n) - \text{Mean}]^2 = (26.13517)^2.$$

Note that these estimates are independent of the sample spacing D .

Periodogram estimates of the power spectrum of this data set and data derived from them are given in the following tables.

Periodogram estimates

Periodogram estimates, $S(m)$, of the preceding data set, calculated using the formalism in Eqs. (1) and (2), are given in Table II. The estimates are proportional to the sample spacing D , and depends on the data window used. The particular results in Table II use $D = 1$ and a rectangular data window, $W(n) = 1$.

Table II - Periodogram Estimates
For D = 1 and W(n) = 1 [Rectangular Window]

m	Type of least-squares data detrending			
	None	Piston	Piston + tilt	Full quadratic
1	1200.500	0	0	0
2	2059.506	2059.506	1825.409	148.1781
3	1421.861	1421.861	1373.580	2043.631
4	569.8463	569.8463	444.7469	530.8546
5	1479.039	1479.039	1396.268	1265.831
6	586.0630	586.0630	556.7305	501.3899
7	2484.120	2484.120	2647.417	2661.277
8	1528.321	1528.321	1586.038	1545.160
9	1851.250	1851.250	1928.103	1900.105
10	288.1090	288.1090	262.2179	254.0131
11	286.1438	286.1438	279.0264	287.4841
12	1531.641	1531.641	1457.812	1452.367
13	3565.711	3565.711	3664.113	3654.717
14	1991.965	1991.965	2026.268	2034.514
15	415.3757	415.3757	402.6207	405.0671
16	1636.550	1636.550	1688.428	1690.258
17	162.000	162.000	178.7214	178.7215

The spatial frequency corresponding to each entry in Table II is given by

$$f_m = (m-1)/ND$$

where m = 1 corresponds to "DC" and m = 1 + N/2 = 17 to the Nyquist frequency.

The mean and RMS values of the original data set can be calculated from these spectral values using the general expressions:

$$(\text{Mean})^2 = (1/ND) S(1)$$

and

$$(\text{RMS})^2 = (1/ND) \sum_{m=1}^{1+N/2} S(m).$$

The Tables below give the sixteen Mean and RMS values calculated from the power spectrum of the test data set S(m) with the four kinds of detrending applied, and each detrended spectrum with the four data-window factors W(n) applied. The detailed spectra of these additional three cases have not been included.

Table III - Mean values

Data window	Type of detrending			
	None	Piston	Piston + tilt	Full quadratic
Rectangular	6.125	0.0	0.0	0.0
Hann	9.090228	4.089187	4.181670	0.8480636
Hamming	8.901183	3.654487	3.737139	0.7579108
Blackman	9.881537	5.220414	5.306613	1.7528740

Table IV - RMS values

Data window	Type of detrending			
	None	Piston	Piston + tilt	Full quadratic
Rectangular	26.13517	26.13517	26.05133	25.34362
Hann	25.49031	24.02299	23.91725	22.31003
Hamming	25.53997	24.26072	24.15406	22.58759
Blackman	25.29220	23.38999	23.30221	21.54917

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Appendix

```

10  '** This is program FFTPSD in simple BASIC.  Version of 10 V 93.
20  '   POC for comments and corrections:
21  '       E.L. Church, 201 724 7317, email: echurch@delphi.com,
22  '       P.Z. Takacs, 516 282-2824, email: takacs@bnlcl6.bnl.gov
30  '
40  ' ** It calculates the periodogram estimate of the power spectral density
50  '   of a uniformly-spaced set of profile data points Z(n) , where
60  '   n = 1 --> NN, and NN is a power of 2.
70  '
80  ' ** The output is the periodogram S(f), where  $f = (m-1)/(NN*D)$  is the
90  '   surface spatial frequency and  $m = 1 \rightarrow 1+NN/2$ .  $m = 1$  corresponds
100 '   to "DC" = the profile average, and  $m = 1+NN/2$  to the Nyquist
110 '   frequency. Note that this is a one-sided spectrum; i.e. there is
120 '   no need to consider negative frequencies since they have been folded
130 '   into the positive-frequency region.
140 '
150 ' ** The inputs required are the number of data points, the profile data,
160 '   the types of detrending and windowing desired, and the sampling
170 '   interval, D . Data can be imported from or exported to external
180 '   files, the names of which are automatically called for.
190 '
200 ' ** If both the profile data and the sampling interval are in micrometers,
    the spectrum is in units of micrometers-cubed **
201 ' ** External data files must have the following format:
202 '       Numbers must be written as data pairs --
203 '       x-val,z-val
204 '       separated by a comma, with each pair on a separate line.
205 '       No header information is allowed before the first number.
206 '       Beware of extra blank space or blank lines at beginning of file.
207 ' **
210 '
220 NN = 64
230 INPUT "Enter number of data points (must be power of 2 !); NN
240 '** NOTE! The value of NN entered at this point applies only to internally-
250 '   generated input data. If data are subsequently imported from an
260 '   external file, the number points in that file will overwrite
270 '   the number NN input at this point **
280 D = 1
290 '** The parameter "mu" is used in the sinusoidal signal generator **
300 MU = 8
310 '
320 DIM Z(NN): DIM ZD(NN): DIM ZW(NN): DIM W(NN)
330 DIM DATQ(2*NN): DIM S(1+NN/2)
340 PI=3.141593
350 '
360 PRINT "What profile data would you like to use?
370 PRINT "      From an internal random-noise generator ..... (1)"
380 PRINT "      An internally-generated sinusoid ..... (2)"
390 PRINT "      A unit-height step ..... (3)"
400 PRINT "      Read from an external file ..... (4)"
410 PRINT
420 INPUT "Enter 1, 2, 3 or 4"; ANS
430 IF ANS = 1 THEN GOSUB 2520 ELSE
440 IF ANS = 2 THEN GOSUB 2600 ELSE

```

```

450 IF ANS = 3 THEN GOSUB 2740 ELSE
460 IF ANS = 4 THEN GOSUB 2430 ELSE
470 '
480 GOTO 520
490 FOR K = 1 TO NN
500 PRINT K, Z(K)
510 NEXT K
520 PRINT
530 PRINT "How would you like to detrend the data?"
540 PRINT "    No detrending ..... (-1)"
550 PRINT "    Piston only ..... (0)"
560 PRINT "    Piston and tilt ..... (1)"
570 PRINT "    Piston, tilt and quadratic curvature ..... (2)"
580 PRINT
590 INPUT "Enter appropriate number"; ANS
600 IF ANS=-1 THEN GOSUB 3170 ELSE
610 IF ANS=0 THEN GOSUB 3230 ELSE
620 IF ANS=1 THEN GOSUB 3340 ELSE
630 IF ANS=2 THEN GOSUB 3480 ELSE
640 GOTO 590
650 '
660 PRINT
670 PRINT "What data window would you like to use?"
680 PRINT "    A rectangular or Daniell window ..... (1)"
690 PRINT "    A cosine-squared or Hann window ..... (2)"
700 PRINT "    A Hamming window ..... (3)"
710 PRINT "    A Blackman window ..... (4)"
720 PRINT
730 INPUT "Enter 1, 2, 3 or 4"; ANS
740 IF ANS=1 THEN GOSUB 2840 ELSE
750 IF ANS=2 THEN GOSUB 2900 ELSE
760 IF ANS=3 THEN GOSUB 2990 ELSE
770 IF ANS=4 THEN GOSUB 3080 ELSE
780 GOTO 730
790 '
800 ' ** Place the windowed data in the format required by the FFT subroutine **
810 FOR I = 1 TO 2*NN-1 STEP 2
820 J = (I+1)/2
830 DATQ(I)=ZW(J)
840 DATQ(I+1)=0
850 NEXT I
860 GOTO 910
870 FOR K = 1 TO 2*NN-1 STEP 2
880 PRINT K, DATQ(K), DATQ(K+1)
890 NEXT K
900 PRINT
910 '
920 ' ** Go to the FFT subroutine **
930 PRINT
940 PRINT
950 PRINT "                      FFT RUNNING"
960 PRINT
970 PRINT
980 GOSUB 1950
990 '
1000 ' ** Calculates the one-sided PSD by applying the end-correction and
1010 '    normalization factors to the square magnitude of the FFT **
1020 ' ** Note that the normalization factor contains the sample spacing D **

```

```

1030 FOR I = 1 TO NN+1 STEP 2
1040 J = (I+1)/2
1050 S(J) = DATQ(I)^2 + DATQ(I+1)^2
1060 NEXT I
1070 S(1)=S(1)/2: S(NN/2+1)=S(NN/2+1)/2
1080 PRINT
1090 PRINT " m", " S(m)"
1100 PRINT
1110 FOR M = 1 TO 1+NN/2
1120 S(M) = 2*D*S(M)/NN
1130 NEXT M
1140 '
1150 FOR M = 1 TO NN/4
1160 PRINT M, S(M), M+NN/4, S(M+NN/4)
1170 NEXT M
1180 PRINT , , , 1+NN/2, S(1+NN/2)
1190 PRINT
1200 PRINT
1210 PRINT
1220 PRINT "Hit key F5 to continue"
1230 STOP
1240 '
1250 '** The next collection of routines calculates various mean and rms values
1260 ' of the profile and displays them for evaluation **
1270 '
1280 ' First in real space
1290 SUM1=0: SUM2=0: SUM3=0: SUM4=0
1300 FOR N = 1 TO NN
1310 SUM1=SUM1+Z(N): SUM2=SUM2+ZW(N)
1320 SUM3=SUM3+Z(N)^2: SUM4=SUM4+ZW(N)^2
1330 NEXT N
1340 ' Now in frequency space
1350 SUM5=0
1360 FOR M = 2 TO 1+NN/2
1370 SUM5=SUM5+S(M)
1380 NEXT M
1390 '
1400 ALPH = SUM1/NN: BETA = SUM2/NN
1410 GAMM = SUM3/NN: DELT = SUM4/NN
1420 '
1430 PRINT
1440 PRINT
1450 PRINT "Mean value of the raw input data", ALPH
1460 PRINT
1470 PRINT "Mean value of the windowed/detrended data", BETA
1480 PRINT "Mean value of the windowed/detrended data"
1490 PRINT " derived from the DC component of its spectrum =", SQR(S(1)/(NN*D))
1500 PRINT
1510 PRINT "Note! The last two values of the mean must be equal."
1520 PRINT " See comments in program for other relationships."
1530 ' ** Note! These three values of the mean, 1 2 3 , satisfy the rules:
1540 ' No detrending, rectangular window X X X
1550 ' No detrending, non-rectangular window X Y Y
1560 ' Detrending, rectangular window X 0 0
1570 ' Detrending, non-rectangular window X Z Z **
1580 PRINT
1590 PRINT "rms value of the raw input data", SQR(GAMM-ALPH^2)
1600 PRINT

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1610 PRINT "rms value of the windowed/detrended data      =", SQR(DELT-BETA^2)
1620 PRINT "rms value of the windowed/detrended data"
1630 PRINT " derived from the non-DC part of its spectrum =", SQR(SUM5/(NN*D))
1640 PRINT
1650 PRINT "Note! The last two values of the rms must be equal."
1660 PRINT "      See comments in program for other relationships."
1670 '*** Note! These three values of the rms, 1 2 3 , satisfy the rules:
1680 '          No detrending, rectangular window      X X X
1690 '          No detrending, non-rectangular window  X Y Y
1700 '          Detrending, rectangular window        X Z Z
1710 '          Detrending, non-rectangular window    X W W  **
1720 PRINT
1730 PRINT "Hit key F5 to continue"
1740 STOP
1750 PRINT
1760 PRINT
1770 PRINT "Would you like to write the NN/2 periodogram values to an"
1780 PRINT "      external file?"
1790 PRINT
1800 PRINT "          No ..... (N) "
1810 PRINT "          Yes ..... (Y) "
1820 PRINT
1830 INPUT "Enter N or Y "; ANSS$
1840 IF (ANSS$ = "n" OR ANSS$ = "N") GOTO 1890 ELSE
1850 IF (ANSS$ = "y" OR ANSS$ = "Y") GOTO 3710 ELSE
1860 GOTO 1830
1870 '
1880 '
1890 PRINT
1900 PRINT
1910 PRINT "          PROGRAM COMPLETED"
1920 END
1930 '
1940 '
1950 '*** This is subroutine FOUR1 from numerical recipes **
1960 '*** Whether we use the direct or inverse FFT (i.e. ISIGN = +1 or -1)
1970 '      is irrelevant for the calculation of the periodogram. **
1980 ISIGN = + 1
1990 N=2*NN
2000 J=1
2010 FOR I=1 TO N STEP 2
2020 IF J>I THEN 2030 ELSE 2090
2030 TEMPR=DATQ(J)
2040 TEMPI=DATQ(J+1)
2050 DATQ(J)=DATQ(I)
2060 DATQ(J+1)=DATQ(I+1)
2070 DATQ(I)=TEMPR
2080 DATQ(I+1)=TEMPI
2090 '
2100 M=INT(N/2)
2110 WHILE M>=2 AND J > M
2120 J=J-M
2130 M=INT(M/2)
2140 WEND
2150 J=J+M
2160 NEXT I
2170 MMAX=2
2180 WHILE N>MMAX

```

```

FOR M=1 TO MMAX STEP 2
FOR I = M TO N STEP ISTEP
J=I+MMAX
TEMPR=CSNG(WR#)*DATQ(J)-CSNG(WI#)*DATQ(J+1)
TEMPI=CSNG(WR#)*DATQ(J+1)+CSNG(WI#)*DATQ(J)
DATQ(J)=DATQ(I)-TEMPR
DATQ(J+1)=DATQ(I+1)-TEMPI
DATQ(I)=DATQ(I)+TEMPR
DATQ(I+1)=DATQ(I+1)+TEMPI
NEXT I
WTEMP#=WR#
WR#=WR#*WPR#-WI#*WPI#+WR#
WI#=WI#*WPR#+WTEMP#*WPI#+WI#
NEXT M
MMAX=ISTEP
WEND
RETURN
'
' ** This subroutine imports the NN profile points from an external file **
PRINT
INPUT "Enter FILENAME, number of data points (power of 2)";FILEIN$,NN
OPEN "i", #1, FILEIN$
FOR N = 1 TO NN
INPUT #1,DUMMY,Z(N)
NEXT N
GOTO 480
'
' ** This subroutine generates (for our version of BASIC) a random profile
uniformly distributed between -0.5 and +0.5; i.e. it has zero mean and
rms = 1/sqr(12) = 0.2886751 **
FOR N = 1 TO NN
Z(N) = RND -.5
Z(N) = 100*Z(N)
Z(N) = INT (Z(N))
NEXT N
GOTO 480
'
' ** This subroutine generates a determinsitic sinusoidal profile of
amplitude H that, for mu = an integer, fits exactly within the
NN-point data window. In that case the periodogram of the
rectangularly-windowed profile falls in the single frequency
bin m = mu + 1 and has the magnitude s(m) = (NN/2)*D*H^2 .
Also, the profile has zero mean and rms = H/sqr(2) = 0.7071068 * H.
'
' For convenience the value of the parameter mu is set at the start
of the program
H=1
FOR N = 1 TO NN
Z(N) = H*SIN(2*PI*MU*(N-1)/NN)
NEXT N
GOTO 480
'

```

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2740  *** This subroutine generates a positive unit-height step between
2750  '   points NN/2 and 1+NN/2 **
2760  FOR N = 1 TO NN
2770  ZD(N) = + .5
2780  IF N <= NN/2 THEN ZD(N) = - ZD(N)
2790  '
2800  NEXT N
2810  '
2820  GOTO 670
2830  '
2840  *** This subroutine applies a Daniell or rectangular window to the
      profile data **
2850  FOR N = 1 TO NN
2860  ZW(N)=ZD(N)
2870  NEXT N
2880  GOTO 790
2890  '
2900  *** This subroutine applies a raised-cosine or Hann window to the profile
      data **
2910  FACT=SQR(2/1728)
2920  FOR N = 1 TO NN
2930  THETA = 2*PI*(N-1)/NN
2940  ARG = 24 - 24*COS(THETA)
2950  ZW(N)=ZD(N)*FACT*ARG
2960  NEXT N
2970  GOTO 790
2980  '
2990  *** This subroutine applies a Hamming window to the profile data **
3000  FACT=SQR(2/1987)
3010  FOR N = 1 TO NN
3020  THETA = 2*PI*(N-1)/NN
3030  ARG = 27 - 23*COS(THETA)
3040  ZW(N)=ZD(N)*FACT*ARG
3050  NEXT N
3060  GOTO 790
3070  '
3080  *** This subroutine applies a Blackman window to the profile data **
3090  FACT=SQR(2/1523)
3100  FOR N = 1 TO NN
3110  THETA = 2*PI*(N-1)/NN
3120  ARG = 21 - 25*COS(THETA) + 4*COS(2*THETA)
3130  ZW(N)=ZD(N)*FACT*ARG
3140  NEXT N
3150  GOTO 790
3160  '
3170  *** This subroutine applies no detrending **
3180  FOR N = 1 TO NN
3190  ZD(N)=Z(N)
3200  NEXT N
3210  GOTO 650
3220  '
3230  *** This subroutine removes piston from the original profile data **
3240  M0 = 0
3250  FOR N = 1 TO NN
3260  M0 = M0 + Z(N)
3270  NEXT N
3280  AA = M0/NN
3290  FOR N = 1 TO NN

```

```

3300 ZD(N) = Z(N) - AA
3310 NEXT N
3320 GOTO 650
3330 '
3340 '** This subroutine removes piston and tilt **
3350 M0 = 0: M1 = 0
3360 FOR N = 1 TO NN
3370 M0 = M0 + Z(N)
3380 M1 = M1 + Z(N)*(N-1)
3390 NEXT N
3400 DD=NN*(NN*NN-1)*(NN*NN-4)
3410 AA=2*((2*NN-1)*M0-3*M1)/(NN*(NN+1))
3420 BB=-6*((NN-1)*M0-2*M1)/(NN*(NN*NN-1))
3430 FOR N = 1 TO NN
3440 ZD(N) = Z(N) - (AA + BB*(N-1))
3450 NEXT N
3460 GOTO 650
3470 '
3480 '** This subroutine removes piston, tilt and quadratic curvature **
3490 M0 = 0: M1 = 0: M2 = 0
3500 DD=NN*(NN*NN-1)*(NN*NN-4)
3510 ONE = 3*NN*NN-3*NN+2
3520 TWO = -6*(2*NN-1)
3530 THR = 3*(NN-1)*(NN-2)*(2*NN-1)
3540 FOU = -2*(2*NN-1)*(8*NN-11)
3550 FIV = 30*(NN-1)
3560 SIX = (NN-1)*(NN-2)
3570 SEV = -6*(NN-1)
3580 FOR N = 1 TO NN
3590 M0 = M0 + Z(N)
3600 M1 = M1 + Z(N)*(N-1)
3610 M2 = M2 + Z(N)*(N-1)^2
3620 NEXT N
3630 AA = 3*(NN-1)*(NN-2)*(ONE*M0+TWO*M1+10*M2)/DD
3640 BB = -6*(THR*M0+FOU*M1+FIV*M2)/DD
3650 CC = 30*(SIX*M0+SEV*M1+6*M2)/DD
3660 FOR N = 1 TO NN
3670 ZD(N) = Z(N) - (AA + BB*(N-1) + CC*(N-1)^2)
3680 NEXT N
3690 GOTO 650
3700 '
3710 '** This subroutine loads the NN values of the computed periodogram
3720 ' into an external file **
3730 PRINT
3740 INPUT "Enter FILENAME"; FILEOUT$
3750 OPEN "o", #1, FILEOUT$
3760 FOR M = 1 TO 1+NN/2
3770 WRITE #1, S(M)
3780 NEXT M
3790 CLOSE #1
3800 GOTO 1880
3810 '***** END OF PROGRAM CODE *****

```