THE LOS ALAMOS PRIMER

The following notes are based on a set of five lectures given by R. Serber during the first two weeks of April 1943, as an "indoctrination course" in connection with the starting of the Los Alamos Project. The notes were written up by E. U. Condon.

1. Object

The object of the project is to produce a practical military weapon in the form of a bomb in which the energy is released by a fast neutron chain reaction in one or more of the materials known to show nuclear fission.

2. Energy of Fission Process

The direct energy release in the fission process is of the order of 170 MEV per atom. This is considerably more than 10^7 times the heat of reaction per atom in ordinary combustion processes.

This is $170 \cdot 10^6 \cdot 4.8 \cdot 10^{-10}/300 = 2.7 \cdot 10^{-4}$ erg/nucleus. Since the weight of 1 nucleus of 25 is $3.88 \cdot 10^{-22}$ gram/nucleus the energy release is

$$7 \cdot 10^{17} \text{ erg/gram}$$

The energy release in TNT is $4 \cdot 10^{10}$ erg/gram or $3.6 \cdot 10^{16}$ erg/ton. Hence

1 kg of 25 $\approx$ 20000 tons of TNT

3. Fast Neutron Chain Reaction

Release of this energy in a large scale way is a possibility because of the fact that in each fission process, which requires a neutron to produce it, two neutrons are released. Consider a very great mass of active material, so great that no neutrons are lost through the surface and assume the material so pure that no neutrons are lost in other ways than by fission. One neutron released in the mass would become 2 after the first fission, each of these would produce 2 after they each had produced fission so in the nth generation of neutrons there would be 2^n neutrons available.

Since in 1 kg. of 25 there are $5 \cdot 10^{25}$ nuclei it would require about n 80 generations ($2^{80} \approx 5 \cdot 10^{25}$) to fish the whole kilogram.

While this is going on the energy release is making the material very hot, developing great pressure and hence tending to cause an exposion.

In an actual finite setup, some neutrons are lost by diffusion out through the surface. There will be therefore a certain size of say a sphere for which the surface losses of neutrons are

THE LOS ALAMOS PRIMER

The following notes are based on a set of five lectures given by R. Serber during the first two weeks of April 1943, as an "indoctrination course" in connection with the starting of the Los Alamos Project. The notes were written up by E. U. Condon.

1. Object

The object of the project is to produce a practical military weapon in the form of a bomb in which the energy is released by a fast neutron chain reaction in one or more of the materials known to show nuclear fission.

2. Energy of Fission Process

The direct energy release in the fission process is of the order of 170 MEV per atom. This is considerably more than 10^7 times the heat of reaction per atom in ordinary combustion processes.

This is $170 \cdot 10^6 \cdot 4.8 \cdot 10^{-10}/300 = 2.7 \cdot 10^{-4}$ erg/nucleus. Since the weight of 1 nucleus of 25 is $3.88 \cdot 10^{-22}$ gram/nucleus the energy release is

$$7 \cdot 10^{17} \text{ erg/gram}$$

The energy release in TNT is $4 \cdot 10^{10}$ erg/gram or $3.6 \cdot 10^{16}$ erg/ton. Hence

$$1 \text{ kg of } 25 \approx 20000 \text{ tons of TNT}$$

3. Fast Neutron Chain Reaction

Release of this energy in a large scale way is a possibility because of the fact that in each fission process, which requires a neutron to produce it, two neutrons are released. Consider a very great mass of active material, so great that no neutrons are lost through the surface and assume the material so pure that no neutrons are lost in other ways than by fission. One neutron released in the mass would become 2 after the first fission, each of these would produce 2 after they each had produced fission so in the nth generation of neutrons there would be 2^n neutrons available.

Since in 1 kg. of 25 there are $5 \cdot 10^{25}$ nuclei it would require about n 80 generations ($2^{80} \approx 5 \cdot 10^{25}$) to fish the whole kilogram.

While this is going on the energy release is making the material very hot, developing great pressure and hence tending to cause an exposion.

In an actual finite setup, some neutrons are lost by diffusion out through the surface. There will be therefore a certain size of say a sphere for which the surface losses of neutrons are

just sufficient to stop the chain reaction. This radius depends on the density. As the reaction proceeds the material tends to expand, increasing the required minimum size faster than the actual size increases.

The whole question of whether an effective explosion is made depends on whether the reaction is stopped by this tendency before an appreciable fraction of the active material has fished.

Note that the energy released per fission is large compared to the total binding energy of the electrons in any atom. In consequence even if but $\frac{1}{2}\%$ of the available energy is released the material is very highly ionized and the temperature is raised to the order of $40 \cdot 10^6$ degrees. If 1% is released the mean speed of the nuclear particles is of the order of 10^8 cm/sec. Expansion of a few centimeters will stop the reaction, so the whole reaction must occur in about $5 \cdot 10^{-8}$ sec otherwise the material will have blown out enough to stop it.

Now the speed of a 1 MEV neutron is about $1.4 \cdot 10^9$ cm/sec and the mean free path between fissions is about 13 cm so the mean time between fissions is about 10^{-8} sec. Since only the last few generations will release enough energy to produce much expansion, it is just possible for the reaction to occur to an interesting extent before it is stopped by the spreading of the active material.

Slow neutrons cannot play an essential role in an explosion process since they require about a microsecond to be slowed down in hydrogenic materials and the explosion is all over before they are slowed down.

4. Fission Cross-sections

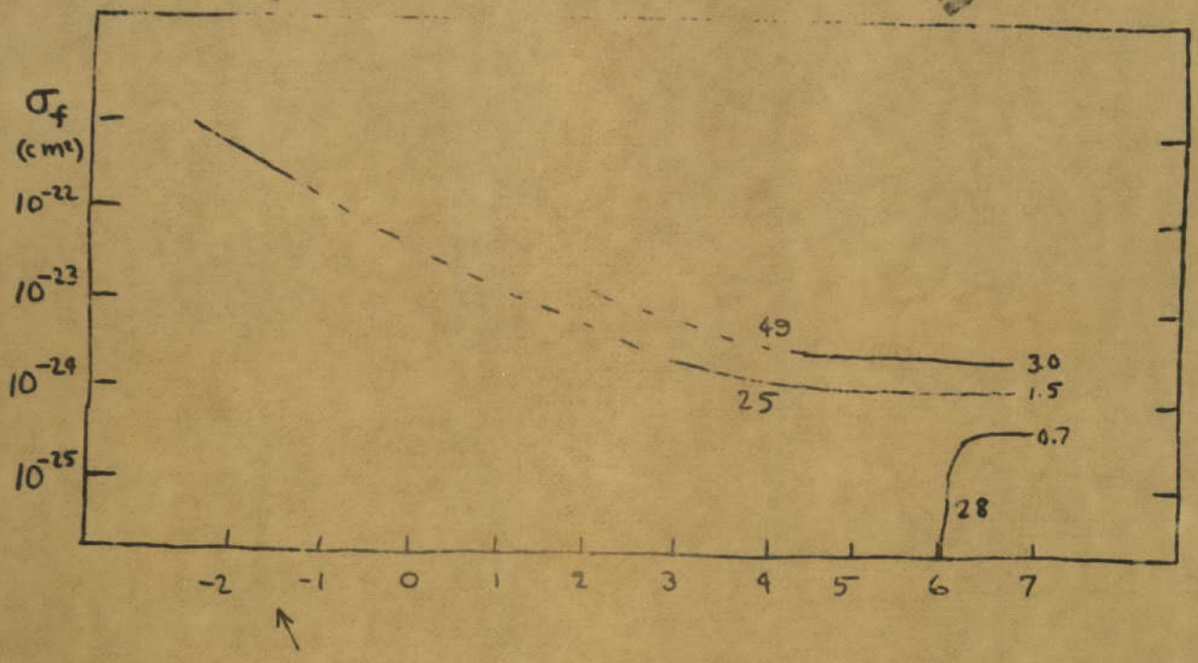
The materials in question are $U_{92}^{235} = 25$, $U_{92}^{238} = 28$ and element $94^{239} = 49$ and some others of lesser interest.

Ordinary uranium as it occurs in nature contains about 1/140 of 25, the rest being 28 except for a very small amount of 24.

The nuclear cross-section for fission of the two kinds of U and of 49 is shown roughly in Fig. 1 where σ_f is plotted against the log of the incident neutron's energy. We see that 25 has a cross-section of about $\sigma_f \approx 1.5 \cdot 10^{-24}$ cm^2 for neutron energies exceeding 0.5 MEV and rises to much higher values at low neutron energies ($\sigma_f = 640 \cdot 10^{-24}$ cm^2 for thermal neutrons). For 28 however a threshold energy of 1 MEV occurs below which $\sigma_f = 0$. Above the threshold σ_f is fairly constant and equal to $0.7 \cdot 10^{-24}$ cm^2.

(Fig. 1 on next page)

Fig. 1.



(thermal) log neutron energy in EV.

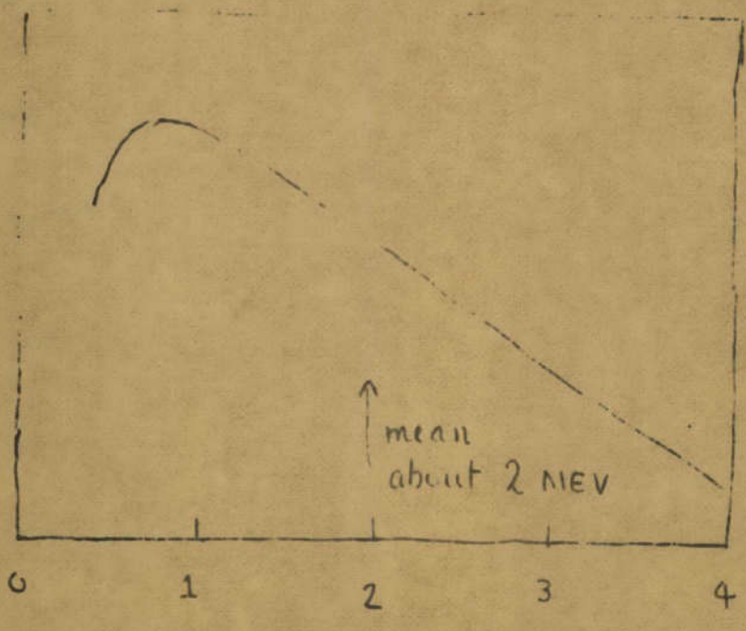
5. Neutron Spectrum

In Fig. 2 is shown the energy distribution of the neutrons released in the fission process. The mean energy is about 2 MEV but an appreciable fraction of the neutrons released have less than 1 MEV of energy and so are unable to produce fission in 28.

One can give a quite satisfactory interpretation of the energy distribution in Fig. 2 by supposing it to result from evaporation of neutrons from the fission product nuclei with a temperature of about ½ MEV. Such a Maxwellian velocity distribution is to be relative to the moving fission product nuclei giving rise to a curve like Fig. 2.

(Fig. 2 on next page)

Fig. 2.



6. Neutron number

The average number of neutrons produced per fission is denoted by ν. It is not known whether ν has the same value for fission processes in different materials, induced by fast or slow neutrons or occurring spontaneously.

The best value at present is

$$\nu = 2.2 \pm 0.2$$

although a value $\nu = 3$ has been reported for spontaneous fission.

7. Neutron capture

When neutrons are in uranium they are also caused to disappear by another process represented by the equation

$$28 + n \rightarrow 29 + \gamma$$

The resulting element 29 undergoes two successive β transformations into elements 39 and 49. The occurrence of this process in 28 acts to consume neutrons and works against the possibility of a fast neutron chain reaction in material containing 28.

It is this series of reactions, occurring in a slow neutron fission pile, which is the basis of a project for large scale production of element 49.

8. Why ordinary U is safe

Ordinary U, containing only 1/140 of 25, is safe against a fast neutron chain because, (a) only 3/4 of the neutrons from a fission have energies above the threshold of 28, (b) only ¼ of the neutrons escape being slowed below 1 MEV, the 28 threshold before they make a fission.
So the effective neutron multiplication number in 28 is

$$\nu \approx 3/4 \times 1/4 \times 2.2 = 0.4$$

Evidently a value greater than 1 is needed for a chain reaction. Hence a contribution of at least 0.6 is needed from the fissionability of the 25 constituent. One can estimate that the fraction of 25 must be increased at least 10-fold to make an explosive reaction possible.

9. Material 49

As mentioned above this material is prepared from the neutron capture reaction in 28. So far only microgram quantities have been produced so bulk physical properties of this element are not known. Also its ν value has not been measured. Its σ_f has been measured and found to be about twice that of 25 over the whole energy range. It is strongly α-radioactive with a half-life of about 20000 years.

Since there is every reason to expect its ν to be close to that for U and since it is fissionable with slow neutrons it is expected to be suitable for our problem and another project is going forward with plans to produce it for us in kilogram quantities.

Further study of all its properties has an important place on our program as rapidly as suitable quantities become available.

10. Simplest Estimate of Minimum Size of Bomb

Let us consider a homogeneous material in which the neutron number is ν and the mean-time between fissions is τ. In Sec. 3 we estimated $\tau = \quad 10^{-8}$ sec. for uranium. Then if N is the number of neutrons in unit volume we have

$$\dot{N} + \operatorname{div} \underline{j} = \frac{\nu - 1}{\tau} N$$

The term on the right is the net rate of generation of neutrons in unit volume. The first term on the left is the rate of increase of neutron density. In the second term on the left $\underline{j}$ is the net diffusion current stream of the neutrons (net number of neutrons crossing 1 cm^2 in 1 sec across a plane oriented in such a way that this net number is maximum).

In ordinary diffusion theory (which is valid only when all dimensions of boundaries are large compared to the mean free path of the diffusing particles - a condition not fulfilled in our case) the diffusion current is proportional to the gradient of N,

$$\underline{j} = -D \operatorname{grad} N$$

where D is the diffusion coefficient (cm^2/sec).

Hence we have

$$\dot{N} = D\Delta N + \left(\frac{\nu-1}{\tau}\right)N$$

Assume a solution whose time dependence is of the form

$$N = N_1(x,y,z)\, e^{\nu' t/\tau}$$

where ν' is called the "effective neutron number". The equation to be satisfied by N_1 is

$$\Delta N_1 + \frac{-\nu' + \nu - 1}{D\tau} N_1 = 0$$

together with a boundary condition. In the simple case in which we are dealing with a sphere of radius R, we may suppose that N_1 is spherically symmetric.

At r = R we would have, on simple theory $N_1 = 0$. (In point of fact $N_1 > 0$ due to the effect of the mean free path's not being small compared with R, but this will not be considered here). For spherical symmetry the equation for N_1 has the solution

$$N_1(r) = \frac{\sin \pi r/R}{r}$$

provided that ν' has the value

$$\nu' = (\nu - 1) - \pi^2 D\tau / R^2.$$

This shows that in an infinitely large sphere the neutron density would build up with the time constant $(\nu-1)/\tau$. Smaller spheres build up less rapidly. Any sphere so small that $\nu' < 0$ is one for which the neutrons leak out the surface so rapidly that an initial density will die out rather than build up. Hence the critical radius is given by

$$R_c^2 = \frac{\pi^2 D\tau}{\nu - 1}$$

Now D is given by $D = \lambda v/3$ where λ is the transport mean free path, $\lambda = 1/n\sigma_t$, n is the number of nuclei per cc and

$$\sigma_t = \left[\sigma_f + \iint \sigma_s (1 - \cos\theta)\, d\omega\right.$$

which brings out the reason for measurements of the angular scattering of neutrons in U. In metallic U we have

$$\sigma_t = 4 \cdot 10^{-24}\ cm^2.$$

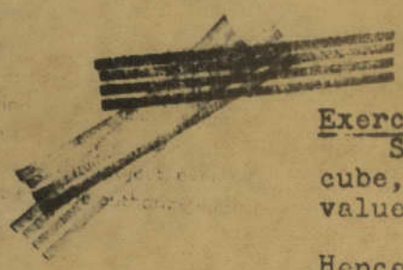
which, for a density of 19 gm/cm^3, gives $\lambda = 5$ cm.
Also

$$\tau = \frac{1}{n\sigma_f v} = \frac{\ell}{v}\frac{\sigma_t}{\sigma_f} \quad \text{so} \quad \pi^2 D\tau = \frac{\pi^2}{3}\ell^2\frac{\sigma_t}{\sigma_f} = 220.$$

Therefore

$$\pi^2 R_c^2 = \frac{220}{1.3} = 183 \quad \text{and} \quad R_c = 13.5\ cm.$$

The critical volume is therefore $10.5 \cdot 10^3\ cm^3$ giving a critical mass of 200 kilograms.



Exercise:
Show that if the gadget has the shape of a cube, $0<x<a$, $0<y<a$, $c<z<a$, that the critical value of a is given by

$$a = \sqrt{3}\, R_c$$

Hence the critical mass for a cubical shape is $3^{5/2}/4\pi = 1.24$ times as great as for a sphere.

The value of the critical mass is, however, considerably overestimated by the elementary diffusion theory. The more exact diffusion theory allowing for the long free path drops Rc by a factor about 2/3 giving

$$R_c \sim 9 \text{ cm} \qquad M_c \sim 60 \text{ kg of } 25.$$

The elementary treatment just given indicates the dependence of M_c on the principal constants

$$M_c \sim \frac{1}{\rho^2} \left[\frac{1}{\sigma_f \sigma_t (\nu-1)}\right]^{3/2}$$

where ρ is the density. For $R \neq R_c$ we have the time dependence of neutron multiplication given by

$$e^{(\nu-1)t\left[1-\left(\frac{R_c}{R}\right)^2\right]/\tau}$$

Hence for a sphere of twice the critical mass the time constant, for multiplication of neutron density by e is 2.4×10^{-8} sec.

11. Effect of Tamper

If we surround the core of active material by a shell of inactive material the shell will reflect some neutrons which would otherwise escape. Therefore a smaller quantity of active material will be enough to give rise to an explosion. The surrounding case is called a tamper.

The tamper material serves not only to retard the escape of neutrons but also by its inertia to retard the expansion of the active material. (The retardation provided by the tensile strength of the case is negligible.) For the latter purpose it is desirable to use the densest available materials (Au, W, Re, U). Present evidence indicates that for neutron reflecting properties also, one cannot do better than use these heavy elements. Needless to say, a great deal of work will have to be done on the properties of tamper materials.

We will now analyze the effect of tamper by the same approximate diffusion theory that was used in the preceding section. Let D' be the diffusion coefficient for fast neutrons in the tamper material and suppose the lifetime of a neutron in the tamper is α/τ. Here $\alpha = n'\sigma'_{c-p}/n\sigma_t$, with n' the nuclear density of the tamper and σ'_{c-p} its capture cross-section. If the tamper material is itself fissionable (U tamper) the absorption coefficient is reduced by a factor $(1-\nu_t)$, with ν_t the number of neutrons produced per capture.

At the boundary between active material and tamper, the diffusion stream of neutrons must be continuous so

$$D\left(\frac{\partial N}{\partial r}\right)_{active} = D'\left(\frac{\partial N_1}{\partial r}\right)_{tamper}$$

In the tamper the equation for neutron density is

$$\dot{N} = D'\nabla N - \frac{\nu'+\alpha}{T} N$$

or for the spatial dependence,

$$\Delta N_1 - \frac{\nu'+\alpha}{T} N_1 = 0$$

As an easy special case suppose the tamper has the same neutron diffusion coefficient as the active material (i.e. the same mean free path) but has no absorption, so $\alpha = 0$. Then under critical conditions ($\nu' = 0$) we have

$$N_1 = A/r + B$$

in the tamper material and

$$N_1 = \frac{\sin kr}{r}$$

in the active material.

At the outer boundary of the tamper, $r = R'$, we must have $N_1 = 0$ hence

$$N_1 = A\left(\frac{1}{r} - \frac{1}{R'}\right)$$

On each side of the boundary $r = R$ between active material and tamper material, the slopes must be equal so, equating the densities and slopes on both sides of the boundary we find the following equation to determine k,

$$kR \cos kR + \frac{R/R'}{1 - R/R'} \sin kR = 0$$

In the limit of a very large tamper radius $R' \to \infty$ this requires that

$$k = \pi/2R$$

which is just half the value it had in the case of the untampered gadget. Hence the critical mass needed is <u>one-eighth</u> as much as for the bare bomb.

Actually on better theory the improvement is not as great as this because the edge effect (correction for long free path) is not as big in this case as in the bare bomb. Hence the improvement of non-absorptive equal diffusion tamper over the critical mass, both handled by more accurate diffusion theory only turns out to be a factor of <u>four</u> instead of <u>eight</u>.

<u>Exercise:</u>

Consider a non-absorptive tamper material for which the diffusion coefficient D' is small compared to D. In the limit if $D' = 0$, no neutrons could escape from the active material by diffusion, so the critical radius would vanish and <u>any</u> amount of active core would be explosive.

To get an idea of the improvement obtainable from tamper material of shorter mean free path than the active material show that if $D' = \frac{1}{2}D$ then the critical mass is 1/2.40 times what it is in the case of thick tamper ($R' = \infty$) if $D' = D$. From this we see that it would be very much worth while to find tamper materials of low diffusion coefficient.

(It turns out that $\chi = kR$ is a root of

$$\chi \cos \chi = (1 - D'/D) \sin \chi$$

which is 1.17 approximately when $D'/D = 0.5$.)

If the tamper material is absorptive then the neutron density in it will fall off like e^{-kr}/r instead of 1/r which tends to make the critical mass greater than if the tamper did not absorb.

The distance the neutrons get into the tamper is $1/k = \ell' \sqrt{\frac{s}{3(1-\nu)}}$ where ℓ' is the mean free path and s the number of collisions before capture. Guessing $s \sim 20$ this gives, with $\ell' = 5$ cm, an effective tamper thickness ~ 13cm. For a U tamper $\nu \sim 0.6$, and the effective thickness is raised to 17 cm. These figures give an idea of the tamper thickness actually required; the weight of the tamper is about a ton.

For a normal U tamper the best available calculations give $R_c = 6$ cm and $M_c = 15$ Kg of 25 while with Au tamper $M_c = 22$ kg of 25.

The critical mass for 49 might be because of its larger fission cross section, less than that of 25 by about a factor 3. So for 49

M 5 Kg for U tamper
M 7.5 Kg for Au tamper.

These values of critical masses are still quite uncertain, particularly those for 49. To improve our estimates requires a better knowledge of the properties of bomb materials and tamper: neutron multiplication number, elastic and inelastic cross sections, overall experiments on tamper materials. Finally however, when materials are available, the critical masses will have to be determined by actual test.

12. Damage

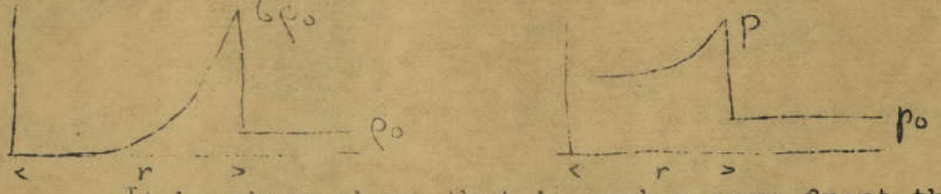
Several kinds of damage will be caused by the bomb.

A very large number of neutrons is released in the explosion. One can estimate a radius of about 1000 yards around the site of explosion as the size of the region in which the neutron concentration is great enough to produce severe pathological effects.

Enough radioactive material is produced that the total activity will be of the order of 10^6 curies even after 10 days. Just what effect this will have in rendering the locality uninhabitable depends greatly on very uncertain factors about the way in

which this is dispursed by the explosion. However the total amount of radioactivity produced, as well as the total number of neutrons, is evidently proportional just to the number of fission processes, or to the total energy released.

The mechanical explosion damage is caused by the blast or shock wave. The explosion starts acoustic waves in the air which travel with the acoustic velocity, c, superposed on the velocity u of the mass motion with which material is convected out from the center. Since $c \sim \sqrt{T}$ where T is the absolute temperature and since both u and c are greater farther back in the wave disturbance it follows that the back of the wave overtakes the front and thus builds up a sharp front. This is essentially discontinuous in both pressure and density.



It has been shown that in such a wave front the density just behind the front rises abruptly to six times its value just ahead of the front. In back of the front the density falls down essentially to zero.

If E is the total energy released in the explosion it has been shown that the maximum value of the pressure in the wave front varies as

$$p \sim E/r^3$$

the maximum pressure varying as $1/r^3$ instead of the usual $1/r^2$ because the width of the strongly compressed region increases proportionally to r.

This behavior continues as long as p is greater than about 2 atmospheres. At lower pressures there is a transition to ordinary acoustic behavior the width of the pulse no longer increasing.

If destructive action may be regarded as measured by the maximum pressure amplitude, it follows that the radius of destructive action produced by an explosion varies as $\sqrt[3]{E}$. Now in a ½ ton bomb, containing ¼ ton of TNT the destructive radius is of the order of 150 feet. Hence in a bomb equivalent to 100000 tons of TNT (or 5 kg of active material totally converted) one would expect a destructive radius of the order of $\sqrt[3]{400000} \times 150 = 11 \times 10^4$ feet or about 2 miles.

This points roughly to the kind of results which may be expected from a device of the kind we hope to make. Since the one factor that determines the damage is the energy release, our aim is simply to get as much energy from the explosion as we can. And since the materials we use are very precious, we are constrained to do this with as high an efficiency as is possible.

13. Efficiency

As remarked in Sec. 3, the material tends to blow apart as the reaction proceeds, and this tends to stop the reaction. In general then the reaction will not go to completion in an actual gadget. The fraction of energy released relative to that which would be released if all active material were transformed is called the efficiency.

Let R_{c0} = critical radius figured for normal density ρ_0, also R_0 initial radius and R = radius at a particular instant. Assume homogeneous expansion. Then the density when expanded is

$$\rho = \rho_0 (R_0/R)^3$$

and the critical radius R_c figured with the actual density ρ is

$$R_c = R_{c0}(\rho_0/\rho)$$

The reaction will proceed until expansion has gone so far that $R_c = R$. Therefore the radius R at which expansion stops is given by

$$R/R_0 = \sqrt{R_0/R_{c0}}$$

Since the ratio of R_0/R_{c0} is equal to the cube root of the ratio of M_0, the actual active mass, to M_c the critical mass we see that

$$R/R_0 = \sqrt[3]{M_0/M_{c0}}$$

and therefore a gadget having twice the critical mass will expand to a radius only $\sqrt[6]{2} = 1.12$ times its original radius before the reaction stops.

The next problem is to find a simple expression for the time taken for this expansion to occur, since we already know how to calculate the time constant ν'/τ of the reaction. Of course ν' is not a constant during the expansion since its value depends on the radius but this point will be ignored at first.

At a place where we have N neutrons/cm^3 there will be N/τ fissions/cm^3 sec and therefore if $\mathcal{E}$ is the energy release in erg/fission the volume rate of energy generation is $(\mathcal{E}/\tau)N$. Hence the total energy released in unit volume between time $-\infty$ and time t is

$$W = (\mathcal{E}/\nu')N e^{\nu' t/\tau}$$

Most of this energy goes at once into kinetic energy of the fission fragments which are quickly brought to rest in the material by communication of their energy largely to thermal kinetic energy of motion to the other atoms of the active stuff.

The course of events is shown in Fig. 3. The units on the scale of abscissas are units of $\nu' t/\tau$. If there was no expansion, and if the rate of reaction toward the end was not slowed down by depletion of active material, then the energy released up to a given time in erg/cm^3 would be given by the values on the upper logarithmic scale. The places on this scale marked 100%, 10% and 1% respectively show the energy released in unit volume for these three values of the efficiency. A second logarithmic scale shows the growth of the neutron density with time under these assumptions.

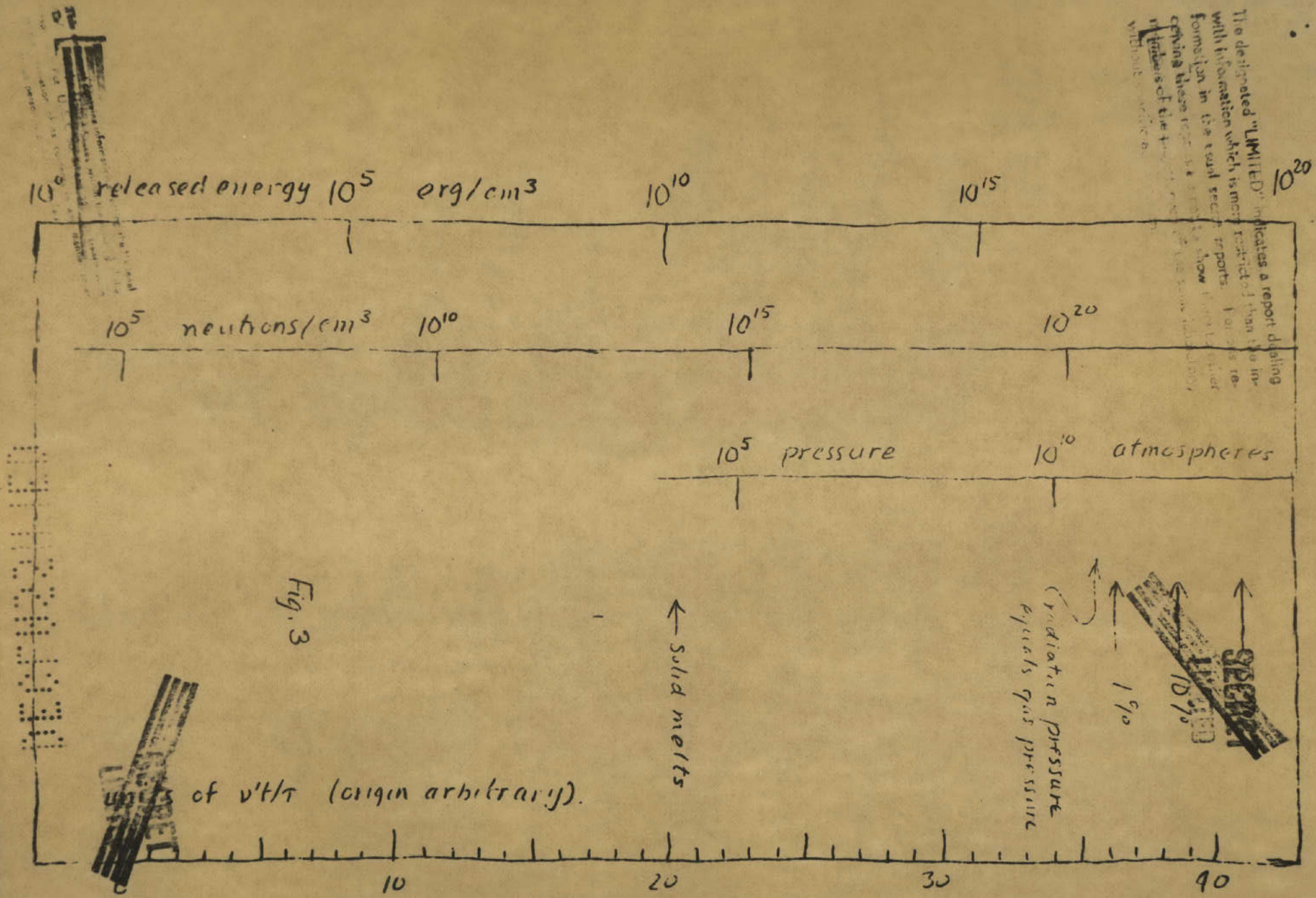

10^0 released energy 10^5 erg/cm^3 10^{10} 10^{15} 10^{20}
10^5 neutrons/cm^3 10^{10} 10^{15} 10^{20}
10^5 pressure 10^{10} atmospheres
← solid melts
(radiation pressure equals gas pressure
1%
10%
units of v't/τ (origin arbitrary).
10
20
30
90

Fig. 3

It can be calculated that the pressure in atmospheres is very roughly like the values given on the third scale. At a point just below 10^{17} erg/cm^3 evolved the radiation pressure is equal to the gas pressure, after that radiation pressure predominates. Near 10^{10} erg/cm^3 is the place where the solid melts so up to this time nothing very drastic has happened - the importtant phenomena occur in the next 20 units of $\nu' t/\tau$.

Very roughly we may estimate, as follows for masses not much larger than the critical mass, the combination of factors on which the efficiency depends: In a time of the order τ/ν' the material moves from R_0 to R so acquires a velocity

$$v \sim (\nu'/\tau)(R - R_0)$$

Writing $R_0 = R_{c0}(1+\Delta)$ we find that

$$R - R_0 \approx \tfrac{1}{2}\Delta R_{c0}$$

The kinetic energy per gram that is acquired by the material is

$$v^2/2 \sim \tfrac{1}{4}(\nu'/\tau)^2 \Delta^2 R_{c0}^2$$

The total energy released is greater in the order $pV \div pdV$ or $2/3\Delta$. Let $\varepsilon = 7.10^{17}$ erg/gram be the energy release for complete conversion then the efficiency is of the order

$$f \approx (\nu'^2/\varepsilon\tau^2)(\Delta^2/4)R_{c0}^2(2/3\Delta)$$

or

$$f \approx (1/6)(\nu'^2/\varepsilon\tau^2)R_{c0}^2\Delta$$

For an untampered gadget

$$\nu' \approx 2(\nu-1)\Delta$$

giving

$$\boxed{f = \frac{2}{3}\,\frac{(\nu-1)^2 R_{c0}^2}{\varepsilon\tau^2}\,\Delta^3}$$

Putting in the known constants

we find $\varepsilon = 7.10^{17}$ $\quad \tau = 10^{-8}$ $\quad R_c = 9$

$$f = K\Delta^3 \quad \text{with} \quad K = 1.1$$

If this very rough calculation is replaced by a more accurate one the only change is to alter the value of the coefficient K. The calculations are not yet complete, but the true value is probably $K \approx \frac{1}{4}$ to $\frac{1}{2}$.

Hence for a mass that is twice the critical mass, $R_0 = \sqrt[3]{2}\,R_c$ so $\Delta \approx 0.25$ and the efficiency comes out less than 1%. We see that the efficiency is extremely low even when this much valuable material is used.

Notice that τ varies inversely as the velocity of the neutrons. Hence it is advantageous for the neutrons to be fast. The efficiency depends on the nuclear properties through the factors

$$f \sim \frac{v^2(\nu-1)}{\varepsilon}\,\frac{\sigma_f}{\sigma_t}\,\Delta^3$$

where v is the mean speed of the neutrons and the other symbols are already defined.

In the above treatment we have considered only the effect of the general expansion of the bomb material. There is an additional effect which tends to stop the reaction: as the pressure builds up it begins to blow off material at the outer edge of the bomb. This turns out to be of comparable importance in

stopping the reaction to the general expansion of the interior. However the formula for the efficiency can be shown to be unchanged in form; the edge expansion manifests itself simply in a reduction in the constant K. The effect of blowing off the edge has been already taken into account in the more accurate estimate of K given above.

14. Effect of Tamper on Efficiency

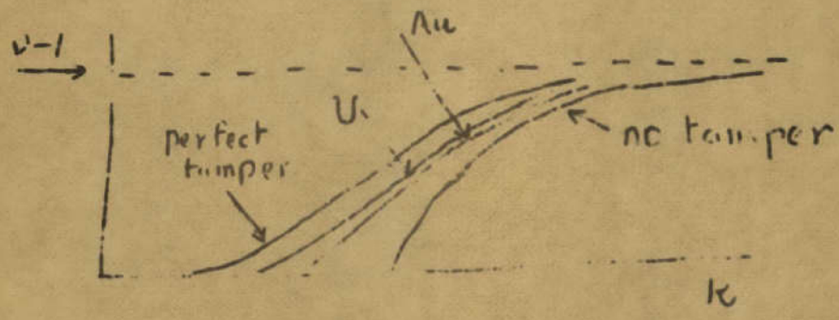
For a given mass of active material, tamper always increases efficiency. It acts both to reflect neutrons back into the active material and by its inertia to slow the expansion thus giving opportunity for the reaction to proceed farther before it is stopped by the expansion.

However the increase in efficiency given by a good tamper is not as large as one might judge simply from the reduction in the critical mass produced by the tamper. This is due to the fact that the neutrons which are returned by diffusion into and back out of the tamper take a long time to return, particularly since they are slowed down by inelastic impacts in the tamper material.

The time scale, for masses near critical where one has to rely on the slowest neutrons to keep the chain going, now becomes effectively the lifetime of neutrons in the tamper, rather than the lifetime in the bomb. The lifetime of neutrons in a U tamper is $\sim 10^{-7}$ sec, ten times that in the bomb. The efficiency is consequently very small just above the critical mass, so to some extent the reduction in critical mass is of no use to us.

One can get a picture of the effect of tamper on efficiency from Fig. 4, in which U ν' is plotted against bomb radius for various tamper materials. The time scale is given by τ/ν' ; the efficiency, as we have seen in the preceeding section, is inversely proportional to the square of the time scale. Thus $f \sim \nu'^2$.

Fig. 4.



If we use good tamper (U) the efficiency is very low near the critical mass due to the small slope of the ν' vs.R curve near $\nu'=0$. When one uses a mass sufficiently greater than the critical to get good efficiency there is not very much difference between U and Au as tamper materials.

It turns out that if one is using 4 K_c and the U tamper, then only about 15% more active material is needed to get the same energy release with a gold tamper, although the critical masses differ by 50%.

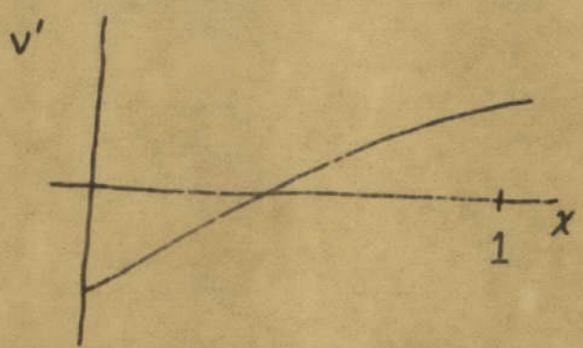
In addition to reflecting neutrons, the tamper also inhibits the tendency of the edge of the bomb to blow off. The edge expands into the tamper material, starting a shock wave which compresses the tamper material sixteenfold. These edge effects as remarked in Sec. 13 always act to reduce the factor K in the formula, $f = K\Delta^3$, but not by as great an amount in the case of tamped bomb as in the case of the untamped bomb.

15. Detonation

Before firing, the active material must be disposed in such a way that the effective neutron number ν' is less than unity. The act of firing consists in producing a rearrangement such that after the rearrangement ν' is greater than unity.

This problem is complicated by the fact that, as we have seen, we need to deal with a total mass of active material considerably greater than the critical in order to get appreciable efficiency.

For any proposed type of rearrangement we may introduce a coordinate χ which changes from 0 to 1 as the rearrangement of parts proceeds from its initial to its final value.



Schematically ν' will vary with χ along some such curve as is indicated in the sketch. Since the rearrangement proceeds at a finite speed there will be a finite time interval during which ν' though positive is much smaller than its final value. As considered in more detail later there will always be some un-

avoidable sources of neutrons in the active material. In any scheme of rearrangement some fairly massive amount of material will have to be moved a distance of the order of $R_c \sim 10$ cm. Assuming a speed of 3000 ft/sec can be imparted with some type of gun this means that the time it takes to put the pieces of the bomb together is $\sim 10^{-4}$ sec. Since the whole explosion is over in a time $\sim 75\, T/\nu' = \frac{10^{-6}}{\nu'}$ sec, we see that, except for very small ν' ($\nu' < .01$), an explosion started by a premature neutron will be all finished before there is time for the pieces to move an appreciable distance. Thus if neutron multiplication happens to start before the pieces reach their final configuration an explosion will occur that is of lower efficiency corresponding to the lower value of   at the instant of explosion.

To avoid predetonation it is therefore necessary to keep the neutron background as low as possible and to effect the rearrangement as rapidly as possible.

16. Probability of Predetonation

Since it will be clearly impossible to reduce the neutron background rigorously to zero, there will always be some chance of predetonation. In this section we try to see how great this chance is in order to see how this affects the firing problem.

The chance of predetonation is dependent on the likelihood of a neutron appearing in the active mass while ν' is still small and on the likelihood that such a neutron will really set off a chain reaction. With just a single neutron released when $\nu' > 0$ it is by no means certain that a chain reaction will start, since any particular neutron may escape from the active material without causing a chain reaction.

The question can be considered in relation to a little gambling problem. In tossing loaded coins suppose p is the probability of winning and q that of losing. Let P_n be the probability of losing all of an initial stock of n coins. On the first toss either one wins and thus has $(n + 1)$ coins or loses and thus has $(n - 1)$ coins. Hence the probability P_n is given by

$$P_n = p P_{n+1} + q P_{n-1}$$

the solution of which is

$$P_n = (q/p)^n$$

Identifying this with the neutron multiplication problem one can show that $q/p = 1 - \nu'$. Hence the probability of not starting a chain reaction with one neutron is $(1 - \nu')$ or ν' is the probability that any one neutron will start a chain reaction.

Suppose now that there is a source of N neutron/sec. Let $P(t)$ be the probability of not getting a predetonation up to the instant t. In the interval dt we have

$$dP = -N\,dt\,\nu' P$$

On the left the first three factors together give the probability of going off in time dt, and the factor P is the probability of not having had a predetonation up to that time.

Near the value $\nu' = 0$ we may suppose that ν' varies linearly with time, $\nu' = ct$. Hence, integrating the differential equation

$$P = e^{-\frac{1}{2} N c t^2} = e^{-\frac{1}{2} \overline{N} \nu'}$$

where $\bar{N} = Nt$ is the number of neutrons expected in the interval between t = 0, when $\nu' = 0$, and the time when the multiplication number has reached the value ν'. Evidently for a particular type of firing rearrangement $\bar{N}$ will vary inversely as the velocity with which the firing rearrangement is carried out.

For example consider a bomb whose mass is between two and three critical masses, for which the final value of ν' is 0.3 and suppose that $N = 10^4$ neutrons/sec from unavoidable sources. Also suppose that one piece must move d = 10 cm from the $\nu' = 0.0$ configuration to the final $\nu' = 0.3$ configuration. Suppose that this piece has a velocity of 10^5 cm/sec then $\bar{N} = 1$ and

$$P = e^{-0.15}$$

so there is approximately a 15% chance of predetonation.

This is the chance of predetonation any time up to that at which the final value of ν' is reached. In this example the exponent is small enough that the chance of predetonation, (1 - P), is given by the linear approximation.

$$(1 - P) = \tfrac{1}{2} \bar{N} \nu'$$

Since the efficiency varies as ν'^3 one will get an explosion of less than ¼ of the maximum if it goes off before ν' has reached the value $0.3/\sqrt[3]{4} = 0.19$. Hence the probability of an explosion giving less than 25% of the maximum value is

$$(.19/.3)^2 \times .15 = 6\%$$

The example serves to indicate the importance of taking great pains to get the least possible neutron background, and of shooting the firing rearrangement with the maximum possible velocity. It seems one should strive for a neutron background of 10000 neutron/sec or less and firing velocities of 3000 ft/sec or more. Bith of these are difficult ~~at~~ of attainment.

17. Fizzles

The question now arises: what if by bad luck or because the neutron background is very high, the bomb goes off when ν' is very close to zero? It is important to know whether the enemy will have an opportunity to inspect the remains and recover the material. We shall see that this is not a worry; in any event the bomb will generate enough energy to completely destroy itself.

It has been remarked in the last section that for very small ν' ($\nu' < .01$), the explosion takes so long that the pieces do have time to move an appreciable distance before the reaction ends. Thus even if a neutron enters and starts a chain just when $\nu' = 0$ there will be time for ν' to rise to a positive value, and give an efficiency small, but greater than zero.

Suppose, then, that a neutron is released when $\nu' = 0$. The number of neutrons builds up according to the equation

$$\dot{N} = (\nu'/\tau) N$$

As a first approximation we may suppose ν' varies linearly with the distance x the pieces move from the point where $\nu' = 0$, so

$$\nu' = \nu_0 (x/d_0)$$

where ν_0 is the value of ν' when the pieces reach their final op-

timal configuration, and d_0 is the distance to reach this configuration. If the velocity of fire is v, we have $x = vt$,

$$\ln N = \int_0^t (\nu'/\tau)\,dt = \frac{1}{2}\frac{\nu_0 v}{d_0 \tau} t^2$$

Suppose the reaction continues until about 10^{22} neutrons are produced, which would correspond to an energy production equivalent to 100 tons of TNT. Then, at the end of the reaction

$$\ln N = \ln 10^{22} \simeq 50.$$

(We can check this assumption after we have completed our estimate of the energy release. However, since the final number of neutrons enters only in the logarithm of a large number, our result is quite insensitive to what we take for N at this point.)

Thus the reaction ends when

$$\frac{1}{2}\frac{\nu_0 v}{d_0 \tau} t^2 = 50 \qquad x' = v t' = 100 \frac{d_0 v \tau}{\nu_0} \qquad \nu' = \nu_0 \frac{x}{d_0}$$

The efficiency is

$$f \sim \frac{1}{2}\nu'^3 = 500\,\nu_0^{3/2}\left(\frac{v\tau}{d_0}\right)^{3/2} = 10\sqrt{\nu_0 \frac{v\tau}{d_0}}$$

Using the same figures as in the preceeding section ($\nu_0 = .3$, $v = 10^5$, $d_0 = 10$) we find

$$f = 8 \times 10^{-5}$$

The mass of 25 in the bomb is about 40 kg. The mass used up is thus $40 \times 8 \times 10^{-5} = .003$ kg, and the energy release is .003 x 20000 = 60 tons of TNT equivalent, ample to destroy the bomb.

18. Detonating Source

To avoid predetonation we must make sure that there is only a small probability of a neutron appearing while the pieces of the bomb are being put together. On the other hand, when the pieces reach their best position we want to be very sure that a neutron starts the reaction before the pieces have a chance to seperate or break. It may be possible to make the projectile seat and stay in the desired position. Failing in this, or in any event as extra insurance, another possibility is to provide a strong neutron source which becomes active as soon as the pieces come into position. For example one might use a Ra + Be source in which the Ra is on one piece and the Be on the other so neutrons are only produced when the pieces are close to the proper relative position.

We can easily estimate the strength of source required. After the source starts working, we want a high probability of detonation before the pieces have time to move more than say 1 cm. This means that N, the neutrons/sec from the source must be large enough that

$$\frac{1}{2}\frac{N d v'}{v} \gg 1 \quad (\text{say} = 10)$$

$$N = 10^7 \text{ neutrons/sec.}$$

This is the yield from 1 gr Ra intimately mixed with beryllium. Hence it might be necessary to ~~rise~~ several grams of radium since it will probably not be used efficiently in this type of source.

Some other substance such as polonium that is not so γ-active as radium will probably prove more satisfactory.

Evidently a source of this strength that can be activated within about 10^{-5} sec and is mechanically rugged enough to stand the shocks associated with firing presents a difficult problem.

19. Neutron Background

There are three recognized sources of neutrons which provide the background which gives rise to danger of predetonation: (a) cosmic ray neutrons, (b) spontaneous fission, (c) nuclear reactions which produce neutrons.

(a) Cosmic Rays. The number of cosmic ray neutrons is about 1 per cm^2 per minute which is too few to be of any importance.

(b) Spontaneous fission. The spontaneous fission rate is known only for 28 which is responsible for the fission activity of ordinary U. At present we have only upper limits for 25 and 49 since the activity of these has not been detected. The known facts are

28 gives 15 neutrons/kg sec.
25 " < 150 "
49 " < 500 "

It is considered probable that the rates for 25 and 49 are much smaller than these upper limits. Even if 25 and 49 were the same as 28, a 40 kg bomb would have a background from this source of 600 neutron/sec. This does not seem difficult to beat.

But if U is used as tamper this will weigh about a ton which gives 15000 neutron/sec. Of course not all of these will get into the active material but one may expect a background of several thousand per second from this source.

Thus with a U tamper one is faced with the problem of high velocity firing. In the range of moderately high efficiencies, say 4 Mc of active material, it might for this reason not be worth while to use a U tamper, since as we have seen, an inactive tamper will cost only about 15% more active material. Or one might use a compromise in which the tamper was an inner layer of U, backed up by inactive material; for masses this large the time scale is so short that neutrons do not have time to penetrate more than about 5 cm into the tamper anyway.

(c) Nuclear reactions. The only important reactions are the (α,n) reactions of light elements which might be present as impurities. The (γ,n) reactions have a negligible yield.

Let us examine what sort of limit on light element impurities in the active material is set by the need of holding down the neutron background from this source.

The problem is particularly bad for 49 since its half-life is only 20000 years. Its mean life is thus 30000 years $= 10^{12}$ sec. Thus 10 kg of 49, containing 2.5×10^{25} nuclei gives 2.5×10^{13} α-particles/sec.

The yield from Ra α's on Be is 1.2×10^{-4} and the shorter range from α's of 49 as compared with those of Ra and its equilibrium products will perhaps cut this figure in half, say $6 \cdot 10^{-5}$. Since the stopping power for α's of these energies is proportional to $\sqrt{A}$ where A is the atomic weight, the stopping power per gram is proportional to $1/\sqrt{A}$.

If the concentration by weight of Be in the active material is C then the yield of neutron/sec is

$$\sqrt{239/9} \cdot c \cdot N_\alpha \cdot y$$

where N_α is the number of α's per second and y is the yield. Hence to get 10000 neutrons/sec one would need to have a concentration given by

$$\sqrt{239/9} \cdot c \cdot 2.5 \cdot 10^{13} \cdot 6 \times 10^{-5} = 10^4$$

that is $c \sim 10^{-6}$ which is, of course, a very low concentration of anything in anything else.

The yield drops rapidly as one goes to elements of higher atomic weight because of the increased Coulomb barrier. So it is unnecessary to consider limits on elements beyond Ca as long as ordinary standards of purity are maintained.

Experiments on the yields with light elements need to be done. One can base some rough guesses on the standard barrier penetration formulas and find the following upper limits on the concentration by weight for several light elements for production of 10^4 neutron/sec/

Li	2×10^{-5}	
Be	~~10^{-7}~~ 10^{-6}	
B	$2 \cdot 10^{-6}$	
C	$2 \cdot 10^{-4}$	*
N	-	**
O	2×10^{-3}	***
F	2×10^{-5}	

* Low yield because only C^{13} contributes.
** $(\alpha\text{-}n)$ reaction not energetically possible.
*** Low yield because only O^{17} contributes.
The effect of several impurities simultaneously present is of course additive.

It is thus recognized that the preparation and handling of the 49 in such a way as to attain and maintain such high standards of purity is an extremely difficult problem. And it seems very probable that the neutron background will be high and therefore high velocity firing will be desirable.

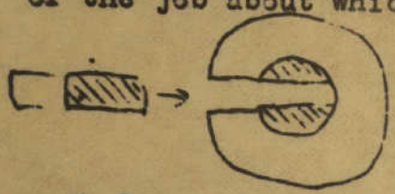
With 25 the situation is much more favorable. The α's come from 24 present in normal U to about 1/10000. If all 24 goes with 25 in the separation from 28 we shall have 1/100 of 24 in the 25. The lifetime of 24 is 100 times that of 49 so the

concentration of impurities in 25 may be 10^4 times that in 49 for the same background, which is not at all difficult of attainment.

To summarize: 49 will be extremely difficult to work with from the stand-point of neutron background whereas 25 without U tamper will be not very difficult.

20. Shooting

We now consider briefly the problem of the actual mechanics of shooting so that the pieces are brought together with a relative velocity of the order of 10^5 cm/sec or more. This is the part of the job about which we know least at present.



One way is to use a sphere and to shoot into it a cylindrical plug made of some active material and some tamper, as in the sketch. This avoids fancy shapes and gives the most favorable shape, for shooting; to the projected piece whose mass would be of the order of 100 lbs.

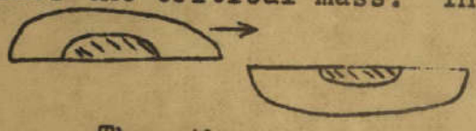
The highest muzzle velocity available in U. S. Army guns is one whose bore is 4.7 inches and whose barrel is 21 feet long. This gives a 50 lb. projectile a muzzle velocity of 3150 ft/sec. The gun weighs 5 tons. It appears that the ratio of projectile mass to gun mass is about constant for different guns so a 100 lb. projectile would require a gun weighing about 10 tons.

The weight of the gun varies very roughly as the cube of the muzzle velocity hence there is a high premium on using lower velocities of fire.

Another possibility is to use two guns and to fire two projectiles at each other. For the same relative velocity this arrangement requires about 1/8 as much total gun weight. Here the worst difficulty lies in timing the two guns. This can be partly overcome by using an elongated tamper mass and putting all the active material in the projectiles so it does not matter exactly where they meet. We have been told that at present it would be possible to synchronize so the spread in places of impact on various shots would be 2 or 3 feet. One serious restriction imposed by these shooting methods is that the mass of active material that can be gotten together is limited by the fact that each piece separately must be non-explosive. Since the separate pieces are not of the best shape, nor surrounded by the best tamper material, one is not limited to two critical masses for the completed bomb, but might perhaps get as high as four critical masses. However in the two gun scheme, if the final mass is to be $\sim$ 4Mc, each piece separately would probably be explosive as soon as it entered the tamper, and better synchronization would be required. It seems worthwhile to investigate whether present performance might not be improved by a factor ten.

Severe restrictions on the mass of the bomb can be circumvented by using pieces of shape more difficult to shoot. For example a flat plate of actual material tamped on only one side, has a mimimum thickness below which it can no longer support a chain reaction, no matter how large its area, because of neutron leakage across the untamped surface. If two such plates were slid together, unpamped surfaces in contact, the resulting arrangement could be well over the critical thickness for a plate tamped on both sides, and the mass would depend only on the area of the plates.

Calculations show that the critical mass of a well tamped spheroid, whose major axis is five times its minor axis, is only 35% larger than the critical mass of a sphere. If such a spheroid 10 cm thick and 50 cm in diameter were sliced in half, each piece would be sub-critical though the total mass, 250 Kg, is 12 times the critical mass. The efficiency of such an arrangement would be quite good, since the expansion tends to bring the material more and more nearly into a spherical shape.



Thus there are many ordnance questions we would like to have answered. We would like to know how well guns can be synchronized. We shall need information about the possibilities of firing other than cylindrical shapes at lower velocities. Also we shall need to know the mechanical effects of the blast wave preceeding the projectile in the gun barrel. Also whether the projectile can be made to seat itself properly and whether a piston of inactive material may be used to drive the active material into place, this being desirable because thus the active material might be kept out of the gun barrel which to some extent acts as a tamper.

Various other shooting arrangements have been suggested but as yet not carefully analyzed.

For example it has been suggested that the pieces might be mounted on a ring as in the sketch. If explosive material were distributed around the ring and fired the pieces would be blown inward to form a sphere.

Another more likely possibility is to have the sphere assembled but with a wedge of neutron-absorbing material built into it, which on firing would be blown out by an explosive charge causing ν' to go from less than unity to more than unity. Here the difficulty lies in the fact that no material is known whose absorption coefficient for fast neutrons is much larger than the emission coefficient of the bomb material. Hence the absorbing plug will need to have a volume comparable to that of the absorber and when removed will leave the active material in an unfavorable configuration, equivalent to a low mean density.

21. Autocatalytic Methods

The term "autocatalytic method" is being used to describe any arrangement in which the motions of material produced by the reaction will act, at least for a time, to increase ν' rather than to decrease it. Evidently if arrangements having this property can be developed they would be very valuably, especially if the tendency toward increasing ν' was possessed to any marked degree.

Suppose we had an arrangement in which for example ν' would increase of its own accord from a low value like 0.01 up to a value 10 to 50 times greater. The firing problem would be simplified by the low initial value of ν', and the efficiency would be maintained by the tendency to develop a high value of ν' as the reaction proceeds. It may be that a method of this kind will be absolutely essential for utilization of 49 owing to the difficulties of high neutron background from (α, n) reactions with the impurities as already discussed.

The simplest scheme which might be autocatalytic is indicated in the sketch where the active material is disposed in a hollow shell. Suppose that when the firing plug is in place one has just the critical mass for this configuration. If as the reaction proceeds the expansion were to proceed only inward it is easy to see from diffusion theory that ν' would increase. Of course in actual fact it will proceed outward (tending to decrease ν') as well as inward and the outward expansion would in reality give the dominant effect. However, even if the outward expansion were very small compared to the inward expansion it has been calculated that this method gives very low efficiency: with 12 Mc an efficiency of only about 10^{-9} was calculated.

A better arrangement is the "boron bubble" scheme. B^{10} has the largest known absorption cross-section for fast neutrons, $1.52 \cdot 10^{-24}$ cm^2. Suppose we take a large mass of active material and put in enough boron to make the mass just critical. The device is then fired by adding some more active material or tamper.

As the reaction proceeds the boron is compressed and is less effective at absorbing neutrons than when not compressed. This can be seen most readily if one considers the case in which the bubbles are large compared to the mean depth in which a neutron goes in boron before being absorbed. Then their effectiveness in removing neutrons will be proportional to their total area and so will drop on compression. Hence ν' will increase as the bubbles are compressed. If the bomb is sufficiently large this tendency is bound to overweigh the opposing one due to the general expansion of the bomb material, since the distance the edge of the bomb must move to produce a given decrease in ν' increases with the radius of the bomb, whereas for a larger bomb the distance the edge of

bubble must move is unchanged, since it is not necessary to increase the radius of the bubbles but only to use more of them.

The density of particles (electrons plus nuclei) in boron is 8.3×10^{23} particle/cm^3 while in uranium it is more than 5 times greater. Therefore as soon as the reaction has proceeded to the point where there is a high degree of ionization and the material behaves as a gas there will be a great action to compress the boron. An opposing tendency to the one desired will be the stirring or turbulence acting to mix the boron uniformly with the uniform, but the time scale is too short for this to be effective.

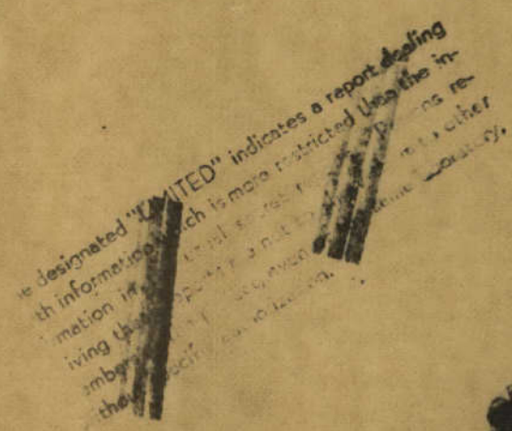
It can be shown that if initially $\nu' = 0$, allowing for the boron absorption, and if no expansion of the outer edge occurs then ν' will rise to $\nu' \sim \frac{1}{9}(\nu - 1)$ by compression of the boron. This scheme requires at least five times the critical mass for no boron, and the efficiency is low unless considerably more is used.

If one uses just that amount of boron which makes twice the no-boron critical mass be just critical, then the efficiency is lower by a factor at least 30.

All autocatalytic schemes that have been thought of so far require large amounts of active material, are low in efficiency unless very large amounts are used, and are dangerous to handle. Some bright ideas are needed.

22. Conclusion

From the proceeding outline we see that the immediate experimental program is largely concerned with measuring the neutron properties of various materials, and with the ordnance problem. It is also necessary to start now studies on techniques for direct experimental determination of critical size and time scale, working with large but sub-critical amounts of active material.



END

DATE FILMED

9 / 19 / 73