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ADDED MASS FOR PLATES PARTIALLY SUBMERGED IN WATER

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Abstract

Added mass values for plates partially submerged in water are obtained from a series of experiments. Both solid and perforated plates are tested. Each plate is circular and is constrained to move as a piston in a horizontal cylindrical container. Water flow past the perimeter of a plate is controlled by the narrow gap between the container and the edges of the plate. The perforated plate has an equilateral triangular pitch pattern of circular holes.

The plate vibrates as a rigid mass on a flexible suspension system. The plate excitation is produced with an impact hammer, although shaker excitation is used to confirm natural frequencies. The natural frequency of the plate is measured for seven water levels in the container ranging from empty to full. For each water level, added mass is computed using the ratio of the natural frequency in air to the natural frequency in water. The added mass for fully submerged plates is compared to published experimental and theoretical results.

Nomenclature

A_s = solid area of plate	m_a = added mass per unit area
A_h = perforated plate hole area	M_a = added mass
A_w = plate solid area covered by water	M_v = mass in vacuum
f_v = natural frequency in vacuum	M_w = apparent mass in water
f_w = natural frequency in water	ΔM = incremental plate mass
f_{Δ} = natural frequency with incremental mass in vacuum	ω_v = angular frequency in vacuum
K_v = stiffness in vacuum	ω_w = angular frequency in water
K_w = stiffness in water	$\omega_{\Delta M}$ = angular frequency with incremental mass in vacuum

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1.0 Introduction

The fact that a structure vibrating in water has lower natural frequencies than it has in a vacuum has long been recognized. DuBuat [1] began investigation of hydrodynamic phenomena began about 1779. The effect of fluid on the frequencies of a structure is typically represented by a quantity called "added mass" in an analysis. Stelson and Mavis [2] experimentally determined the added mass for several simple structural elements. Numerous researchers have since published added mass data for other structural geometries. Compendiums of added mass formulas and values for a variety of geometries has been published in references such as [3]. DeSanto [4] published added mass values for perforated plates vibrating in water, and Lindholm [5] treated cantilever plates partially submerged in water. Until now, added mass values for partially submerged plates which move as rigid bodies or which are perforated have not been published.

This paper presents the added mass determined for solid and perforated plates partially submerged in water. The added mass is based on experimental measurements. Results for the frequency effects of fluid on a plate are represented by added mass per unit surface area.

The method of exciting the plates for the present investigation is a departure from the typical method used. Traditionally, the vessel which contains the water is restrained, and the structure for which the added mass is being determined is excited directly. In this case, the excitation is applied to the vessel, and the response of the structure inside is studied. Comparison of the excitation techniques is addressed briefly in this paper.

2.0 Calculating Added Mass

Added mass values can be calculated from several readily measured structural response parameters, providing the structure vibrates as a single mass oscillator. The necessary parameters are the rigid body mass of the structure, the fundamental frequency in a vacuum, and the frequency in fluid. For relatively dense materials, the frequency in air is sufficiently close to the frequency in a vacuum to permit use of the in-air frequency for added mass calculations. An alternative to using the rigid body mass of the structure in the calculations is discussed later.

The undamped frequency of a single mass oscillator in a vacuum (or air) is expressed as

$$\omega_v = \sqrt{K_v/M_v} \quad (1)$$

and the frequency of the same oscillator in water is

$$\omega_w = \sqrt{K_w/M_w} \quad (2)$$

For typical structures, the immersion of the oscillator in water does not change the stiffness of the system, so

$$K_v = K_w \quad (3)$$

The difference in frequencies is attributed to the difference in oscillator masses, M_v versus M_w . The apparent mass of the water, or added mass M_a , is defined as

$$M_a = M_w - M_v \quad (4)$$

From equations (1) through (4), it can be shown that

$$M_a = M_v [(\omega_v/\omega_w)^2 - 1] \quad (5)$$

If the mass of the oscillator in a vacuum (or air) is not readily measurable, the technique used in [4] provides an alternative. An additional mass, ΔM , can be attached to the oscillator, and the frequency is measured again.

$$\omega_{\Delta M} = \sqrt{K_v/(M_v + \Delta M)} \quad (6)$$

Solution of equations (1) through (4) and (6) for M_a in terms of ΔM gives

$$M_a = \Delta M [(\omega_v/\omega_w)^2 - 1] / [(\omega_v/\omega_{\Delta M})^2 - 1] \quad (7)$$

Now the added mass can be calculated from the natural frequencies of the oscillator in air and water plus either a) the mass of the oscillator or b) the incremental mass and the frequency of the oscillator with the incremental mass attached.

3.0 Experiments

The parameters used in equations (5) or (7) to calculate the added mass can be measured in experiments which satisfy certain assumptions either explicit or implicit in the equations. First, the plates must vibrate as single mass oscillators. The frequencies of the plates must be clearly identifiable. The vibrating system must have essentially linear elastic properties. And, the fluid environment of the plates must be well defined so that the applicability of the calculated added mass values is clear.

The experimental apparatus for this investigation is designed to provide plate vibration as a rigid body and only in the direction normal to the plane of the plate. The fundamental frequency of each plate on its support system is designed to be far removed from the natural frequencies of any other component of the apparatus. Water is inhibited from flowing around the edges of the plates by a close fit between the edges of the plate and the tank containing the plate and water. The boundary conditions of the tests allow the added mass values to represent any size plate on an added mass per unit area basis.

Apparatus - The tests are conducted in a closed cylindrical vessel having circular caps. The vessel is 50.2 cm long and 25.3 cm in diameter (inside dimensions). Wall thickness of the vessel and end caps is 1.27 cm. The vessel is formed from clear acrylic material. All other components of the test vessel are stainless steel. The design of the vessel and plate support structure is represented by Figures 1 and 2. The vessel is oriented horizontally during the tests.

The test plate geometries are shown in Figure 3. Both plates are 25.1 cm in diameter and 0.64 cm thick. There are 68 holes 1.59 cm in diameter in the perforated plate. The ratio of hole area to the area of solid material remaining is 0.37.

The support structure for a plate in the test vessel consists of a threaded rod 22.9 cm long by 1.9 cm diameter mounted axially through the center of the plate and supported at each end by three radial flexure rods connected from the center rod to the cylinder wall. The flexure rods (0.80 cm dia.) are spaced at 120 degree intervals circumferentially. Collars at each end of the threaded rod provide threaded connections between the center rod and the flexure rods. Lock nuts on each side of the plate fix the plate on the center rod. And reinforcing fittings shown in detail in Figure 4 provide the connection between the flexure rods and the cylinder wall.

Two circumferential steel bands are installed outside the vessel. The bands stiffen the cylinder against shell mode vibration and provide support connections for the vessel. Each band is located about one-fifth the distance from an end of the vessel. A long pair of cables connects each band to a ceiling-mounted frame. The pairs of cables attached to opposite sides of a band are closer to each other at the elevation of the vessel than at the ceiling elevation. This trapezoidal configuration inhibits lateral motion of the vessel. The two cables on either side of the vessel are parallel so that the vessel can move freely in the axial direction.

Test Parameters - Several parameters were varied in the tests. Seven different water elevations in the test vessel were investigated. Tests were conducted with a solid plate and with a perforated plate. Finally, two methods of exciting the plates were used, impact hammer and electrodynamic shaker. Impact was the principal method of excitation. Shaker excitation was used as a check on results.

Instrumentation and Test Methods - The fundamental frequencies of plate vibration as a function of submerged plate area were measured with uniaxial accelerometers attached to the plate. Three Endevco model 2220-C miniature accelerometers were attached to the plate at the perimeter locations shown in Figure 3, and were oriented to sense vibration in the direction normal to the plane of the plate. These top, mid-height, and bottom locations were selected to verify rigid body motion, particularly for the partially submerged test cases. The installed accelerometers and associated cables and connectors were waterproofed by coating them with Silastic 732 RTV Sealant. Endevco model 2735 charge amplifiers were used to condition the accelerometer signals and filter the signals at 500 Hz lowpass.

Two excitation techniques were employed to obtain the plate vibration response data. The primary technique involved the use of a non-instrumented 71 gram hammer to provide low level excitation in the direction normal to the plate. The vessel was subjected to repeated impacts by tapping at the center of one of the end caps at a rate of about one tap per second. The corresponding accelerometer signals were input to a Genrad Time/Data Fourier Analyzer which processed the measurements in the form of frequency spectrum and phase plots, from which the fundamental plate frequency was obtained. The duration of the repeated impacts was sufficient to perform a 16-average analysis of the signals.

For the second test technique, sinusoidal excitation was imparted to the vessel using a floor-mounted Ling shaker. The fundamental plate response was determined by the sine-dwell method. The excitation was imposed along the axis of the vessel by connecting the shaker to a steel harness which encased the entire vessel. Vibrating the vessel in this manner, as a rigid body, avoided the introduction of possible extraneous excitation that could result from pumping the water if the shaker were attached to the vessel end cap or the plate itself. Response signals from the three accelerometers were monitored on a four-channel oscilloscope while sweeping through a frequency range expected to include the fundamental plate frequency. The fundamental frequency was identified as the shaker excitation frequency for which the three signals were at the same acceleration amplitude and phase.

4.0 Results

Identification of the fundamental plate frequency was readily accomplished due to the distinct separation of the plate frequency from any other system frequency. Power spectral density plots for each water level clearly show the influence of the water on the plate natural frequencies.

Solid Plate - The power spectral density plots are shown in Figure 5 for the series of seven water levels ranging from empty (in-air plate response) to full. Beginning with the empty condition in Figure 5-a, where the fundamental plate frequency is represented by the peak to the left in the plot, the frequency shifts from the 103.75 Hz value in air to a value less than 47 Hz for the 100% water level response of Figure 5-g. The frequency measured for each water level is presented in Table 1.

The frequency peaks to the right in each plot of Figure 5 are shell modes of the vessel. This was confirmed by radially impacting the side of the vessel cylinder and noting that the relative amplitude of the higher frequencies in each power spectral density plot became greater than for the fundamental frequency.

The fundamental frequency for the 100% water level case could not be defined because, as can be seen from Figure 5g, the response acceleration level dramatically decreased. The mode is evidently being suppressed by the nearly incompressible water surrounding the plate. For all lower water levels, the free surface of the water apparently provides paths for acceleration of the water near the vibrating plate. But for the filled condition, the water is highly constrained except for leakage around the edges of the plate and the small volume changes which the shell flexibility may provide.

Perforated Plate - The response as a function of water level is similar to that for the solid plate with two exceptions: 1) the range of frequencies from the empty condition to the 100% full condition is much narrower for the perforated plate, and 2) the fundamental frequency for the 100% submerged perforated plate is clearly identifiable. Figure 6 shows the power spectral density plots for the perforated plate. Frequencies are given in Table 1. The overall range of frequencies decreases from 114.0 Hz to 81.25 Hz as water level is increased from empty to full (Figure 6-a through 6-g).

5.0 Analysis

The frequencies in Table 1 are used to define the added mass for the solid and the perforated plates at each of the seven water levels. The added mass is calculated by both equation (5) and equation (7) for comparison.

The incremental mass, ΔM , for these experiments is obtained by using the difference in mass between the solid and the perforated plates. Assuming that the air has negligible effect on the vibrating plates, and that the support system has the same stiffness for either plate, then the solid plate vibrates at the same frequency in air as would the perforated plate with a mass of ΔM attached. Applying equation (7) to the perforated plate, $\omega_{\Delta M}$ is taken as the frequency of the solid plate in air.

The added mass values per square centimeter of solid plate area for the solid and the perforated plates are plotted in Figure 7. It is evident that the added mass is highly sensitive to water level as the level nears 100% full. The data for the perforated plate results per equations (5) and (7) are within 1% of each other, so a single set of data points is used to represent both solutions. For the solid plate, however, the data for the calculations by equation (5) is about 18% lower than by the ΔM method of equation (7). Therefore, both sets of results are plotted. As a check on the results for the perforated plate, an equation from [4],

$$m_a = \rho A_e^2 [1 + (8d/3\pi)(1 - d/2b)] / A_h$$

which applies to fully submerged plates, is applied to the plate used for this experiment. The added mass value is plotted in Figure 7. The agreement is excellent.

6.0 Conclusions

The added mass for both solid and perforated plates submerged partially or fully in water has been determined. The values of added mass are suitable for natural frequency calculations of plates vibrating out-of-plane. The perforated plate values apply to plates for which a) the hole area is about 37% of the area of solid material, b) the plate thickness is 40% of the hole diameter, and c) the hole pattern is similar to the triangular pitch pattern used on the perforated test plate. Adjustments to the added mass values may be necessary for variations in these parameters. The method of reference [4] may be appropriate for adjustments based on the ratios of thickness over hole diameter and pitch over hole diameter. For the fully submerged perforated plate, the added mass from the equation in [4] agrees very closely with the results from this experiment.

Excitation of the vessel instead of the plate being evaluated for added mass has been shown to be successful. This alternative eliminates the problem of installing flexible fluid seals around the connection between a shaker and the plate.

6.0 Conclusions (cont.)

The different added mass values for the solid plate, depending on the method of calculation, needs to be resolved. The two value sets are in close enough agreement that their use in calculations of partially submerged plates will provide reasonable results regardless of the solid plate value set selected.

Acknowledgements

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Figure 3. Test Plates

Figure 4. Reinforcing Fittings

Section
Figure 1. Longitudinal View of Test Apparatus

Figure 2. End View of Test Apparatus
(End Cap Removed)

Figure 5.
Solid Plate

Figure 6.
Perforated Plate

Percent Plate Area Submerged	Natural Frequency (Hz)	
	Solid Plate	Perforated Plate
0.0	103.8	114.0
16.6	98.3	111.5
50.0	80.0	103.5
83.4	65.5	94.5
94.0	57.5	90.0
99.9	46.3	84.5
100.	--	81.3

Table 1. Natural Frequencies of Solid and Perforated Plates Versus Percent of Plate Submerged

Figure 7. Added Mass Per Unit Area of Plate Versus Percent Submerged Plate Area

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