Visualization and Simulation of Immiscible Displacement in
Fractured Systems Using Micromodels: I. Drainage

Topical Report

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Contents

1	ABSTRACT	1
2	INTRODUCTION	1
3	EXPERIMENTAL	3
4	THEORY	6
5	SIMULATION	11
6	RESULTS	12
6.1	Critical Capillary Number	12
6.2	Displacement Patterns	18
6.3	Displacement Efficiency and Relative Permeability Curves	21
7	CONCLUSIONS	29
8	ACKNOWLEDGEMENTS	31

List of Figures

1	Schematic of the Experimental Apparatus.	4
2	Experimental Steady-State Distribution Before Matrix Penetration: (a) Kerosene-Water, $Ca = 8 \times 10^{-5}$ (b) Air-Water, $Ca = 9 \times 10^{-5}$	7
3	Experimental Steady-State Distribution After Matrix Penetration: (a) Air-Water, $Ca = 7 \times 10^{-3}$ (b) Kerosene-Water, $Ca = 3 \times 10^{-3}$ (c) Kerosene-Water, $Ca = 8 \times 10^{-3}$	8
4	Schematic of Steady-State Distribution After Matrix Penetration.	10
5	Comparison Between Experimental and Numerical Results for Kerosene-Water, $Ca = 3.25 \times 10^{-4}$	13
6	Comparison Between Experimental and Numerical Results for Kerosene-Water, $Ca = 3.9 \times 10^{-4}$	14
7	Comparison Between Experimental and Numerical Results for Kerosene-Water, $Ca = 6.1 \times 10^{-4}$	15
8	Comparison Between Experimental and Numerical Results for Kerosene-Water, $Ca = 7.81 \times 10^{-4}$	16
9	Steady-State Wetting Saturation vs. Capillary Number. (Dots are Experimental Points, Solid Line is Numerical).	17
10	Scaling of Critical Capillary Number (Dots are Numerical Simulations, Dashed Solid Line is the Theoretical Prediction).	19
11	Numerical Simulation of Steady-State Pressure Profiles (Full Dots Denote Pressure at Front, Open Circles Denote Pressure in Fracture).	20
12	Numerical Simulation of Steady-State Distribution for $M = 66$ and $Ca = 1.67 \times 10^{-3}$ (Based on Fracture Area): (a) Injection Through Fracture, (b) Injection Through Entire Inlet Face.	22
13	Steady-State Saturation Profiles for the Condition of Figures 12a, 12b.	23
14	Numerical Simulation of Steady-State Distribution for $M = 0.066$ and $Ca = 1.01 \times 10^{-3}$ (Based on Fracture Area): (a) Injection Through Fracture, (b) Injection Through Entire Inlet Face.	24
15	Steady-State Saturation Profiles for the Condition of Figures 14a, 14b.	25
16	Steady-State Wetting Saturation Vs. Capillary Number for $M = 66, 6.6, 0.66$ and 0.066	27
17	Steady-State Wetting Saturation Vs. Capillary Number for $M = 66, 0.66$ and 0.066 . Injection in the Absence of Fracture.	28
18	Steady-State Relative Permeabilities for $M = 66, 0.66$ and 0.066 and $\phi_f = 0.005$: (a) $\lambda = 126$, (b) $\lambda = 22$	30

List of Tables

1	Micromodel characteristics.	5
2	Experimental conditions for drainage.	5

1 ABSTRACT

We consider drainage processes in model geometries that represent a matrix block-fracture system. Flow visualization in etched glass micromodels was carried out for various pairs of fluids, injection rates (capillary numbers) and viscosity ratio values. The experiments were then modeled with the use of a pore network simulator based on meniscus displacement. It was found that displacement occurs only in the fracture as long as the flow rate is below a critical value. Invasion of the matrix block occurs after this critical value (capillary number threshold) is exceeded. Numerical and experimental results were compared and found in good agreement. A theory for the invasion process and the critical capillary number was then developed. Displacement efficiencies were evaluated as a function of the capillary number and the mobility ratio. The process is reminiscent of a capillary pressure-saturation curve, with the notable exception that the role of capillary pressure is here played by the capillary number, and that the process is dynamic rather than quasi-static. Finally, effective relative permeabilities for the matrix-fracture system were calculated. Contrary to homogeneous systems, these curves depend on the mobility ratio.

2 INTRODUCTION

Naturally fractured systems represent a significant fraction of oil and gas producing reservoirs worldwide. Although the internal reservoir structure varies greatly among them, the basic element commonly assumed for each consists of a porous block called the matrix, which is the storage of the natural resource, and of a connected network of fractures, which act to transport injected or produced fluids [1]. Various models on the geometrical properties of the matrix and the connectivity and transport of the fracture network have been proposed. They vary from structures orderly and periodic to random and fractal [5], [20], [23], [24]. The internal structure is of fundamental importance to two processes, the transport of fluids in the fracture network and the matrix-fracture interaction. The characteristics of the fracture network, particularly its correlation and connectedness, affect single-phase transport, requiring non-trivial characterization and modeling methods. Fracture connectivity is paramount to tracer dispersion and pressure transients in single-phase flow [4], [14]. Matrix-fracture interaction affects the distribution of immiscible fluids, which determines to a large degree the effectiveness of fluid displacement from the matrix. At this level, it is the matrix block geometry and pore and fracture size distributions that dictate the displacement. It is the latter problem that we are concerned with in this paper.

Immiscible displacement in heterogeneous, but uncorrelated, porous media proceeds by different mechanisms, depending on the nature and history of the displacement (drainage or imbibition, primary or secondary process). These mechanisms have been clearly elucidated by various authors, notably by Lenormand who conducted careful experimental studies in micromodel geometries [16]. Local displacement mechanics are not expected to change when a fracture-matrix system is considered. Thus, primary displacement will occur by a frontal

advance of the menisci, the conditions for which depend on whether the process is drainage or imbibition, the latter including possible displacement along the surface roughness. Secondary imbibition may also involve film flow, film thickening and snap-off, resulting in displacements different than frontal. A fracture-matrix system displays additional large scale features, due to the existence of fractures, which affect greatly the large scale efficiency. In particular, the presence of a fracture adds an element of large-scale correlation to the system. As a result, rate effects which reflect the composite interaction between viscous (or gravity) and capillary forces become important. Several studies have been reported on this topic, all of which concern imbibition, however.

Typically, theoretical studies on displacement in fractured systems are concerned with deriving some representation of the imbibition rate into a matrix block, by either solving an appropriate initial boundary value problem or by introducing various simplifying assumptions [7]. Although routinely used in simulations, such purely numerical approaches offer little to further our understanding of the process. Experimental studies have been more useful in this regard. In one of the first visualization studies in fractured systems, Handy and Datta [12] provided evidence of water imbibing into artificially-fractured sandstones and heterogeneous sandpacks. They observed water advancement preferentially through the fracture in the first case, and imbibition of the finer sand in the second case. Mattax and Kyte [21] introduced a critical rate, defined as the limiting value above which the front advances faster in the fracture than in the matrix. The latter can be detrimental to the process efficiency, as the advancing front may reach the end of the matrix block and prevent further recovery. In a more recent study, Babadagli [2] conducted a series of forced imbibition experiments in model fractured sandstones, visualized by CT scanning, which demonstrated the importance of rate effects on the properties of the composite system, such as relative permeabilities. As also shown below, rate effects are fundamental to the large scale behavior of displacement in fractured systems, in contrast to homogeneous media. The above were restricted to imbibition of strongly wet systems. In particular, no drainage experiments have been reported.

While these studies have furnished valuable information, our understanding of displacement in fractured systems is still based on phenomenology. Notably lacking are local flow visualization and pore-scale simulation of the displacement. Flow visualization with transparent micromodels has been very useful in elucidating flow mechanisms in many processes [6], [18], [19], [22] despite the inherent limitation of two dimensions. To our knowledge, analogous displacement studies in micromodels that mimic a fracture-matrix block geometry and corresponding pore level simulation have not been undertaken to date. This paper reports on both these topics. Using a glass micromodel representing a microscale replica of a fracture-matrix system we have visualized drainage and imbibition processes. The displacement was then simulated with the use of a pore network simulator. Because of the difference between drainage and imbibition, we shall present the results in separate publications. This part deals with drainage.

The paper is organized as follows: We, first, present experimental results on drainage in micromodel geometries that mimic a matrix-fracture system. A critical rate (capillary number) is identified, below which penetration of the matrix is not possible, but above which

the displacing phase also invades the matrix block. This is opposite to the imbibition reported by Mattax and Kyte [21], where at rates below critical the displacing wetting phase penetrates mostly the matrix, and where the critical value must be exceeded for an advance in the fracture. A theoretical model based on gradient percolation is developed to predict the onset of matrix penetration. The experimental results are subsequently compared with pore network simulations of drainage. A sensitivity analysis of the effects of various parameters, particularly the capillary number and viscosity ratio, on the displacement efficiency is presented. Finally, based on the pore level simulation, steady-state relative permeabilities for a matrix fracture system are computed. The results are compared to the behavior of heterogeneous but uncorrelated systems.

3 EXPERIMENTAL

The schematic of the experimental apparatus is shown in Figure 1. The glass micromodels were fabricated using 3" x 5" glass plates on which the desired patterns were etched. Experiments were performed in two micromodels: The first consisted of a triangular network of pores of coordination number 6, representing the matrix block, and of a uniform channel of 2 mm width through the middle of the micromodel representing the fracture. The total pore volume of the micromodel was estimated as 0.77 cm³. Pore throat and pore body sizes were estimated at 400 μ m and 800 μ m, respectively. The matrix of the second micromodel consisted of a square lattice of coordination number 4. Additional characteristics are described in Table 1. During fabrication, the fracture was etched to a depth estimated as twice as large that of the matrix pores.

Inlet and outlet ports were connected to the fracture. All displacements were run at constant rate using a syringe pump, delivering at the rates of 0.054, 0.108, 0.2 and 0.26 cc/min. These values correspond to capillary numbers in the range where penetration of the matrix can occur. For primary drainage experiments we used the pairs air-water and oil-water. Kerosene or viscous mineral oil of viscosities 1.5 cp or 100 cp, respectively, was used as the non-wetting fluid in the latter case. The micromodel was first saturated with water dyed with Methylene Blue, at a low rate to completely saturate the matrix. The model was then placed horizontally for the drainage experiment to be conducted. Each run was carried out by injecting the non-wetting fluid at a constant rate until steady-state was established. Then, the flow rate was increased to the next setting and injection resumed. A total of 12 runs, corresponding to three different mobility ratio values each at four different capillary numbers, were conducted (Table 2). The frontal movement was recorded using a video camera with a close-up kit or a microscope.

Typical visualization experiments using the first micromodel are shown in Figures 2-6. In all experiments, the pore level displacement mechanism is the typical "stop and go" rule of drainage, where the interface advances only after the capillary pressure threshold in the throat is exceeded. It was observed that for capillary number values below a critical, estimated to be approximately 2×10^{-4} , the displacing phase penetrates the fractures only, breaks fast through the production port and reaches a steady-state flow without subsequent

Figure 1: Schematic of the Experimental Apparatus.

Network	Triangular	Square
Length	12.5 cm	11.2 cm
Width	7.5 cm	5.7 cm
Network size	75×45	51×26
Maximum bond width	0.04 cm	0.1 cm
Minimum bond width	0.04 cm	0.04 cm
Average bond width	0.04 cm	0.07 cm
Fracture width	0.2 cm	0.2 cm
Fracture depth	0.01 cm	0.018 cm
Network depth	0.01 cm	0.009 cm

Table 1: Micromodel characteristics.

Fluid Pair	Displacing Fluid Viscosity (cp)	Mobility Ratio	Volumetric Rate (cc/min)	Capillary Number
Air-Water	0.018	55.56	0.00052	9×10^{-8}
“	”	”	0.052	9×10^{-6}
“	”	”	5.2	9×10^{-4}
“	”	”	40	7×10^{-3}
Oil-Water	1.5	0.66	0.0027	8×10^{-5}
“	”	”	0.054	1.59×10^{-4}
“	”	”	0.108	3×10^{-3}
“	”	”	0.27	8×10^{-3}
Oil-Water	100	0.01	0.00052	1×10^{-4}
“	”	”	0.0052	1×10^{-3}
“	”	”	0.052	1×10^{-2}
“	”	”	0.52	1×10^{-1}

Table 2: Experimental conditions for drainage.

invasion of the matrix (Figure 2). However, for values above the critical, the matrix was also invaded shortly after penetration of the fracture and before breakthrough of the displacing phase (Figure 3). Simultaneous displacement was observed in both the matrix and the fracture, until a steady-state is reached, where the front in the matrix becomes stationary. The pattern is characterized by percolation-like features in cases where the displacing fluid is air (Figure 3a) and by a compact pattern in cases of higher viscosity fluids, such as mineral oil (Figures 3b and 3c). At even higher rates, invasion of the matrix commences at about the same time as in the fracture and results into progressively higher displacement efficiencies, until a limiting efficiency is reached at sufficiently large rates.

These experiments were readily reproducible. Qualitatively similar results, but different critical values, were obtained for experiments involving other micromodel geometries, including those where the entire inlet of the matrix was exposed to drainage. It was concluded that penetration of the matrix requires that the capillary number exceeds a critical value, which depends on the geometry of the micromodel. This process is reminiscent of capillary invasion, e.g. during mercury porosimetry [8], where a critical threshold must be exceeded before penetration takes place, with the important difference, however, that in the present case the critical quantity is the capillary number, which is rate-dependent, rather than the capillary pressure. As discussed later in the paper, an equivalent “capillary pressure curve” can be constructed by plotting capillary number vs. matrix saturation. Unfortunately, and contrary to an ordinary invasion process, which is locally controlled by capillary forces alone and it is thus amenable to percolation theory [25], invasion in the fracture-matrix system is also influenced by viscous forces, which impart patterns different than percolation. To understand these results, theoretical analysis and numerical simulation of the process were conducted.

4 THEORY

It is straightforward to calculate the critical capillary number. Consider flow in the fracture only, which can be approximated by a conduit of length L , constant cross section A_c and flow conductance g . Application of Poiseuille’s law yields the pressure drop across injection and production ends (A and B , respectively, Figure 4) where Q is volumetric flow rate, μ is viscosity, and we used subscript o to denote the displacing fluid. The fracture permeability k_f can be approximated with the use of a hydraulic diameter

$$k_f = \frac{D_h^2}{32} \quad (1)$$

where $D_h = \frac{4A_c}{\Pi}$, Π being the wetted perimeter. Making the reasonable assumption that $P_{o,B}$ is the pressure of the immiscible matrix fluid, equation (1) shows that a constant gradient in the capillary pressure develops as flow in the fracture occurs. Since penetration of the matrix requires that the capillary threshold be exceeded, this problem is very similar, although not identical, to gradient percolation [10]. Gradient percolation is a percolation process, where

Figure 2: Experimental Steady-State Distribution Before Matrix Penetration: (a) Kerosene-Water, $Ca = 8 \times 10^{-5}$ (b) Air-Water, $Ca = 9 \times 10^{-5}$.

Figure 3: Experimental Steady-State Distribution After Matrix Penetration: (a) Air-Water, $Ca = 7 \times 10^{-3}$ (b) Kerosene-Water, $Ca = 3 \times 10^{-3}$ (c) Kerosene-Water, $Ca = 8 \times 10^{-3}$.

the percolation probability varies in space following a constant gradient, e.g.

$$\frac{dp}{dx} = -B \quad (2)$$

where x is expressed in terms of lattice spacing units and B is the Bond number ($B = \frac{k\Delta\rho g x}{\gamma}$ in gravity driven drainage). For such processes, it was shown in [11] that a connected cluster (front) forms, with mean front position x_f at the location where the percolation threshold of the network is reached. The front has a width which increases as the Bond number decreases following the scaling

$$\xi \sim B^{-\frac{1}{1+\nu}} \quad (3)$$

where $\nu = \frac{4}{3}$ in 2-d and 0.88 in 3-d [11]. To find the Bond number for the present problem we first need to connect the percolation probability p to the capillary pressure P_c . Using the relation

$$P_c = \frac{2\gamma}{r_d} \quad (4)$$

where γ is the interfacial tension, the mean radius of curvature r_d can be related to p via

$$p = \int_{r_d}^{\infty} \alpha(r) dr \quad (5)$$

where $\alpha(r)$ is the size distribution in the matrix. By combining (8) and (2), we obtain

$$B \simeq \frac{r^2 C'a}{2k_f} \quad (6)$$

where we defined the capillary number $C'a = \frac{Q\mu_o}{\gamma A}$ and where r is the mean radius in the matrix. Thus, the Bond number for this problem is directly proportional to the injection rate, suggesting dynamic effects. Using gradient percolation theory arguments, we next make the postulate that the matrix is penetrated when the front location is at the beginning of the matrix block, namely when $p = p_c$ at $x = 0$, where p_c is the percolation threshold of the matrix. The critical capillary number follows by combining (3) and (7)

$$C'a^* = \frac{2k_f}{Lr_{dc}} \quad (7)$$

If it is assumed that the fracture cross-section is rectangular and that its depth a is comparable to the critical radius of curvature, equation (7) further suggests $a \simeq 8LC'a^*$. This prediction was checked against experimental and numerical results. Using experimental values for L and $C'a^*$ yields the estimate $a \simeq 131\mu m$, which compares favorably with the measured value of $120\mu m$. Comparison with numerical results is discussed below.

$$P_{o,A} - P_{o,B} = \frac{Q\mu_o L}{A_c k_f} \quad (8)$$

Figure 4: Schematic of Steady-State Distribution After Matrix Penetration.

Equation (8) shows that the critical capillary number depends on geometric properties only, particularly on the fracture length L , to which it decreases in direct proportion. Thus, for a fixed capillary number, longer blocks are penetrated earlier. This conclusion also holds when it is the total flow rate per unit area in the system, rather than the capillary number through the fracture, which is fixed. Indeed, as can be easily shown, in the latter case the flow velocity through the fracture would scale in proportion to the linear dimension L of each matrix block, assumed cubic, hence the penetration condition would show a capillary number scaling proportional to L^{-2} , which is stronger than (7).

Subsequent to the onset of penetration and a further increase in the capillary number, the matrix starts to be invaded. It is tempting to also use gradient percolation results to model the matrix saturation. Gradient percolation predicts that x_f satisfies the relation

$$\frac{L - x_f}{L} = \frac{Ca^*}{Ca} \quad (9)$$

based on which the saturation profile can be predicted from the scaling described in [11]. However, such an approach is oversimplified. Although the degree of penetration is gradual and the front at steady-state is stationary, the problem differs substantially from the standard gradient percolation process. Even at steady-state conditions, fluid flow takes place both in the invading and in the displaced fluids in the matrix, thus rendering the process rate-dependent and of a dynamic nature. This cannot be modeled by gradient percolation. To understand this process a pore level numerical simulation is necessary.

5 SIMULATION

To simulate the process, we used a drainage pore network simulator, where the porous medium is represented as a square lattice of pore throats and pore bodies. Its basic aspects are similar to those of Lenormand et al [17] and Blunt and King [3]: Menisci separating the two phases are assumed to reside in pore bodies only, the saturation of which can be partial (less than 1). Interfaces are allowed to penetrate throats adjacent to a pore body only if the capillary pressure threshold of the throat is exceeded. Pressure drops are associated with flow in the pore throats only, and it is only pore bodies that contribute to saturation and volume. At any given time the front consists of connected interfaces residing in pore bodies. During a time step, the front advances by either penetrating a bond adjacent to it or by filling a pore body partly saturated, depending on the prevailing flow conditions. Among many partly filled pores, the pore requiring the smallest time increment to become fully saturated is filled first. All pressure drops are calculated based on a Successive Over Relaxation (SOR) technique, where an additional iteration cycle is required to solve problems at a specified flow rate. In the simulations, the fracture was represented as an equivalent system of consecutive pores of large size and conductance. To mimic the experimental approach, injection at the tip of the fracture only with production at the downstream end of the model were first considered. Subsequently, however, we also considered injection over the entire inlet face of the matrix.

The simulation was carried out by specifying the geometrical characteristics of the experimental micromodel, converted to equivalent cylindrical or spherical segments for pore throats or pore bodies, respectively. Although the numerical lattice was patterned exactly after the original pattern used to etch the micromodel, numerical and experimental models were not identical, due to non-uniformities incurred during the processes of etching and fusion, which could not be precisely controlled due to glass imperfections. Numerical results corresponding to the experiments in the square lattice micromodel are shown in Figures 5-9. For the two experimental capillary number values 1.12×10^{-4} and 3.25×10^{-4} , the penetration of the matrix is insignificant, the injected fluid flowing through the fracture only as also observed experimentally (Figure 5). When the capillary number was raised to the value of 3.9×10^{-4} , however, the matrix becomes invaded (Figure 6), suggesting a critical capillary number of approximately 3.5×10^{-4} . This value also compares favorably with the experimental value for the triangular network of Figures 2 and 3, if the difference in the percolation thresholds in the two lattices, $p_c = 0.5$ and $p_c = 0.33$, respectively, is taken into consideration (such difference would result in an increase of the critical number by a factor of $\frac{0.50}{0.33}$). Subsequent increases in the capillary number result in larger occupancy of the matrix (Figure 7) until a limiting saturation is reached (Figure 8).

Another comparison is shown in Figure 9, where the steady-state matrix wetting saturation is plotted vs. the capillary number. As shown, the comparison between simulation and experiment is very good, suggesting that the simulated mechanism describes well the experimental observations. As previously suggested, the curve of Figure 5 can be interpreted as an equivalent “capillary pressure” curve, where a capillary threshold and an effective residual saturation can be identified. However, the distribution of fluids is dictated here by dynamic rate effects, which also depend on the viscosity ratio as shown below. This distribution will be subsequently used to construct effective permeabilities.

6 RESULTS

The pore network simulator was next used to investigate the critical capillary numbers, to compute displacement patterns and relative permeabilities, and to analyze the effect of the process parameters.

6.1 Critical Capillary Number

The experimental results for the critical capillary number were found in agreement with predictions based on gradient percolation theory. The scaling of the critical capillary number on fracture permeability, length and average pore size was predicted to satisfy (8). It would be interesting to further test these predictions with the use of the numerical simulator. Figure 10 shows a comparison between theoretical and computed critical capillary numbers for various values of k_f , L , and r in a range covering several orders of magnitude for each variable. The relation of $\bar{r}$ to r_d^* is through

Figure 5: Comparison Between Experimental and Numerical Results for Kerosene-Water, $Ca = 3.25 \times 10^{-4}$.

Figure 6: Comparison Between Experimental and Numerical Results for Kerosene-Water, $Ca = 3.9 \times 10^{-4}$.

Figure 7: Comparison Between Experimental and Numerical Results for Kerosene-Water, $Ca = 6.1 \times 10^{-4}$.

Figure 8: Comparison Between Experimental and Numerical Results for Kerosene-Water, $Ca = 7.81 \times 10^{-4}$.

Figure 9: Steady-State Wetting Saturation vs. Capillary Number. (Dots are Experimental Points, Solid Line is Numerical).

$$p_c = \int_{r_{dc}}^{\infty} \alpha(r) dr \quad (10)$$

For a uniform size distribution and for $p_c = 0.5$ for bond percolation in a square lattice, (10) yields $r_{dc} = r$. As shown in Figure 10, the theoretical scaling (7) is very well satisfied. Actually, the numerical results suggest a minor modification to the prefactor, which should be 2.5 instead of the theoretical prediction of 2, probably due to finite size effects [9].

While (8) is useful if $\alpha(r)$ is known, it would be perhaps more practical to relate the critical capillary number to other directly measurable properties of matrix and fracture. In a seminal paper, Katz and Thompson [15] related the permeability of a homogeneous material to the percolation length l_c , here equal to $2r_h$, and to the formation factor F , through

$$k_m = \frac{1}{266F} l_c^2 \quad (11)$$

Substitution of (11) in (7) yields the more general result

$$\frac{Ca^* L \sqrt{k_m F}}{k_f} = 0.26 \quad (12)$$

This formula allows one to relate the critical capillary number to fracture and matrix permeabilities. It must be pointed out that this capillary number is much larger than the corresponding to flow in the matrix, Ca_m (when penetration occurs). The two values are related with the relationship $Ca = Ca_m / \phi_f$, where ϕ_f is the fracture volumetric fraction (porosity).

The success of gradient percolation in correctly predicting the scaling of the initial capillary number is not surprising. Before penetration, the pressure gradient in the fracture is constant, thus lending itself to gradient percolation theory. After the onset of penetration, the patterns obtained differ significantly from those corresponding to gradient percolation. Due to the flow of the displacing fluid an additional pressure drop occurs in the matrix, which reduces the effective pressure gradient at the front and results in patterns different from gradient percolation. This is demonstrated in Figure 11, where the pressures at the front and in the fracture are plotted as a function of the streamwise coordinate. A pressure difference exists between the fracture and the matrix in the upstream part, indicating transverse fluid flow from the fracture to the matrix. The pressure gradient is constant only at the downstream portion of the displacement, and varies significantly near the injection point. This difference makes invalid the use of the gradient percolation predictions in the regime following the onset of penetration.

6.2 Displacement Patterns

Because viscous are the key forces in the invasion of the matrix, important parameters that affect the process performance are the viscosity ratio, M , and the boundary conditions, such as the mode of injection. In typical phenomenological models for fractured systems it is

Figure 10: Scaling of Critical Capillary Number (Dots are Numerical Simulations, Dashed Solid Line is the Theoretical Prediction).

Figure 11: Numerical Simulation of Steady-State Pressure Profiles (Full Dots Denote Pressure at Front, Open Circles Denote Pressure in Fracture).

implicitly assumed that all displacement occurs from menisci that enter the matrix from the fracture side only, thus resembling lateral displacement, and that frontal displacement is not important. To check this hypothesis, we simulated processes where displacement from the inlet face was also allowed. The critical capillary number for the matrix penetration was the same in either case, and conforms with the theoretical predictions (8). The mechanism of displacement, however, differed significantly in the two cases.

Figure 12 shows displacement patterns for the two different injection conditions for unfavorable displacement ($M = 66$), where viscous instability and fingering are very pronounced as expected for an air-liquid systems. The two patterns exhibit certain differences. When the injection is only in the fracture (Figure 12a), the displacement is initiated and spread in the portion of the matrix adjacent to the fracture only, thus resembling a lateral invasion. In contrast, a mostly frontal advance is observed when injection occurs over the entire inlet face (Figure 12b). In the latter case, the role of the fracture lies primarily in contributing the critical pressure drop for the onset of invasion. The corresponding saturation profiles of Figure 13 reflect this difference near the downstream end, where the fractional penetration is deeper for the former case. Because of the need of connectivity with the fracture, the front in the first case is likely to be somewhat more compact and less fingered than in the second. Qualitatively similar trends are also displayed in the opposite case of favorable, compact displacement ($M = 0.066$, Figures 14,15). When injection occurs in the fracture only, the displacement is first confined to a region adjacent to the fracture (Figure 14a), although it spreads over the entire block at sufficiently high rates. With injection over the inlet face, on the other hand, the displacement is mostly frontal, the fracture only serving to provide the necessary critical capillary number for penetration (Figure 14b). The saturation plots of Figure 15 confirm these trends. While the two patterns differ depending on the mode of injection, however, the importance of the fracture should not be discounted. The fracture geometrical characteristics set the critical capillary number for invasion and the steady-states after invasion, features which are both absent for injection in the absence of fracture (see also below). The existence of a fracture constrains the capillary number range to a small interval of about a decade within which the displacement from the matrix is completed. Consequently, plots such as Figure 9 remain qualitatively the same for either injection boundary condition, particularly for $M \gg 1$, the main difference being that the slope of the curve is steeper in the case of injection through the fracture only.

6.3 Displacement Efficiency and Relative Permeability Curves

The preceding demonstrated important features associated with drainage of a fractured system. The existence of a critical rate for invasion of the matrix block is uniquely due to the fracture (or for that matter any other highly permeable streak) and resembles the invasion associated with mercury porosimetry, with the important exceptions that it is a threshold in the capillary number rather than the capillary pressure that needs to be exceeded, and that non-local, viscous forces are fundamental in the displacement in the case of a fracture. As previously noted, plots such that of Figure 9 can be considered as equivalent to a "capillary

Figure 12: Numerical Simulation of Steady-State Distribution for $M = 66$ and $Ca = 1.67 \times 10^{-3}$ (Based on Fracture Area): (a) Injection Through Fracture, (b) Injection Through Entire Inlet Face.

Figure 13: Steady-State Saturation Profiles for the Condition of Figures 12a, 12b.

Figure 14: Numerical Simulation of Steady-State Distribution for $M = 0.066$ and $Ca = 1.01 \times 10^{-3}$ (Based on Fracture Area): (a) Injection Through Fracture, (b) Injection Through Entire Inlet Face.

Figure 15: Steady-State Saturation Profiles for the Condition of Figures 14a, 14b.

pressure-saturation” curve, where the capillary number is substituted for the capillary pressure. The key role of viscous effects in setting the displacement efficiency, however, makes impossible the development of a simple local theory, such as percolation, to analytically describe this curve, as is commonly done for the capillary pressure curve. In particular, these curves depend on the viscosity ratio. Shown in Figure 16 are plots of the displacement efficiency vs. the capillary number for various values of the viscosity ratio. Ultimate efficiencies are high for favorable viscosity ratio, and low for unfavorable viscosities, as expected. In terms of the residual saturation terminology of the capillary pressure curve, we may state that the residual saturation increases with an increase in the mobility ratio. The effect of the fracture is important at low values of the capillary number. Plotted in Figure 17 are the corresponding to Figure 16 displacement efficiencies in the absence of a fracture (frontal displacement). Note the absence of a threshold value. Theoretically, the low capillary number efficiency, where capillary forces predominate, should be very small. The numerical value is different due to finite size effects [9] and to the slow dependence of the mass of percolation cluster on size ($S \sim L^{-\beta}$, where $\beta = 0.138$ for 2-D and $L = 45$ in these simulations). Before the onset of matrix invasion, the two sets of curves differ substantially, the difference being greater for $M \gg 1$ (compare Figures 16 and 17). The curves asymptotically approach each other, however, when the displacement in the matrix has been completed.

The one-to-one relationship between the steady-state saturation of the matrix block-fracture system and the corresponding capillary number as shown in Figures 9 and 16 allows for steady-state relative permeabilities for the composite system to be constructed. The issue of effective relative permeabilities is key to the simulation of displacement in fractured systems, which are commonly represented by an effective continuum. In a recent study, Babadagli [2] obtained pseudo-relative permeabilities pertaining to transient displacement in imbibition process which exhibited a flow rate dependence. We recall that relative permeabilities that are only functions of saturation can be defined in uncorrelated systems, as long as the capillary number is sufficiently low such that the distribution of phases is under capillary control [13]. Following the assignment of occupancy, the conductivity of each phase can next be calculated (or approximated by scaling laws) and the relative permeability determined. Heiba et al [13] used such an approach in Bethe lattices to obtain curves which reproduced important experimental features. Under such conditions ($Ca \ll 1$), the occupancy of the pore space is solely dictated by capillarity, and percolation theory rules can be directly applied.

In fractured systems (or in systems with long-range correlations) capillarity does not alone control the distribution of phases, however. Instead, it is a combination of capillary and viscous forces that dictates the occupancy of the pore space, this combination resulting in the steady-state diagrams of Figure 16. Nonetheless, composite relative permeabilities can be computed numerically from the individual phase conductances after the establishment of steady-state. Results obtained are shown in Figure 18 for various values of the mobility ratio and for two different values of the ratio of fracture to matrix permeability, $\lambda = \frac{k_f}{k_m}$. Overall, the composite system appears strongly “water-wet”, in that the point of intersection of the relative permeability curves is significantly to the right of the mid-point saturation value.

Figure 16: Steady-State Wetting Saturation Vs. Capillary Number for $M = 66, 6.6, 0.66$ and 0.066 .

Figure 17: Steady-State Wetting Saturation Vs. Capillary Number for $M = 66, 0.66$ and 0.066 . Injection in the Absence of Fracture.

Non-wetting permeabilities are high throughout most of the saturation range, while wetting permeabilities rapidly reach a small value as the saturation decreases. It is interesting to note that because of the existence of the fracture, simultaneous two-phase flow is possible over a substantial saturation range, despite the 2-D geometry. This feature is absent in uncorrelated systems, where the simultaneous flow of two percolating phases is impossible. The effect of the viscosity ratio on the non-wetting permeability is strong, particularly at not too high values of λ (Figure 18b), and results in an increase of the non-wetting relative permeability and a shift of the set to the right as M decreases. By contrast, no significant effect of M on the wetting permeability is noted. The sensitivity to M is stronger as the ratio of permeability λ decreases. It is interesting to note that in such steady-state composite permeabilities, there is no direct effect of rate (or capillary number), although dynamic effects enter indirectly in setting saturation levels and the phase occupancy. The shape of these curves is drastically affected by the geometry, pore size and fracture characteristics of the matrix-fracture system and by the viscosity ratio. The permeabilities pertain to steady-state flow conductivity after the displacement pattern has reached a steady-state and it is not clear at this point if they can be used to model transient large scale displacements. It is worth also pointing out that effects of rate and viscosity ratio on relative permeabilities in uncorrelated media have been pursued with vigor over the past years, but still remain a somewhat unsettled issue. Due mainly to the role played by the fracture in dictating penetration, these effects are much more apparent in a fracture-matrix system.

7 CONCLUSIONS

In this paper we studied the constant rate drainage of a model fractured system, based on micromodels. It was shown that regardless of the injection conditions, namely whether it is from the fracture or from the entire inlet face, there exists a critical capillary number below which the matrix is not invaded. The onset of penetration can be predicted well from gradient percolation arguments which lead to the correct scaling of the critical capillary number on geometrical characteristics. The scaling is inversely proportional to the length of the fracture and the square root of the matrix permeability and proportional to the fracture permeability. The existence of fractures lead to steady-state displacement fronts, a feature that is absent in homogeneous or mildly heterogeneous systems, where a constant rate displacement may reach steady-state only after breakthrough and completion of the process. Displacement in the matrix terminates after one or two decades of further increases in the capillary number, an unfavorable mobility ratio requiring a larger increase. Once this regime is reached, the effect of the fracture becomes unimportant. While gross features are independent of the particular invasion mode, displacement patterns are significantly affected. Frontal displacement modes prevail when injection over the matrix inlet face is allowed as opposed to a lateral displacement mode when only injection in the fracture is assumed. The latter assumption is commonly made in existing simulators that model the interaction between matrix and fracture. Based on the present findings, this assumption appears in need of re-examination. Steady-state relative permeabilities were computed and found to

Figure 18: Steady-State Relative Permeabilities for $M = 66, 0.66$ and 0.066 and $\phi_f = 0.005$:
(a) $\lambda = 126$, (b) $\lambda = 22$.

depend on the geometry and the viscosity ratio, but not on the rate. These results find applications to fractured systems and to correlated heterogeneous porous media containing streaks of high permeability paths. In light of the present work, the modeling of drainage processes in the latter case also needs to be revisited.

A final comment pertains to effects of gravity. In many practical applications a vertical matrix block containing a fluid, such as oil, is surrounded by gas which is the non-wetting phase, and the pressure of which can be taken as constant. Penetration of the block by gas and subsequent drainage of oil is now driven by the hydrostatic pressure gradient in the oil. Although this problem is different from the previous, a critical value on a suitably defined Bond number can also be derived to describe the onset of penetration. Using the same arguments as before we find

$$\frac{\rho g L \sqrt{k_m F}}{\gamma} \geq 0.266$$

Because of the different mechanism of the drainage process, which is driven here by hydrostatic pressure differences, and where rate effects are not involved, however, gradient percolation theory should be successful in describing this displacement problem.

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