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MODIFICATION OF CHEMICAL AND PHYSICAL FACTORS IN
STEAMFLOOD TO INCREASE HEAVY OIL RECOVERY

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ABSTRACT

This report covers work performed in the various physicochemical factors for the improvement of oil recovery efficiency. In this context, three general areas were studied: (i) The understanding of vapor-liquid flow in porous media, whether the flow is internal (boiling), external (steam injection) or countercurrent (as in vertical heat pipes); (ii) The effect of reservoir heterogeneity, particularly as it regards fractured systems; (iii) The flow properties of additives for the improvement of recovery efficiency, in particular the injection of caustic and foams. The studies completed under this contract involved all three research tools, analysis, computation and experiments. We have focused on pore level modeling using pore networks and on flow visualization using Hele-Shaw cells. Experiments involving core samples were conducted for the chemical additives investigation. Finally, simulation at the pore scale, pore network scale and reservoir scale were also undertaken. Part of the work has been detailed in five DOE Technical Reports as shown at the end of this report.

The study of vapor-liquid flow in porous media attempted to address the following questions: to what degree do the dynamics of steam-water or steam-oil interfaces differ from a typical isothermal, immiscible, two-component displacement, and how do temperature gradients and heat transfer affect the displacement pattern? Pore network simulation of moving interfaces and visualization experiments helped to provide some insight on these questions. Ultimately, the aim is at scaling up pore network level phenomena to model vapor-liquid flow at the macroscale. It is for this reason that simple theories, such as percolation and gradient percolation or other variations, may be of great utility. Two aspects were addressed: (i) The development of pore network models for phase change (boiling and steam injection) in porous media; (ii) Visualization of steam injection in Hele-Shaw cells and glass-bead packs. We developed a pore network model to describe the growth of vapor phase in a porous medium. The model incorporates the basic pore-level mechanisms typical of immiscible displacement as well as phase equilibria and heat transfer. The latter occurs by conduction in the solid phase and by conduction and convection in the liquid phase. The model is a substantial improvement of our previous work employing percolation theory. Based on the pore network

model, we are able to simulate bubble growth at the pore level and to address the issues of steam zone growth and the relevance of a relative permeability description, depending on injection rate, temperature and other properties of the process. Two specific examples are discussed.

Visualization experiments continued for the study of steam injection in Hele-Shaw cells and glass micromodels. During the past year, we focused on displacement in glass-bead packed cells. To isolate various mechanisms, steam injection into a water-saturated cell was also considered. Heat losses were found to affect the stability of the steam front, the latter being more compact when heat losses increase. The previously reported steam front dynamics and oscillations were also present in the steam-water experiments, provided that injection was updip. In the experiments involving the packed Hele-Shaw cells, displacement patterns similar to the previous experiments conducted in regular Hele-Shaw cells were observed. The effect of the pore structure, however, enters in setting the width of unstable fingers and the rate of steam front propagation.

Significant progress was made in the study of reservoir heterogeneity and its effects on flow processes. During last year, we considered two areas: (i) The representation of naturally fractured systems and displacement processes; (ii) The displacement in macroscopically heterogeneous systems.

Present models for the representation of naturally fractured systems rely on the double-porosity Warren-Root model or on random arrays of fractures. However, many field observations have demonstrated the existence of multiple length scales in naturally fractured media. Present models fail to capture this important property of self-similarity. We have used concepts from the theory of fragmentation and from fractal geometry to construct numerically a network of fractures that exhibits self-similar behavior over a range of scales. The method is based on the IFS (Iterated Functions System) approach and allows for great flexibility in the development of patterns. The numerical simulation of unsteady single phase flow in such networks was next developed. It was found that the pressure transient response of finite fractals behaves according to previous analytical predictions provided that there

exists a power law in the mass-radius relationship around the test well location. Finite size effects can become significant and interfere with the identification of the fractal structure. Examples from actual well tests in fractured systems were analyzed using fractal pressure transient theory.

Our interest in fractured systems arises because of the existence of considerable heavy oil deposits in fractured reservoirs, particularly in the diatomite reservoirs of California. While our ultimate interest is on steam displacement, we have realized that for a meaningful progress, the simpler, immiscible, isothermal displacement in fractured systems needs to be analyzed first. To this end, we considered drainage processes in model geometries that represent a matrix block-fracture system. Flow visualization in etched glass micromodels was carried out for various pairs of fluids, injection rates (capillary numbers) and viscosity ratio values. The experiments were modeled with the use of a pore network simulator involving meniscus displacement in the matrix and in the fracture. For drainage, it was found that displacement occurs only in the fracture as long as the flow rate is below a critical value. Invasion of the matrix block occurs after a critical value in the capillary number is exceeded. The resulting invasion is reminiscent of a capillary pressure-saturation curve, with the notable exception that the role of capillary pressure is here played by the capillary number, and that the process is dynamic rather than quasi-static. Numerical and experimental results were compared and found in good agreement. Relative permeabilities for a fracture-matrix system were also developed. This work illustrates the matrix-fracture interaction at the pore-level without the use of phenomenology and allows for capillary pressure and relative permeabilities to be obtained.

Effects of capillary heterogeneity were addressed in two other contexts. First, we investigated heterogeneity induced by variations in permeability in the direction of displacement under drainage conditions. The investigation used macroscopic simulation, based on the standard continuum equations, experiments with the use of an acoustic technique conducted with the collaboration of scientists at the University of Paris, and pore network numerical models. It was found that heterogeneity affects significantly the saturation profiles, the effect

being stronger at lower rates. A good agreement was found between the continuum model predictions and the experimental results based on which it was concluded that capillary heterogeneity effects in the direction of displacement act much like a body force (e.g. gravity). A qualitative agreement was found between the continuum approach and the pore network numerical models, which is expected to improve when finite size effects in the pore network simulations diminish. The results were interpreted with the use of invasion percolation concepts.

The third area of research in this report involves chemical additives for the improvement of recovery efficiency. We have been studying the following three aspects: (i) Caustic additives at elevated temperatures; (ii) Injection of foam additives; and (iii) Non-Newtonian flow and displacement in porous media. Specifically, the study of caustic injection at elevated temperatures involves silica dissolution and caustic consumption.

During steamfloods, due to gravity override, steam moves to the upper portion of the reservoir while a hot water zone is formed in the lower portion. Therefore, a large portion of the oil may not be recovered. Injection of caustic along with steam was proposed to reduce the interfacial tension (IFT) between the oil phase and the water phase in the hot water zone. In order for caustic to be effective in reducing interfacial tension it must preserve the pH above an effective level. The major consideration involved in practical applications of caustic flooding has been whether or not caustic effectively sweeps the major portion of the reservoir. Caustic loses pH and alkalinity through its interaction with the reservoir rock and brine. To evaluate caustic as an enhanced oil recovery (EOR) agent, it is necessary to determine the extent of the caustic/rock interaction and the rate of the reactions involved. Two studies were conducted on this topic, one experimental and one theoretical. Both studies were supported by previous DOE contracts, but they were completed during the period covered by the present contract.

In the experiments, a long porous medium which provided extended residence times was used. It was necessary to set up the porous medium such that changes in the pH and chemical composition of the solution could be monitored. Four Berea sandstone cores placed in series

provided the desired length and the opportunity for sampling in-between cores. This enabled the establishment of pH and chemical composition profiles. The experiments were run at temperatures up to 180° C, while the flow rates varied from 4.8 to 0.2 ft/day. The samples were analyzed for pH, Si and Al concentrations. The results show that caustic consumption is insignificant for temperatures up to 100° C. Above 100° C, consumption increases and is accompanied by a significant decrease in pH. The sharp decline in pH also coincides with a sharp decline in the concentration of silica in solution. The removal of silica from solution is attributed to the irreversible caustic/rock interaction. This interaction is in the form of a chemisorption reaction in which silica is adsorbed onto the rock surface consuming hydroxyl ion. Once these reactions are satisfied, caustic breakthrough occurs at a high pH. However, significant pore volumes of caustic must be injected for completion of the chemisorption. This work is summarized in a Topical Report (Aflaki and Handy, 1992) and will not be presented in the bulk of this report.

The theoretical study addressed questions on the kinetics of silica dissolution and the propagation of alkaline and temperature fronts in porous media. A mathematical model simulating the kinetics of silica dissolution and hydroxyl ion consumption in a typical alkaline flooding environment was developed. The model is based on the premise that dissolution occurs via hydrolysis of active sites through the formation of an intermediate complex, which is in equilibrium with the silicic acid in solution. Both static (batch) and dynamic (core flood) process were simulated to examine the sensitivity of caustic consumption and silica dissolution to process parameters, and to determine rates of propagation of pH values. The model provides a quantitative description of the quartz-alkali interaction in terms of pH, salinity, ion exchange properties, temperature and contact time, which are of significant importance in the design of soluble silicate flooding processes. The latter are shown to minimize or eliminate silica dissolution and caustic consumption, in agreement with recent suggestions. The modeling of a hot waterflood assisted by the simultaneous injection of a chemical additive was also studied. The model is applicable to the hot alkaline flooding under the two conditions of negligible adsorption of the generated anionic surfactant and

of the hydroxide adsorption being Langmuirian. The theory of generalized simple waves was used to develop solutions of the temperature, concentration, oil saturation profiles, and oil recovery curves. It was shown that the chemical resides in the heated region of the reservoir if its injection concentration is below a critical value, and in the unheated region if its concentration exceeds this critical value. Typical results for chemical slug injection in a tertiary recovery process indicate that recovery performance is maximized when the chemical resides entirely in the heated region of the reservoir. This study is to be published as a DOE Topical Report (Saneie and Yortsos, 1993) and will not be presented in the main body of this report.

Another issue related to chemical additives involves the modeling of non-Newtonian fluid flow in porous media. Most EOR fluids, including foams, exhibit non-Newtonian behavior. The effect of microstructure on the flow rate-pressure drop relation is well known for Darcy flow of Newtonian fluids. Not surprisingly, the state of the art is far from complete in this area. A particularly important question is how does the pore structure affect the macroscopic flow behavior, and, thus, the macroscopic models. We have examined certain aspects relevant to Bingham plastics and to power-law fluids. For the first case, we developed a numerical approach to calculate single-phase properties, such as permeability. Aspects of two-phase flow were analyzed for power-law and Bingham fluids. Numerical simulation of the flow of Bingham plastics, taken to represent the behavior of some heavy oils, showed the existence of a macroscopic yield stress for the onset of flow. This behavior resembled somewhat that of power-law fluids at small values of the exponent n . We subsequently described simulation for the displacement (drainage) of one fluid by another of different power-law indices. Displacement patterns similar to the Newtonian case, namely percolation, viscous fingering and compact displacement were found. For the case of fluids of equal power-law indices, but of different consistency indices, we were able to classify the patterns according to a phase diagram for power-law fluids.

A final area of interest to chemical additives involves the use of foams for mobility control. In order to complement experimental work previously conducted in our laboratory, a numer-

ical model was developed to model the formation and mobilization of foam in pore networks. The mechanisms of leave-behind and snap-off were included in the model. It was found that the minimum pressure gradient for foam mobilization is much smaller than that previously reported when the basic mechanism considered for foam generation is leave-behind. In this case, the minimum pressure gradient reduces to zero at large sizes. When the dominant mechanism is snap-off, however, the minimum pressure gradient is quite large in agreement with previous theoretical studies.

1 NETWORK MODELING OF HEAT FLUX DRIVEN VAPOR-LIQUID FLOW IN POROUS MEDIA

C. Satik and Y. C. Yortsos

1.1 INTRODUCTION

The heat flux-driven vapor-liquid flow in porous media finds many applications in various fields of engineering. Oil recovery by steam injection, geothermal systems, boiling phenomena, cavitation, drying, design of heat pipes, underground nuclear waste disposal and post-accident heat removal are the most common among them. Due to this importance, the subject has attracted considerable attention in different contexts. However, most of the studies are at the phenomenological level, based on Darcy's law and relative permeability and capillary pressures functions developed for gas-liquid displacement processes.

At the microscopic level, phenomena that are involved in the heat flux-driven vapor-liquid flow in porous media are nucleation, phase equilibria, phase change and phase growth. Many efforts have been taken, notably by Russian researchers [68], [111], [114] to describe some of these issues in the context of boiling on porous surfaces. Additional studies were reported by Udell [116] and by Parlar and Yortsos [90]. However, these studies are rather incomplete: in the first, the pore structure is largely ignored, while in the second, heat transfer is not considered. Important progress was recently made by Yortsos and Parlar [125] in the context of solution gas-drive. These authors studied a two-component, two-phase system where the bubble growth is controlled by solubility, pressure decline and mass transfer and they also consider processes such as nucleation and phase equilibria. A percolation model was proposed for the bubble growth from many nucleation sites.

In a recent paper [106] we applied the general methodology developed by Yortsos and Parlar [125] to the case of boiling in porous media. We discussed issues of nucleation and phase equilibria in porous media and formulated the bubble growth problem. In particular, we considered bubble growth from multiple nucleation sites at low superheats, where the

growth process is of the percolation type. At low superheats, we proposed three such models: (i) Ordinary percolation with nucleation; (ii) Percolation with superheat depletion; and (iii) Gradient percolation with nucleation. The problem was formulated in terms of a set of dimensionless parameters that may control the growth regime. However, that study was also restricted, in that the heat transfer was not really taken into consideration, except at very low superheat rates. At low superheat, the temperature fields are quasi-static, the solution of which is obtained by solving the Laplace equation. At moderate and large superheats, temperature fields are no longer quasi-static, and unsteady state and convection terms must be included. In this report, we extend our previous work to account for unsteady state and convection in the temperature field. For this, a numerical pore level model is developed. Here, we shall describe our model and formulate the problem of bubble growth. Then, we examine the application of the model to two different cases and present some simulation results.

1.2 BUBBLE GROWTH

To study bubble growth in a porous medium, we use a pore network model [44], [67], [75], [110], [112] by representing the porous medium as a two-dimensional network of interconnected pore bodies and pore throats randomly distributed according to a uniform distribution. We consider growth from a single vapor bubble in a horizontal porous medium of finite size by allowing only one nucleation event to take place at a given location. Effects of multiple nucleation events and gravity forces will be examined in the future. Furthermore, density, specific heat and conductivity values for solid and liquid, liquid viscosity, latent heat of evaporation and surface tension are all kept constant.

1.2.1 Pore Network Model

To model bubble growth in porous media, we make the following assumptions:

- (i) Pore bodies may contain both vapor and liquid phases, while interconnecting bonds contain only one phase (either vapor or liquid).

(ii) Pressure drops occur only in the connections (bonds) between pore bodies while the total volume of both phases (vapor or liquid) is calculated by summing the volume of either phases in a pore body only.

(iii) The vapor phase is non-viscous, therefore no pressure gradients exist in the vapor-occupied pore space.

(iv) The liquid phase is incompressible.

(v) The vapor phase is at saturated conditions while the liquid phase is superheated. The pressure of the vapor phase is obtained by using the Clasiuss-Clapeyron equation :

$$P_v = P_{vo}(T_v) \quad (1)$$

(vi) Ideal gas law applies, such that changes in the total mass of the vapor phase (Δm) during time increment Δt are calculated as

$$\Delta m = \frac{M}{R} \left(\frac{P_v^{n+1}}{T_v^{n+1}} \Delta V + \frac{P_v^n V^n}{T_v^{n+1}} \left(\frac{\Delta P}{P_v^n} - \frac{\Delta T}{T_v^n} \right) \right) \quad (2)$$

where,

$$\Delta P = P_v^{n+1} - P_v^n, \quad (3)$$

$$\Delta T = T_v^{n+1} - T_v^n, \quad (4)$$

$$\Delta V = V^{n+1} - V^n - \quad (5)$$

and P_v , T_v , V , M and R denote vapor pressure, vapor temperature, vapor volume, molecular weight and the ideal gas constant, respectively. Superscripts n and $n+1$ indicate time levels separated by an increment Δt . The change in the total mass of the vapor phase, Δm , is related to the total heat flow rate from liquid to vapor :

$$\Delta m = \frac{\sum Q}{L_v} \Delta t, \quad (6)$$

where Q , L_v , Δt are heat flow rate, latent heat of evaporation and time increment, respectively, the summation is over the perimeter of the vapor bubble.

(vii) The capillary pressure drop across interfaces between vapor and liquid phases in the pore body is neglected (flat interface) when pore bodies are partially occupied by vapor and

liquid phases, and are inversely proportional to the radii of the connections between the pore bodies when they are completely occupied by the vapor phase. Therefore, the condition for an interface to move from a completely vapor-occupied pore j to a neighboring completely liquid-occupied pore i is given by

$$P_v^j - P_l^i + \frac{1}{\rho_l L_v} \frac{G^{ji}}{g^{ji}} (T_l^i - T_v^j) \geq \frac{2\gamma}{R_b^{ji}} \quad (7)$$

where P_l^i , T_l^i , ρ_l , G^{ji} , g^{ji} , γ and R_b^{ji} are liquid pressure, temperature, density, flow conductance, heat conductance, surface tension and radius of the connection between pore bodies j and i , respectively.

(viii) The liquid phase may be trapped. The volume of liquid in the trapped pores does not change during the bubble growth since the ratio of liquid to vapor densities is extremely large, but the heat flow from the trapped liquid to the vapor bubble is included in the total heat flow when calculating the total mass involved in phase change.

To proceed, we identify two different steps which exist during the bubble growth in porous media: (a) Pressurizing step, (b) Pore body-filling step. During the first step, the capillary pressure is larger than the viscous pressure drop, all menisci of the vapor bubble are static and the total vapor volume is constant during this step. All pores on the perimeter of the vapor bubble are completely filled, except those containing trapped liquid. Since there is no change in the total volume of vapor during this step, the total mass generated causes the vapor pressure (and temperature) to increase. The vapor pressure at the end of the step is calculated by combining equations (2) and (6). This pressurizing step continues until the capillary condition in (7) is satisfied. At the end of the pressurizing step, the pressure in the vapor phase is high enough for at least one interface to jump to a neighboring pore body. Then, the capillary pressure difference is set to zero across such interfaces. The next step involves filling of the pore body, during which liquid is displaced from the pore body, while the vapor phase is occupying it as dictated by the evaporation rate. During this step, both the total volume of the vapor bubble and the pressure (and temperature) change. At the end of this step, the pore body is completely filled with the vapor phase. If the capillary condition given in (7) is not satisfied, the pressurizing step begins, otherwise meniscus jumps

may occur elsewhere and other pore bodies are filled with vapor.

1.2.2 Balances

To formulate mass balances in the liquid-occupied pore space, we use Poiseuille's law. Then, the flow rate Q^{ji} through the bond which connects pore bodies j and i is given by

$$Q^{ji} = \frac{\pi(R_b^{ji})^4}{8\mu_l L_b} \Delta P^{ji} = g^{ji} \Delta P^{ji} \quad (8)$$

where $\Delta P^{ji} = P_l^j - P_l^i$. To solve the liquid pressure field we apply the mass balance $\sum_i Q^{ji} = 0$ at each liquid-occupied pore since the liquid phase is incompressible. We use the point SOR (successive over relaxation) method [86] which is most common in solving this type of problems. An expression for the liquid pressure at the pore body j is given by :

$$P_l^j = (1 - \omega)P_{lg}^j + \omega \frac{\sum_i (1 - K_e^{ji})g^{ji}P_l^i - \frac{1}{\rho_l L_v} \sum_i K_e^{ji}G^{ji}(T_l^j - T_v^i)}{\sum_i (1 - K_e^{ji})g^{ji}} \quad (9)$$

where ω and P_{lg}^j are overrelaxation parameters for the SOR method and the guessed liquid pressure at the pore body j , respectively. K_e^{ji} equals one at the liquid-vapor interfaces and zero otherwise. In the above equation, the summation is made over all the pore bodies neighboring the pore body j .

To formulate the energy balance in both liquid-occupied pore space and the solid part of the porous medium, we first express heat transfer rates by including both heat conduction and convection terms but neglecting any radiation heat transfer. The heat transfer rate Q_h^{ji} from a liquid-occupied pore body j to any neighboring liquid occupied pore body i is

$$Q_h^{ji} = G^{ji}[(T_l^i - T_l^j) - Pe^{ji} \frac{\hat{h}_l^{ji}}{C_{pl}}] \quad (10)$$

where C_{pl} is the liquid specific heat and Pe^{ji} is the local Peclet number for heat transfer through the bond between pore bodies j and i , defined as

$$Pe^{ji} = \frac{\rho_l C_{pl} L_b}{\lambda_l} U_l^{ji} \quad (11)$$

where U_l^{ji} denotes velocity through a bond. Finally,

$$\hat{h}_l^{ji} = \frac{h_l^j + h_l^i}{2} \quad (12)$$

where $h_l^j = h_{ref} + C_{pl}(T_l^j - T_{ref})$ and h_{ref} and T_{ref} are reference enthalpy and temperature values, respectively. We allow for a coupling between the temperatures in the solid body and the liquid-occupied pore space by defining a heat transfer rate Q_{sl}^{ji} as follows :

$$Q_{sl}^{ji} = G_{sl}^{ji}(T_s^i - T_l^j) \quad (13)$$

where G_{sl}^{ji} is the thermal conductance for heat transfer between the liquid-occupied pore body j and any neighboring solid node i , and T_s^i is the solid body temperature at solid node i . In the above equation, we have also discretized the solid body into a network of nodes, each of which is coupled to four liquid-occupied pore bodies.

An energy balance at a pore body j with the use of SOR method gives

$$(T_l^j)^{(n+1)} = (1 - \omega)T_{lg}^j + \omega \frac{\frac{\rho_l C_{pl} V_l^j}{\Delta t} (T_l^j)^n + \sum_i G_{li}^{ji} ((T_l^i)^{(n+1)} - A_{conv}^{ji}) + \sum_i G_{sl}^{ji} (T_s^i)^{(n+1)} - \sum_i K_p^{ji} Q_p^{ji}}{\frac{\rho_l C_{pl} V_l^j}{\Delta t} + \sum_i G_{li}^{ji} + \sum_i G_{sl}^{ji}} \quad (14)$$

where

$$A_{conv}^{ji} = Pe^{ji} \frac{\hat{h}_l^{ji}}{C_{pl}}, \quad (15)$$

$$Q_p^{ji} = Pe^{ji} G^{ji} \frac{\hat{h}_l^j}{C_{pl}}, \quad (16)$$

K_p^{ji} equals one at the outer boundary and zero otherwise and V_l is the node volume.

At the liquid-vapor interfaces, heat transfer occurs by both convection and heat conduction. The flow rate (Q_c^{ji}) due to evaporation at the liquid-vapor interfaces is given by

$$Q_c^{ji} = \frac{1}{\rho_l L_v} \frac{\lambda_l \pi (R_b^{ji})^2}{L_b} \Delta T^{ji} = \frac{1}{\rho_l L_v} G^{ji} \Delta T^{ji} \quad (17)$$

where $\Delta T^{ji} = T_l^j - T_v^i$.

To derive an expression for the solid temperature, we employ the following expression for the heat transfer rate Q_s^{ji} from a solid node j to a neighboring solid node i

$$Q_s^{ji} = G_s^{ji}(T_s^i - T_s^j) \quad (18)$$

where G_s^{ji} is the thermal conductance for the heat transfer occurring between the solid nodes j and i . Using the same method as before, an energy balance for a solid node gives

$$(T_s^j)^{(n+1)} = (1 - \omega)T_{sg}^j + \omega \frac{\frac{\rho_s C_{ps} V_s^j}{\Delta t} (T_s^j)^n + \sum_i G_s^{ji} (T_s^i)^{(n+1)} + \sum_i G_{sl}^{ji} (T_l^i)^{(n+1)}}{\frac{\rho_s C_{ps} V_s^j}{\Delta t} + \sum_i G_s^{ji} + \sum_i G_{sl}^{ji}} \quad (19)$$

where ρ_s , C_{ps} and V_s denote solid density, solid specific heat and node volume respectively. We conclude by pointing out that because we assume that the vapor is non-conductive, the heat transfer from solid to vapor is considered negligible.

1.2.3 The simulation scheme

As mentioned above, two different steps exist during bubble growth in porous media. The corresponding algorithms are described below :

Pore body-filling step:

- (1) Guess P_v^{n+1} and calculate T_v^{n+1} from the Classius-Clapeyron equation;
- (2) Calculate liquid pressures by using equation (9);
- (3) Calculate the minimum time Δt_g required to completely fill a pore body by using the current liquid pressure field;
- (4) Calculate liquid and solid temperatures using equations (14) and (19);
- (5) Calculate the total mass evaporation rate ($\sum Q/L_v$) at the vapor-liquid interfaces;
- (6) Calculate the change in the total mass of the vapor phase (Δm) by using equation (2);
- (7) Calculate the new time increment value (Δt_c) by using $\Delta t_c = (\Delta m L_v) / \sum Q$;
- (8) Check whether condition $(\Delta t_g - \Delta t_c) / \Delta t_g \leq \epsilon$ is satisfied, where ϵ is the relative error allowed. If yes, then the acceptable Δt is found, and we continue to step 10, otherwise we go to step 9;
- (9) Calculate a new value for P_v^{n+1} and repeat steps 2-8;
- (10) Update the liquid and vapor volumes at each node from the liquid pressure field.

Pressurizing step :

- (1) Calculate P_v^{n+1} from which the movement of an interface will be calculated;
- (2) Guess Δt_g ;

- (3) Calculate liquid pressures from (9);
- (4) Calculate liquid and solid temperatures using (14) and (19);
- (5) Calculate the total mass evaporation rate ($\sum Q/L_v$) at the vapor-liquid interfaces;
- (6) Calculate the change in the total mass of the vapor phase (Δm) by using (2) with $\Delta V = 0$;
- (7) Calculate the new time increment (Δt_c) by using $\Delta t_c = (\Delta m L_v) / \sum Q$;
- (8) Check whether condition $(\Delta t_g - \Delta t_c) / \Delta t_g \leq \epsilon$ is satisfied, where ϵ is the relative error allowed. If yes, an acceptable Δt is found and we continue to step 9, otherwise we repeat steps 3-8;

These algorithms were coded into a computer program which was run to investigate various cases.

1.3 RESULTS

We examined two different cases: (i) Bubble growth with a uniform superheat imposed initially and (ii) Bubble growth with a prescribed heat flux at one boundary. In the first case, the pore space in the porous medium is completely filled with a superheated liquid. Initial temperatures in the liquid and solid of the porous medium are spatially constant and at the same temperature value. The geometry of the porous medium is square and all four boundaries are open to the atmosphere. Thus, the pressure in the liquid-occupied pore space is initially constant and at atmospheric pressure. In the second case, the geometry of the medium is rectangular. A heat flux is imposed on the right hand side of the medium. No-heat flux boundary conditions are imposed at the remaining boundaries. A constant pressure (atmospheric) boundary condition is imposed on the left-hand side boundary, while no-flux boundary conditions are imposed on all other boundaries of the medium. The initial nucleation site is located at the center of the medium in the first case and at the center of the side where heat flux is imposed in the second case.

Depending on the system parameters, bubble growth may reach two states. In the first state, the bubble grows until it touches the outer boundaries. In the second state, the

superheat in the system is completely depleted so that the bubble cannot grow further. At this steady state, bubble growth stops and temperature and pressure in the network stabilize to a spatially constant value. In our simulation results, the final growth stage is reached when the vapor bubble reaches either of the two states.

During bubble growth, two dimensionless parameters control the growth regime : (i) The Jacob number, Ja , and (ii) the modified capillary number, Ca^* . Ja is related to the superheat imposed

$$Ja = \frac{\rho_l C_{pl} \Delta T}{\rho_v L_v} \quad (20)$$

$$\Delta T = T_{init} - T_{vo}(P_{init}) \quad (21)$$

where ΔT , T_{init} , $T_{vo}(P_{init})$ and ρ_v denote superheat, initial temperature of the liquid, saturation temperature of the liquid at the initial pressure and density of the vapor, respectively. The modified capillary number is expressed as

$$Ca^* = \frac{\lambda_l \mu_l}{\rho_l C_{pl} R_b^* \gamma} \quad (22)$$

where R_b^* is the average bond radius.

1.3.1 Bubble Growth in a Uniform Initial Superheat

To model this problem, we used a square lattice (51x51) network. Throat (bond) sizes were randomly assigned from a uniform distribution, while pore body (site) sizes were kept constant. Simulation results at three different stages of bubble growth for the typical parameters given in Table 1 are shown in Figure 1. In the Figure, white or black colors represent liquid only or vapor only occupancy, respectively, while gray color denotes partial liquid occupancy. The final stages of bubble growth for three different Ja values are shown in Figure 2. As seen from the figure, fingering-type growth patterns form at large Ja values, whereas percolation-type patterns are obtained as Ja is smaller. In our previous work [106], we have presented an analysis of the effect of the various dimensionless parameters that control bubble growth in porous media. At low superheat conditions, $Ja \ll 1$, the temperature fields in both liquid and solid are quasi-static and the growth is controlled by capillarity, such that viscous

(a)

(b)

(c)

FIGURE 1. Simulation of three different stages of bubble growth

(a)

(b)

(c)

Figure 2: Final stages of bubble growth for $Ja = 0.845$, 5.075 and 233.4

$$\begin{aligned}
\Delta T &= 4.4 \text{ } ^\circ\text{C} \\
\rho_l &= 960.85 \text{ kg/m}^3 \\
C_{pl} &= 4.2092 * 10^3 \text{ J/kg} - K \\
\lambda_l &= 0.6808 \text{ W/m} - K \\
\mu_l &= 2.4799 * 10^{-4} \text{ N} - \text{s/m}^2 \\
L_v &= 2.2568 * 10^6 \text{ J/kg} \\
\gamma &= 0.1963 \text{ N/m}^2 \\
L_b &= 1320 \text{ } \mu\text{m} \\
R_b^* &= 385 \text{ } \mu\text{m} \\
\rho_s &= 2082.40 \text{ kg/m}^3 \\
C_{ps} &= 8.3732 * 10^2 \text{ J/kg} - K \\
\lambda_s &= 6.808 \text{ W/m} - K
\end{aligned}$$

Table 1: Typical Parameter Values.

forces are negligible. Under these conditions, the pattern should be of the percolation type. At moderate and large superheat values, where $Ja \sim O(1)$ or larger, the growth process is still controlled by heat transfer, but the temperature fields are no longer quasi-static and both convection and unsteady state terms must be considered. At very large superheat values, $Ja \gg 1$, convection is very important in bubble growth. In Figure 2, Ja values are 0.8458, 5.075 and 233.4 corresponding to moderate, large and very large superheats. We point out that inertia was neglected in all our simulations. We also simulated a process with $Ja = 0.1692$, which corresponds to a low superheat, although in this case, the bubble reached a steady state very early due to fast depletion of superheat. As mentioned above, the bubble is still ramified when Ja is moderate, indicating that capillarity still plays an important role. As Ja increases, the cluster acquires many fingering characteristics. The final growth pattern for two different values of Ca^* (0.0214 and 0.214) at $Ja = 0.4229$ is shown in Figure 3. We observe a compact bubble at low Ca^* value and a less compact bubble with thin fingers when Ca^* is increased tenfold. As expected, the importance of capillary forces diminishes as Ca^* increases.

(a)

(b)

Figure 3: Final stages of bubble growth for $Ca^* = 0.0214$ and 0.214 at $Ja = 0.4229$.

1.3.2 Bubble Growth with a Prescribed Heat Flux

To model this process, we used a square lattice (21x42) network with the same bond and site distributions as in the previous case. Now, initial liquid and solid temperatures are the same, while a heat flux (q_h) is imposed over the first column of the network. Initial liquid temperature and pressure were set to 100 °C and 14.696 psi, respectively. The initial nucleation site was arbitrarily located at the node (11,1). Onset of nucleation takes place when the temperature of the node (11,1) reaches the nucleation temperature. In [106] we discussed the issue of nucleation in great detail and proposed a nucleation model for bubble growth in porous media. A sensitivity analysis was conducted to evaluate the effects of various parameters. These include q_h , Ca^* , λ_s/λ_l and b^* , defined as the ratio of the thermal conductances for solid-to-liquid and liquid-to-liquid heat transfer. This parameter provides a measure of the thermal coupling between solid and liquid, the thermal interaction between solid and liquid increasing as b^* increases.

Figure 4 shows results at three different q_h values (94.61, 946.1, 9461.0 W/m²). The bubble is of the percolation type when the heat flux imposed is small and it becomes fingering-like as the flux increases. Since the superheat in the liquid increases when the imposed flux is increased, this flux plays the same role as Ja . Therefore, we can use the same analysis above to infer the effect of higher superheat. Final growth patterns for three different Ca^* values at $q_h = 3153.58$ W/m² are shown in Figure 5. As can be seen, the results are similar to those for bubble growth with an initially imposed superheat.

We next used the numerical model to discuss the effect of solid contribution to the heat transfer, namely effects of λ_s/λ_l and b^* . The final growth patterns at $q_h = 3153.58$ W/m² and $Ca^* = 0.00214$ for three different λ_s/λ_l values are shown in Figure 6. The effect of two different values of b^* is shown in Figure 7. All final growth patterns in these figures are similar. We are led to the conclusion that under the present assumption the solid contribution of heat transfer to the growth regime is insignificant. However, inclusion of a thin liquid film in the vapor zone may significantly change the heat transfer. This is currently under investigation.

(a)

(b)

(c)

Figure 4: Final stages of bubble growth for $q_h = 94.61$, 946.1 and 9461.0 W/m², respectively.

(a)

(b)

(c)

Figure 5: Final stages of bubble growth at $q_h = 3153.58 \text{ W/m}^2$ for $Ca^* = 0.0214$, 0.00214 and 0.000214 , respectively.

(a)

(b)

(c)

Figure 6: Final stages of bubble growth at $q_b = 3153.58 \text{ W/m}^2$ and $Ca^* = 0.00214$ for $\lambda_s/\lambda_l = 10, 100$ and 1000 , respectively.

(a)

(b)

Figure 7: Final stages of bubble growth for $b^* = 0.1$ and 1.0 at $q_h = 3153.58 \text{ W/m}^2$ and $C^*a^* = 0.00214$.

1.4 CONCLUSIONS

In this section, we have presented a pore network model that describes heat flux-driven vapor-liquid flow in porous media. The numerical model accounts for liquid convection and solid conduction, in addition to heat conduction. Two different steps, one due to filling the pore body and one to pressurizing, exist during a bubble growth process. During the filling step, the liquid phase is continuously being discharged from a pore, while the vapor phase is filling. During the pressurizing step, the interfaces between liquid and vapor are static and the pressure in the vapor phase continuously increases. We have analyzed two different types of bubble growth: (i) Bubble growth with a uniform superheat initially imposed, and (ii) Bubble growth with a constant heat flux imposed. A sensitivity analysis for both problems was presented. In the first problem, we examined the effects of Ja and Ca^* . Simulation results show that the bubble growth patterns change from percolation to viscous fingering as Ja and Ca^* increase. The effects of Ja in the first problem and of q_h in the second problem are conjectured to be the same. Effect of solid contribution to the heat transfer in the second problem was found to be less important than expected. We are currently continuing the numerical sensitivity studies.

2 STUDY OF STEAM INJECTION IN A HELE-SHAW CELL

X. Kong

2.1 INTRODUCTION

In last year's report [124] we presented studies involving the steam displacement of various oils in Hele-Shaw cells. Many novel phenomena regarding the interaction between steam, oil and water were found. However, due to the involvement of three phases, it was difficult to isolate many important mechanisms involved. Much more work is needed for these phenomena to be understood physically and quantified mathematically. To concentrate on steam condensation and front displacement, we conducted a systematic study of water displacement by steam in a Hele-Shaw cell. From this system we expect to obtain a much better understanding of the various steam displacement mechanisms. In particular, we can study effects of heat losses from the cell to the surroundings. In this report we summarize observations such on Hele-Shaw cell experiments involving only steam and water. In addition, experiments were performed involving steam, water and oil in glass bead packs, in order to compare the performance in Hele-Shaw cells and in porous media. We used the same experimental parameters as in our previous study [66] in order to allow for a direct comparison. These experiments are also reported here.

2.2 STEAM DISPLACEMENT IN A HELE-SHAW CELL

2.2.1 Steam Injection in a Water-Filled Hele-Shaw Cell

A total of 16 experiments were conducted. General conditions involved a rate of 3 *cc/min* and a steam temperature of 230° *F* in order to allow for a direct comparison with our previous experiments. Effects of heat losses were studied by adding different thickness of insulation on the top and bottom of the cell while measuring the steam front position as a function of

Figure 8: Snapshot of displacement of water by steam.

time. The base run corresponds to a Hele-Shaw cell without any additional insulation.

Initial observation are similar to those when we used steam injection to displace oil [66], namely, a condensed water zone forms ahead of the cold water in place (Figure 8), which is displaced ahead of the steam zone. As before, a steam front oscillation was observed but only when the orientation of the cell was in the downdip direction, in contrast to the case of oil displacement, where a substantial frontal oscillation was observed even in the absence of gravity. Furthermore, no steam fingering was detected in the present case of water displacement, most likely due to the smaller viscosity ratio between displacing and displaced fluids. This suggests that for large mobility ratio values (for example in the case of Dutrex 739 displaced by steam), the front may not be stabilized by steam condensation.

During steam injection in horizontal cells, the movement of the front is quite stable. The front is flat and the displacement has a piston-like appearance (Figure 9). Contrary to the case involving viscous oil, steam condensation was not followed by violent oscillations, instead, the steam condensed slowly inside the steam zone, the steam front being in an almost stationary state. Various capillary phenomena play an important role during this process. For example, we observed a growing water region as steam continuously condensed at its periphery. Figures 10 and 11 show consecutive shots indicating the growing of this region

Figure 9: Stable frontal displacement

within the steam zone. After the region becomes large enough, a sudden replacement by steam takes place. Then this process repeats again. As the region growing replacing of this region is continued, the steam zone grows stably. For another experiment, a much slower steam condensation rate was noticed compared to that of stable displacement. Considering that steam rate, temperature and heat insulation are the same as stable displacement experiments, the reduced steam condensation rate implies smaller heat transfer rate from the steam to the displaced liquid. The higher heat losses incurred during unstable displacement are a result of steam fingering into the old displaced oil because of the higher steam condensation rate. Steam fingering in the oil phase increases the heat transfer area, thus increasing a faster heat loss rate.

2.2.2 Effects of Lateral Heat Losses

The Hele-Shan cell is ideally suited to study isothermal flow. When it is used to study non-isothermal problems, such as steamflooding, however, difficulties arise as to how to avoid excessive heat losses to the surroundings. This problem is difficult to solve because insulation will affect the condensation quality. A partial solution to the problem of the effect of heat losses on the performance of steam displacement is by controlling the degree of insulation. For the

Figure 10: Steam condensation inside the steam zone: stage 1.

Figure 11: Steam condensation inside the steam zone: stage 2

Figure 12: Steam zone size variation with time.

end, we conducted a series of experiments by adding different layers of glass plates on the Hele-Shaw cell. We focus on the effects of insulation on the steam zone size, and compared the effects of different insulation. The same injection parameter were used as before, namely, 3 cc/min injection rate and 230°F steam temperature.

Figure 12 shows a comparison of the change of steam zone size with time between a run without insulation and a run with one layer of 0.5 in thickness insulation. In both cases, the steam zone initially increases with time at about the same rate. There is virtually no difference in steam zone size and steam displacement performance during the initial 30 min. of steam injection. After some time, however, the steam zone size increases faster in the case of increased insulation. This indicates that the effect of additional insulation is felt only after some time has elapsed. This result is consistent with heat conduction theory. Differences were observed following this point. Steam fronts in the better-insulated case were found to be more fingered. This observation is in good agreement with [85] and [123], where it was shown that heat loss acts to stabilize steam fronts. Experiments with additional layers of insulation were also conducted. All experiments showed the same trend, namely, the larger the number of insulation layers, the faster the rate of steam front advance and the more unstable the steam front.

Figure 13: Correlation between insulation thickness and time.

A simple quantitative analysis based on heat conduction can be derived to estimate the effects of finite thickness insulation layers. We can obtain a simple equation to describe the difference in heat transfer rates between one-dimensional heat conduction in a finite slab and in a semifinite medium, both subject to a constant step increase in temperature at the boundary [24]. If we assume that perceptible differences between the two problems set in when the ratio of the two fluxes is ϵ , the following equation is obtained [24]:

$$\sum_{m=1}^{\infty} ierfc \frac{mh}{\sqrt{\alpha_c t}} - \frac{\epsilon}{2\sqrt{\pi}} = 0 \quad (23)$$

where $ierfc$ is the integral of complementary error function

$$ierfc(x) = \frac{1}{\sqrt{\pi}} e^{-x^2} - xerfc(x) \quad (24)$$

α_c is the thermal diffusivity of insulation material and h is the thickness of the insulation.

The above equation can be solved to yield h as a function of the time, at which the heat flux reaches a given fraction of that in the semi-infinite medium. Figure 13 shows the results for $\epsilon = 0.01$. From this graph we find that 1" of insulation thickness corresponds to approximately 36 minutes, a time which is in good agreement with our experimental observation.

2.3 STEAM DISPLACEMENT OF OILS IN A GLASS BEAD PACKED SHELL

The second part of this section deals with steam injection in an oil-saturated glass-bead packed cell. The purpose of these experiments was to examine similarities and differences between Hele-Shaw cell flow and porous media flow. Various packing methods were used to construct the porous media model and observations at different magnification scales were also made. For a meaningful comparison with the previous work on Hele-Shaw cell the same experimental conditions were employed.

2.3.1 Construction of the Model

Both unconsolidated and consolidated models were used. The former have the advantage of uniform wettability distribution, and the obvious disadvantage that the glass beads tend to move with the fluids, especially when a very viscous liquid is involved. The latter can be constructed by using epoxy to fix the glass bead on the surface of the plate. However, the epoxy may also change the media wettability and introduce additional uncertainty.

Glass beads with a diameter of $300\ \mu\text{m}$ and $1.5\ \text{mm}$ were used. The dimension of the model varied from $9\ \text{in.} \times 4\ \text{in.}$ to $24.5\ \text{in.} \times 12\ \text{in.}$. Guidelines for the construction of the glass bead packed model are summarized below. The procedure for the unconsolidated pack is as follows:

1. Uniformly distribute one layer of glass beads on top of the glass plate. Place another glass plate on top of the layered glass beads.
2. Make careful alignment between the two glass plates. Use a transparent high strength tape to seal the three sides of the model.
3. Pack the model with glass beads from the open side until it is full. Shake the model during filling.

4. Use epoxy to seal all four sides of the model and make sure there is no free space for the glass beads to move.

The procedure for constructing the consolidated model is slightly different:

1. Carefully brush a thin layer of epoxy onto the surface of one glass plate. Make sure that the epoxy layer has a uniform thickness.
2. Place one layer of glass beads on top of the epoxy layer for the bead to stick to the surface. Make the space between the glass beads as small as possible.
3. Repeat steps 1 and 2 and prepare another glass-bead-surfaced plate.
4. Put the two plates together and seal the four sides with epoxy.

Because of the harsh conditions within the steam zone, the glass beads gradually loosen and become movable. The life of an epoxy model is typically 4-5 experimental trials.

2.3.2 Results

The same experimental conditions used in Hele-Shaw experiments were also used in the glass bead packed models. Three oils were tested as before, mineral oil, Dutrex 739 and a Long Beach crude oil, which were displaced by steam or water at various injection rates. The typical steam temperature was $230^{\circ}F$. The results can be summarized as follows:

- The initial stage of injection is characterized by the formation of fingers of condensed water which displace the oil. As injection continues, the steam injected follows the path created by the condensed water (Figure 14). This observation is the same with the corresponding injection in the Hele-Shaw cell. Important differences observed are that the interface between oil and condensed water is less smooth than in the Hele-Shaw cell, and takes patterns approaching percolation. The reason is obviously due to the restricted pore space and the resulting capillary effects.

Figure 14: Initial stage of steam injection in the glass bead packed model.

Figure 15: Finger thinning in the glass bead packed model.

Figure 16: Steam gravity override in the glass bead packed model.

- Similar to the previous Hele-Shaw cell case, as the steam front advances in the porous media, a rapid condensation of part of the steam zone takes place as a result of heat transfer. This is immediately followed by a vaporization process which restores the steam zone to its original size. The condensation-vaporization cycle is repeated continuously as the injection continues. Its frequency, however, is less than that in the Hele-Shaw cell. Considering that these phenomena may be due to heat transfer, one reason for this difference may involve the larger heat capacity of the porous media.
- As in the previous experiments, a viscoelastic response of both the Dutrex 739 and the Long Beach crude oils was observed after the first finger first reached the production end (Figure 15). The resulting finger thinning was attributed to a non-Newtonian response of these fluids.
- Experiments with all the oils used in the Hele-Shaw cell displacement show similar displacing patterns and steam-water-oil interaction. The visualization quality, however, is considerably poorer here, and many details of the interface cannot be readily recognized.

- Gravity effects were introduced by rotating the model to the direction parallel to the gravity vector. Figure 16 shows a typical steam gravity override with steam injected into the model. The pattern is very similar to that observed in the vertical Hele-Shaw cell.
- As expected from displacements in porous media, the pore geometry affects substantially the characteristics of the displacement, in particular the setting of the finger wavelengths in the case of unstable displacement. These features were clearly observed in our case as well.

2.4 CONCLUSIONS

Many of the features of the displacement in the Hele-Shaw cell were reproduced in the experiments involving glass bead packed Hele-Shaw cells. In particular, aspects related to heat transfer were found essentially the same in either case. On the other hand, phenomena related to surface, such as film flow, gravity drainage and three phase-flow are different in the two configurations. Current research is aimed at understanding these effects.

3 APPLICATION OF FRACTAL GEOMETRY TO THE STUDY OF NETWORKS OF FRACTURES AND THEIR PRESSURE TRANSIENT

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3.1 ABSTRACT

This section describes the application of fractal geometry to the study of networks of fractures and their pressure transient response. Present models for the representation of naturally fractured systems rely on the double-porosity Warren-Root model or on random arrays of fractures. However, many field observations have demonstrated the existence of multiple length scales in several naturally fractured media. Present models fail to capture this important property of self-similarity. In this section, we first use concepts from the theory of fragmentation and from fractal geometry to construct numerically of a network of fractures that exhibits self-similar behavior over a range of scales. The method is based on the IFS (Iterated Functions System) approach and allows for great flexibility in the development of patterns. Numerical simulation of unsteady single phase flow in such networks is next described. It is found that the pressure transient response of finite fractals behaves according to the analytical predictions of Chang and Yortsos [29] provided that there exists a power law in the mass-radius relationship around the test well location. Finite size effects can become significant and interfere with the identification of the fractal structure. The section is completed by providing examples from actual well tests in fractured systems which are analyzed using fractal pressure transient theory.

3.2 INTRODUCTION

Recent developments in the theory of fractal geometry [45], [83] have prompted extensive and novel applications to many problems in science and engineering, particularly to those

resulting from unstable phenomena. In general, fractal objects are the result of the repetition of a given algorithmic rule or pattern over a cascade of different scales. When coupled with random noise, the resulting fractal images have complexity that characterizes a variety of natural objects. Specifically, the application of fractals to porous media is very promising. The review by Sahimi and Yortsos [103] classifies many fractal patterns that result from various porous media processes, such as percolation, viscous fingering and fracturing. Networks of fractures in a rock are also natural candidates for a fractal description. This particular alternative is explored here.

Conventionally, naturally fractured systems have been represented by the Warren and Root double porosity model [117] or by a random array of fractures [21], [82]. Although capturing important properties, neither of the two geometries can account for fractal characteristics recently attributed to naturally fractured systems [14], [79], [95]. The relation of fractals to fracture networks dates back to 1985, with the study of a nuclear waste disposal site [14]. That study revealed that fracture patterns at Yucca Mountain, NV, were self-repetitive over a range of scales within which several generations of fractures were reported. Additional support for the fractal character of fracture networks can be found in recent studies of the fracture pattern of the Monterey formation [47] and of the Geysers geothermal field [79], prominent features of which are the cascade of fracture scales and the self-similar structure.

It was recently proposed that the fracturing of disordered media, such as natural rocks, can be modeled using fractals [56], [115]. Fractal structures have been related to the fracture resistance of materials and to the particular fracturing process [115]. For example, fragmentation with substantial shearing, which appears to be a dominant mechanism for many fracture networks, leads to fractal dimension values ranging between 1.2 and 1.8 [20]. Sammis et al. [65] reported fractal dimensions between 1.5 and 1.7. The fractal characterization of real networks of fractures has offered a promising alternative for the solution of pressure transient problems, which still remain challenging. Motivated by the analogy with diffusion on fractals, investigators have recently analyzed the hydraulic response of networks

of fractures using fractal theories (Beier [15], Chang and Yortsos [29]). These studies, whose theoretical underpinning is based on anomalous diffusion in fractals [53], [88], demonstrated that the traditional solutions for transient single-phase flow are particular cases of a more general solution, where the dimensionality (reflected in the fractal dimension) is a key variable. Significantly, this dimensionality takes non-integer values and dominates the fractal response.

To validate and test the theoretical predictions requires numerical simulation in model fractals and comparison with field data. Numerical simulation is particularly important, because in real systems fractals exhibit finite size effects that cannot be captured analytically. Earlier numerical studies in fractal of fractured systems relied on the use of Sierpinski carpets and percolation networks [11]. In certain cases, simulation of these objects has shown the expected for a fractal transient response. However, such networks represent rather special and idealized cases. Naturally fractured systems do not necessarily correspond to percolation only, in fact they must be consistent with various fracturing mechanisms, such as shear fracturing, extension fracturing, etc., which differ from percolation [60]. Model networks should also honor available data, such as fracture length distribution, fracture orientation and density, which demands additional flexibility in model parameters. It must be pointed out that at present a single, universal theory for fracturing in naturally fractured systems is not available. Therefore, rigorous methods for the construction of networks of fractures must await the successful development of fracturing theories. Recent advances in this area have been many and significant and they hold promise that a unified theory may soon emerge [56].

A practical alternative, which is meant to be used for sensitivity studies and not as a general theory for fracturing, exists for systems that may be described by a fragmentation process. The latter is known to lead to fractal size distributions [115]. Indeed, key aspects of fragmentation can be simulated with the application of the IFS (Iterated Function System) approach introduced by Barnsley [13]. This is a technique of great flexibility and yields networks of the required properties, for instance in the orientation of the fractures and in

the shapes of the fragments. The approach of combining fragmentation and IFS to create fractal networks is presented here. These networks are then used to simulate transient, single-phase flow of a slightly compressible fluid and its pressure transient response, and to conduct sensitivity studies.

Two are the main objectives of this section: (i) The simulation of pressure transients in synthetic fractal networks of fractures to test existing theories; and (ii) the identification of the fractal properties of well data, if appropriate. We first describe the possible relevance of fractals to networks of fractures, based on which we propose a heuristic method for creating synthetic fractal networks. We subsequently review the theoretical response of a fractal network. Theoretical predictions are tested against the numerical simulation of pressure transients in the synthetic networks. The last section is devoted to the interpretation of well test data and the identification of fractal parameters.

3.3 NUMERICAL SYNTHESIS OF FRACTAL NETWORKS OF FRACTURES

3.3.1 Fragmentation and Fractals

In many fragmentation processes, the distribution of fragment sizes can be described by a power law. This fact has been known since the early 1940s, when Schuhmann's law [49] was introduced to describe fragment distribution in grinding operations. Turcotte [115] documents many fracturing applications, where the size distribution is described by a power law. More recently, Poulton et al. [95] postulated a power-law behavior not only for the fragment size but also for the length and spacing of discontinuities in the rock. As shown in [1], these distributions are interrelated. In a related study, a power-law distribution of fracture trace lengths was discovered by Barton [14].

Fragmentation was modeled in the classical work of Gilvarry [49] who used an exponential distribution that describes repetitive fracturing. The key assumption in his work is the existence of randomly distributed imperfections in the rock, which can initiate fractures. A

basic parameter in the analysis is the probability $p_f(l)\delta l$ that a given fragment of size l in the interval between l and $l + \delta l$ will be fragmented. Recently, Turcotte [115] has showed that when p_f is constant, a power-law distribution of fragment sizes is obtained. For an idealized fragmentation processes, where the fragments of a given generation are all of the same relative size, the size distribution is of the power law type, with an exponent related to the probability of fracturing p_f . For example, in space of dimension d , if each block creates an average of Sp_f new blocks of relative size $1/S$, the resulting fragment size distribution can be described by a power law with exponent

$$D = \frac{d \ln(Sp_f)}{\ln S} \quad (25)$$

This exponent is also equal to the box fractal dimension of the mosaic made of the same pieces. Since an unfractured block represents a missing subset of fractures, the fracture length distribution will also be power-law distributed with the same exponent. Equation (25) suggests that non-trivial fractals ($D < d$) are obtained for $p_f < 1$.

The box counting fractal dimension is one measure of a fractal structure. It is typically calculated by superposing a grid of a given cell-size on the fracture pattern and by counting the number of occupied cells. The power law relation

$$N(r) \sim r^{-D} \quad (26)$$

between the number $N(r)$ of occupied cells and the scale r , yields the dimension D . Box counting has been routinely used to characterize the fractal properties of real networks [14], [65].

For more realistic fragmentation processes, blocks of a given generation are not all of the same relative size and a given size group may be composed of blocks of different generations. Nonetheless, a power law distribution in fragment size and fracture length still persists. We should add that fractal behavior typically holds only in a finite range between upper and lower cutoff scales. The upper cutoff is defined by the maximum size fragment (the “largest hole” in the network). The lower cutoff is more arbitrary, usually decided from practical

considerations and in a synthetic usually corresponds to the size of the last generation to be added. As shown below, finite cutoffs play an important role in the hydraulic response.

The way by which patterns are created by fragmentation, namely the initiation from a large scale and the propagation towards successively smaller scales in an organized and systematic manner, has a close analogy with the IFS technique.

3.3.2 Synthetic Fractal Networks of Fractures Using IFS

Barnsley [13] has recently proposed the method of Iterative Function System (IFS) to construct fractal images. With this technique, a fractal is obtained from an initial simple shape (initiator) by applying in an iterative fashion a set of numerical transformations (propagator). Each iteration creates multiple sets of smaller, transformed images that occupy the place of the previous image. After several generations, the set converges to a fractal. The technique creates fragments of various sizes and shapes and it is well suited for the development of synthetic networks of fractures.

For the creation of two-dimensional patterns, two transformations are used. Each consists of the quadratic expressions

$$x_n = ax_{n-1} + by_{n-1} + cx_{n-1}y_{n-1} + d$$

$$y_n = ex_{n-1} + fy_{n-1} + gx_{n-1}y_{n-1} + h$$

where x_n, y_n are the coordinates of a given point of the n th generation, and a, b, c, d, e, f, g, h are the coefficients of the transformation. When $c = g = 0$ the transformation is linear (generally self-affine). If, in addition, $a = f$ and $b = -e$, it gives rise to self-similar fractals. The non-linear terms (c, g) are useful in controlling the geometry of the final pattern. However, control of the fractal characteristics and the fractal dimension is obtained by varying the probability p_f , as described below.

Figure 17a shows a pattern of eleven generations of fractures initiated from a quadrilateral shape with a single initial fracture. In this Figure all fractures are allowed to exist,

corresponding to $p_f = 1$. The pattern thus formed has a box counting fractal dimension equal to 2 (and, thus, corresponds to a Euclidean object). As additional generations are included, the fracturing process continues until the medium completely disintegrates. To obtain non-trivial fractals the probability of fracturing must be lower. As in fragmentation, we may specify that a fraction $(1 - p_f)$ of newly generated blocks is not allowed to further subdivide. Lower values of p_f result in lower values of the box fractal dimension D , as predicted by equation (25). Figures 17b and 17c were constructed as in Figure 17a, except that a fraction $(1 - p_f)$ of randomly selected fragments were left unfractured after the fourth generation. Different selections of fragments result in different networks, although all realizations have the same box fractal dimension. To illustrate the versatility of this technique three typical examples of fracture networks related to Sierpinski gaskets are shown in Figure 18. The networks are obtained with the application of self-similar, self-affine and non-linear transformations, respectively. A fracturing probability $p_f = 0.75$ was applied every second generation. It must be pointed out that because the fractures are retained, the above objects are neither self-similar nor self-affine. In this regard, the networks of Figure 18 represent modifications of the well-known Sierpinski gasket [83]. Nevertheless, they do possess similar fractal characteristics. Figure 19 shows plots of the box counting, fracture length and fragment size distributions corresponding to the networks of Figure 17. Power laws in the fragment size and fracture length distributions are observed.

The networks of Figure 17 were constructed by the application of two transformations to a two-dimensional initial pattern, composed by a single fracture that splits a quadrilateral shape. The technique allows for an infinite variety of networks produced by the application of two or more transformations in two or three dimensions. Figure 20 shows examples of the generalization of the technique to create fractal networks of fractures in three dimensions. The sequence of transformations that creates a fracture also specifies its address. For example, if an IFS of two transformations is used, denoted by 0 and 1 respectively, typical addresses of fractures are 001, 1010, 01101, etc. The number of digits in the sequence equals that of the generation to which the fracture belongs. The numbers also determine the posi-

Figure 17: Two-dimensional networks of fractures with (a) $p_f = 1$, (b) $p_f = 0.95$ and (c) $p_f = 0.90$.

Figure 18: Modified Sierpinski gasket (11 generations). (a) Self-similar IFS, (b) Self-affine IFS, (c) Non-linear IFS.

Figure 19: Box counting (*), fragment size (x) and fracture length (o) distributions for the networks of Figure 17. From the slope of the box counting curve: (a) $D = 2$, (b) $D = 1.78$ and (c) $D = 1.65$.

tion of a particular fracture. This systematic fracture identification is especially appropriate for fluid flow simulation as will be shown below. Before we proceed, however, we shall review some theoretical aspects of pressure transients in fractals.

3.4 THEORY OF PRESSURE TRANSIENT RESPONSE IN FRACTAL NETWORKS

A key property of fractals is that their mass density decreases following a power-law fashion when an increasingly larger region is considered [45], [83]. For example, consider the network of Figure 21a, containing a single fracture along which we have arbitrarily selected an origin, and take the mass M contained in a circle of radius r and area A . Obviously, the relationships $M \propto r$, $A \propto r^2$, apply, resulting into a mass density $\rho = \frac{M}{A}$ that behaves as $\rho \propto r^{-1}$. More generally, for a fractal of a mass fractal dimension D embedded in dimension d , the density obeys the scaling

$$\rho(r) \propto r^{D-d} \quad (27)$$

Thus, for the homogeneous network of Figure 21c the mass density is constant ($D = 2$, $d = 2$) while the fractal network of Figure 21b has a mass density decreasing with distance, as described by (27) with $1 < D < 2$ and $d = 2$. The scale dependence of properties in fractals introduces new issues that are absent in Euclidean objects. If the networks of Figure 21 were infinite in size, equation (27) would be obeyed independently of the selection of the origin in the network. For the network of Figure 21b this would require an infinite cascade of fragment sizes. Finite size limitations, either computational for synthetic networks or physical for real networks, result in the density approaching a constant value when the upper or lower cutoffs are reached.

Consider, next, a fractal network of fractures embedded in an impermeable rock matrix. The point value of porosity is either 0 or 1 depending on whether the point is located in the matrix or in the fracture, respectively. The macroscopic fracture porosity, however, is defined over a volume of size r , and therefore it must be proportional to the above defined

Figure 20: Three-dimensional networks of fractures (a) $D = 2.78$, (b) $D = 2.65$ and (c) $D = 2.47$.

Figure 21: Power law properties of fractal networks. (a) Linear system $D = 1$. (b) Fractal network $1 < D < 2$ and (c) Radial system $D = 2$.

mass density. For the case of a Euclidean network this value is constant. For a fractal object, however, it should decrease with distance following the scaling

$$\phi(r) = \phi_0 \left(\frac{r}{r_0} \right)^{D-d} \quad (28)$$

where $\phi(r)$ is the macroscopic porosity of a region of size r and ϕ_0 its value at $r = r_0$. Here r_0 is the lower cutoff scale (smallest block size in the fracture network above which a fractal behavior is obeyed).

While ϕ is related to storage, another key parameter is permeability which is related to transport. Consider the steady-state permeability in single-phase flow in a fractal object (e.g. the fracture network). Contrary to the Euclidean case, if a pressure drop Δp is applied across a region of size r , the overall permeability is not constant, but it scales with the distance r . As reviewed by Sahimi and Yortsos [103], the scaling is

$$K = K_0 \left(\frac{r}{r_0} \right)^{D-d-\theta} \quad (29)$$

where θ is a transport exponent. For percolation networks, θ is related to two key fractal exponents by the relationship $\theta = (\mu - \beta)/\nu$. For example, in 3-D percolation networks $\theta = 1.784$. Exponent θ is also related to the dynamic random walk on a fractal lattice. For example, in diffusion in fractal networks the mean-square displacement obeys the scaling [53], [88]

$$\langle R^2 \rangle \propto t^{\frac{2}{d_w}} \quad (30)$$

where the dimension of the random walk d_w is given by $d_w = 2 + \theta$. Thus, exponent θ is related to the network connectivity and describes the deviation from an ordinary random walk in the fractal network. For the Euclidean networks of Figures 21a and 21c, diffusion is Fickian, $\theta = 0$, and the familiar diffusion growth $R \propto \sqrt{t}$ is obtained. For the fractal network Figure 21b, however, the many traps of various scales in the network causes diffusion to slow down. This anomalous behavior is reflected in $\theta > 0$. Were a diffusion coefficient to be defined, $D \sim \frac{\langle R^2 \rangle}{t}$, the coefficient would decrease to zero as time increases $D \sim t^{-\frac{\theta}{d_w}}$. An

important consequence of this is that the hydraulic diffusivity η is not constant, but varies with position, $\eta \propto r^{-\theta}$.

The qualitatively different behavior exhibited by diffusion in fractals compared to Euclidean objects, should also be reflected in pressure transients in fractal networks of fractures. Two independent models to describe this behavior, both written in terms of partial differential equations were recently developed. Barker [12] derived a generalized diffusivity equation involving fractional dimension, which one can identify as the mass fractal dimension D , but which does not account for diffusion slowdown due to the fractal nature of the network. Following a different approach, Chang and Yortsos [29] independently proposed an alternative formalism based on the theory of fractals and following the work of O'Shaughnessy and Procaccia [89]. They showed that the response of a well producing at constant rate is given by the general expression

$$p(r, t) = \frac{r^{(2+\theta)(1-\delta)}}{\Gamma(\delta)(2+\theta)} \Gamma\left(1 - \delta, \frac{r^{2+\theta}}{(2+\theta)^2 t}\right) \quad (31)$$

where $p(r, t)$ is the pressure drop at distance r at time t , $\delta = \frac{D}{(2+\theta)}$ (also equal to $\frac{d_s}{2}$, where d_s is the spectral dimension [83]), $\Gamma(x)$ is the Gamma function and $\Gamma(x, y)$ is the incomplete Gamma function. For small values of the argument $\frac{r^{2+\theta}}{(2+\theta)^2 t}$, which occurs at the wellbore $r = r_w$ after a short time, we may use only the first two terms of the series expansion to obtain the approximation

$$p(r_w, t) = \frac{\Gamma(\delta - 1)r_w^{(2+\theta)(1-\delta)}}{\Gamma(\delta)(2+\theta)} + \frac{(2+\theta)^{1-2\delta}}{(1-\delta)\Gamma(\delta)} t^{1-\delta} \quad (32)$$

In a recent note, Chang and Yortsos [29] revisited their formalism and suggested that a Green's function approach based on an integro-differential rather than a differential formulation should be followed. The latter would be particularly useful for multiple wells, where the differential equation approach is of no use. Nonetheless, it was concluded that single-well pressures should obey the same scaling as found in [29], which therefore remains valid as a diagnostic tool of a fractal reservoir.

The behavior of equation (32) depends on whether $\delta < 1$ or $\delta > 1$. Since $\delta = \frac{D}{2+\theta}$ the two cases correspond respectively to $D < 2$ (flow behavior intermediate between linear and radial) and $D > 2$ (flow behavior intermediate between radial and spherical). The two types of behavior are shown graphically in Figure 22. When $\delta < 1$ (Figure 22a), the time-independent term in equation (32) becomes negligible compared to the time-dependent term after a short period of time. For practical purposes, therefore, the well pressure can be approximated by the expression $p_w(t) = Ct^{1-\delta}$, where C is a constant and where no wellbore skin was assumed. The logarithmic pressure derivative, defined as $\frac{dp}{d\log t}$, follows the scaling $C(1 - \delta)t^{1-\delta}$. From these two expressions, it follows that log-log plots of pressure and pressure derivative versus time will appear as two straight parallel lines of slope $1 - \delta$ separated by a distance equal to $\log\left(\frac{1}{1-\delta}\right)$.

When $\delta > 1$ (Figure 22b), the time-dependent term becomes negative and the well pressure behaves according to $p_w(t) = A - Bt^{1-\delta}$, where A and B are positive constants. Constant A is the asymptotic value of pressure at long time. The pressure derivative $\frac{dp}{d\log t} = B(1 - \delta)t^{1-\delta}$, however, is still a power-law function of time, and in a log-log plot will appear as a straight line with a negative slope equal to $1 - \delta$. To estimate the value of A from the pressure plot we may draw a straight line parallel to the pressure derivative curve and separated from below by the distance $\log\left(\frac{1}{\delta-1}\right)$. This line representing $A - p_w(t)$, intersects the pressure curve at the point $p_w(t) = \frac{A}{2}$, from which A can be estimated.

The preceding theory suggests that the fractal properties of a network of fractures could be inferred from the scaling of the well pressure. To test these theoretical predictions, and before any comparisons with actual well data are made, we proceeded with numerical simulations in the synthetic networks generated by IFS.

3.5 PRESSURE TRANSIENT SIMULATION IN SYNTHETIC NETWORKS OF FRACTURES

Fractal objects have meaningful fractal properties as long as their size spans several orders of magnitude. This requires a fine-scale discretization. Consequently, the traditional mass

Figure 22: Characteristic fractal pressure transient behavior for (a) $\delta < 1$, and (b) $\delta > 1$.

balance approach used in the simulation of fluid flow in discrete systems leads to large systems of equations that exhaust computer resources after a small number of generations. In our simulations two different approaches were taken depending on the size of the system and the importance of the early data. The first one is the traditional mass conservation approach, which imposes large computational requirements, but allows a better resolution of the early data. The second approach was based on the exact-enumeration technique used in the theory of random walks [16] and facilitates considerably the calculations, thus allowing for a large number of generations to be considered.

In both approaches the following assumptions were made: The fluid is single phase and slightly compressible, the flow rate in each fracture element is given by Darcy's law, all fractures have the same permeability, the system is isothermal and the matrix is impermeable and incompressible. Along each fracture, the usual expression applies:

$$Q = CW^\lambda \left[\frac{\Delta p}{l} \right] \quad (33)$$

where Q is the volumetric flow rate, W is the width and l is the length of the fracture, Δp is the pressure drop along the fracture, and λ is the conductivity exponent, usually taken equal to three.

For the direct numerical simulation the flow conductivity matrix must be constructed. To proceed, the address of each node must be specified. The two end nodes of the initial fracture, of known coordinates, were numbered nodes 1 and 2. Each new fracture subdivides a block and adds two new nodes to the system. The general expression for the address of the end nodes of a fracture will be α and $\alpha + 1$, respectively, where

$$\alpha = 2^{j+1} - 1 + 2 * (ad) \quad (34)$$

Here, j is the generation to which the given fracture belongs, and ad is the decimal value of the binary address. The coordinates of each such node are obtained by applying the sequence of transformations described to the coordinates of each node in the network. The final step is to calculate the conductivity of each fracture. A true fractal pattern of fractures contains

an infinite number of generations. As pointed out above, however, in practice only a small number of generations can be considered, since the size of the conductivity matrix doubles with every new generation. This limitation is very significant on the response of the fractal.

We considered the simulation of the pressure transient during drawdown. A single well is assumed, producing at constant rate, with no-flux boundary conditions imposed at the sides of the pattern. The flow between nodes was evaluated using (33), while an appropriately weighted volume was assigned to each node. Denoting by g_{ij} the conductance between neighboring nodes i and j , we obtain the following for the mass balance in node i

$$c_f \rho_i V_i \frac{\Delta p_i}{\Delta t} = \sum_j \frac{\rho_i g_{ij} (p_j - p_i)}{\mu \ell_{ij}} - Q \delta_{im} \quad (35)$$

where subscript j denotes all nodes connected to node i , c_f is the fluid compressibility and m is the node number of the well. We have also used the Kronecker delta, $\delta_{ij} = 1$ for $i \neq j$ and $\delta_{ij} = 0$ for $i = j$. For further simplicity, we take the approximation $V_i = \frac{1}{2} \sum_j A \ell_{ij}$, and $g_{ij} = A k$, where k is the fracture permeability. Defining dimensionless variables $p_D = \frac{2\pi(p_{in}-p)\rho_0 k A}{Q \ell \mu}$, $\ell_{D,i,j} = \frac{\ell_{ij}}{\ell}$ and $t_D = \frac{k t}{\ell^2 c_f \mu}$, where ℓ is the average size of fractures of the last generation and $\rho_D = \frac{\rho}{\rho_0}$, where ρ_0 is a reference density, we finally obtain

$$\frac{\rho_{D,i}}{2\Delta t_D} \sum_j \ell_{D,i,j} \cdot \Delta p_{D,i} = \sum_j \rho_{D,i,j} \left(\frac{p_{D,i} - p_{D,j}}{\ell_{D,i,j}} \right) + \delta_{im} \quad (36)$$

This set of equations was solved using a fully implicit algorithm which iterates on the density term at each time step.

Of crucial importance to the solution of the above is the size ratio $N = \frac{L}{\ell}$, where L is the physical size of the domain. N is also related to the total number of nodes (approximately proportional to $\frac{2}{Z} N$, where Z is the coordination number of the network). A theoretical fractal corresponds to $N \gg 1$ (obtained for infinitely many generations). However, practical considerations restrict N to a much smaller value ($O(100)$). It is important to discuss the effect of ℓ . When the number of generations of fractures in the network increases for a fixed L , this is equivalent to a decrease in ℓ and to an increase in N . In the dimensionless

notation above, the effect will be equivalent to an increase in the size of the system, or to a delay of the boundary effect. It follows that, working with as large a number of generations as possible is beneficial in two ways: It not only assigns stronger fractal characteristics to the network, it also serves to delay the effect of the boundary on the pressure transient. On the other hand, by decreasing ℓ , the real pressure and real time corresponding to fixed dimensionless values also decrease, the real time following an ℓ^2 dependence. It is possible that this may render more difficult the actual identification of the fractal structure because of the demand for an increased resolution of the diagnostic instruments, particularly when the fracture permeability is high.

The solution of (36) can be expressed solely in terms of N and t_d

$$p_{D,m} = f(t_D; N) \quad (37)$$

In the limit ($N \gg 1$), the above should reduce to the power law of Chang and Yortsos [29]

$$p_{D,0} \sim t_D^{1-\delta} \quad (38)$$

The advantage of the direct simulation method is that it allows a for finer discretization in areas of high pressure gradients, such as those close to the well site, as well as for smaller time steps at early times. The main disadvantage is that the iterative solution of the linear system may be prohibitively time consuming for problems of large size.

The second approach utilizes random walks, which can be used for the solution of diffusion problems [16], [118]. It is well know that in a random walk, the probability $p(r, t)dr$ of finding the walker at time t at a distance from the origin located between r and $r + dr$ is governed by the diffusion equation. Until recently, the procedure to calculate the probability density required a large number of random walks. Recently, the exact-enumeration approach [16] was proposed as an alternative for a large number of random walks at much less computational requirements. The principle of the method is as follows: The probability of a random walker to be at any site j at some time t is determined solely by the probability of being at the nearest neighbors of site j at time $t - 1$ and by the probability $W(i, j)$ of hopping from node

i to node j in a single step. If Z nodes are connected to node i and the conductivity of all the fractures is the same, $W(i, j)$ equals $\frac{1}{Z}$ for each of the Z nodes. A zero value is assigned to all other elements of the matrix W . If the vectors M_t and M_{t+1} of size n represent the probability of being at a node at times t and $t + 1$, respectively, then

$$M_{t+1}(i) = \sum_{j=1}^n M_t(j)W(i, j) \quad (39)$$

where the size of the network is $n \times n$. Since the random walk is a discrete approximation to continuous diffusion, convergence requires that sufficient steps are taken. To improve the resolution at early times, a smaller step size is required and a smaller discretization has to be performed. Also, because the network contains fractures of different length, longer fractures have to be discretized in segments of the same length. For problems that require good resolution at early times, a short simulation with small step size was performed prior to the longer simulation.

The exact-enumeration approach gives the pressure transient response to a point source acting at the well site at time $t = 0$. The solution to the constant flow rate problem is obtained by integrating in time. Indicative of the considerable advantages of this approach is that it is possible to obtain the response of a three-dimensional system with more than 25,000 nodes, a size too large to be solved by direct flow simulation. Yet another advantage is that it offers a direct approach to estimating the exponent θ , as shown below.

3.6 NUMERICAL RESULTS

We consider, first, the simulation of the pressure transient response of a Euclidean homogeneous network, using the direct simulation approach. For this network, both slopes should approach zero after an early transient. This is indeed the case as shown in Figure 23. The early response with slope of $\frac{1}{2}$ is present in all our simulations, indicating flow in a single (or a few) fractures adjacent to the well. It is interesting to note that it is the second curve (second derivative) that allows a clearer identification of the underlying structure (it approaches a constant value faster). For a fractal network, the theoretical fractal behavior is

well displayed in Figure 24, which shows the transient response of a well at the center of the modified Sierpinski gasket of approximately 1500 nodes. After an early transient, the two slopes remain constant for a significant interval of time, until boundary effects are felt (Figure 24b). The constant slope is a clear indication of an underlying fractal structure. We obtain $m = 0.26$, which gives $\delta = 1 - m = 0.74$. Because $D = 1.59$ (the fractal dimension of the regular Sierpinski gasket) the resulting value of θ is 0.16. This value can also be independently estimated with the exact-enumeration approach. Figure 25a shows a log-log plot of $\langle r^2 \rangle$ and its derivative $\frac{d(\langle r^2 \rangle)}{d \log(t)}$ versus time for the network of Figure 24a. The two slopes shown graphically in Figure 25b equal approximately 0.93 indicating $\theta = 0.15$ (which compares well with the value obtained from the pressure transient).

The fractal response is a consequence of the well-developed fractal structure around the well under consideration. True fractals comply with this requirement at any point in the network. In finite size systems, however, a radial fractal structure around the well is necessary for a fractal pressure response. Figure 26 was constructed such that there is a power law distribution of fractures around the well site with an exponent $D = 1.70$. The corresponding pressure response indicates a fractal with a value of δ equal to 0.77. Networks with different exponent in the fracture density distribution can be readily constructed and fractal-like pressure transient responses be obtained. Figure 27 shows the transient response of random fractal networks with a fractal dimension $D = 1.78$. Although the pressure response is very different from the homogeneous case, the test does not conform exactly to the theoretical expectations for a fractal due to finite size effects. Nonetheless the response tends to a constant slope, this behavior being consistent with the radial mass plot, resulting in $\delta = 0.86$ and $\theta = 0.07$. The low value of θ reflects a good connectivity in this network.

In general, the response of finite systems varies significantly with the arrangement of the matrix blocks (computationally obtained by taking different realizations). If the blocks are arranged so that the radial structure around the well possesses fractal characteristics, the response is also fractal. This is consistent with the theoretical results which assume radially fractal characteristics [29], and with the findings of [11]. It should be stressed

Figure 23: Pressure transient response for a homogeneous network. (a) Pressure (solid line) and pressure derivative (dashed line). (b) Corresponding slopes. (c) Radial fracture mass plot.

Figure 24: (a) Self-similar fractal network of fractures (modified Sierpinski gasket) and (b) Pressure transient response.

Figure 25: Exact enumeration approach. (a) Distance versus time ($\langle r^2 \rangle$ vs. t) (*) and its derivative (o). (b) Slope of $\langle r^2 \rangle$ vs. t (solid) and its derivative (dashed).

Figure 26: (a) Network constructed to be power-law distributed around the well and (b) Pressure transient response.

Figure 27: (a) Fractal network of fractures ($D = 1.78$) and (b) Pressure transient response for well located at the position indicated.

that the departure from the theoretical expectations is only a result of the limited number of generations allowed in the numerical computations. Finite size effects can also be introduced by a rearrangement of the well. In such simulations we observed that the early transient of $\frac{1}{2}$ slope is prolonged, while the period of constant fractal slope appears later. Departure from the theoretical response depends largely on how well a power law fits the radial mass relationship around the test well. For a perfect fractal, this relationship would be of the power law type with an exponent equal to D . This may not be the case for our finite networks, however, and accounts for the longer lasting period of the early transients.

As mentioned previously, the technique for constructing networks and simulating their transient response can be extended to three dimensions. Figure 28 shows a three-dimensional network with a fractal dimension $D = 2.8$ and its pressure response for a producing well in the center of the network. Because of the large number of nodes (26969) the simulation was made with the exact-enumeration technique. From the pressure derivative curve we obtain $\delta = 1.40$ for this network.

The numerical simulation of fluid flow in the numerical networks has confirmed the previous theory, provided that a significant number of generations is allowed, or, equivalently, that the object is fractal over a large enough range of scales. Otherwise, finite size effects and randomness may dominate the pressure response and make difficult the identification of the structure. This behavior serves to emphasize the importance of cutoff scales in fractals.

3.7 APPLICATION TO WELL TESTS

The preceding theory finds application to many published well test data in fractured systems [2]. Figure 29 shows a real set of buildup and drawdown curves from the same well in a fractured reservoir in Western Venezuela [2]. Horner time was used in the buildup test, thus the positive slope appears inverted. From the data we infer $\delta = 0.65$. Figure 30 shows two examples corresponding to $\delta < 1$ and $\delta > 1$, from wells in The Geysers geothermal field [2]. Fractal response is displayed in both cases. It should be pointed out that constant terms that add to the pressure response, such as skin factors or a faulty value of the flowing pressure,

Figure 28: (a) Three-dimensional network and (b) Pressure response.

can be filtered out by differentiation of the signal. Thus, the slope is more accurate in the pressure derivative curve. The separation between lines, especially at late times, may be helpful when the derivative curve is noisy.

A fractal dimension less than 2 in a three-dimensional network can be interpreted in several ways. For example, this could be the case of a network of fractures which is fractal in the plane but has a uniform distribution in the transverse direction as described by Beier [15]. In this case, the production is uniform along the producing interval. Another interpretation can be the equivalent of the line source solution for fractal systems. Consider a three-dimensional network of fractures with $D > 2$. The pressure response of a well that intersects that network will consist of the superposition of infinitely many fractal point sources of different strength distributed along the well. It is known [45], [83] that the sources themselves form a fractal of dimension $D - 2$. Each fractal point source responds according to equation (31). For a boundary condition of equal pressure for all point sources, it can be shown that the product (number) times (strength) of sources is power-law distributed with respect to distance, with an exponent equal to 1. Then, the overall pressure of the source line would behave as

$$p(t) \propto t^{1 - \frac{D-1}{2+\theta}} \quad (40)$$

Thus a flow dimension $D - 1$ can be observed in a well through a fractal network of fractures of dimension $D > 2$ if the production of the individual point sources is distributed in such a way that it compensates for the fractal spatial distribution of the sources.

Most fractured reservoirs tend to have a dimension less than 2 either because the productive thickness is small when compared to the areal extent or due to the presence of impermeable layers. Initially, however, the reservoir may behave with a dimension greater than 2, because such conditions may not have been felt. The result is a “V” shape transition in the pressure derivative curve. The semilog plot of this pressure response can be associated to the shape given by the double porosity model during the transition period. Contrary to the double porosity model, however, this dimensional transition does not correspond to

Figure 29: Pressure transient response for (a) Drawdown and (b) Buildup for a well in a fractured reservoir in Western Venezuela.

Figure 30: Pressure transient responses from wells in The Geysers geothermal field (a) $\delta < 1$ and (b) $\delta > 1$.

semilog straight lines at early or late times. Several examples of double porosity cases with no recognizable semilog straight line exist in the literature (see [37]). The fractal model with dimensional transition accounts for the lack of linear behavior and can be considered this type of pressure test.

3.8 CONCLUSIONS

A method to create fractal fracture networks with fractal characteristics was developed. The method is based on a combination of the IFS technique for constructing fractal images [13] and of a probability rule consistent with a fragmentation process. This technique allows for the synthetic generation of two-dimensional fractal networks with controlled fragment shape and size, upper and lower cutoffs, and fractal dimension. The method can be applied to two- and three-dimensional networks.

The simulation of pressure transients showed that the identification of a fractal network with the help of existing theoretical methods is possible. The theoretical expectations of fractal behavior [29] were confirmed numerically by the simulation of fluid flow in these fractal networks of fractures. Real pressure transient data in many fractured reservoirs were also found to display fractal behavior as predicted. It was shown that fractal analysis can be used to characterize such networks of fractures. At present, however, the existing theory does not allow to determine separately the two fractal parameters D and θ from a single well test. For further progress in this direction, additional work is necessary.

4 CAPILLARY EFFECTS IN DRAINAGE IN HETEROGENEOUS POROUS MEDIA: CONTINUUM MODELING, EXPERIMENTS AND PORE NETWORK SIMULATIONS

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4.1 INTRODUCTION

Displacement processes in porous media, such as oil reservoirs, soils and water aquifers, find important engineering applications in oil recovery and in subsurface remediation and cleanup. Of particular significance to such processes is the inherent heterogeneity of medium properties, key among which are permeability and porosity. While recognized in early studies, a systematic investigation of the effects of heterogeneity has been undertaken mostly during the last decade [45, 70]. In immiscible displacement, pore-scale studies led to the development of novel statistical tools to characterize displacement patterns in random media, such as Invasion Percolation (IP) and Diffusion-Limited-Aggregation (DLA) [72]. In miscible displacement, large-scale investigations on effects of randomness and correlation have led to classifications of flow regimes in terms of correlation length, degree of heterogeneity and mobility ratio [26]. For example, for a fixed correlation length, an increase in the heterogeneity index leads successively to a transition from a viscous fingering (VF) to a dispersive and to a channeling regime.

The preponderance of large-scale heterogeneity investigations on miscible displacement (for an exception, see [10]) reflects mainly the premise that on a large scale, differences between immiscible and miscible displacements are minimal. However, this hypothesis is not unconditionally nor uniformly valid, and depends on both geometric and flow properties. At sufficiently low capillary numbers (more precise estimate for which are developed later

in the text), for example, permeability gradients have effects similar to gravity, leading to gradient percolation patterns (both stable and unstable) [32]; sharp changes in permeability in the direction of flow significantly change steady-state saturation profiles [33]; random distribution of permeabilities attribute large-scale percolation features (such as large-scale trapping) to displacements [126]; while correlated fields drastically alter the percolation aspects of the displacement [107]. All these are capillary heterogeneity effects driven by permeability heterogeneity.

A simple, but convenient, first-step to explore capillary heterogeneity effects is by using the continuum equations that are presumed to model immiscible displacement. This was undertaken in recent studies, where we investigated the solutions of these equations in 1-D flows (where capillary heterogeneity effects are maximized) under both steady-state (concurrent or counter-current) and transient conditions [30, 31, 105]. In such displacements, capillary and permeability (and wettability) heterogeneity are interrelated via the Leverett relationship $P_c = \frac{\gamma}{\sqrt{k}} J(S)$, where γ is the interfacial tension and J a dimensionless function of the saturation S . It was found that during drainage in a medium where the permeability k suddenly changes, the response of the non-wetting saturation is a well-pronounced kink in the saturation profile, pointing downwards for a permeability increase and upwards in the opposite case. The size of the kink was shown to decrease with an increase in the capillary number, Ca . At steady-state flow, the continuum predictions were tested with experiments and found to be in excellent agreement [33]. Good experimental support was also found for transient drainage in experiments independently performed, and with no intention to test the theory, in eolian sandstones which involve spatial permeability oscillations [30, 31, 105].

The preliminary agreement between continuum theory and experiments is somewhat surprising. Capillary heterogeneity predominates at low Ca , where capillary effects lead to fractal percolation patterns, at least over the length scale determined by the viscous correlation length. In these regimes the continuum approach, and particularly the Leverett J -function, is least likely to apply. More generally, the validity of the continuum equations for drainage in heterogeneous media has not been investigated. To resolve these questions

and to provide a rigorous test of the existing theory we conducted a series of experiments involving a well-characterized heterogeneity and we developed alternative models based on a pore-network simulator. Experimental and pore-network results were then compared to the continuum theory. In addition, a pore-level analysis was employed to interpret experimental, pore-network model and continuum results.

Under the conditions tested, the experimental results were found to be in good agreement with the continuum equations. Pore network simulations also showed the qualitative trends expected from the continuum theory, although a quantitative agreement was not attempted due to the strong finite size effects and fluctuations in the pore network model. The paper is organized as follows: We first proceed with a brief review of the continuum theory and its implications. The experimental procedures and results are subsequently presented and compared to the theory. Then, a pore network simulator is developed and used to simulate these processes in various heterogeneous media. The final section focuses on interpretation of the various findings based on percolation theory and on attempts to unify the results obtained.

4.2 THEORY

We summarize briefly the results obtained from the continuum models. Consider the constant rate, immiscible, two-phase displacement (drainage) in an 1-D system with properties that vary in the direction of displacement. We assume that the non-wetting phase (“oil”) displaces the wetting phase (“water”) and that the usual Darcy’s law applies to yield the equation

$$\phi \frac{\partial S_o}{\partial T} + q \frac{\partial}{\partial X} \left[f_o(S_o) - \frac{(\rho_w - \rho_o)}{\mu_w q} g_x k_{rw}(S_o) f_o(S_o) - \frac{k k_{rw}(S_o)}{\mu_w q} f_o(S_o) \frac{\partial P_c}{\partial X} \right] = 0 \quad (41)$$

where standard notation has been employed [30, 31, 105], gravity effects were included and the permeability may also be variable in space. Capillary heterogeneity is contained in the capillary term P_c and can be a result of spatial variations in wettability, pore space configuration, or pore size. All these variations are understood in a macroscopic sense, so

that the continuum formulation is relevant. Although wettability heterogeneity may directly be included, here we concentrate on effects associated with pore size alone or equivalently on effects of permeability heterogeneity.

The connection between permeability and capillarity is well known. It is implicitly contained in the Laplace-Young equation ($P_c = 2\gamma\cos\theta/r$), a particularly useful form of which is the Leverett relationship

$$P_c = \frac{\gamma\cos\theta}{\sqrt{k/\phi}} J(S_w) \quad (42)$$

where θ is the contact angle, γ the interfacial tension and r a characteristic pore size. A derivation of (42) using percolation theory on random pore networks in the absence of gradients (such as induced by gravity or viscous effects) can be obtained, e.g. by following approaches such as that of Heiba et al. [54]. Additional support of the relation between permeability and capillarity can be found in [63]. Although pore space configuration and geometry may exert additional effects, they are secondary so that dimensionless quantities, such as porosity, or the functions of relative permeabilities k_{ro}, k_{rw} and capillary pressure J may be taken independent of the permeability change. We must stress, however, that the ensuing theory remains qualitatively the same even if a weak permeability dependence is ascribed to these functions. Taking a reference characteristic value k^* , the dimensionless function $\tau = \sqrt{k/k^*}$, which roughly follows the variation of an average pore size, is the relevant dimensionless variable that measures capillary heterogeneity in our problem.

In the usual dimensionless notation, $x = /L$, $t = \frac{q}{\phi L(1-S_{wr})}T$, where L is a macroscopic characteristic length (e.g. core length), denoting $u = \frac{S_o}{1-S_{wr}}$, where $M = \frac{\mu_w}{\mu_o}$ is the viscosity ratio, $D(u) = k_{rw}f_o$, $G(u) = k_{rw}Jf_o$, $H(u) = k_{rw}f_oJ'$, $\varepsilon = \frac{\gamma\cos\theta\sqrt{k^*\phi}}{L\mu_wq}$, $\varepsilon_g = \frac{k^*(\rho_w-\rho_o)g\bar{x}}{\mu_wq}$ and $\tau'(x) = d\tau(x)/dx$, equation (41) is rewritten as follows:

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x} \left[F(u) - \varepsilon_g D(u) \tau^2(x) + \varepsilon G(u) \tau'(x) \right] = \varepsilon \frac{\partial}{\partial x} \left[\tau(x) H(u) \frac{\partial u}{\partial x} \right] \quad (43)$$

In the above, important variables are the inverse macroscopic capillary number ε , the gravity number ε_g , and the permeability heterogeneity Bond number $\tau'(x)$, the latter obtained from

the heterogeneity function $\tau(x)$.

While permeability heterogeneity enters in both sides of (43), its main effect to the saturation profile is through the term $\epsilon G(u)\tau'(x)$ on the left hand side. This novel, position-dependent addition to the fractional flow function F carries the main thrust of heterogeneity in the present context. The modification of the fractional flow is akin to the effect of gravity in homogeneous systems (compare with $\epsilon D(u)$ in (43)), with which it mathematically coincides when τ is a linear function of position. This similarity was exploited in [32], where a modification of gradient percolation was proposed to describe slow drainage. The combination $\epsilon\tau'(x)$ is the ratio of two length scales, the mean-field viscous correlation length, $l_c \sim \sqrt{k}Ca^{-1}$, over the permeability heterogeneity length, $l_k \sim \sqrt{k}/\left(\frac{d\sqrt{k}}{dx}\right)$. Capillary heterogeneity effects are expected to be important as long as this ratio is not small. It is also worth noting that the role of capillarity (ϵ) in the present context is a subtle one. Theoretically, as long as sharp, step-like discontinuities exist in permeability (τ), the smallest value of ϵ should be sufficient to trigger a response. However, in our simulations, and certainly in natural rocks, the permeability has a bounded variation, thus the response amplitude would be expected to diminish as ϵ decreases.

Before we proceed with the solution of the initial-boundary value problem (43) we shall briefly discuss the steady-state solution for a linear change in τ , which is physically instructive. Consider the heterogeneity model (Figure 31a)

$$\tau = \begin{cases} \tau_- & x < x_- \\ a(x - x_-) + \tau_- & x_- < x < x_+ \\ \tau_+ & x > x_+ \end{cases} \quad (44)$$

where $a = (\tau_+ - \tau_-)/\Delta x$, $\Delta x = x_+ - x_-$, and subscripts $+$ and $-$ refer to downstream and upstream regions, respectively. Then, the steady state version of (43) is

$$F(u) + \lambda G(u) - \epsilon\tau(x)H(u)\frac{du}{dx} = F_i \quad (45)$$

where we defined $\lambda = a\epsilon$, the gravity term has been omitted for simplicity and F_i is the flow

fraction of the non-wetting phase in the vicinity of the permeability jump (where F_i could be different from its inlet value). Equation (45) was solved in a self-consistent manner in [33] and [30, 31, 105]. It was found that for decreasing permeability ($a < 0$) the saturation builds up from its upstream value u_i ($F(u_i) = F_i$) in the high permeability region, $x < x_-$, to a maximum value u_- (at $x = x_-$), before it declines continuously inside the region of permeability variation ($x_- < x < x_+$) to its asymptotic value u_i (Figure 31b). For $x \leq x_-$ the saturation remains constant at u_i until another heterogeneous region is encountered. For increasing permeability ($a > 0$), a similar analysis leads to a “mirror image” of the previous (rightmost part of Figure 31b). (Yortsos and Chang, 1990). The physical interpretation is that for the non-wetting fluid to flow at steady conditions in a progressively lower permeability medium its saturation must decrease in accordance with capillary continuity. Since, however, in the continuum theory both asymptotic saturations must be equal, this can only be accomplished if the non-wetting fluid builds up a high enough saturation in the high permeability part. One should note the difference with the picture dictated from the static capillary continuity criterion, which requires a step-like change in saturation across the jump from high permeability to low permeability. Both the amplitude $\Delta u = |u_- - u_i|$ of the saturation peak and its width in the $x < x_-$ zone are increasing functions of λ , which can be rearranged to read

$$\lambda = \frac{\gamma\sqrt{\phi}}{\mu_o q} \left(\frac{\sqrt{k_+} - \sqrt{k_-}}{2\Delta X} \right) \quad (46)$$

Similar results are also obtained when the injected fluid is wetting but only under conditions of primary imbibition, which involves meniscus displacement. Secondary imbibition proceeds by the additional mechanisms of film flow and may not be amenable to the simple continuum description above. Finally, we should note that for the case of drainage, where $F_i = 1$, Δu should vanish at a decreasing permeability jump, contrary to an increasing jump where Δu should attain a maximum value. The steady-state predictions were tested experimentally in [33] for the case of drainage and found to be in good agreement.

To test the continuum theory under the general unsteady-state conditions, we performed

Figure 31: Schematic saturation response to heterogeneity in permeability.

displacement experiments on model porous media, which consisted of slices of packed glass beads of different sizes, thus different permeabilities, and we simulated the process using pore network simulation. In this paper we shall focus on drainage processes only. Due to hyperdiffusion phenomena [6, 7] secondary imbibition displays qualitatively different features and must be analyzed separately.

4.3 EXPERIMENTS

An acoustic technique [5] was used to determine the saturation profile inside the porous medium. Saturation measurements were derived from the velocity variations of a sound wave along the sample (of length $L = 30\text{ cm}$ and cross-sectional area $A = 4 \times 2\text{ cm}^2$). From the calibration curve [8] and the accuracy of the relative velocity measurements (10^{-4}), the overall resolution in saturation is estimated at 1%. The acoustic device has been improved so that we can obtain saturation profiles along the main core direction at different times t with a spatial resolution of 2 mm (150 measurements along the length of the sample) in order to track precisely the saturation variations.

The porous media consists of packs of glass beads of different diameter d . The sample was patiently filled with slices of various diameters of glass beads, the close packing ensured by shaking after each filling. We describe experiments on six-slice samples of alternating permeability (large/small) $k_l/k_s/k_l/k_s/k_l/k_s$, each slice of 5 cm thickness. To impart the desired heterogeneity, two bead sizes of diameters $d_s = 50\text{ }\mu\text{m}$ and $d_l = 180\text{ }\mu\text{m}$, respectively, were used. Two other diameter pairs ($80, 280\text{ }\mu\text{m}$) and ($50, 90\text{ }\mu\text{m}$) were also used. The measured permeability was found in agreement with the Kozeny-Carman value $k = \frac{\phi^3 d^2}{180(1-\phi)^2}$ for the porosity value $\phi \simeq 0.4$. The permeability map of the sample (corresponding to simple jumps) is sketched in Figure 32. The fluids used were water as a wetting fluid ($\mu_w = 1 \times 10^{-3}\text{ Pa.s}$, $\rho_w = 10^3\text{ kgm}^{-3}$) and nonane as the non-wetting fluid ($\mu_o = 0.7 \times 10^{-3}\text{ Pa.s}$, $\rho_o = 780\text{ kgm}^{-3}$), with interfacial tension $\gamma = 25\text{ mJm}^{-2}$ and $\cos\theta = 1$. Experiments corresponded to primary drainage processes. The sample was initially saturated

with water and was subject to injection of the lighter fluid (oil) in the direction from top to bottom. The flow rate q varied from 0.3 cm/h to 150 cm/h corresponding to a wide variation in the capillary number values, from 3×10^{-8} to 1.5×10^{-4} . We investigated six different flow rates ($0.3, 6, 15, 30, 60$ and 150 cm/h) corresponding to the following ϵ values (700, 35, 14, 7, 3.5, and 1.4). The data are shown as oil saturation versus reduced sample length at different times, expressed in terms of the pore volume fraction t .

Figure 33 shows two such profiles for various values of time. First, we shall comment on the qualitative features of the experimental results. As the displacement progresses, it is affected substantially by the heterogeneities encountered. The profiles rather rapidly assume a steady-state to which they become locked, suggesting that the steady-state interpretation should be of relevance. The experiments show that a large amount of the wetting-phase remains trapped behind regions of permeability increase. Although the degree of trapping and the profiles themselves are affected by the capillary number, several features are common at steady-state: Sharp saturation jumps (within the experimental resolution of 2 mm) occur at the positions of permeability changes (corresponding to $\xi = 0.17, 0.55$ and 0.88 for a permeability decrease and to $\xi = 0.35, 0.72$ and 1.00 for a permeability increase, respectively, in the schematic of Figure 32). In the regions of high permeability the saturation profile becomes approximately flat and assumes a high value, without displaying any remarkable features. By contrast, significant dips in the profile occur in the low permeability regions, with the saturation decreasing continuously before the region of sharp permeability increase is encountered. The size of the dip ΔS varies with the flow rate, decreasing with an increase in the latter. Heterogeneity effects are mitigated as the flow rate increases, although a weak imprint was still left at the highest numbers tested.

The features are in qualitative agreement with the previous steady-state theory, which predicts sharp changes in saturation when regions of permeability changes are traversed, a significant response of the saturation profile as a permeability change region is approached (with a downwards kink in the oil saturation before a permeability increase, such as at $\xi = 0.35$ and 0.72), an insignificant response in the high permeability region for the pure

Figure 32: Permeability map of experimental set up.

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Figure 33: Experimental saturation profile at various times (a) $\varepsilon = 35.3$, (b) $\varepsilon = 3.53$.

drainage case ($F_i = 1$) and the dependence of all these effects on the capillary number. For a quantitative comparison, however a numerical simulation is needed. We first simulated the steady-state results by solving (43). A particularly useful measure of the saturation response is the variation ΔS across a region of permeability increase. Plotted in Figure 34 are theoretical and experimental results for ΔS as a function of λ . Because the latter contains the unknown parameter ΔX , which is difficult to estimate a priori, this parameter is adjustable. All other functions were evaluated using the simple expressions of [30, 31, 105]. The theoretical predictions agree reasonably well with the experiments at all flow rates for a width Δx of the permeability jump region equal to 0.015 (corresponding to $\Delta X = 4 \text{ mm}$). This estimate suggests that during the packing large and small sizes interpenetrated in a small region extending over a few millimeters. When the other bead size pairs were used (50, 90 μm and 180, 280 μm) the value of λ obtained was the same for constant ε , implying that the slope of the permeability jump ($a \sim (\sqrt{k_l} - \sqrt{k_s}) / \Delta X$) can be considered as constant. Since $\sqrt{k} \sim d$, ΔX must be proportional to the bead size ($\Delta X \sim d$) in order for this to be valid, in which case we can postulate $a \sim (d_+ - d_-) / (d_+ + d_-)$. This assumption was used in all our subsequent calculations. Having determined ΔX we proceeded to simulate the transient profiles. Figures 35 shows numerical results corresponding to the experiments of Figure 33. Despite the simplicity of the model, in particular as it regards the relative permeability and capillary pressure expressions, which were not independently evaluated, quite a reasonable agreement between simulation and experiment is found both for steady-state and transient profiles. At this point it should be remarked that as long as the relative permeability and capillary pressure functions were approximated by expressions qualitatively consistent with percolation trends the sensitivity to the particular shape of the functions was weak. The simulations reproduced all large-scale features of the experiments at all capillary numbers tested. Certain small scale features were not successfully reproduced, either because of device artifacts due to local porosity or because of wettability variations, which also affect capillarity. However, such fine-tuning can be performed, if desired. In particular, it should be possible to relate saturation to permeability fluctuations, and thus

to extract statistical characteristics of the permeability field, particularly as it regards fractal statistics and correlations. This possibility is currently under investigation.

4.4 PORE NETWORK SIMULATIONS

The good agreement between experiments and the simple continuum model is somewhat unexpected. Capillary heterogeneity effects are significant at low values of the capillary number (large values of λ). During transient displacement, the advancing front is dominated by capillary effects leading to fractal structures, where continuum models are least likely to apply. Since the ultimate saturation distribution is affected by the transient process, it is interesting to inquire whether the continuum predictions can also be supported by a pore-network level analysis, where the advance of the interfaces is followed from pore to pore. To accomplish this task, a drainage pore network simulator was developed.

The basic structure of the simulator parallels that of Blunt et al. [19]] and it is based on the following premises and rules. Interfaces between the two phases reside only in pore bodies (sites), which can be partially or fully occupied. In the former case the capillary pressure across the interface is set to zero. After it fully occupies a site, the interface can move to an adjacent site provided that the capillary pressure threshold of the connecting pore throat (bond) is exceeded. If many sites on the front are partially occupied, the one to be filled next is that with the minimum time required for pore filling. Pressure drops during fluid flow are only associated with pore throats, where a Poiseuille-type law is used to relate pressure drop to flow rates. An *SOR* technique was used to calculate pressure fields in the two fluids. The main difference between the present model and that of [19] is that the simulator solved displacement at constant flow rate, rather than constant pressure drop, a process which required additional iteration of the flow field. Often, and particularly at low capillary number values, the interfaces of the front would lock at all places and convergence could not be reached. In such cases, we followed Invasion Percolation rules and advanced the interface neighboring the largest pore. Simulations were carried in 2-D square lattices of size 100×21 and 100×60 , for capillary number values ranging between 1.5×10^{-6} and

Figure 34: Saturation jump vs. ϵ .

Figure 35: Numerical simulations for the conditions of Figure 33.

1.5×10^{-3} , three different values of the viscosity ratio ($M = 0.1, 1$ and 10) and for various forms of heterogeneity. Because of the connection between permeability and average pore size, the simulation of permeability heterogeneity can be accomplished by a variation of the pore size. The latter included a sudden increase or decrease in the average pore size in the direction of displacement by a factor of 2 or 4, a smooth variation from one zone to another, as well as a sinusoidal variation. Pore sizes were randomly and uniformly distributed around their mean value. Typical schematics of the lattices simulated are shown in Figure 36, where the pore sizes are exaggerated. It should be pointed out that because of the small size of the network (100×21), strong finite size effects in the form of strong fluctuations in the saturation profile exist in our simulations. These effects persist for the case of a homogeneous sample as well, where the average pore size is constant, but tend to diminish as the size or the capillary number increase.

We shall concentrate mostly on simulations with equal viscosities ($M = 1$). Simulations results for a pore size ramp increase of five lattice spaces are shown in Figures 37 and 38 for two different values of the capillary number. A response similar to that predicted by the continuum model and observed in the experiments, namely the reduction of the non-wetting phase saturation as the permeability increase region is approached, followed by a sharp increase, is evident. The downwards kink diminishes as the capillary number increases, although not as smoothly as in the experiments, mostly we suspect as a result of finite size effects. Also, because of the overlap with the boundary end effect at the outlet of the lattice, the saturation level does not rise, following the jump, to the high level observed in either experiments or continuum simulations. A better agreement was found in the simulations involving a larger lattice, however. In all cases, the response diminished substantially as the capillary number increased to levels higher than 10^{-3} . To illustrate the effect of heterogeneity (λ) on the profile, we simulated a heterogeneous system where the ramp extent is 20 lattice spacings (Figure 39). It is apparent that the effect of heterogeneity here is all but lost. Certain differences with the “homogeneous” case (Figure 40) do exist, however, as a result of the differences in viscous effects because of the different permeabilities

in the two regions. These viscous effects exist in our previous simulations as well, and can partly explain the not so satisfactory match of the saturation response with the continuum theory or the experiments downstream of the heterogeneity. We expect that this difference would diminish and eventually disappear as the lattice size is increased.

Simulations involving a permeability decrease were also conducted. Figure 41 shows results obtained for a decreasing ramp of 5 lattice spacings. The response is qualitatively similar to the theoretically expected. The non-wetting saturation rises as the permeability decline region is approached and subsequently decreases substantially as the low permeability region is entered. The upwards kink in the saturation profile predicted from the theory is thus observed. As in the previous case, the effect of increasing capillary numbers is to smooth out the heterogeneity. The combined effects of alternating regions of increasing and decreasing permeability can be quite dramatic. Figure 42 shows pore network simulations with a sinusoidal variation in the average pore size of a wavelength equal to $1/5$ of the system length. According to the continuum theory [30, 31, 105], the saturation response should also oscillate with a slight phase lag depending on the capillary number. Both predictions are clearly observed in the pore network simulations. We must point out that similar effects were recently observed in a related simulation on multifractal lattices [74].

We conclude by presenting two illustrative examples of the effect of the viscosity ratio. Figure 43 shows saturation profiles corresponding to a sudden permeability increase in the middle of the sample at a favorable viscosity ratio, $M = 0.1$. Because viscous effects are here favorable, the homogeneous displacement is compact in all regions, leading to a high efficiency and high saturation values. As the region of permeability increase is encountered, the corresponding saturation kink is very small and the capillary effect is insignificant. In contrast, for unfavorable viscosity ratio values ($M = 10$), the displacement is much less efficient and heterogeneity effects are quite noticeable in the saturation profile (Figure 44). Both effects of M were anticipated in the continuum study of Chang and Yortsos [30, 31, 105].

To interpret the pore network simulation results, it is necessary to consider the application of percolation theory. Let us first discuss invasion percolation (IP), namely displacement at

Figure 36: Typical lattices used in pore network simulations.

Figure 37: Saturation profile and displacement patterns for permeability increase (1:2) and for $Ca = 1.5 \times 10^{-5}$.

Figure 38: Saturation profile and displacement patterns for permeability increase (1:2) and for $Ca = 1.5 \times 10^{-4}$.

Figure 39: Saturation profile and displacement patterns for permeability increase (1:2) in a ramp of 20 lattice spacings for $Ca = 1.5 \times 10^{-4}$.

Figure 40: Saturation profile and displacement patterns for uniform permeability and $Ca = 1.5 \times 10^{-4}$.

Figure 41: Saturation profile and displacement patterns for permeability de
 $Ca = 1.5 \times 10^{-5}$.

Figure 42: Saturation profile and displacement patterns for sin
 1.5×10^{-4} .

Figure 43: Saturation profile and displacement patterns for the conditions of Figure 38 and $M = 0.1$.

Figure 44: Saturation profile and displacement patterns for the condition of Figure 38 and $M = 10$.

constant rate at low Ca , in a medium involving a sudden decrease in permeability at the boundary of two regions 1 and 2. According to the IP rules, the front advances by occupying the largest pore available to it. Before the front reaches the boundary, the pattern is given by percolation corrected by finite size effects (Figure 45a). In the upstream region 1 of high permeability, pores of size greater than r_d can be penetrated, where r_d is another measure of the capillary pressure level, $P_c = \frac{2\gamma}{r_d}$. The corresponding percolation probability p_1 is related to the pore distribution in this region, $\alpha_1(r)$, by

$$p_1 = \int_{r_d}^{\infty} \alpha_1(r) dr \quad (47)$$

Given the width of the lattice W , p_1 can be obtained from finite size scaling which involves the percolation threshold of the lattice p_c

$$|p_1 - p_c|^{-\nu} \sim W \quad (48)$$

The extent of the fractal regime is quenched by the width of the lattice, which sets the characteristic length of the system. The corresponding saturation in region 1 satisfies the percolation scaling

$$S_1 \sim (p_1 - p_c)^\beta \quad (49)$$

Upon encountering the boundary between the two regions, the front would penetrate only a small part of the low permeability region 2 (Figure 45b). The extent is determined from the corresponding percolation probability

$$p_2 = \int_{r_d}^{\infty} \alpha_2(r) dr \quad (50)$$

which is related to p_1 via (47). Clearly $p_2 < p_1$ for all r_d . The extent of frontal advance in region 2 satisfies the scaling law

$$l_2 \sim |p_2 - p_c|^\nu \quad (51)$$

As long as $l_2 < W$, penetration of region 2 is limited to a short tail. As the displacement proceeds, the probability p_1 and the saturation of region 1 steadily increase with only small changes in S_2 and l_2 (Figure 45c-45d). Percolation of region 2 is reached (Figure 45e) when $l_2 \sim W$, which occurs at the critical value p_2^* , where

$$|p_2^* - p_c|^{-\nu} \sim W \quad (52)$$

Upon percolation, subsequent increase in the saturation of either region requires that either another region of lower permeability is encountered (for example if the outlet end is a semi-permeable membrane) or that viscous effects are significant to provide the necessary pressure increase (see also below). The relationship between the saturation values S_1 and S_2 at any time requires knowledge of the size distribution functions α_1 and α_2

$$\frac{S_2}{S_1} = \left(\frac{p_2 - p_c}{p_1 - p_c} \right)^\beta \quad (53)$$

Because the capillary pressure is also related to r_d , the above is another representation of the Leverett J-function. It is apparent that in this case the Leverett J-function formalism remains valid and that the continuum approach should be applicable as long as W is large enough. Experimental confirmation of this sequence is shown in Figure 46, where the non-wetting (oil) saturation profile sequence is plotted for difference values of time during drainage in a medium involving a step change of bead size from 280 μm to 90 μm . In this experiment, the capillary number is 3×10^{-7} , which for a core length of 30 cm is sufficiently small for percolation theory to be applicable. The events described above can also be interpreted in terms of (stable) gradient percolation but at an infinitely large Bond number, where front widths (or tails) are of infinitesimal extent.

Consider next IP in a medium involving a sharp increase in permeability (Figure 47). The percolation of region 2 begins immediately when the front contacts the heterogeneity, at which time p_1 and S_1 are still given by (47) and (49). However, S_2 will not follow (53). Instead, as long as there is no subsequent region of lower permeability downstream or viscous pressure drops are not accounted for, the probability percolation in region 2 remains fixed at

Figure 45: Invasion percolation with an abrupt decrease (2:1) in permeability at various stages.

Figure 46: Experimental saturation profiles at various times corresponding to a sudden jump in permeability at $Ca = 3 \times 10^{-7}$.

the value p_2^* given by (52), implying that the two saturations S_1 and S_2 are momentarily the same. Capillary pressure continuity can only be enforced in region 1 by a re-adjustment and partial withdrawal of its menisci towards pore bodies, to bring about a drop in the capillary pressure and a possible (although small) drop in the saturation S_1 . During this transient, which however depends strongly on W , we expect the Leverett J-function formalism to fail. It is only after contact with a lower permeability region or if some viscous pressure drops develop, and only after the capillary pressure has increased to the original level to satisfy (48), that further penetration of region 1 takes place. This problem is somewhat analogous to capillary fingering in unstable gradient percolation, where percolation fractal regimes exist throughout the process. Figure 47 shows the sequence of events in this case. Experimental confirmation of this displacement process can be found in [33].

The preceding suggests the possibility of hysteresis in the capillary pressure-saturation diagram, depending on the direction of displacement, namely whether it is from high-to-low permeability or inversely, and the possibility that the Leverett formalism is only conditionally valid. Fortunately, this would be the case only if finite size effects are strong (small values of W). Indeed, while the displacement pattern follows the schematics of Figures 45 and 47, the transients where continuum approaches would fail correspond to fractal regimes where saturation values are expected to be negligibly small for large W . For such cases, the saturation in the low permeability region 1 would remain infinitesimally small before its capillary pressure threshold is reached, and would increase thereafter following the typical continuum approach. We conclude that hysteresis, if it exists, is infinitesimal for large enough widths W , thus the Leverett J-function formalism applies locally in each permeability region regardless of the position of the permeability block or the direction of displacement (as also assumed in our recent study of large scale percolation of drainage [126]).

The previous analysis provided a description of the displacement under static conditions. Consider, next, displacement in the presence of viscous effects, which introduce a viscous correlation length ξ_v . In a displacement in a homogeneous region, ξ_v measures the extent upstream of the front where fractal concepts apply, and beyond which continuum descriptions

Figure 17: Invasion percolation with an abrupt increase of P_c in permeability and at various stages.

are applicable. Two different expressions for the dependence of ξ_ν on Ca were developed by Wilkinson [119], and by Blunt et al. [18], independent of the viscosity ratio M . In a separate publication we shall consider the dependence of ξ_ν on Ca and M following Lenormand's work [71], where it will be shown that the two different cases $M \ll 1$ and $M = O(1)$ or higher need to be considered. We find $\left(\frac{\xi_\nu}{r^*}\right) \sim Ca^{-\frac{\nu}{1+t+\nu}}$ for $M \ll 1$, and $\left(\frac{\xi_\nu}{r^*}\right) \sim Ca^{-\frac{\nu}{1+\nu}}$ for $M = O(1)$ or higher, where t and ν are conductivity and correlation length percolation exponents and r^* the average pore size. These expressions differ from those in either [119] or [18].

Consider, first, the problem involving a sudden permeability decrease. Here, an analysis similar to the previous applies, with the important exception that the variable r_d is now space-dependent. For convenience, we shall use the boundary position as the origin of the space coordinate, which increases in the direction of displacement. Clearly, r_d is an increasing function of x . When the front reaches the boundary, the extent of the frontal advance into region 2 is still given by (51), where p_2 is evaluated at $r_d(0)$. As previously, penetration would be insignificant, until condition (52) is reached. Before this, the displacement is confined in the region 1 of high permeability, and the heterogeneity boundary behaves as a no-flow boundary for the non-wetting phase. Because of the insignificant frontal advance, the only possibility for further displacement is by lateral advancement of the menisci in region 1. The result is that the non-wetting pressure becomes almost uniform at least in the (percolation) region near the boundary

$$p_o(-\xi_\nu, t) \sim p_o(0, t) \quad (54)$$

while, at the same time, the wetting phase continues to flow

$$p_w(-\xi_\nu, t) - p_w(0, t) \sim q\xi_\nu\mu_w \quad (55)$$

A capillary pressure gradient rapidly develops

$$P_c(0, t) - P_c(-\xi_\nu, t) \sim q\xi_\nu\mu_w \quad (56)$$

The existence of a capillary pressure gradient in the direction opposite to the viscous pressure gradient and flow causes the non-wetting saturation to increase preferentially in the region closest to the heterogeneity boundary, thus the initially concave saturation profile progressively changes curvature and approaches the shape of an upwards kink. This process continues until percolation of region 2 is reached, namely when condition (52) is satisfied. Because the capillary pressure gradient is in a direction opposite to viscous pressure drop, the regime ceases to be fractal the moment equation (56) becomes in effect, which roughly corresponds to the time the front reaches the boundary for the first time. It follows that soon after the boundary is encountered the macroscopic equations can reasonably well predict the subsequent saturation evolution. Thus, the extent of the region of the saturation response to the heterogeneity would scale with the mean field result Ca^{-1} rather than with ξ_v . The effect of M can be similarly interpreted. For favorable ratio, $M \gg 1$, the saturation pattern following the fractal regime is much more compact resembling a piston-like displacement, thus any saturation response to the heterogeneity has no room to develop and the profile acquires, no discernible features. By contrast, when $M \ll 1$, the pattern is much sparser, thus the saturation response is strong and lasts for a considerable time.

The situation is qualitatively different in the case of a sudden decrease in permeability. Upon encounter of the heterogeneity boundary, penetration of region 2 commences immediately, much like in the case where viscous effects are absent. However, because of the viscous pressure drops, saturation profiles are now concave and not flat. As a result, the upstream saturation in region 2 (and particularly the region on the downstream side of the heterogeneity boundary) begins to rise following the rules of a regular displacement. Until such time that the capillary pressure at the boundary, $P_c(0, t)$, has reached the percolation level for region 1 (equation (48) with p_1 evaluated at $r_d(0)$), no substantial change in the saturation in region 1 takes place. The familiar downwards kink of the saturation response thus emerges. During the period of a frozen saturation in region 1, a re-adjustment of the menisci may be necessary to balance the lower capillary pressure experienced during the invasion of the high permeability region 2. Neither the continuum nor the pore network model capture this fea-

ture. However, made by both models by excluding the menisci movement are not significant. In particular, the two models do predict the downwards kink in the saturation profile. Upon the subsequent onset of penetration of region 1 at the point near the boundary, the fractal regime ceases to exist and a continuum description should be a reasonably good alternative. As before, the spatial extent of the dip in the profile should scale inversely proportional to Ca . If the permeability contrast is so high that the percolation threshold for region 1 cannot be reached even when region 2 becomes fully occupied, the spatial extent of the dip should also scale as Ca^{-1} , provided that region 1 is long enough to accomodate it. Steady-state profiles at variable rates across a discontinuity were used in the experimental study of [33]. The effect of M is similar to the previous case.

4.5 CONCLUSIONS

In this section, experiments and pore network simulations were used to study primary drainage in a heterogeneous porous medium involving changes in the permeability in the direction of displacement and to test the validity of macroscopic, continuum models. Both experiments and pore level model confirmed the validity of the macroscopic predictions, at least as far as the basic features of the saturation response is concerned. An analysis of the results based on percolation arguments provided additional support to the adequacy of continuum models in describing the response of the saturation to the heterogeneity. It should be pointed out that these results apply only for drainage and not for imbibition, which is qualitatively different due to the different flow mechanisms. In particular, for secondary imbibition, the standard continuum models do not reproduce well preliminary experiments suggesting that improved continuum models are needed. This subject is currently under study. The sensitivity of the saturation to heterogeneity at low displacement rates suggests the possibility of heterogeneity identification from saturation maps. This idea, originally suggested in [30, 31, 105], is also currently under investigation.

5 VISUALIZATION AND SIMULATION OF IMMISCIBLE DISPLACEMENT IN FRACTURED SYSTEMS USING MICROMODELS: I. DRAINAGE

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5.1 INTRODUCTION

Naturally fractured systems represent a significant fraction of oil and gas-producing reservoirs worldwide. Although the internal reservoir structure varies greatly among them, the basic element commonly assumed for each consists of a porous block called the matrix, which is the storage of the natural resource, and of a connected network of fractures, which act to transport injected or produced fluids [3]. Various models on the geometrical properties of the matrix and the connectivity and transport of the fracture network have been proposed. They vary from structures orderly and periodic to random and fractal [29], [81], [98], [117]. The internal structure is of fundamental importance to two processes, the transport of fluids in the fracture network and the matrix-fracture interaction. The characteristics of the fracture network, particularly its correlation and connectedness, affect single-phase transport, requiring non-trivial characterization and modeling methods. Fracture connectivity is paramount to tracer dispersion and pressure transients in single-phase flow [22], [41]. Matrix-fracture interaction affects the distribution of immiscible fluids, which determines to a large degree the effectiveness of fluid displacement from the matrix. At this level, it is the matrix block geometry and pore and fracture size distributions that dictate the displacement. It is the latter problem that we are concerned within this section.

Immiscible displacement in heterogeneous, but uncorrelated, porous media proceeds by different mechanisms, depending on the nature and history of the displacement (drainage or imbibition, primary or secondary process). These mechanisms have been clearly elucidated

by various authors, notably by Lenormand who conducted careful experimental studies in micromodel geometries [72]. Local displacement mechanics are not expected to change when a fracture-matrix system is considered. Thus, primary displacement will occur by a frontal advance of the menisci, the conditions for which depend on whether the process is drainage or imbibition, the latter including possible displacement along the surface roughness. Secondary imbibition may also involve film flow, film thickening and snap-off, resulting in displacements different than frontal. A fracture-matrix system displays additional large scale features, due to the existence of fractures, which affect greatly the large scale efficiency. In particular, the presence of a fracture adds an element of large-scale correlation to the system. As a result, rate effects which reflect the composite interaction between viscous (or gravity) and capillary forces become important. Several studies have been reported on this topic, all of which concern imbibition, however.

Typically, theoretical studies on displacement in fractured systems are concerned with deriving some representation of the imbibition rate into a matrix block, by either solving an appropriate initial boundary value problem or by introducing various simplifying assumptions [39]. Although routinely used in simulations, such purely numerical approaches offer little to further our understanding of the process. Experimental studies have been more useful in this regard. In one of the first visualization studies in fractured systems, Handy and Datta [52] provided evidence of water imbibing into artificially-fractured sandstones and heterogeneous sandpacks. They observed water advancement preferentially through the fracture in the first case, and imbibition of the finer sand in the second case. Mattax and Kyte [84] introduced a critical rate, defined as the limiting value above which the front advances faster in the fracture than in the matrix. The latter can be detrimental to the process efficiency, as the advancing front may reach the end of the matrix block and prevent further recovery. In a more recent study, Babadagli [4] conducted a series of forced imbibition experiments in model fractured sandstones, visualized by CT scanning, which demonstrated the importance of rate effects on the properties of the composite system, such as relative permeabilities. As also shown below, rate effects are fundamental to the large scale behavior of displacement in fractured systems,

in contrast to homogeneous media. The above were restricted to imbibition of strongly wet systems. In particular, no drainage experiments have been reported.

While these studies have furnished valuable information, our understanding of displacement in fractured systems is still based on phenomenology. Notably lacking are local flow visualization and pore-scale simulation of the displacement. Flow visualization with transparent micromodels has been very useful in elucidating flow mechanisms in many processes [38], [76], [77], [96] despite the inherent limitation of two dimensions. To our knowledge, analogous displacement studies in micromodels that mimic a fracture-matrix block geometry and corresponding pore level simulation have not been undertaken to date. This paper reports on both these topics. Using a glass micromodel representing a microscale replica of a fracture-matrix system we have visualized drainage and imbibition processes. The displacement was then simulated with the use of a pore network simulator. Because of the difference between drainage and imbibition, we shall present the results in separate publications. This part deals with drainage.

The section is organized as follows: We, first, present experimental results on drainage in micromodel geometries that mimic a matrix-fracture system. A critical rate (capillary number) is identified, below which penetration of the matrix is not possible, but above which the displacing phase also invades the matrix block. This is opposite to the imbibition reported by Mattax and Kyte [84], where at rates below critical the displacing wetting phase penetrates mostly the matrix, and where the critical value must be exceeded for an advance in the fracture. A theoretical model based on gradient percolation is developed to predict the onset of matrix penetration. The experimental results are subsequently compared with pore network simulations of drainage. A sensitivity analysis of the effects of various parameters, particularly the capillary number and viscosity ratio, on the displacement efficiency is presented. Finally, based on the pore level simulation, steady-state relative permeabilities for a matrix fracture system are computed. The results are compared to the behavior of heterogeneous but uncorrelated systems.

5.2 EXPERIMENTAL

The schematic of the experimental apparatus is shown in Figure 48. The glass micromodels were fabricated using 3" x 5" glass plates on which the desired patterns were etched. Experiments were performed in two micromodels: The first consisted of a triangular network of pores of coordination number 6, representing the matrix block, and of a uniform channel of 2 mm width through the middle of the micromodel representing the fracture. The total pore volume of the micromodel was estimated as 0.77 cm^3 . Pore throat and pore body sizes were estimated at $400 \text{ }\mu\text{m}$ and $800 \text{ }\mu\text{m}$, respectively. The matrix of the second micromodel consisted of a square lattice of coordination number 4. Additional characteristics are described in Table 2. During fabrication, the fracture was etched to a depth estimated as twice as large that of the matrix pores.

Inlet and outlet ports were connected to the fracture. All displacements were run at constant rate using a syringe pump, delivering at the rates of 0.054, 0.108, 0.2 and 0.26 cc/min . These values correspond to capillary numbers in the range where penetration of the matrix can occur. For primary drainage experiments we used the pairs air-water and oil-water. Kerosene or viscous mineral oil of viscosities 1.5 cp or 100 cp, respectively, was used as the non-wetting fluid in the latter case. The micromodel was first saturated with water dyed with Methylene Blue, at a low rate to completely saturate the matrix. The model was then placed horizontally for the drainage experiment to be conducted. Each run was carried out by injecting the non-wetting fluid at a constant rate until steady-state was established. Then, the flow rate was increased to the next setting and injection resumed. A total of 12 runs, corresponding to three different mobility ratio values each at four different capillary numbers, were conducted (Table 3). The frontal movement was recorded using a video camera with a close-up kit or a microscope.

Typical visualization experiments using the first micromodel are shown in Figures 49-53. In all experiments, the pore level displacement mechanism is the typical "stop and go" rule of drainage, where the interface advances only after the capillary pressure threshold in the throat is exceeded. It was observed that for capillary number values below a critical,

1. Video Camera
2. Water Reservoir
3. Syringe Pump
4. Steam Generator
5. Oil Reservoir
6. Syringe Pump
7. Pressure Transducer
8. Thermocouple
9. Micromodel

Figure 48: Schematic of the experimental apparatus.

Network	Triangular	Square
Length	12.5 cm	11.2 cm
Width	7.5 cm	5.7 cm
Network size	75×45	51×26
Maximum bond width	0.04 cm	0.1 cm
Minimum bond width	0.04 cm	0.04 cm
Average bond width	0.04 cm	0.07 cm
Fracture width	0.2 cm	0.2 cm
Fracture depth	0.01 cm	0.018 cm
Network depth	0.01 cm	0.009 cm

Table 2: Micromodel characteristics.

Fluid Pair	Displacing Fluid Viscosity (cp)	Mobility Ratio	Volumetric Rate (cc/min)	Capillary Number
Air-Water	0.018	55.56	0.00052	9×10^{-8}
"	"	"	0.052	9×10^{-6}
"	"	"	5.2	9×10^{-4}
"	"	"	40	7×10^{-3}
Oil-Water	1.5	0.66	0.0027	8×10^{-5}
"	"	"	0.054	1.59×10^{-4}
"	"	"	0.108	3×10^{-3}
"	"	"	0.27	8×10^{-3}
Oil-Water	100	0.01	0.00052	1×10^{-4}
"	"	"	0.0052	1×10^{-3}
"	"	"	0.052	1×10^{-2}
"	"	"	0.52	1×10^{-1}

Table 3: Experimental conditions for drainage.

estimated to be approximately 2×10^{-4} , the displacing phase penetrates the fracture only, breaks fast through the production port and reaches a steady-state flow without subsequent invasion of the matrix (Figure 49). However, for values above the critical, the matrix was also invaded, shortly after penetration of the fracture and before breakthrough of the displacing phase (Figure 50). Simultaneous displacement was observed in both the matrix and the fracture, until a steady-state is reached, where the front in the matrix becomes stationary. The pattern is characterized by percolation-like features in cases where the displacing fluid is air (Figure 50a) and by a compact pattern in cases of higher viscosity fluids, such as mineral oil (Figures 50b and 50c). At even higher rates, invasion of the matrix commences at about the same time as in the fracture and results into progressively higher displacement efficiencies, until a limiting efficiency is reached at sufficiently large rates.

These experiments were readily reproducible. Qualitatively similar results, but different critical values, were obtained for experiments involving other micromodel geometries, including those where the entire inlet of the matrix was exposed to drainage. It was concluded that penetration of the matrix requires that the capillary number exceeds a critical value, which depends on the geometry of the micromodel. This process is reminiscent of capillary invasion, e.g. during mercury porosimetry [40], where a critical threshold must be exceeded before penetration takes place, with the important difference, however, that in the present case the critical quantity is the capillary number, which is rate-dependent, rather than the capillary pressure. As discussed later in the paper, an equivalent “capillary pressure curve” can be constructed by plotting capillary number vs. matrix saturation. Unfortunately, and contrary to an ordinary invasion process, which is locally controlled by capillary forces alone and it is thus amenable to percolation theory [120], invasion in the fracture-matrix system is also influenced by viscous forces, which impart patterns different than percolation. To understand these results, theoretical analysis and numerical simulation of the process were conducted.

Figure 49: Experimental Steady State Distribution Before Matrix Penetration: (a) Kerosene Water, $Ca = 8 \times 10^{-5}$ (b) Air Water, $Ca = 9 \times 10^{-5}$.

Figure 50: Experimental Steady State Distribution After Matrix Penetration: (a) Air-Water, $C'a = 7 \times 10^{-3}$ (b) Kerosene-Water, $C'a = 3 \times 10^{-3}$ (c) Kerosene-Water, $C'a = 8 \times 10^{-3}$.

5.3 THEORY

It is straightforward to calculate the critical capillary number. Consider flow in the fracture only, which can be approximated by a conduit of length L , constant cross section A_c and flow conductance g . Application of Poiseuille's law yields the pressure drop across injection and production ends (A and B , respectively, Figure 51)

$$P_{o,A} - P_{o,B} = \frac{Q\mu_o L}{A_c k_f} \quad (57)$$

where Q is volumetric flow rate, μ is viscosity, and we used subscript o to denote the displacing fluid. The fracture permeability k_f can be approximated with the use of a hydraulic diameter

$$k_f = \frac{D_h^2}{32} \quad (58)$$

where $D_h = \frac{4A_c}{\Pi}$, Π being the wetted perimeter. Making the reasonable assumption that $P_{o,B}$ is the pressure of the immiscible matrix fluid, equation (1) shows that a constant gradient in the capillary pressure develops as flow in the fracture occurs. Since penetration of the matrix requires that the capillary threshold be exceeded, this problem is very similar, although not identical, to gradient percolation [50]. Gradient percolation is a percolation process, where the percolation probability varies in space following a constant gradient, e.g.

$$\frac{dp}{dx} = -B \quad (59)$$

where x is expressed in terms of lattice spacing units and B is the Bond number ($B = \frac{k\Delta\rho g x}{\gamma}$ in gravity-driven drainage). For such processes, it was shown in [51] that a connected cluster (front) forms, with mean front position x_f at the location where the percolation threshold of the network is reached. The front has a width which increases as the Bond number decreases following the scaling

$$\xi \sim B^{-\frac{\nu}{1+\nu}} \quad (60)$$

where $\nu = \frac{4}{3}$ in 2-d and 0.88 in 3-d [51]. To find the Bond number for the present problem we first need to connect the percolation probability p to the capillary pressure P_c . Using the

relation

$$P_c = \frac{2\gamma}{r_d} \quad (61)$$

where γ is the interfacial tension, the mean radius of curvature r_d can be related to p via

$$p = \int_{r_d}^{\infty} \alpha(r) dr \quad (62)$$

where $\alpha(r)$ is the size distribution in the matrix. By combining (57) and (59), we obtain

$$B \simeq \frac{\bar{r}^2 Ca}{2k_f} \quad (63)$$

where we defined the capillary number $Ca = \frac{Q\mu_o}{\gamma A}$ and where $\bar{r}$ is the mean radius in the matrix. Thus, the Bond number for this problem is directly proportional to the injection rate, suggesting dynamical effects. Using gradient percolation theory arguments, we next make the postulate that the matrix is penetrated when the front location is at the beginning of the matrix block, namely when $p = p_c$ at $x = 0$, where p_c is the percolation threshold of the matrix. The critical capillary number follows by combining (3) and (7)

$$Ca^* = \frac{2k_f}{Lr_{dc}} \quad (64)$$

If it is assumed that the fracture cross-section is rectangular and that its depth a is comparable to the critical radius of curvature, equation (64) further suggests $a \simeq 8L Ca^*$. This prediction was checked against experimental and numerical results. Using experimental values for L and Ca^* yields the estimate $a \simeq 134\mu m$, which compares favorably with the measured value of $120\mu m$. Comparison with numerical results is discussed below.

Equation (8) shows that the critical capillary number depends on geometric properties only, particularly on the fracture length L , to which it decreases in direct proportion. Thus, for a fixed capillary number, longer blocks are penetrated earlier. This conclusion also holds when it is the total flow rate per unit area in the system, rather than the capillary number through the fracture, which is fixed. Indeed, as can be easily shown, in the latter case the flow velocity through the fracture would scale in proportion to the linear dimension L of each matrix block, assumed cubic, hence the penetration condition would show a capillary number scaling proportional to L^{-2} , which is stronger than (64).

Figure 51: Schematic of steady-state distribution after matrix penetration.

Subsequent to the onset of penetration and a further increase in the capillary number, the matrix starts to be invaded. It is tempting to also use gradient percolation results to model the matrix saturation. Gradient percolation predicts that x_f satisfies the relation

$$\frac{L - x_f}{L} = \frac{Ca^*}{Ca} \quad (65)$$

based on which the saturation profile can be predicted from the scaling described in [51]. However, such an approach is oversimplified. Although the degree of penetration is gradual and the front at steady-state is stationary, the problem differs substantially from the standard gradient percolation process. Even at steady-state conditions, fluid flow takes place both in the invading and in the displaced fluids in the matrix, thus rendering the process rate-dependent and of a dynamic nature. This cannot be modeled by gradient percolation. To understand this process a pore level numerical simulation is necessary.

5.4 SIMULATION

To simulate the process, we used a drainage pore network simulator, where the porous medium is represented as a square lattice of pore throats and pore bodies. Its basic aspects are similar to those of Lenormand et al [75] and Blunt and King [19]: Menisci separating the two phases are assumed to reside in pore bodies only, the saturation of which can be partial (less than 1). Interfaces are allowed to penetrate throats adjacent to a pore body only if the capillary pressure threshold of the throat is exceeded. Pressure drops are associated with flow in the pore throats only, and it is only pore bodies that contribute to saturation and volume. At any given time the front consists of connected interfaces residing in pore bodies. During a time step, the front advances by either penetrating a bond adjacent to it or by filling a pore body partly saturated, depending on the prevailing flow conditions. Among many partly filled pores, the pore requiring the smallest time increment to become fully saturated is filled first. All pressure drops are calculated based on a Successive Over Relaxation (SOR) technique, where an additional iteration cycle is required to solve problems at a specified flow rate. In the simulations, the fracture was represented as an equivalent system of consecutive

pores of large size and conductance. To mimic the experimental approach, injection at the tip of the fracture only with production at the downstream end of the model were first considered. Subsequently, however, we also considered injection over the entire inlet face of the matrix.

The simulation was carried out by specifying the geometrical characteristics of the experimental micromodel, converted to equivalent cylindrical or spherical segments for pore throats or pore bodies, respectively. Although the numerical lattice was patterned exactly after the original pattern used to etch the micromodel, numerical and experimental models were not identical, due to non-uniformities incurred during the processes of etching and fusion, which could not be precisely controlled due to glass imperfections. Numerical results corresponding to the experiments in the square lattice micromodel are shown in Figures 52-56. For the two experimental capillary number values 1.12×10^{-4} and 3.25×10^{-4} , the penetration of the matrix is insignificant, the injected fluid flowing through the fracture only as also observed experimentally (Figure 52). When the capillary number was raised to the value of 3.9×10^{-4} , however, the matrix becomes invaded (Figure 53), suggesting a critical capillary number of approximately 3.5×10^{-4} . This value also compares favorably with the experimental value for the triangular network of Figures 49 and 50, if the difference in the percolation thresholds in the two lattices, $p_c = 0.5$ and $p_c = 0.33$, respectively, is taken into consideration (such difference would result in an increase of the critical number by a factor of $\frac{0.50}{0.33}$). Subsequent increases in the capillary number result in larger occupancy of the matrix (Figure 54) until a limiting saturation is reached (Figure 55). Another comparison is shown in Figure 56, where the steady-state matrix wetting saturation is plotted vs. the capillary number. As shown, the comparison between simulation and experiment is very good, suggesting that the simulated mechanism describes well the experimental observations. As previously suggested, the curve of Figure 52 can be interpreted as an equivalent “capillary pressure” curve, where a capillary threshold and an effective residual saturation can be identified. However, the distribution of fluids is dictated here by dynamic rate effects, which also depend on the viscosity ratio as shown below. This distribution will be subsequently used to construct

effective permeabilities.

5.5 RESULTS

The pore network simulator was next used to investigate the critical capillary numbers, to compute displacement patterns and relative permeabilities, and to analyze the effect of the process parameters.

5.5.1 Critical Capillary Number

The experimental results for the critical capillary number were found in agreement with predictions based on gradient percolation theory. The scaling of the critical capillary number on fracture permeability, length and average pore size was predicted to satisfy (8). It would be interesting to further test these predictions with the use of the numerical simulator. Figure 57 shows a comparison between theoretical and computed critical capillary numbers for various values of k_f , L , and $\bar{r}$ in a range covering several orders of magnitude for each variable. The relation of $\bar{r}$ to r_d^* is through

$$p_c = \int_{r_{dc}}^{\infty} \alpha(r) dr \quad (66)$$

For a uniform size distribution and for $p_c = 0.5$ for bond percolation in a square lattice, (66) yields $r_{dc} = \bar{r}$. As shown in Figure 57, the theoretical scaling (64) is very well satisfied. Actually, the numerical results suggest a minor modification to the prefactor, which should be 2.5 instead of the theoretical prediction of 2, probably due to finite size effects [45].

While (8) is useful if $\alpha(r)$ is known, it would be perhaps more practical to relate the critical capillary number to other directly measurable properties of matrix and fracture. In a seminal paper, Katz and Thompson [63] related the permeability of a homogeneous material to the percolation length l_c , here equal to $2r_{dc}$, and to the formation factor F , through

$$k_m = \frac{1}{266F} l_c^2 \quad (67)$$

Figure 52: Comparison between experimental and numerical results for kerosene-water, $Ca = 3.25 \times 10^{-4}$.

Figure 53: Comparison between experimental and numerical results for kerosene-water, $Ca = 3.9 \times 10^{-4}$.

Figure 54: Comparison between experimental and numerical results for kerosene-water, $Ca = 6.1 \times 10^{-4}$.

Figure 55: Comparison between experimental and numerical results for kerosene-water, $Ca = 7.81 \times 10^{-4}$.

Figure 56: Steady-state wetting saturation vs. capillary number. (Dots are experimental points, solid line is numerical).

Substitution of (67) in (64) yields the more general result

$$\frac{Ca^* L \sqrt{k_m F}}{k_f} = 0.26 \quad (68)$$

This formula allows one to relate the critical capillary number to fracture and matrix permeabilities. It must be pointed out that this capillary number is much larger than the corresponding to flow in the matrix, Ca_m (when penetration occurs). The two values are related with the relationship $Ca = Ca_m / \phi_f$, where ϕ_f is the fracture volumetric fraction (porosity).

The success of gradient percolation in correctly predicting the scaling of the initial capillary number is not surprising. Before penetration, the pressure gradient in the fracture is constant, thus lending itself to gradient percolation theory. After the onset of penetration, the patterns obtained differ significantly from those corresponding to gradient percolation. Due to the flow of the displacing fluid an additional pressure drop occurs in the matrix, which reduces the effective pressure gradient at the front and results in patterns different from gradient percolation. This is demonstrated in Figure 58, where the pressures at the front and in the fracture are plotted as a function of the streamwise coordinate. A pressure difference exists between the fracture and the matrix in the upstream part, indicating transverse fluid flow from the fracture to the matrix. The pressure gradient is constant only at the downstream portion of the displacement, and varies significantly near the injection point. This difference makes invalid the use of the gradient percolation predictions in the regime following the onset of penetration.

5.5.2 Displacement Patterns

Because viscous are the key forces in the invasion of the matrix, important parameters that affect the process performance are the viscosity ratio, M , and the boundary conditions, such as the mode of injection. In typical phenomenological models for fractured systems it is implicitly assumed that all displacement occurs from menisci that enter the matrix from the fracture side only, thus resembling lateral displacement, and that frontal displacement is not

Figure 57: Scaling of critical capillary number. (Dots are numerical simulations, dashed solid line is the theoretical prediction).

Figure 58: Numerical simulation of steady-state pressure profiles. (Full dots denote pressure at front, open circles denote pressure in fracture).

important. To check this hypothesis, we simulated processes where displacement from the inlet face was also allowed. The critical capillary number for the matrix penetration was the same in either case, and conformed with the theoretical predictions (8). The mechanism of displacement, however, differed significantly in the two cases.

Figure 59 shows displacement patterns for the two different injection conditions for unfavorable displacement ($M = 66$), where viscous instability and fingering are very pronounced as expected for an air-liquid systems. The two patterns exhibit certain differences. When the injection is only in the fracture (Figure 59a), the displacement is initiated and spread in the portion of the matrix adjacent to the fracture only, thus resembling a lateral invasion. In contrast, a mostly frontal advance is observed when injection occurs over the entire inlet face (Figure 59b). In the latter case, the role of the fracture lies primarily in contributing the critical pressure drop for the onset of invasion. The corresponding saturation profiles of Figure 60 reflect this difference near the downstream end, where the fractional penetration is deeper for the former case. Because of the need of connectivity with the fracture, the front in the first case is likely to be somewhat more compact and less fingered than in the second. Qualitatively similar trends are also displayed in the opposite case of favorable, compact displacement ($M = 0.066$, Figures 61, 62). When injection occurs in the fracture only, the displacement is first confined to a region adjacent to the fracture (Figure 61a), although it spreads over the entire block at sufficiently high rates. With injection over the inlet face, on the other hand, the displacement is mostly frontal, the fracture only serving to provide the necessary critical capillary number for penetration (Figure 61b). The saturation plots of Figure 62 confirm these trends. While the two patterns differ depending on the mode of injection, however, the importance of the fracture should not be discounted. The fracture geometrical characteristics set the critical capillary number for invasion and the steady-states after invasion, features which are both absent for injection in the absence of fracture (see also below). The existence of a fracture constrains the capillary number range to a small interval of about a decade within which the displacement from the matrix is completed. Consequently, plots such as Figure 56 remain qualitatively the same for either

injection boundary condition, particularly for $M \gg 1$, the main difference being that the slope of the curve is steeper in the case of injection through the fracture only.

5.5.3 Displacement Efficiency and Relative Permeability Curves

The preceding demonstrated important features associated with drainage of a fractured system. The existence of a critical rate for invasion of the matrix block is uniquely due to the fracture (or for that matter any other highly permeable streak) and resembles the invasion associated with mercury porosimetry, with the important exceptions that it is a threshold in the capillary number rather than the capillary pressure that needs to be exceeded, and that non-local, viscous forces are fundamental in the displacement in the case of a fracture. As previously noted, plots such that of Figure 56 can be considered as equivalent to a “capillary pressure-saturation” curve, where the capillary number is substituted for the capillary pressure. The key role of viscous effects in setting the displacement efficiency, however, makes impossible the development of a simple local theory, such as percolation, to analytically describe this curve, as is commonly done for the capillary pressure curve. In particular, these curves depend on the viscosity ratio. Shown in Figure 63 are plots of the displacement efficiency vs. the capillary number for various values of the viscosity ratio. Ultimate efficiencies are high for favorable viscosity ratio, and low for unfavorable viscosities, as expected. In terms of the residual saturation terminology of the capillary pressure curve, we may state that the residual saturation increases with an increase in the mobility ratio. The effect of the fracture is important at low values of the capillary number. Plotted in Figure 64 are the corresponding to Figure 63 displacement efficiencies in the absence of a fracture (frontal displacement). Note the absence of a threshold value. Theoretically, the low capillary number efficiency, where capillary forces predominate, should be very small. The numerical value is different due to finite size effects [45] and to the slow dependence of the mass of percolation cluster on size ($S \sim L^{-\beta}$, where $\beta = 0.138$ for 2-D and $L = 45$ in these simulations). Before the onset of matrix invasion, the two sets of curves differ substantially, the difference being greater for $M \gg 1$ (compare Figures 63 and 64). The curves asymptotically approach each

Figure 59: Numerical simulation of steady-state distribution for $M = 66$ and $Ca = 1.67 \times 10^{-3}$ (based on fracture area): (a) injection through fracture, (b) injection through entire inlet face.

Figure 60: Steady-state saturation profiles for the condition of Figures 59a, 59b.

Figure 61: Numerical simulation of steady-state distribution for $M = 0.066$ and $Ca = 1.01 \times 10^{-3}$ (based on fracture area): (a) injection through fracture, (b) injection through entire inlet face.

Figure 62: Steady-state saturation profiles for the condition of Figures 61a, 61b.

other, however, when the displacement in the matrix has been completed.

The one-to-one relationship between the steady-state saturation of the matrix block-fracture system and the corresponding capillary number as shown in Figures 56 and 63 allows for steady-state relative permeabilities for the composite system to be constructed. The issue of effective relative permeabilities is key to the simulation of displacement in fractured systems, which are commonly represented by an effective continuum. In a recent study, Babadagli [4] obtained pseudo-relative permeabilities pertaining to transient displacement in imbibition process which exhibited a flow rate dependence. We recall that relative permeabilities that are only functions of saturation can be defined in uncorrelated systems, as long as the capillary number is sufficiently low such that the distribution of phases is under capillary control [55]. Following the assignment of occupancy, the conductivity of each phase can next be calculated (or approximated by scaling laws) and the relative permeability determined. Heiba et al [55] used such an approach in Bethe lattices to obtain curves which reproduced important experimental features. Under such conditions ($Co \ll 1$), the occupancy of the pore space is solely dictated by capillarity, and percolation theory rules can be directly applied.

In fractured systems (or in systems with long-range correlations) capillarity does not alone control the distribution of phases, however. Instead, it is a combination of capillary and viscous forces that dictates the occupancy of the pore space, this combination resulting in the steady-state diagrams of Figure 63. Nonetheless, composite relative permeabilities can be computed numerically from the individual phase conductances after the establishment of steady-state. Results obtained are shown in Figure 65 for various values of the mobility ratio and for two different values of the ratio of fracture to matrix permeability, $\lambda = \frac{k_f}{k_m}$. Overall, the composite system appears strongly “water-wet”, in that the point of intersection of the relative permeability curves is significantly to the right of the mid-point saturation value. Non-wetting permeabilities are high throughout most of the saturation range, while wetting permeabilities rapidly reach a small value as the saturation decreases. It is interesting to note that because of the existence of the fracture, simultaneous two-phase flow is possible over a

Figure 63: Steady-state wetting saturation vs. capillary number for $M = 66, 6.6, 0.66$ and 0.066 .

Figure 64: Steady-state wetting saturation vs. capillary number for $M = 66, 0.66$ and 0.066 . Injection in the absence of fracture.

substantial saturation range, despite the 2-D geometry. This feature is absent in uncorrelated systems, where the simultaneous flow of two percolating phases is impossible. The effect of the viscosity ratio on the non-wetting permeability is strong, particularly at not too high values of λ (Figure 65b), and results in an increase of the non-wetting relative permeability and a shift of the set to the right as M decreases. By contrast, no significant effect of M on the wetting permeability is noted. The sensitivity to M is stronger as the ratio of permeability λ decreases. It is interesting to note that in such steady-state composite permeabilities, there is no direct effect of rate (or capillary number), although dynamic effects enter indirectly in setting saturation levels and the phase occupancy. The shape of these curves is drastically affected by the geometry, pore size and fracture characteristics of the matrix-fracture system and by the viscosity ratio. The permeabilities pertain to steady-state flow conductivity after the displacement pattern has reached a steady-state and it is not clear at this point if they can be used to model transient large scale displacements. It is worth also pointing out that effects of rate and viscosity ratio on relative permeabilities in uncorrelated media have been pursued with vigor over the past years, but still remain a somewhat unsettled issue. Due mainly to the role played by the fracture in dictating penetration, these effects are much more apparent in a fracture-matrix system.

5.6 CONCLUSIONS

We studied the constant rate drainage of a model fractured system, based on micromodels. It was shown that regardless of the injection conditions, namely whether it is from the fracture or from the entire inlet face, there exists a critical capillary number below which the matrix is not invaded. The onset of penetration can be predicted well from gradient percolation arguments which lead to the correct scaling of the critical capillary number on geometrical characteristics. The scaling is inversely proportional to the length of the fracture and the square root of the matrix permeability and proportional to the fracture permeability. The existence of fractures lead to steady-state displacement fronts, a feature that is absent in homogeneous or mildly heterogeneous systems, where a constant rate displacement may reach

Figure 65: Steady-state relative permeabilities for $M = 66, 0.66$ and 0.066 and $\phi_f = 0.005$:
(a) $\lambda = 126$, (b) $\lambda = 22$.

steady-state only after breakthrough and completion of the process. Displacement in the matrix terminates after one or two decades of further increases in the capillary number, an unfavorable mobility ratio requiring a larger increase. Once this regime is reached, the effect of the fracture becomes unimportant. While gross features are independent of the particular invasion mode, displacement patterns are significantly affected. Frontal displacement modes prevail when injection over the matrix inlet face is allowed as opposed to a lateral displacement mode when only injection in the fracture is assumed. The latter assumption is commonly made in existing simulators that model the interaction between matrix and fracture. Based on the present findings, this assumption appears in need of re-examination. Steady-state relative permeabilities were computed and found to depend on the geometry and the viscosity ratio, but not on the rate. These results find applications to fractured systems and to correlated heterogeneous porous media containing streaks of high permeability paths. In light of the present work, the modeling of drainage processes in the latter case also needs to be revisited.

A final comment pertains to effects of gravity. In many practical applications a vertical matrix block containing a fluid, such as oil, is surrounded by gas which is the non-wetting phase, and the pressure of which can be taken as constant. Penetration of the block by gas and subsequent drainage of oil is now driven by the hydrostatic pressure gradient in the oil. Although this problem is different from the previous, a critical value on a suitably defined Bond number can also be derived to describe the onset of penetration. Using the same arguments as before we find

$$\frac{\rho g L \sqrt{k_m F}}{\gamma} \geq 0.266$$

Because of the different mechanism of the drainage process, which is driven here by hydrostatic pressure differences, and where rate effects are not involved, however, gradient percolation theory should be successful in describing this displacement problem.

6 ASPECTS OF NON-NEWTONIAN FLUID FLOW AND DISPLACEMENT IN POROUS MEDIA

Charul Shah and Yanis C. Yortsos

6.1 INTRODUCTION

The flow of fluids through a variety of porous media is common in many petroleum engineering applications. A partial list includes filtration, ground water flow, production of oil and gas from underground reservoirs, ion exchange and adsorption [36, 109]. Most of these involve either single- or multi-phase flow of fluids in order to achieve desired objectives. In some of these applications, non-Newtonian fluids are extensively involved. Particular examples include heavy oils and Enhanced Oil Recovery (EOR). The rheology of many heavy oils has been shown to be non-Newtonian, Bingham plastics being one manifestation of heavy oil flow. In EOR applications, non-Newtonian fluids such as low concentration polymer solutions, emulsions, gels, etc. are simultaneously injected to increase the viscosity of driving agents that displace oil. Such rheologically complex fluids are used to improve sweep efficiencies, divert displacing fluids and block swept zones.

While the flow of Newtonian fluids through porous media has been well understood through a variety of experimental and theoretical techniques, the state of the art in the flow of non-Newtonian fluids, either single- or multiphase, in porous media is far from complete. Reasons for this include the complex rheological behavior in combination with the complicated flow geometry of porous media. The macroscopic description for the flow of non-Newtonian fluid through porous media is therefore a difficult task [104, 113]. With few exceptions, laws equivalent to Darcy's law for the flow of non-Newtonian fluids through porous media have not been developed.

The present study has been undertaken to understand the flow of non-Newtonian fluids through porous media. The work considered involves the numerical (pore network) modeling of both single- and multiphase flow of power-law and Bingham plastic fluids in network-like

porous media. We consider aspects of both single- and multi-phase flow and displacement. The behavior of single-phase power-law fluid flow in porous media was discussed in our previous DOE Report [108]. Here we shall emphasize single-phase flow of Bingham plastics and the immiscible displacement of power-law fluids.

6.2 FLOW OF BINGHAM PLASTICS IN POROUS MEDIA

Bingham plastics exhibit a finite yield stress at zero shear rate and have been used to model the flow of some heavy oils. Laboratory investigations and field tests have indicated that flow of heavy oil takes place only after the applied pressure gradient exceeds a certain minimum value [99, 122]. Also, flow of groundwater in certain clayey soils, flow of foam in porous media, flow of certain drilling and hydraulic-fracturing fluids, all exhibit rheological behavior that in certain instances can be described as a Bingham plastic.

The simulation procedure for the single-phase flow of Bingham plastic in a network-like porous media is the same as outlined previously in [108], except that the governing equations for the flow are different and more difficult to solve. The expression for the flow of Bingham plastic in a capillary is given by [17]

$$q = \frac{\pi R^4}{8\mu_o L} \left(1 - \frac{4}{3} \left(\frac{\tau_o}{\tau_R}\right) + \frac{1}{3} \left(\frac{\tau_o}{\tau_R}\right)^4\right) \Delta P \text{ when } \tau_R > \tau_o, \quad (69)$$

and by

$$q = 0 \text{ when } \tau_R < \tau_o \quad (70)$$

where R and L denote radius and length of the capillary respectively, q is the volume flow rate in a pore, across which there is an applied pressure drop ΔP and τ_R is the wall shear stress.

Appropriately rescaled, the dimensionless form of the equations are

$$q^* = R^{*4} \left(1 - \frac{4}{3} \left(\frac{\tau_o}{\tau_R}\right) + \frac{1}{3} \left(\frac{\tau_o}{\tau_R}\right)^4\right) \Delta P^* = g^* \Delta P^* \text{ when } \tau_R > \tau_o, \quad (71)$$

Figure 66: Flow pattern for $\tau_o = 0$ [0.01-1.99].

Figure 67: Flow pattern for $\tau_o = 0.01$ [0.01-1.99].

Figure 68: Flow pattern for $\tau_o = 0.05$ [0.01-1.99].

and

$$q^* = 0 \text{ when } \tau_R < \tau_o \quad (72)$$

where g^* is a dimensionless conductance for a single capillary and

$$\tau_R = \frac{\Delta P R}{2L} \quad (73)$$

The pressure field in the network was calculated by applying a successive relaxation iteration method. Results for the flow and conductance distributions have been obtained for Bingham plastic fluids in networks of size 21×21 . Two uniform size distributions, a narrow and a wide were used.

Figure 66 shows the flow pattern of a Newtonian fluid ($\tau_o = 0.0$) in a uniform distribution of bond sizes in the interval [0.01-1.99]. We observe that the flow is distributed over the entire network, and we may identify a connected cluster, resembling a percolating cluster, through which most of the flow is taking place. Figure 67 represents the corresponding flow pattern

Figure 69: Flow pattern for $\tau_o = 0.065$ [0.01-1.99].

Figure 70: Critical yield stress [0.01-1.99].

Figure 71: Flow pattern for $\tau_o = 0$ [0.9-1.1].

for Bingham plastic fluid with the value of yield stress equal to 0.01. We observe additional empty pockets in the network as more bonds in the network become non-conducting due to the higher value of yield stress. As it is not possible to accommodate all the simulation results in this report, two typical examples are shown in Figures 68 and 69, where the values of yield stress are 0.05 and 0.065, respectively. In Figure 68, flow occurs only through a limited network of connecting bonds. We identify large empty pockets in the network, where no flow occurs. We observe more pronounced effects when $\tau_o = 0.065$, where the entire flow occurs through a very small number of connected bonds. The remaining network has a limited participation to flow and behaves much like an empty pocket. These results are similar to those obtained for shear thinning fluids when the value of power-law index is small [108]. Figure 70 shows a plot of the effective conductance versus yield stress. We can identify a macroscopic critical yield stress beyond which the flow is zero.

The other case involves a narrow size distribution [0.9, 1.1]. Figure 71 shows flow patterns for the Newtonian fluid ($\tau_o = 0$). The flow is quite uniform throughout the entire network and we do not observe many empty pockets. Flow is taking place mainly along the pores

Figure 72: Flow pattern for $\tau_o = 0.01$ [0.9-1.1].

Figure 73: Flow pattern for $\tau_o = 0.045$ [0.9-1.1].

Figure 74: Flow pattern for $n=0.07$ [0.01-1.99].

in the flow direction and very limited flow is observed in the transverse direction. As we increase the value of the yield stress (Figure 72) all flow occurs basically through straight paths, each connecting the same number of pores. Figure 73 shows flow patterns for larger values of the yield stress $\tau_o=0.045$, where flow takes place through only a few horizontal paths and the rest of the network is non-conducting.

We suggested earlier that the flow behavior of shear thinning fluids with small n would be similar to the flow of a Bingham plastic. To test this argument, typical flow patterns for the single-phase flow of shear thinning fluid, with $n=0.07$, for the same network size and distribution used for Bingham plastic, are shown in Figure 74. We observe a striking similarity in the flow patterns with the patterns of Figure 68 corresponding to a Bingham plastic with $\tau_o=0.05$. This analogy, however, needs to be tested further.

6.3 IMMISCIBLE DISPLACEMENT INVOLVING POWER-LAW FLUIDS

The previous sections dealt with single phase flow. In this section, we consider two phase flow and displacement. First, we will simulate the displacement of a Newtonian fluid by a power-law fluid in a drainage process. The long-term goal of this study is to examine the displacement patterns obtained for various types of non-Newtonian fluids and proceed with scale-up. We have mainly concentrated on shear thinning fluids. In the case of Newtonian fluids, it is common practice to relate displacement patterns to the viscosity ratio of the two fluids (stable vs. unstable). However, the viscosity of power-law fluids changes depending on the applied shear rate and it is varying from bond to bond in the network. Therefore, the previous results do not apply and must be modified. In modeling such immiscible displacement through the network, we have used the following standard assumptions [75]: (i) Volume calculations apply to pore bodies only, the pressure drop occurring in pore throats. (ii) Throats are filled by either invading or displaced fluids, while bodies may contain both fluids. (iii) The capillary pressure in a throat is inversely proportional to the tube radius, while the capillary pressure drop in a node is negligible. Two cases were considered, one involving constant pressure and another involving constant flow rate.

In the simulations we have considered the drainage of a wetting Newtonian fluid by injecting a non-wetting power-law fluid. When the bonds in the network are occupied by the Newtonian fluid, Poiseuille's law for the flow rate Q_{ij} across each bond connecting adjacent nodes i and j applies

$$Q_{ij} = \frac{\pi R_{ij}^4 (P_i - P_j)}{8\mu L_{ij}} = g_{ij}(P_i - P_j) \quad (74)$$

where P_i is the nodal pressure, R_{ij} and L_{ij} are the radius and length of the bond, respectively, and μ is the viscosity of the Newtonian fluid.

The pressure field in the network is calculated by applying the mass balance equation at each node as described in the simulation of single-phase flow [108]. The non-wetting fluid (non-Newtonian in this case) cannot enter a bond unless the pressure difference across that

bond exceeds a threshold pressure P_c , given by Laplace's law

$$P_c = \frac{2\gamma}{R_{ij}} \quad (75)$$

where γ is the interfacial tension. Hence, a particular bond is invaded if there is no trapping and the capillary pressure condition is satisfied. Once the bond is invaded, the governing flow equation in the bond changes to

$$Q_{ij} = \frac{\pi R_{ij}^{(3n+1)/n} n (P_i - P_j)^{1/n}}{(2L_{ij}K)^{1/n} (3n+1)K} = g_{ij} \Delta P_{ij}^{1/n} \quad (76)$$

hence, the value of g_{ij} depends on both the power-law index n and the consistency index K . In the subsequent steps for pressure calculation, the same iterative method is applied, except that we use a combination of linear and non-linear flow equations depending on the type of the fluid present in each bond of the network. The node saturation is updated at each time step using (76). In time Δt the node saturation $S_i(t)$ increases as follows

$$S_i(t + \Delta t) = S_i + \Delta t V_i \sum_j Q_{ij} \quad (77)$$

where V_i is the volume of node i . The time interval Δt is calculated such that only one node is filled at each time step. For the next time step, the calculations are repeated and the flow of the two immiscible fluids is solved. Two dimensionless groups emerge depending on the boundary conditions and they are defined as follows

$$N_{caP} = \frac{R^2 \Delta P}{N L_{ij} \gamma} \quad (78)$$

where N_{caP} is the capillary number for constant pressure displacement and

$$Ca = \frac{Q_D N N_{caP}}{8M} \quad (79)$$

where Ca is the capillary number for constant rate displacement. Here, Q_D is the dimensionless flow rate calculated for an $N \times M$ network.

Figure 75: Immiscible displacement for $n = 1$, $N_x = N_y = 100$ at four different time steps: (a) 90 (b) 150 (c) 300 (d) 390.

6.3.1 Constant Pressure Drop Displacement

Simulation results were used to understand the effect of power-law index n on displacement patterns. For convenience, we have chosen the value of consistency index K to be the same as the viscosity of the Newtonian fluid for all the values of n used. As the simulation procedure involves highly nonlinear iterative calculations, networks of small size (25×25) were selected. The results were verified with larger networks of size 100×100 in few cases, however, to ensure their validity. The simulation patterns reported below are for two values of N_{caP} . In all Figures, black color refers to the Newtonian fluid and white color represent the invading non-Newtonian fluid.

Figure 75 shows displacement patterns for $n=1$ (Newtonian fluid) which serves as a basis of comparison with different values of n (with $N_{caP}=0.02$). The mobility ratio for this case is unity and we observe more or less stable displacement in this range of capillary number (10^{-3}) in agreement with Lenormand et al. [75]. Typical results of two-phase flow in the network for both shear thinning ($n = 0.8, 0.4$ and 0.2) and shear thickening ($n = 1.5$) are shown in Figures 76-79 for the same capillary number. In all Figures, four displacement patterns (a)-(d) are shown at different time steps.

Figure 76 shows displacement patterns for shear thinning fluids with $n = 0.8$. We observe patterns quite similar to the Newtonian case although we see little tendency for fingering and trapping during displacement. The pattern is very close to that of unstable displacement. Further lowering of the power-law index shows instability (Figure 77, $n = 0.4$), the interface advancing with large fingering and bypassing. Viscous fingers, developed right after the onset of flow, penetrate rapidly through the Newtonian fluid. For $n=0.2$, the flow of the invading fluid increases rapidly through the network and we see faster movement of the fingers. In general, we observe excessive tendency for fingering, trapping and bypassing in the shear thinning case, particularly at low values of n . These results are consistent with the previous flow patterns of single-phase flow [108]. Indeed, one would anticipate that the shear thinning fluid would select a narrow flow path, and thus show a pronounced tendency for fingering. The opposite picture develops for the shear thickening case (Figure 79, $n=1.5$).

Figure 76: Immiscible displacement for $\alpha = 0.8$ ($N = 0.02$) at four different time steps: (a) 90, (b) 180, (c) 270, (d) 330.

Figure 17: Thin cable displacement for $n = 0.4$, $N = 7$, $\gamma = 0.02$ at four different times: (top-left) 60 (b) 120 (c) 150 (d) 180.

Figure 8: Images of the parameter θ and λ obtained at different times from a given initial condition.

Figure 79: Minimum cable displacement for $\alpha = 1.5 \times 10^{-4}$, $\beta = 0.02$ at four different time steps (a) 90, (b) 150, (c) 300, (d) 420.

Figure 80: Change in Ca with time step for $n < 1$.

It is observed that the interface is quite compact with a very small amount of trapping at the indicated capillary number. In these constant pressure drop simulations, the capillary number Ca increases as the shear thinning fluids advances through the network (Figure 80). The increase is very steep for smaller values of n as can be predicted from the macroscopic law for power-law fluids. For the shear thickening case, on the other hand, Ca decreases with time.

In the above, we found excessive fingering and tendency for unstable displacement as n decreases. As is the case with Newtonian fluids, this can be explained in terms of the mobility ratio. We decrease the value of n , the shear thinning tendency of the fluids is more visible, the mobility ratio, being the ratio of the viscosity of the Newtonian fluid to the average effective viscosity of shear thinning fluid, increasing dramatically. Typical simulation results for $n=0.2$ but with different viscosity for the Newtonian fluid (0.001 poise) are shown in Figure 81 for a 50×50 network. We observe that the tendency for fingering and bypassing is still unabated. In all these results, the capillary number N_{caP} is equal to 0.1 for the displacement to be purely viscous. For such small values of n the pattern is one of unstable

displacement dominated by viscous fingering, irrespective of the displaced phase viscosity in the observed range. After a certain time, which varies with N_{caP} , the finger accelerates and a very high value of flow rate through the network is obtained.

6.3.2 Constant Flow Rate Displacement

In the constant flow rate problem, the invading fluid is injected at a constant rate through the inlet face, while the pressure at the outlet face is constant. This type of displacement is of great practical interest. Values of the power-law parameters K and n based on [23] were used in the numerical simulation. Before we proceed with the numerical simulations, we shall describe some macroscopic results.

Pascal [91, 92, 93, 94] reported a theoretical work on the stability of a moving macroscopic interface in porous media in the case of immiscible displacement at a constant rate for power-law fluids. In the absence of gravitational and capillary effects, the stability condition reads

$$\frac{\mu_2}{k_2}V - \frac{\mu_{eff}}{k_1}V^n < 0 \quad (80)$$

and if we assume $k_1 = k_2$,

$$\frac{\mu_2}{\mu_{eff}}V^{1-n} < 1 \quad (81)$$

Thus, stability requires V to be low for shear thinning fluids ($n < 1$) and high for shear thickening fluids ($n > 1$). This prediction is to be tested in the pore network displacement process.

To characterize the non-linear immiscible displacement, the following dimensionless number is introduced

$$N_P = \frac{q}{R\left(\frac{K}{\mu}\right)^{\frac{1}{1-n}}} \quad (82)$$

where q is the flow velocity, R is a characteristic radius of the network and K and n are the usual power-law parameters. The simulation procedure is as described in the case of

Figure 1: Four square images showing a noisy, black and white texture. Each image contains a jagged, white, stepped line (resembling a staircase or a series of connected horizontal and vertical segments) that runs diagonally across the frame. The lines are positioned in the top-left, top-right, bottom-left, and bottom-right corners of the images.

n	K ($Psec^{n-1}$)
0.75	0.17
0.60	0.43
0.48	1.95
0.35	6.20

Table 4: Power-law parameters (from [23]).

constant pressure. The major difference is that to maintain a constant injection rate. The following mass balance at the inlet face must be solved

$$\sum_{k=1}^M g_k (P_{in} - P_k)^{1/n} = Q \quad (83)$$

where P_{in} is the pressure at the inlet face, P_k is the pressure at the nodes adjacent to the inlet nodes and Q is the prescribed constant flow rate. P_{in} is calculated iteratively until convergence. We typically use a network of size 25×25 .

Numerical simulations were carried out for four different types of shear thinning fluids for the parameters of Table 4. The displacement patterns are for two types of shear thinning fluids and for a wide range of capillary numbers. The power-law parameters n and K are 0.35 and 6.20 $Psec^{n-1}$ for the first invading fluid, which is highly shear thinning in nature. A similar set of results is also reported for the other fluid with $n = 0.6$ and $K = 0.43$ $Psec^{n-1}$, which can be considered as moderately shear thinning. The simulations were made to understand the effects of capillary number and power-law parameters on the displacement patterns.

Figures 82 to 83 show displacement patterns for the first case. Figure 82 shows the displacement at four different time steps for a high value of the capillary number ($Ca=3.33 \times 10^{-1}$). At such high velocity, the displacement is similar to the viscous fingering domain of a Newtonian fluid. As the injection rate decreases to $Ca=3.33 \times 10^{-4}$, (Figure 83), we observe less tendency for fingering and bypassing and percolation-like patterns are emerging. We

1. The first image shows a dark, noisy background with a white, pixelated, stepped pattern on the left side. The pattern is composed of small, white, rectangular blocks arranged in a jagged, vertical line.

(a)

(b)

(c)

(d)

Figure S3: Constant rate (0.01 cm/sec) immiscible displacement for $n = 0.35$ and $K = 6.20$ ($P_{sc} = 1$, $\phi C_0 = 3.33 \times 10^{-4}$ and $N_0 = 0.0171$) at four different time steps (a) 120 (b) 210 (c) 270 (d) 360.

thus observe the transition from viscous fingering as the capillary forces increase. Further decrease in the capillary number leads to the capillary fingering domain. Figures 84 and 85 show displacement patterns for the other shear thinning fluid. Although, the tendency for viscous fingering, channeling etc. has decreased, the general trend in the two patterns remains the same. It will be interesting to verify the theoretical stability condition by the numerical simulation of the microdisplacement. Simulation results are shown below for a displacement in the absence of capillary forces.

Figures 86 to 88 show displacement patterns for the shear thinning fluid with power-law parameters $n = 0.35$ and $K = 6.20 \text{ Psec}^{n-1}$ at various injection rates. As required by the macroscopic analysis, we have progressively reduced the injection velocity in order to study the transition from instability to stability. Figure 86 shows the displacement patterns for a high injection rate of 1 cm/sec at four different time steps. We observe pronounced viscous fingering with the invading non-Newtonian fluid rapidly trying to break-through. As we decrease the injection velocity to 0.01 cm/sec (Figure 87), the tendency for viscous fingering has disappeared and the interface advances uniformly. Figure 88 represents displacement patterns for the much lower injection velocity of 0.0001 cm/sec. We now observe almost piston-like displacement of the interface and the tendency for viscous fingering no longer exists for such low velocity. The opposite results were obtained for shear thickening fluids with power-law parameters $n = 2.0$ and $K = 0.1 \text{ Psec}^{n-1}$ as shown in Figures 89 and 90. Now, macroscopic stability requires high interfacial velocities. Figure 89 shows displacement patterns for an injection velocity of 0.00001 cm/sec. We observe unstable interface and fingering. As we increase the injection rate to 0.01 cm/sec piston-like movement of the interface sets in (Figure 90). Both these results confirm the validity of the macroscopic predictions of Pascal [93].

6.4 DRAINAGE INVOLVING TWO POWER-LAW FLUIDS

In the previous section, we discussed drainage patterns when the displacing fluid is non-Newtonian but the displaced fluid is Newtonian. It is also important to understand the

Figure S5: Constant rate 0.01 cm/sec nanoparticle displacement for $\epsilon = 0.06$ and $K = 0.43$. $D_{\text{sec}} = 1$, $C_{\text{in}} = 3.44 \times 10^{-3}$ and $N = 0.26$ at four different times: top-left 180, top-right 240, bottom-left 360 and bottom-right 420.

Figure 86: Contact rates of column cable displacement for $\alpha = 0.35$ and $K = 0.20$ $Psec^{-1}$ at four different times: (top-left) 60, (top-right) 90, (bottom-left) 120, and (bottom-right) 150.

Figure 1: (a) and (b) are the original images, (c) and (d) are the images after $\gamma = 0.5$ and $\gamma = 0.20$. $P_{\text{max}} = 0.0001$ and $\sigma = 0.0001$ are used in the proposed algorithm.

Figure 88: Constant rate (0.0001 cm/sec) minimum cable displacement for $\nu = 0.35$ and $K = 6.20$ $Psec^{-1}$ at four different time steps: (a) 60, (b) 180, (c) 240, (d) 360.

(a)

(b)

(c)

(d)

Figure 90. Constant rate (0.01 cm/sec) immiscible displacement for $n = 2.0$ and $K = 0.1$ $Psec^{-1}$ at four different time steps (a) 60 (b) 180 (c) 240 (d) 330.

patterns when both displacing and displaced fluids are of the power-law type [94]. The principle of the simulation is as described previously except that the viscosities of both displacing and displaced fluids are now varying with the shear rate. We shall study the case where the power-law index n is the same for both displacing and displaced fluids, but the consistency index K differs. The two appropriate dimensionless numbers are the modified capillary number Ca and the ratio of consistency indices M , defined as

$$Ca = \frac{K_1 q^n}{R^{n-1} \gamma} \quad (84)$$

and

$$M = \frac{K_1}{K_2} \quad (85)$$

where K_1 and K_2 denotes the consistency indices for displacing and displaced fluids, respectively, and R is a characteristic radius of the network. The network size was 25×25 . We carried out a sensitivity analysis for the effect of capillary number (at constant M) and the effect of M (at constant Ca). Simulations were run until a steady state was reached. In all simulations, black denotes pores occupied by the invading fluid. The power-law index n was chosen as 0.6 for the two non-Newtonian fluids.

In the first series, the value of M was set to 14.7. At high flow rates, the patterns are characteristic of viscous fingering (Figures 91), while at low Ca , the displacement patterns is very similar to capillary fingering (Figures 92 and 93). In the second series, M was set equal to 0.07. The resulting flow patterns are shown in Figure 94 for a low capillary number. We can observe a stable displacement as in the Newtonian case when a more-viscous fluid is injected. In all simulations, we observe the transition from one pattern to the other depending on the relative ratio of viscous to capillary effects.

6.4.1 Domain Boundaries

Comparison of the above results for $n=0.6$ with other simulations for $n=0.4$ indicated that all the three displacement domains are observed, although the boundaries of the domains

Figure 1. The four images show the same page of text, but with different levels of noise and distortion. The images are arranged in a 2x2 grid. The top-left image is the original scan, the top-right image is a noisy version, the bottom-left image is a distorted version, and the bottom-right image is a noisy and distorted version.

 

Figure 10: Capillary fingering at $\phi_c = 0.001$ and $\phi_c = 0.01$ with side displacement for $\phi_p = 0.001$ (top left), $\phi_p = 0.01$ (top right), $M = 1$ at two different time steps: $\tau = 0.001$ (bottom left) and $\tau = 0.01$ (bottom right).

Figure 91: Stable displacement at constant rate (0.001 cm/sec) immiscible displacement for $n_1 = n_2 = 0.6$, $C/a = 0.005$ and $M = 0.07$ at four different time steps (a) 30 (b) 90 (c) 150 (d) 210.

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Figure 95: Schematic phase diagram for immiscible displacement for non-newtonian fluids.

change depending on the power-law index n . The three basic domains can be mapped on a plane with axes Ca and M as shown in Figure 95. A detailed discussion of these three domains for Newtonian fluids was presented by Lenormand [71]. Here, we shall extend his analysis to power-law fluids. The principle of the calculation is the same as in the Newtonian case [71]. We define the limit Ca^* or M^* of each domain such that

$$|S(M^*) - S(\text{plateau})| = \epsilon \quad (86)$$

where S is the saturation at breakthrough and ϵ is a small constant. The analysis proceeds as follows:

Consider a pore network of size $L \times L$ of bond length equal to a . The bonds are uniformly distributed. For single-phase flow of a power-law fluid, the pressure gradient $\Delta P/L$ across the network is expressed in terms of the flow rate q , the Newtonian permeability k and the consistency index of invading fluid K_2 as mentioned in our previous DOE report

$$q^n = \frac{k_1 A k_1^{(n-1)/2} \Delta P}{mL} \quad (87)$$

where A is related to the formation factor σ/σ_o and the exponent n

$$A = [(\sigma/\sigma_o)^{0.436} \phi^{0.0638}]^{n-1} \left[\frac{(10.62)^{1-n} 3^{1+n} (23.51 t_n)^{n t_n}}{2^{n+2} \left(\frac{1+3n}{n}\right)^n (3 + t_n)^{1.12+2.76n}} \right] \quad (88)$$

The mean conductance g_o for a power-law fluid in a single capillary is

$$g_o = \frac{\pi^n}{2K_2 a} R^{3n+1} \left(\frac{n}{3n+1} \right)^n \quad (89)$$

Following the arguments of Lenormand [71] and based on percolation theory we can express

$$L/a \propto (p_c - p^*)^{-\nu} \quad (90)$$

$$N_b/L^2 \propto (p_c - p^*)^\beta \quad (91)$$

and the conductance of the percolation cluster is expressed by

$$G/g_o \propto (p_c - p^*)^{n t_n - \nu(n-1)} \quad (92)$$

or G can be expressed exactly as

$$G = (p_c - p^*)^{n t_n - \nu(n-1)} \frac{a^{2n-1} R^{n+1}}{2K_2} \left(\frac{n}{3n+1} \right)^n \quad (93)$$

where p_c and p^* denote the percolation threshold for a network of size L and for an infinite network, respectively. Here, exponent t_n is also dependent on the power-law index n . Now, due to viscous effects, the pressure will increase at the entrance. When we inject the fluid with the higher consistency index, we shall have

$$\Delta P = \frac{Q^n}{G} \quad (94)$$

The corresponding variation of size of the accessible throats is

$$\frac{\Delta R}{R_o} = R_o \Delta P / \gamma \quad (95)$$

while the variation Δp of the fraction of accessible bonds is

$$\Delta p = \Delta R / 2\sigma R_o \quad (96)$$

where σ is the standard deviation for the distribution of bonds. The corresponding variation of bonds in the cluster is obtained by taking the derivative of equation (91)

$$\Delta N / L^2 \propto (p_c - p^*)^{\beta-1} \Delta p \quad (97)$$

Substituting the values of Δp , ΔR and g_o from equations (96), (95) and (92) respectively, we get

$$\frac{\Delta N}{L^2} \propto \frac{(p_c - p^*)^{(\beta-1)-(nt_n-\nu(n-1))} Q^n K_1 \left(\frac{3n+1}{n}\right)^n}{\sigma \gamma a^{2n-1} R^n} \quad (98)$$

We define capillary number as

$$C_a = \frac{K_1}{4R^{n-1}\gamma} \left(\frac{3n+1}{n}\right)^n q^n \quad (99)$$

To obtain the limiting value Ca^* on the boundary we equate the relative variation $\Delta N/N_b$ in the number of bonds in the percolation cluster to ϵ

$$\left(\frac{L}{a}\right)^{(1+nt_n+\nu)/\nu} \frac{4a}{R} \frac{Ca^*}{\sigma} = \epsilon \quad (100)$$

to obtain the result

$$Ca^* = B\epsilon\sigma \left(\frac{L}{a}\right)^{-(1+nt_n+\nu)/\nu} \frac{R}{a} \quad (101)$$

where B is a prefactor and can be evaluated from numerical simulation for a particular value of power-law index.

When we inject the power-law fluid with a lower consistency index, the fraction of accessible channels decreases by a value of Δp due to fingering. The decrease in pressure is given from equation (87) by assuming that the displaced fluid flows in the entire network [71]

Following the same approach as before, we obtain the corresponding variation of bonds in the network as

$$\frac{\Delta N}{L^2} \propto \frac{(p_c - p^*)^{\beta-1} q^n K_2 L a^{n+1}}{2(\frac{\pi}{8})^{(n+1)/2} R^{2n+1} A \sigma \gamma} \quad (102)$$

or, after substitution,

The relative variation $\Delta N/N_b$ of the number of bonds in the percolation cluster is given by

$$\frac{\Delta N}{N_b} = \left(\frac{L}{a}\right)^{(1+\nu)/\nu} \frac{2a^{n+2} C a^*}{A \sigma M R^{n+2} (\frac{\pi}{8})^{(n+1)/2}} = \epsilon \quad (103)$$

and, finally

$$C a^* = C \epsilon \sigma A \left(\frac{\pi}{8}\right)^{\frac{n+1}{2}} \left(\frac{L}{a}\right)^{-(1+\nu)/\nu} \frac{R^{n+2}}{a^{n+2}} M \quad (104)$$

The value of the prefactor C can be estimated from simulation results for specific values of the power-law index.

We may also derive boundary equations pertaining to the viscous and capillary limits for the viscous fingering domain. For the viscous limit we get

$$(M^*)^{1/n} = \epsilon \left(\frac{a}{L}\right) \frac{(\frac{n}{3n+1}) a^{\frac{n+1}{n}}}{R^{\frac{n+1}{n}} A^{1/n} (\frac{\pi}{8})^{\frac{n+1}{2n}}} \quad (105)$$

while for the capillary limit we get

$$C a^* = \frac{D \sigma}{\epsilon A (\frac{\pi}{8})^{\frac{n+1}{2}}} \left(\frac{a}{L}\right)^n \left(\frac{R}{a}\right)^{n+2} \left(\frac{n}{3n+1}\right)^n M \quad (106)$$

where D is a prefactor to be determined from simulation results.

Finally, the boundaries for the stable displacement are obtained. For the capillary limit we have

$$Ca^* = \frac{E\sigma A}{\epsilon} \left(\frac{R}{a}\right)^{n+2} \left(\frac{3n+1}{n}\right)^n \left(\frac{\pi}{8}\right)^{\frac{n+2}{2}} \quad (107)$$

while for the viscous limit the simple result is found

$$M^* = F\epsilon \quad (108)$$

where E and F are the prefactors to be determined from simulation. These results are different from the Newtonian case, with which they coincide in the limit $n = 1$.

6.5 CONCLUSIONS

In this report we described our ongoing research related to the flow of non-Newtonian fluids in porous media. Most of the work presented pertained to power-law fluids in pore networks, where extensional flow effects were neglected. Extension of the research to cover more general viscoelastic fluids is currently under consideration. For power-law fluids we considered the comparison of the numerical simulation results with previous theoretical expressions based on EMT and CPA. Except for very small values of n , the latter was found to be in excellent agreement with the simulations pointing out to the usefulness of CPA-derived estimates for power-law fluid flow in porous media. Numerical simulation of the flow of Bingham plastics, taken to represent the behavior of some heavy oils, showed the existence of a macroscopic yield stress for the onset of flow. This behavior resembled somewhat that of power-law fluids at small values of the exponent n . We subsequently described simulation for the displacement (drainage) of one fluid by another of different power-law indices. Displacement patterns similar to the Newtonian case, namely percolation, viscous fingering and compact displacement were found. For the case of fluids of equal power-law indices, but of different consistency indices, we were able to classify the patterns extending Lenormand's phase diagram to power-law fluids.

7 MINIMUM PRESSURE GRADIENT PREDICTION FOR THE ONSET OF FOAM MOBILIZATION

H. Kharabaf

7.1 INTRODUCTION

During gas injection in EOR (Enhanced Oil Recovery) poor sweep efficiency as a result of gravity override or viscous fingering, sharply reduces the oil production. Foams can improve reservoir sweep by reducing gas mobility. In porous media, foam is a dispersed gas (or immiscible liquid) trapped by interconnected thin liquid films called lamellae. These thin films are stabilized by conjoining/disjoining pressure introduced by surfactants. Mobility reduction in foam injection is mainly possible by the bubble-pore wall interactions which cause lamellae to deform, divide, break or regenerate, and as a result block the high permeability gas pathways.

The major effects of using surfactants in EOR are as follows:

- Formation of series of lamellae in the gas pathway, leading to an increase in the pressure gradient;
- Blockage of streaks and fractures or high permeability regions, thereby reducing gravity override and viscous fingering, which are the results of large differences between densities and viscosities of gas and liquid phases, and obtaining better sweep efficiency;
- Reduction of the interfacial tension (IFT), between oil and aqueous phase, thus increasing the ease of oil phase mobilization.

Realizing the potential for these purposes needs a better understanding of foam properties and the mechanisms by which they act. A literature survey shows a variety of empirical and theoretical models of foam mobility. The studies may be divided into two groups, high and low quality foams [101].

The first group of studies refers to low or moderate pressures and high qualities of foam (high gas fractional flow). In these strong foams, lamellae are stable enough to move from pore to pore without breaking. Whereas the displacement of a lamella from pore throat needs sufficiently high pressure drop across the lamella ($\Delta P^{max} = 4\gamma/r_{min}$, where γ is interfacial tension between wetting and non-wetting phases and r_{min} is considered here to be equal to pore throat radius, r_t), there is a minimum pressure gradient ∇P^{min} , or minimum gas velocity v_g^{min} , for both the propagation and the generation of the foam. These studies [25, 46, 57, 58, 59, 62, 69, 80] are usually conducted in high permeability porous media, dry foams and low liquid saturation. Rossen and Gauglitz [101] presented network models to show the various problems involved with a strong foam that has a yield stress. In their model the onset of foam flow depends strongly on a small fraction of largest pore throats in the porous media. They also predict that the onset of foam flow in a Bethe lattice may lead to a new kind of percolation phenomena.

The second group of studies [35, 42, 43, 64] refer to higher pressures and lower foam qualities. These studies suggest that lamellae do not travel in the porous media, but break and regenerate at certain points. Hence, there may not exist a minimum pressure gradient for either generation or propagation of the foam. Chou [35] points out that foam could be readily formed by co-injecting gas and surfactant solution whenever the core is presaturated by surfactant solution regardless of flow rate or pressure gradient. He has also presented [34] a network model and used percolation theory to describe foam behavior in porous materials.

Jimenez and Radke [61] presented studies concerning the stability of lamellae during displacement from pore throats. They have given a relation between critical velocity and critical capillary pressure P_c^* , at which the film breaks in a given medium of fixed wetting liquid saturation. According to this criterion for static foam, P_c^* depends on S_w , absolute permeability and surfactant formulation. For a dynamic foam, it would also depend on

Figure 96: A conjoining/disjoining pressure isotherm for the constant-potential and weak overlap electrostatic model (from [60]).

velocity and pore geometry. This condition implies that there exists a critical wetting phase saturation below which foam cannot remain stable. In dynamic foams, if the flow rate is low, there is enough time for the liquid to fill in the lamella. This causes the film thickness h to remain higher than the minimum value h_{min} where the lamella ruptures (Figure 96). Given a value P_c^* , there is a specific flow rate at which the lamellae break, thus P_c^* is flow-rate dependent. The model contains two terms that account for the film thickness change, one due to pore geometry and the other due to capillary-pressure-driven influx/efflux. The second term is inversely proportional to the capillary number, N_{ca} . If N_{ca} becomes large, the second term is negligible, namely the time for the lamellae to pass through the pore throat are too small. Whereas, if N_{ca} becomes small, the second term becomes more important and P_c^* becomes equal to Π^{max} , where Π^{max} is the maximum of conjoining/disjoining pressure.

Some experimental results cannot be explained by the available theories. Baghdikian and Handy [9] have shown that it is possible to generate foam in porous media at relatively low flow velocities after long periods of time have elapsed, referred to as “incubation time”. This time period indicates the stage during which there is no significant change in the pressure gradient. Following this stage, there is a sudden increase of ∇P due to foam generation

Figure 97: Pressure gradient history with simultaneous flow of gas and surfactant solution with prefoamer at low gas velocity (from [95]).

(Figure 97). However, in these experiments it takes several hundred pore volumes of fluid injection before the pressure build-up. At present, there is no compelling theory that explains the mechanisms involved in such experiments.

We study foam behavior and particularly the *onset* of foam mobilization in porous media by simulating the problem using a numerical network model. The model is capable of calculating ∇P^{min} for mobilizing the first lamella at different initial percolation probabilities p^o , as a representative of different initial liquid saturation S_w^o . To make the comparison easier, most of the physical data and parameters chosen here are the same as reported in literature [102].

7.2 MODEL AND ASSUMPTIONS

The porous media is simulated by two- and three-dimensional lattices having coordination numbers 4 and 6. Bonds and nodes are representing pore throats and sites, respectively. For size distribution of either throats or sites, a Rayleigh distribution with different average

radii ($\overline{r}_i$) was used

$$\alpha_i(r) = \frac{\pi r}{2\overline{r}_i^2} \exp\left[-\frac{\pi r^2}{4\overline{r}_i^2}\right] \quad (109)$$

where i denotes throat or site, and $\overline{r}_s = 1.5\overline{r}_t$. A periodic boundary is used to reduce boundary effects in the network.

7.2.1 Drainage

Initially the system is assumed to be fully occupied by liquid, such as surfactant solution. This liquid is next displaced by injecting gas from one end. At low rates this process is described by invasion percolation model. Drainage is stopped at a percolation probability p_b^o , representing a certain saturation value S_w^o , where

$$p_b = \int_{r_{min}}^{\infty} \alpha_i(r) dr \quad (110)$$

and r_{min} is the minimum pore throat size in invaded phase. A given point p_b^o can be related to S_w^o by simply considering the relation of saturation to the occupied site fraction, $P(p)$.

Invasion percolation model was first proposed by Chandler *et al.* [28] and Wilkinson and Willemsen [121] to describe capillary controlled fluid flow in a porous medium. For the non-wetting phase to displace a wetting phase, pressure drop ΔP across the interface must be exceeded. For a cylindrical throat of radius r (after neglecting gravitational forces), this capillary pressure is

$$\Delta P = \frac{2\gamma}{r} \cos\theta \quad (111)$$

where θ is the contact angle, which depends on fluids, type of rock and roughness of the surface. We shall assume $\theta = 0$, and estimate r by its smallest value r_t (at the pore throat). Then,

$$\Delta P = \frac{2\gamma}{r_t} \quad (112)$$

In this model we consider trapping for each phase. Trapping is a mechanism which needs to be considered during invasion percolation, whenever flow in the roughness is negligible. Under such conditions, this part of the wetting phase which has been surrounded by invading phase cannot escape or be swept. Figure 98 shows trapping of wetting phase in a 100×100 network at breakthrough (where $p = p_c$, and p_c is the percolation threshold). Drainage may continue to the desired value of p° or until all sites in the system are trapped or occupied (Figure 99).

Lamellae Generation

There are three mechanisms for lamellae generation in porous media, *Leave-Behind*, *Snap-Off* and *Lamella-Division* (Figure 100). Since we are interested in the onset of foam mobilization, all lamellae are stationary, hence we need not consider *Lamella-Division*, which is due to film division caused by foam movement. However, *Leave-Behind* and *Snap-Off* mechanisms will be retained. These are explained below:

Leave-Behind. During drainage, there are many places in the network, where two neighboring pore bodies are drained but the throat between them is not. Such trapped liquids at the pore throats form lenses. In case the entry capillary pressure is high enough (but not larger than P_c^*) the trapped liquid can drain through the solid corners and roughness to thin the lenses and generate lamellae. The mechanism occurs during the drainage process whenever pore throat trapping is permitted.

Snap-Off. Another important way of foam generation in porous media is snap-off, which is mainly a function of pore geometry [27], and like *Leave-Behind* mechanism, it may occur even in the absence of surfactant in the system. However, the role of surfactant is fundamental, as mentioned earlier, in stabilizing thin liquid films by exerting disjoining pressure and stopping liquid drainage, through (high capillary pressure suction) Plateau borders (see Figure 101). Snap-off is the result of an instability due to the curvature difference between pore throats and pore bodies which provides higher capillary pressure or lower liquid pressure at the pore

Figure 98: A 100×100 Network at breakthrough ($p^o = 0.5$). Displacement is from left to right.

Figure 99: Pore level occupancy after drainage ($p^o = 0.99, p = 0.9$). Displacement is from left to right.

"Leave-Behind" Mechanism

"Snap-Off" Mechanism

"Lamella Division" Mechanism

Figure 100: Principal mechanisms of foam generation in porous media (after Friedman and Jensen [46]).

Figure 101: A foam lamella terminated at Plateau borders [48].

throats (assuming uniform pressure over the gas phase). This draws more liquid towards the throat (via solid corners) and eventually generates a liquid film at the point (Figure 102). For this mechanism to occur, the pore body-to-the-pore throat size ratio (*aspect ratio*) needs to be higher than a specific value, believed to be between 1.5-2.5 [97, 73, 87, 78]. Snap-off is possible during both drainage and imbibition. However, because of the need for sufficient wetting phase saturation to allow liquid to return to the pore throat for snap-off, we neglect snap-off during drainage and consider this mechanism only during imbibition. The assumption is that the liquid phase is interconnected throughout the system, therefore at each step the smallest pore throat will snap-off.

7.3 SIMULATION

The process we attempt to simulate is the following: the porous medium is drained to a certain level denoted by p^o , and it is subsequently subject to imbibition during which p is decreased. During the latter process we allow for snap-off at the throats. The IFT is assumed to be $30mN/m$ (*dyne/cm*). This value applies for Nitrogen and a variety of

Figure 102: Drainage displacement, film deposition and snap-off of a wetting liquid in a constricted cylindrical pore [48].

surfactant solutions. The average radius for the bonds ($\bar{r}_t$) is set to $50\mu m$ and for the sites ($\bar{r}_s$) to $75\mu m$, respectively.

To calculate ∇P^{min} , we use the equation

$$\nabla P^{min} = \min\left(\frac{\Delta P}{L}\right) \quad (113)$$

where L is the network length given by

$$L = N\ell \quad (114)$$

ℓ is the average pore length assumed to be equal to $150\mu m$, N is the network size and ΔP is the pressure drop across the network

$$\Delta P = P_{in} - P_{out} \quad (115)$$

In calculating ΔP , we assume that pore throats are cylindrical of radius r_t , where Poiseuille's law applies,

$$Q_{ij} = g \frac{\Delta P_{ij}}{\mu_g} \quad (116)$$

Here, Q_{ij} and ΔP_{ij} are flow rate and pressure difference between two neighboring pores (i and j), μ_g is gas viscosity (here taken as $0.02cp$) and g is the conductance,

$$g = \frac{\pi r_t^4}{8\ell} \quad (117)$$

Assuming incompressible fluid and using mass conservation for any invaded pore site (m), one can write

$$\sum_{k=1}^n Q_{mk} = 0 \quad (118)$$

where k denotes neighboring site. The above provides a system of equations which can be solved for pressure by using any iterative or exact matrix solution method. Here we use point *SOR* (Successive Over Relaxation). In the equations, the throat conductance g is assumed to be zero whenever the pore throat is not invaded or contains a lamella.

After the pressure at each node is found, the program locates the particular throat (with radius r_t^*), the lamellae of which is capable of being mobilized. Then ∇P^{min} is calculated from

$$\nabla P^{min} = \frac{4\gamma}{L(r_t \Delta P_{ij})^*} \quad (119)$$

It must be pointed out that all these calculations are in the continuous gas flow regime, where $p > p_{c,b}$ ($p_{c,b}$ is the bond percolation threshold, equal to 0.5 for square and 0.25 for cubic lattices). In this case, the invading phase is fully connected across the network and there always exist a pressure gradient in the phase. If lamellae generation by snap-off mechanism continues, the gas phase becomes discontinuous and a pressure gradient in the system no longer exists. In the porous medium the gas phase is comprised of three fractions: trapped gas, backbone and dead-ends. In finding ∇P^{min} , the simulator checks to find the lamella which resides on the backbone.

7.4 RESULTS AND DISCUSSION

Figure 103: Plot of ∇P^{min} as a function of the network size ($p^o = 0.99$ and snap-off is stopped at $p = 0.54$).

Figures 103 and 104 show ∇P^{min} for a variety of network sizes (in 2-D and up to 200×200), for the two cases of $p = 0.54$ and $p = 0.60$. In contrast to what was expected, the magnitude of ∇P^{min} is continuously decreasing by increasing the network size. Figure 105 shows the results of the same experiments for ∇P^{min} as a function of p , for different sizes in 3-D. Similar trends exist in both 2-D and 3-D lattices.

Another interesting observation is that the value of ∇P^{min} is constant with respect to variations of p (Figure 105). This is also true for 2-D networks (Figure 106). Evidently, for any given size, ∇P^{min} is almost constant at different snap-off (percolation) probabilities. It is interesting to know which lamellae are involved in calculating ∇P^{min} . Two cases will be considered, one in which lamellae generation occurs only by the snap-off mechanism and another one in which it occurs only by leave-behind. These results are plotted in Figures 107 and 108.

Figure 107, where ∇P^{min} is calculated only by considering snap-off mechanism in a

Figure 104: Plot of ∇P^{min} as a function of the network size. In all the cases ($p^o = 0.99$ and snap-off is stopped at $p = 0.60$).

Figure 105: Effect of size on ∇P^{min} .

Figure 106: Effect of size on ∇P^{min} in 2-D.

Figure 107: Variation of ∇P^{min} , when only snap-off mechanism is allowed in a $25 \times 25 \times 25$ Network.

Figure 108: Variation of ∇P^{min} , when only leave-behind mechanism is allowed in a $25 \times 25 \times 25$ Network.

$25 \times 25 \times 25$ network, shows that the magnitude of ∇P^{min} is increasing very sharply by increasing p . This trend is consistent with Rossen and Gauglitz' theory [102], where a simple percolation model in the continuous gas regime is used to calculate ∇P^{min} . In their model, the projection of the length between two points where the path meets the backbone L' is given by

$$L' \cong \frac{3}{2} \bar{n} \ell \quad (120)$$

where, the average number of pores in a gas flow channel blocked by a single pore throat, $\bar{n}$, was found by Rossen [100] to be

$$\bar{n} = \frac{d \ln B(p)}{d \ln p} \quad (121)$$

and $B(p)$ is the backbone fraction of the network. Then, ∇P^{min} is calculated from the following equation

$$\nabla P^{min} = \frac{4\gamma/\bar{r}_{min}}{L'} \quad (122)$$

Figure 108 shows ∇P^{min} values by Leave-Behind lamellae. Here, ∇P^{min} is infinite right before breakthrough. This results from the assumption that there exist no pressure gradient

Figure 109: Minimum pressure gradient when snap-off and leave-behind mechanisms are considered separately ($25 \times 25 \times 25$ Network).

in the gas phase before the percolation threshold, so that $\Delta P_{ij} = 0$ or $\nabla P^{min} \rightarrow \infty$. After breakthrough, the value of ∇P^{min} decreases rapidly and very soon it becomes constant. To understand the reason why ∇P^{min} values are constant, we plot Figure 109, obtained from a combination of Figures 107 and 108, in which both mechanisms for lamellae generation are allowed. One expects that the minimum of the two curves would be found. This is indeed the case as shown in Figure 110.

Comparison between numerical data for various cases shows that r_t and ΔP_{ij} are almost constant for each size (Table 5). This seems reasonable, because leave-behind lamellae are being generated by bond trapping during the invasion percolation process. Any throat, regardless of its size, may be trapped by this mechanism and if the network is large enough, its size statistically would be the average throat size of the network. In addition, as shown in Table 5 the normalized pressure drop across the lamellae is always close to 1. This means that dendrites at two sides of the lamella are meeting at the two opposite ends of the network. For this to be valid, not only such long dendrites must exist, but additional lamellae

Figure 110: Minimum pressure gradient when the two mechanisms are considered together ($25 \times 25 \times 25$ Network).

should not be generated in their clusters during the subsequent imbibition. In other words, these dendrites should be generated right after breakthrough.

To investigate this hypothesis, the fractions of bonds having leave-behind lamellae and the backbone fraction were plotted vs. p in a $25 \times 25 \times 25$ network (Figures 111 and 112, respectively). As shown in Figure 111 there exist sufficient numbers of leave-behind lamellae right after breakthrough. For comparison, the percolation fraction of the bonds $P(p)$ is also plotted. As expected, shortly after the percolation threshold is exceeded, dendrites are quite likely to be as long as the length of the network. As a result, one may conclude that it is possible to have a lamella which separates two long dendrites, each of which is connected to different ends of the system. To illustrate this, a schematic plot in a 54×54 network is shown in Figure 113. The arrow points to the first lamella which would be mobilized in the network.

From the above reasoning, it also becomes evident why the minimum pressure gradient

<i>NetworkDimension</i>	<i>N</i>	<i>p</i>	$r \times 10^5[m]$	ΔP_{ij}
<i>2 - D</i>	54×54	0.93	4.606	0.67
		0.824	4.6068	0.67
		0.505	4.6068	0.668
	65×65	0.651	4.76	0.786
		0.56	4.76	0.791
		0.509	4.601	0.805
	100×100	0.987	4.801	0.771
		0.656	4.801	0.771
	150×150	0.60	4.649	0.872
		0.54	4.649	0.872
	200×200	0.60	4.455	0.936
		0.54	4.455	0.936
<i>3 - D</i>	$15 \times 15 \times 15$	0.551	6.6423	0.968
		0.456	6.6423	0.968
		0.259	6.6423	0.9701
	$20 \times 20 \times 20$	0.892	6.139	0.998
		0.47	6.139	0.998
		0.278	6.139	0.998
	$25 \times 25 \times 25$	0.597	6.521	0.996
		0.375	6.521	0.996
		0.315	6.521	0.995
	$30 \times 30 \times 30$	0.928	6.323	0.999
		0.7276	6.323	0.999
		0.325	6.323	0.999

Table 5: Pressure Drop Across Initially Mobilized Lamella

Figure 111: Fraction of bonds having leave-behind lamellae, and percolation fraction in a $25 \times 25 \times 25$ Network.

Figure 112: Backbone and percolation fractions ($25 \times 25 \times 25$ Network).

Figure 113: Pore level occupancy for a 54×54 Network. Arrow indicates the leave-behind lamella which is mobilized first.

Figure 114: ∇P^{min} and $\nabla P^{min} \cdot N$ vs. p in 3-D Networks $N= 5 , 10 , 15 , 20 , 25 , 30$.

for foam mobilization is not a function of p when both mechanisms are taken into account. This reasoning also explains the dependence of ∇P^{min} on system length. Indeed, if ΔP_{ij} is approximately 1 and r_i equals the average size of the network, then it is clear that the following scaling

$$\nabla P^{min} \propto L^{-1} \quad (123)$$

should apply. Figure 114 shows ∇P^{min} and $\nabla P^{min} \cdot N$ vs. p for different network sizes. It is evident that all the data collapse to the same value, suggesting that $\nabla P^{min} \cdot N$ is constant, which confirms the above argument.

7.5 CONCLUSIONS

The network model introduced here shows that at the onset of foam mobilization, the first lamella displaced from pore throats is the one that has been generated by a leave-behind mechanism. The reason for this lies in the fact that, in invasion percolation, dendrites as long as the network length are being generated right after the percolation threshold and leave-behind lamellae formed at this stage are the best candidates for the onset of mobi-

lization. Because the pressure differences across these films are almost equal to the overall pressure drop of the networks, even small pressure gradients may initiate foam mobilization. This conclusion is opposite to the minimum pressure gradient arguments based on snap-off mechanisms.

TECHNICAL REPORTS

The following technical reports resulted from the research conducted in the previous year.

“Chemical Composition Profiles During Alkaline Flooding at Different Temperatures and Extended Residence Times” by Roshanak Aflaki and Lyman L. Handy (September 1992)

“Flow and Displacement of Power-Law Fluids in Porous Media” by Charul Shah and Yanis C. Yortsos (October 1992)

“Capillary Effects in Drainage in Heterogeneous Porous Media: Continuum Modeling, Experiments and Pore Network Simulations” by Baomin Xu and Yanis C. Yortsos (March 1993)

“Alkaline Assisted Thermal Oil Recovery: Kinetic and Displacement Studies” by Shahrouzeh Saneie and Yanis C. Yortsos (April 1993)

“Visualization and Simulation of Immiscible Displacement in Fractured Systems Using Micromodels: I. Drainage” by Manouchehr Haghighi, Baomin Xu and Yanis C. Yortsos (April 1993)

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