Proton-Proton Scattering Contribution to Emittance Growth

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1776276-113

JUN 20 1993

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T. Garavaglia

Superconducting Super Collider Laboratory[†]
2550 Beckleymeade Avenue
Dallas, TX 75237

May 1993

* To be presented at the 1993 IEEE Particle Accelerator Conference on May 17-20, Washington, D.C.

[†] Operated by the Universities Research Association, Inc., for the U.S. Department of Energy under Contract No. DE-AC35-89ER40486.

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Proton-Proton Scattering Contribution to Emittance Growth

T. Garavaglia

Superconducting Super Collider Laboratory*
2550 Beckleymeade Ave., Dallas, TX 75237

Abstract

Proton-proton scattering contributes to the emittance growth of the SSC. A formulation is given and used to estimate the mean scattering angle, which is used to determine the contribution to SSC emittance growth resulting from elastic pp scattering. The method is based upon Lorentz invariants, and it permits the determination of the cross-section for scattering in the center of mass (c.m.) system, as well as scattering from a fixed target (f.t.). Also an example is given for the case of electromagnetic pp scattering, which results from single virtual photon exchange.

I. INTRODUCTION

The contribution from pp elastic scattering to transverse emittance growth is determined for the SSC. Elastically scattered protons with a small scattering angle will remain within the Collider proton beam. These scattered particles contribute to the growth of the beam's transverse emittance. Numerical results for emittance growth resulting from pp scattering and other sources are given in [1]. In this paper, a summary is given of the analytical methods that are used to determine the mean scattering angle resulting from pp elastic scattering. Lorentz invariants and cross-sections are defined in Appendix A.

II. PROTON-PROTON ELASTIC SCATTERING

The contribution to transverse emittance growth, for one degree of freedom, resulting from pp elastic scattering is given by

$$\left(\frac{d\epsilon_x}{dt}\right) = (\beta_{1x}^* \mathcal{L}_1 + \beta_{2x}^* \mathcal{L}_2) \sigma_{el} \langle \theta_x^2 \rangle / (MN_B), \quad (2.1)$$

where β_i^* and $\mathcal{L}_i$ are, respectively, the beta function and the luminosity at the i^{th} interaction point. In this expression N_B is the number of protons per bunch, M is the number of bunches, σ_{el} is the pp elastic scattering cross-section, and $\sqrt{\langle \theta_x^2 \rangle}$ is the rms value of the pp elastic scattering angle in the center of mass system, which is projected onto the transverse x -direction. A similar expression occurs for the transverse y -direction. The mean scattering angle can

be estimated from the differential elastic scattering cross-section

$$\left(\frac{d\sigma_{el}}{d\Omega}\right)_{c.m.} = \frac{S}{4\pi} \left(\frac{d\sigma_{el}}{dT}\right)_{c.m.}, \quad (2.2)$$

where S and T are Lorentz invariants. These invariants, defined in (A1), are $S \approx 2E_{c.m.}$, and $T \approx -S \sin^2(\theta/2) \approx -S(\theta_x^2 + \theta_y^2)/4$. The invariant differential cross-section for pp elastic scattering is [2]

$$\frac{d\sigma_{el}}{dT} \approx \frac{\sigma_T^2}{16\pi} (1 + \rho^2) e^{bT}, \quad (2.3)$$

where σ_T is the total pp cross-section, b is the slope parameter, and $\rho \approx 0$ is the ratio of the real part to the imaginary part of the scattering amplitude. Assuming that the slope parameter is a constant, one can integrate the differential cross-section to obtain $b \approx \sigma_T^2 / 16\pi \sigma_{el}$. With the approximation $\sigma_{el} \approx (1/4)\sigma_T$, one finds $b \approx \sigma_T / 4\pi$.

Using the approximations above, the differential cross-section in the c.m. system becomes

$$\left(\frac{d\sigma_{el}}{d\Omega}\right)_{c.m.} \approx \frac{S\sigma_T^2}{64\pi^2} e^{\left(\frac{\theta_x^2}{2\sigma_{\theta_x}^2} + \frac{\theta_y^2}{2\sigma_{\theta_y}^2}\right)}, \quad (2.4)$$

where

$$\sigma_{\theta_x} = \sqrt{\langle \theta_x^2 \rangle} = (bS/2)^{-1/2}. \quad (2.5)$$

This is the expression to be substituted into (2.1) to find $d\epsilon_x/dt$. For colliding proton beams with $\sqrt{S} = 40$ TeV and $\sigma_T \approx 130$ mb, one finds $b \approx 26.6$ GeV $^{-2}$ and

$$\sigma_{\theta_x} = \sqrt{\langle \theta_x^2 \rangle} = 6.9 \mu\text{rad}. \quad (2.6)$$

In the above, the value of the total pp scattering cross-section is determined from

$$\sigma_T = 38.5 + 1.33 \ln^2\left(\frac{\sqrt{S}}{10 \text{ GeV}}\right), \quad (2.7)$$

which is obtained from cosmic ray data [3]. Theoretical models giving values for the total and elastic pp cross-sections can be found in [4].

Using the above techniques, one can estimate the rms scattering angle $\sqrt{\langle \theta^2 \rangle}$ for the scattering of a proton in a 20 TeV beam from a fixed proton. The scattering angle in the c.m. system for high energy pp scattering is found from

$$\cos(\theta) \approx \frac{2T}{S} + 1. \quad (2.8)$$

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The scattering angle for a proton of energy $\omega = S/2m$ from a proton at rest is found from

$$\cos(\tilde{\theta}) \approx 1 - \frac{2\sin^2(\theta/2)}{(S/m^2)} - \frac{4}{(S/m^2)^2}. \quad (2.9)$$

Using $\cos \tilde{\theta} \approx 1 - (1/2)\sin^2 \tilde{\theta}$, the fixed target scattering angle is related to the c.m. scattering angle θ through

$$\tilde{\theta} \approx \frac{\sqrt{2}}{(\omega/m)} \sqrt{\left(\frac{\theta^2(\omega/m)}{4} + 1\right)}. \quad (2.10)$$

For the scattering of a 20 TeV proton from a proton at rest, the rms scattering angle in the c.m. system is found from (2.5), with $\sqrt{S} = 193$ GeV, $\sigma_T = 50.2$ mb and $b \approx 10.3 \text{ GeV}^{-2}$, to be $\sqrt{\langle \theta^2 \rangle} \approx 2.3$ mrad. The corresponding angle in the fixed target system is found to be $\sqrt{\langle \tilde{\theta}^2 \rangle} \approx 67 \mu\text{rad}$.

III. PROTON-PROTON ELECTROMAGNETIC SCATTERING

In this example, pp scattering is treated as an electromagnetic event, and the scattering of two fermions of initial four-momenta a and b to a final state of four-momenta c and d results from the exchange of a single virtual photon. The system of units $\hbar = c = m = 1$ is used. Since both the initial and final states involve identical particles, these states must be antisymmetrical. The initial state $|I\rangle$ and the final state $|F\rangle$ are represented as

$$|I\rangle = \frac{|ab\rangle - |ba\rangle}{\sqrt{2}} \quad |F\rangle = \frac{|cd\rangle - |dc\rangle}{\sqrt{2}}. \quad (3.1)$$

The matrix element for this process is

$$\begin{aligned} \langle F|M|I\rangle &= [(cd|M|ab) - (cd|M|ba) + \\ &\quad (dc|M|ba) - (dc|M|ab)]/2 \\ &= e^2[J^\mu(d,b)D_{\mu\nu}(a-c)J^\nu(c,a) - (d \leftrightarrow c)], \end{aligned} \quad (3.2)$$

where the fermion current is $J^\mu(c,a) = \bar{u}_c \gamma^\mu u_a$. The photon propagator is $D_{\mu\nu}(a-c) = 4\pi g_{\mu\nu}/((a-c)^2 + i\epsilon)$. We use the fermion density matrix $\rho_{ij}(a) = u_{ai}\bar{u}_{aj}$, which has the property $\text{Tr}\rho(a) = 2$.

The invariant differential cross-section for this process is

$$\frac{d\sigma_{el}}{dT} = \frac{1}{64\pi f(S, a, b)} \mathcal{M}(S, T), \quad (3.3)$$

where $\mathcal{M}(S, T) = |\langle F|M|I\rangle|^2$. For the scattering of unpolarized fermions when the polarization of the final state fermions is not observed, the initial state spin density matrices for a and b are of the form $\rho_0(a) = (\not{1} + 1)/2$. For the final states $|c\rangle$ and $|d\rangle$, which include a summation over the final spin states, the density matrices are multiplied by

two. The invariant differential cross-section for this case is now found to be

$$\begin{aligned} \frac{d\sigma_{ee}}{dT}(ab \rightarrow cd) &= \frac{\pi e^4}{64S(S/4 - 1)} [A(S, T, U) + A(S, U, T) \\ &\quad - B(S, T, U) - B(S, U, T)]. \end{aligned} \quad (3.4)$$

The invariant functions are

$$A(S, T, U) = \frac{4}{T^2} T^{\mu\nu}(d, b) T_{\mu\nu}(c, a) \quad (3.5)$$

$$B(S, T, U) = \frac{4}{TU} T^{\mu\nu}_{\mu\nu}(b, c, a, d), \quad (3.6)$$

where

$$T^{\mu\nu}(c, a) = \text{Tr}[(\not{c} + 1)\gamma^\mu(\not{a} + 1)\gamma^\nu], \quad (3.7)$$

$$T^{\mu\nu\lambda\sigma}(b, c, a, d) = \text{Tr}[\gamma^\mu(\not{b} + 1)\gamma^\nu(\not{c} + 1)\gamma^\lambda(\not{a} + 1)\gamma^\sigma(\not{d} + 1)]. \quad (3.8)$$

Upon evaluation of the traces, the invariant functions become

$$A(S, T, U) = \frac{32}{T^2} [S^2 + U^2 + 8T - 8] \quad (3.9)$$

$$B(S, T, U) = -\frac{32}{TU} [S^2 - 8S + 12]. \quad (3.10)$$

In the high energy limit when S becomes large, one finds

$$\frac{d\sigma_{el}}{dT}(ab \rightarrow cd) \approx \frac{2\pi e^4}{T^2}. \quad (3.11)$$

The rms value of the c.m. scattering angle associated with (3.11) can be found using (2.8) in the form

$$\langle \cos \theta \rangle \approx 1 - \langle \theta^2 \rangle / 2 = 2 \langle T/S \rangle + 1, \quad (3.12)$$

where

$$\langle T \rangle = \int_{T_{min}}^{T_{max}} T(d\sigma_e/dT) dT / \sigma_e, \quad (3.13)$$

and $\sigma_e = \int (d\sigma_e/dT) dT$. The rms value of the scattering angle is written in terms of the projections on the transverse directions as $\sqrt{\langle \theta^2 \rangle} = \sqrt{(\langle \theta_x^2 \rangle + \langle \theta_y^2 \rangle)/2}$.

The integration limits are found from $T \approx \theta^2/4$, where θ_{max} is found from the condition that the proton is scattered from the beam, and where θ_{min} is found from the uncertainty principal, $\Delta 2\sigma_{beam} \Delta \theta \approx \hbar/p$. One finds $\sqrt{\langle \theta_x^2 \rangle} \approx 1. \mu \text{ rad}$.

APPENDIX A: Kinematics and Cross-Sections

In this appendix, the kinematical variables and cross-sections used in the analysis are given. Particles characterized by four-momenta a and b interact elastically to yield particles characterized by four-momentum c and d . For this process, energy-momentum conservation is represented as $a + b = c + d$, where a typical four-vector is represented as $a = (a^0, \mathbf{a})$, such that $a^2 = (a^0)^2 - \mathbf{a} \cdot \mathbf{a} = m_a^2$. The

interaction channels are defined according to the Lorentz invariants

$$S = (a+b)^2, T = (a-c)^2, U = (a-d)^2, \quad (A1)$$

which satisfy $(S+T+U) = a^2 + b^2 + c^2 + d^2$.

In the c.m. system, one finds the invariant expressions for energy, momentum, and scattering angle

$$\mathcal{E}_a = \mathcal{E}(S, a, b) = (S + a^2 - b^2)/4(S/4)^{1/2}, \quad (A2)$$

$$\mathcal{E}_b = \mathcal{E}(S, b, a), \mathcal{E}_c = \mathcal{E}(S, c, d), \mathcal{E}_d = \mathcal{E}(S, d, c),$$

$$|a| = |b| = [f(S, a, b)/4S]^{1/2}, |c| = |d| = [f(S, c, d)/4S]^{1/2}, \quad (A3)$$

and

$$\cos \theta_{ac} = (T - a^2 - c^2 + 2\mathcal{E}_a\mathcal{E}_b)/2|a||c|, \quad (A4)$$

with

$$f(S, a, b) = [S - (m_a + m_b)^2][S - (m_a - m_b)^2]. \quad (A5)$$

In the fixed target system, the corresponding relations are

$$\omega_a = \omega(S, a, b) = (S - a^2 - b^2)/2m_b,$$

$$\omega_b = m_b, \omega_c = -\omega(U, c, b), \omega_d = -\omega(T, d, b), \quad (A6)$$

$$|a| = f^{1/2}(S, a, b)/2m_b, |b| = 0,$$

$$|c| = f^{1/2}(U, c, b)/2m_b, |d| = f^{1/2}(T, b, d)/2m_b, \quad (A7)$$

and

$$\cos \tilde{\theta}_{ac} = [2b^2(T - a^2 - c^2) - (S - a^2 - b^2)(U - b^2 - c^2)]/[f(S, a, b)f(U, c, b)]^{1/2}. \quad (A8)$$

The differential cross-sections are found from the definition of the invariant total cross-section for the interaction of two particles initially in the states $|a\rangle$, and $|b\rangle$ and the subsequent production of an n -particle final state, where each particle is characterized by a momentum state $|p_i\rangle$.

This cross-section is defined as

$$\sigma(S, T) = \frac{1}{2f^{1/2}(S, a, b)(2\pi)^{3n-4}} \times \int dp_1 dp_2 \dots dp_n \prod_{i=1}^n \delta(p_i^2 - m_i^2) \theta(p_i) \times \delta(a + b - \sum_{i=1}^n p_i) \mathcal{M}(S, T, U), \quad (A9)$$

with $\theta(p) = [(p^0/\omega) + 1]/2$, $\omega = (|\mathbf{p}|^2 + m^2)^{1/2}$, and

$$\mathcal{M}(S, T, U) = |\langle f | \mathcal{M} | a, b \rangle|^2,$$

where $\langle f | \mathcal{M} | a, b \rangle$ is the transition amplitude from the initial to the final state. In (A9), one uses an invariant definition of the flux, which is represented as the magnitude

of the relative velocity $|\mathbf{v}_a - \mathbf{v}_b|$ in the c.m. system. The flux becomes $F = f^{1/2}(s, a, b)/2\mathcal{E}_a\mathcal{E}_b$.

Particular differential cross-sections may now be obtained from (A9). Of special interest is the differential cross-section defined formally as

$$\frac{d\sigma}{dT} = \sigma(S, T) \delta[T - (a - c)^2]. \quad (A10)$$

For scattering into the solid angle $d\Omega_{ac}$, one finds for elastic scattering in the c.m. system

$$\frac{d\sigma}{d\Omega_{c.m.}} = \frac{1}{4\pi S} f^{1/2}(S, a, b) f^{1/2}(S, c, d) \frac{d\sigma}{dT}. \quad (A11)$$

The corresponding differential cross-section in the f.t. system may be found from (A8) and (A10) to be

$$\frac{d\sigma}{d\Omega_{f.t.}} = \frac{-2f^{1/2}(S, a, b)f^{3/2}(U, c, b)}{\pi g(S, T, 1, m, 1, m)} \frac{d\sigma}{dT}. \quad (A12)$$

For the elastic scattering of a particle of unit mass with a particle of mass m , one finds the expression

$$g(S, t, 1, m, 1, m) = 128m^2[(S/4)^2 + ST/16 - (S/4)(m^2 + 1)/2 - (T/4)(m^2 - 1)/4 + (m^2 - 1)^2/16]. \quad (A13)$$

The integration indicated in (A9) and (A10), when there is a two-particle final state characterized by four-momenta c and d , is performed in the c.m. system using the momentum-space measure

$$dcdd = |c|^2 d|c| d\Omega_c \frac{dc^2}{2c^0} dd \frac{dd^2}{2d^0} \quad (A14)$$

to find

$$\frac{d\sigma}{dT} = \frac{1}{16\pi f(S, a, b)} \mathcal{M}(S, T). \quad (A15)$$

IV. REFERENCES

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