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Moments of Ambient Doppler Spectra

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Moments of Ambient Doppler Spectra*

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Abstract

We studied the first four moments (center of mass, standard deviation, skew, and kurtosis) of the Doppler spectra in ambient regions of LLNL-Hughes real aperture radar data collected during WCSEX91-92. Our goal was to correlate trends in the moments with wind velocity and direction.

Although the center of mass appears to increase when the wind is blowing into the radar antenna, no other conclusions have been drawn from the higher order moments.

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1 Definition of Continuous and Discrete Moments

Let $g(x)$ be a continuous function defined over the support $[a, b]$ with $g(x) \geq 0$ for $x \in [a, b]$. The k^{th} moment of $f(x)$ is defined as:

$$\mathcal{M}_k = \int_a^b x^k g(x) dx \quad (1)$$

The zeroth moment is the mass:

$$\mathcal{M}_0 = \int_a^b g(x) dx \quad (2)$$

The first moment is the center of mass, μ :

$$\mu = \frac{1}{\mathcal{M}_0} \int_a^b x g(x) dx \quad (3)$$

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The central moments are defined as the moments with respect to the center of mass:

$$C_k = \frac{1}{\mathcal{M}_0} \int_a^b (x - \mu)^k g(x) dx \quad (4)$$

where

$$C_0 = \frac{1}{\mathcal{M}_0} \int_a^b g(x) dx = 1$$

$$C_1 = \frac{1}{\mathcal{M}_0} \int_a^b (x - \mu) g(x) dx = 0$$

In the work presented here, all moments above the first (center of mass) are central moments.

For the discrete case, let $D(n)$ be a discrete function defined over $n \in [0, N-1]$ with $D(n) \geq 0$. Let $D(n)$ have sample interval Δx and origin x_0 . The moments of $D(n)$ are defined as:

$$\mathcal{M}_k = \frac{\Delta x}{\mathcal{M}_0} \sum_{n=0}^{N-1} (x_0 + n\Delta x)^k D(n) \quad (5)$$

$$C_k = \frac{\Delta x}{\mathcal{M}_0} \sum_{n=0}^{N-1} (x_0 + n\Delta x - \mu)^k D(n) \quad (6)$$

$$(7)$$

$$\mathcal{M}_0 = \Delta x \sum_{n=0}^{N-1} D(n) \quad (8)$$

$$\mu = \frac{\Delta x}{\mathcal{M}_0} \sum_{n=0}^{N-1} (x_0 + n\Delta x) D(n) \quad (9)$$

2 Doppler Spectrum

The Doppler spectrum is estimated over a few second's worth of data, and is the mean intensity of the Fourier transform of consecutive slices of size N_{dft} through the data for a fixed range. Let $r(m, n) = I(m, n) + iQ(m, n)$ be the complex IQ data where $0 \leq m \leq M-1$ is the range index, and $0 \leq n < \infty$ is the time index. Then the definition of the Doppler spectrum is:

$$D(m, l) = \frac{1}{K} \sum_{k=0}^{K-1} |\mathcal{F}_t \{r_k(m, n) w(n)\}|^2 \quad (10)$$

where K is the number of time slices through the data, $w(n)$ is a smoothing window, l is the discrete frequency index, and $r_k(m, n)$ is the k^{th} slice through the data, defined as:

$$r_k(m, n) = r(m, n + kN_{\text{dft}}) \quad 0 \leq n \leq N_{\text{dft}} - 1 \quad (11)$$

and $\mathcal{F}_t\{\}$ is the DFT of r_k with respect to time:

$$\mathcal{F}_t \{r_k(m, n)\} = \frac{1}{N_{\text{dft}}} \sum_{n=0}^{N_{\text{dft}}-1} r_k(m, n) (-1)^n \exp\left(-2\pi i \frac{nl}{N_{\text{dft}}}\right) \quad (12)$$

Note the multiplication by -1 in the DFT. This technique is known as "sign modulation" and has the effect of moving the origin in the frequency domain to the center of the data set. See [1] for a description of this procedure which we call "togglng".

In the discrete frequency domain, the sample interval is:

$$\Delta f = \frac{1}{N_{\text{dft}} \Delta t}$$

and, as a result of the toggling, the origin is:

$$f_0 = \frac{-1}{2 \Delta t}$$

With the Doppler spectrum defined in (10), the moments were calculated from a mean spectrum in an ambient region. Thus, picking an m' in an ambient region, we computed a mean spectrum:

$$\hat{D}(l) = \frac{1}{W} \sum_{m=m'-W/2}^{m'+W/2-1} D(m, l)$$

and estimated the moments from $\hat{D}(l)$:

$$\mathcal{M}_k = \frac{\Delta f}{\mathcal{M}_0} \sum_{l=0}^{N_{\text{in}}-1} (f_0 + l\Delta f)^k \hat{D}(l) \quad (13)$$

$$\mathcal{C}_k = \frac{\Delta f}{\mathcal{M}_0} \sum_{l=0}^{N_{\text{in}}-1} (f_0 + l\Delta f - \mu)^k \hat{D}(l) \quad (14)$$

$$(15)$$

$$\mathcal{M}_0 = \Delta f \sum_{l=0}^{N_{\text{in}}-1} \hat{D}(l) \quad (16)$$

$$\mu = \frac{\Delta f}{\mathcal{M}_0} \sum_{l=0}^{N_{\text{in}}-1} (f_0 + l\Delta f) \hat{D}(l) \quad (17)$$

Note if $\hat{D}(l)$ went negative, that is $\min_l[\hat{D}(l)] < 0$, we shifted it positive by subtracting $\min_l[\hat{D}(l)]$:

$$\hat{D}(l) \rightarrow \hat{D}(l) - \min_l[\hat{D}(l)]$$

For the data presented here, the Doppler spectra were estimated over 30 seconds of data with a time slice size of $N_{\text{dft}} = 128$ returns. The number of range points over which the mean spectra were computed were $W = 5$.

Unfortunately, due to the relatively long temporal sample interval in the LLNL-Hughes RAR, some of the Doppler spectra are aliased. To eliminate this problem, we judiciously “unwrapped” the spectra, taking into account the wind direction with respect to the radar and using the fact the spectra are band limited. That is, we estimated where the upper limit of the aliased spectrum ended, then cut and barrel shifted the data from that frequency. Figure(1) shows data from the 1991 experiment overlayed with its unwrapped spectrum. The cut was made around -53 Hz as indicated by the arrow. Appendix A provides a *Mathematica* if not mathematical justification for this procedure.

3 Results and Conclusions

The graphs in Figures (2) through (5), show the first four moments (center of mass, μ ; standard deviation, $\sigma = \sqrt{\mathcal{C}_2}$; cubed root of skew, $\sqrt[3]{\mathcal{C}_3}$; and the fourth root of kurtosis, $\sqrt[4]{\mathcal{C}_4}$) for HH and VV as a function of wind direction with respect to the radar and wind velocity. Wind direction is across the horizontal axis and velocity is on the vertical. A direction of 0° is into the radar while 180° is from the back. The third axis is the specified moment represented by the colored dots. The lighter the color, the higher the value of the moment. The mapping of the color scale is given at the bottom of each plot.

For comparison, we show two cases: 1) we have included unwrapped spectra (Figures (2) and (3)), and, 2) we have no unwrapped (thus no aliased) spectra (Figures (4) and (5)). The plots tend to agree thus supporting the unwrapping.

Figure 1: Example of aliased and unwrapped spectra.

Includes Unwrapped Spectra

Figure 2

Includes Unwrapped Spectra

Figure 3

No Unwrapped Spectra

Figure 4

Figure 5

The easiest to interpret is the center of mass. We expect to have the highest center of mass (in frequency) when the wind is blowing into the radar and the lowest or most negative when the wind is from behind. This causes wind driven currents toward and away from the radar respectively, and is confirmed in the graphs.

We are currently developing models for the Doppler spectra and further interpretation of the data will have to be performed when these have been implemented.

A Justification of Doppler Unwrapping

In working with the Doppler spectra of the LLNL-Hughes RAR, we were obliged to “unwrap” spectra aliased because of the under sampling of the radar. Under the normal Nyquist sampling constraint, aliased data cannot be recovered. However, using the additional knowledge of wind direction, the band-limited nature of the spectra, and the location of the centroid is within a band of ± 150 Hz, we chose an appropriate place to cut and unwrapped the spectra. A question arose to the validity of this procedure.

We present here a *Mathematica* if not mathematical justification why the aliased Doppler spectra from WCSEX91 and WCSEX92 may be unwrapped, in fact “unaliaed”, using prior knowledge.

A.1 Manufactured Doppler Spectrum

We manufactured the following Doppler spectrum:

$$D(f) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left[-\frac{1}{2} \left(\frac{f - f_c}{\sigma} \right)^2 \right] \quad (18)$$

where $f_c = 100$ and $\sigma = 25$ are the Doppler centroid and standard deviation respectively and are based upon the actual values taken from the 1991-92 data for the case of the wind blowing into the radar.

The analytical Fourier transform of $D(f)$ is:

$$S(t) = \exp [-2\pi i f_c t] \exp \left[-2(\sigma \pi t)^2 \right] \quad (19)$$

The Fourier transform pair, $D(f)$ and $S(t)$, are plotted in Figure 6.

In 1991, the time sample interval, Δt , was approximately 6 milliseconds or 167 Hz. In 1992, it was 3.5 milliseconds or 286 Hz. Notice a choice for the cutoff frequency of the manufactured Doppler spectrum is above 200 Hz. Thus, to sample the time domain signal correctly, we would require a sample frequency above 400 Hz based upon the Nyquist criteria.

In Doppler processing the LLNL-Hughes RAR data, time slices of $N = 128$ points were used. Thus, sampling with a time interval of Δt results in a sample rate of $f_s = 1/\Delta t$ and a sample interval in the frequency domain of $\Delta f = 1/(N\Delta t)$. Sampling the analytical time domain signal $S(t)$ symmetrically about the origin results in a start time of $t_0 = -N\Delta t/2$. Before performing the DFT, the time samples are sign modulated (or “toggled”) which moves the frequency domain origin to the center of the series. Thus the starting frequency of the sampled Doppler spectrum is $-1/(2\Delta t)$.

We present here three cases:

- $S(t)$ is sampled at the 1991 rate of 6 ms;
- $S(t)$ is sampled at the 1992 rate of 3.5 ms;
- $S(t)$ is sampled at 2.5 ms = 400Hz (twice a cutoff frequency of 200Hz), which we call a “Sanity Check” and is used to verify the correct functioning of the Mathematica code.

Table 1 lists the various parameters discussed above for each case.

Figure 7 shows the real and imaginary parts of $S(t)$ and its sampled version at the 1991 sample interval of 6 milliseconds. Note the undersampling. Figure 8 shows the DFT of the sampled signals in Figure 7 plotted with the manufactured Doppler spectrum. The aliasing is obvious. With the knowledge the true spectrum is band limited and the wind is blowing into the radar, we know the negative frequency components are aliased

Figure 6: Manufactured Doppler spectrum (top) and the magnitude of its Fourier transform (bottom). The centroid and standard deviation of the manufactured Doppler spectrum were chosen to match actual data.

PARAMETER	1991	1992	Sanity Check
f_s (Hz)	167	286	400
Δt (ms)	6	3.5	2.5
t_0 (s)	-0.384	-0.224	-0.16
Δf (Hz)	1.3	2.2	3.125
f_0 (Hz)	-83	-143	-200

Table 1: Sampling frequency, f_s , sampling interval, Δt , sampling origin, t_0 , frequency interval, Δf , and frequency origin, f_0 , of the test cases. For all cases there are 128 sample points.

and should be moved into positive frequency bins. Thus, we chose to cut the sampled spectrum where it is close to zero, at around 11 Hz, and “unwrap” the spectrum. Figure 9 shows the true and unwrapped spectra. The match is very good. If we define an error as:

$$\epsilon = \sqrt{\sum_{n=0}^{N-1} [D(f_w + n\Delta f) - U(n)]^2} \quad (20)$$

where $U(n)$ is the unwrapped spectrum and f_w is the frequency about which the spectrum is unwrapped, then for 1991, the error is 2.6×10^{-4} .

Figures 10, 11, and 12, show the same procedure applied using the 1992 sample interval of 3.5 milliseconds. For this case the unwrapping cut was made at 0 Hz, and the error is 7.4×10^{-6} .

Finally, the sanity check is in Figures 13 and 14. We are relieved to see there is no aliasing and the discrete and sampled spectra match very well. Thus the Mathematica procedure functions as expected. The error is 6.7×10^{-6} , and is defined as:

$$\epsilon = \sqrt{\sum_{n=0}^{N-1} [D(f_u + n\Delta f) - D_d(n)]^2} \quad (21)$$

where $D_d(n)$ is the discrete, correctly sampled spectrum.

A.2 Conclusion

We presented a simple analytical model for a Doppler spectrum with parameters based upon actual data, and its time domain counterpart. We sampled the spectrum at the rates used in the LLNL-Hughes RAR for the WCSEX91-92 experiments and demonstrated the ability to recover aliased data by “unwrapping” the discrete spectrum. Finally, we presented a “sanity check” on the procedure to verify its functioning.

References

- [1] E. M. Johansson and J. P. Fitch. *Remarks on Translation by Sign Modulation*, IEEE Trans. Signal Processing, April 1991

Figure 7: Real and imaginary parts of the analytic time domain signal, $S(t)$, and the discrete signal sampled at the 1991 interval of 6 ms. Note: only the center part of the signal is shown, there are actually 128 points which have been truncated from the graph because they are zero.

Figure 8: The manufactured Doppler spectrum and 1991 aliased discrete counterpart.

Figure 9: The manufactured Doppler spectrum and 1991 unwrapped sampled spectrum. The cut was made around 11 Hz.

Figure 10: Real and imaginary parts of the analytic time domain signal, $S(t)$, and the discrete signal sampled at the 1992 interval of 3.5 ms. Note: only the center part of the signal is shown, there are actually 128 points which have been truncated from the graph because they are zero.

Figure 11: The manufactured Doppler spectrum and 1992 aliased discrete counterpart.

Figure 12: The manufactured Doppler spectrum and 1992 unwrapped sampled spectrum. The cut was made at 0 Hz.

Figure 13: Real and imaginary parts of the analytic time domain signal, $S(t)$, and the sanity check sampled at 400Hz. The signal is well sampled. Note: only the center part of the signal is shown, there are actually 128 points which have been truncated from the graph because they are zero.

Figure 14: The manufactured Doppler spectrum and 1992 aliased discrete counterpart.

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