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The Evolution of Tensor Polarization *

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Abstract

Using the equation of motion for the vector polarization, the spin transfer matrix for spin tensor polarization is derived. We study the evolution equation for the tensor polarization in the presence of an isolated spin resonance and in the presence of a spin rotator, or snake.

I. INTRODUCTION

Polarized beam is used very often as a research tool to study the fundamental interactions between elementary particles. Much has been written about both the theoretical and experimental aspects of spin polarization in particle accelerator for spin- $\frac{1}{2}$ particles. But it is believed that there has been no systematic study on the possibility of accelerating polarized spin-1 particle (e.g., deuteron) beams. The complexity of spin-1 particle comes from the tensor polarization¹. Recently, two of the authors (Lee and Ratner) studied the possibility of polarized deuteron beams in the AGS and RHIC². To get a basic understanding of the polarized deuteron beam, it is worthwhile to derive the evolution equation of tensor polarization.

Spin motion in circular accelerators follows the Thomas-BMT equation³. In a synchrotron, the Thomas-BMT equation can be rewritten in terms of particle coordinates in the synchrotron, i.e., the Frenet-Serret curvilinear coordinates⁴. Expressing the spin vector in terms of its components,

$$\vec{S} = S_1 \hat{x} + S_2 \hat{y} + S_3 \hat{z}, \quad (1)$$

and defining $S_{\pm} = S_1 \pm iS_2$, the equation of spin motion becomes,

$$\frac{dS_{\pm}}{d\theta} = \pm iG\gamma S_{\pm} \pm iF_{\pm} S_3; \quad \frac{dS_3}{d\theta} = \frac{i}{2}(F_- S_+ - F_+ S_-) \quad (2)$$

where G is the anomalous gyromagnetic g -factor, γ is the relativistic Lorentz factor, and $F_{\pm}$ characterize the spin depolarization kick. The spin tune, $G\gamma$, is the number of

spin precessions per orbit turn. Eq.(2) is the general spin equation of motion for a particle moving in a synchrotron. Given the periodic nature of circular accelerator, $F_{\pm}$ can be Fourier analyzed as

$$F_{\pm} = \sum \epsilon_j e^{-iK_j \theta} \quad (3)$$

where the Fourier amplitude ϵ_j is called the resonance strength, the corresponding frequency K_j is called resonance tune, and θ is azimuthal orbit rotation angle. When particle is moving at constant $G\gamma$, for a single resonance, i.e., $F_{\pm} = \epsilon e^{-iK\theta}$, the equations can be solved analytically. In this paper, we only consider the single resonance case. Transforming the equation of motion into the resonance precessing frame, the eigenvalues σ can be determined as,

$$\sigma = 0, \pm\lambda, \quad \lambda = \sqrt{\delta^2 + |\epsilon|^2} \quad (4)$$

where $\delta \equiv (K - G\gamma)$. The spin closed orbit is given by

$$\hat{n}_{co} = \frac{1}{\sqrt{|\epsilon|^2 + \delta^2}} [\delta \vec{e}_3 + \epsilon_R \vec{e}_1 - \epsilon_I \vec{e}_2] \quad (5)$$

where the unit vectors $\vec{e}_1, \vec{e}_2, \vec{e}_3$ coincide with $\hat{x}, \hat{y}, \hat{z}$ at any azimuth in the ring.

II. TENSOR POLARIZATION

For particles of spin S the polarization state can be completely described by $(2S+1)^2 - 1$ parameters. For spin- $\frac{1}{2}$ particles, three parameters are required for the specification of the most general mixed state. This is referred to as vector polarization. It is just the average value of the particle spin $\vec{S}$, where the average is taken over all the particles in the beam and can be written as $\langle \vec{S} \rangle$. $\langle \vec{S} \rangle$ follows the similar equation as eq.(2) with every spin quantity being averaged. For spin-1 particles, apart from 3 components of the vector polarization, there are in general 5 components of tensor polarization. The explicit forms of the tensor operators in terms of the spin operator $\vec{S}$ are¹;

$$\begin{aligned} \langle T_0 \rangle &= \frac{1}{\sqrt{2}} (3\langle S_3^2 \rangle - 2), \\ \langle T_{\pm 1} \rangle &= \mp \frac{\sqrt{3}}{2} (\langle S_{\pm} S_3 + S_3 S_{\pm} \rangle), \\ \langle T_{\pm 2} \rangle &= \frac{\sqrt{3}}{2} \langle S_{\pm}^2 \rangle \end{aligned} \quad (6)$$

*Work supported by the U.S. DOE under contract number DE-FG02-92ER40747.

Using the spin vector equation of motion, eq.(2), the spin tensor equation of motion for particles moving in a synchrotron can be derived. The bracket notation is dropped for simplicity.

$$\begin{aligned}\frac{dT_0}{d\theta} &= \frac{1}{\sqrt{2}}(3\frac{dS_3}{d\theta}S_3 + 3S_3\frac{dS_3}{d\theta}) \\ &= -\frac{\sqrt{6}}{2}i(F_-T_1 + F_+T_{-1})\end{aligned}\quad (7)$$

$$\begin{aligned}\frac{dT_{\pm 1}}{d\theta} &= -\frac{\sqrt{3}}{2}(\frac{dS_{\pm}}{d\theta}S_3 + S_{\pm}\frac{dS_3}{d\theta} + \frac{dS_3}{d\theta}S_{\pm} + S_3\frac{dS_{\pm}}{d\theta}) \\ &= \pm iG\gamma T_{\pm 1} - \frac{\sqrt{6}}{2}iF_{\pm}T_0 - iF_{\mp}T_{\pm 2}\end{aligned}\quad (8)$$

$$\begin{aligned}\frac{dT_{\pm 2}}{d\theta} &= \frac{\sqrt{3}}{2}(\frac{dS_{\pm}}{d\theta}S_{\pm} + S_{\pm}\frac{dS_{\pm}}{d\theta}) \\ &= \pm 2iG\gamma T_{\pm 2} - iF_{\pm}T_{\pm 1}\end{aligned}\quad (9)$$

These equations can be rewritten in matrix form as,

$$\frac{d}{d\theta} \begin{pmatrix} T_2 \\ T_1 \\ T_0 \\ T_{-1} \\ T_{-2} \end{pmatrix} = A \begin{pmatrix} T_2 \\ T_1 \\ T_0 \\ T_{-1} \\ T_{-2} \end{pmatrix} \equiv AT \quad (10)$$

where A is given by

$$A = \begin{pmatrix} 2iG\gamma & -iF_+ & 0 & 0 & 0 \\ -iF_- & iG\gamma & -\frac{\sqrt{6}}{2}iF_+ & 0 & 0 \\ 0 & -\frac{\sqrt{6}}{2}iF_- & 0 & -\frac{\sqrt{6}}{2}iF_+ & 0 \\ 0 & 0 & -\frac{\sqrt{6}}{2}iF_- & -iG\gamma & -iF_+ \\ 0 & 0 & 0 & -iF_- & -2iG\gamma \end{pmatrix} \quad (11)$$

III. SPIN TENSOR CLOSED ORBIT

For an imperfection resonance, the spin closed orbit for the vector polarization is well-defined. Since the evolution of tensor polarization can be derived from the equation of motion of vector polarization, the tensor polarization should not change when vector polarization is on closed orbit. Then corresponding tensor polarization is spin tensor closed orbit. From section 1, spin closed orbit for vector polarization is

$$\begin{pmatrix} S_3 \\ S_+ \\ S_- \end{pmatrix}_{co} = \frac{1}{\sqrt{2|\epsilon|^2 + \delta^2}} \begin{pmatrix} \delta \\ \epsilon^* \\ \epsilon \end{pmatrix} \quad (12)$$

where ϵ^* is the complex conjugate of ϵ . From the definition of tensor polarization, the corresponding tensor polarization closed orbit is

$$\begin{pmatrix} T_2 \\ T_1 \\ T_0 \\ T_{-1} \\ T_{-2} \end{pmatrix}_{co} = \begin{pmatrix} \frac{\sqrt{3}}{2} \frac{\epsilon^* \delta}{2|\epsilon|^2 + \delta^2} \\ -\frac{\sqrt{3}\epsilon^* \delta}{2|\epsilon|^2 + \delta^2} \\ \frac{1}{\sqrt{2}} \left(\frac{3\delta^2}{2|\epsilon|^2 + \delta^2} - 2 \right) \\ -\frac{\sqrt{3}\epsilon \delta}{2|\epsilon|^2 + \delta^2} \\ \frac{\sqrt{3}}{2} \frac{\epsilon^2}{2|\epsilon|^2 + \delta^2} \end{pmatrix} \quad (13)$$

IV. TRANSFER MATRIX FOR TENSOR POLARIZATION

When particles are moving at constant $G\gamma$, and a single resonance is considered, the equation of motion for tensor polarization can be solved analytically. At a constant spin tune $G\gamma$, the equation of motion can be transformed into the resonance precessing frame by following transformation:

$$T = \begin{pmatrix} e^{i2K\theta} & 0 & 0 & 0 & 0 \\ 0 & e^{iK\theta} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & e^{-iK\theta} & 0 \\ 0 & 0 & 0 & 0 & e^{-2iK\theta} \end{pmatrix} \begin{pmatrix} \tilde{T}_2 \\ \tilde{T}_1 \\ \tilde{T}_0 \\ \tilde{T}_{-1} \\ \tilde{T}_{-2} \end{pmatrix} \equiv \Theta \tilde{T} \quad (14)$$

Then the equation of motion becomes

$$\frac{d\tilde{T}}{d\theta} = \Theta \frac{d}{d\theta} \tilde{T} + \frac{d\Theta}{d\theta} \tilde{T} \quad (15)$$

which can be rewritten in matrix form as,

$$\frac{d}{d\theta} \begin{pmatrix} \tilde{T}_2 \\ \tilde{T}_1 \\ \tilde{T}_0 \\ \tilde{T}_{-1} \\ \tilde{T}_{-2} \end{pmatrix} = R \begin{pmatrix} \tilde{T}_2 \\ \tilde{T}_1 \\ \tilde{T}_0 \\ \tilde{T}_{-1} \\ \tilde{T}_{-2} \end{pmatrix} \quad (16)$$

where

$$R = \begin{pmatrix} -2i\delta & -i\epsilon^* & 0 & 0 & 0 \\ -i\epsilon & -i\delta & -i\frac{\sqrt{6}}{2}\epsilon^* & 0 & 0 \\ 0 & -i\frac{\sqrt{6}}{2}\epsilon & 0 & -i\frac{\sqrt{6}}{2}\epsilon^* & 0 \\ 0 & 0 & -i\frac{\sqrt{6}}{2}\epsilon & i\delta & -i\epsilon^* \\ 0 & 0 & 0 & -i\epsilon & 2i\delta \end{pmatrix} \quad (17)$$

Now we can find the eigenvalues of R. The five eigenvalues are

$$\sigma_{\pm 2} = \pm 2\lambda, \quad \sigma_{\pm 1} = \pm \lambda, \quad \sigma_0 = 0. \quad (18)$$

where λ is given by eq.(4). The system has five eigenvalues, 0, $\pm i\lambda$ and $\pm i2\lambda$ which correspond to five eigensolutions describing the spin tensor along the spin closed orbit and the spin tensors precessing right/left with respect to $\hat{n}_{co}$ with precessing tunes λ and 2λ .

Given the eigenvalues, the equation of motion could have following solutions

$$T_i = A_i^{(\pm)} e^{\pm i2\lambda\theta}; \quad T_i = B_i^{(\pm)} e^{\pm i\lambda\theta}; \\ T_i = C_i, \quad i = 0, \pm 1, \pm 2. \quad (19)$$

where $A_i^{(\pm)}$, $B_i^{(\pm)}$, and C_i are arbitrary coefficients. For each eigenvalue, the 5 coefficients are not independent, they are constrained by the differential equation (10). Because 4 of the 5 equations are independent, the 5 coefficients are reduced to 1 independent coefficient. For 5

eigenvalues, there are 5 independent coefficients which are determined by the initial condition $T_i(\theta = \theta_0)$, where θ_0 is the initial orbit rotation angle. By solving the equation of motion, we can get the spin transfer matrix $\tilde{M}(\theta, \theta_0)$

$$\tilde{T}(\theta) = \tilde{M}(\theta, \theta_0) \tilde{T}(\theta_0). \quad (20)$$

To simplify the formalism, let $\theta_0 = 0$. Define two sets of functions m_i and a_{ij}

$$\begin{aligned} m_{-2}(\sigma_i) &= 1 \\ m_{-1}(\sigma_i) &= -(\sigma_i - 2\delta) \\ m_0(\sigma_i) &= (\sigma_i - \delta)(\sigma_i - 2\delta) - |\epsilon|^2 \\ m_1(\sigma_i) &= -\sigma_i m_0(\sigma_i) - \frac{3}{2}|\epsilon|^2(\sigma_i - 2\delta) \\ m_2(\sigma_i) &= -\frac{1}{\sigma_i + 2\delta} m_1(\sigma_i) \end{aligned} \quad (21)$$

$$\begin{aligned} a_{j2} &= \frac{\epsilon^4}{16\lambda^4} (m_j(2\lambda)e^{i2\lambda\theta} + m_j(-2\lambda)e^{-i2\lambda\theta}) - \\ &\quad \frac{\epsilon^4}{4\lambda^4} (m_j(\lambda)e^{i\lambda\theta} + m_j(-\lambda)e^{-i\lambda\theta}) + \frac{3\epsilon^4}{8\lambda^4} m_j(0) \\ a_{j1} &= -\frac{\epsilon^3}{8\lambda^4} [(\delta + \lambda)m_j(2\lambda)e^{i2\lambda\theta} + (\delta - \lambda) \\ &\quad m_j(-2\lambda)e^{-i2\lambda\theta}] + \frac{\epsilon^3}{4\lambda^4} [(2\delta + \lambda)m_j(\lambda)e^{i\lambda\theta} + \\ &\quad (2\delta - \lambda)m_j(-\lambda)e^{-i\lambda\theta}] - \frac{3\delta\epsilon^3}{4\lambda^4} m_j(0) \\ a_{j0} &= \frac{\sqrt{6}\epsilon^2}{16\lambda^4} [(\delta + \lambda)^2 m_j(2\lambda)e^{i2\lambda\theta} + (\delta - \lambda)^2 m_j(-2\lambda) \\ &\quad e^{-i2\lambda\theta}] - \frac{\sqrt{6}\epsilon^2}{16\lambda^4} [(\delta + \lambda)m_j(\lambda)e^{i\lambda\theta} + \\ &\quad (\delta - \lambda)m_j(-\lambda)e^{-i\lambda\theta}] - \frac{\sqrt{6}\epsilon^2}{8\lambda^4} (|\epsilon|^2 - 2\delta^2) m_j(0) \\ a_{j,-1} &= -\frac{\epsilon}{8\lambda^4} [(\delta + \lambda)^3 m_j(2\lambda)e^{i2\lambda\theta} + (\delta - \lambda)^3 m_j(-2\lambda) \\ &\quad e^{-i2\lambda\theta}] - \frac{\epsilon}{4\lambda^4} [(\lambda - 2\delta)(\lambda + \delta)^2 m_j(\lambda)e^{i\lambda\theta} + \\ &\quad (-\lambda - 2\delta)(-\lambda + \delta)^2 m_j(-\lambda)e^{-i\lambda\theta}] + \frac{3\delta|\epsilon|^2\epsilon}{4\lambda^4} m_j(0) \\ a_{j,-2} &= \frac{1}{16\lambda^4} [(\delta + \lambda)^4 m_j(2\lambda)e^{i2\lambda\theta} + (\delta - \lambda)^4 \\ &\quad m_j(-2\lambda)e^{-i2\lambda\theta}] + \frac{|\epsilon|^2}{4\lambda^4} [(\delta + \lambda)^2 m_j(\lambda)e^{i\lambda\theta} \\ &\quad + (\delta - \lambda)^2 m_j(-\lambda)e^{-i\lambda\theta}] + \frac{3|\epsilon|^4}{8\lambda^4} m_j(0) \end{aligned} \quad (22)$$

Then the transfer matrix for the tensor polarization $\tilde{M}(\theta, 0)$ is,

$$\begin{pmatrix} \frac{2\epsilon^2}{3\epsilon^3} a_{22} & \frac{2\epsilon^2}{3\epsilon^3} a_{21} & \frac{2\epsilon^2}{3\epsilon^3} a_{20} & \frac{2\epsilon^2}{3\epsilon^3} a_{2-1} & \frac{2\epsilon^2}{3\epsilon^3} a_{2-2} \\ \frac{2}{3\epsilon^3} a_{12} & \frac{2}{3\epsilon^3} a_{11} & \frac{2}{3\epsilon^3} a_{10} & \frac{2}{3\epsilon^3} a_{1-1} & \frac{2}{3\epsilon^3} a_{1-2} \\ \frac{2}{\sqrt{6}\epsilon^2} a_{02} & \frac{2}{\sqrt{6}\epsilon^2} a_{01} & \frac{2}{\sqrt{6}\epsilon^2} a_{00} & \frac{2}{\sqrt{6}\epsilon^2} a_{0-1} & \frac{2}{\sqrt{6}\epsilon^2} a_{0-2} \\ \frac{1}{\epsilon} a_{-12} & \frac{1}{\epsilon} a_{-11} & \frac{1}{\epsilon} a_{-10} & \frac{1}{\epsilon} a_{-1-1} & \frac{1}{\epsilon} a_{-1-2} \\ a_{-22} & a_{-21} & a_{-20} & a_{-2-1} & a_{-2-2} \end{pmatrix} \quad (23)$$

V. TRANSFER MATRIX FOR SNAKE

A snake is characterized by the spin rotating angle, ϕ , and the snake axis angle, ϕ_s , with respect to $\hat{z}$ (radially outward direction). A full snake rotates particle spin by π . The transfer matrix for snake with arbitrary ϕ and ϕ_s can be derived easily. To save length, we only consider an accelerator with two snakes with $\phi_s = \frac{\pi}{2}$ and $\phi_s = \pi$, respectively, located at an orbital angle of π from each other. For

$\phi_s = \frac{\pi}{2}$, the spin components at a snake are transformed according to

$$\begin{pmatrix} S_3(\theta^+) \\ S_+(\theta^+) \\ S_-(\theta^+) \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} S_3(\theta^-) \\ S_+(\theta^-) \\ S_-(\theta^-) \end{pmatrix} \quad (24)$$

where the $\theta^\pm$ depict azimuthal orbit rotation angles just before and after the snake.

The transfer matrix of the tensor polarization for a snake with $\phi_s = \frac{\pi}{2}$ can be derived to be:

$$\tilde{T}(\theta^+) = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \tilde{T}(\theta^-) \equiv S(\frac{\pi}{2}) \tilde{T}(\theta^-) \quad (25)$$

Similarly, the tensor polarizations are transformed by the following mapping equation for a snake with $\phi_s = \pi$:

$$\tilde{T}(\theta^+) = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \tilde{T}(\theta^-) \equiv S(\pi) \tilde{T}(\theta^-) \quad (26)$$

Then the one turn map matrix is given by

$$\tilde{T}(2\pi) = S(\frac{\pi}{2}) M(2\pi, \pi) S(\pi) M(\pi, 0) \tilde{T}(0) \quad (27)$$

Since

$$T = \Theta \tilde{T}, \quad \tilde{T}(\theta) = \tilde{M}(\theta, 0) \tilde{T}(0), \quad (28)$$

Finally, $T(\theta)$ is transformed as

$$\begin{aligned} T(2\pi) &= \Theta(2\pi) S(\frac{\pi}{2}) \tilde{M}(2\pi, \pi) \cdot \\ &\quad S(\pi) \tilde{M}(\pi, 0) \Theta^{-1}(0) T(0) \end{aligned} \quad (29)$$

VI. CONCLUSION

We have discussed tools needed for tracking the spin tensor polarization of spin-1 particles in synchrotrons. The tensor transfer matrix was derived for a single resonance and in the presence of snakes. The evolution of tensor polarization in synchrotron can therefore be studied along with the vector polarization.

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