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Lasers**

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Analytic Models for Beam Propagation and Far-Field Patterns in Slab and Bow-Tie X-Ray Lasers

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Abstract. Simplified analytic models for beam propagation in slab and bow-tie x-ray lasers yield convenient expressions that provide both a framework for guidance in computer modeling and useful approximates for experimenters. In unrefracted bow-tie lasers, the laser shape in conjunction with the nearly-exponential weighting of rays according to their length produces a small effective aperture for the signal. We develop an analytic expression for the aperture and the properties of the far-field signal. Similarly, we develop the view that the far-field pattern of refractive slab lasers is the result of effective apertures that are created by the interplay of refraction and exponential amplification. We present expressions for the size of this aperture as a function of laser parameters as well as for the intensity and position of the far-field lineout. This analysis also yields conditions for the refraction limit in slab lasers and an estimate for the signal loss due to refraction.

INTRODUCTION

This work develops a general understanding of the origins of the far-field beam pattern from within the laser. We have used a very simple analytic model as an approximation to investigate the optics of slab and bow-tie x-ray lasers. While this approach is not a substitute for detailed computer models of these systems, the author has found it useful to have a set of approximate analytic expressions to guide her in computer modeling. These expressions may also be useful in roughing-out experimental designs.

In slab lasers, there is a large variation of the index of refraction in the ablation direction (see Fig. 1) that causes refraction of the rays in this direction. In the other directions, the propagation and "vertical" directions, the measured refraction is small and one assumes that the index of refraction is relatively invariant. Recently, in an effort to improve coherence, investigators have changed the shape of the laser in the vertical direction to produce active apertures in that direction; one shape under study is the "bow-tie" (1). We begin by analyzing the aperture for the unsaturated bow-tie and the shape and strength of the signal resulting from this geometry.

Similar calculations for a refracting slab reveal that only part of the slab laser participates in the production of the far-field signal at any given angle near the signal peak, resulting in an internal aperture for this signal. How the aperture

changes as the laser is lengthened or as the slope of the index of refraction changes is a question we address analytically. Related to this question is the issue of how much signal is lost from a slab laser because of refraction.

SLAB AND BOW-TIE LASERS WITH NO REFRACTION

Let us consider first the simplest slab and bow-tie models, with no refraction. To make the analytic problem tractable, we assume that the laser is unsaturated with a small-signal gain g_0 that is uniform throughout. Since we are interested in the far-field image, we calculate the intensity of all rays leaving the laser at a particular angle. For each incremental ray tube comprising the signal at this angle, the strength is just the integral of the equation

$$\frac{dI}{dl} = g_0 I + \epsilon$$

where l is the length along the ray and the source term ϵ accounts for spontaneous emission. The initial value is also taken to be ϵ . The integrated intensity I for a ray tube of length Λ in the laser is then

$$I_\Lambda = \epsilon \frac{e^{g_0 \Lambda} - 1}{g_0} \quad (1)$$

The two geometries are shown in Fig. 1. The x -direction is taken as the direction of ablation while the z -direction is the propagation direction. We study the pattern produced in the far-field by calculating the signal along a line-out at the mid plane of the pattern and parallel to x for the slab. When we neglect refraction in the slab, the problem is symmetrical for the two non-propagating directions; but we also wish to study refraction, and its effect is to deflect the pattern in the x -direction. For the bow-tie, we are interested in the effect of the waist in the vertical direction, so the appropriate line-out is calculated perpendicular to x . We sum all

FIGURE 1. The geometries of a slab (upper sketch) and bow-tie laser (lower sketch) are simplified for this analytic study. The large arrow indicates the direction of the drive laser, x is the ablation direction, and z is the propagation direction.

rays emerging at each angle along the lineout direction with zero inclination in the perpendicular direction using a length-dependent weight given by Eq. (1). The geometric angle ϑ_g is defined by $\tan \vartheta_g = d/L$. Typically, the lasing region is 100–200 μm wide, while the length is several cm, so that $d/L \cong 3 \text{ mrad}$. Clearly, with no refraction, ϑ_g is a natural measure for the limit of the pattern in the far-field. In fact, the calculations show that the intensity of the pattern $I(\vartheta)$ peaks at $\vartheta = 0$ and, in the slab, falls almost linearly to $I(\vartheta_g)$ where

$$\text{Slab:} \quad I(\vartheta_g) = \frac{2}{g_o L} I(0), \quad (2)$$

so that the signal is less peaked for low-gain-length lasers. The bow-tie signal is flatter at angles smaller than ϑ_g and falls precipitously at ϑ_g :

$$\text{Bow-tie:} \quad I(\vartheta_g) = \frac{1}{g_o L} I(0). \quad (3)$$

The ratio of the peaks of the bow-tie and the slab indicates the degree of signal loss in the bow-tie:

$$I(0)_{\text{bow-tie}} = \frac{1}{g_o L} I(0)_{\text{slab}}. \quad (4)$$

The loss is expected to be offset by an improvement in the coherence of the signal as the source size is decreased to an effective aperture at the waist. From Eq. (4) we estimate the size of the aperture to be $\sim d/g_o L$ full-width. In fact, calculations show that 70% of the 0° signal passes through this aperture. The aperture decreases in size as the gain-length increases.

Similarly, in the unrefracted slab at ϑ_g there is an effective aperture of full width $\sim 2d/g_o L$ which accounts for $\sim 70\%$ of the signal at this angle. Of course, at 0° with no refraction, the signal comes from the full width of the slab laser.

SLAB GEOMETRY WITH REFRACTION

We employ a model of the index of refraction $n(x)$, $n(x) = n_0 \exp(bx)$, that allows us to perform the integrations needed to analyze the signal analytically (2). Since b is typically of size $\sim 7 \times 10^{-3} \text{ cm}^{-1}$ and n_0 is close to 1.0 in value, this expression is essentially a linear approximation of the index of refraction profile,

$$n(x) = n_0(1 + bx). \quad (5)$$

Computer simulations of the ablating process made using the LASNEX code show a variation that is better fit to $n(x) = n_0 - n_1 \exp(-cx)$ which, while being an

Figure 2. Schematics of the paths of rays within the refracting slab laser. In our approximations all rays that exit the laser at a particular angle are parallel throughout. The rays that comprise the peak of the far-field pattern are shown in the upper sketch; they are limited to an aperture $A_p(\vartheta_m)$ which is larger than the apertures for rays at greater or lesser angles. Note the symmetries of these rays.

increasing function of x in the lasing region, has a noticeable negative second derivative. However, n_0 and b in Eq. (5) can be chosen to span the total variation in the index of refraction; by the mean-value theorem (3), b will match the actual derivative at some place along the x axis within the laser, which we take to be representative of the laser as a whole. To further simplify the calculations in the refracting laser, we initially ignore all rays but those traversing the full length of the laser.

The variation of the index of refraction shifts the far-field maximum to ϑ_m ,

$$\vartheta_m = bL/2, \quad (6)$$

which increases linearly with both b and the laser length, L . For $b = 7 \times 10^{-3} \text{ cm}^{-1}$ and $L = 3 \text{ cm}$, Eq. (6) gives $\vartheta_m = 10 \text{ mrad}$ which is a typical experimental result.

The paths of full-length rays that contribute to the signal at ϑ_m are symmetrical about the midpoint of the laser as shown in Fig. 2. They are parallel because of the linear index of refraction profile, and they have an internal aperture $A_p(\vartheta_m)$:

$$A_p(\vartheta_m) = d(1 - bL^2/8d). \quad (7)$$

Full-length rays with other exit angles have smaller apertures.

The term in parentheses in Eq. (7) decreases quadratically as L is increased. For $b = .007 \text{ cm}^{-1}$, $d = .015 \text{ cm}$, and $L = 3 \text{ cm}$, $A_p(\vartheta_m) = .007 \text{ cm}$ or 48% of the width d . Because the rays are parallel, $A_p(\vartheta_m)$ is the size of the exit aperture as well.

The refraction limit is reached when the internal aperture of the rays contributing to ϑ_m disappears. This condition defines a critical angle ϑ_{cr} and critical length L_{cr} determined by Eqs. (6) and (7),

$$\vartheta_{cr} = \sqrt{2bd} \quad \text{and} \quad L_{cr} = \sqrt{8d/b}. \quad (8)$$

For the conditions above, $\vartheta_{cr} = 14.5 \text{ mrad}$ and $L_{cr} = 4.14 \text{ cm}$.

Refractive losses are due to the narrowing of the internal apertures for rays of every angle as the length increases or the index of refraction variation becomes steeper. Eq. (7) allows us to calculate the fractional loss to the peak amplitude due to refraction effects.

As a check on our calculations, we have performed a series of Monte Carlo simulations with the LLNL code BEAM3 (1) which generates photons spontaneously throughout the laser and follows them as they pick up stimulated emission. In these simulations, g_0 and L are held constant while b is changed. As b increases, the signal moves out to higher angle and the peak power falls. We scale the peak signal from these simulations with the aperture formula in Eq. (7). The scaling works well until near the refraction limit where rays of almost-full length contribute appreciably to the weakened signal. By including rays of almost-full length in a modification to this calculation near ϑ_{cr} , we can account for the difference.

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