AIM

Association for Information and Image Management

1100 Wayne Avenue, Suite 1100
Silver Spring, Maryland 20910
301/587-8202

Centimeter

Inches

MANUFACTURED TO AIM STANDARDS
BY APPLIED IMAGE, INC.

1 of 1

AN ADAPTIVE FINITE ELEMENT METHODOLOGY FOR 2D SIMULATION OF TWO-PHASE FLOW THROUGH POROUS MEDIA *

D.J. Morton J.M. Tyler A.T. Bourgoyne P.A. Schenewerk

Louisiana State University

CONF-9403132--1

Keywords: finite elements, two-phase flow, adaptive methods

Abstract

A scheme for the accurate simulation of two-phase flow through porous media, utilizing adaptive finite element methods is presented. The theoretical equations and their approximation using Galerkin's method is covered, followed by a discussion of a dynamically refined mesh which preserves piecewise solutions across transition elements. Finally, comparisons are made between results of computed simulations and laboratory experiments.

Introduction

The accurate computer simulation of multi-phase fluid flow in porous media continues to be a difficult endeavor. Such problems feature near-discontinuities in the solution which are not sufficiently resolved by standard domain discretization procedures without extensive grid refinement. With the use of adaptive finite element methods, it is possible to efficiently model these problems by concentrating the computations in regions with sharp gradients, while reducing the computations in regions with relatively smooth gradients [6, 16]. An additional benefit of using finite elements is the versatil-

Figure 1: Water Coning at a Production Well

ity which they provide in modelling heterogeneous and irregularly shaped problem domains [18].

Presented in this paper is a report on the first stage in the development of an adaptive finite element procedure designed to simulate the processes occurring in a *water coning* scenario - a problem of particular interest to petroleum engineers [9, 15, 12]. A petroleum reservoir is composed of porous media saturated with water and oil. The oil, being less dense than water, occupies more of the upper regions of the reservoir while water occupies more of the lower regions, often driven by an aquifer. When production at a well begins, oil is initially removed, with the resulting pressure gradient drawing water upwards (see Figure 1); this water may eventually be produced along with the oil. This is an undesirable effect, and it may be minimized with an appropriate choice of production parameters. The ability to simulate this process provides petroleum engineers with an economical tool for experimenting with the production parameters *in vitro*.

This paper begins with a description of the mathematical model and solution used to express the physical processes of two-phase fluid flow through porous media, followed by a discussion of the adaptive grid technique which results in more accurate solutions for this type of problem. Finally, the results of computer simulations

*The support of the work under contract number DE-AC22-92BC14831 from DOE is gratefully acknowledged

and the results of laboratory experiments are compared.

Equations of Two-Phase Flow

The equations which describe the flow of two immiscible fluids through porous media (see [17, 1]) are derived from the mass conservation equations

$$-\nabla \cdot \rho_w \bar{v}_w - q_w = \frac{\partial}{\partial t} (\phi \rho_w S_w) \quad (1)$$

$$-\nabla \cdot \rho_n \bar{v}_n - q_n = \frac{\partial}{\partial t} (\phi \rho_n S_n) \quad (2)$$

and the relations

$$S_n + S_w = 1 \quad (3)$$

$$p_c = p_n - p_w \quad (4)$$

In the above, subscripts w and n designate wetting and non-wetting fluids, respectively (in a water-oil system, water is the wetting fluid and oil is the non-wetting fluid). For a fluid ℓ , ρ_ℓ is the density, $\bar{v}_\ell$ is the mass velocity vector, q_ℓ is the sink/source term, p_ℓ is the pressure, and S_ℓ is the saturation, or fraction of pore volume occupied by the fluid. p_c is the capillary pressure, treated as an empirically derived function of the fluid saturation, and ϕ is the porosity of the medium.

Darcy's Law is used to expand the mass velocity vectors to

$$\bar{v}_\ell = -K \frac{k_{r_\ell}}{\mu_\ell} \nabla [p_\ell - \rho_\ell g z] \quad (5)$$

where K is the absolute permeability tensor of the porous media, k_{r_ℓ} is the relative permeability of fluid ℓ , also an empirically derived function of saturation, and μ_ℓ is the viscosity of fluid ℓ . g is the acceleration due to gravity, and z is the depth in the porous media. The absolute permeability tensor is combined with the relative permeability and viscosity to make up a mobility tensor for fluid ℓ , which is designated as $\hat{M}_\ell$.

These equations are recast to form a coupled system of equations in the unknown variables p_n and S_n

$$\begin{aligned} \nabla \cdot \hat{M}_w \nabla p_n - \nabla \cdot \hat{M}_w \frac{dp_c}{dS_n} \nabla S_n - \\ \nabla \cdot \hat{M}_w \nabla (\rho_w g z) - \frac{q_w}{\rho_w} = \\ -\phi \frac{\partial S_n}{\partial t} + c_w \phi (1 - S_n) \frac{\partial p_n}{\partial t} \end{aligned} \quad (6)$$

$$\begin{aligned} \nabla \cdot \hat{M}_n \nabla p_n - \nabla \cdot \hat{M}_n \nabla (\rho_n g z) - \\ \phi \frac{\partial S_n}{\partial t} + c_n \phi S_n \frac{\partial p_n}{\partial t} \end{aligned} \quad (7)$$

Note that the recasting of the equations has introduced the fluid compressibilities c_w and c_n . Also, note that some parameters may be expressed as functions of p_n or S_n , making these equations nonlinear.

Numerical Solution of Equations

An approximate solution to eqs. (6) and (7) is obtained by discretizing the problem domain Ω into a set of finite elements and applying Galerkin's Method (see [2, 10]). First, all terms in eqs. (6) and (7) are moved to the left-hand side, and the resulting equations are defined as the *residual* equations

$$R_w = 0 \quad (8)$$

$$R_n = 0 \quad (9)$$

Assuming that an appropriate set of shape functions N_i , where $i = 1, \dots, n$, has been defined, the *weighted residual* statements

$$\int_{\Omega_e} R_w N_i d\Omega_e = 0 \quad (10)$$

$$\int_{\Omega_e} R_n N_i d\Omega_e = 0 \quad (11)$$

are minimized over each element domain Ω_e , and the contributions of each element are *assembled* into a global system matrix. Finally, the resulting system of equations is solved to obtain the approximate solution.

The second-order derivative terms in the weighted residual statements (eqs. (10) and (11)) are integrated by parts to reduce the order of the derivatives to one, and recast as the elemental equations [7]

$$\begin{aligned} [A] \{p_n\} + [CW] \{p_n\} + \\ [B] \{S_n\} + [C] \{\dot{S}_n\} = \\ - \{Q_w\} - \{D\} \end{aligned} \quad (12)$$

$$\begin{aligned} [H] \{p_n\} + [CN] \{p_n\} - [C] \{\dot{S}_n\} = \\ - \{Q_n\} - \{U\} \end{aligned} \quad (13)$$

where coefficients are

$$A_{ij} = - \int_{\Omega_e} (\nabla N_i)^T \hat{M}_w \nabla N_j d\Omega_e \quad (14)$$

$$\begin{aligned} CW_{ij} = \phi c_w \left[\sum_{k \in \Omega_e} \left(S_{n_k}^T \int_{\Omega_e} N_i N_j N_k d\Omega_e \right) \right. \\ \left. - \int_{\Omega_e} N_i N_j d\Omega_e \right] \end{aligned} \quad (15)$$

$$B_{ij} = \frac{dp_e}{dS_n} \int_{\Omega_e} (\nabla N_i)^T \hat{M}_w \nabla N_j d\Omega_e \quad (16)$$

$$C_{ij} = \phi \int_{\Omega_e} N_i N_j d\Omega_e \quad (17)$$

$$H_{ij} = - \int_{\Omega_e} (\nabla N_i)^T \hat{M}_n \nabla N_j d\Omega_e \quad (18)$$

$$CN_{ij} = -\phi c_n \sum_{k \in \Omega_e} S_{n_k}^t \int_{\Omega_e} N_i N_j N_k d\Omega_e \quad (19)$$

$$D_i = \sum_{j \in \Omega_e} \left\{ \int_{\Omega_e} (\nabla N_i)^T \hat{M}_w \nabla N_j d\Omega_e \right\} \rho_{w,j}^t g z_j \quad (20)$$

$$U_i = \sum_{j \in \Omega_e} \left\{ \int_{\Omega_e} (\nabla N_i)^T \hat{M}_n \nabla N_j d\Omega_e \right\} \rho_{n,j}^t g z_j \quad (21)$$

$$Q_w = \int_{\Omega_e} \frac{q_w}{\rho_w^t} N_i d\Omega_e \quad (22)$$

$$Q_n = \int_{\Omega_e} \frac{q_n}{\rho_n^t} N_i d\Omega_e \quad (23)$$

In the above, summations are performed over the nodes which define an element. Superscripts of t denote values obtained from the most recent timestep.

Integration-by-parts of the second-order derivative terms in eqs. (10) and (11) also produces flux terms which may be used for the application of Neumann boundary conditions. In the authors' model, the flux terms obtained from the first terms in eqs. (6) and (7) are utilized to specify flow rates and it is assumed that the other flux terms have negligible effect on the solution. However, more rigorous treatment of these terms might improve the accuracy of the solution and is being considered for future work.

The $\dot{p}_n$ and $\dot{S}_n$ in eqs. (12) and (13) are time derivatives, which may be approximated with a backwards difference scheme

$$\dot{p}_n \approx \frac{p_n^{t+\Delta t} - p_n^t}{\Delta t} \quad (24)$$

$$\dot{S}_n \approx \frac{S_n^{t+\Delta t} - S_n^t}{\Delta t} \quad (25)$$

Incorporating these approximations and re-arranging eqs. (12) and (13) leads to the equations

$$\begin{aligned} & \left([A] + \frac{1}{\Delta t} [CW] \right) \{p_n^{t+\Delta t}\} + \\ & \left([B] + \frac{1}{\Delta t} [C] \right) \{S_n^{t+\Delta t}\} = \\ & -\{Q_w\} - \{D\} + \frac{1}{\Delta t} [CW] \{p_n^t\} + \\ & \frac{1}{\Delta t} [C] \{S_n^t\} \end{aligned} \quad (26)$$

$$\begin{aligned} & \left([H] + \frac{1}{\Delta t} [CN] \right) \{p_n^{t+\Delta t}\} - \\ & \frac{1}{\Delta t} [C] \{S_n^{t+\Delta t}\} = \\ & -\{Q_n\} - \{U\} + \frac{1}{\Delta t} [CN] \{p_n^t\} - \\ & \frac{1}{\Delta t} [C] \{S_n^t\} \end{aligned} \quad (27)$$

Elemental equations (26) and (27) are assembled into the global system of equations

$$[M] \{u\} = \{b\} \quad (28)$$

where

$$\{u\} = [p_{n_1}, S_{n_1}, \dots, p_{n_k}, S_{n_k}]^T \quad (29)$$

Due to the nonlinearity of this problem, these equations must be solved iteratively over each time step, re-evaluating the coefficients of $[M]$ and $\{b\}$ with each new solution $\{u\}$. The approach used by the authors utilizes a variant of Richardson's extrapolation to accelerate convergence, coupled with an adaptive scheme to adjust the value of Δt for optimality.

Adaptive Spatial Discretization

The success of finite element methods rests on the assumption that functions are "smooth" [3]. Since the water-oil interface in the problem represents an almost discontinuous saturation, spurious oscillations tend to contaminate the solution. Although on a sufficiently small scale the transition from water to oil is smooth, a high degree of mesh refinement in the vicinity of the front is necessary to capture this behavior. Further complicating the situation is the fact that the front will move through the problem domain over time, requiring the refinement of all regions through which the front passes, resulting in a prohibitive problem size.

Figure 2: 1-irregular 2-D Adaptive Mesh

Figure 3: 1-irregular transition

Recently, adaptive refinement techniques have been developed, automating the process of refining the mesh where it is needed most, and in transient problems, allowing mesh refinement to occur in different regions as the solution evolves through the problem domain [16, 6]. Devloo, et. al. [5] provide a straight-forward algorithm for the adaptive solution of two dimensional problems, using h -refinement on 1-irregular meshes (an element in a 2D 1-irregular mesh will have either one or two elements adjacent to each side [4] - see Fig. 2).

The use of 1-irregular meshes generally results in a transition such as that displayed in Figure 3, where the three nodes along the bottom of element (e) define quadratic shape functions, whereas the two nodes along the top of each of elements (f) and (g) define linear shape functions, violating the requirement for interelement continuity. Many implementations, including that of Devloo, et. al. [5, 4], force interelement continuity by modifying the appropriate shape functions so that, in effect, the solution at the transition node (node 2 in figure 3) is constrained to be the average of the solution at the edge nodes (nodes 1 and 3). Although this has the computational advantage of reducing the number of nodes in the solution, it smooths the solution along the transition boundaries, and removes degrees of freedom that may be needed for higher accuracy.

Figure 4: Physical Model

Gupta [8, 11] introduces a two dimensional transition element which does not rely on constraints to produce interelement continuity. Rather, he modifies the shape functions of the transition element to produce piecewise linear solutions along the interelement boundary, modifying the quadrature integration scheme to account for discontinuities in the shape function derivatives. An advantage of the Gupta element is the preservation of the piecewise linear solution on the interelement boundary, but at the expense of using more degrees of freedom than the constraint method would entail. In this work, Gupta's element has been incorporated into the refinement scheme of Devloo, et. al. [5] in order to retain the piecewise solution along these transition boundaries.

Simulation of Laboratory Model Experiments

Initial tests of the model's effectiveness were performed by simulating the experiments performed on a laboratory model designed to study water coning near a horizontal well in a bottom-drive reservoir [9]. The physical model, packed with a porous media, had dimensions of $50\text{cm} \times 35\text{cm} \times 0.9\text{cm}$ (see figure 4). Water and oil analogs filled the pore volume, and a constant pressure was maintained at the bottom, simulating a bottom water-drive reservoir. The physical model tests commenced with the opening of a valve at the "well", and a constant flow rate was maintained through the valve. As the oil analog was removed from the upper regions of the system, the water analog formed a cone. Plexiglass sides on the model permitted viewing of the water-oil interface, which was recorded by tracing its outline at different times. These traces were used to compare experimental results with computer results.

Due to the small size of the physical model and a desire to maintain a sharp water-oil interface, miscible fluids were used. The use of the immiscible fluid equations (eqs. (6) and (7)) has been justified by Lantz [13], under

Physical Properties	
ρ_w	= 0.904 g/cm ³
ρ_n	= 0.756 g/cm ³
μ_w	= 1.02 centipoise
μ_n	= 1.02 centipoise
ϕ	= 0.28
K	= 12.0 Darcies (Isotropic)
D	= 1.0×10^{-5} cm ² /s (Isotropic)
k_{r_n}	= S_n
k_{r_w}	= $1 - S_n$
g	= $\frac{980.665 \text{ cm/s}^2}{1.0133 \times 10^5 \text{ dynes/(cm}^2 \cdot \text{atm)}}$
z_o	= 8.89 cm (Depth of initial interface)
q	= 2.793×10^{-2} cm ³ /sec (Production rate)
p_b	= 1.1102 atm (Bottom pressure)

Table 1:

Figure 5: Computed vs. Actual Coning Trace, $t = 3$ min.

the assumptions that the fluids are incompressible ($c_w = c_n = 0$), relative permeabilities of fluid ℓ are equal to the saturation (or concentration in miscible flow) of fluid ℓ and p_c (see eq. (4)) is replaced by

$$p_c = -\frac{D\phi\mu}{K} \ln\left(\frac{S_n}{1-S_n}\right) + (\rho_w - \rho_n)gz \quad (30)$$

where D is the diffusion-dispersion tensor and μ is the viscosity of the two fluids in the mixing region.

Fluid and media properties used are given in Table 1.

Comparisons were made at different flow rates by plotting the finite element mesh at different times, coloring all elements in which $S_n \geq 0.50$. It was then assumed that the front was in the vicinity of the boundary between colored and uncolored elements (see figs 5, 6, 7). Due to numerical diffusion, the computed front was not as sharp as the actual front, and it was found that this effect could be reduced by increasing the amount of refinement near the front, adding significantly to the computational load. In general, more accurate results were obtained by introducing more refinement and tightening the convergence criteria, but the computer resources needed were prohibitive. Work is currently underway

Figure 6: Computed vs. Actual Coning Trace, $t = 10$ min.

Figure 7: Computed vs. Actual Coning Trace, $t = 15$ min.

to increase the size of problems solved with the model through the use of iterative solvers for the linear systems of equations, and parallelization of the code.

Conclusion

An adaptive finite element method utilizing a Gupta element has been presented, and verification with experimental data has provided the confidence needed for further development. Although good matches in coning traces have been attained, further work needs to be done for accurate simulation of "post-breakthrough" processes. The matching of additional data - most notably watercut curves - in Hebert's model [9] will confirm the high accuracy of this approach. Certainly, improvements in the program efficiency will allow for more detailed modelling. In addition, it may be necessary to consider additional terms in the theoretical equations (eqs. (6) and (7)) to sufficiently describe the physical processes occurring at the well.

Subsequent work will be focused at the extension of the 2D adaptive FEM software to 3D domains utilizing a new element analogous to the Gupta element [14]. The immediate application areas will be in petroleum engineering and groundwater research, but a broad spectrum of disciplines can make beneficial use of this work.

REFERENCES

- [1] AZIZ, K., AND SETTARI, A. *Petroleum Reservoir Simulation*. Applied Science Publishers, Ltd., London, 1979.
- [2] BURNETT, D. S. *Finite Element Analysis - From Concepts To Applications*. Addison-Wesley, Reading, Mass., 1987.
- [3] CAREY, G. F., AND ODEN, J. T. *Finite Elements: Computational Aspects*, vol. 3 of *The Texas Finite Element Series*. Prentice-Hall, Inc., 1984.
- [4] DEMKOWICZ, L., ODEN, J. T., RACHOWICZ, W., AND HARDY, O. Toward a universal h - p adaptive finite element strategy, part 1. constrained approximation and data structure. *Computer Methods In Applied Mechanics and Engineering* 77 (1989), 79-112.
- [5] DEVLOO, P., ODEN, J. T., AND STROUBOULIS, T. Implementation of an adaptive refinement technique for the SUPG algorithm. *Computer Methods In Applied Mechanics and Engineering* 61 (1987), 339-358.
- [6] EWING, R. E. Adaptive mesh refinements in large-scale fluid flow simulation. In *Accuracy Estimates and Adaptive Refinements in Finite Element Computations*, I. Babuška, O. Zienkiewicz, J. Gago, and E. de A. Oliveira, Eds. John Wiley & Sons Ltd., 1986.
- [7] GOTTARDI, G., AND MESINI, E. A two-phase finite-element program for displacement simulation processes in porous media. *Computers & Geosciences* 12, 5 (1986), 667-695.
- [8] GUPTA, A. K. A finite element for transition from a fine grid to a coarse grid. *International Journal For Numerical Methods In Engineering* 12 (1978), 35-45.
- [9] HEBERT, L. E. An experimental study of the effects of water coning on a horizontal well in a bottom drive reservoir. Master's thesis, Louisiana State University, Baton Rouge, Louisiana, May 1990.
- [10] HUGHES, T. J. *The Finite Element Method*. Prentice-Hall, 1987.
- [11] KAZDA, I. *Finite Element Techniques In Ground-water Flow Studies: With Applications In Hydraulic And Geotechnical Engineering*. Elsevier Science Publishing Company, 1990. translated from Czech. by Zdeňka Jeníková.
- [12] KHAN, A. R. A scaled model study of water coning. *Journal Of Petroleum Technology* (June 1970), 771-776.
- [13] LANTZ, R. B. Rigorous calculation of miscible displacement using immiscible reservoir simulators. *Trans. SPE of AIME* 249 (1970), 192-202.
- [14] MORTON, D. J., TYLER, J., AND DORROH, J. R. A new 3d finite element for adaptive h -refinement in 1-irregular meshes. Tech. Rep. 93-004, Louisiana State University, Department of Computer Science, July 1993.
- [15] MUNGAN, N. A theoretical and experimental coning study. *Society of Petroleum Engineers Journal* (June 1975), 247-254.
- [16] ODEN, J. T. Adaptive finite element methods for problems in solid and fluid mechanics. In *Finite Elements Theory and Application* (1988), D. Dwoyer, M. Hussaini, and R. Voigt, Eds., Springer-Verlag.
- [17] PEACEMAN, D. W. *Fundamentals of Numerical Reservoir Simulation*. Elsevier, Amsterdam, 1977.
- [18] TEMAM, R. Survey of the status of finite element methods for partial differential equations. In *Proceedings of the ICASE Finite Element Theory and Application Workshop* (Hampton, Virginia, 1986), D. L. Dwoyer, M. Y. Hussaini, and R. G. Voigt, Eds., Springer-Verlag, pp. 1-33.

Abstract

A scheme for the accurate simulation of two-phase flow through porous media, utilizing adaptive finite element methods is presented. The theoretical equations and their approximation using Galerkin's method is covered, followed by a discussion of a dynamically refined mesh which preserves piecewise solutions across transition elements. Finally, comparisons are made between computed simulations and laboratory experiments.

DATE

FILMED

8 / 9 / 94

END