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Life Prediction for Bridged/Fatigue Cracks

presented at a Symposium on "Life Prediction Methodology for Titanium Matrix Composites", Hilton Head, South Carolina, March 1994, to be published in an ASTM STP.

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Life Prediction for Bridged Fatigue Cracks

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Abstract

The growth rates of matrix fatigue cracks bridged by sliding fibers are now commonly predicted by models based on the micromechanics of frictional interfaces. However, there exist many reasons, both theoretical and experimental, for suspecting that the most popular micromechanical models are probably wrong in detail in the context of fatigue cracks. Furthermore, a review of crack growth data reveals that the validity of the micromechanics-based predictive model has never been tested and may never be tested. In this paper, two alternative approaches are suggested to the engineering problem of predicting the growth rates of bridged cracks without explicit recourse to micromechanics. Instead, it is shown that the material properties required to analyze bridging effects can be deduced directly from crack growth data. Some experiments are proposed to test the validity of the proposals.

1. Introduction

One of the more promising classes of composites touted for high temperature applications, and certainly the most available,[†] is that of relatively brittle matrices, either ceramic or intermetallic, reinforced by strong, aligned, continuous fibers. Under cyclic loading in the fiber direction, these materials develop matrix cracks that often run perpendicular to the fibers, while the fibers remain intact in the crack wake, supplying bridging tractions across the fracture surfaces [1-7]. The bridging tractions shield the crack tip from the applied load, dramatically reducing the crack velocity from that expected in an unreinforced material subjected to the same value, ΔK_a , of the cyclic applied stress intensity factor. An important issue in reliability is the prediction of the growth rates of the bridged cracks.

Many people have worked on this topic in recent years [5-14] and it appears that the rudiments of the bridging problem are well understood. In particular, considerable effort has been expended on relating bridging effects to the microstructural characteristics of the composite, focusing especially on the role of the fiber-matrix interface. This research has been invaluable in guiding composite design, both for fatigue crack resistance and enhanced toughness under monotonic loading. Micromechanics is indeed the foundation of rational materials design.

However, predicting the reliability of a given composite in an engineering application is a problem of an entirely different kind. A design engineer requires a predictive methodology that is as simple as possible, i.e. containing as few parameters as possible, while remaining true to the material. It is unnecessary and, as argued below, misleading to base predictions on literal applications of micromechanical models.

This paper reviews the uncertainties of applying popular micromechanical models of bridged cracks to the estimation of fatigue lifetime. Two alternative

[†] Cylindrical monofilaments, by the chief virtue of being to hand, have dominated the field of continuously reinforced brittle matrix composites. Yet damage tolerance, transverse and through-thickness strength, and creep resistance are probably better served by reinforcements with different geometry, including three dimensionally woven or braided filament tows, interpenetrating phases, or even continuously formed laminates. But this is another subject.

approaches are proposed, based on the simplest parametric models that are likely to capture the significant features of crack growth data, including notch and specimen size effects and rate dependent bridging phenomena. While both approaches preserve certain functional features that ensue from micromechanical models, they remain noncommittal in regard to the details of such models.

2. Idealizations From Micromechanics

Early work on fibrous ceramic composites demonstrated the advantages for toughness of weak fiber-matrix interfaces. This condition promotes fiber bridging following matrix cracking and obviates fiber failure and catastrophic cracking [15]. Indentation tests on individual fibers in some glass/SiC composites could be fitted very nicely by a simple shear lag model of stress transfer. In this particular case, the data supported the assumption of a uniform friction stress, τ , along the zone of interfacial sliding [16]. Both model and experiment showed a quadratic relation between the indentation load, F , and the displacement of the fiber end (Fig. 1).

An analogous model of fiber sliding was used to develop a relation between the spatially averaged bridging tractions, p , acting in the wake of a bridged crack and the crack opening displacement, u [17]. From [17],

$$p = \beta u^{1/2} \quad (1)$$

where arguments of force balance led to

$$\beta = 2f \left[\frac{\tau E_f E}{(1-f) \rho E_m} \right]^{1/2} \quad (2)$$

with f the fiber volume fraction, ρ the fiber radius, E_f and E_m the fiber and matrix moduli, and $E = f E_f + (1-f) E_m$. Conservation of energy or considerations of force balance coupled with a kinematic displacement condition lead alternatively to a coefficient β' in Eq. (1), where [18,19]

$$\beta' = \frac{E_f}{(1-f) E_m} \beta \quad (3)$$

over Eq. (2)

From the point of view of theoretical mechanics, Eq. (3) should be preferred for the models posed in [17]; but ignorance of τ and other modeling questions (see below) can reduce the distinction to a moot matter. In [17], the dependence of the matrix cracking stress, σ_c , on the matrix crack length, a , was computed for monotonic loading. The criterion for crack advance was assumed to be the attainment of a critical value, K_c , of the net crack tip stress intensity factor, K_{tip} :

$$K_{tip} = K_a - K_b = K_c \quad , \quad (4)$$

where K_a and K_b are the applied stress intensity factor and its reduction due to bridging. For fully bridged cracks and increasing functions $p(u)$, like that of Eq. (1), σ_c is a monotonically decreasing function with the asymptote $\sigma_c = \sigma_1$, a constant, at large a [17,20]. The condition $\sigma_c \approx \sigma_1$ signifies the attainment of steady state cracking, echoing the earlier work of Aveston, Cooper, and Kelly [21].

Under cyclic loading, the same shear lag theory that underlies Eqs. (1) and (2) leads to

$$|p - p_r| = \bar{\beta} |u - u_r|^{1/2} \quad (5)$$

where $\bar{\beta} = \beta/\sqrt{2}$ and u_r and p_r are the values taken at the last load reversal [6,20]. Given a putative relationship between crack growth rate and the cyclic range, ΔK_{tip} , of K_{tip} , e.g. a Paris law

$$da/dN = A(\Delta K_{tip})^m \quad , \quad (6)$$

predicting crack growth rates amounts to tracking the evolution of ΔK_{tip} with crack length. As long as fracture surface contact does not occur during reverse loading, computing ΔK_{tip} is simplified by the following theorem: the cyclic ranges of the crack opening displacement, $\Delta u(x)$, the bridging tractions, $\Delta p(x)$, and the stress intensity factor, ΔK_{tip} , for loading under cyclic stress $\Delta \sigma_a$ are exactly twice the profile $u(x)$, tractions $p(x)$, and value K_{tip} that would be computed for loading a pristine crack under monotonic loading to stress $\sigma_a = \Delta K_{tip}/2$ [5].

Following from these basic ideas, numerous papers have demonstrated apparently successful analysis of fatigue crack growth data, particularly in titanium and titanium aluminide alloy matrices reinforced by SiC fibers [3,5,10-14]. Some of the crack growth data analyzed in these works revealed a regime of steady state crack growth, where, in analogy to the ACK limit, ΔK_{tip} and da/dN are approximately independent of crack length. Other data and modeling have shown the influence of notches and finite specimen geometries - see [9] and below. Extensions of the same modeling precepts to the problem of fiber rupture at maximum load (following [22]; but see also a superior treatment in [23]) have addressed residual composite strength after matrix crack growth [24].

In the analysis of fatigue crack growth rates, several groups have gone so far in applying micromechanical models as to discuss the values of the friction stress, τ , implied by their data [10-12,14]. By and large, good fits of models to data were evidently obtained (e.g. Fig. 2), although the best fit values of τ bore only loose correspondence with values measured by independent experiments on individual fibers. Phenomenological analyses of residual strength data have also now appeared [14].

3. Some Difficulties with the Micromechanical Models

The apparent success of fiber sliding models in calculating crack growth rates is fortuitous. It will be shown in the next section that most fatigue experiments analyzed to date have *not* been tests of the model. Where a few experiments have provided partial tests, fiber sliding models have fared poorly. Yet even before crack growth data are considered, several fundamental difficulties with fiber sliding models and their relevance to crack bridging raise a strong warning.

3.1 Force-Displacement Relations for Sliding Fibers

Both theoretical and experimental results show that the quadratic relation between force and displacement (e.g. Fig 1) cannot be generally correct for the fiber sliding phenomenon.

Theoretical studies have dealt chiefly with an ideally cylindrical fiber moving alone in a semi-infinite elastic matrix. Sliding is most often assumed to be opposed by Coulomb friction. The coefficient of friction and radial residual stresses are taken to be uniform along the sliding zone. Even so, Poisson's effect leads to nonuniformity in the net normal interface stresses when the fiber slides. The relation between the end load on the fiber and its axial displacement is no longer quadratic. In one extreme, Poisson's contraction when the fiber is pulled causes loss of contact between some part of the fiber and the matrix [25,26]. The contact-free length of fiber then strains linearly.

Nonuniformity in interfacial stresses will also arise from variations in residual stresses associated with the presence of the traction-free surface of the matrix. This complication increases in importance with fiber volume fraction, as does breaking of axial symmetry in the residual stresses due to fiber-fiber interactions. Indeed, in a material with fiber volume fraction high enough to optimize properties, the residual stress field is very difficult to compute or measure in any detail. This is a problem even for composites as received and at room temperature. After thermal cycling or mechanical cycling at constant high temperature, predictions cannot be considered more than qualitative guides.

In reality, the sliding interfaces are much more complicated geometrically than they are ever depicted as being in micromechanical models. The popular SCS6 SiC fiber, for example, contains several distinct interfaces in its carbon rich coating [27]. When composited with titanium based alloys or intermetallics, debonding and sliding can occur at any one or more of these or at separate interfaces in reaction products formed during consolidation; multiple interfaces can be active around each fiber simultaneously [8,28]. The implications for laws of frictional sliding are hard to fathom.

Interfaces are also rough. Figure 3 (from [29]) provides a cogent demonstration of how roughness can dominate sliding mechanics. It refers to an experiment in which a short piece of fiber embedded in a slab of matrix was pushed first in one direction and then in the other. Figure 3 shows the load in the reversing step as a function of displacement. The key feature is the large dip in the load when the fiber nears its original position. The load drop corresponds to reseating of matching irregularities on the opposing faces of the rough, sliding

interface. Over the width of the dip, i.e. over displacements of a few microns, roughness effects dominate the load-displacement curve. Yet this order of magnitude of displacements is precisely that of crack opening displacements in fatigue cracks several mm in length, a critical length in many applications.

The effects of roughness are exceedingly difficult to quantify. Precise profilometry of surfaces of materials similar to those expected at common interfaces reveals the fractal nature of roughness; roughness exists on length scales spanning many orders of magnitude [30,31]. The relation between contact force and contact area is not linear, but follows a power law that depends on the fractal dimension [32]. This implies that the assumption of Coulomb friction may be invalid. Unfortunately, the relation between contact area and critical shear stress for sliding is not yet understood, preventing completion of the analysis of friction from contact data. Furthermore, characteristic length scales are usually introduced by microstructure, violating the scale invariance assumed in evaluating fractal dimensions. The problem becomes very forbidding.

3.2 Sliding Fibers as Bridging Entities

For all their fascinating complexity, none of the models or experiments about fibers sliding in and out of half spaces or slabs solves the problem of sliding fibers as bridging ligaments.

Large Scale Sliding

Consider the schematic of a bridged crack in Figure 4. The dashed curves bound the zone of fiber sliding during a loading cycle. Shear lag modeling suggests a relationship similar to Eq. (1) between the stress in a fiber and the crack opening displacement for the matrix crack at the fiber's location. In the usual modeling procedure, the problem of Fig. 4a is then replaced by that of Fig. 4b: the fibrous composite is replaced by a homogeneous (usually orthotropic) continuum; and the bridging fibers are replaced by continuous bridging tractions, p , which act *on the fracture surface*. The tractions, p , are usually taken to be the stress in the bridging fibers reduced by a factor, f , the fiber volume fraction.

The problems of Figs. 4a and 4b are clearly *not* equivalent for sufficiently small cracks. First, the stress at the fracture plane in the fibers in Fig. 4a is transferred to the matrix as a closure stress not just at the fracture surfaces, but over the entire sliding zone and beyond. Now the closure effect (i.e., the reduction of K_{tip}) due to a point load of fixed magnitude diminishes as the position at which the load acts moves away from the crack plane [33]. Thus the model of Fig. 4b overestimates the bridging effect of the fibers in Fig. 4a when p follows Eqs. (1) to (3) or (5) [34,35]. The tractions required in Fig. 4b should more appropriately be those required to produce the same crack profile in Fig. 4b as the matrix crack has in Fig. 4a. The error in modeling Fig. 4a by Fig. 4b leads to severe overestimates of the shielding effect ΔK_b of the bridging fibers unless the sliding distance l_s of Fig. 4a is much smaller than the crack length a [34]. Fortunately, in this same regime the bridging zone is immature and $\Delta K_b \ll \Delta K_a$, so that the error in the net ΔK_{tip} remains moderate; nevertheless, it can still be significant. As a becomes very large, the models of Figs. 4a and 4b become equivalent, with p in Fig. 4b given by Eqs. (1) to (3) or (5). For typical modern composites of SiC fibers in titanium alloy matrices, the crack length must exceed ~ 1 mm for this to be the case.

The limit where $l_s \gtrsim a$ has been dubbed "large scale sliding" in recent literature [36].

Effects of Discreteness of Bridging Fibers

Other errors arise when tractions supplied by discrete fibers are represented by *continuous* surface tractions. These errors should be significant only when the crack spans just a few fibers. With $150\mu\text{m}$ SiC fibers, for example, this means crack lengths < 1 mm. For such small cracks, dips in records of da/dN vs. a have indeed been attributed to the discreteness of fibers in the crack wake [11].

Replacing the fibrous composite by an orthotropic continuum will introduce other errors in describing near-tip fields for cracks of all lengths. The matrix stresses in the composite will always fluctuate as the crack front passes from one fiber to the next, an effect that is lost in the continuum of Fig. 4b. The details of the fluctuations in the crack tip fields are incalculable: they involve elastic

heterogeneity, complex residual stresses, deflection of the crack at fiber/matrix interfaces, irregular pinning along the crack front, plasticity constrained by the presence of fibers, etc. In determining the magnitude of resulting fluctuations in da/dN , only empirical observations will be credible. Whether fluctuations in tip fields or fluctuations in wake effects due to fiber bridging are responsible for dips in da/dN such as those reported in [11] is problematic. However, since large scale sliding, which diminishes bridging effects, will be in force for fibers very near the crack tip, one might quibble with the author of [11] and plump for the former.

3.3 Geometrical Irregularity

Fiber sliding models have dealt almost exclusively with axially symmetric stress distributions induced by purely axial applied loads. Given careful specimen alignment, this may well be a fair basis for analyzing experiments consisting of the pushing or pulling of single fibers. However, whether results of such analyses are then valid for a fibrous composite is another question. Fibers are inevitably wavy in a composite. Fluctuations in fiber alignment due to waviness are commonly $\sim 3^\circ$ in well manufactured unidirectional composites [37]; while much larger variations must occur in woven or braided reinforcement (e.g. [38,39]). Even in a composite with ideally aligned fibers, misalignment of bridging tractions with the fibers' axes (i.e. mode II tractions) will still occur when a matrix crack fails to remain normal to the fiber direction. In reality, fiber irregularity and non-normal crack surfaces will always generate complex, non axisymmetric stresses in bridging fibers. While these effects have been recognized and modeled qualitatively [40], the unpredictability and complexity of crack and fiber geometry will keep quantitative predictions out of reach.

Another manifestation of fiber misalignment is variance in fiber separation. Residual stresses due to thermal mismatch depend strongly on fiber separation (e.g., [41]). In titanium matrix composites, for example, variations of residual hoop stresses with fiber separation are strongly correlated with matrix cracking under transverse loads [42]. In the context of sliding fibers, variable residual stresses around individual fibers imply further unknowns in the expected bridging traction law.

4. Remarks on Prior Analyses of Crack Growth Data

Yet in spite of all misgivings about the correctness of the underlying micromechanical models, the available crack growth data for matrix fatigue cracks in SiC/titanium alloys often appear to be fitted well by models based on Eqs. 5 and 6 (e.g. [3-5,12-14]). Does this prove that Eqs. (5) and (6) are right after all?

The answer lies in understanding the sensitivity of crack growth data to the characteristics of the bridging traction law, which can be elicited from an analysis of various contributions to the net stress intensity factor ΔK_{tip} .

For simplicity, the relevant concepts are developed first for monotonic loading. The crack configuration in the steady state under uniform remote stress σ_a is shown schematically in Fig. 5a. In the absence of a notch ($c_0 = 0$), p and σ_a must be in equilibrium over the entire far crack wake and therefore the contribution to K_{tip} from the far crack wake must be zero. For $c_0 = 0$, a nonzero value of K_{tip} thus arises only if $|\sigma_a| > |p|$ near the crack tip. If $p(u)$ exists, *it must be an increasing function for the steady state to exist*. Application of Rice's J integral then yields [43]

$$\frac{K_{ss}^2}{E'} = 2 \left(\sigma_a u_a - \int_0^{u_a} p du \right) \quad (\text{steady state}) \quad (7)$$

for the steady state value of K_{tip} , where $p(u_a) = \sigma_a$ and E' is a combination of elastic constants defined in [44].

In the absence of a notch, the steady state is preceded by a transient regime in which K_{tip} rises with crack length for fixed applied load. Here crack opening displacements over most of the crack are not yet large enough for p to balance σ_a . This "weak bridging" transient regime persists for a characteristic interval a_m of crack growth in monotonic loading, the "bridging length scale," which depends on the shape of $p(u)$.

For cyclic loading following a power law

$$|p-p_d| = \bar{\beta} |\mu - \mu_d|^\alpha \quad (8)$$

(for which Eq. (5) is the special case $\alpha = 1/2$), scale and transformation invariances lead from Eq. (7) to [20,45]

$$\Delta K_{ss} = \left[\frac{2\alpha^2}{\alpha+1} \frac{E'}{\beta^{1/\alpha}} \Delta \sigma_a^{\frac{\alpha+1}{\alpha}} \right]^{1/2} \quad (9)$$

$$= 2 \sqrt{\frac{2\alpha a_f}{\alpha+1}} \cdot \Delta \sigma_a \quad (10)$$

for the cyclic stress intensity factor in the steady state regime,

where
$$a_f = \frac{\pi \alpha E'}{4\beta^{1/\alpha}} (\Delta \sigma_a)^{\frac{1-\alpha}{\alpha}} \quad (11)$$

is the bridging length scale in fatigue. In the absence of a notch, ΔK_{tip} rises monotonically to ΔK_{ss} at crack length $a \sim a_f$.

Now consider the range of influence of a notch of length c_0 . Ignoring its perturbation of the steady state crack profile for $x \geq c_0$, its contribution ΔK_n to ΔK_{tip} is approximately that of the load $\Delta \sigma_a$ acting over the interval $[0, c_0]$ (e.g. [46]):

$$\Delta K_n \approx 2 \sqrt{\frac{a}{\pi}} \Delta \sigma_a \sin^{-1} \frac{c_0}{a} \approx \frac{2}{\sqrt{\pi a}} \Delta \sigma_a c_0 \quad (12)$$

This function diminishes as $a^{1/2}$ for $a \gg c_0$. As the steady state is approached, the notch loses its influence when $\Delta K_n \ll \Delta K_{ss}$; i.e. when

$$\frac{a}{c_0} \gg \frac{\alpha+1}{2\pi\alpha} \frac{c_0}{a_f} \quad (13)$$

For likely values of α (for energy conserving hysteresis, $\alpha \leq 1$), the factor $(\alpha + 1)/2\pi\alpha$ is ~ 1 . If $c_0 \ll a_f$, the notch will have only a small influence on the relation

$\Delta K_{tip}(a)$, since bridging will be weak when $a \sim c_0$ (Fig. 5b). However, if $c_0 \leq a_f$ and a is too small to satisfy Eq. (13), the crack will be in a "notch dominated" transient regime, in which ΔK_{tip} falls with increasing crack length (Fig. 5c) (see also [20,45]).

In discussing crack growth data, it is imperative to distinguish the regimes of the steady state ($a \gg a_f$ and $a/c_0 \gg c_0/a_f$) from the weak bridging transient regime ($a \lesssim a_f$ and $c_0 \ll a_f$) and the notch dominated transient regime ($c_0 \gtrsim a_f$ and $a/c_0 \lesssim c_0/a_f$).

4.1 Data from the Steady State Regime

Consider the analysis of crack growth data from the steady state regime $a \gg a_f$ and $a/c_0 \gg c_0/a_f$ (e.g. data in [3] and [12]). The information available about bridging laws from this regime is very limited. More generally than Eq. (9), the asymptotically constant value ΔK_{ss} of ΔK_{tip} has the form:

$$\Delta K_{ss} = f_k [\Delta \sigma_a, g(p(u))] \quad (14)$$

where g represents the dependence of ΔK_{tip} on the shape of the rising part of the hysteretic bridging relation $p(u)$ ¹. Given $p(u)$, f_k can be evaluated by application of the J integral [43]; reciprocally, the rising part of $p(u)$ follows from complete knowledge of $\Delta K_{ss}(\Delta \sigma_a)$. The problem is to determine ΔK_{ss} . In the absence of load ratio dependence, the velocity is related to ΔK_{ss} by

$$da/dN = f_v(\Delta K_{ss}) \quad (15)$$

where f_v might represent a Paris law and incorporate a fatigue threshold.

Unfortunately, the function f_v must in general be regarded as unknown. It cannot be equated to the law deduced from data for the monolithic alloy. Parameters in fatigue crack growth laws, e.g. the fatigue exponent and the multiplicative factor in a Paris law or a fatigue threshold, depend on all the

¹ In continuing to write $p(u)$ for the bridging relation in cyclic loading, it is understood that this is now virtually never a one-to-one correspondence as it often is in monotonic loading, but a multivalued function exhibiting hysteresis, e.g. Eq. (5).

nonlinear processes - plasticity, crack deflections, etc. - that act at or near the crack tip. Because of the fiber-related mechanisms listed at the end of Section 3.2, tip processes in a composite are *not* generally the same as those found in (or hypothesized for) the alloy alone.²

Thus records of da/dN in the steady state for different values of applied load, $\Delta\sigma_a$, cannot test the correctness of a particular form that might be assumed for $p(u)$. To illustrate this point further, consider, for example, a Paris law

$$f_V(\Delta K_{ss}) \equiv A(\Delta K_{ss})^m \quad (16)$$

where the exponent m in an unreinforced alloy or intermetallic might range from 2 to 10. Combining this with the power law bridging of Eq. (8) gives

$$\left(\frac{da}{dN}\right)_{ss} = A \sqrt{\frac{2\alpha^2}{\alpha+1} \frac{E'}{\beta^{1/\alpha}}} (\Delta\sigma_a)^{m \cdot \frac{\alpha+1}{2\alpha}} \quad (17)$$

for the crack velocity in the steady state. If the parameters A and m are unknown, measurements of $(da/dN)_{ss}$ vs. $\Delta\sigma_a$ cannot test the correctness of the popular assignment $\alpha = 1/2$: the powers α and m appear together.

4.2 Data from the Transient Regime

Independent information is potentially entailed in data for the transient regime preceding the steady state.

Weak Bridging Transient Regime

This is the regime $a_0 \ll a_f$ and $a \lesssim a_f$. The *shape* of records of da/dN vs. a in this regime should in principle be somewhat sensitive to the shape of the relation $p(u)$. Unfortunately, a_f is usually quite small. It can be estimated via Eq. (2) from values of the interface friction stress τ deduced from crack growth data

² Crack growth in a composite in the absence of bridging can be measured if all fibers are broken or cut in the crack wake. In some cases, velocities so measured are within a factor of two of those inferred from crack growth data for the corresponding monolithic alloy [47]. However, this simple correspondence cannot be *assumed*; and performing these tests for all temperatures and load levels may not always be practicable.

by analysis based on Eq. (5). For typical titanium alloys reinforced by SiC fibers, a_f ranges from $\sim 0.1\text{mm}$ to $\sim 1\text{mm}$ when the cyclic load amplitude is several hundred MPa [20]¹. For such small cracks, the discreteness of fibers leads to relatively large oscillations in da/dN and the limit of large scale sliding is found. Thus noise and modeling difficulties prevent using data for $a < a_f$ to confirm Eq. (5).

Notch Dominated Transient - Uniform Bridging Stress Models

This is the regime $c_0 \geq a_f$ and $a/c_0 < c_0/a_f$. In the notch dominated regime, the contribution to ΔK_{tip} of $\Delta\sigma_a$ acting over the notch $[0, c_0]$ is much larger than that of $\Delta\sigma_a - \Delta p$ near the crack tip, while σ_a and p approximately cancel over most of $[0, c_0]$. The crack growth data are therefore insensitive to the near tip bridging fields and cannot test the traction law $p(u)$.

This is well illustrated by some of the most recent work on modeling fatigue crack growth data in Ti alloy/SiC composites. In [8,10,14], bridging tractions were simply assumed to be uniform over the entire crack (excluding the notch).² In [10], the bridging tractions were assumed equal and opposite to the applied load: $p = -\sigma_a$. In [14], higher values ($|p| > |\sigma_a|$) were derived empirically by fitting the resulting predictions of da/dN to data. In either case, the measured and predicted growth rate curves followed one another within the (quite considerable) experimental noise.

In all of these tests, da/dN was an initially decreasing function, implying that $c_0 > a_f$. Furthermore, da/dN was not observed to approach a constant asymptote, implying that the steady state was not reached. Thus the entire recorded crack growth fell in the notch dominated regime.

Assuming that uniform tractions of magnitude $|p| \geq |\sigma_a|$ act over the bridged crack will give poor results for crack growth in other regimes. If $c_0 < a_f$, bridging tractions should be weak for small cracks ($a < a_f$); and ΔK_{tip} will be under-

¹ While the laws of Eqs (2) and (5) might be incorrect, the resulting estimates of a_f are very likely to be representative orders of magnitude.

² These models are very similar to Marissen's earlier studies of fatigue crack growth in "ARALL," a laminate of aluminum and an aramid/epoxy composite [48,49].

estimated by the assumption $|p| \geq |\sigma_a|$. If the crack grows beyond the notch dominated regime ($a/c_0 \gg c_0/a_f$), ΔK_{tip} should attain its steady state value ΔK_{ss} . The dominant contribution to ΔK_{tip} should now come from the near-tip zone where $|p| < |\sigma_a|$, which is not present in a uniform traction model. Thus the assumption of uniform tractions $|p| \geq |\sigma_a|$ is again nonconservative. Weak shielding for small fully bridged cracks and a positive definite steady state crack tip stress intensity factor ΔK_{ss} can be predicted only if $|\Delta p|$ is an increasing function of $|\Delta u|$.

Notch Dominated Transient - Nonuniform Bridging Models

Further confirmation of the insensitivity of the notch dominated transient to the details of the bridging law is found in modeling work using nonuniform bridging laws. While the quadratic law of Eq. (5) has been used most often and generally successfully to fit da/dN vs crack length data [5,12,14], one recent paper tested out a *linear* law instead [50]. For crack growth extending ~0.6 mm from a 3.5 mm notch in a SiC/Ti-6-4 composite, excellent agreement was obtained between model and experiment. Thus in similar materials quadratic, linear, and uniform traction laws all work well in the notch-dominated transient. Crack growth taken data from this regime alone do *not* test the correctness of the assumed bridging law.

5. Analyzing Crack Opening Profiles

Given data with sufficiently low noise levels, the bridging tractions p (or Δp), can be determined experimentally as a function of position x within the crack from measurements of the crack opening profile. In general, this may be done without assuming any particular form for $p(u)$ or even that the relation $p(u)$ is a constant material property [51]. The existence and form of a putative load-independent relation $p(u)$ can be established by examining $\Delta p(x)$ and $\Delta u(x)$ for various load levels and crack lengths. Unfortunately, this method has been precluded so far by the noise levels in profile data.

Progress has been attempted instead by seeing how well profiles can be fitted if model forms are assumed for $p(u)$. Profile data in [14] appeared to be better fitted when Δp was assumed independent of Δu , rather than following the parabolic law of Eq. (5). However, the distinction was unconvincing: in the notch-

dominated regime, the crack profile is not especially sensitive to $p(u)$, except near the crack tip. Profile data near the crack tip tend to be noisy. In [10], profiles predicted for $\Delta p = \text{constant}$ and $|\Delta p| \propto |\Delta u|^{1/2}$ fitted crack opening data equally well.

Collecting crack profile data is certainly worthwhile. The data in [8,10,14], for example, clearly confirm that bridging fibers provide strong crack tip shielding. However, profile data currently available have not yet provided a test of the bridging law.

6. Engineering Approaches for Rate Independent Bridging

Thus neither crack growth data nor crack profile measurements in the literature to date have tested the correctness of the micromechanical model of Eqs. (1) to (6). Indeed, the details of the model are unlikely ever to be confirmed.

Therefore, it is highly desirable that a predictive model be formulated that makes no explicit reference to the micromechanics underlying bridging. Instead, the simplest possible representation of bridging effects is sought that will allow crack growth rates to be predicted for any load level or specimen configuration.

It will be assumed at first that the bridging law is rate independent, i.e. attrition in the crack wake is negligible. This may not be true, but the assumption is at least helpful in developing concepts and appears in some cases to be supported by experiment. The effects of cycle or other rate dependences are considered in Section 7.

Two approaches will be considered. The first is based on evaluating the bridging relation $p(u)$ in parametric form from crack growth data; $p(u)$ becomes the defining material property. The second approach is the most empirical possible. Instead of seeking to determine $p(u)$, certain crack growth data are themselves taken as the defining material properties.

6.1 Method Based on a Parametric Bridging Law

Some simplifying assumptions appear warranted from prior work.

From the success of existing models in fitting data, it appears that crack growth is governed by ΔK_{tip} and that cyclic bridging tractions can be written as a hysteretic relation $p(u)$. In fact, nothing in existing crack growth data suggests that hysteresis in bridging should not be modeled as a one-to-one correspondence between $|p-p_i|$ and $|u-u_i|$; in other words, assuming symmetry between loading and unloading. The power law form of Eq. (8) probably offers a sufficiently rich range of hysteresis loops (Fig. 6). The linear law $p \propto u$ (no hysteresis) and the popular quadratic law $|p-p_i| \propto |u-u_i|^{1/2}$ are special cases. These basic assumptions greatly simplify bridging analysis and make available useful analytical and asymptotic results.

The power law for bridging tractions is fully determined by the stiffness parameter β and the power α . In the absence of load ratio effects (see below), the only other unknown material property is the relation between da/dN and ΔK_{tip} ; i.e. the function f_v of Eq. (15). If β , α , and f_v are known, fatigue crack growth rates can be calculated for any specimen or notch geometry, e.g. by integral equation or finite element methods. The necessary computer codes are becoming widely available.

The key to evaluating β , α , and f_v is to acquire data from different crack growth regimes.

(i) The existence (or absence) of the steady state regime must be established. This requires specimens whose width w satisfies $w \gg a_f$ and $w/c_0 \gg c_0/a_f$.

(ii) Crack growth rates in the steady state $(da/dN)_{ss}$ should be measured as a function of applied load amplitude. The resulting relation

$$(da/dN)_{ss} = f_s (\Delta \sigma_a) \quad (18)$$

is a material property. It is sensitive to the bridging relation $p(u)$. Through the analytical result Eq. (9), it also links da/dN and ΔK_{tip} via α and β . In other words, if α and β are known from independent data, f_v of Eq. (15) is determined by Eq. (18) and vice versa.

(iii) The shape of da/dN vs. a in the weak bridging transient regime ($a < a_f$ and $c_0 \ll a_f$) is in principle sensitive to $p(u)$. For example, the length scale a_f itself, which characterizes the interval of growth required to reach steady state, depends on $\Delta\sigma_a$ (Eq. (11)). Comparison of transient crack growth records for different values of $\Delta\sigma_a$ yields an implicit relation between ΔK_{tip} , f_v , β , and α . This fitting step must involve numerical solutions to the bridged crack problem. Unfortunately, data for small cracks are likely to be quite noisy (see Sections 3.2 and 4.2).

(iv) For the reasons discussed in the preceding section, data from the notch dominated transient regime ($c_0 > a_f$ and $a/c_0 < c_0/a_f$) show the least dependence on details of $p(u)$. However, the transition from notch-dominated to steady state crack growth involves long cracks ($a \gg a_f$) and should therefore be relatively noise-free. Determination of this transition for various $\Delta\sigma_a$ will yield the relation $a_f(\Delta\sigma_a)$; whence β and α via Eq. (11). Or equivalently, analysis of sets of crack growth data exhibiting both notch-dominated and steady state behavior at different $\Delta\sigma_a$ will yield β and α . The point of the transition itself need not be explicitly identified. It is enough to know that the data will contain sufficient information if the transition is present.

The precision with which α can be determined will depend on how well the notch-dominated to steady state transition can be mapped as a function of $\Delta\sigma_a$. If noise in the data makes the uncertainty large enough, a value for α might as well be chosen for convenience. Canonical results allowing simple evaluation of ΔK_{tip} and estimation of the onset of fiber failure have been made available for $\alpha = 1/2$ [9,52]; while analytical approximations serve the same purpose for $\alpha = 1$ (linear springs) [53-55]. If α is so set, only β and f_v remain to be determined.

Once α , β , and f_v are evaluated, *no interpretation of them in terms of the micromechanics of bridging need be made*. Predictions of lifetime follow from these parameters alone by now standard computational methods.

6.2 A Highly Empirical Strategy

The second suggested method dispenses altogether with the bridging relation $p(u)$. Characterization of steady state crack growth requires only the function f_s given by

$$\left(\frac{da}{dN}\right) = f_s(\Delta\sigma_a) \quad (19)$$

which is measured directly and can be considered a material property. It is not necessary in predicting steady state crack growth to separate out the relations f_k and f_v of Eqs. (14) and (15).

It remains to deal with the transient regimes. Transient crack growth for small notches can be treated not too conservatively as being infinitely rapid. In other words, the process of crack initiation could be assumed to create a crack of length a_i , which would grow thenceforth in the steady state. Cycles to initiation are predicted by methods unrelated to bridging phenomena (e.g.[56]).

On the other hand, writing $da/dN \rightarrow \infty$ for notch dominated transient growth will be too conservative, especially if a crack of length $a/c_0 \leq c_0/a_i$ is considered fatal. However, in the notch dominated transient the assumption of uniform bridging tractions $p = -\sigma_a$ or $p = -\sigma_{net}$ (where σ_{net} is the net section stress [14]) is probably good enough. Again, knowledge of $p(u)$ is not needed.

Nevertheless, for this most empirical strategy to be complete, some knowledge is required of a_i ($\Delta\sigma_a$) to bound the weak bridging and notch dominated transient regimes. Since a_i generally increases with $\Delta\sigma_a$, a conservative estimate of the small notch transient is given by the measured crack length at the attainment of the steady state for the highest load amplitude expected in service. Delimiting the notch dominated transient is more troublesome. A conservative model would require data showing the attainment of the steady state for every load level. Recall from Section 4.2 that failure to recognize the existence of the steady state leads to nonconservative predictions in uniform traction models when $a/c_0 > c_0/a_i$.

7. Finite Specimens

When the specimen width w fails to satisfy $w \gg a_f$ (uncommon for Ti/SiC composites) or $w/c_0 > c_0/a_f$ (common for many materials), steady state crack growth cannot be observed. Using such small specimens to qualify a composite for service risks nonconservative estimates for larger structures. On the other hand, once bridging has been analyzed for both the transient and steady state regimes using large specimens, predictions must also be made for relatively small specimens and various load distributions. For predictions based on the relation $p(u)$, no difficulty exists. Calculations can immediately be performed for any case using the same relation $p(u)$ (e.g. [9,44]).

In contrast, the highly empirical strategy provides no method of accurately computing finite specimen effects. Nevertheless, this may not be a fatal deficiency. While the applied stress intensity factor, ΔK_a , is a very different function of a for given $\Delta\sigma_a$ in finite and infinite specimens [57-59], the distributions of bridging tractions and the crack opening profiles in the two cases are not so dissimilar. Consequently, when the specimen width $w \gg a_f$, as it must be for bridging effects to be significant, da/dN in a specimen of finite width under uniform load $\Delta\sigma_a$ is either close to or below¹ that expected at the same notch size and crack length in an infinite specimen (Fig. 7). Further, for bending that develops a maximum stress $\Delta\sigma_a$, da/dN is always less than it would be under uniform load $\Delta\sigma_a$ [59]. Thus the empirical strategy will lead to conservative predictions for finite specimens and bending loads - not too conservative for uniform loads, perhaps highly conservative for bending.

8. Cycle or Other Rate Dependences

Cycling at room temperature degrades the fiber/matrix interface near matrix fatigue cracks. Evidence of degradation includes observation of debris [8,60]; load-displacement records of push-pull tests on individual fibers [61], and groups of fibers crossing matrix cracks etched right through specimens [62]; and some analyses of crack growth data [14]. All of this evidence indicates a decrease with cycles in the shear load capacity of the interface in the zone of

¹ On the same grounds, crack growth data for small specimens are unlikely to contribute any information that is not available from larger specimens.

fiber slip, implying softening of the bridging stiffness parameter β . Some push-pull tests on groups of fibers at constant load amplitude have suggested that β declines rapidly at first, but then steadies at a constant value over most of the expected fatigue life [62]. In contrast, the analysis of crack growth data in [14] led to the conclusion that β declines perceptibly throughout the fatigue record.

Whether bridging softening, $d\beta/dN$, should be regarded as slow or rapid in the context of crack growth depends on how it compares to the rate of crack advance, da/dN . In other words, writing

$$d\beta/dN = R_1 f_\beta(\beta, \Delta p) \quad (20)$$

the shape of da/dN vs. a for given notch size and load amplitude depends on the *ratio* of the rate constants R_1 and A [63]. As $R_1/A \rightarrow 0$, bridging degradation becomes undetectable in crack growth; as $R_1/A \rightarrow \infty$, steady-state crack growth cannot be attained even approximately.

To date, no physical model of degradation is available to guide the choice of the function f_β . In view of the difficulty of determining bridging parameters for rate independent bridging, a sensible starting point is a law with a single parameter, e.g.

$$\frac{d\beta}{dN} = -R_1 \beta \Delta p \quad (21)$$

A case has also been made for replacing Δp in this law by $\Delta p - \Delta p_0$, where Δp_0 is a threshold for damage [63]. One signature of such a threshold would be long cracks ($a/c_0 \gg c_0/a_f$) for which bridging tractions are uniform while the crack opening profile is not uniform (as for a steady state crack), but approximately elliptical.

Even when rate dependence exists, the length scale a_f remains useful, now defined in terms of the initial value β_0 of β . (If β declines very rapidly in the first few cycles and slowly thereafter, β_0 would be better defined as the value after the first few cycles.) A second length scale, a_i , is introduced by decay in β . This length scale indicates the interval of crack growth required for the stiffness

of newly created bridging ligaments to decay to a relatively small value. It defines the interval of crack growth over which memory of the initial crack configuration or the effects of a change in load amplitude persist. Further, crack growth approximating the steady state will be found if $a_t \gg a_i$; and not otherwise. Failure to achieve steady state even in a large specimen ($w \gg c_0$) will indicate significant decay in β .

For given laws for $d\beta/dN$ and da/dN , the length a_t depends on ΔK_{tip} ; and therefore, like a_i , on $\Delta\sigma_a$. Analytical expressions for a_t have not yet been found in explicit terms of R_1 , R_2 , β , and α . However, very good approximations are available for calculating ΔK_{tip} as the only unknown in a transcendental equation when R_1/A is large [64]. For general cases, numerical solutions must be employed. An efficient code has been developed for this by rendering an integral equation formulation into vector form [63].

Of the two strategies laid out in Section 6, the approach based on using $p(u)$ as a material property allows the easiest incorporation of degradation in β . Even using this approach, it has not yet been determined what experiments would clearly define a law such as Eq. (21). Far more experimental data are required.

9. Residual Stresses

Residual stresses influence bridging in two ways. 1) Radial compression at fiber/matrix interfaces influences the stiffness parameter β . For fatigue at fixed temperature, this effect need not be modeled to predict crack growth rates. In the absence of rate effects, it is incorporated in the stiffness parameter β . If interfacial attrition and the loss of debris diminish radial compression, the effect will be subsumed in the rate parameter R_1 of Eq. (21). 2) Axial tension in the matrix will cause larger mean crack opening displacements. This *might* lead to greater rates of interface attrition and higher values of R_1 ; and it promotes fiber failure. However, it has no immediate effect on the *cyclic* crack opening displacement $\Delta u(x)$ or on ΔK_{tip} .

If the temperature is cycled, residual stresses have a more prominent role. Relief of compressive interface stresses at higher temperatures will cause the

stiffness parameter β to cycle with T . Simultaneous cyclic relaxation of axial residual stresses will cause cyclic variation of the crack opening with T , contributing directly to ΔK_{tip} . The resultant effects of these phenomena on crack growth have been modeled [65] but not yet measured.

10. Load Ratio Effects - Fracture Surface Contact

For tension-tension fatigue, fracture surface contact will probably never occur in fibrous composites, except perhaps for relatively long cracks in bending applications [59]. However, for load ratios $R < 0$, it may. Let σ_c be the compressive applied load at which contact occurs. An accurate prediction of ΔK_{tip} will be that estimated for $\Delta\sigma_a = \sigma_{max} - \sigma_c$. The problem is to predict σ_c .

The contact stress σ_c depends generally on σ_{max} , α , β , and the magnitude of the axial residual stress σ_{ax} in the matrix. It can in principle be predicted once α , β , and σ_{ax} are known. However, uncertainty in values of α and β deduced from crack growth data and the unpredictability of σ_{ax} suggest that σ_c should be measured. For loads other than bending, fracture surface contact will nearly always occur first near the notch or, in the absence of a notch, in the farthest crack wake [59]. Closer to the crack tip, the crack is propped open more successfully by hysteresis in $p(u)$. This suggests measuring σ_c for cracks that have been grown into the study state at various values of $\Delta\sigma_a$ with $\sigma_{min} > 0$. Given values for α and β , a value of σ_{ax} could be deduced and the consistency of the modeling checked by the degree to which it is independent of σ_{max} .

11. Conclusions

Future experiments on bridged fatigue cracks should focus on verifying the existence of a steady state regime and measuring the transition to the steady state from the notch dominated regime. Larger specimens are required than used hitherto.

Methods for predicting lifetime have been proposed that minimize the detail incorporated from micromechanical modeling. While the essential trends in crack growth that are both observed and anticipated from micromechanics are preserved, they are computed with the greatest possible economy in modeling.

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References

- [1] R.A. Naik and W.S. Johnson, "Observations of Fatigue Crack Initiation and Damage Growth in Notched Titanium Matrix Composites," *Composite Materials: Fatigue and Fracture (Third Volume)*, ASTM STP 1110, T.K. O'Brien, Ed., ASTM, Philadelphia, 1991, pp. 753-71.
- [2] B.N. Cox, M. R. James, D.B. Marshall, W.L. Morris, C.G. Rhodes, and M. Shaw, in *Proceedings 11th International SAMPE Conference*, Birmingham, United Kingdom, 1989 (ed. S. Benson, T. Cook, E. Trewin, and R.M. Turner) pp. 313-20 (Elsevier, Amsterdam, 1989).
- [3] M.D. Sensmeier and P.K. Wright, "The Effect of Fiber Bridging on Fatigue Crack Growth in Titanium Matrix Composites," in *Fundamental Relationships Between Microstructure and Mechanical Properties of Metal Matrix Composites*, ed. M.N. Gungor and P.K. Liaw, *Proceedings TMS Fall Meeting*, Indianapolis (TMS, 1989).
- [4] E.Y. Luh, R.H. Dauskardt, and R.O. Ritchie, "Cyclic Fatigue-Crack Growth Behaviour of Short Cracks in SiC-Reinforced LAS Glass-Ceramic Composite," *Journal of Materials Science Letters*, Vol. 9, 1990, pp. 719-25.
- [5] R.M. McMeeking and A.G. Evans, "Matrix Fatigue Cracking in Fiber Composites," *Mechanics of Materials* Vol. 9, 1990, pp. 217-27.
- [6] B.N. Cox and D.B. Marshall, "Crack Bridging in the Fatigue of Fibrous Composites," *Fatigue and Fracture of Engineering Materials and Structures* Vol. 14, 1991, pp. 847-61.
- [7] References in *Fatigue of Advanced Materials*, ed. R.O. Ritchie, R.H. Dauskardt, and B.N. Cox (MCE Publ., Birmingham, England, 1991).
- [8] D.L. Davidson, "The Micromechanics of Fatigue Crack Growth at 25°C in Ti-6Al-4V Reinforced with SCS-6 Fibers," *Metallurgical Transactions A*, Vol. 23A, 1992, pp. 865-79.

- [9] B.N. Cox and C.S. Lo, "Simple Approximations for Bridged Cracks in Fibrous Composites," Acta Metallurgica et Materialia Vol. 40, 1992, pp. 1487-96.
- [10] L.J. Ghosn, P. Kantzos, and J. Telesman, "Modeling of Crack Bridging in a Unidirectional Metal Matrix Composite," International Journal of Fracture Vol. 54, 1992, pp. 345-57.
- [11] P. Bowen, "Characterization and Micromodelling of Fatigue Crack Growth in Unidirectional Continuous Fiber Reinforced Titanium Alloy Composites," in IRC '92, Birmingham, England, September, '92, ed. M.H. Loretto and C.J. Beevers (MCE Publ., Birmingham, 1992, pp. 697-718.
- [12] D.P. Walls, G. Bao, and F.W. Zok, "Mode I Fatigue Cracking in a Fiber Reinforced Metal Matrix Composite," Acta Metallurgica et Materialia Vol. 41, 1993, pp. 2061-71.
- [13] J.G. Bakuckus, Jr., and W.S. Johnson, "Application of Fiber Bridging Models to Fatigue Crack Growth in Unidirectional Titanium Matrix Composites," Journal of Composites Technology and Research, Vol. 15, 1993, pp. 242-55.
- [14] R. John, S.G. Kaldon, and N.E. Ashbaugh, "Applicability of Fiber Bridging Models to Describe Crack Growth in Unidirectional Titanium Matrix Composites," in Proceedings Titanium Metal Matrix Composite Workshop II, La Jolla, California, June, 1993, ed. W.C. Revelos and P.R. Smith (USAF, Dayton, Ohio, 1993).
- [15] A.G. Evans and D.B. Marshall, "Failure Mechanisms in Ceramic-Fiber/Ceramic-Matrix Composites," Journal of the American Ceramics Society Vol. 68, 1985, pp. 225-31.
- [16] D.B. Marshall and W.C. Oliver, "Measurement of Interfacial Mechanical Properties in Fiber-Reinforced Ceramic Composites," Journal of Materials Science and Engineering Vol. A126, 1987, pp. 95-103.

- [17] D.B. Marshall, B.N. Cox, and A.G. Evans, "The Mechanics of Matrix Cracking in Brittle-Matrix Fiber Composites," Acta Metallurgica Vol. 33, 1985, pp. 2013-211.
- [18] L.N. McCartney, "Mechanics of Matrix Cracking in Brittle-Matrix Fiber-Reinforced Composites," Proceedings of the Royal Society Vol. A409, 1987, pp. 329-50.
- [19] L.N. McCartney, "Mechanics for the Growth of Bridged Cracks in Composite Materials: Part I. Basic Principles," Journal of Composites Technology and Research Vol. 14, 1992, pp. 133-154.
- [20] B.N. Cox and D.B. Marshall, "Concepts for Bridged Cracks in Fracture and Fatigue," Acta Metallurgica et Materialia Vol. 42, 1994, pp. 341-63.
- [21] J. Aveston, G.A. Cooper, and A. Kelly, "Single and Multiple Fracture," in The Properties of Fiber Composites. Conf. Proc. (National Physical Laboratory, IPC Sci. Technol. Press, 1971) pp. 15-26.
- [22] D.B. Marshall and B.N. Cox, "Tensile Properties of Brittle Matrix Composites. Influence of Fiber Strength," Acta Metallurgica et Materialia Vol. 35, 1987, pp. 2607-19.
- [23] B. Budiansky and Y.L. Cui, "On the Tensile Strength of a Fiber-Reinforced Ceramic Composite Containing a Crack-Like Flaw," Journal of Mechanics and Physical Solids Vol. 42 ,1994, pp. 1-19.
- [24] G. Bao and R.M. McMeeking, "Fatigue Crack Growth in Fiber-Reinforced Metal-Matrix Composites," submitted to Acta Metallurgica et Materialia.
- [25] Y.-C. Gao, Y.-W. Mai, and B. Cotterell, "Fracture of Fiber Reinforced Materials", Journal of Applied Mathematics and Physics (ZAMP) Vol. 39, 1988, pp. 550-72.

- [26] J.W. Hutchinson and H.M. Jensen, "Models of Fiber Debonding and Pullout in Brittle Composites with Friction," Mechanics of Materials Vol. 9, 1990, pp. 139-63.
- [27] X.J. Ning and P. Pirouz, "The Microstructure of SCS6 SiC Fiber," Journal of Materials Research Vol. 6, 1991, pp. 2234-48.
- [28] M.R. James, W.L. Morris, and B.N. Cox, "Local Stress Relaxation in Ti-6Al-4V-2Fe/SCS6 Composite During Thermal Cycling," in Proc. ICF7, Houston, Texas, March, 1989, ed. K. Salama, K. Ravi-Chandar, D.M.R. Taplin, and P. Rama Rao (Pergamon, New York, 1989) pp. 3029-37.
- [29] P.D. Jero and R.J. Kerans, "The Contribution of Interfacial Roughness to Sliding Friction of Ceramic Fibers in a Glass Matrix," Scripta Metallurgica et Materialia Vol. 24, 1990, pp. 2315-18.
- [30] T.J. Mackin, P.D. Warren, and A.G. Evans, "Effects of Fiber Roughness on Interface Sliding in Composites," Acta Metallurgica et Materialia Vol. 40, 1992, pp. 1251-7.
- [31] T.L. Warren, A. Majumdar, and D. Krajcinovic, "Atomic Force Microscope Imaging of the Surface Roughness of SCS and TiB₂ coated SiC Fibers and Uncoated Sapphire Fibers," submitted to Composites.
- [32] A. Majumdar and B. Bhushan, "Characterization and Modeling of Surface Roughness for Microtribology," in Microtribology and its Applications in Magnetic Storage Systems, ed. B. Bhushan (CRC Press, Boca Raton, Florida, 1994).
- [33] P. Paris, R.M. McMeeking, and H. Tada, "The Weight Function Method for Determining Stress Intensity Factors," in Cracks and Fracture, ASTM STP 601 (ASTM, Pennsylvania, 1976) pp. 471-89.
- [34] W.L. Morris, D.B. Marshall, M.S. Dadkhah, B.N. Cox, and M.R. James, "Short Cracks Bridged by Discrete Long Fibers," submitted to Acta Metallurgica et Materialia.

- [35] G. Meda and P.S. Steif, "A Detailed analysis of Cracks Bridged by Fibers. Part I: Limiting Cases of Short and Long Cracks," submitted to Journal of Mechanics and Physics of Solids.
- [36] Z.C. Xia, J.W. Hutchinson, A.G. Evans, and B. Budiansky, "On Large Scale Sliding in Fiber-Reinforced Composites," submitted to Journal of the Mechanics and Physics of Solids.
- [37] S.W. Yurgatis, "Measurement of Small Angle Fiber Misalignment in Continuous Fiber Composites," Composites Science and Technology Vol. 30, 1987, pp. 279-93.
- [38] B.N. Cox, M.S. Dadkhah, T. Kniveton, and W.L. Morris, "Simple Models for Triaxially Braided Composites," submitted to Composites.
- [39] B.N. Cox and M.S. Dadkhah, "The Macroscopic Elasticity of 3D Woven Composites," submitted to Journal of Composite Materials.
- [40] V.C. Li, Y. Wang, and S. Backer, "Effect of Inclining Angle, Bundling, and Surface Treatment on Synthetic Fiber Pull-Out from a Cement Matrix," Composites Vol. 21 , 1990, pp. 132-40.
- [41] D. Kouris, "Stress Concentration Due to the Interaction Between Two Imperfectly Bonded Fibers in a Continuous Fiber Composite," Journal of Applied Mechanics Vol. 60, 1993, 203-6.
- [42] D.B. Marshall, W.L. Morris, B.N. Cox, J. Graves, J.R. Porter, D. Kouris, and R.K. Everett, "Transverse Strengths and Failure Mechanisms in Ti₃Al Matrix Composites," Acta Metallurgica et Materialia, in press.
- [43] D.B. Marshall and B.N. Cox, "A J-Integral Method for Calculating Steady-State Matrix Cracking in Composites," Mechanics of Materials Vol. 7, 1988, pp. 127-33.

- [44] B.N. Cox and D.B. Marshall, "Stable and Unstable Solutions for Bridged Cracks in Various Specimens," Acta Metallurgica et Materialia Vol. 39, 1991, pp. 579-89.
- [45] B.N. Cox, "Scaling for Bridged Cracks," Mechanics of Materials Vol. 15, 1993, pp. 87-98.
- [46] H. Tada, "The Stress Analysis of Cracks Handbook," (Paris Prod. Inc., St. Louis, Missouri, 1985) article 5.13.
- [47] J.M. Larsen, J.R. Jira, R. John, and N.E. Ashbaugh, "Crack Bridging Effects in Notch Fatigue of SCS-6/Timetal 21S Composite Laminates" in Proc. Int. Conf. on Life Prediction for Titanium Matrix Composites, Hilton Head, South Carolina, March, 1994, ed. W.S. Johnson, J. Larsen, and B.N. Cox, ASTM STP, to be published.
- [48] R. Marissen, "Fatigue Crack Growth Predictions in Aramid Reinforced Aluminum Laminates (ARALL)," in Proc. 15th ICAS Conference, London, England, Sept. 1986 (ICAS, 1986) pp. 801-807.
- [49] R. Marissen, "Fatigue Crack Growth in Aramid Reinforced Aluminum Laminates (ARALL). Mechanisms and Predictions," Report No. DFVLR-FB-84-37, Wissenschaftl. Berichtswesen DFVLR, Cologne, German, 1984.
- [50] T.-H. B. Nguyen and J.-M. Yang, "Elastic Bridging for Modelling Fatigue Crack Propagation in a Fiber-Reinforced Titanium Matrix Composite," Fatigue and Fracture of Engineering Materials and Structures Vol. 17, 1994, pp. 119-31.
- [51] B.N. Cox and D.B. Marshall, "The Determination of Crack Bridging Forces," International Journal of Fracture Vol. 49, 1991, pp. 159-76.
- [52] M. Begley and R.M. McMeeking, unpublished work.
- [53] L.R.F. Rose, "Crack Reinforcement by Distributed Springs," Journal of the Mechanics and Physics of Solids Vol. 35, 1987, pp. 383-405.

- [54] B.N. Cox and L.R.F. Rose, "A Self-Consistent Approximation for Crack Bridging by Elastic/Perfectly Plastic Ligaments," to be published.
- [55] J.R. Willis and S. Nemat-Nasser, "Singular Perturbation Solution of a Class of Singular Integral Equations," Quarterly Journal of Applied Mathematics Vol. 48, 1990, pp. 741-53.
- [56] B.M. Hillberry and W.S. Johnson, "Prediction of Matrix Fatigue Crack Initiation in Notched SCS6/Ti-15-3 Metal Matrix Composites," NASA Technical Memorandum 104141, 1991.
- [57] J. Bowling and G.W. Groves, "The Propagation of Cracks in Composites Consisting of Ductile Wires in a Brittle Matrix," Journal of Materials Science Vol. 14, 1979, pp. 443-9.
- [58] F. Zok and C.L. Hom, "Large Scale Bridging in Brittle Matrix Composites," Acta Metallurgica et Materialia Vol. 38, 1990, pp. 1895-1904.
- [59] B.N. Cox and C.S. Lo, "Load Ratio, Notch, and Scale Effects for Bridged Cracks in Fibrous Composites," Acta Metallurgica et Materialia Vol. 40, 1992, pp. 69-80.
- [60] J.M. Jira and J. Larsen, "Fatigue of Unidirectional SCS-6/Ti-24Al-11Nb Composite Containing a Circular Hole (Part I)," Metallurgical Transactions, in press.
- [61] D.B. Marshall, M.C. Shaw, and W.L. Morris, "Measurement of Interfacial Debonding and Sliding Resistance in Fiber Reinforced Intermetallics," Acta Metallurgica et Materialia Vol. 40, 1992, pp. 443-54.
- [62] D. Walls and F. Zok, unpublished work.
- [63] B.N. Cox and L.R.F. Rose, "Time or Cycle Dependent Crack Bridging," Mechanics of Materials, in press.

- [64] B.N. Cox and L.R.F. Rose, unpublished work.
- [65] G. Bao and R.M. McMeeking "Thermal-Mechanical Fatigue Cracking in Fiber Reinforced Metal Matrix Composites," submitted to Journal of the Mechanics and Physics of Solids.

Fig. 1 Load-displacement data for indentation tests of SiC fibers in a glass matrix and best fit curves from shear lag modeling. (From [16]).

Fig. 2 Typical fatigue crack growth data for a SiC fiber reinforced titanium alloy [12], showing the characteristic steady-state regime (constant slope); and best fit curves from micromechanics based modeling (from [23]).

Fig. 3 Load-displacement data for reversed sliding of a single fiber, exhibiting reseating phenomenon (from [28]).

Fig. 4 Schematic of a bridged crack: (a) showing interfacial slip in an inhomogeneous composite; and (b) as commonly represented in formulations of the problem as a cracked orthotropic continuum. (Following [33].)

Fig. 5. (a) Schematic of a bridged crack in the steady state regime. (b) The weak bridging transient regime and steady state regime for small notches. (c) The notch dominated transient regime and steady state regime for large notches.

Fig. 6. Hysteresis loops corresponding to a power law relation between $|p-p_r|$ and $|u-u_r|$.

Fig. 7. The effects of finite specimen geometry shown in predictions of $\Delta K_{tip}(a)$ (solid curves) for the bridging law of Eq. (5) (adapted from [54]). The long-dashed curves show the steady state value of ΔK_{tip} in an infinite specimen. The short-dashed curves show $\Delta K_{tip}(a)$ in the finite specimen in the absence of bridging.

(b) $c_0 \ll a_f$

(c) $c_0 \gg a_f$

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