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**GEOPHYSICAL AND TRANSPORT PROPERTIES OF RESERVOIR
ROCKS**

**Final Report for Task 4:
Measurements and Analysis of Seismic Properties**

**By
Neville G.W. Cook**

May 1993

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**University of California
Berkeley, California**

**Bartlesville Project Office
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Prepared for
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CHAPTER 1

INTRODUCTION AND OVERVIEW

1.1 Introduction

The principal objective of research on the seismic properties of reservoir rocks is to develop a basic understanding of the effects of rock microstructure and its contained pore fluids on seismic velocities and attenuation. Ultimately, this knowledge would be used to extract reservoir properties information such as the porosity, permeability, clay content, fluid saturation, and fluid type from borehole, cross-borehole, and surface seismic measurements to improve the planning and control of oil and gas recovery.

One of the main challenges in research on the seismic properties of reservoir rocks is to understand the details of the micromechanical processes which operate at the grain level during the passage of a compressional or shear wave. Laboratory measurements made over the last twenty years have revealed that velocities and attenuation are often complicated functions of the state of stress in the rock, as well as the pore fluid properties, fluid saturation, and clay content (e.g., see Tóksöz and Johnston, 1981; Bourbié et al., 1987; Nur and Wang, 1988). At present, many of these observations can only be partly explained by existing theoretical models.

With the rapid development of seismic imaging techniques over the last decade, a trend in rock properties research has been to focus less on basic research aimed at understanding the micromechanics of seismic wave propagation in granular rocks. Instead, the focus has shifted to providing the reservoir geophysicist with empirical relationships between seismic properties and rock properties without necessarily having a complete understanding of the mechanisms which affect seismic velocities and attenuation at the grain level. This is a necessary but perhaps dangerous endeavor since it is well known that relationships between reservoir properties and seismic properties can

be strongly non-unique. Without an understanding of the processes which influence the seismic properties, it is difficult to assess the accuracy of these empirical relationships when, for example, the frequency or the magnitudes of the principal stresses are altered from those used in the laboratory.

This thesis presents laboratory ultrasonic measurements for three granular materials and attempts to relate the microstructural properties and the properties of the pore fluids to P- and S-wave velocities and attenuation. These experimental results show that artificial porous materials with sintered grains and a sandstone with partially cemented grains exhibit complexities in P- and S-wave attenuation that cannot be adequately explained by existing micromechanical theories. It is likely that some of the complexity observed in the seismic attenuation is controlled by details of the rock microstructure, such as the grain contact area and grain shape, and by the arrangement of the grain packing. To examine these effects, a numerical method was developed for analyzing wave propagation in a grain packing. The method is based on a dynamic boundary integral equation and incorporates generalized stiffness boundary conditions between individual grains to account for viscous losses and grain contact scattering.

1.2 Definitions and Terminology

Several basic concepts will be used throughout this thesis and will be introduced here.

In an attenuating material, the amount of energy removed from a seismic wave at a particular angular frequency, ω , is described by a dimensionless quantity known as the *quality factor*, $Q(\omega)$. The reciprocal of the quality factor, $Q^{-1}(\omega)$, is the *attenuation* and is defined as the energy loss per cycle, ΔE , normalized by the peak energy stored per cycle, E ,

$$Q^{-1}(\omega) = \frac{\Delta E}{2\pi E} \quad (1.1)$$

When the material is attenuating, the wavenumber becomes a complex quantity, $\hat{k} = k_r + i\alpha$, where $k_r = \omega/c_p(\omega)$ is the wavenumber and α is the *attenuation coefficient*. The displacement of a wave of angular frequency, ω , propagating through this material in the x-direction is described by

$$\hat{u}(\omega, x) = \hat{S}(\omega) \hat{G}(\omega, x) e^{i(\hat{k}x - \omega t)} = \hat{S}(\omega) \hat{G}(\omega, x) e^{-\alpha x} e^{i(k_r x - \omega t)} \quad , \quad (1.2)$$

where $\hat{S}$ is the source spectrum and $\hat{G}$ is the geometrical spreading of the material.

For an attenuating material satisfying $Q \gg 1$, the complex wavenumber is related to Q in the following manner,

$$\alpha = \frac{\omega Q^{-1}(\omega)}{2c_p(\omega)} \quad , \quad (1.3)$$

where c_p is the phase velocity of the material. Note that the attenuation, Q^{-1} , and the attenuation coefficient, α , have different dependencies on frequency. This difference arises because Q^{-1} is the energy loss per cycle while α is the energy loss per distance. The distinction between the two quantities is clear when examining Eq. (1.3) for the particular case when Q^{-1} and c_p are nearly constant so that α is roughly proportional to the frequency. The attenuation coefficient, α , continues to increase with frequency because the number of wave cycles in a fixed propagation distance increases with increasing frequency. This behavior is immediately apparent in the differences between Q^{-1} vs. frequency curves which tend to be peaked and α vs. frequency curves which increase monotonically.

A distinction can be made between attenuation processes which dissipate energy in the form of heat losses such a viscous shearing and scattering attenuation where energy is removed from the direct wave and redirected into scattered waves. The total attenuation in a medium which contains both types of attenuation can be approximated as

$$Q_{total}^{-1}(\omega) = Q_{intrinsic}^{-1} + Q_{scattering}^{-1} \quad (1.4)$$

For the wave to be causal, the attenuation and the phase velocity must satisfy the Kramers-Kronig attenuation-dispersion relation (Aki and Richards, 1980)

$$\frac{\omega}{c_p(\omega)} = \frac{\omega}{c_p(\omega \rightarrow \infty)} + Hilbert\ Transform\left\{ \frac{\alpha(\omega)}{\omega} \right\}. \quad (1.5)$$

This is a convenient theoretical constraint which can be used in theoretical modeling to develop an analytic expression for the attenuation coefficient from a already known expression for the phase velocity or to develop an analytic expression for the phase velocity from a known expression for the attenuation coefficient (Beltzer, 1983; Angel and Achenbach, 1991). In principal, this relationship could also be used in the laboratory and field to extract attenuation from measured velocities or to extract velocities from attenuation measurements. In most experiments, however, the frequency range of accurate velocity or attenuation measurements is limited by the bandwidth of the source, as well as various other factors. Since the Hilbert Transform requires an integration from zero frequency to infinite frequency, the Kramers-Kronig relation cannot be applied without making some assumptions about the behavior of the attenuation or phase velocity for frequencies outside the bandwidth of the source (Lee et al., 1990).

Attenuation processes typically exhibit strong dependencies on frequency. A variety of laboratory techniques exist for measuring attenuation and velocities in laboratory samples, however, no single technique is capable of measuring seismic properties from exploration to borehole logging frequencies. Fig. 1.1 illustrates the frequency range of operation of the various laboratory and in situ methods which are used to measure the seismic properties of rocks. It should be noted that the ultrasonic pulse methods are the only laboratory techniques that measure true uniaxial strain (P-wave) seismic wave propagation. The waveform inversion method (Tang et al., 1992) utilizes waves which

Frequency Range of Seismic Waves Used in Investigations of Rock Properties

Figure 1.1 Laboratory and in situ techniques for measuring the seismic properties of rocks at different frequencies.

propagate along a cylindrical rod, and the resonant bar technique utilizes standing waves. The stress-strain method applies a quasi-static axial stress or torsion to the sample and does not involve wave propagation altogether. All methods except the latter will measure scattering attenuation.

1.3 Review of Seismic Properties of Granular Rocks

At present, a considerable volume of research on the seismic properties of reservoir rocks exists in the form of conference proceedings (e.g., from the annual meetings of the Society of Exploration Geophysicists, American Geophysical Union, and Acoustical Society of America), published papers (e.g., in *Geophys.*, *J. Geophys. Res.*, and *J. Acoust. Soc. Am.*), and text books (e.g., by White, 1965, 1983; Bourbié et al., 1987). This research consists of both laboratory and theoretical work on a variety of sedimentary rocks which include sandstones, limestones, shales, and sediments. A basic conclusion that can be drawn from this research is that seismic velocities and attenuation are strongly influenced by stress-induced changes to the grain microstructure and by the presence of pore fluids. The strong effects of pore fluids on seismic attenuation has been attributed to viscous fluid-flow and mineral-fluid interactions that take place in the contacts between neighboring grains.

To date, there have been a considerable number of investigations on the velocities of reservoir rocks (e.g., Nur and Wang, 1988, 1992). Velocities are simple to measure and have been found to be useful indicators of reservoir properties and contained pore fluids. The changes in velocity, however, are usually accompanied by changes in attenuation which are many times greater. This makes attenuation a very sensitive parameter for monitoring changes in reservoir properties. Attenuation information is potentially more difficult to extract from seismic data because it requires data corrections (e.g., for the source radiation pattern, geometrical spreading, and geophone coupling) and more

sophisticated data processing algorithms. However, rapid improvements in field equipment and computer processing capabilities made in the last decade have resulted in a renewed interest in seismic attenuation.

1.3.1 Laboratory Attenuation Measurements

A wealth of laboratory attenuation measurements exist and are collected and analyzed in a recent text, *Acoustics of Porous Media*, by Bourbié, Coussy and Zinszner (1987). Most laboratory attenuation measurements are conducted using ultrasonic methods or the resonant bar technique with a limited number of studies employing the stress-strain method (Fig.1.1). These measurements have illustrated that attenuation is a function of strain amplitude (Winkler and Nur, 1979), fluid saturation (Murphy, 1982a), confining stress (Winkler and Nur, 1979), frequency (Winkler, 1983), and the properties of the pore fluid such as viscosity (Jones, 1986; Vo-Thanh, 1990), surface tension (Spencer, 1981), dipole moment per unit volume and the polar nature of the fluid molecules (Tittmann et al., 1980). Despite the considerable volume of laboratory data, the current understanding of the mechanisms of P- and S-wave attenuation is incomplete and many laboratory measurements cannot be explained adequately by existing theoretical models..

The experiments presented in this thesis will attempt to control as many of these variables while examining the effects of stress and pore fluids in granular materials with different microstructures. Strain amplitudes are kept below 10^{-6} to eliminate the possibility of frictional sliding between grains, fluid saturation is kept close to 100% to avoid attenuation effects associated with partial saturation, and pore fluids were selected with known viscosities and chemical properties.

1.3.2 Theoretical Attenuation Models

There have been a number of attenuation mechanisms which have been proposed for porous, granular, sedimentary rocks such as sandstones, limestones, and shales as illustrated in Fig. 1.2, however, only a few have survived the scrutiny of investigation. For granular sedimentary rocks, these mechanisms can be classified into three general categories: (1) scattering attenuation, (2) fluid-flow attenuation, and (3) fluid-matrix attenuation. Attenuation in saturated granular rocks is most likely a combination of all three types of mechanisms although in many instances one mechanism may dominate the others.

1.3.2.1 Scattering Attenuation

The attenuation effects described in the following sections occur in rocks fully or partially saturated with pore fluids. In dry rocks, these attenuation mechanisms are inoperative and attenuation will primarily result from scattering. Scattering from grain scale heterogeneities such as pores and partially cemented grain contacts removes energy from the direct P- and S-waves and redistributes this energy into scattered waves which appear in the coda.

The state of the present knowledge on scattering effects in dry granular materials is presented in a series of papers by Winkler (1983, 1985) and Winkler and Murphy (1984). These papers demonstrate that ultrasonic attenuation coefficients for unconsolidated and fused glass beads and a variety of low porosity sandstones exhibit attenuation coefficients proportional to the fourth power of frequency (Fig. 1.3). Fourth power attenuation is consistent with the theoretical predictions of Rayleigh scattering theory which was derived under the assumptions of single scattering off spherical particles with radii much smaller than the wavelength (i.e., $ka \ll 1$). In the analysis of his experimental results, Winkler (1983) concludes that while the theoretical scattering models for spherical pores imbedded

SUMMARY OF PROPOSED ATTENUATION MECHANISMS IN DRY AND FLUID-SATURATED SEDIMENTARY ROCKS

References:

- ¹ Plona & Johnson (1981), Winkler (1983), Winkler & Murphy (1984).
- ² Biot (1956a, 1956b, 1962)
- ³ Mavko & Nur (1979), Palmer & Traviola (1980), Murphy et al. (1984, 1986), Miksis (1988).
- ⁴ Walsh (1968, 1969), O'Connell & Budiansky (1977).
- ⁵ Spencer (1981), Murphy et al. (1984).
- ⁶ Tittmann et al. (1980).
- ⁷ Spencer (1981).
- ⁸ Clark et al. (1980).
- ⁹ Martin (1972), Spencer (1981).
- ¹⁰ Walsh (1966), Winkler & Nur (1982).
- ¹¹ Norwick & Berry (1972), Karato & Spetzler (1990).
- ¹² Savage (1966).

Figure 1.2 Proposed mechanisms of seismic wave attenuation in sedimentary rocks.

Scattering in Granular Rocks

Figure 1.3 Frequency-dependent scattering in granular rocks. (a) The magnitude and radiation characteristics of scattering is controlled by the product of the wavenumber, k , and the size of the scatterer, a . Strong scattering occurs when the wavelength approaches the size of the scatterer (i.e., $ka=1$). Scattering attenuation in reservoir rocks results from discontinuities at the same scale as the wavelength.

in a solid material (Sayers, 1981) predict the correct frequency dependence, they underestimate the magnitude of the observed attenuation by an order of magnitude. Winkler suggests that current theoretical scattering models are not applicable to his experiments because *they do not consider interactions between the grains*.

As the frequency of the wave is decreased below the ultrasonic range, the wavelength becomes many times larger than the dimensions of the grains and the attenuation predicted by Rayleigh scattering becomes negligible. Resonant bar tests performed on dry sandstones and limestones in the 1-10 kHz range exhibit attenuation larger than the Rayleigh scattering predictions that is characterized by an attenuation coefficient proportional to the first power of frequency (Fig. 1.3b). This attenuation has been attributed to scattering off centimeter scale heterogeneity (Murphy, 1984; Blair, 1990; Burns et al., 1991; Lucet and Zinszner, 1992). Centimeter scale heterogeneity may include features such as grain clustering, regions of cementation overgrowths, and microfractures. At present it is not completely understood why this type of scattering attenuation exhibits a low power frequency dependence but one possible explanation is that the size of the heterogeneities is uniformly distributed so that approximately the same magnitude of scattering occurs over a range of frequencies (Frankel and Clayton, 1986; Frankel, 1989; Jannaud et al., 1991). At even lower frequencies, it is not possible to obtain laboratory measurements of scattering attenuation. Wavelengths at these frequencies sample reservoir scale heterogeneities such as sand lenses, faults, and gas pockets in addition to the grain scale properties.

As the frequency is increased above 1 MHz, the wavelength begins to approach the dimensions of the individual grains and strong (forward) scattering ensues. This type of large angle scattering is *resonance* or *Mie* scattering (Wu and Aki, 1988) and it results from intergranular resonances as the wave becomes trapped within individual grains (Fig. 1.3). At even higher frequencies, the wavelength becomes much smaller than the size of

the grains, and the wave propagates through the packing in accordance with geometrical optics.

The parameter which controls the magnitude and the radiation characteristics of the scattering mechanism is the product of the wavenumber, k , and the size of the scatterer, a (Fig. 1.3a). When the wavelength approaches the size of the scatterer (i.e., $k \approx a$), the wave is strongly scattered resulting in a large attenuation of the direct wave. Non-welded interfaces such as grain contacts, fractures, joints and faults can also produce substantial scattering losses despite their almost negligible thickness. A wave incident upon a non-welded interface is scattered off of the individual voids within the discontinuity producing scattered wavelets which are roughly in phase. The superposition of the scattered wavelets defines a coherent wavefront in accordance with Huygen's Principle. There is no obvious physical equivalent of the size of the scattering body, a , for scattering off a non-welded interface. The magnitude of the reflected energy is dependent on the stiffness characteristics of the discontinuity which is determined by the microgeometry of the asperities and voids.

1.3.2.2 Fluid-Flow Attenuation

A model for attenuation in fully saturated rock was first proposed by Biot (1956a, 1956b, 1962). In Biot's model, the pore space is idealized as a collection of interconnected constant width tubes of diameter, b , embedded in a solid material. A fluid with a viscosity, η , and a density, ρ_f , is assumed to fill the entire pore space. Seismic energy is dissipated as the transient displacement of the pore wall induces viscous flow of the pore fluid. The attenuation is maximum at a critical frequency (Johnson and Plona, 1982)

$$\omega_c = 2 \eta / (\rho_f b^2) \quad (1.6)$$

where the shear stresses just couple across the pore to produce Poiseuille flow within the pores. For frequencies below ω_c , the viscosity of the fluid binds the fluid to the pore and there is very little relative motion between the fluid and the pore walls. For frequencies above ω_c , the shear stresses cannot diffuse across the width of the pore and the fluid and the solid move out of phase. The P-wave Biot attenuation for a packing of identical spherical grains in the high frequency limit of the theory (i.e., $\omega \gg \omega_c$) is described by the following equation (Salin and Schön, 1981),

$$\alpha_{Biot} = B_{Biot} \eta \omega^{1/2} \quad (1.7)$$

where B_{Biot} is a constant which depends on the porosity, grain diameter, total phase velocity dispersion, and fluid density. The P-wave attenuation coefficient increases linearly with viscosity and with the square root of frequency.

While this model is rigorously formulated, it tends to underestimate the observed level of attenuation in sedimentary rocks when realistic values are used for the 16 parameters of the model (Mochizuki, 1982). Various modifications to the theory have been proposed. The effects of variable pore radii were examined by McCann and McCann (1985), Yamamoto and Turgut (1988), and Pride and Morgan (1990). Recently, Berryman (1988) has suggested that a localized permeability rather than the global permeability be used in the theory.

Biot recognized the possibility of other sources of seismic attenuation and in a later paper (1962) he generalized his original model to include additional attenuation mechanisms. Among these was a simple rheologic model of viscous damping between grains produced during the compression of a viscous fluid surrounding the grain contact. This attenuation mechanism was later labeled *squirt* or *local* flow to distinguish it from Biot's earlier model in which flow took place on a global scale.

Mavko and Nur (1979) developed an analytic solution for local flow in a wedge shaped pore and demonstrated that low aspect ratio pores could produce local flow attenuation of the same order as the attenuation observed in partially saturated rocks. In the high frequency limit of their theory (i.e., $\omega \gg \omega_c$ and incompressible flow), P-wave attenuation for a parallel-walled pore exhibits the following functional dependence on viscosity and frequency:

$$\alpha_{local} = \frac{B_{local}}{c_p(\omega)} (\eta \omega)^{1/2} \quad (1.8)$$

where B_{local} is a constant which depends on the fluid properties, the pore geometry, and the shear modulus of the rock and c_p is the phase velocity of the rock. The local flow theory predicts that the P-wave attenuation will increase with the square root of viscosity and frequency.

Palmer and Traviolia (1980) developed a similar local flow attenuation model for a simple cubic packing of elastic spheres in Hertzian contact. This idealized model of a sand pack underestimates the attenuation observed in water-saturated sandstone at 1 kHz by more than a factor of ten. Higher attenuation is predicted by this model if a thin film of fluid is allowed to fill the grain contacts.

Murphy et al. (1986) pursued the idea of local flow within the contacts between grains by developing the *contact relaxation* model. In this model, pore fluid is squeezed out of the regions of high pressure and sucked back into the regions of low pressure during the passage of a wave. The predictions of this model for a 2 nm thick fluid-filled grain contact showed reasonable agreement with resonant bar shear wave attenuation measurements made on Berea sandstone.

Walsh (1968, 1969) and O'Connell and Budiansky (1977) have proposed that *viscous shear relaxation* across thin pores filled with a Newtonian fluid can result in substantial

attenuation. The P-wave attenuation predicted for a thin penny-shaped ellipsoidal inclusion filled with a viscous fluid for frequencies satisfying $\omega \ll 3K_{fluid}/(2\eta)$ is described by

$$\alpha_{relax} = \frac{B_{relax}}{c_p(\omega)} \omega^2 \eta \quad (1.9)$$

where B_{relax} is a constant which depends on the properties of the fluid and the rock. This model also predicts an increase in attenuation with increasing frequency and viscosity.

1.3.2.3 Fluid-Matrix Attenuation

There have been only a limited number of studies investigating the influence of surface chemistry on the attenuation of seismic waves. Using the resonant bar technique, Tittmann et al. (1980) and Clark et al. (1980) observed that attenuation in initially dry sedimentary rocks increased dramatically with increasing monolayer coverage of adsorbed volatiles. At low partial pressures, Tittmann et al. noted a correlation between the attenuation and the fluid dipole moment per unit volume or the strength of the hydrogen bond, the polar nature of the molecules, and the molecular size and proposed that seismic attenuation is caused by thermally activated interactions in the adsorbed film of fluid.

Spencer (1981) measured attenuation in sandstone and limestone using a stress-strain apparatus and concluded that small amounts of water interact with the quartz grains reducing the surface free energy. Murphy et al. (1984b) pointed out that a reduction in the surface free energy reduces the cohesive Van der Waals forces between the grains. To quantify this effect, they computed the reduction in the intergranular contact stiffness associated with the presence of a fluid using the Gibb's adsorption equation. The magnitude of the reduction in grain contact stiffness was reported to be negligible above normal loads in excess of about 1 mN.

It should be noted that most fluid chemistry related attenuation was observed in rocks at low fluid saturations, typically less than 10 monolayers of fluid coverage. Clark et al. (1980) state that as the fluid saturation increases, *bulk fluids condense in fine capillaries and fluid flow attenuation becomes the dominant attenuation mechanism*. Most of the experiments on sedimentary rocks were performed without confining stress.

The predictions of several of the P-wave attenuation mechanisms discussed in the previous subsections are plotted as a function of frequency in Fig. 1.4. Also shown in this figure, are the upper and lower bounds of attenuation inferred from the spread in the in situ attenuation coefficients compiled by Attewell and Ramana (1966). The laboratory attenuation measurements in the 1 kHz and 1 MHz frequency ranges were taken from Jones (1986) and this study (Chapter 3), respectively. This plot illustrates that local flow can explain the attenuation peak observed in laboratory measurements made at 1 kHz. At ultrasonic frequencies, the attenuation resulting from local flow is negligible and Rayleigh scattering and viscous relaxation become the dominant sources of attenuation.

1.4 Thesis Objectives and Overview

The primary objective of this research is to quantify experimentally and analyze theoretically how stress and pore fluids in various parts of the pore space influence seismic attenuation and velocities in granular sedimentary rocks. This research will focus on a particular class of sedimentary rocks which are regarded as having granular or clastic textures and examine the effects of grain contacts on seismic wave propagation.

This chapter has summarized the important discoveries on the seismic properties of granular rocks that have been made in the laboratory and in theoretical modeling. In Chapter 2, the microstructural properties of granular rocks are identified and their effects on seismic waves are analyzed. Ultrasonic P- and S-wave measurements on two artificial porous materials and Berea sandstone for four different pore fluids and a range

Frequency Dependence of Theoretical and Observed P-wave Attenuation in Water-Saturated Sandstone

Figure 1.4 P-wave attenuation plotted as a function of frequency for water-saturated sandstone. Laboratory measurements are plotted as circles and the predictions of the theoretical models are plotted as solid lines. The shaded zone indicates the upper and lower bounds of attenuation inferred from the spread in the in situ attenuation coefficients compiled by Attewell and Ramana (1966).

of stresses are presented in Chapter 3. Tests are also performed on Berea sandstone partially saturated with low-melting point solids. These measurements yield insight into the mechanisms of seismic wave attenuation and place constraints that are useful when attempting to understand the micromechanical processes. A numerical method for simulating wave propagation in granular materials is presented in Chapter 4. The method uses a dynamic boundary integral equation method with generalized stiffness boundary conditions across the grain contacts. The method can be used to investigate the effects of grain packing microstructure, grain contact properties, and grain shape on seismic wave propagation in granular rocks. The final chapter summarizes the important conclusions of this research and provides suggestions for future research.

CHAPTER 2

ROCK MICROSTRUCTURE AND ITS EFFECTS ON SEISMIC WAVES

2.1 Introduction

There is a wealth of experimental data which shows that the seismic properties of granular rocks are related to rock composition and microstructure. However, the details of the dynamic processes which operate at the grain level during the passage of a seismic wave are not well understood, particularly as the seismic frequency is increased from near zero to ultrasonic frequencies. The objective of this chapter is to illustrate how grain composition and microstructure can influence the seismic properties of granular rocks. The first section presents an overview of the microstructural properties of granular rocks. Section 2.3 presents scanning electron microscope (SEM) photomicrographs of specimens used in the ultrasonic pulse transmission measurements of Chapter 3 and identifies microstructural features which may influence seismic wave propagation. Section 2.4 analyzes the micromechanics of seismic wave propagation in granular rocks and, in particular, focuses on the effects of grain contacts.

2.2 Microstructural Properties of Granular Rocks

The overall mechanical response of a granular rock to an applied static or dynamic stress takes place through mechanical interactions operating at the grain level. Rock microstructure, such as grain texture and fabric, can influence the manner in which stresses and displacements are transmitted from grain-to-grain. Thus, to understand the mechanical processes which operate at the grain level in rocks, it is useful to first examine its microstructural properties.

At the grain scale, sedimentary rocks are a complex assemblage of grains, cementing

material, and, in some cases, clay. Individual grains are connected over a finite contact area producing the solid skeleton and the pore space of the rock. The topology of the pore space has been studied extensively (Caruso, Simmons, and Wilkens, 1985), in part, because of its profound effects on fluid permeability (Bernabe, 1991). The topology of the solid skeleton has also been investigated to determine its effects on the overall strength and moduli (Bruno and Nelson, 1991) and on the seismic velocities and attenuation (Murphy, 1982b) of rock.

From the point of view of seismic wave propagation, the dynamic stresses and displacements are transmitted through the rock almost entirely from grain-to-grain. Therefore, it seems plausible that the following microstructural properties will influence the wave as it propagates through the grain structure:

- (1) grain packing geometry,
- (2) grain contact properties (e.g., contact area, contact roughness, cementation, clay in contacts, fluid in contacts),
- (3) grain shape and size,
- (4) variability in grain size and shape.

The packing geometry describes the spatial arrangement of grains and grain contacts and how they are assembled to form the rock. This property, which is conventionally referred to as the rock fabric, is a function of the properties of the individual grains, such as shape, size, and roughness. The rock fabric can be described in terms of the number of contacts per grain (coordination number) among other quantitative measures (Pettijohn, Potter, and Siever, 1987). Grain contacts have been classified into four types according to their geometries by Blatt et al. (1980): (1) tangential, (2) long, (3) concavoconvex, and (4) sutured (Fig. 2.1). Grain contacts encountered in the three granular materials used in this study are predominantly of the long and tangential types. In addition to grain contact geometry, grain contact properties such as contact area, roughness, cementation, clay, and fluid can alter the stiffness of the contact and introduce contact rheology which can have a

Classification of Grain Contact Geometries

Figure 2.1 Classification of grain contacts according to their geometries (after Blatt et al., 1980).

profound effect on the velocities and attenuation of seismic waves.

2.3 Scanning Electron Microscope Observations

Microstructural properties of granular rocks can be directly observed using a scanning electron microscope (SEM) in backscatter (secondary electron emission) mode. In addition to providing information on the texture and fabric of rocks, many SEM's are equipped with an energy dispersive X-ray spectrometer (EDS), which can provide a relative measurement of the elemental composition of the rock specimen.

We used an SEM/EDS to inspect the microstructural details of our laboratory specimens. Specimen selection was limited to Berea sandstone, a widely studied sandstone, and two artificial granular materials, sintered glass beads and porous alundum. Our approach was to focus on a very limited number of specimens and to isolate the effects of microstructure on ultrasonic wave propagation. The basic assumption of this type of approach is that the results of this study will allow us to draw connections between microstructural properties and wave propagation that will be applicable or extendable to a wider class of granular rocks.

Berea sandstone was selected because it is considered a typical clean quartz sandstone. Sintered glass beads and porous alundum were selected because they have microstructures which are much simpler than most sedimentary rocks. Both of these porous sintered materials are free of clays and have well-sintered grain contacts. The grain shapes in the three materials range from angular (alundum) to sub-angular (Berea sandstone), to well-rounded (glass beads). The grain sizes in the three materials are typical of most sandstones: 100 μm in glass beads, 200 μm in Berea sandstone, and 300 μm in porous alundum.

2.3.1 Sintered Glass Beads

Cylindrical specimens of sintered glass beads were fabricated at the Lawrence Berkeley Laboratory Ceramics Shop by heating spherical soda lime glass beads (Potter Industries Ballotini™ Impact Beads, Anaheim, CA) in a carbon mold under low confining pressure. The temperature history was controlled by a programmable oven and was designed to avoid the generation of microcracks. The composition of soda lime glass is 75% SiO₂, 15% Na₂O and 10% CaO. The bead diameters ranged from about 75 μm to 150 μm. The physical properties of the sintered glass bead specimen are given in Table 2.1. The measured porosity, $\phi = 0.37$, is consistent with a random packing of spherical particles. Fig. 2.2 is an SEM micrograph of the sintered glass bead specimen used in the ultrasonic tests of Chapter 3. Aside from the saw damage produced during the preparation of the SEM sample, the packing structure appears to be well bonded and before testing, there was no visual evidence of microcracks. Individual beads have retained their spherical shape and the surfaces of the beads are smooth. Individual beads are bonded together mainly by tangential type grain contacts which are circular in cross section, and, in fewer instances, by narrow necks.

2.3.2 Porous Alundum

Cylindrical specimens of porous alundum were cored from commercially available aluminum oxide filter plates (Norton Industries #P260 Filter Plates, Worchester, MA). The alumina grains are angular and range from about 200 to 500 μm in diameter. The physical properties of the alundum are given in Table 2.1. The measured porosity (0.45) and permeability ($> 20 D$) are considerably larger than the measured values for the sintered glass beads and the Berea sandstone. Most grain contacts are of the tangential and long types (Fig. 2.3). In most cases, neighboring alumina grains are fully bonded together by a vitrified ceramic, although there is some evidence of incomplete bonding (Fig. 2.3).

Table 2.1 Properties of the Granular Materials

Material	Density	Porosity	Permeability	Grain Shape	Grain Size	Contact Radius	Coordination Number
Sintered Glass Beads	1570 kg/m ³	0.37	> 4.5 D	spherical	75 - 150 μm	20 - 30 μm	~ 6
Porous Alundum	2200	0.45	> 20 D	angular	200 - 500	100 - 200	~ 5
Berea Sandstone	2100	0.21	350 mD	subangular	150 - 300	50 - 150	~ 6

Figure 2.2 SEM photomicrographs of sintered glass beads. The surface damage was produced by saw cutting during the preparation of the sample for SEM analysis. The lower photomicrograph displays the details of a tangential grain contact. 100 micron bars are displayed in the lower right hand corners of each photomicrograph for scale.

Figure 2.3 SEM photomicrographs of porous alundum. Most of the alundum grains are completely bonded together by a vitrified ceramic, although there is some evidence of unbonded angular contacts as shown in the lower photomicrograph. 100 micron bars are displayed in the lower right hand corners of each photomicrograph for scale.

2.3.3 Berea Sandstone

Most sedimentary rocks can be described as a random packing of mineral grains of similar size and varying levels of intergranular cementation. The rock is formed from a series of geological processes which include grain transport, deposition, and diagenesis. Grains are sorted during the transport and deposition processes. During diagenesis, the loose grains are compacted and consolidated as they slip, rotate, and deform. In an environment with enhanced pressure and temperature, pressure solution may occur in grain contacts resulting in the precipitation of cement in the grain contact and overgrowths on the exposed grain surfaces. Typical cementing materials in many sandstones are quartz (Fig. 2.4), calcite (Fig. 2.5), and clay. As a result of the processes of diagenesis, the rock can develop a substantial cohesive strength.

The microstructural properties of Berea sandstone (Table 2.1) are regarded by many as being representative of a wide class of clean quartz sandstones. It is frequently used in laboratory studies of fluid flow, hydraulic fracturing, and seismic wave propagation because of its availability and cost. Berea sandstone is quarried from Devonian-Mississippian rocks by the Cleveland Quarries Company in Amherst, OH.

Berea sandstone is considered a typical clean quartz arenite which is relatively homogeneous and isotropic when viewed on a scale larger than about 5 grain diameters (Fig. 2.6). The granular framework of Berea sandstone is composed of subangular, well-sorted quartz grains with small amounts of microcrystalline quartz, feldspar, calcite, and clay (primarily kaolinite and illite). Grain contacts are planar features with considerable micro-roughness (Fig. 2.7). The grains are covered with quartz and calcite overgrowths (Fig. 2.4 and 2.5) which add roughness to the grain surfaces and partial cementation between individual grains.

Figure 2.4 SEM photomicrographs of quartz grains in Berea sandstone. The lower photomicrograph is a close up of quartz overgrowths coating a quartz grain. The lower photomicrograph illustrates some of the finer detail of the overgrowth. Scales are displayed in the lower right hand corners of each photomicrograph.

Figure 2.5 SEM photomicrographs of Berea sandstone which shows calcite overgrowths encompassing several grains. 100 micron bars are displayed in the lower right hand corners of each photomicrograph for scale.

Figure 2.7 SEM photomicrographs showing the details of a grain contact in Berea sandstone. 100 micron bars are displayed in the lower right hand corners of each photomicrograph for scale.

Our SEM/EDS analysis did not reveal the presence of clays. Detailed examination of the distribution of clays in Berea sandstone by Tosaya and Nur (1982) revealed that the grain contacts are *predominantly clay free, with authigenic kaolinite and illite tending to occupy free pore space*. This observation is important when analyzing the seismic properties of Berea sandstone since solid-to-solid grain contacts are expected to have higher normal and shear stiffnesses than clay-filled grain contacts and, hence, higher seismic transmission and velocities.

2.4 Effects of Microstructure on Seismic Wave Propagation

From the SEM analysis of the previous section, it is visually apparent that a typical sandstone is a complex, multicomponent system composed of grains, overgrowths, partial intergranular cementation, clay, and open void spaces. These elements combine to form the structure of the rock and define its overall mechanical properties.

When considering seismic wave propagation, a granular rock can be viewed as a waveguide which is characterized by a very complex geometry and the presence of structural discontinuities in the form of partially cemented grain contacts. When a wave encounters a grain packing arrangement, as shown in Fig. 2.8, part of the wave will scatter off the traction-free surfaces of the grain. This type of scattering is typically referred to as *pore scattering* and is well described by Rayleigh scattering theory for spherical inclusions when the wavelength is much larger than the grain size. Pore scattering attenuation is proportional to the fourth power of frequency (i.e., $\alpha = C f^4$) and, therefore, will be negligible below ultrasonic frequencies.

If the grains are only partially cemented together, there will also be scattering off of the grain contacts. When viewed in this manner, grain contacts are non-welded interfaces very

Grain Contact and Pore Scattering

Figure 2.8 Schematic illustration of scattering off open pores and partially cemented grain contacts in a granular rock. The wavefront may be non-planar at a scale of several grain diameters but is roughly planar at the scale of about $5d$. The state of stress for a P-wave at this level is not uniaxial strain as in an elastic continuum.

similar in nature to large scale fractures and joints (Pyrak-Nolte et al., 1990). The magnitude of the grain contact scattering attenuation, as predicted by the theory for wave transmission across non-welded interfaces (Shoenberg, 1980), increases as the stiffness of the grain contact is reduced or as the frequency is increased. The major difference between grain contact scattering and pore scattering is that for frequencies used in the field (10 Hz to 10 kHz) the former may be non-negligible for low grain contact stiffnesses (i.e., for poorly consolidated rocks). In addition, fluid-filled grain contacts may serve as the sites for rheological losses. The properties of grain contacts and their effects on seismic waves are discussed in further detail in the next section.

2.4.1 Lumped Properties of Grain Contacts

Grain contacts in many granular rocks are regions of considerable complexity. For example in sandstones, the process of diagenesis often leaves individual grains only partially cemented together. In addition, the surfaces of the grains in the contact region may have considerable micro-roughness and appear similar to the surfaces of a large scale fracture or joint. Clays and pore fluid may fill the microvoids in the grain contact, adding complex rheology and surface chemistry to this interface.

At first sight, capturing the seismic properties of a grain contact might appear to be a hopelessly complicated task. However, if it is recognized that the seismic wavelength is almost always much larger than the thickness of the grain contact, then the properties of the rough interface can be *lumped* into a single parameter which describes the elastic stiffness of the interface. Lumped properties approximations of this sort have been used extensively to model transmission in acoustic (Kinsler et al., 1982) and electrical networks (Feynman et al., 1964).

For seismic wave transmission across fractures and joints, the ultrasonic measurements of Pyrak-Nolte et al. (1990) and the crosshole measurements of Myer et al. (1990) were

found to exhibit the same amplitude reduction (Shoenberg, 1980) and velocity change (Pyrak-Nolte, 1990) predicted by the displacement-discontinuity model of a non-welded interface. This model has also been used extensively by researchers in non-destructive testing of welds and adhesive bonds (e.g., Rohklin and Wang, 1991) with considerable success. The theoretical justification for the range of validity of this model was presented by Angel and Achenbach (1985). Using the boundary integral equation method, they computed the transmission and reflection characteristics for an infinite array of equally spaced, coplanar cracks. The results of their simulations show that the displacement-discontinuity model is a valid approximation for a row of cracks provided that the wavelength is several times greater than the crack spacing. For a grain contact, this requirement is almost always satisfied even at ultrasonic frequencies since the wavelength is much larger than the spacing of the asperities within the grain contact (Fig. 2.9).

In the displacement-discontinuity model, a non-welded interface is described by boundary conditions on the displacements and tractions:

$$u_2 - u_1 = t_1 / \hat{\kappa} \quad (2.1)$$

$$t_1 = -t_2 \quad (2.2)$$

where the subscripts 1 and 2 refer to the grain on either sides of the grain contact, u is the displacement, t is the traction, and $\hat{\kappa}$ is the interface stiffness. For large values of $\hat{\kappa}$, the contact is very stiff and the displacements are nearly continuous as in the case of a welded interface. For small values of $\hat{\kappa}$, the traction becomes zero and the interface becomes a traction-free surface.

By generalizing the stiffness such that it becomes a complex quantity, it is possible to model the effects of interface rheology (Rohklin and Wang, 1991):

$$\hat{\kappa} = \kappa_r + i \kappa_i \quad (2.3)$$

Lumped Properties of Grain Contacts

Figure 2.9 Partially cemented grain contacts are non-welded interfaces which can be modeled using the displacement-discontinuity boundary condition provided that the wavelength is much larger than the spacing between the asperities of contact.

Rheological Models for Grain Contacts - I

Generalized Stiffness: $\hat{\kappa} = \kappa_r + i\kappa_i$

- **dry grain contacts:**

- **fluid saturated grain contacts:**

Figure 2.10 Rheological models for grain contacts expressed in terms of the real and imaginary parts of the complex stiffness, $\hat{\kappa}$.

In this form, any grain contact loss mechanisms (e.g., local flow, viscous relaxation) that can be modeled by a combination of springs and dashpots by choosing the appropriate values for κ_r and κ_i , as shown in Fig. 2.10. For example, a purely viscous contact (Newton model) can be modeled by setting $\kappa_r = 0$ and $\kappa_i = \omega\eta$, where ω is the angular frequency and η is the specific viscosity of the fluid (viscosity per unit thickness). Substituting these values into Eq. (2.1) and noting that $v = i\omega u$ gives the velocity-discontinuity boundary condition for a viscous fluid (Shoenberg, 1980; Pyrak-Nolte et al., 1990):

$$v_2 - v_1 = t_i / \eta \quad . \quad (2.4)$$

Several proposed models for dry and fluid saturated grain contacts are given in Fig. 2.11. A dry, partially cemented grain contact is modeled by normal and shear stiffnesses. A thin fluid-filled gap can be represented by a combination of a Maxwell model for the normal stiffness, $\hat{\kappa}_n$, and a Newton model for the shear stiffness, $\hat{\kappa}_i$, provided that the thickness of the gap is less than the thickness of the hydrodynamic boundary layer, $\delta = [\rho/(\omega\eta)]^{1/2}$, where ρ is the fluid density. A partially cemented grain contact filled with fluid can be modeled using a Kelvin-Voigt model. Suárez-Rivera (1992) performed S-wave transmission measurements across thin clay films under the presence of different fluids and found that the transmission losses were very well described by the Maxwell model. It should be noted that while the basic form of these models may be appropriate to describe the rheologic losses occurring in the grain contact, the values of the fluid viscosity may or may not correspond to the actual viscosity of the fluid. For example, in the case of clay-filled grain contacts, the clay itself constitutes a porous, chemically-reactive structure which has an effective viscosity that can be several orders of magnitude higher than that of the fluid itself (Suárez-Rivera, 1992).

Rheological Models for Grain Contacts - II

- dry grain contacts:

- fluid-filled grain contacts:

- clay-filled grain contacts:

Figure 2.11 Rheological models for the grain contact properties of partially cemented grain contacts with and without viscous fluids.

The idea of using rheological models to describe loss mechanisms in granular rocks is by no means a new one (e.g., see Biot, 1960; Jones and Nur, 1983). However, to date, analysis of attenuation resulting from grain contacts has ignored transmission losses which can occur in partially cemented grain contacts and has not considered the cumulative effects of an assemblage of grain contacts on the overall transmission losses. In Chapt. 4, a dynamic boundary integral equation formulation is developed for a packing of grains coupled together by the displacement-discontinuity boundary condition. The numerical code based on this formulation is capable of simulating wave propagation in a packing of grains with arbitrary shapes and contact rheologies.

2.4.2 Zero Order Model of P-wave Attenuation

The idea of discontinuities in rock occurring on all scales is an appealing one since it would allow for a single theory to describe the seismic properties for any frequency of interest. Field and laboratory measurements have verified that the displacement-discontinuity model accurately describes the transmission losses across fractures under varying stress conditions. The previous section illustrated that this model may also be used to model imperfectly cemented and fluid-filled grain contacts.

However, deriving a closed form expression for the transmission coefficient and velocity of a granular rock is not a tractable exercise as in the case for a single fracture. Nevertheless, it is possible to develop a zero order approximate model for P-wave propagation through a granular rock by making some simple approximations. If the actual rock microstructure is idealized in the fashion displayed in Fig. 2.12 into a simple array of parallel interfaces, then it is possible to use the expression for normal incidence transmission coefficient for a single non-welded interface (Shoenberg, 1980),

Contact Stiffness Scattering Model

Figure 2.12 Idealized model of a granular rock as a series of contact planes with effective stiffnesses used to estimate the attenuation in dry or saturated rocks.

$$|T(\omega)| = \left[1 + \frac{\omega^2 Z^2}{4\kappa^2} \right]^{-\frac{1}{2}}, \quad (2.5)$$

where Z is the impedance (density $\times$ velocity) and κ is the stiffness of each contact plane in units of [Pa/m]. Ignoring multiple scattering, the transmission coefficient for propagation across n contact stiffness planes can be approximated by raising Eq. (2.5) to the n^{th} power,

$$|T(\omega)| = \left[1 + \frac{\omega^2 Z^2}{4\kappa^2} \right]^{-\frac{n}{2}}. \quad (2.6)$$

The P-wave attenuation predictions of this highly idealized model of a granular rock are compared with ultrasonic P-wave attenuation measurements on a sample of oven dry Berea sandstone for a uniaxial stress of 19.7 MPa in Fig. 2.13. The two adjustable parameters in the model are n and κ . The values of the number of contact planes, n , was taken as the number of 200 μm diameter grains necessary to form a column equivalent to the length of the Berea sandstone sample (i.e., by assuming a cubic packing grain microstructure). The value of the contact plane stiffness, κ , was estimated from the effective P-wave modulus, $\bar{M}$, obtained from the measured P-wave velocity,

$$c_p = \sqrt{\frac{\bar{M}}{\rho}} \rightarrow \bar{M} = c_p^2 \rho \quad (2.7)$$

using the approximate relation between the effective and intrinsic moduli and the stiffness,

$$\bar{M}^{-1} = M^{-1} + \frac{n}{\kappa} \rightarrow \kappa = \frac{n}{(\bar{M}^{-1} - M^{-1})} \quad (2.8)$$

The theoretical curve shown in Fig. 2.13 was computed by substituting Eq. (2.6) into the following equation

$$Q^{-1}(\omega) = \frac{-2c_p}{\omega} \ln(|T(\omega)|) \quad (2.9)$$

Figure 2.13 Comparison of P-wave attenuation in dry Berea sandstone measured in the laboratory with the predictions of the contact-stiffness model for a stiffness of $3 \times 10^{14} \text{ Pa/m}$ and 175 contact interfaces.

The agreement between theory and measurement is reasonably good considering the simplicity of the model. The fall-off of the theoretical curve at lower frequencies in Fig. 2.13 implies that this grain scale scattering attenuation will be negligible at crosshole frequencies for a dry sample of Berea with an axial stress of 19.7 MPa.

The attenuation from ultrasonic transmission experiments on single natural fractures performed by Pyrak (1988) and for an in situ transmission experiment across approximately three natural fractures (Myer and Majer, 1984) at the Basalt Waste Isolation Project (BWIP) site in Washington are plotted in Fig. 2.14 with the corresponding predictions of the displacement-discontinuity model. The results indicated that discontinuities occurring from the grain scale to the field scale behave as non-welded interfaces which can be accurately represented by a displacement-discontinuity type boundary condition.

The effects of changing the stiffness and the number of contact planes is shown in Fig. 2.15. Increasing the number of contact planes shifts the attenuation curve up, while increasing the stiffness shifts the curve up and over to lower frequencies. The effects of the simple Newton (dashpot only) model are also given in Fig. 2.15. Increasing the specific viscosity, η , has the effect of shifting the attenuation curve towards lower frequencies without increasing the peak attenuation. These results indicate that grain contact scattering attenuation effects can be significant below ultrasonic frequencies if the stiffnesses of the grain contacts are low (e.g., in a poorly consolidated sediment) and when the grain contacts are filled with a viscous fluid.

2.5 Summary and Conclusions

This chapter has illustrated how grain microstructure can influence the seismic

Figure 2.14 Comparison of laboratory and in situ attenuation measurements and theoretical attenuation predictions of the contact-stiffness model. Laboratory measurements on dry granite with a single fracture were taken from Pyrak (1988). In situ measurements across 3 fractures at the Basalt Waste Isolation Project were provided by L. R. Myer (Lawrence Berkeley Laboratory).

Figure 2.15 Theoretical attenuation predictions of the contact-stiffness model illustrating the effects of varying the stiffness, the number of contact planes, and the viscosity of the fluid.

properties of granular rocks. SEM/EDS analysis was used to visually inspect the microstructures of two artificial porous granular materials (sintered glass beads and alundum) and Berea sandstone. The two artificial materials were found to contain mostly tangential type grain contacts and individual grains appeared to be well sintered in most cases. Berea sandstone exhibited more complexity. Grain contacts were observed to be only partially cemented together. Quartz and calcite overgrowths coated the surfaces of many of the grains and, in some instances, were observed to span a region encompassing three to four grain diameters. The elastic and viscoelastic properties of grain contacts were defined in terms of a displacement-discontinuity boundary condition and a complex grain contact stiffness. The predictions of a simple model for scattering losses in a dry granular rock based on the displacement-discontinuity boundary condition was compared to measured P-wave attenuation in Berea sandstone and found to show basic agreement.

CHAPTER 3

EFFECTS OF PORE FLUIDS AND STRESS ON THE ULTRASONIC PROPERTIES OF GRANULAR MATERIALS

3.1 Experimental Set-up and Data Analysis

3.1.1 Sample Preparation

Ultrasonic pulse transmission measurements were performed on cylindrical samples of sintered glass beads, porous alundum, and Berea sandstone with diameters of 5.08 cm and lengths ranging from 1.27 to 5.08 cm as specified in Table 3.1. In order to minimize the effects of anisotropy, the samples of Berea sandstone were cored perpendicular to the bedding planes. The ends of the samples were ground flat and parallel to within 10 μm to ensure uniform axial loading.

Samples were oven dried at 110° C for more than 12 hours prior to the testing. The primary purpose of the oven drying was to remove any bulk water occupying the fine pores within the granular samples. It should be noted that oven drying at 110° C is not sufficient to remove the mono-layer of water absorbed on the surfaces of the silica grains. Upon removal from the oven, the samples were placed in either plastic or rubber jackets to shield the sample from atmospheric moisture.

Samples which were tested under saturated conditions were vacuum impregnated with a fluid by placing the sample into a fluid-filled beaker, placing the beaker into a vacuum jar, and applying a vacuum of 1 kPa absolute pressure. The vacuum saturation was stopped and the sample was allowed to equilibrate at atmospheric pressure when there were no visible signs of air bubbles emanating from the surfaces of the sample.

Table 3.1 Physical Properties of the Granular Materials

Specimen	Sintered Glass Beads	Porous Alundum	Berea Sandstone
Description	Fused soda lime glass beads.	Alumina bonded by a vitrified ceramic.	Medium quartz grain sandstone with some feldspar and calcite.
Grain Size and Shape	75 to 150 μm in diameter; spherical in shape.	200 to 500 μm in diameter; angular in shape.	150 to 300 μm in diameter; sub-angular in shape.
Density	1570 kg/m^3	2200 kg/m^3	2100 kg/m^3
Porosity	0.37	0.45	0.21
Permeability	> 4.5 D	> 20 D	350 mD
Bulk Modulus (dynamic)	6.5 GPa	40.3 GPa	13.1 GPa (@ 25 MPa)
Shear Modulus (dynamic)	4.6 GPa	23.8 GPa	12.8 GPa (@ 25 MPa)
Uniaxial Compressive Strength	72 MPa	48 MPa	---
Specimen Dimensions	5.08 cm diameter 5.08 cm length	5.08 cm diameter 1.27 cm length	5.08 cm diameter 4.32 cm length

3.1.2 Experimental Set-up

The experimental set-up used to perform the P- and S-wave ultrasonic measurements reported throughout this chapter is shown schematically in Fig. 3.1. Two identical ultrasonic transducers were fabricated using cast iron for the electrodes and the backing and aluminum for the face plate and the housing. Each transducer contains both compressional and a shear piezoelectric (PZT) crystals, with resonance frequencies of approximately 1 MHz. The transducers were designed to withstand moderate stresses (up to 40 MPa).

The jacketed sample was placed between the two transducers. Thin, chemically clean lead foil disks 30 μm in thickness were inserted between the transducers and the ends of the sample to improve the mechanical coupling and repeatability. The transducer-sample assembly was placed in an aligned loadframe. The sample was preloaded with a hydraulic jack and a calibrated pump up to an axial stress 19.7 MPa to establish the lead foil coupling between the transducer-sample interfaces and then unloaded to 1 MPa. Ultrasonic measurements were made on samples subjected to either a uniaxial or a hydrostatic state of stress at 2.5 MPa intervals as the stress on the sample was increased. For the hydrostatic tests, confinement was provided by a water pressurized confining cell.

Either a P- or S-wave was generated by one of the two transducers by exciting one of the piezoelectric crystals with a square wave pulse with an amplitude of 500 V and a width of 640 ns supplied by a Cober[®] 605P high voltage pulser. The transmitted wave was received by one of the piezoelectric crystals inside of the second transducer and then digitized and stored by a computer controlled Lecroy[®] 8013A digital acquisition system. Waveforms were averaged 100 times to improve the signal-to-noise ratio.

Figure 3.1 Schematic of the experimental apparatus used to measure ultrasonic P- and S-wave transmission through cylindrical samples under varying stress. Hydrostatic tests were performed with a confining cell placed around the sample (not shown).

3.1.3 Repeatability Tests

The uncertainties in the group velocities and attenuation coefficients computed from the recorded P- and S-waves are influenced by a number of factors which include variability in the sample preparation and assembly, in the signal generation and acquisition system, and in the data analysis techniques. While it is difficult to estimate the uncertainties of the individual components of the experiment, an estimate of the overall uncertainty can be obtained by comparing measurements obtained in multiple experiments on the same sample or similar samples taken from the same block of material.

P-wave peak-to-peak amplitudes for four aluminum calibration tests are plotted as a function of the uniaxial stress in Fig. 3.2. To ensure a repeatable signal, the pulser was monitored at various stages during each test to detect possible drifts in the width, magnitude, and general shape of the square wave pulse. In addition, calibrations on aluminum standards were performed periodically to monitor possible changes in the transducers and the data acquisition system and for use in the attenuation calculation described in the following section.

Also shown in Fig. 3.2 are the P-wave peak-to-peak amplitudes for three samples of oven dry Berea sandstone taken from the same block of rock. There is a small difference between the peak-to-peak amplitudes of the aluminum tests which decreases with increasing stress. There is very little difference in the peak-to-peak amplitudes of the Berea sandstone tests. These results suggest that the coupling between the aluminum sample and the transducers is sensitive to small misalignments in the assembly since the high modulus of the aluminum prevents it from accommodating any misalignment in the loading. In contrast, the lower modulus Berea sandstone is able to deform to accommodate any misalignments in the loading, thus, giving more repeatable sample-transducer

Experimental Repeatability for P-waves

Figure 3.2 P-wave peak-to-peak amplitudes from four separate tests on aluminum and oven dry Berea sandstone. The small amount of scatter in the amplitudes is an indicator of the repeatability of the experimental apparatus and procedures used as well as the homogeneity of the block of Berea sandstone from which the four samples were taken.

coupling. The small variability in the amplitudes of the ultrasonic waves in Berea sandstone for the three different samples (approximately $\pm 2\%$) is evidence for the stability of the experimental equipment and procedures and also illustrates the homogeneity of the block of sandstone from which the samples were cored.

3.1.4 Data Analysis

Peak-to-peak amplitudes and first break travel times were measured directly from the recorded P- and S-waves. These measured values were monitored during each test. The attenuation coefficients and group velocities were computed after the completion of each test from spectral analysis on the waveforms.

Spectral analysis consisted of windowing the P- or S-wave arrival with a boxcar, padding the tail of the extracted wave with the appropriate number of zeros to bring the total length of the signal to either 512 or 1024 points, and taking the Fast Fourier Transform (FFT). Tapering was not used because the P- and S-waves were observed, in almost all cases, to be distinct arrivals with identifiable onsets and tails. The magnitude of the Fourier Transformed waveform was used to compute the frequency spectrum, $|\hat{u}(\omega)|$, and the attenuation coefficient, α . The phase, ϕ , was used to compute the group velocity dispersion. The procedure for computing the group velocity and attenuation are discussed in the next section.

3.1.4.1 Group Velocity

In principal, it is possible to compute the phase and group velocities as a function of frequency (i.e., the velocity dispersion) from the phase angle, ϕ , obtained from the real and imaginary parts of the Fourier Transform of the measured waveform, $u(t)$,

$$\phi(\omega) = \tan^{-1} \left[\frac{u_i(\omega)}{u_r(\omega)} \right] , \quad (3.1)$$

where ω is the angular frequency. The phase angle is typically supplied by most FFT (Fast Fourier Transform) routines in *wrapped* form $\phi \in [-\pi, \pi]$, as displayed in Fig. 3.3. The phase is *unwrapped* by shifting each linear segment of the sawtooth phase upward by a factor of 2π , as shown in Fig. 3.3.

The phase and group velocities can be computed directly from the unwrapped phase using

$$\begin{aligned} c_{ph}(\omega) &= \frac{L}{t_{ph}(\omega)} , \\ c_{gr}(\omega) &= \frac{L}{t_{gr}(\omega)} , \end{aligned} \quad (3.2)$$

where L is the length of the sample and t_{ph} and t_{gr} are the phase and group travel times through the sample which are given by

$$\begin{aligned} t_{ph}(\omega) &= \frac{\phi(\omega)}{\omega} , \\ t_{gr}(\omega) &= \frac{\partial \phi}{\partial \omega} . \end{aligned} \quad (3.3)$$

Using the above equations, it is a straightforward process to compute the group velocity dispersion from the recorded waveforms. However, it is difficult to obtain an accurate estimate of the phase velocity because of inaccuracies in the phase at lower frequencies (i.e., below 100 kHz) where the signal-to-noise ratio is low. These phase inaccuracies at low frequencies are propagated to higher frequencies since the phase velocity is computed from the secant of the $\phi - \omega$ curve. Since the group velocity is computed from the tangent of the $\phi - \omega$ curve, it is not affected by the phase at lower frequencies. Techniques for estimating the phase velocity exist (e.g., see Sachse and Pao,

Procedure for Computing the Phase and Group Velocities

Figure 3.3 Procedure for computing the phase and group velocities from the Fast Fourier Transform supplied phase angle, ϕ_w .

1978) but require that certain assumptions be made about the behavior of the phase angle at low frequencies. For this reason, this study will present group velocities only. It should be noted that the group velocities reported here have not been corrected for dispersion produced by a finite piston source (Papadakis, 1966; Winkler and Plona, 1982). The magnitude of this dispersion is anticipated to be small and not to affect the overall trends in the group velocities.

3.1.4.2 Attenuation Coefficient

Attenuation coefficients were computed from the spectra of the transmitted waves using the *method of spectral ratio's* with an aluminum reference (Toksöz et al., 1979). This simple technique eliminates the need for a knowledge of the source spectrum, and, by employing the assumption that the geometrical spreading of the rock and aluminum standard of equivalent dimensions are similar, enables the attenuation coefficient of the rock to be computed directly. Recently, the accuracy of the technique has been questioned (Winkler and Plona, 1982; Tang et al., 1990; Klimientos, 1991) as it became recognized that the geometrical spreading in the rock and aluminum are often sufficiently different to produce first order errors in the computed attenuation coefficients. A correction factor for the differences in the geometrical spreading of the rock and aluminum samples can be found in Tang et al. (1990) and is used in the attenuation coefficients presented in the following section. The method for computing the geometrical spreading corrected attenuation coefficient is described below.

The magnitude of the spectrum at an angular frequency, ω , for a wave travelling in the x -direction is described by

$$|\hat{u}_r(x, \omega)| = \hat{S}(\omega) \hat{G}_r(x, \omega) e^{-\alpha_r L} \quad (3.4)$$

where L is the propagation distance, $\hat{G}_r$ is the geometrical spreading in the rock, and α_r is the attenuation coefficient of the rock sample.

Similarly, for an aluminum sample of equivalent dimensions, the magnitude of the frequency spectrum at an angular frequency, ω , is

$$|\hat{u}_a(x, \omega)| = \hat{S}(\omega) \hat{G}_a(x, \omega) e^{-\alpha_a L} \quad , \quad (3.5)$$

where the subscript a denotes the properties of the aluminum. Dividing Eq. (3.5) by Eq. (3.4) produces the following equation,

$$\ln \left[\frac{|\hat{u}_a(\omega)|}{|\hat{u}_r(\omega)|} \right] = \ln \left[\frac{\hat{G}_a(\omega)}{\hat{G}_r(\omega)} \right] + [\alpha_r(\omega) - \alpha_a(\omega)] L \quad (3.6a)$$

$$\rightarrow \alpha_r(\omega) = \alpha_a(\omega) + \frac{1}{L} \ln \left[\frac{|\hat{u}_a(\omega)|}{|\hat{u}_r(\omega)|} \right] + \frac{1}{L} \ln \left[\frac{\hat{G}_r(\omega)}{\hat{G}_a(\omega)} \right] \quad . \quad (3.6b)$$

No approximations have been made in the derivation of Eq. (3.6b). Two commonly employed assumptions are that the geometrical spreading of the aluminum and the rock sample are nearly identical ($\hat{G}_r \approx \hat{G}_a$) and that the attenuation coefficient of the aluminum is negligible compared to that of the rock ($\alpha_a \approx 0$), so that Eq. (3.6b) can be simplified as follows,

$$\alpha_r(\omega) \approx \frac{1}{L} \ln \left[\frac{|\hat{u}_a(\omega)|}{|\hat{u}_r(\omega)|} \right] \quad [\text{Nepers / m}] \quad , \quad (3.7)$$

or in units of [decibels / meter],

$$\alpha_r(\omega) \approx \frac{20}{L} \log \left[\frac{|\hat{u}_a(\omega)|}{|\hat{u}_r(\omega)|} \right] \quad [\text{db / m}] \quad . \quad (3.8)$$

The attenuation coefficient can be estimated directly from Eq. (3.7) or (3.8) by specifying the length of the sample and the amplitude spectra of the transmitted waves for the aluminum and the rock.

Tang et al. (1990) pointed out that the assumption of equal geometric spreading in the rock and aluminum samples (i.e., $\hat{G}_r \approx \hat{G}_a$) may lead to an overestimate of the attenuation coefficient since in many cases the acoustic impedance of the aluminum is larger than that of the rock. They derived analytic expressions for the geometrical spreading of P- and S-waves which are generated by a circular piston source and received by a coaxial circular piston receiver separated by a distance, z . These expressions for the P- and S-wave geometric spreading involve complex integrals which must be solved numerically. For source-receiver separations which are large compared to the wavelength (i.e., large $k_r z$), they demonstrated that the elastic expressions are well approximated by the corresponding acoustic expression for geometrical spreading (Bass, 1958),

$$\hat{G}_c(z, k_r) = \frac{1}{ik_r} \left[1 - \left(1 - \frac{\xi^2}{2k_r^2 a^2} \right) \left[J_0(\xi) + iJ_1(\xi) \right] e^{-i\xi} - \frac{\xi^2}{k_r^2 a^2} \left(\frac{iJ_1(\xi)}{\xi} \right) e^{-i\xi} \right] , \quad (3.9)$$

with the following modifications for a P-wave,

$$\hat{G}_p(z, k_p) = s^{-2} \hat{G}_c(z, k_p) , \quad (3.10)$$

and for an S-wave,

$$\hat{G}_s(z, k_s) = 2 \hat{G}_c(z, k_s) , \quad (3.11)$$

where z is the separation between source and receiver, k_p and k_s are the P- and S-wavenumbers, a is the radius of the circular piston source, $s = c_p / c_s$ is the ratio of P- to S-wave phase velocities and

$$\xi = \frac{k_r}{2} \left(\sqrt{4a^2 + z^2} - z \right) . \quad (3.12)$$

The requirement that $k_y z$ is large in order for Eq. (3.9) to be valid is satisfied for most of the ultrasonic measurements reported in this chapter since $8 \leq k_p \leq 160$ and $10 \leq k_s \leq 106$. An estimate of the geometric spreading for large $k_y z$ can be computed from Eq. (3.9)-(3.12) provided that the phase velocities, radii of the piezoelectric crystals, and the separation between the source and receiver crystals are known. In principal, if the geometric spreading, $\hat{G}_r$, and the source spectrum, $\hat{S}$, are both known, the attenuation coefficient can be computed directly without using an aluminum standard from Eq. (3.4). However, in the experiments reported here, the source spectrum was not measured so the attenuation coefficients were computed by the method of spectral ratio's using Eq. (3.6b) with the geometrical spreading correction specified in Eq. (3.9).

To demonstrate the magnitude of the geometric spreading correction, the geometric spreading for P- and S-wave transmission through $z = 5.08$ cm long samples of aluminum and sintered glass beads were computed using Eq. (3.9) and (3.10). The radius of the piezoelectric crystals are $a=1.27$ cm. Because the transmitting and receiving piezoelectric crystals are embedded inside of the transducer, the actual separation between the two crystals is actually larger than the thickness of the sample. The geometrical spreading associated with this additional propagation distance through the aluminum face plate of the transducer was assumed to be negligible and the length of the sample given above was used for z in Eq. (3.12).

A comparison of the attenuation coefficients computed with and without the geometric spreading correction are shown in Fig. 3.4. This figure illustrates that ignoring the differences in the geometric spreading of the aluminum and rock samples can result in a significant overestimate of the magnitude and an underestimate of the frequency dependence of the attenuation coefficient. The differences between the geometric spreading of the aluminum and rock samples will tend to become most significant when

Figure 3.4 P- and S-wave attenuation coefficients computed with and without the geometrical spreading correction.

the velocities are very different, when the attenuation in the rock is small, when the source-receiver separation is large, and when the transducer radius is small.

3.2 Experimental Results

Ultrasonic P- and S-wave group velocities and attenuation coefficients were computed for sintered glass beads, porous alundum, and Berea sandstone under varying stress conditions and for a variety of pore fluids. The pore fluids selected for this study were distilled water, n-decane, glycerol, and silicone oil. The properties of the three granular materials are summarized in Table 3.1, and the properties of the fluids are given in Table 3.2.

The four fluids were selected based on their viscosities and chemical properties. Water (1 cp) and n-decane (0.92 cp) have similar low viscosities, but water is a polar fluid and n-decane is a non-polar fluid. Glycerol (1490 cp) and silicone oil (960 cp) both have high viscosities, but glycerol is a polar fluid while silicone oil is non-polar.

The ultrasonic measurements for each of the three materials are presented in the following sections. The estimated experimental uncertainties for the attenuation coefficients and the group velocities are ± 60 db/m and $\pm 1.5\%$, respectively.

3.2.1 Effects of Stress and Pore Fluids on Velocities and Attenuation

The effects of stress on both P- and S-waves were examined by progressively increasing the stress on the sample to a peak stress which is approximately one third of the estimated uniaxial compressive strength of the material. Because the two artificial granular materials have sintered grain contacts, they were anticipated to exhibit little or no stress dependence on the measured velocities and attenuation. These materials were tested under uniaxial stress. Berea sandstone has partially cemented grain contacts and,

Table 3.2 Fluid Properties

Fluid	Density (g/cm³)	Bulk Modulus (GPa)	P-wave Velocity (m/s)	Viscosity (cP)	Surface Tension (mJ/m²)
Water (distilled)	1.0	2.21	1488	1.0	72.75 @ 20 °C 71.97 @ 25 °C
n-Decane†	0.73	1.15	1257	0.92	28
Silicone Oil‡	0.96	0.99	1015	960	20.7
Glycerol*	1.26	4.75	1943	1490 @ 20 °C 954 @ 25 °C	63.4 @ 20 °C

† 30560 Decan, Fluka Chemie AG, Switzerland

‡ 200 Fluid dimethylpolysiloxane, Dow Corning®, San Jose, CA.

* 2136-01 Glycerol, Baker Analyzed®, Phillipsburg, NJ.

therefore, was anticipated to exhibit stress-dependent ultrasonic properties. To examine the effects of stiffening different populations of grain contacts in Berea sandstone, ultrasonic measurements were performed under both uniaxial and hydrostatic states of stress, as illustrated in Fig. 3.5.

The waveforms of the transmitted P- and S-waves at different stresses are displayed in Fig. 3.6 and 3.7. The P- and S-waves for the two artificial granular materials exhibit only a small increase in amplitude with increasing stress. In contrast, the waveforms for Berea sandstone for both states of stress show a strong increase in P- and S-wave amplitudes as well as a notable decrease in P- and S-wave travel times with increasing stress. The basic shape and frequency content of the P- and S-waves for the three materials are similar.

The effects of the different pore fluids on the transmitted P- and S-waves are displayed in Fig. 3.8 and 3.9. For all granular materials, the pore fluids have a moderate to strong effect on the amplitudes and arrival times of the P- and S-waves. The effects of the fluids are less apparent in the waveforms for the alundum because the alundum samples are about one quarter of the length of the sintered glass bead and Berea sandstone samples. A general trend which is apparent in both the P- and S-waves is an increase in attenuation with increasing fluid viscosity. This viscosity-dependent attenuation is present in all samples except for the Berea sandstone samples tested under uniaxial stress. Attenuation in these samples better correlates with the surface tension of the pore fluid.

3.2.2 P- and S-wave Group Velocities

P- and S-wave group velocities were computed using the procedure described in Sec. 3.1.4.1 and are presented in Fig. 3.10 and 3.11 for a particular frequency which is specified on each plot. The group velocities for the two artificial materials exhibit little or

Uniaxial Stress

Hydrostatic Stress

Figure 3.5 Preferential stiffening of a certain population of partially cemented grain contacts in a granular or clastic material using the applied stress field.

Figure 3.6 Transmitted P-waves observed in dry granular materials are displayed at different stresses. Measurements on Berea sandstone were computed for both hydrostatic and uniaxial states of stress.

Figure 3.7 Transmitted S-waves observed in dry granular materials are displayed at different stresses. Measurements on Berea sandstone were computed for both hydrostatic and uniaxial states of stress.

Figure 3.8 Transmitted P-waves observed granular materials are shown for dry samples and fluid-saturated samples for a constant values of stress.

Figure 3.9 Transmitted S-waves observed granular materials are shown for dry samples and fluid-saturated samples for a constant values of stress.

Figure 3.10 P-wave group velocities are plotted as a function of stress for sintered glass beads (uniaxial), alundum (uniaxial), and Berea sandstone (uniaxial and hydrostatic). Each curve corresponds to a different pore fluid.

Figure 3.11 S-wave group velocities are plotted as a function of stress for sintered glass beads (uniaxial), alundum (uniaxial), and Berea sandstone (uniaxial and hydrostatic). Each curve corresponds to a different pore fluid.

no dependence on stress. This stress independence is attributed to the sintered contacts which do not increase in contact area as the stress on the sample is increased. In contrast, the group velocities for Berea sandstone samples for both states of stress increase by as much as 19% as the stress varied from 0 - 25 MPa. This stress dependence is attributed to the stiffening of the partially cemented grain contacts with increasing stress. The increase in group velocity with stress is greatest in the dry samples and is reduced when the pore space of the sandstone is filled with a fluid, particularly if the viscosity of the fluid is large, as in the case of silicone oil and glycerol.

The P-wave group velocities in sintered glass beads are largest for glycerol which has a bulk modulus over twice that of water (Table 3.2). The presence of this low compressibility fluid in the parts of the pore space surrounding the grain contacts (Fig. 2.2) effectively stiffens the solid frame during the passage of a high frequency wave. In alundum, the P-wave group velocities for the different fluids are very similar and are slightly lower than the dry velocity. The high level of bonding between the alumina grains has produced grain contacts which are well rounded (Fig. 2.3). The lower velocities for the fluid saturated samples of alundum result because the fluid occupying these low-aspect ratio gaps which separate the grains increases the effective densities of the grain contacts without significantly increasing the stiffness of the solid frame.

The P-wave group velocities in Berea sandstone for hydrostatic and uniaxial states of stress exhibit similar overall trends for the different fluids. The hydrostatic velocities are 1 to 2% higher than the uniaxial velocities. For hydrostatic loading, the velocities are largest for glycerol which has the highest bulk modulus followed by silicone oil which has a bulk modulus which is about one-half the bulk modulus of water (Table 3.2). This suggests that at ultrasonic frequencies the viscosity of the fluid can effectively stiffen the grain contacts resulting in higher velocities. For uniaxial loading, group velocities and attenuation coefficients for silicone oil were measured prior to transducer modifications

and are not presented. The uniaxial group velocities for n-decane, water, and glycerol can be interpreted as increasing with either the bulk modulus or viscosity of the pore fluid.

The S-wave group velocities are displayed in Fig. 3.11. Both artificial granular materials have lower velocities for the fluid-saturated samples than the corresponding dry samples. The reduction in S-wave velocity upon saturation with a pore fluid has been observed in many sedimentary rocks and is usually ascribed to the *density effect*. The density effect attributes lower S-wave velocities in fluid-saturated samples to an increase in the effective density of the sample due to the presence of the pore fluid without a corresponding increased in the effective shear modulus of the sample:

$$\left. \begin{array}{l} \mu_{sat} \approx \mu_{dry} \\ \rho_{sat} > \rho_{dry} \end{array} \right\} \rightarrow c_{S_{sat}} = [\mu_{sat} / \rho_{sat}]^{1/2} < c_{S_{dry}} \quad (3.13)$$

According to the density effect, the difference between the shear wave velocities of the fluid-saturated and dry samples should be larger for alundum ($\phi=0.45$) than for sintered glass beads ($\phi=0.37$). This is not observed in Fig. 3.11 which shows differences between the S-wave velocities of the fluid-saturated and dry samples of roughly 5% for the alundum and 15% for the sintered glass beads. Thus, S-wave velocities are not simply controlled by the bulk porosity of the material as predicted by the density effect but appear to be more sensitive to the geometry of the grain contacts. This notion is consistent with the ideas of Anstey (1991) and Mavko and Jizba (1991).

The S-wave group velocities in Berea sandstone for hydrostatic and uniaxial loading show considerably different behavior. For hydrostatic loading, the velocities for water and n-decane are not significantly different from the dry velocities. This result suggests that the volume of fluid contained in the grain contacts is very small so that the increase in the effective density of the grain contact is negligible. The velocities for silicone oil

and glycerol are several hundreds meters per second higher than the dry velocities, indicating that the viscosity of the fluid is effectively stiffening the grain contacts. For uniaxial loading, the velocities are considerably lower than the corresponding hydrostatic velocities, except for glycerol which has roughly the same velocity for both types of loading. This behavior was completely unexpected and does not show any obvious correlation with the mechanical properties of the pore fluids.

The P- and S-wave group velocity dispersion are shown in Fig. 3.12 and 3.13 for a particular stress which is specified on each plot. In general, P- and S-wave group velocities were observed to exhibit only a weak dependence on frequency. Alundum was the only material to exhibit any significant amount of dispersion. The magnitudes of the group velocity dispersion for sintered glass beads and Berea sandstone samples are typically less than 5% over the frequency range from 200 to 800 kHz. P- and S-wave group velocities exhibit both positive and negative dispersion.

3.2.3 P- and S-wave Attenuation Coefficients

P- and S-wave attenuation coefficients were computed using the procedure described in Sec. 3.1.4.2 and are displayed in Fig. 3.14 and 3.15 for a particular frequency which is specified on each plot. In contrast to the group velocities, the attenuation coefficients for sintered glass beads and alundum samples exhibit some stress-dependence, particularly at the lower stresses. The attenuation coefficients for Berea sandstone for both uniaxial and hydrostatic states of stress show a strong dependence on stress which becomes less pronounced at stresses above 20 MPa.

The presence of pore fluids in the three granular materials produces an increase in the P- and S-wave attenuation coefficients. The sintered glass beads samples show negligible attenuation for the two low viscosity fluids (n-decane and water) and large attenuation for the two high viscosity fluids (silicone oil and water). The attenuation coefficients for

Figure 3.12 P-wave group velocities are plotted as a function of frequency for sintered glass beads (uniaxial), alundum (uniaxial), and Berea sandstone (uniaxial and hydrostatic). Each curve corresponds to a different pore fluid.

Figure 3.13 S-wave group velocities are plotted as a function of frequency for sintered glass beads (uniaxial), alundum (uniaxial), and Berea sandstone (uniaxial and hydrostatic). Each curve corresponds to a different pore fluid.

Figure 3.14 P-wave attenuation coefficients are plotted as a function of stress for sintered glass beads (uniaxial), alundum (uniaxial), and Berea sandstone (uniaxial and hydrostatic). Each curve corresponds to a different pore fluid.

Figure 3.15 S-wave attenuation coefficients are plotted as a function of stress for sintered glass beads (uniaxial), alundum (uniaxial), and Berea sandstone (uniaxial and hydrostatic). Each curve corresponds to a different pore fluid.

alundum samples increase with increasing pore fluid viscosity. For hydrostatically confined Berea sandstone at stresses below 15 MPa, the attenuation coefficients decrease with increasing fluid viscosity. However, above 20 MPa the reverse behavior is observed and the attenuation coefficients increase with increasing fluid viscosity. For both alundum and hydrostatically confined Berea sandstone samples, there is only a small difference between the magnitudes of the attenuation coefficients despite the over 1000 times difference between the viscosities of the pore fluids.

The attenuation coefficients for Berea sandstone loaded uniaxially displays a completely different dependence on pore fluid properties. For uniaxially loaded Berea sandstone, the P- and S-wave attenuation coefficients are largest for water (polar fluid) and smallest for n-decane (non-polar fluid). A similar effect was reported by Tittmann et al. (1980) in their S-wave attenuation measurements on unconfined, partially-saturated samples of Coconino sandstone at 10 kHz. Tittmann et al. suggested that the polar nature and the small molecular size of water may result in adsorption of greater amounts of volatiles, stronger bonds to the surface, and access to narrowest parts of the pore space. Spencer (1981) observed (extensional) attenuation in unconfined, fully-saturated samples of Navajo sandstone at 4 to 400 Hz which was greatest for pore fluids which produce the largest reduction in the surface free energy of silica (e.g., water). Murphy et al. (1984b) quantified this effect by computing the reduction in the normal component of the grain contact stiffness produced by an adsorbed fluid on a quartz grain using the Gibb's adsorption equation. The reduction in the stiffness of the grain contact was reported to be negligible above 1 mN, and they concluded that *the chemical effect should be important only at low effective pressures*. The dependence of attenuation coefficients on the pore fluid chemistry for uniaxially loaded samples of Berea sandstone is at least qualitatively consistent with Murphy et al.'s analysis since a large population of grain contacts are exposed to low stresses in uniaxial loading.

It should be noted that the attenuation coefficients for Berea sandstone under uniaxial loading were computed using the geometrical spreading correction specified in Eq. (3.9) - (3.12) which assumes that the material is isotropic. However, during uniaxial loading of the sandstone, grain contacts perpendicular to the direction of the axial stress will be preferentially stiffened (Fig. 3.5) leading to the development of transverse anisotropy. It is likely that a small error in the attenuation coefficients will result from applying isotropic geometrical spreading corrections to a transversely anisotropic material.

The observed trends in the P-wave attenuation coefficients for the various pore fluids are consistent with the trends observed in the S-wave attenuation coefficients, although the magnitude of the fluid-related S-wave attenuation is larger for all three granular materials. This similarity in the P- and S-wave attenuation coefficients may be interpreted as a common mechanism that is more dissipative in shear than in compression. The trends observed in the P- and S-wave group velocities of Fig. 3.10 and 3.11 are not entirely consistent with the trends observed in the P- and S-wave attenuation coefficients of Fig. 3.14 and 3.15. This difference is direct evidence that velocities and attenuation are not necessarily controlled by the same pore fluid properties.

P- and S-wave attenuation coefficients are displayed in Fig. 3.16 and 3.17 as a function of frequency on a log-log scale for a particular stress specified on each plot. Lines with slopes of two (f^2) and four (f^4) are shown on these plots for reference. These attenuation coefficients were computed using 1024 point FFT's (Fast Fourier Transform). The attenuation coefficients shown in Fig. 3.14 and 3.15 were computed using 512 point FFT's. The attenuation coefficients from the separate analyses agree to within 20 db/m for most of the tests.

In contrast to the group velocities, which were only weakly dependent on frequency, the P- and S-wave attenuation coefficients show a strong increase with frequency. In almost all cases, the attenuation curves for the fluid-saturated granular materials have a

Figure 3.16 P-wave attenuation coefficients are plotted as a function of frequency for sintered glass beads (uniaxial), alundum (uniaxial), and Berea sandstone (uniaxial and hydrostatic). Each curve corresponds to a different pore fluid.

Figure 3.17 S-wave attenuation coefficients are plotted as a function of frequency for sintered glass beads (uniaxial), alundum (uniaxial), and Berea sandstone (uniaxial and hydrostatic). Each curve corresponds to a different pore fluid.

lower slope than the corresponding dry curves. The reduction in the slope and the increase in the magnitude of the attenuation coefficient is direct evidence that a fluid-related attenuation mechanism dominates over scattering when the pore space is filled with a fluid for frequencies below 1 MHz.. At the highest frequencies, the attenuation curves for the alundum, uniaxially loaded Berea sandstone, and hydrostatically loaded Berea sandstone for both dry and fluid-saturated tests approach a slope of four, which corresponds to the characteristic f^4 Rayleigh scattering attenuation. Attenuation coefficients for the sintered glass beads samples display a weaker dependence on frequency that is closer to a slope of two for both dry and fluid-saturated samples (with the exception of the four slope P-wave attenuation observed in the water-saturated sample).

A comparison of the P- and S-wave attenuation coefficients for the dry materials is shown in Fig. 3.18. As pointed out above, all the granular materials exhibit fourth power attenuation with the exception of the sintered glass beads, which displays second power attenuation. The ranking of the attenuation curves for the different granular materials is not the same for P- and S-waves. For example at 700 kHz, alundum exhibits the highest P-wave attenuation while sintered glass beads exhibits the highest S-wave attenuation. There does not appear to be a direct correlation between either the grain size, the pore size, or the grain contact area that is consistent with both the P- and S-wave attenuation coefficients.

3.3 Tests with Low-Melting Point Solids

Tests with low-melting point solids were performed with the purpose of investigating the effects of the relative contributions of the small and large parts of the pore space to the seismic properties of Berea sandstone (Fig. 3.19). Wetting fluids such as sulfur, paraffin, and epoxy can be imbibed into the rock until a prescribed saturation is

Figure 3.18 P- and S-wave attenuation coefficients in dry sintered glass beads (4.9 MPa), dry alundum (9.8 MPa), and dry Berea sandstone (19.7 MPa).

Tests with Low-Melting Point Solids

Experimental Technique:

- Wetting fluids are introduced into the narrow pores by imbibition.
- Non-wetting fluids with a surface tension, γ , and a contact angle, θ , are forced in pores of radius, R , with an injection pressure, P_c , described by Young's equation,

$$P_c = \frac{2\gamma}{R} \cos \theta \quad .$$

Figure 3.19 Low-melting point wetting and non-wetting fluids can be used to preferentially fill the narrow pores or the large pores and then solidified in place to effectively weld these features.

achieved and then allowed to solidify in place to effectively weld the grain contacts and other small parts of the pore space. Non-wetting fluids such as Wood's metal can be forced under pressure to achieve a prescribed saturation and then allowed to solidify to reduce pore scattering off the larger pores.

The capillary pressure, P_c , required to force a non-wetting fluid with a surface tension, γ , and a contact angle, θ , into a pore with a radius of curvature, R , is determined by Laplace's Equation:

$$P_c = \frac{2\gamma}{R} \cos \theta \quad . \quad (3.14)$$

Two tests were performed using low-melting point solids. The first test was designed to isolate the location of fluid-related attenuation by progressively forcing molten Wood's metal into the narrowest parts of the pore space while monitoring P- and S-wave ultrasonic velocities and attenuation. This test is unlike conventional tests performed with wetting fluids such as water, brines, and hydrocarbons since Wood's metal is a non-wetting fluid and will therefore only begin to fill the narrowest parts of the pores after the largest pores have been filled.

The second test was designed to examine the primary locations of scattering losses in dry Berea sandstone by selectively welding the large pores with solid Wood's metal and the narrow pores with solid sulfur.

3.3.1 Berea Sandstone with Molten Wood's Metal

The experimental apparatus used to measure ultrasonic P- and S-wave transmission through Berea sandstone saturated with molten Wood's metal is shown schematically in Fig. 3.20. The particular Wood's metal alloy used in this study is marketed under the Cerrosafe® name and is composed of 42.5% Bi, 37.7% Pb, 11.3% Sn, and 8.5% Cd with

Figure 3.20 Experimental apparatus used to measure ultrasonic P- and S-wave transmission across samples partially saturated with molten Wood's metal.

a density close to that of mercury and a melting point of about 88 °C. Table 3.3 summarizes the properties of molten and solid Wood's metal. The bulk and shear moduli of the molten Wood's metal were not measured but a study by Mizutani and Kanamori (1964) on an alloy of similar composition reported molten bulk and shear moduli of approximately 41 GPa and 0 GPa, respectively.

The dimensions of the sample are the same as in the tests of the previous section and are specified in Table 3.1. The sample was oven dried at 110 °C for 10 hours, vacuum dried at 1 kPa absolute pressure for 4 hours., and then placed in a copper inner jacket and a rubber outer jacket. Ultrasonic transmission measurements were made using two identical steel transducers designed to withstand temperatures in excess of 200 °C. Each transducer contained both P- and S-wave piezoelectric crystals with resonance frequencies of 1 MHz. Stainless steel endcaps were placed between the transducers and the ends of the sample to enable the use of a pre-existing confining cell without having to make any modifications to the cell. The sample-transducer assembly was placed in an aligned loadframe and then surrounded by temperature controlled electric heaters.

The assembly was heated and a vacuum of 1 kPa absolute pressure was applied to sample. When the temperature of the rock reached 110 °C, a temperature well above the 88 °C melting point of the Cerrosafe® Wood's metal, the molten metal was forced into the sample using nitrogen pressure. The confining stress was progressively increased while keeping the Wood's metal pressure constant.

Transmitted P- and S-waves were monitored during all stages of the test. Representative waveforms observed at an effective stress of 17 MPa are shown in Fig. 3.21 for a dry sample and a sample with 92% of its pore space filled with Wood's metal (molten and solid). For the molten Wood's metal, both waves display an over four times reduction in amplitude and a significant travel time delay. The velocities (computed from the first arrivals) and attenuation coefficients are displayed in Fig. 3.22 and 3.23. The

Table 3.3 Properties of Sulfur and Wood's Metal

Fluid	Density (g/cm³)	Bulk Modulus (GPa)	Shear Modulus (GPa)	Melting Point (°C)	Viscosity (cP)	Surface Tension (mJ/m²)
Sulfur†	1.96	---	---	119 °C	8.66 @ 135 °C	60.9 @ 119 °C
Wood's Metal*	9.67	42.8 (solid)	10.8 (solid)	88 °C	---	400

† SX1226-1 Sulfur, precipitated, EM Science, Gibbstown, NJ.

* Cerrosafe® Alloy, Cerro Metal Products Co., Bellefonte, PA.

P-wave, Berea Sandstone, 17 MPa Hydrostatic Stress

S-wave, Berea Sandstone, 17 MPa Hydrostatic Stress

Figure 3.21 Transmitted P- and S-waves in Berea sandstone under dry conditions and partially saturated with molten and solid Wood's metal are displayed for a hydrostatic stress of 17 MPa.

Figure 3.22 P- and S-wave velocities are displayed for Berea sandstone tested dry and with molten and solid Wood's metal filling the pore space. Velocities were computed from the break in slopes of the first transmitted arrivals.

Figure 3.23 P- and S-wave attenuation coefficients are displayed for Berea sandstone tested dry and with Wood's metal filling the pore space.

molten Wood's metal has a low compressibility and a low viscosity but a very high density. The presence of 67% and 92% Wood's molten metal in the pores reduces the P- and S-wave velocities by over 10%. Filling the pores with 92% solid Wood's metal increases the P- and S-wave velocities by about 10%.

P- and S-wave attenuation coefficients were computed using Eq. (3.8). Geometric spreading corrections were not applied to these values because of the complexities introduced by the presence of the two steel endcaps. The presence of the molten Wood's metal in the pore space results in large P- and S-wave attenuation which decreases as the saturation is increased from 67% to 92% (Fig. 3.23). The attenuation is reduced by over 10% when the pore space is filled with 92% solid Wood's metal.

From the injection pressure, the contact angle and the surface tension of molten Wood's metal (i.e., $P_c = 8.6$ MPa, $\theta \approx 180^\circ$ and $\gamma = 400$ mJ/m²), the minimum pore radius that will be filled with molten Wood's metal can be calculated from Eq. (3.14). For these tests, the minimum pore radius is estimated to be 0.1 μm . The strong P- and S-wave attenuation must therefore be taking place inside pores larger than 0.1 μm in radius.

The large P- and S-wave attenuation in the samples of Berea sandstone partially saturated with molten Wood's metal can be interpreted as viscous pumping losses in the grain contacts as proposed by Mavko and Nur (1979) and Murphy et al. (1986) or the movement of the contact line as proposed by Miksis (1988). Because the molten Wood's metal is a non-wetting fluid, the pore fluid is forced from the larger parts of the grain contact into the smaller regions. As the saturation approaches 100%, the high bulk modulus of the molten Wood's metal restricts the movement of the fluid resulting in less attenuation.

3.3.2 Berea Sandstone with Solid Sulfur and Solid Wood's Metal

Ultrasonic measurements were performed on samples of Berea sandstone with 18% solid sulfur and 55% solid Wood's metal. The primary objective of these tests was to determine the relative contribution of the large pores and the narrow pores to the scattering attenuation in dry Berea sandstone. In its molten state, sulfur is a wetting fluid which is drawn by capillary pressure into the narrowest pores. Molten Wood's metal is a non-wetting fluid which must be injected under pressure into the pore space and will only occupy pores with radii larger than the minimum radius specified by Eq. (3.14). Upon solidification, the solid sulfur will effectively weld the narrowest pores while the solid Wood's metal will effectively reduce the scattering produced by the large pores. The properties of the two low-melting point solids are given in Table 3.3.

The Berea sandstone sample with 18% sulfur was prepared by placing about 4 grams of sulfur powder (melting point of 119 °C) on the top of the sample and placing the sample in an oven set at 130 °C. The sides of the sample were wrapped with teflon tape to prevent the liquid sulfur from seeping down the sides. The molten sulfur was allowed to imbibe into the sample and after several days in the oven at this temperature, the sample was turned over the process was repeated to allow uniform saturation.

The sample with 55% Wood's metal was prepared by vacuum immersion in molten Wood's metal. This process consists of immersing the sample in a heated vacuum cell containing molten Wood's metal, applying a vacuum of 1 kPa absolute pressure for several minutes, and then removing the vacuum and allowing the sample to equilibrate at atmospheric pressure while immersed in the molten Wood's metal.

Ultrasonic measurements were conducted with the same experimental apparatus used in the measurements of Sec. 3.2.1 and displayed in Fig. 3.1. Both samples were tested under uniaxial stress. P- and S-waves for a dry sample and samples saturated with 18%

solid sulfur and 55% solid Wood's metal are shown in Fig. 3.24. Both low-melting point solids enhance the P- and S-wave transmission relative to the dry sample.

P- and S-wave velocities computed from the first arrivals and are exhibited in Fig. 3.25. P- and S-wave velocities for the 55% Wood's metal sample are not significantly different from the dry velocities. However, the velocities for the 18% sulfur sample are over 20% faster for both P- and S-waves and virtually stress-independent. This increase in velocity for the sulfur sample is consistent with the notion that the seismic properties are controlled to a large extent by the grain contacts (Sec. 2.4).

P- and S-wave attenuation coefficients were computed using Eq. (3.8) and are displayed in Fig. 3.26. Geometric spreading corrections were not applied to the attenuation or velocities and are not anticipated to influence the overall trends in the data. Both low-melting point solids reduce the overall P- and S-wave attenuation and also reduce the stress-dependence of the attenuation at the lower stresses. The magnitudes of the P- and S-wave attenuation coefficients are similar for the two low-melting point solids, despite the fact that the volume of Wood's metal filling the pores is three times higher than the volume of sulfur. This large reduction in the attenuation of the sulfur sample is direct evidence that grain contact scattering may be the dominant source of attenuation in dry sandstones.

3.4 Discussion

The introduction of pore fluids in the two artificial granular materials and Berea sandstone at hydrostatic stresses above 15 MPa resulted in P- and S-wave attenuation coefficients which were largest for the two high viscosity pore fluids. The attenuation of the fluid-saturated samples (with the corresponding dry sample attenuation subtracted out) are plotted as a function of pore fluid viscosity in Fig. 3.27 for a stresses and frequencies specified in the figure caption. The P- and S-wave attenuation is small for

Figure 3.24 Transmitted P- and S-waves in Berea sandstone under dry conditions, with 18% solid sulfur and with 55% solid Wood's metal are displayed for a uniaxial stress of 19.7 MPa.

Figure 3.25 P- and S-wave velocities are displayed for Berea sandstone tested dry, with 18% solid sulfur and with 55% solid Wood's metal. Velocities were computed from the break in slopes of the first transmitted arrivals.

Figure 3.26 P- and S-wave attenuation coefficients are displayed for Berea sandstone tested dry, with 18% solid sulfur and with 55% solid Wood's metal.

Figure 3.27 P- and S-wave residual attenuation coefficients (i.e., $\alpha_{sat} - \alpha_{dry}$) for samples of sintered glass beads (5 MPa, 635 kHz P-wave, 439 kHz S-wave), alundum (10 MPa, 732 kHz), and Berea sandstone (20 MPa, 732 kHz) are plotted as a function of the pore fluid viscosity, η . Attenuation curves proportional to $\eta^{0.5}$ and $\eta^{1.0}$ are shown for reference.

sintered glass beads and alundum samples saturated with the low viscosity fluids (i.e., n-decane and water). The attenuation coefficients for the two high viscosity fluids shown in Fig. 3.27 exhibit functional dependences on viscosity between $\eta^{0.5}$ (local flow) and $\eta^{1.0}$ (Biot). The large P- and S-wave attenuation coefficients for the sintered glass beads sample at 1000 cp was not anticipated. It is postulated that this large attenuation is produced by viscous fluid-flow losses which take place within the wedge-shaped grain contacts. The local flow attenuation model of Palmer and Traviolia (1980) treats this type of grain contact geometry but the theoretical attenuation predictions were found to be negligible unless fluid was allowed to form a continuous liquid film between the grains.

For Berea sandstone, the confining stress stiffens the grain contacts resulting in reduced grain contact scattering attenuation and higher velocities. The presence of a low-melting point solid, such as sulfur (which is a wetting fluid in its molten state) in the fine pores of the sandstone can also be used to effectively stiffen the grain contacts and reduce the grain contact scattering attenuation.

Fluid-saturated samples of Berea sandstone exhibited different attenuation and velocities for the two states of stress. Hydrostatically confined sandstone samples at stresses above 15 MPa displayed attenuation which increases very slightly with increasing pore fluid viscosity (Fig. 3.27). Uniaxially loaded sandstone samples are characterized by attenuation which shows better correlation with the surface tension of the pore fluid as shown in Fig. 3.28.

Tests on hydrostatically confined samples of Berea sandstone partially saturated with molten Wood's metal, a non-wetting fluid, suggest that the fluid-related attenuation is taking place in pores with radii larger than 0.1 μm . At present, the geometry and dimensions of the asperities and voids of the grain contacts in Berea sandstone are not well characterized so it is not possible to make any direct statement about locations of the

Figure 3.28 P- and S-wave residual attenuation coefficients (i.e., $\alpha_{sat} - \alpha_{dry}$) for samples of Berea sandstone loaded uniaxially (20 MPa, 732 kHz) are displayed as a function of the pore fluid surface tension.

Wood's metal in the grain contacts. In principle, this information could be obtained by sectioning and viewing the specimen using a scanning electron microscope.

3.5 Conclusions

P- and S-wave ultrasonic group velocities and attenuation in sintered glass beads, alundum, and Berea sandstone were examined as a function of stress, fluid properties, and frequency. The experimental measurements supports the view that the sites of attenuation in granular materials are the compliant grain contacts. In dry samples, frequency-dependent attenuation can result from scattering attenuation at the grain contacts. In fluid-saturated samples, viscous fluid-flow losses can take place in these regions although the details of the attenuation mechanism is not fully understood at present.

Attenuation coefficients were observed to be more sensitive than group velocities to stress-induced changes to the grain packing microstructure and the presence of pore fluids. P- and S-wave attenuation coefficients exhibited similar trends for samples saturated with four different pore fluids, but the magnitude of the S-wave attenuation was, in most cases, larger.

The basic trends in the experimental measurements for the three granular materials is at least qualitatively consistent with a local flow attenuation mechanism. It is unlikely, however, that the present local flow models can explain the large viscosity-dependent attenuation observed in sintered glass beads samples which do not have very narrow grain contacts. Future laboratory work is needed to further quantify the effects of pore fluids on the attenuation of granular materials.

CHAPTER 4

MODELING SEISMIC WAVE PROPAGATION IN GRANULAR ROCKS

4.1 Grain Dynamics Modeling

At the grain level, sedimentary rocks often appear as a packing of irregularly shaped grains with various degrees of intergranular cementation. The complicated geometry and the properties of the grain packing and its constituents make it a challenge to model elastic wave propagation without introducing some approximations.

Simplifications of the microstructure of granular rocks into a geometry which is amenable to mathematical description usually proceeds in one of two directions, as shown in Fig. 4.1. The conventional approach is to focus attention on the pore space, as if one were viewing a negative image (i.e., a pore cast) of the rock. This view leads to the idealization of the rock as a solid material permeated with isolated inclusions with simple geometric shapes (e.g., elliptical pores and cracks). While this might be a realistic model for crystalline rocks, it is not obvious that the inclusion model adequately captures the essential microstructural properties which control elastic wave propagation in granular sedimentary rocks. The second approach is to focus attention on the continuous solid grain phase and, through some simplifications of the grain shapes (e.g., spherical grains) and grain contact geometry, develop a grain packing model. One possible advantage of the grain packing model is that it preserves the connectedness of both the grain and pore phases and the basic details of the grain packing structure.

Both the inclusion and grain packing models have been used extensively to study the effective static elastic properties of granular materials (e.g., see Zimmerman, 1991 and Stoll, 1989 for a discussion and references) and both of these models appear capable of reproducing the basic stress-strain behavior observed in laboratory measurements on

Strategies for Modeling Seismic Wave Propagation Granular Rocks

Figure 4.1 Two approaches for modeling seismic wave propagation in granular rocks which follow from different conceptualizations of the microstructure.

sandstones. Most investigations of the seismic properties of granular rocks have used inclusion models and either single or multiple scattering wave theories (Devaney, 1980; Hudson, 1981, 1990; Varadan et al., 1989) to compute the phase velocity and attenuation. The inclusion models used in these studies typically assume spherical pores (filled with air or a viscoelastic material) of equal radii distributed randomly in an elastic matrix. Pore shapes approaching the geometrical complexity of those observed in granular rocks are difficult to implement and have not yet been considered in these types of models.

One of the contentions of this thesis, which is supported by the visual observations of Chapter 2 and the ultrasonic measurements of Chapter 3, is that the microstructure of most granular rocks typically is better described as a packing of grains in non-welded contact. Recent research has recognized that seismic velocities and attenuation are largely controlled by the stiffness and the rheology of the grain contacts (e.g., Murphy et al., 1982b; Dvorkin et al., 1991). For example, as a seismic wave passes from grain-to-grain, partial scattering losses may occur at the grain contacts as a result of imperfect cementation. In addition, if fluids or clays are present in the grain contacts, viscous losses may also result (Suárez-Rivera, 1992).

Modeling the dynamic processes which operate at the grain level during the passage of a seismic wave is a relatively new area of research. Dynamic photoelastic simulations of elastic wave propagation in assemblies of disks have been reported in a series of papers by Shukla et al. (1987, 1988) and Zhu et al. (1991). These simulations have been used to determine the velocities, the reduction in strain amplitudes, the details of the dynamic stress field inside the disks, and the effects of packing geometry and heterogeneity on elastic wave propagation.

The development of numerical methods for simulating seismic wave propagation in granular materials is a relatively new area of research with much of the work appearing in the field of geomechanics. Methods such as the Distinct Element Method (Cundall and

Hart, 1989) and Discontinuous Deformation Analysis (Shi, 1990) were originally developed to study the deformation and dynamic response of soils and jointed rock systems in which there may be frictional sliding and large separations between the soil particles or blocks of rock. These methods, which fall under the general heading of *discrete element methods*, solve the quasi-static block deformation problem: (1) the deformations within the blocks are assumed to be static, and (2) the blocks are allowed to have a finite acceleration which is considered to be lumped in the grain's center of mass. At present, these discrete element methods are limited to modeling the quasi-static response of blocky systems; they have not been adapted to model wave propagation within individual blocks.

4.2 Analysis of Numerical Methods for Modeling Grain Dynamics

Numerical methods for simulating the propagation of a dynamic disturbance through a granular packing can be categorized into dynamic methods and quasi-static methods. Dynamic methods, such as the approach which is developed in this chapter, allow for wave propagation within each grain (Fig. 4.2). Quasi-static methods can be further subdivided into two categories, both which assume that the mass of the grain is concentrated or lumped into a point mass at the grain's center when considering the acceleration or kinetic energy of the grain. The distinction between the higher and lower order quasi-static methods is shown schematically in Fig. 4.2. Higher order quasi-static methods allow for an arbitrary static deformation of the grain so that the stiffness of the grain is distributed throughout the entire grain. Lower order methods lump the entire stiffness of the grain into a single spring element (e.g., Sadd et al., 1992). This spring is analogous to the static stiffness described by the Hertz-Mindlin contact theory for two spheres subjected to a normal or tangential stress (Johnson, 1987).

Numerical Methods for Modeling Seismic Wave Propagation in Granular Rocks

• <u>Frequency Range:</u>	Dynamic	Quasi-Static ($\lambda \gg \text{grain size}$)

• <u>Numerical Method:</u>	Dynamic BIEM	Dynamic FEM w/ a static shape function	DEM, Dynamic FEM w/ point masses.
• <u>System Properties:</u>	distributed mass, distributed stiffness	lumped mass, distributed stiffness	lumped mass, lumped stiffness
• <u>Wave Propagation Inside Grains ?</u>	yes	no	no

Figure 4.2 Classification of numerical approaches for modeling seismic wave propagation in granular rocks. Dynamic approaches such as the dynamic boundary integral equation method (BIEM) are valid for all frequencies. Quasi-static methods such as the dynamic finite element method (FEM) with a static shape function and the distinct element methods (DEM) are valid at lower frequencies where $\lambda \gg \text{grain size}$.

As was pointed out in Chapter 2, the process of diagenesis often leaves grains only partially cemented together, thus, offering the possibility of scattering losses off the non-welded contacts. The advantage of using a fully dynamic approach is that it will allow for wave propagation within each grain and hence correctly model scattering off of the partially cemented contacts. The quasi-static approaches do not allow for propagation within the grains and hence will only approximately model the effects of grain contact scattering.

Clearly, it is likely that at low frequencies, where the wavelength is much larger than the diameter of the grains, wave motion within the grains can be ignored and the total mass of each grain can be assumed to be moving at the velocity of its center of mass (Johnson, 1987), as is assumed in the two quasi-static approaches. The advantage of the fully dynamic approach presented in this chapter is that it can be used to test the frequency range of validity of these lower-order methods.

The underlying reason which makes it possible to use modeling methods based on linear elasticity is that elastic wave propagation typically involves only small strains ($\epsilon < 10^{-6}$). Strains of this magnitude are too small to produce frictional sliding between grains (Winkler and Nur, 1982) unless the grain contact stiffnesses are very low as a result of poor intergranular cementation or low contact stresses. Since the objective of this chapter is to investigate the micromechanics of wave propagation in sedimentary rocks, the numerical approach developed in this chapter is based on the assumption that there is a finite cohesion between the grains.

4.3 Multi-grain Boundary Integral Equation Method

This section presents the formulation of a dynamic boundary integral equation method for modeling seismic wave propagation in grain packings. Individual grains are coupled together using the displacement-discontinuity boundary condition with a

complex stiffness (Sect. 2.4.1). The formulation is developed for the special case of SH-wave (i.e., anti-plane strain) propagation for the sake of illustrating the method. The extension of the formulation to P-SV (i.e., in plane) propagation and propagation in three dimensions is straightforward. The multi-grain BIEM is capable of modeling wave propagation in grains packings composed of grains with arbitrary shapes and grain contact stiffness and rheologies.

The sections following the formulation of this method outline the basic strategies for simulating wave propagation in granular packings and presents several numerical examples. The final section outlines several aspects of grain dynamics modeling which deserve further attention.

4.3.1 Elastodynamic Integral Equation

An integral equation for the displacement can be directly obtained from the Betti-Rayleigh reciprocal theorem and the fundamental solution for a unit point force. The derivation of this integral equation is straightforward and can be found in many classic papers and texts on elastic wave propagation (e.g., Pao and Varatharulu, 1976; Aki and Richards, 1980). In indicial notation, the elastodynamic integral equation for a monochromatic wave of angular frequency ω can be written as

$$u_i(r, \omega) = \int_{\Gamma} \left[G_{ij}(r, r^*, \omega) t_j(r^*, \omega) - \Sigma_{ij}(r, r^*, \omega) u_j(r^*, \omega) \right] d\Gamma(r^*) + \int_{\Omega} G_{ij}(r, r^p, \omega) f_j(r^p, \omega) d\Omega \quad (4.1)$$

where u_i is the displacement, t_j is the traction, G_{ij} is the Green's function, Σ_{ij} is the Green's stress tensor, f_j is the volumetric body force located in the volume, Ω , r is the location of the receiver, r^* is the position of a point on the surface, Γ , and r^p is the location of the body force (Fig. 4.3).

Figure 4.3 Geometry used in the evaluation of the elastodynamic integral representation.

Physically, Eq. (4.1) can be viewed as a mathematical statement of Huygen's principle. The body force, f_j , generates a primary wave which produces displacements and tractions on the surface, Γ . The surface displacements and tractions act as sources of secondary wavelets which propagate away from the surface. The displacement at a position, r , is obtained by constructive and destructive interference (i.e., summation) of the secondary wavelets.

4.3.1.1 Green's Function

Expressions for G_{ij} and Σ_{ij} for two- and three-dimensional elastodynamics can be found in Achenbach (1973) and Manolis and Beskos (1988). Physically, G_{ij} and Σ_{ij} represent the displacements and tractions in the j -direction at the location r resulting from a unit point force in the i -direction at the location r^* . In this thesis, we will consider the simplest problem of two-dimensional, anti-plane strain (i.e., SH-wave) elastodynamics for which G_{ij} and Σ_{ij} are the scalar quantities:

$$\begin{aligned} G_{ij}(r, r^*, \omega) &= G_{33}(r, r^*, \omega) = \frac{i}{4\mu} H_0^{(1)}(k_T R) \\ \Sigma_{ij}(r, r^*, \omega) &= \Sigma_{33}(r, r^*, \omega) = -\frac{ik_T}{4} (\hat{n}_1 \gamma_1 + \hat{n}_2 \gamma_2) H_1^{(1)}(k_T R) \end{aligned} \quad (4.2)$$

where μ is the shear modulus, $k_T = \omega/c$ is the wavenumber, $R = (r - r^*)$ is the distance from the receiver to the source, $\gamma_i = (x_i^* - x_i)/R$ are the direction cosines, and $\hat{n}_i$ are the components of the unit normal at position r^* . These expressions are identical to the two-dimensional acoustic expressions for G_{ij} and Σ_{ij} except for a constant multiplier.

The asymptotic forms of G_{ij} and Σ_{ij} as $R \rightarrow 0$ (i.e., $r \rightarrow r^*$) are of importance in the boundary integral equation method because they determine the form of the singularity encountered when Eq. (4.1) is converted into a boundary integral equation. The limiting

forms of Eq. (4.2) for $r \rightarrow r^*$ can be obtained by taking the leading terms in the series expansions for $H_0^{(1)}$ and $H_1^{(1)}$ (Abramowitz and Stegun, 1970),

$$\begin{aligned} G_{33}(r \rightarrow r^*, \omega) &\rightarrow -\frac{1}{2\pi\mu} \left[\ln(k_T R / 2) + \zeta \right] + \frac{i}{4\mu} \\ \Sigma_{33}(r \rightarrow r^*, \omega) &\rightarrow -\frac{1}{2\pi\mu} (\hat{n}_1 \gamma_1 + \hat{n}_2 \gamma_2) \end{aligned} \quad (4.3)$$

where $\zeta=0.57721566$ is Euler's Constant.

4.3.1.2 Volumetric Source Term

The second integral in Eq. (4.1) is a volumetric source term which describes the displacement, u^p , produced by a body force f_j located at r^p

$$u_i^p(r, \omega) = \int_V G_{ij}(r, r^p, \omega) f_j(r^p, \omega) d\Omega \quad (4.4)$$

For an SH-type line source with a complex source spectrum, $s(\omega)$, the source term is

$$f_j(r^p, \omega) = f_3(r^p, \omega) = \mu s(\omega) \delta(r - r^p) \quad , \quad (4.5)$$

where f_j has the units of [force/volume] for three-dimensional problems and [force/area] for two-dimensional problems. Substituting Eq. (4.5) into Eq. (4.4) and using the sifting property of the Dirac delta function gives the displacement at r produced by a volumetric source located inside Ω ,

$$\begin{aligned} u_i^p(r, \omega) &= \mu s(\omega) \int_{\Omega} G_{ij}(r, r^p, \omega) \delta(r - r^p) d\Omega \\ &= \mu s(\omega) G_{ij}(r, r^p, \omega) \quad . \end{aligned} \quad (4.6)$$

4.3.2 Boundary Integral Equation

The elastodynamic integral equation of Eq. (4.1) can only be used to compute the displacements, u_i , at a location, r , provided that the boundary displacements and tractions

are known everywhere on Γ . In general, these quantities are not known. It is of interest, therefore, to recast Eq. (4.1) into a *boundary integral equation* which can be used to solve for the unknown boundary displacements and stresses. Once these quantities are known, they can be substituted into Eq. (4.1) and the displacement and traction can be computed at any location, r , away from the boundary, Γ .

The procedure for obtaining a boundary integral equation from Eq. (4.1) involves a limiting analysis as the point r approaches boundary, Γ . Particular care must be taken for the situation when r approaches r^* and the kernels G_{ij} and Σ_{ij} become singular. The contribution of the singularity can be evaluated by isolating the singularity and replacing the full integrals over Γ by their principle values. Although the derivation of the boundary integral equation can be found in many standard references such as Brebbia et al. (1984) and Cole (1978) and it will be outlined here for the sake of completeness.

To begin this process, first consider a body bounded by a surface, $\Gamma = \bar{\Gamma}_\epsilon + \Gamma_\epsilon$, where Γ_ϵ is a small spherical surface of radius, ϵ (Fig. 4.4). Eq. (4.1) becomes

$$u_i(r, \omega) = \int_{\Gamma - \Gamma_\epsilon + \Gamma_\epsilon} G_{ij}(r, r^*, \omega) t_j(r^*, \omega) d\Gamma(r^*) - \int_{\Gamma - \Gamma_\epsilon + \Gamma_\epsilon} \Sigma_{ij}(r, r^*, \omega) u_j(r^*, \omega) d\Gamma(r^*) + u_i^p(r, \omega) \quad (4.7)$$

Next, consider the limiting form of Eq. (4.7) as $\epsilon \rightarrow 0$ (or equivalently, $r \rightarrow r^*$). The limiting form of the first integral contains no special singularities and can be evaluated by standard integration. The limiting form of the second integral, however, contains a strong singularity. The singular contribution of this integral is evaluated in Appendix A. Using the results of Appendix A, Eq. (4.7) can be rewritten as

$$c(r) u_i(r, \omega) = - \oint_{\Gamma} \Sigma_{ij}(r, r^*, \omega) u_j(r^*, \omega) d\Gamma(r^*) + \int_{\Gamma} G_{ij}(r, r^*, \omega) t_j(r^*, \omega) d\Gamma(r^*) + u_i^p(r, \omega) \quad (4.8)$$

Figure 4.4 Singular point at $\vec{r}$ surrounded by a small spherical surface of radius ϵ .

where the bar on the first integral is used to denote the Cauchy principle value of the integral and the *free term*, c , is the contribution of the singularity (Appendix A)

$$c(r) = \left\{ \delta(r - r^*) + \lim_{r \rightarrow r^*} \int_{\Gamma_e} \Sigma_{ij}(r, r^*, \omega) d\Gamma(r^*) \right\} . \quad (4.9)$$

Eq. (4.8) is a *singular boundary integral equation* for the displacement, u_i . This equation holds for both two and three-dimensional problems provided that the appropriate Green's function and stress tensor are specified. The procedure for solving this equation numerically is outlined in the following section.

4.3.3 Numerical Evaluation of the Boundary Integral Equation (BIE)

Solving the BIE analytically is a formidable task that is only possible for problems with simple geometries. The BIE method (BIEM) is a numerical technique for solving BIE's for boundaries with arbitrary geometries. The BIEM consists of the following steps (Brebbia, 1984):

- (1) Discretize the boundary, Γ , into a series of elements. In each element, interpolation functions (e.g., constant, linear, quadratic) are used to describe the displacements and tractions at nodal points on the element.
- (2) The discretized form of the BIE is formulated for each nodal point of the on the boundary, Γ . The integrals are evaluated by numerical quadrature. The result is a system of N linear algebraic equations for N unknown nodal displacements and N unknown tractions (system of N linear equations with $2N$ unknowns).
- (3) Apply the boundary conditions on displacements and tractions at each nodal point. This step reduces the number of unknowns to N . Solve the linear system of equations for the unknown tractions and displacements.

(4) Displacements and stresses away from the boundary can be computed by substituting the computed surface displacements and tractions into the original integral equation (Eq. (4.1)).

The following sections will present the details of each of the above steps for a packing of grains which are jointed together by generalized stiffnesses.

4.3.4 Boundary Elements

For two-dimensional problems, the boundary of the body, Γ , is divided into M line elements as shown in Fig. 4.5. The discretized form of the boundary integral equation Eq. (4.8) can be written for each angular frequency, ω , as

$$c_{mn}(r_n)u_i(r_n) + \sum_{m=1}^M u_j(r_m^*) \oint_{\Gamma_m} \Sigma_{ij}(r_n, r_m^*) d\Gamma(r_m^*) = \sum_{m=1}^M t_j(r_m^*) \int_{\Gamma_m} G_{ij}(r_n, r_m^*) d\Gamma(r_m^*) + u_i^p(r_n) \quad (4.10)$$

Eq. (4.10) represents the simplest approximation of Eq. (4.8) where the unknown boundary displacements are assumed constant over each element. Boundary elements which allow for variability in the shape of each element (e.g., linear and quadratic elements) offer improved accuracy, but are slightly more involved to implement numerically. Since the main concern at this stage is to illustrate the multi-grain BIEM the simple constant element will be used.

For constant elements, the free term, c_{mn} is (Brebbia et al., 1984, p. 193; Gipson, 1987, p. 112-115)

$$c_{mn}(r_n) = \begin{cases} 1, & \text{for } r_n \text{ inside } \Omega \\ \delta_{mn}/2, & \text{for } r_n \text{ on } \Gamma \\ 0, & \text{for } r_n \text{ outside } \Omega \end{cases} \quad (4.11)$$

Figure 4.5 Discretization of the boundary, Γ , into M line elements.

For r_n on the surface, Γ , the discretized boundary integral equation (4.10) for constant line elements is expressed as

$$\frac{\delta_{nn}}{2} u_i(r_n) + \sum_{m=1}^M u_j(r_m^*) \oint_{\Gamma_m} \Sigma_{ij}(r_n, r_m^*) d\Gamma(r_m^*) = \sum_{m=1}^M t_j(r_m^*) \int_{\Gamma_m} G_{ij}(r_n, r_m^*) d\Gamma(r_m^*) + u_i^p(r_n) \quad . \quad (4.12)$$

4.3.5 Matrix Equation

Eq. (4.12) represents a linear system of equations with unknowns u_j and t_j . The coefficients of the unknowns are the integrals in Eq. (4.12)

$$\begin{aligned} g_{mn} &= \int_{\Gamma_m} G_{ij}(r_n, r_m^*) d\Gamma(r_m^*) \\ h_{mn} &= \oint_{\Gamma_m} \Sigma_{ij}(r_n, r_m^*) d\Gamma(r_m^*) \end{aligned} \quad (4.13)$$

Rewriting Eq. (4.12) in terms of Eq. (4.13) gives

$$\frac{\delta_{nn}}{2} u_i(r_n) + \sum_{m=1}^M u_j(r_m^*) h_{mn} = \sum_{m=1}^M t_j(r_m^*) g_{mn} + u_i^p(r_n) \quad , \quad (4.14)$$

which can be expressed in vector form as

$$\frac{\delta_{nn}}{2} \bar{u}_n + \sum_{m=1}^M \bar{u}_m h_{mn} = \sum_{m=1}^M \bar{t}_m g_{mn} + \bar{u}_n^p \quad . \quad (4.15)$$

Eq. (4.15) can be written in matrix form as

$$\begin{pmatrix} \bar{h}_{11} & h_{12} & \cdots & h_{1M} \\ h_{21} & \bar{h}_{22} & \cdots & h_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ h_{M1} & h_{M2} & \cdots & \bar{h}_{MM} \end{pmatrix} \begin{pmatrix} \bar{u}_1 \\ \bar{u}_2 \\ \vdots \\ \bar{u}_M \end{pmatrix} = \begin{pmatrix} g_{11} & g_{12} & \cdots & g_{1M} \\ g_{21} & g_{22} & \cdots & g_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ g_{M1} & g_{M2} & \cdots & g_{MM} \end{pmatrix} \begin{pmatrix} \bar{t}_1 \\ \bar{t}_2 \\ \vdots \\ \bar{t}_M \end{pmatrix} + \begin{pmatrix} \bar{u}_1^p \\ \bar{u}_2^p \\ \vdots \\ \bar{u}_M^p \end{pmatrix} \quad (4.16)$$

$$\rightarrow (h)\{\bar{u}\} = (g)\{\bar{t}\} + \{\bar{f}\} \quad , \quad (4.17)$$

where the diagonal terms of $\{h\}$ are

$$\bar{h}_{mm} = h_{mm} + 1/2 \quad . \quad (4.18)$$

4.3.6 Evaluation of g_{mn} and h_{mn} Integrals

The integrals g_{mn} and h_{mn} of Eq. (4.13) can be evaluated numerically using standard Gaussian quadrature. To use the Gaussian quadrature formulas, the limits of integration must first be transformed from $\Gamma_m \in [0, L_m]$ to $\eta \in [-1, +1]$, where L_m is the length of the m^{th} element. This transformation can be accomplished by defining

$$\eta = \left(\frac{2\Gamma_m}{L_m} - 1 \right), \quad d\eta = \frac{2}{L_m} d\Gamma_m \quad . \quad (4.19)$$

Substituting Eq. (4.19) into Eq. (4.13) results in the following equations

$$\begin{aligned} g_{mn} &= \int_{\Gamma_m} \tilde{G}(r_n, r_m^*) d\Gamma(r_m^*) = \frac{L_m}{2} \int_{-1}^{+1} \tilde{G}(r_n, r_m^*(\eta)) d\eta \\ h_{mn} &= \int_{\Gamma_m} \tilde{\Sigma}(r_n, r_m^*) d\Gamma(r_m^*) = \frac{L_m}{2} \int_{-1}^{+1} \tilde{\Sigma}(r_n, r_m^*(\eta)) d\eta \end{aligned} \quad (4.20)$$

Approximating these integrals using standard Gaussian quadrature gives for $m \neq n$ (Stroud and Secrest, 1966)

$$\begin{aligned} g_{mn} &\approx \frac{L_m}{2} \sum_{k=1}^K \tilde{G}(r_n, r_m^*(\eta_k)) w_k, \quad m \neq n \\ h_{mn} &\approx \frac{L_m}{2} \sum_{k=1}^K \tilde{\Sigma}(r_n, r_m^*(\eta_k)) w_k, \quad m \neq n \quad , \end{aligned} \quad (4.21)$$

where w_k are the *Gauss weighting functions* and η_k are the *Gauss integration points*.

For the integrals over the singular elements (i.e., $m=n$) higher order integration formulas are required to preserve accuracy. Alternatively, these integrals can be evaluated analytically by using the asymptotic forms of $\tilde{G}$ and $\tilde{\Sigma}$ for $r_n \rightarrow r_m^*$ given in Eq. (4.3).

Substituting Eq. (4.3) into Eq. (4.13) for g_{mn} , $m=n$, gives

$$g_{mn} = \int_{\Gamma_m} \left\{ -\frac{1}{2\pi\mu} \left[\ln\left(\frac{k_T R}{2}\right) + \zeta \right] + \frac{i}{4\mu} \right\} d\Gamma \quad (4.22)$$

Transform the limits of integration to the interval $[-1, 1]$ by letting

$$R = \frac{L_m}{2} \eta, \quad dR = \frac{L_m}{2} d\eta \quad (4.23)$$

Substituting Eq. (4.23) into Eq. (4.22) results in the following equation,

$$\begin{aligned} g_{mn} &= \frac{L_m}{2} \int_{-1}^{+1} \left\{ -\frac{1}{2\pi\mu} \left[\ln\left(\frac{k_T \eta L_m}{4}\right) + \zeta \right] + \frac{i}{4\mu} \right\} d\eta \\ &= L_m \int_0^{+1} \left\{ -\frac{1}{2\pi\mu} \left[\ln\left(\frac{k_T \eta L_m}{4}\right) + \zeta \right] + \frac{i}{4\mu} \right\} d\eta \quad (4.24) \\ &= \frac{L_m}{2\pi\mu} \left[\ln\left(\frac{4}{k_T L_m}\right) + 1 - \zeta \right] + i \frac{L_m}{4\mu} \end{aligned}$$

Eq. (4.13) can be evaluated for h_{mn} , $m=n$, by substitution of Eq. (4.3) for Σ_{ij} . However, examination of the terms which constitute Σ_{ij} for the case when $m=n$ reveals that the normal vector and the direction cosine are orthogonal so that $\hat{n}_i \cdot \gamma_i = 0$ making both $\Sigma_{ij} = 0$ and

$$h_{mn} = \oint_{\Gamma_m} \Sigma_{ij}(r_m, r_m^*) d\Gamma_m = 0 \quad (4.25)$$

4.3.7 Boundary Conditions

Eq. (4.17) represents a system of M linear equations with M unknown boundary displacements and M unknown boundary tractions. For a unique solution, M boundary conditions must be specified on the displacements or tractions or a combination of both.

For the purpose of modeling elastic wave propagation in a granular packing, we will use the *generalized stiffness boundary condition* described in Sec. 2.4.1:

$$(i) \quad t^a = 0 = t^b, \quad \text{outside the grain contact} \quad (4.26)$$

$$(ii) \quad t^b = -t^a \quad \text{and} \quad t^a = \hat{\kappa} (u^b - u^a), \quad \text{inside the grain contact,} \quad (4.27)$$

where the superscripts a and b are used to distinguish the individual grains (Fig. 4.6) and $\hat{\kappa}$ is the complex stiffness of the grain contact.

The first boundary condition represents the vanishing of the stresses at the solid-air interface. If the pores are filled with a viscous fluid, then an additional boundary condition on the velocity and fluid pressure would be required to describe the coupling of the normal and shear stresses from the surface of the grain into the fluid. This solid-fluid interaction would produce viscous fluid-flow losses of the type proposed by Biot (1956a,b, 1960). This type of viscous fluid coupling will not be considered in this thesis.

By allowing the stiffness, $\hat{\kappa}$, to be complex,

$$\hat{\kappa} = \kappa_r + i \kappa_i \quad (4.28)$$

the second boundary condition can represent a variety of rheologic models, as discussed in Sec. 2.4.1. The values for κ_r and κ_i for several grain contact rheology models are given in Fig. 2.10.

4.3.8 Multiple Grains

Consider the problem of two grains in contact (Fig. 4.6). This simple example will serve to illustrate the basic details of the multi-grain boundary integral equation formulation.

Boundary Conditions for the Grain Dynamics Model

Figure 4.6 Boundary conditions on the surfaces of grain outside and within the grain contact. Zero traction boundary conditions are assumed outside of the grain contact, and displacement-discontinuity boundary conditions with a generalized stiffness are used to represent the conditions within the grain contact.

The matrix equation for the two grain problem is obtained by first writing the separate matrix equations, for each grain and then applying the boundary conditions on the surfaces of the grains. To demonstrate this procedure in full detail, consider two grains, each modeled by two constant line elements, which are in non-welded contact as depicted in Fig. 4.7. A traction source $t_1 = f$ is applied to node 1. The system of linear equations for grain a in matrix form is

$$\begin{pmatrix} \bar{h}_{11} & h_{12} \\ h_{21} & \bar{h}_{22} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} \begin{pmatrix} t_1 \\ t_2 \end{pmatrix} \quad (4.29)$$

Similarly, the matrix equation for grain b is

$$\begin{pmatrix} \bar{h}_{33} & h_{34} \\ h_{43} & \bar{h}_{44} \end{pmatrix} \begin{pmatrix} u_3 \\ u_4 \end{pmatrix} = \begin{pmatrix} g_{33} & g_{34} \\ g_{43} & g_{44} \end{pmatrix} \begin{pmatrix} t_3 \\ t_4 \end{pmatrix} \quad (4.30)$$

The two matrix equations (4.29) and (4.30) can be combined to form a single matrix equation with twice the dimensions

$$\begin{pmatrix} \bar{h}_{11} & h_{12} & 0 & 0 \\ h_{21} & \bar{h}_{22} & 0 & 0 \\ 0 & 0 & \bar{h}_{33} & h_{34} \\ 0 & 0 & h_{43} & \bar{h}_{44} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix} = \begin{pmatrix} g_{11} & g_{12} & 0 & 0 \\ g_{21} & g_{22} & 0 & 0 \\ 0 & 0 & g_{33} & g_{34} \\ 0 & 0 & g_{43} & g_{44} \end{pmatrix} \begin{pmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \end{pmatrix} \quad (4.31)$$

Applying the boundary condition specified in Eq. (4.26) on the external nodes 1 and 4 results in

$$t_1 = f \quad \text{and} \quad t_4 = 0 \quad (4.32)$$

Applying the boundary conditions of Eq. (4.27) on the grain contact nodes 2 and 3 gives

$$t_2 = -t_3 = t_c \quad (4.33)$$

and

Discretization Scheme Used to Illustrate the Basic Form of the Multigrain Matrix Equation

Figure 4.7 Idealized discretization scheme used to illustrate the form of the multigrain matrix equation. Each grain has been divided into two nodes. A traction source is applied on the top of grain a. The two grains are coupled together at nodes 2 and 3 through a complex stiffness, $\hat{\kappa}$. The traction is zero at node 4.

$$(u_3 - u_2) = \frac{t_c}{\hat{\kappa}} \quad (4.34)$$

The column vector $\{t\}$ on the right hand side of Eq. (4.31) can be re-expressed in terms of displacements by using the boundary conditions specified in Eq. (4.32) - (4.34)

$$\text{R.H.S. \{Eq. (4.31)\}} = \begin{pmatrix} g_{11} & g_{12} & 0 & 0 \\ g_{21} & g_{22} & 0 & 0 \\ 0 & 0 & g_{33} & g_{34} \\ 0 & 0 & g_{43} & g_{44} \end{pmatrix} \begin{pmatrix} f \\ t_c \\ -t_c \\ 0 \end{pmatrix} \quad (4.35)$$

$$= \begin{pmatrix} g_{11} & g_{12} & 0 & 0 \\ g_{21} & g_{22} & 0 & 0 \\ 0 & 0 & g_{33} & g_{34} \\ 0 & 0 & g_{43} & g_{44} \end{pmatrix} \begin{pmatrix} f \\ \hat{\kappa}(u_3 - u_2) \\ \hat{\kappa}(u_2 - u_3) \\ 0 \end{pmatrix} \quad (4.36)$$

$$= \begin{pmatrix} 0 & -\hat{\kappa}g_{12} & \hat{\kappa}g_{12} & 0 \\ 0 & -\hat{\kappa}g_{22} & \hat{\kappa}g_{22} & 0 \\ 0 & \hat{\kappa}g_{33} & -\hat{\kappa}g_{33} & 0 \\ 0 & \hat{\kappa}g_{43} & -\hat{\kappa}g_{43} & 0 \end{pmatrix} \begin{pmatrix} 0 \\ u_2 \\ u_3 \\ 0 \end{pmatrix} + \begin{pmatrix} g_{11}f \\ g_{21}f \\ 0 \\ 0 \end{pmatrix} \quad (4.37)$$

Substituting Eq. (4.37) for the right hand side of Eq. (4.3.1) and rearranging produces a matrix equation with only unknown displacements

$$\begin{pmatrix} \bar{h}_{11} & (h_{12} + \hat{\kappa}g_{12}) & -\hat{\kappa}g_{12} & 0 \\ h_{21} & (\bar{h}_{22} + \hat{\kappa}g_{22}) & -\hat{\kappa}g_{22} & 0 \\ 0 & -\hat{\kappa}g_{33} & (\bar{h}_{33} + \hat{\kappa}g_{33}) & h_{34} \\ 0 & -\hat{\kappa}g_{43} & (h_{43} + \hat{\kappa}g_{43}) & \bar{h}_{44} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix} = \begin{pmatrix} g_{11}f \\ g_{21}f \\ 0 \\ 0 \end{pmatrix}, \quad (4.38)$$

or in short hand notation,

$$\rightarrow [a]\{u\} = \{b\} \quad (4.39)$$

Eq. (4.38) is the multi-grain boundary integral matrix equation. The basic form of this matrix consists of two 2x2 submatrices centered about the diagonal with off-diagonal coupling terms. The matrix is non-symmetric, and it will be sparsely populated for large

grain packings since each grain is coupled to only a very limited number of neighboring grains. Since the equation was formulated in the frequency domain, a matrix of the form of Eq. (4.38) must be solved for the boundary displacements for a range of frequencies $[\omega_1, \omega_2, \omega_3, \dots, \omega_{\max}]$, where $\omega_{\max} = 4\pi/\Delta t$ is the maximum frequency in the synthetic seismogram. Once the boundary displacements are known for all the required frequencies, these displacements can be inverse Fourier Transformed to obtain the displacement in the time domain.

Eq. (4.38) contains only the unknown boundary displacements. After the boundary displacements have been determined, the tractions in the grain contacts can be computed from the displacements using the boundary condition specified in Eq. (4.34). It should be noted that Eq. (4.38) has the same form as the matrix equation derived by Kitahara et al. (1988) to study the scattering of elastic waves off an imperfectly bonded inclusion imbedded in an infinite medium.

The multi-grain boundary integral equation method can be used to numerically simulate elastic wave propagation in grain packings with grains of arbitrary shape, size, and contact properties. In the multi-grain method, individual grains interact only through grain contacts. This simplifies the numerical formulation and reduces the number of coupling terms that need to be computed. As the wavelength approaches the grain diameter, strong intragranular multiple scattering will produce resonances within each grain and the corresponding attenuation will be high.

In principle, the multi-grain method may better capture the basic microstructural details which influence seismic wave propagation than the more familiar inclusion (i.e., pore, crack) scattering models. The inclusion models typically embed pores and cracks in a solid matrix without paying much attention to the placement of the inclusions. Many inclusions are free to communicate with one another resulting in a difficult multiple

scattering problem which becomes increasingly intractable as the number of inclusions is increased.

4.3.9 Computing the Stresses and Displacements Inside the Grains

The displacement inside of a grain can be computed once the boundary displacements and tractions have been determined by directly substituting these quantities into the discretized elastodynamic integral equation (4.1):

$$\bar{u}(r) = \sum_{m=1}^M \left\{ \bar{t}_m(r_m^*) \left[\int_{\Gamma_m} \bar{G}(r, r_m^*) d\Gamma(r_m^*) \right] - \bar{u}_m(r_m^*) \left[\int_{\Gamma_m} \bar{\Sigma}(r, r_m^*) d\Gamma(r_m^*) \right] \right\} + \bar{u}^p(r) \quad (4.40)$$

The traction term appearing in Eq. (4.40) is zero everywhere except within the grain contact regions. The non-zero tractions in the grain contacts can be directly computed from the grain contact displacements using the boundary condition specified in Eq. (4.34)

$$\bar{t}_c = \hat{k} (\bar{u}_2 - \bar{u}_1) \quad (4.41)$$

where the subscripts 1 and 2 refer to the grain 1 and 2.

Stresses inside the grain can also be evaluated once the boundary tractions are known by applying the stress operator,

$$\bar{\tau}(r) = \lambda \nabla \cdot \bar{u} + \mu (\nabla \bar{u} + \bar{u} \nabla) = L\{\bar{u}\} \quad (4.42)$$

to Eq. (4.40) to obtain the following equation,

$$\bar{\tau}(r) = \sum_{m=1}^M \left\{ \bar{t}_m(r_m^*) \left[\int_{\Gamma_m} \bar{\Sigma}(r, r_m^*) d\Gamma(r_m^*) \right] - \bar{u}_m(r_m^*) \left[\int_{\Gamma_m} L\{\bar{\Sigma}(r, r_m^*)\} d\Gamma(r_m^*) \right] \right\} + \bar{\tau}^p(r) \quad (4.43)$$

where $L\{\bar{G}\} = \bar{\Sigma}$ and $L\{\bar{\Sigma}\}$ represent tensors produced by the application of the stress operator of Eq. (4.42) to the Green's function and stress tensor, respectively.

4.3.10 Techniques for Computing the Attenuation and Velocity

Seismic attenuation and velocities can be computed by numerical simulation using the same basic strategies used in the laboratory. Three commonly employed laboratory techniques for measuring seismic properties of rock are the pulse transmission, resonant bar, and stress-strain methods (Fig. 1.1 and 4.8). The pulse transmission method measures the amplitude reduction and the travel time of a wave propagating across a sample of length, L . The width of the sample must be larger than the wavelength for a plane P-wave to propagate in true uniaxial strain. In addition, the length of the sample must also be larger than the wavelength to prevent reflections from the bottom of the sample from interfering with the direct wave.

Restrictions identical to those encountered in the laboratory application of the resonant bar and stress-strain methods apply to the numerical application of these techniques and are shown schematically in Fig. 4.8.

Numerical simulation offers an additional technique for measuring attenuation and velocities that does not have a laboratory equivalent. The technique is valid for materials with periodicity in the microstructure, and it consists of computing the modal response of a *unit cell*, which is defined as the basic building block or representative elementary volume of the periodic material. By employing periodic boundary conditions to link the opposite the sides of the unit cell together and solving the modal response of the system, this technique is capable of providing the attenuation and dispersion of the various modes that would exist in an infinite medium composed of the periodic material (Ruffa, 1992).

The numerical simulations presented in this chapter employ the simple pulse transmission technique described above. The simulations are presented for a simple cubic packing of grains of identical elastic properties, shapes, and sizes. The number of grains that need to be modeled in a regular grain packing with a plane wave source can be

Techniques for Computing Attenuation and Velocity

Figure 4.8 Numerical techniques for computing frequency-dependent velocity and attenuation. The first three techniques are directly analogous to techniques used in the laboratory. The last technique does not have a laboratory analog.

significantly reduced by exploiting the symmetry of the packing. As shown in Fig. 4.9, plane wave propagation in a cubic packing can be reduced to a single grain column with traction-free boundary conditions on the sides of the grain column. The length of the column must always be made larger than the wavelength to prevent the reflection off the bottom of the column from interfering with the directly transmitted wave. The ends of the grain column are coupled by high stiffness springs to the surrounding infinite medium to allow the scattered waves to radiate out of the grain column. If the ends are completely decoupled from the infinite medium, strong resonances develop within the grain column.

4.3.11. Computational Considerations

The multi-grain boundary integral equation formulation was implemented numerically in a FORTRAN 77 code, BIEMSH. BIEMSH is a two-dimensional, SH-wave code with line source excitation which solves the multi-grain boundary integral equation (4.38) in the frequency domain for a range of frequencies and then performs an inverse Fourier Transform to obtain the displacements and stresses in the time domain. At present, no attempts have been made to streamline the efficiency of the code and the maximum grain packing size which can be modeled is primarily limited by the execution time required to perform each matrix inversion using standard LU decomposition. A considerable reduction in execution time can be gained by using a sparse matrix inversion method, although this has not been attempted. Formulation of the method in the time domain could significantly increase the computational speed since the time domain formulation requires only a single matrix inversion (Cole, 1978; Manolis and Beskos, 1988), in contrast to the frequency domain formulation which requires a matrix inversion for a number of frequencies. However, a potential drawback of the time domain approach is that grain contact rheology is somewhat more difficult to model.

Exploiting the Symmetry of the Packing Geometry

Figure 4.9 For regular grain packings the symmetry of the packing can be used to reduce the size of the computation to a single grain column with traction-free sides.

At present, the maximum size job which can be completed in less than 8 hours of CPU on an IBM® RISC 6000 workstation with 16 MB of memory is about 300 nodes (i.e., a 600 by 600 matrix since the displacements are complex) with 128 frequencies. This limits the size of the grain packing to less than 20 grains if each grain is discretized into 15 nodes. It is anticipated that greater computational efficiency can be achieved by a variety of improvements to the numerical algorithm and by increasing the memory of the computer.

4.3.12 Validation of the Numerical Code BIEMSH

Convergence tests were performed to determine the minimum number of nodes per wavelength that is required to obtain accurate results with little or no numerical dispersion and attenuation. These tests indicate that surfaces with moderate curvature could be accurately modeled with about 10 nodes per wavelength. This value is consistent with the values recommended by Cole et al. (1978) and Schuster (1985) for constant elements.

Self-consistency and accuracy tests were performed to verify that the numerical code was functioning properly. Two self-consistency tests were performed. The first test was a reciprocity check with the geometry shown in Figure 4.10. The square region was given the same elastic properties as the surrounding infinite medium. The source was a three-loop Ricker wavelet with a spectrum peaked at about 5 MHz. Zero stiffness boundary conditions were specified on the two vertical sides and 8×10^{14} Pa/m stiffnesses were assigned on the top and bottom of the square. Synthetic waveforms (Fig. 4.10) were computed when the source was in the center of square and the receiver was in the infinite medium and for the situation when the source and receiver were interchanged. The BIEMSH computed waveforms for the two source-receiver configurations show very good agreement verifying that the numerical code obeys reciprocity.

receiver
(source)

Figure 4.10 A reciprocity test was performed using a square grain imperfectly bonded to the surrounding infinite medium. The model is shown on the left. Synthetic waveforms generated using the BIEMSH code were computed for the source inside/receiver outside and source outside/receiver inside configurations and are displayed on the left. The waveforms show good agreement for both source/receiver configurations.

The objective of the second test was to verify that a wave would propagate in accordance with the Green's function solution for an infinite medium when all the interfaces within the medium are completely welded. The geometry of the model is given in Fig. 4.11. The source was placed at the top of a column of four square grains. Receivers were placed at the tops of each of the grains. All stiffnesses were assigned very high values (1×10^{22} Pa/m) to effectively weld the grains to the surrounding infinite medium. The elastic properties were assigned the same values for the grains and the surrounding medium. The BIEMSH waveforms for receivers 1 and 4 are compared to the infinite space Green's function (Eq. (4.2)) in Fig. 4.11. The comparison between the BIEMSH and the Green's function waveforms shows very good agreement, indicating that the code is functioning properly.

The accuracy of the code was evaluated by performing several tests with non-welded interfaces. The numerical results were compared to the solutions developed by Shoenberg (1980) for plane wave reflection and transmission across non-welded interfaces. The first test examined normal incidence reflection and transmission across a viscous (velocity-discontinuity) interface (Fig. 4.12). The source and receiver were placed about 4 wavelengths from the viscous interface and the same elastic properties were assigned on both sides of the interface. A comparison of the BIEMSH and Shoenberg's predictions for the reflected and transmitted waves are displayed in Fig. 4.12. The *Shoenberg* results were obtained by applying the plane wave transmission and reflection coefficients for a viscous interface to the infinite space Green's function of Eq. (4.2). These coefficients are expressed as

$$\begin{aligned} T(\omega) &= \frac{1}{1 + Z/(2\eta)} \\ R(\omega) &= \frac{1}{1 + 2\eta/Z} \end{aligned} \tag{4.44}$$

Figure 4.11 An infinite medium test was performed by coupling four square grains to the surrounding infinite medium with high stiffness springs. The synthetic waveforms generated using the BIEMSH code for receivers 1 and 4 are displayed on the right. Also plotted are the waveforms computed from the infinite space Green's function for the same source-receiver separations. The waveforms show good agreement for both receivers.

Figure 4.12 Normal incidence transmission and reflection off a viscous interface were computed for the model displayed on the left. The waveforms generated using BIEMSH and Shoenberg's (1980) plane wave reflection and transmission coefficients for a viscous interface are displayed on the right and exhibit good agreement.

where $Z=(\rho c)\cos\phi$ is the acoustic impedance (density x velocity) of the elastic media multiplied by the cosine of the angle of incidence and η is the specific viscosity of the grain contact (viscosity per unit thickness).

A second test with the same geometry as Fig. 4.12 was performed but with a stiffness interface in which the stiffness was varied from 0 to 10^{20} Pa/m. The synthetic waveforms for the transmitted and reflected waves are compared with Shoenberg's predictions for a normal incidence reflection and transmission across a stiffness interface in Fig. 4.13. The plane wave transmission and reflection coefficients for an interface with a stiffness, κ , are described by the following expressions:

$$\begin{aligned} T(\omega) &= \frac{2}{2 - i\omega Z/\kappa} \\ R(\omega) &= \frac{-i\omega Z/\kappa}{2 - i\omega Z/\kappa} \end{aligned} \quad (4.45)$$

The waveforms display a gradual reduction in the reflected wave and a corresponding increase in the transmitted wave as the stiffness of the interface is increased from zero to a very high value.

The final test, shown in Fig. 4.14, examined the accuracy of the BIEMSH result for oblique incidence reflection and transmission off a non-welded interface with a stiffness of 5×10^{12} Pa/m. The comparison of the waveforms computed by the BIEMSH code with Shoenberg's solution shown good agreement except at the tails of the transmitted and reflected waveforms where diffractions off the ends of the interface are present in the BIEMSH result. In the grain dynamics simulations presented in the next section, these diffractions are not present since all the grains are closed bodies.

Variable Stiffness Test

Figure 4.13 Normal incidence transmission and reflection off a stiffness interface were computed using the same model geometry as shown in Fig. 4.12. The interface was assigned a stiffness which varied from 0 Pa/m (bottom) to 1×10^{20} Pa/m (top). The waveforms computed using Shoenberg's theory (1980) for a non-welded interface are also plotted and show good agreement with the BIEMSH waveforms.

Figure 4.14 Oblique incidence transmission and reflection were computed using the model shown on the left. The waveforms computed using the BIEMSH code and Shoenberg's (1980) reflection and transmission coefficients exhibit good agreement and are shown on the right.

4.3.13 Grain Dynamics Simulations

At present, grain dynamics simulations have only been performed for regular grain packings. These simple models are the logical starting point for understanding the basic behavior of granular media before more complex random packings with variable grain size and shape are attempted.

Plane wave propagation in a simple cubic packing of square grains was modeled using the geometry shown in Fig. 4.15. As discussed in Sec. 4.10, the symmetry of the cubic packing can be used to reduce the size of the model to a single column of grains with traction-free sides. The wavelength of the source is roughly three times the grain diameter (i.e., $ka \approx 1$) so strong scattering in the form of intragranular resonances was anticipated. The synthetic waveforms for an intergranular stiffness of 1×10^{22} Pa/m is shown in Fig. 4.16. For this high value of stiffness, the grain contacts are essentially welded and the grain column becomes a solid rectangular bar. As Fig. 4.16 illustrates, the wave progresses down the packing with very little reduction in amplitude. The waveforms show the arrivals of the direct SH-wave followed by a reflection off the bottom (corners) of the grain column and by a later reflection of this bottom reflected wave off the top (corners) of the column.

The effect of having finite intergranular stiffnesses is shown in Fig. 4.17. The stiffness for this simulation was 4×10^{14} Pa/m which corresponds to a normal incidence plane wave transmission coefficient (Eq. (4.47)) equal to 0.95. The synthetic waveforms exhibit considerable distortion of the direct wave as well as the presence of scattered waves in the coda. Scattered waves appearing at the end of waveforms wrap-around during the FFT process and slightly contaminate the beginning portion of the waveforms (waveforms for receivers 2, 3 and 4 appear slightly non-causal). The effect of having

Figure 4.15 Model geometry and parameters for a simple cubic packing of square grains. A total of 250 nodes were used in the model. The elastic properties are the same for the grains and the surrounding infinite medium. The stiffnesses on all the vertical sides are set at zero (i.e., zero traction sides). The value of ka (wavenumber $\times$ grain diameter) is 1.9.

Figure 4.16 Transmitted waves were computed at four receiver locations for the model shown in Fig. 4.15 for the case when the grain contact stiffnesses are high. The transmitted waveforms display only a very small decay in amplitude with increasing propagation distance. The arrivals following the direct wave are reflections off the bottom and top of the grain column.

Figure 4.17 Transmitted waves were computed at four receiver locations for the model shown in Fig. 4.15 for the case when the grain contact stiffnesses are equal to 4×10^{14} Pa/m.

grain contacts with 3/5's of the contact welded (1×10^{22} Pa/m) and the other two-fifths completely unwelded (0 Pa/m) is shown in Fig. 4.18. The amplitude drops between the first and second grains but decays much less rapidly below the second grain. Scattered waves appear in the coda of the direct waves.

The effect of having purely viscous interfaces (Newton model) is shown in Fig. 4.19 for a viscosity of 4.03×10^7 kg/m²/s. This viscosity corresponds to a plane wave transmission coefficient of 0.91 as computed from Eq. (4.44). The amplitude decays slightly as the wave progresses down the grain column. The effect of the viscosity is to dampen the intragranular (resonances) scattered waves in the coda.

Identical simulations were performed for a simple cubic packing of circular grains. The properties of the grains were taken to be the same as those specified in Fig. 4.15. The grain radii were 100 μ m. Two simulations were performed for a 5 MHz source ($ka \approx 1.0$) and for a 3 MHz source ($ka \approx 0.6$) and are displayed in Fig. 4.20. The synthetic waveforms for the 5 MHz source are similar to the basic shapes of the waveforms for square grains with intergranular stiffnesses of 4×10^{14} Pa/m (Fig. 4.17). The transmitted waves for the 3 MHz source have considerably less distortion and scattered waves in accordance with the scattering theory transition from strong (resonance) scattering to weak (Rayleigh) scattering.

4.4 Future Work on Grain Dynamics Modeling

Only a limited analysis of the effects of various properties of granular rock have been addressed in the numerical simulations presented in the preceeding section. Future work should investigate the effects of variable grain size, shape, and contact geometry on wave propagation in granular rocks. When these effects have been characterized and understood, the details of wave propagation in a random packing of grains should also be

Figure 4.18 Transmitted waves were computed at four receiver locations for the model shown in Fig. 4.15 for the case when center three fifths of the grain contact have stiffnesses equal to 1×10^{22} Pa/m and the outer two fifths have stiffnesses equal to 0 Pa/m.

Figure 4.19 Transmitted waves were computed at four receiver locations for the model shown in Fig. 4.15 for the case when the grain contact viscosity is equal to $4.03 \times 10^7 \text{ kg/m}^2/\text{s}$.

Figure 4.20 Transmitted waves were computed at four receiver locations for the model similar to that shown in Fig. 4.15 except that the grains are circular. The material properties are the same as specified in Fig. 4.15. The waveforms were computed for a source spectrum peaked at (a) 5 MHz and (b) 3 MHz. Notice the strong scattering and distortion of the direct wave for the higher frequency source which is not present for the 3 MHz source.

examined and the effects of grain packing microstructure on the attenuation and velocities should be quantified. By systematically varying certain attributes of the grain packing while controlling the remainder, the key features which influence seismic wave propagation in granular rock can be identified and characterized.

The simulations presented in this chapter were for SH-wave (anti-plane) propagation. The multi-grain boundary integral approach for modeling grain dynamics can be used to model elastic wave propagation in two- and three-dimensional grain packings by replacing the SH-wave Green's function with the two- and three-dimensional elastic Green's functions. At present, a graduate student in our research group is in the process of developing a three-dimensional elastic multi-grain boundary integral equation code. This code will be used to investigate the details of P- and S-wave propagation in granular materials.

Finally, it should be noted that the simple numerical method presented in this chapter represents a first effort in developing a simulator of wave propagation in granular media. It is anticipated that future research in this area will result in methods of greater computational efficiency which are capable of handling large grain packings and which include solid-fluid viscous coupling between the grains and the pore fluid (to investigate Biot type losses). It is also anticipated that existing numerical methods for simulating the static (large strain) deformation of highly jointed rock masses such as DDA (Shi, 1990) will be extended to provide grain contact stiffnesses and geometrical properties of the grain packing for any particular state of stress. This information can be fed into the grain dynamics simulator to determine the relationship between seismic properties and the corresponding static elastic and inelastic properties..

4.5 Summary and Conclusions

This chapter has classified existing numerical methods for simulating wave propagation in granular materials according to the type of stress field within the grains. Fully dynamic approaches allow for wave propagation within each grain while quasi-static approaches assume that the entire grain moves with a finite velocity associated with the grain's center of mass. A fully dynamic approach (multi-grain boundary integral equation method) was introduced for modeling wave propagation in granular rocks. The method uses a dynamic boundary integral equation and stiffness boundary conditions to couple the grains together. Examples for SH-wave propagation in regular packings of square and circular grains illustrated the effects of contact stiffness, rheology, and frequency on the transmitted wave. The final section proposed future work in the area of grain dynamics research that may further advance our understanding of the micromechanics of seismic wave propagation in granular rocks.

CHAPTER 5

SUMMARY AND CONCLUSIONS

5.1 Summary and Conclusions

This thesis has examined the influence of microstructure and pore fluids on seismic wave propagation in granular materials. The details of the microstructures of two artificial granular materials and Berea sandstone were inspected visually using a scanning electron microscope. Grain contacts were identified as the primary sites of attenuation in dry and fluid-saturated rocks. In many sedimentary rocks such as sandstones and limestones, the process of diagenesis leaves the grains only partially cemented together. When viewed at the micron scale, grain contacts are non-welded interfaces similar in nature to large scale joints and faults. Using a lumped properties approximation, the macroscopic properties of partially cemented grain contacts can be modeled using a displacement-discontinuity boundary condition. This model was used to estimate the magnitude and the frequency dependence of the grain contact scattering attenuation for an idealized grain packing geometry.

Ultrasonic P- and S-wave group velocity and attenuation measurements on samples of sintered glass beads, alundum, and Berea sandstone were performed to determine the effects of stress, frequency, and pore fluid properties in granular materials with sintered and partially sintered grain contacts. P- and S-wave group velocities were less sensitive to the presence of pore fluids and the magnitude of the confining stress than the attenuation coefficients. P- and S-wave group velocities were observed to be only weakly dispersive in the 200 to 800 kHz frequency range.

P- and S-wave attenuation coefficients displayed the same overall trends for tests on samples saturated with n-decane, water, silicone oil, and glycerol. The magnitudes of the attenuation coefficients were, in general, higher for S-waves. The experimental

measurements reveal that viscosity-dependent attenuation dominates in materials with sintered grain contacts. Viscosity-dependent attenuation was also observed in Berea sandstone samples but only at hydrostatic stresses in excess of 15 MPa, where the grain contacts are highly stiffened. The P- and S-wave attenuation coefficients for the limited number of fluids tested can be interpreted as exhibiting a 0.5 to 1.0 power dependence on fluid viscosity, which is consistent with the predictions of the local flow model of Mavko and Nur (1979) in the high frequency limit of their model. Fluid surface chemistry-related attenuation was observed in Berea sandstone samples which were loaded under uniaxial stress. These measurements suggest that attenuation in fluid-saturated rocks with partially cemented grain contacts is dependent on both the fluid properties and the state of stress at the grain contacts.

Ultrasonic measurements on hydrostatically confined Berea sandstone samples partially saturated with molten Wood's metal were performed to determine the sites of fluid-related attenuation. For sandstone samples saturated with 67% and 92% molten Wood's metal, the P- and S-wave velocities decreased and the attenuation coefficients increased relative to the dry velocities and attenuation. The P- and S-wave attenuation coefficients decreased as the saturation of the molten Wood's metal was increased from 67% to 92%. The maximum pore radius filled with Wood's metal at a capillary pressure corresponding to 92% saturation was estimated to be about 0.1 μm . This suggests that the fluid-related attenuation mechanism is taking place in pores larger than 0.1 μm in radius.

Ultrasonic measurements on Berea sandstone partially saturated with 18% solid sulfur and 55% solid Wood's metal were performed to examine the relative contributions of the large and small pores to scattering attenuation. The addition of 18% sulfur into the pore space produced a 20% increase in the P- and S-wave velocities and significantly reduced the stress-dependence of the velocities. The addition of 55% Wood's metal produced no significant change in the P- and S-wave velocities. The P- and S-wave attenuation

coefficients were found to decrease by the same amount for samples with 18% sulfur and 55% Wood's metal. The similarity in the attenuation coefficients despite the over 3 times larger Wood's metal saturation is direct evidence that grain contact scattering losses may be the dominant source of scattering attenuation in Berea sandstone.

A numerical method for simulating seismic wave propagation in granular materials was developed to examine the influence of the grain packing geometry, grain shape, and grain contact properties on seismic velocities and attenuation. This method differs from quasi-static methods, which are based on lumped mass dynamics, in its ability to model wave propagation inside individual grains. The *multi-grain method* is formulated using a dynamic boundary integral equation method in which individual grains are coupled together using a displacement-discontinuity boundary condition. Viscous fluid-flow losses which take place within the grain contacts are modeled using a complex interfacial stiffness. Preliminary simulations demonstrate the utility of the method for investigating the micromechanical processes of seismic wave propagation in granular rocks.

5.2 Recommendations for Future Work

Careful laboratory measurements of P- and S-wave attenuation and velocities provide a wealth of information that can be used to refine existing models and develop new models. It is anticipated that novel experimental techniques which are capable of isolating particular parts of the pore space may allow both the sites and mechanisms of attenuation to be identified and characterized. Ultrasonic tests with low-melting point solids should be performed for a range of saturations with and without pore fluids. Scanning electron microscope analysis should be conducted in parallel to characterize the locations and distributions of the low-melting point solids. These tests should be carefully corrected for geometrical spreading effects and possible reflections off the transducer-specimen interfaces.

Numerical simulation of wave propagation in granular packings is rapidly becoming a feasible task because of advances in computer hardware and software. It is recommended that a grain dynamics simulator be developed which can be used to model both the static (large strain) deformations of the grain packing as well as (small strain) elastic wave propagation. The grain statics simulator will provide the geometries and stiffnesses of the grain contacts that will be used in the grain dynamics simulations. The grain dynamics simulator could be used to investigate the effects of microstructure on scattering and to test the applicability of effective moduli theories at seismic frequencies. Ultimately, the grain dynamics simulator would allow for fully coupled fluid-grain interaction and dynamic frictional sliding so Biot type fluid-flow attenuation and dispersion and non-linear wave propagation could be investigated.

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APPENDIX A

Evaluation of $\lim_{\varepsilon \rightarrow 0} \int_{\Gamma - \Gamma_\varepsilon + \Gamma_\varepsilon} \Sigma_{ij}(r, r^*, \omega) u_j(r^*, \omega) d\Gamma(r^*)$

The limiting form of the second integral in Eq. (4.7) in the limit as $\varepsilon \rightarrow 0$ (or equivalently, as $r \rightarrow r^*$) contains a strong singularity and can be evaluated using the procedure outlined in Brebbia et al. (1984, p. 191-194).

The first step is to isolate the part of the surface encompassing the singularity by rewriting the second integral in Eq. (4.7) as follows,

$$\begin{aligned} \lim_{\varepsilon \rightarrow 0} \int_{\Gamma - \Gamma_\varepsilon + \Gamma_\varepsilon} \Sigma_{ij}(r, r^*, \omega) u_j(r^*, \omega) d\Gamma(r^*) &= \lim_{\varepsilon \rightarrow 0} \int_{\Gamma_\varepsilon} \Sigma_{ij}(r, r^*, \omega) u_j(r^*, \omega) d\Gamma(r^*) \\ &+ \lim_{\varepsilon \rightarrow 0} \int_{\Gamma - \Gamma_\varepsilon} \Sigma_{ij}(r, r^*, \omega) u_j(r^*, \omega) d\Gamma(r^*) \quad . \end{aligned} \quad (A.1)$$

The first integral on the right hand side of Eq. (A.1) can be integrated by parts to obtain,

$$\begin{aligned} \lim_{\varepsilon \rightarrow 0} \int_{\Gamma_\varepsilon} \Sigma_{ij}(r, r^*, \omega) u_j(r^*, \omega) d\Gamma(r^*) &= \lim_{\varepsilon \rightarrow 0} \int_{\Gamma_\varepsilon} \Sigma_{ij}(r, r^*, \omega) [u_j(r^*, \omega) - u_j(r, \omega)] d\Gamma(r^*) \\ &+ \lim_{\varepsilon \rightarrow 0} \left\{ u_j(r, \omega) \int_{\Gamma_\varepsilon} \Sigma_{ij}(r, r^*, \omega) d\Gamma(r^*) \right\} \end{aligned} \quad (A.2)$$

$$= \lim_{\varepsilon \rightarrow 0} \left\{ u_j(r, \omega) \int_{\Gamma_\varepsilon} \Sigma_{ij}(r, r^*, \omega) d\Gamma(r^*) \right\} \quad . \quad (A.3)$$

where the first term in Eq. (A.2) is zero because of the continuity of the displacements.

The second integral on the right hand side of Eq. (A.1) is the Cauchy Principal Value of the original integral over the surface, Γ ,

$$\lim_{\varepsilon \rightarrow 0} \int_{\Gamma - \Gamma_\varepsilon} \Sigma_{ij}(r, r^*, \omega) u_j(r^*, \omega) d\Gamma(r^*) = \oint_{\Gamma} \Sigma_{ij}(r, r^*, \omega) u_j(r^*, \omega) d\Gamma(r^*) \quad (A.4)$$

where the bar on the integral sign denotes the Cauchy Principal Value of the integral.

Eq. (A.1) can be rewritten in terms of Eq. (A.3) and (A.4) as

$$\lim_{\epsilon \rightarrow 0} \int_{\Gamma - \Gamma_\epsilon - \Gamma_\epsilon} \Sigma_{ij}(r, r^*, \omega) u_j(r^*, \omega) d\Gamma(r^*) = \oint_{\Gamma} \Sigma_{ij}(r, r^*, \omega) u_j(r^*, \omega) d\Gamma(r^*) + \lim_{\epsilon \rightarrow 0} \left\{ u_j(r^*, \omega) \int_{\Gamma_\epsilon} \Sigma_{ij}(r, r^*, \omega) d\Gamma(r^*) \right\} . \quad (A.5)$$

A singular boundary integral equation for r on Γ is obtained by rewriting Eq. (4.7) using the results of Eq. (A.5)

$$u_i(r, \omega) = \int_{\Gamma} G_{ij}(r, r^*, \omega) t_j(r^*, \omega) d\Gamma(r^*) - \oint_{\Gamma} \Sigma_{ij}(r, r^*, \omega) u_j(r^*, \omega) d\Gamma(r^*) - \lim_{\epsilon \rightarrow 0} \left\{ u_j(r^*, \omega) \int_{\Gamma_\epsilon} \Sigma_{ij}(r, r^*, \omega) d\Gamma(r^*) \right\} . \quad (A.6)$$

$$\rightarrow u_i(r, \omega) + \lim_{\epsilon \rightarrow 0} \left\{ u_j(r^*, \omega) \int_{\Gamma_\epsilon} \Sigma_{ij}(r, r^*, \omega) d\Gamma(r^*) \right\} = \int_{\Gamma} G_{ij}(r, r^*, \omega) t_j(r^*, \omega) d\Gamma(r^*) - \oint_{\Gamma} \Sigma_{ij}(r, r^*, \omega) u_j(r^*, \omega) d\Gamma(r^*) . \quad (A.7)$$

The left hand side of Eq. (A.7) can be rewritten as follows,

$$L.H.S.\{Eq. (A.7)\} = c(r) u_i(r, \omega) , \quad (A.8)$$

where

$$c(r) = \delta(r - r^*) + \lim_{r \rightarrow r^*} \int_{\Gamma_\epsilon} \Sigma_{ij}(r, r^*, \omega) u_j(r^*, \omega) d\Gamma(r^*) \quad (A.9)$$

is the *free term*.

The final step is to use Eq. (A.8) to rewrite Eq. (A.7) as

$$\begin{aligned}
c(r) u_i(r, \omega) = & \int_{\Gamma} G_{ij}(r, r^*, \omega) t_j(r^*, \omega) d\Gamma(r^*) \\
& - \oint_{\Gamma} \Sigma_{ij}(r, r^*, \omega) u_j(r^*, \omega) d\Gamma(r^*) + u_i^p(r, \omega) .
\end{aligned}
\tag{A.10}$$

Eq. (A.10) is the singular boundary integral equation for elastodynamics and elastostatics.