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PREDICTIVE MODELLING OF PARTICLE-LADEN,
TURBULENT FLOWS

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Gas-solid flows are encountered in a wide variety of engineering applications. These processes include coal injection into entrained-flow gasifiers, solid transport lines, and circulating fluidized beds. The development of appropriate models that can predict system performance can aid in the design of these systems. For example, in circulating fluidized beds, the radial non-uniformity of the basic variables over the tube cross-section results in high gas-solid slip velocities and may result in a downflow of solids along the tube walls even in the case of a net cocurrent upflow of gas and solids (Weinstein *et al*, 1986). Hence, the design and scale-up of circulating fluidized beds with excellent gas-solid contacting and high solid throughputs requires understanding of the complex hydrodynamics observed in the flow of gas-solid mixtures.

A mechanism responsible for the lateral segregation of solids over the cross-section, based on particle-particle collisions, has been developed by Sinclair and Jackson (1989), who employ methods from the kinetic theory of dense gases to derive constitutive relations describing solid-phase stresses. They developed a model for fully-developed flow in a vertical pipe taking into account this particle phase interaction and interaction between the mean velocity components represented by a drag force. This treatment was shown to be extremely successful qualitatively in that a variety of complex flow patterns observed in practice were predicted. However, an extensive comparison of model predictions with experimental data was not made. Pita and Sundaresan (1991) recently performed scale-up studies with a variation of the Sinclair and Jackson model (1989). In addition, they compared the predictions of their model with experimental data generated by Bader *et al* (1988) using 76 μm particles in a 10.2 m tall riser with a 15.2 cm radius. Their agreement with the data was excellent. In Figure 1 we show our comparison of the predictions from the Sinclair and Jackson (1989) model with this data with chosen values for the coefficient of restitution for particle-particle collisions ($e = 1.0$), the coefficient of restitution for particle-wall collisions ($e_w = 0.9$), and the specularity factor ($\phi = 0.5$). (For details concerning the Sinclair and Jackson (1989) model, the reader is referred to the original paper due to space limitations here.) It should also be noted that the change in the predictions using the Ding and Gidaspow (1990) drag model rather than the Richardson and Zaki (1954) drag model were imperceptible on the scale of Figure 1. Part of the aim of this present study was to make a more extensive comparison of model predictions using the Sinclair and Jackson (1989) model with experimental data obtained over a range of operating conditions.

The model of Sinclair and Jackson neglected the influence of gas-phase turbulence which should have a marked influence on the particle concentration

distribution. Louge *et al* (1991) have recently presented a model for vertical gas-solid upflow in a pipe which treats the particle phase as a rapid granular flow. The kinetic theory of dense gases was extended to describe the momentum and energy transferred by the velocity fluctuations of the particles (Lun *et al*, 1984). This description for the particle phase is similar to the one used by Sinclair and Jackson (1989) and Pita and Sundaresan (1991). Louge *et al* (1991), however, have presented a more detailed analysis of dilute gas-solid turbulent flow considering velocity fluctuations in both phases. They use a "rather rudimentary" one-equation gas turbulence closure model with wall functions and compare their predictions with the measurements of Tsuji *et al* (1984). They also modify an expression proposed by Koch (1990) for the correlation between the velocity fluctuations of the gas and those of the particles. They found the model predictions to be sensitive to the magnitude of the particle shear and flux of granular energy at the wall.

In the second part of the paper we expand on the effort of Louge *et al* (1991) by using a two-equation gas turbulence closure model (transport equations for the fluctuation energy and its dissipation rate) without wall functions. From the results of the Sinclair and Jackson (1989) and Pita and Sundaresan (1991) studies, it is known that the model predictions for the particle concentration distribution are sensitive to the value of the coefficient of restitution e . Also, in practice, e is not equal to one. A correct description for the correlation of the velocity fluctuations between the two phases may resolve this sensitivity issue. When the coefficient of restitution is decreased, particles lose more fluctuation energy when they collide. In these regions, the predicted solid volume fraction increases in an unrealistic fashion. Interactions between the fluctuating components of the two velocity fields may sustain the particle fluctuations. It is well known that small particles can dampen gas phase turbulence and massive particles enhance it (Tsuji *et al*, 1984). Predictions are made for the mean and fluctuating velocity profiles for both phases, as well as the particle concentration profile and pressure drop, in the case of fully-developed flow and compared with the experimental data of Tsuji *et al* (1984).

SCIENTIFIC DISCUSSION

Dispersed two-phase turbulent flows can be analyzed by using either the Lagrangian or Eulerian approach. The Lagrangian scheme, while having the advantage of following the history of the particle trajectories for multiple particle sizes by solving the Newtonian equations of motion, requires a large computational effort in order to describe the two-way coupling between the gas and particle velocity fields.

The Eulerian method circumvents this problem by regarding the fluid phase and the dispersed phase as separate interpenetrating continua. Hence, for example, the particle momentum balance can be obtained by local averaging the equation of motion for the center of mass of a single particle through a region large enough to contain many particles but small with respect to the bounding container. The averaging procedure leads to terms which describe the variety of interactions associated with the velocity fields of the two phases. Since both the fluid and the particles have local average and random velocity components, these interactions include:

1. Interaction between particles and gas that result from the difference between their mean velocity fields, and gives rise to the drag force that propels the nonrandom part of the particle motion.
2. Interactions between the fluctuating part of the particle motion and the mean particle motion through interaction forces between the particles. These generate stresses in the particle assembly and give rise to its apparent viscosity.
3. Interactions between the fluctuating part of the gas velocity and the mean motion of the gas, which generate the Reynolds stresses in the gas phase.
4. Interactions between the fluctuating component of the gas velocity and the fluctuating component of the particle velocity. These interactions may cause a flux of kinetic energy in either direction, damping the fluctuations in the gas and stimulating fluctuations in the particles, or vice versa.

A complete model formulation should include descriptions for all of these interactions. We begin our model discussion by considering a turbulent gas containing dispersed particles in dilute concentration.

Governing Equations - Gas Phase

For axisymmetric, steady, fully-developed flow in a vertical pipe, the vertical component of the momentum equation becomes

$$0 = -\frac{\partial p}{\partial z} - \beta(v)(v_z - u_z) + \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \tau_{rz}^T}{\partial r} \right) \quad (1)$$

$$\tau_{rz}^T = \mu \frac{\partial v_z}{\partial r} - \rho \overline{v'_r v'_z}$$

We adopt a closure of the Reynolds stress by assuming that the stress is related to the mean rate of strain through the eddy viscosity μ_t .

$$-\rho \overline{v'_r v'_z} = \mu_t \frac{\partial v_z}{\partial r}$$

It is also assumed that the eddy viscosity is uniquely determined by k , the turbulent kinetic energy per unit mass of the gas, where

$$k = \frac{1}{2} \overline{v'_i v'_i}$$

and ϵ , the rate of dissipation of k , via the Kolmogorov-Prandtl relation

$$\mu_t = \frac{f_\mu \rho_g k^2}{\epsilon}$$

where f_μ is a function of the turbulent Reynolds number R_T and y^+ .

$$f_\mu = 0.09 \left\{ \left(1 - \exp \left[-\frac{y^+}{70} \right] \right) (1 + 3.45/\sqrt{R_T}) \right\}$$

In single phase flows ($v = 0$) the intensity of the turbulent fluctuations can be found from the following transport equation for k .

$$0 = \frac{1}{r} \frac{\partial}{\partial r} \left[r \left(\mu + \frac{\mu_t}{\sigma_k} \right) \frac{\partial k}{\partial r} \right] + \mu_t \left(\frac{\partial v_z}{\partial r} \right)^2 - \rho_g \epsilon$$

This equation describes the balance between the diffusive transport by turbulent motion (first term), the production by interaction of turbulent stresses and mean velocity gradients (second term), the destruction by dissipation (third term). In one-equation closure models the dissipation rate is determined by specified turbulent mixing length relations. However, in two-equation models the dissipation rate is given by a separate transport equation.

$$0 = \frac{1}{r} \frac{\partial}{\partial r} \left[r \left(\mu + \frac{\mu_t}{\sigma_\epsilon} \right) \frac{\partial \epsilon}{\partial r} \right] + f_1 \frac{\epsilon}{k} \mu_t \left(\frac{\partial v_z}{\partial r} \right)^2 - f_2 \rho_g \frac{\epsilon^2}{k}$$

The terms in the above equation represent physical processes similar to those discussed already for the k equation; that is, the equation represents a balance of diffusive transport, production, and destruction.

The particular base two-equation closure model given here (which will be modified for the presence of solids) is the low-Reynolds number Myong and Kasagi (1990) model. Since this base closure model is applicable near walls, the model allows integration to the wall and, hence, the viscous sublayer is not bridged by wall functions. Four empirical constants/functions are contained in the transport equations for k and ϵ . Their values/forms are given as

$$f_1 = 1.4$$

$$f_2 = 1.8 \left\{ 1 - \frac{2}{9} \exp \left[-\left(\frac{R_T}{6} \right)^2 \right] \left[1 - \exp \left(-\frac{y^+}{5} \right) \right]^2 \right\}$$

$$\sigma_k = 1.4$$

$$\sigma_\epsilon = 1.3$$

The constants σ_k and σ_ϵ in the equations have the significance of turbulent Prandtl numbers.

In order to validate the "base" turbulence model, the system of equations (gas momentum balance, k-equation, and ϵ -equation) was first solved with the following boundary conditions for pure gas ($v = 0$) and compared with the experimental data of Schildknecht *et al*, 1979)

$$r = 0 \quad \frac{\partial \epsilon}{\partial r} = \frac{\partial k}{\partial r} = \frac{\partial v_z}{\partial r} = 0$$

$$r = R \quad k = v_z = 0$$

$$r = R \quad \epsilon = \frac{\mu}{\rho_g} \frac{\partial^2 k}{\partial r^2}$$

Figures 2 and 3 compare the predictions from our single phase model (no particles) with the single-phase turbulence measurements of Schildknecht *et al*, 1979). The measurements of Schildknecht *et al*, 1979) were made for the fully-developed flow of air in a circular pipe. The primary purpose of these predictions was to gauge the success of the single-phase turbulence closure model for the gas phase which serves as a base model for the gas-phase in two-phase gas-solid flow. In these plots the frictional velocity was matched with the measured value by adjusting the input pressure gradient. The predicted profile for the mean gas velocity (not shown) was in exact agreement with the measured profile as is the case for most turbulence models. The prediction for k is shown in Figure 2. The model predicts the core and peak turbulent intensities very well. We have performed a comparative study of all of the low-Reynolds number turbulence models to date and have found that the Myong and Kasagi (1990) model clearly yields the best predictions for k . We use an isotropic turbulence model here since Martinuzzi and Pollard (1989) showed that the results from another low-Reynolds number model (Lam and Bremhorst, 1981) were superior to the results from the algebraic stress and Reynolds stress models (higher order closure models) because of the pronounced improvement in predicting the k profile in the wall region. Figure 3 shows a comparison of model predictions for the axial fluctuating gas velocity component with the experimental data of Schildknecht *et al*, 1979). The agreement is not good, particularly in the region of maximum intensity of the turbulent fluctuations. This disagreement between model predictions and experimental data for one fluctuating gas velocity component is expected and due to the lack of perfect isotropic turbulence for the gas phase even in circular pipe flow. The axial fluctuating component is largest near the pipe wall. However, agreement at the level of the turbulent kinetic energy is more critical, as the value for the turbulent

kinetic energy is used to determine the eddy viscosity in the gas phase. Agreement at the level of the axial fluctuating velocity component in isotropic turbulence models, such as that obtained by Louge *et al* (1991) in their rudimentary closure model, will result in an overprediction of the eddy gas viscosity in single-phase and two-phase flow. Alternatively, a more complex turbulence model which does not rely on the assumption of isotropic turbulence can be employed, but again based on the results from the comparative study of Martinuzzi and Pollard (1989), this modelling alternative was not pursued.

For two-phase flow the particle phase turbulent kinetic energy equation and its dissipation rate equation must be modified to include the presence of the particles. By adopting the closures of Elghobashi and Abou-Arab (1983), and making similar order of magnitude arguments as Louge *et al* (1991), we obtain the gas-phase turbulent kinetic energy equation

$$0 = \frac{1}{r} \frac{\partial}{\partial r} \left\{ r (1 - v) \left(\mu + \frac{\mu_t}{\sigma_k} \right) \frac{\partial k}{\partial r} \right\} + (1 - v) \mu_t \left(\frac{\partial v_z}{\partial r} \right)^2 - \rho_g \epsilon (1 - v) - \beta(v) (1 - v) \overline{v_i' (v_i' - u_i')} \quad (2)$$

The last term in the turbulent kinetic energy equation accounts for the interactions associated with the fluctuating components of the gas and particle velocity fields. Using the definition of k , it can also be expressed as

$$- \beta(v) (1 - v) \overline{v_i' (v_i' - u_i')} = - \beta(v) (1 - v) (2k - \overline{u_i' v_i'})$$

The equation for the rate of dissipation of particle phase turbulent kinetic energy is given as

$$0 = \frac{1}{r} \frac{\partial}{\partial r} \left\{ r (1 - v) \left(\mu + \frac{\mu_t}{\sigma_\epsilon} \right) \frac{\partial \epsilon}{\partial r} \right\} + f_1 \frac{\epsilon}{k} (1 - v) \mu_t \left(\frac{\partial v_z}{\partial r} \right)^2 - f_2 \rho_g (1 - v) \frac{\epsilon^2}{k} - f_3 \beta(v) (1 - v) \overline{v_i' (v_i' - u_i')} \frac{\epsilon}{k} \quad (3)$$

where

$$f_3 = 1.2 \left\{ 1 - \frac{2}{9} \exp \left[- \left(\frac{R_T}{6} \right)^2 \right] \left[1 - \exp \left(- \frac{y^+}{5} \right) \right]^2 \right\}$$

following Elghobashi and Abou-Arab (1983)

Governing Equations - Particle Phase

In this section, we outline the particle phase hydrodynamics describing the steady-state, fully-developed, isothermal upflow of a gas containing cohesionless solid particles in a vertical pipe. The particles are assumed to be uniform in size. Rotational

motions of the particles are neglected in the following treatment. The z- and r-component of the momentum equations are given as

$$0 = -\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \sigma_{rz}}{\partial r} \right) + \beta(v)(v_z - u_z) - \rho_s v g \quad (4)$$

$$0 = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \sigma_{rr}}{\partial r} \right) - \frac{\sigma_{\theta\theta}}{r}$$

The form for the drag coefficient is adopted from the expression used by Ding and Gidaspow (1990) and Pita and Sundaresan (1991).

In order to formulate a model capable of describing particle-laden, turbulent flows, it is necessary to treat the mutual interactions of the particles. The kinetic energy of the random, fluctuating component of the particle motion associated with these collisions, the pseudo-thermal energy E_{PT} , is analogous to that of the thermal motion of molecules in a gas, and, correspondingly, can be characterized by a pseudo-thermal temperature T (or granular temperature) proportional to the mean square of the random component of velocity. Hence,

$$E_{PT} = \frac{1}{2} \langle u'^2 \rangle = \frac{3}{2} T$$

where u' is the random, fluctuating component of the particle velocity. Stresses within the particle assembly and the effective viscosity of the particle phase depend strongly on the pseudo-thermal temperature; thus, this must be found by solution of a separate differential equation representing a balance for the pseudo-thermal energy.

$$0 = -\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial q_{PT}}{\partial r} \right) - \sigma_{rz} \frac{\partial u_z}{\partial r} - \gamma + \beta(v)(1-v) \overline{u'_1(v'_1 - u'_1)} \quad (5)$$

Pseudo-thermal energy is conducted as a result of gradients in the pseudo-thermal temperature (analogous to heat conduction due to gradients in the thermodynamic temperature), generated by the working of the effective shear stresses in the particle phase (second term), dissipated by the inelasticity of collisions between particles (third term). The last term represents a flux of kinetic energy to or from the particle phase due to interactions between the fluctuating components of the two velocity fields. The last term may also be rewritten using the definition of the pseudo-thermal temperature.

$$\beta(v)(1-v) \overline{u'_1(v'_1 - u'_1)} = \beta(v)(1-v) (\overline{u'_1 v'_1} - 3T)$$

In order to close these equations, the constitutive relations of Lun *et al* (1984) with a few modifications are used for the particle phase stress tensor σ , the pseudo thermal

energy flux vector q , and the dissipation rate of pseudo thermal energy γ (Sinclair and Jackson, 1989).

In summary, the dilute particle-laden turbulent flow model is comprised of five coupled partial differential equations (equations 1-5) which describe the relationship between the mean and fluctuating components of the gas and solid velocities and the pressure gradient. (The r -component of the particle momentum balance yields an algebraic relationship between the pseudo-thermal temperature and the solid volume fraction). The complete formulation of the model requires boundary conditions at the tube wall and centerline. At the axis of the pipe, symmetry clearly demands that the gradients of all the variables be zero. At the wall, boundary conditions are needed for the particle velocity and the pseudo thermal temperature. These are generated following the discussion in Sinclair and Jackson (1989) and Johnson and Jackson (1987).

RESULTS

First, predictions from the Sinclair and Jackson model (1989) which neglects gas phase turbulence (hence, equations (2) and (3) did not appear in this model and the gas phase eddy viscosity was zero) were compared with several sets of experimental data (Rhodes *et al*, 1992; Monceaux *et al*, 1986); Weinstein *et al*, 1986). None of the comparisons with experimental data were in as good agreement with the model predictions as in the case of the Bader *et al* (1988) comparison. However, none of the other experimental data for solid flux as a function of tube radius were obtained at as high of a length-to-diameter ratio (L/D) as the Bader *et al* (1988) data. Hence, the flow in the data of Bader *et al* (1988) may be fully-developed while in the other cases, the flow is still in a developing state. Of the data sets investigated thus far, the highest measurements were made at an L/D ratio of 28. The solid flux profiles are flatter than the experimental measurements which is consistent with the observations of a developing gas-solid flow (Miller and Gidaspow, 1992). In the case of the Bader *et al* (1988) data, the particles may be dampening the gas-phase turbulence to such an extent that a laminar description for the gas phase is modelling the situation adequately. However, for gas-solid flows with dilute particle concentrations, a simple laminar description for the gas phase is no longer sufficient.

The dilute particle-laden, turbulent flow model has been implemented and can predict profiles for the mean and fluctuating components of the gas and particle velocities for different Reynolds numbers and solids loading. In the following predictions we employ the description for the correlation between the fluctuating

velocity components of the two velocity fields as proposed by Louge *et al* (1991). Figure 4 presents model predictions for the mean gas and solid velocity profiles with $e=0.9$ (coefficient of restitution for particle-particle collisions), $e_w=0.75$ (coefficient of restitution for particle-wall collisions), and $\phi=0.25$ (specularity factor). These predictions are compared with the experimental data of Tsuji *et al* (1984) for 200 μm polystyrene spheres. Since no independent measurements are available for the specularity factor, we arbitrarily chose the value of 0.25. The specularity factor indicates the fraction of tangential momentum lost when a particle collides with the wall. Hence, the large particle slip measured at the wall indicates that the fraction of tangential momentum lost is very small and our chosen value is too large. This prediction highlights the importance of an independent measure of the specularity factor. Nevertheless, we obtain predictions of particle velocities exceeding those of the gas in the region near the wall which is consistent with the experimental observations. In models which neglect particle-particle interactions and the corresponding stresses this crossing of the gas and solid velocity profiles can not be predicted. In addition, simplified models can not predict segregation of solids and predict a constant gas-particle slip, which is approximately equal to the particle terminal velocity, throughout the cross-section.

Since no independent measurement for the specularity factor is available and its value is unknown, an approximate value was determined by adjusting it such that the model predicted wall slips in accordance with the experimental data at a solids loading of 1.0 (Figure 5). This value for the specularity coefficient was found to be approximately 0.002. Such a small value is not unreasonable as the measured wall slip is very large. This value for the specularity factor was then held constant for the other comparisons at different solids loading. It should be emphasized that this coefficient can, in principle, be measured and is dependent on the wall roughness. In addition, since the predicted slip throughout the pipe was significantly different in Figures 4 and 5, it is clear that knowledge of the magnitude of the particle momentum flux at the wall, by an independent measurement of the specularity coefficient, is critical to flow prediction.

In Figure 5 we observe that the model results overpredict the gas-solid slip at the centerline. For dilute flows the kinetic contribution to the particle phase stress tensor increases in an unbounded fashion for unconfined flows. For confined flows, we modify this stress description following the discussion in Sinclair and Jackson (1989) so that the stress vanishes as the solid volume fraction tends to zero. The overprediction of the slip at the pipe centerline may be due to an incorrect dampening

of the kinetic contribution for very dilute flows. We have also employed the correction for the kinetic contribution adopted by Louge *et al* (1991) which resulted in a negligible difference in model predictions.

Figure 6 shows model predictions and experimental data for the mean velocities at a higher solids loading. We predict a flattening effect for the gas velocity profile as the loading ratio increases as observed by Tsuji *et al* (1984). In addition, we predict a maximum gas velocity which deviates from the pipe centerline as was also observed. Finally, we predict that the particle velocities increase with increasing loading as seen by Tsuji *et al* (1984). The predicted slip at the centerline is more in line with the data at higher loadings as the mean free path between particle collisions decreases, and the necessity to dampen the kinetic contribution to the stress decreases, as well as the sensitivity to the proposed form of dampening.

Figures 7 and 8 show the predicted axial gas fluctuations compared with the experimental data of Tsuji *et al* (1984). Qualitatively the predictions show a dampening of the gas phase turbulence in the presence of 200 μm particles as was experimentally observed. In addition, at higher loadings, the extreme flattening of the fluctuation profile was predicted - a result not seen with model of Louge *et al* (1991). Quantitative comparisons will improve with a more accurate description for the correlation between the fluctuating velocity components of the two phases. This is the focus of our present effort.

SUMMARY

The excellent comparison between the data of Bader *et al* (1988) and the predictions using the fully-developed flow model of Sinclair and Jackson (1989) have not yet been repeated with any other experimental data set for riser flow. This may be due to the fact that the data sets investigated thus far have reflected gas-solid flow in a developing state (low L/D ratios). In addition, the model of Sinclair and Jackson (1989) neglected gas phase turbulence which should have an effect on the solid concentration distribution. Hence, a dilute, particle-laden, turbulent flow model is developed here. Model predictions highlight the importance of an independent measurement of the specularly factor, an appropriate representation of the kinetic contribution to the total particle phase stress for very low solid loadings in confined flows, and a correct representation of the interaction between the fluctuating velocity components for the two phases.

NOTATION

e	coefficient of restitution for particle-particle collisions
e_w	coefficient of restitution for particle-wall collisions
f_μ, f_1, f_2, f_3	functions in k- ϵ model
k	gas phase turbulent kinetic energy
p	pressure
q_{PT}	flux vector of pseudo-thermal energy for the particle phase
r	radial coordinate
R	tube radius
Re	Reynolds number based on the centerline velocity
Re_T	turbulent Reynolds number
T	pseudo-thermal temperature
u	solid velocity
u'	fluctuating solid velocity component
U_τ	frictional velocity
v	gas velocity
v'	fluctuating gas velocity component
V_{CL}	centerline gas velocity
y	distance from wall
y^+	$y U_\tau \rho_g / \mu$
z	axial coordinate
$\beta(v)$	drag coefficient
ϵ	rate of dissipation of gas phase turbulent kinetic energy
ϕ	specularity coefficient
γ	rate of dissipation of pseudo thermal energy per unit volume
ρ_g	gas density
ρ_s	solid density
$\underline{\sigma}$	particle collisional stress tensor
σ_k	turbulent Prandtl number for k
σ_ϵ	turbulent Prandtl number for ϵ
$\underline{\tau}^T$	total stress tensor for the gas phase
$\underline{\tau}^t$	turbulent stress tensor for the gas
v	solid volume fraction
μ	intrinsic gas viscosity
μ_t	turbulent or eddy gas viscosity

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Comparison of the Sinclair and Jackson (1989) model predictions with the experimental data of Bader *et al.* (1988)

Figure 1

Calculated profile of turbulent kinetic energy for a clear gas
with a pressure gradient of -8.2096 Pa/m

Figure 2

Calculated profile of axial fluctuating gas velocity for a clear gas with a pressure gradient of -8.2096 Pa/m

Figure 3

Comparison of model predictions for the mean gas and solid velocities with the experimental data of Tsuji *et al.* (1984) (Specularity coefficient $\phi = 0.25$)

Figure 4

Comparison of model predictions for the mean gas and solid velocities with the experimental data of Tsuji *et al.* (1984)
 (Specularity coefficient $\phi = 0.002$)

Figure 5

Comparison of model predictions for the mean gas and solid velocities with the experimental data of Tsuji *et al.* (1984)
 (Specularity coefficient $\phi = 0.002$)

Figure 6

Comparison of model predictions for the axial fluctuating gas velocity with the experimental data of Tsuji *et al.* (1984)
 (Specularity coefficient $\phi = 0.002$)

Figure 7

Comparison of model predictions for the axial fluctuating gas velocity with the experimental data of Tsuji *et al.* (1984)
 (Specularity coefficient $\phi = 0.002$)

Figure 8

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