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Preliminary Study of Discharge Characteristics of Slim Holes Compared to Production Wells in Liquid-Dominated Geothermal Reservoirs

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Abstract

There is current interest in using slim holes for geothermal exploration and reservoir assessment. A major question that must be addressed is whether results from flow or injection testing of slim holes can be scaled to predict large diameter production well performance. This brief report describes a preliminary examination of this question from a purely theoretical point of view. The WELBOR computer program was used to perform a series of calculations of the steady flow of fluid up geothermal boreholes of various diameters at various discharge rates. Starting with prescribed bottomhole conditions (pressure, enthalpy), the WELBOR code integrates the equations expressing conservation of mass, momentum and energy (together with fluid constitutive properties obtained from the steam tables) upwards towards the wellhead using numerical techniques. This results in computed profiles of conditions (pressure, temperature, steam volume fraction, etc.) as functions of depth within the flowing well, and also in a forecast of wellhead conditions (pressure, temperature, enthalpy, etc.). From these results, scaling rules are developed and discussed.

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TABLE OF CONTENTS

Section	Page
Acknowledgment	ii
List of Figures	iv
List of Tables	vi
1. Introduction	1
2. Results Neglecting Reservoir Flow Resistance	3
3. The Influence of Pipe Friction	6
4. The Influence of Heat Losses	9
5. The Difficulty of Initiating Slim-Hole Discharge	19
6. Incorporation of Reservoir Flow Resistance	24
7. Estimation of Productivity Index: Cylindrical Flow Approximation	25
8. Estimation of Productivity Index: Spherical Flow Approximation	28
9. Results for Finite Reservoir Flow Resistance	31
10. Conclusions and Recommendations	42
11. References	44
Appendix	
A. Mathematical Formulation of Two-Phase Fluid Flow in a Wellbore	A-1

LIST OF FIGURES

Figure	Page
1 Influence of borehole diameter on borehole performance characteristics (no reservoir flow resistance).	4
2 Maximum discharge rate (at one bar absolute wellhead pressure) as function of borehole diameter in the absence of reservoir flow resistance (bottomhole flowing pressure = 80 bars absolute).	7
3 Minimum discharge rate (at one bar absolute wellhead pressure) as function of borehole diameter in the absence of reservoir flow resistance (bottomhole flowing pressure = 80 bars absolute).	10
4 Typical flowing conditions in production well. Note that reservoir flow resistance is assumed to be zero (bottomhole flowing pressure = 80 bars). Dotted line indicates formation temperature profile.	12
5 Relative effects of heat losses to the formation on flowing characteristics of production wells (25 cm) and slim holes (10 cm).	17
6 Effective heat transfer coefficient multiplier as a function of time after flow startup for various values of borehole diameter (centimeters).	20
7 Ranges of area-scaled discharge rate within which discharge is possible for 10-cm slim holes and 25-cm production wells, for various values of heat loss multiplier. Reservoir flow resistance neglected (flowing bottomhole pressure = 80 bars).	21
8 Time required for heat loss multiplier to drop to 1/4 of critical value for 10-cm diameter slim holes and 25-cm diameter production wells.	23
9 Productivity index scaling exponent (p) for various drainage radii (r_D) and skin factor (S) values.	27
10 Productivity index scaling exponent (p) under various conditions based on results reported by Tang (1988).	30
11a Wellhead pressure as function of area-scaled well discharge rate for slim hole (10 cm diameter) and production well (25 cm diameter) with no reservoir flow resistance. ...	35

Figure	Page
11b	Wellhead pressure as function of area-scaled well discharge rate for slim hole (10 cm diameter) and production well (25 cm diameter) with production-well productivity index equal to 6 kilograms per second per bar.36
11c	Wellhead pressure as function of area-scaled well discharge rate for slim hole (10 cm diameter) and production well (25 cm diameter) with production-well productivity index equal to 4 kilograms per second per bar.37
11d	Wellhead pressure as function of area-scaled well discharge rate for slim hole (10 cm diameter) and production well (25 cm diameter) with production-well productivity index equal to 3 kilograms per second per bar.38
11e	Wellhead pressure as function of area-scaled well discharge rate for slim hole (10 cm diameter) and production well (25 cm diameter) with production-well productivity index equal to 2 kilograms per second per bar.39
11f	Wellhead pressure as function of area-scaled well discharge rate for slim hole (10 cm diameter) and production well (25 cm diameter) with production-well productivity index equal to 1 kilogram per second per bar.40

LIST OF TABLES

Table	Page
1	Reservoir temperature distribution assumed for WELBOR calculations.2
2	Wellhead pressure as a function of borehole diameter (D) and area-scaled discharge rate (M^*) neglecting flow resistance of reservoir (bottomhole pressure = 80 bars).5
3	Maximum discharge rate as function of borehole diameter for no reservoir flow resistance.8
4	Minimum discharge rate as function of borehole diameter for no reservoir flow resistance.11
5	Wellhead fluid enthalpy decrease (relative to bottomhole flowing enthalpy) as a function of borehole diameter (D) and area-scaled discharge rate (M^*) neglecting flow resistance of reservoir (bottomhole pressure = 80 bars).13
6	Increase in fluid kinetic energy from bottomhole to wellhead as function of discharge rate for slim hole and production well.14
7	Influence of heat loss to formation on production well and slim hole performance.18
8	Performance characteristics of 25-cm diameter production well as a function of productivity index.32
9	Performance characteristics of 10-cm diameter slim hole using $p = 0$ productivity index scaling rule.33
10	Performance characteristics of 10-cm diameter slim hole using $p = 1$ productivity index scaling rule.34

1. INTRODUCTION

During the exploration of a geothermal prospect, so-called "slim holes" offer an attractive alternative to full-size production well drilling for preliminary characterization of the reservoir, owing both to lower cost and reduced drilling time requirements. The most important problem in reservoir exploration is to establish the underground temperature distribution and the presence of a thermal resource, and large-diameter wells are not required for downhole temperature measurements. Slim holes can also serve to help characterize geological structure, and many geothermal well logging tools can be run in relatively small-diameter holes.

The fluid discharge capacity of a typical slim hole will generally be significantly less than that of a corresponding production-size hole, however. This means, of course, that if a productive region is discovered using slim-hole exploration, it will be necessary to subsequently drill production-size wells to exploit the resource. Also, due to their relatively low fluid discharge capacity, slim holes are not usually useful as signal sources for pressure interference testing (they may, however, be used as shut-in observation wells in interference tests in which the flowing wells are of conventional diameter).

If a slim hole is drilled into a geothermal prospect and can be induced to discharge, the rate of steam production from the slim hole is presumably much less than would have occurred had the well been of conventional diameter. Accordingly, one question of interest is how one might estimate the steam delivery capacity of future production-size wells based upon results from testing slim holes. The discharge characteristics of a geothermal well involve the relationships among stable wellhead flowing pressure, total mass discharge rate, and wellhead fluid enthalpy (i.e., steam/water ratio). If these characteristics are known for one or more slim holes in a prospective geothermal field, is it possible to estimate the corresponding characteristics of future production wells in any meaningful way?

This brief report describes a preliminary examination of this question from a purely theoretical point of view. The WELBOR computer program (Pritchett, 1985, see Appendix A) was used to perform a series of calculations of the steady flow of fluid up geothermal boreholes of various diameters at various discharge rates. Starting with prescribed bottomhole conditions (pressure, enthalpy), the WELBOR code integrates the equations expressing conservation of mass, momentum and energy (together with fluid constitutive properties obtained from the steam tables) upwards towards the wellhead using numerical techniques. This results in computed profiles of conditions (pressure, temperature, steam volume fraction, etc.) as functions of depth within the flowing well, and also in a forecast of wellhead conditions (pressure, temperature, enthalpy, etc.). Pipe friction (for both single- and two-phase flow regions) is treated using the formulation of Dukler, Wicks and Cleveland (1964). Liquid holdup (that is, the relative motion between the liquid and vapor phases) is treated using Hughmark's (1962) correlation. Heat transfer through the casing between the fluid

within the wellbore and the formation outside is treated using a method derivable from results presented by Minkowycz and Cheng (1976).

For all calculations performed in this preliminary study, it was assumed that:

- (1) the reservoir fluid may be treated as pure H₂O (negligible dissolved solids and/or noncondensable gases),
- (2) the thermal conductivity of the rock outside the borehole is equal to 4 Watts/meter/degree Celsius (2.31 BTU/hour/foot/degree Fahrenheit),
- (3) the borehole is vertical, of uniform interior diameter (D), and its interior surface is hydraulically smooth,
- (4) the borehole has a single feedzone, at a depth of 1500 meters (4921 feet),
- (5) the undisturbed reservoir pressure at 1500 meters depth is 80 bars absolute (1160 psia), and
- (6) the distribution of reservoir temperature with depth may be obtained by linear interpolation among the values listed in Table 1.

Table 1. Reservoir temperature distribution assumed for WELBOR calculations.

Depth		Temperature	
0 m	(0 feet)	10°C	(50°F)
200 m	(656 feet)	100°C	(212°F)
500 m	(1640 feet)	180°C	(356°F)
1000 m	(3281 feet)	230°C	(446°F)
1500 m	(4921 feet)	250°C	(482°F)

The borehole produces fluid from 1500 meters depth in the formation at a temperature of 250°C (482°F). Note that the bubble point pressure for water at 250°C is only 39.8 bars (compared to 80 bars reservoir pressure) so the reservoir fluid is all single-phase liquid near the borehole's feedpoint. For most of the calculations reported herein, the borehole's inside diameter D was taken to be either 10 centimeters (3.94 inches; typical of a "slim hole") or 25 centimeters (9.84 inches; a typical "production well"). Several calculations were also performed for other values of D .

2. RESULTS NEGLECTING RESERVOIR FLOW RESISTANCE

It is first instructive to examine the fluid carrying capacity of the well as a function of borehole diameter without consideration of the resistance of the reservoir to fluid discharge. Therefore, for this first series of calculations, it was assumed that the bottomhole flowing pressure is the same as the reservoir pressure (80 bars). In reality, of course, the bottomhole flowing pressure will be somewhat lower than the stable reservoir pressure—the effects of finite reservoir flow resistance are taken up later in this report.

The simplest approximation for scaling fluid carrying capacity between boreholes of different diameters is to assume that the fluid flow rate (for a given pressure difference between wellhead and bottomhole) will vary in proportion to the cross-section area of the interior of the borehole. Therefore, we define the “area-scaled borehole discharge rate” M^* by:

$$M^* = M \times (25 \text{ cm}/D)^2 \quad (1)$$

where M is the actual borehole discharge rate. Thus, if the borehole diameter D is equal to 25 centimeters (that of our “typical” production well), the “area-scaled” discharge rate will be equal to the actual discharge rate. If $D > 25$ cm, $M^* < M$ and if $D < 25$ cm, $M^* > M$. In particular, if $D = 10$ cm (our “slim hole”), then

$$M^*(D = 10 \text{ cm}) = 6.25M \quad (2)$$

Calculations were carried out for a variety of borehole diameters (D) varying between 5 centi-meters (1.97 inches) and 35 centimeters (13.78 inches). Figure 1 shows how the stable flowing wellhead pressure varies with area-scaled discharge rate (M^*). Numerical values are listed in Table 2. Several features of these curves are noteworthy. First, in each case, a “minimum” and “maximum” value of the discharge rate is present. For flowrates outside these bounds, the borehole cannot spontaneously discharge (with wellhead pressure above one bar absolute pressure). At some intermediate flowrate, the wellhead flowing pressure takes on a maximum value. Second, the flowrate/wellhead pressure relationships for the various borehole diameters do not collapse to a single curve, even when flow rates are adjusted to account for differences in cross-sectional area. As borehole diameter increases, (1) the maximum area-scaled discharge rate increases, (2) the minimum area-scaled discharge rate decreases, and (3) the maximum attainable flowing wellhead pressure increases, and occurs at a higher area-scaled discharge rate.

Figure 1. Influence of borehole diameter on borehole performance characteristics (no reservoir flow resistance).

Table 2. Wellhead pressure as a function of borehole diameter (D) and area-scaled discharge rate (M^*) neglecting flow resistance of reservoir (bottomhole pressure = 80 bars).

Entries are Flowing Wellhead Pressure, Absolute Bars							
Area-Scaled Flow Rate (kg/s)	Borehole diameter (D), in centimeters						
	5.0	7.5	10.0	12.5	15.0	17.5	20.0
10	—	4.27	7.20	8.71	9.57	10.11	10.40
20	3.56	7.96	9.85	10.82	11.38	11.75	12.00
30	4.93	8.60	10.28	11.19	11.75	12.13	12.39
40	—	7.39	9.58	10.74	11.46	11.93	12.28
50	—	2.93	7.70	9.54	10.60	11.30	11.79
60	—	—	2.85	7.30	9.11	10.19	10.92
70	—	—	—	—	6.50	8.42	9.58
80	—	—	—	—	—	5.26	7.49
90	—	—	—	—	—	—	—
100	—	—	—	—	—	—	—
110	—	—	—	—	—	—	—
120	—	—	—	—	—	—	—
Area-Scaled Flow Rate (kg/s)	Borehole diameter (D), in centimeters						
	20.0	22.5	25.0	27.5	30.0	32.5	35.0
10	10.40	10.74	10.93	11.08	11.19	11.28	11.36
20	12.00	12.19	12.32	12.43	12.52	12.59	12.65
30	12.39	12.59	12.74	12.86	12.96	13.04	13.11
40	12.28	12.53	12.73	12.89	13.02	13.12	13.22
50	11.79	12.15	12.43	12.65	12.83	12.97	13.10
60	10.92	11.45	11.85	12.16	12.41	12.62	12.80
70	9.58	10.38	10.97	11.42	11.78	12.07	12.31
80	7.49	8.80	9.70	10.36	10.87	11.29	11.62
90	—	6.24	7.81	8.85	9.62	10.21	10.69
100	—	—	4.29	6.52	7.80	8.72	9.42
110	—	—	—	—	—	6.41	7.59
120	—	—	—	—	—	—	—

Area-scaled flow rate $M^* = M \times (25 \text{ cm}/D)^2$.

M is actual discharge rate.

3. THE INFLUENCE OF PIPE FRICTION

One reason why the discharge characteristics (scaled by cross-section area) depend upon borehole diameter is the influence of frictional forces in the pipe. The effects of friction are most important at high flow rates. The pressure gradient within the borehole consists of three components: (1) the hydrostatic pressure gradient due to gravity, (2) an acceleration term associated with fluid expansion, and (3) pipe friction. In the special case of single-phase flow, the pressure gradient due to pipe friction may be written (in the pertinent high Reynolds number range):

$$\left(\frac{dP}{dz}\right)_{friction} = 2f\left(\frac{M}{A}\right)^2 / \rho D \quad (3)$$

where f is a "friction factor" which depends weakly on Reynolds number, A is the cross-section area of the pipe, D is pipe diameter and ρ is fluid density. For two-phase flow, Dukler, *et al.*, (1964) provide modifications to this formula, but the general form is the same. Thus, if we consider the frictional pressure gradient in two wells of different diameters (D_1, D_2) discharging at the same "area-scaled" flow rate M^* , we obtain approximately:

$$\frac{\left[\left(\frac{dP}{dz}\right)_{friction}\right]_1}{\left[\left(\frac{dP}{dz}\right)_{friction}\right]_2} = \frac{D_2}{D_1} \quad (4)$$

In other words, the frictional pressure gradient will be more important for the smaller-diameter core holes than for the larger-diameter well. Since the frictional pressure gradient increases with the square of the discharge rate, this effect is most important at high flow rates. In Figure 2, we show how the maximum attainable flow rate depends upon borehole diameter for diameters between 2.5 cm (0.98 inch) and 50 cm (19.69 inches). Note that the borehole would not discharge at all for a diameter of 2.5 centimeters. Numerical values are provided in Table 3. The dotted line in Figure 2 is a straight line connecting the values for $D = 10$ cm and $D = 25$ cm. Essentially, the maximum flow rate (M_{max}) increases with borehole diameter (D) raised to the power ~ 2.5 . Therefore, the "area-scaled" maximum attainable flow rate (M^*_{max}) increases approximately with the square root of borehole diameter.

Figure 2. Maximum discharge rate (at one bar absolute wellhead pressure) as function of borehole diameter in the absence of reservoir flow resistance (bottomhole flowing pressure = 80 bars absolute).

Table 3. Maximum discharge rate as function of borehole diameter for no reservoir flow resistance.

<i>D</i> (cm)	Maximum Discharge Rate	
	Actual (kg/s)	Area-Scaled (kg/s)
2.5	—	—
3.75	0.669	29.7
5	1.55	38.8
6.25	2.83	45.3
7.5	4.56	50.7
8.75	6.83	55.8
10	9.65	60.3
12.5	17.2	68.8
15	27.4	76.1
17.5	40.7	83.1
20	57.2	89.4
22.5	77.2	95.3
25	100.6	100.6
27.5	127.9	105.7
30	159	110.4
35	234	119.4
40	326	127.3
45	436	134.6
50	565	141.3

4. THE INFLUENCE OF HEAT LOSSES

At relatively low flow rates, the effect of frictional pressure gradient is less important. We note, however, that the discharge performance curves shown in Figure 1 do not converge at low area-scaled flow rates either. For any value of the borehole diameter D , the discharge rate must lie between a particular maximum value and a particular minimum value for spontaneous flow to be possible. The variation of the minimum permissible flow rate with wellbore diameter is indicated in Figure 3; Table 4 contains numerical values. For small borehole diameters (less than 10 cm or so) the minimum permissible discharge rate is nearly independent of diameter and equal to about 0.5 kilogram per second. For larger pipe diameters, the minimum flow rate is greater, but the "area-scaled" minimum flow rate decreases monotonically with increasing borehole diameter.

This behavior is a consequence of the effects of heat losses from the rising fluid within the wellbore to the formation outside. A typical situation within a flowing well under stable conditions as computed by the WELBOR program is illustrated in Figure 4. The well diameter is 25 centimeters (production-size well), and the discharge rate is 90 kilograms per second (near the maximum flow rate attainable from the well). Conditions are single-phase within the well from the feedpoint (at 1500 meters depth) up to about 980 meters depth. Above this point, two-phase flow takes place. Note that the temperature within the well is nearly constant below the boiling level, but that temperature increases with depth within the shallower two-phase region since temperatures and pressures in two-phase flow are constrained to lie on the saturation curve for water. Also shown (as a dotted line) is the formation temperature profile assumed for these calculations. Note that the temperature inside the well exceeds that outside, particularly at shallow depths. Thus, it is to be expected that heat will be lost laterally from the fluid within the well to the cooler rock formations outside.

In Table 5, we show the decrease in specific enthalpy of the fluids discharged from the well relative to the initial bottomhole flowing enthalpy (1086.3 Joules per gram) for the same combinations of area-scaled discharge rates and borehole diameters considered in Table 2. Wellhead fluid enthalpies are lower than bottomhole enthalpies due to three effects: (1) increases in kinetic energy due to fluid expansion, (2) work done against gravity raising the fluid from the feedpoint level (1500 meters deep) to the wellhead, and (3) lateral conductive heat losses to the formation. The work done against gravity is the same in all cases, and is equal to 14.7 Joules per gram. The effects of kinetic energy changes are even smaller, and reach maximum values for each borehole diameter at the maximum possible discharge rate; these maxima are, however, typically only a few Joules per gram. For flow rates significantly below the maximum value, kinetic energy changes amount to less than one Joule per gram (see Table 6). Most of the energy changes listed in Table 5 are, therefore, due to lateral heat loss to the formation.

Figure 3. Minimum discharge rate (at one bar absolute wellhead pressure) as function of borehole diameter in the absence of reservoir flow resistance (bottomhole flowing pressure = 80 bars absolute).

Table 4. Minimum discharge rate as function of borehole diameter for no reservoir flow resistance.

<i>D</i> (cm)	Minimum Discharge Rate	
	Actual (kg/s)	Area-Scaled (kg/s)
2.5	—	—
3.75	0.476	21.16
5	0.465	11.63
6.25	0.461	7.38
7.5	0.460	5.11
8.75	0.465	3.80
10	0.478	2.99
12.5	0.525	2.10
15	0.588	1.63
17.5	0.662	1.35
20	0.744	1.16
22.5	0.832	1.03
25	0.928	0.93
27.5	1.031	0.85
30	1.141	0.79
35	1.382	0.71
40	1.653	0.65
45	1.952	0.60
50	2.280	0.57

Figure 4. Typical flowing conditions in production well. Note that reservoir flow resistance is assumed to be zero (bottomhole flowing pressure = 80 bars). Dotted line indicates formation temperature profile.

Table 5. Wellhead fluid enthalpy decrease (relative to bottomhole flowing enthalpy) as a function of borehole diameter (D) and area-scaled discharge rate (M^*) neglecting flow resistance of reservoir (bottomhole pressure = 80 bars).

Entries are specific enthalpy decrease, Joules/gram							
Area-Scaled Flow Rate (kg/s)	Borehole diameter (D), in centimeters						
	5.0	7.5	10.0	12.5	15.0	17.5	20.0
10	—	431.2	280.3	194.4	143.2	110.9	89.2.
20	468.8	258.2	160.6	110.9	82.5	65.1	53.8
30	342.4	182.7	113.8	79.9	60.7	48.8	41.1
40	—	139.3	88.8	63.5	49.1	40.3	34.4
50	—	109.9	72.4	53.0	41.9	34.9	30.4
60	—	—	63.1	45.8	36.9	31.3	27.6
70	—	—	—	—	33.6	28.8	25.6
80	—	—	—	—	—	27.8	24.4
90	—	—	—	—	—	—	—
100	—	—	—	—	—	—	—
110	—	—	—	—	—	—	—
120	—	—	—	—	—	—	—
Area-Scaled Flow Rate (kg/s)	Borehole diameter (D), in centimeters						
	20.0	22.5	25.0	27.5	30.0	32.5	35.0
10	89.2	74.2	63.2	55.0	48.7	43.8	39.9
20	53.8	45.7	40.1	35.7	32.5	29.9	27.9
30	41.1	35.6	31.8	28.9	26.8	25.0	23.6
40	34.4	30.4	27.5	25.4	23.7	22.4	21.4
50	30.4	27.2	24.9	23.2	21.9	20.8	20.0
60	27.6	25.0	23.1	21.6	20.6	19.7	19.0
70	25.6	23.4	21.9	20.6	19.7	19.0	18.4
80	24.4	22.4	21.0	19.9	19.1	18.4	18.0
90	—	22.3	20.6	19.5	18.8	18.2	17.7
100	—	—	23.6	19.9	18.8	18.1	17.6
110	—	—	—	—	—	19.0	18.0
120	—	—	—	—	—	—	—

Area-scaled flow rate $M^* = M \times (25 \text{ cm}/D)^2$.

M is actual discharge rate.

Table 6. Increase in fluid kinetic energy from bottomhole to wellhead as function of discharge rate for slim hole and production well.

Borehole Diameter = 10 cm (Slim Hole)		
Area Scaled Flow Rate (kg/s)	Actual Hole Flow Rate (kg/s)	Specific Kinetic Energy Increase (Joules/gram)
10	1.6	0.0007
20	3.2	0.0051
30	4.8	0.0169
40	6.4	0.0478
50	8.0	0.1614
60	9.6	3.5907
60.3*	9.65*	5.8237

*Maximum well discharge rate

Borehole Diameter = 25 cm (Production Well)		
Area Scaled Flow Rate (kg/s)	Actual Well Flow Rate (kg/s)	Specific Kinetic Energy Increase (Joules/gram)
10	10	0.0023
20	20	0.0082
30	30	0.0186
40	40	0.0354
50	50	0.0620
60	60	0.1053
70	70	0.1812
80	80	0.3358
90	90	0.7631
100	100	4.4146
100.6*	100.6*	6.0277

*Maximum well discharge rate

In the WELBOR program, heat losses to the formation are treated by assuming an outward-directed heat flux from each element of the well surface given by

$$\text{Heat flux} = U(T_{\text{fluid}} - T_{\text{rock}}) \quad (5)$$

where T_{fluid} is the local temperature of the fluid within the well, T_{rock} is the undisturbed rock formation temperature distant from the borehole at the same depth, and U is a heat transfer coefficient (i.e. Watts per square meter per degree Celsius). This heat transfer coefficient, in turn, may be approximated by:

$$U = 0.6 K / D \quad (6)$$

(where D is borehole diameter and K is the effective thermal conductivity of the rock formations outside the borehole—taken as 4 W/m°C for the present calculations) as shown by Pritchett (1981) based upon the work of Minkowycz and Cheng (1976). In effect, heat conduction takes place through a “thermal boundary layer”, with temperature equal to the borehole temperature at the inner edge of the boundary layer and equal to the reservoir temperature outside the boundary layer. Beyond the conductive boundary layer, heat transfer is dominated by convection in the porous reservoir rock. Minkowycz and Cheng (1976) showed that the thickness of the boundary layer, at equilibrium, will be proportional to the borehole diameter. Based upon the above expression, it may easily be shown that the total power (i.e., Watts) lost by the fluid within the borehole is given by:

$$\text{Total Power} = 0.6 \pi K Z \overline{\Delta T} \quad (7)$$

where Z is the depth of the borehole and $\overline{\Delta T}$ is the vertically-averaged temperature difference between the borehole and the surrounding formation. Dividing the heat loss rate (from Eq. 7) by the rate of fluid production (M) yields the specific fluid enthalpy decrease due to heat losses:

$$\Delta H = 0.6 \pi K Z \overline{\Delta T} / M \quad (8)$$

For two boreholes of equal depths but of different diameters (D_1, D_2), the ratio of the enthalpy change due to heat losses at the same area-scaled discharge rate (M^*) will be:

$$\frac{\Delta H_1}{\Delta H_2} = \frac{\overline{\Delta T}_1}{\overline{\Delta T}_2} \left(\frac{D_2}{D_1} \right)^2 \quad (9)$$

In other words, heat losses will always be more significant for the slimmer of the two boreholes; the enthalpy loss will vary inversely with “area-scaled” discharge rate and also inversely with the square of the borehole diameter. As shown in Table 5, the greatest enthalpy declines occur for relatively small-diameter boreholes operating at low discharge rates.

This difference in relative heat loss is responsible for the fact that "area-scaling" fails to produce the same discharge characteristics for production-size wells and for slim holes, even at low flow rates. The computed wellhead pressure is shown in Figure 5 as a function of area-scaled flow rate (M^*) for 10-cm slim holes and for 25-cm production wells both for the calculations described above (solid lines) and for a special series of calculations in which the heat-transfer coefficient was set to zero (dotted lines). Numerical values are provided in Table 7. The following are noteworthy: (1) for all flow rates, the wellhead pressure is larger if heat losses are neglected, (2) the difference in wellhead pressure between the "no-heat-loss" and "heat-loss" cases (for a fixed borehole diameter) is greatest for the lowest discharge rates, (3) the influence of heat losses upon wellhead pressure is much greater for the 10-cm slim hole than for the 25-cm production well, and (4) in the absence of heat losses (compare the two dotted curves) the wellhead pressure becomes independent of borehole diameter and depends only on area-scaled discharge rate for low flow rates (where the effects of pipe friction are relatively unimportant).

The mechanism by which heat loss influences wellhead pressure may be understood by consideration of the flowing pressure distribution indicated by the green line in Figure 4. Note that, below the boiling interface, the vertical flowing pressure gradient is approximately hydrostatic (~80 bars per kilometer) but that within the two-phase zone above the boiling level the flowing pressure gradient is significantly lower (~35 bars per kilometer in Figure 4). If heat losses increase, then the liquid temperature below the boiling surface will decline. As a result, the boiling level will occur at a shallower depth within the borehole, at the boiling-point pressure for the lower temperature. Therefore, a greater portion of the borehole will contain the high-pressure-gradient single-phase liquid region and a smaller portion will contain the low-pressure-gradient two-phase region. As a consequence, the average pressure gradient for the borehole as a whole will increase. Since the bottomhole flowing pressure is fixed, this means that the wellhead pressure must decline with increasing heat loss.

Figure 5. Relative effects of heat losses to the formation on flowing characteristics of production wells (25 cm) and slim holes (10 cm).

Table 7. Influence of heat loss to formation on production well and slim hole performance.

Area Scaled Flow Rate M^* (kg/s)	Slim Hole ($D = 10$ cm)			Production Well ($D = 25$ cm)		
	Actual Flow Rate M (kg/s)	Wellhead Pressure (bars)		Actual Flow Rate M (kg/s)	Wellhead Pressure (bars)	
		No Heat Loss	With Heat Loss		No Heat Loss	With Heat Loss
1	0.16	4.30	—	1	4.72	1.37
2	0.32	7.09	—	2	7.41	4.96
3	0.48	8.40	1.02	3	8.70	6.84
4	0.64	9.22	2.17	4	9.51	8.02
5	0.80	9.80	3.31	5	10.09	8.85
7	1.12	10.58	5.24	7	10.88	9.94
10	1.6	11.26	7.20	10	11.62	10.93
15	2.4	11.81	8.98	15	12.31	11.83
20	3.2	11.96	9.85	20	12.69	12.32
25	4.0	11.87	10.23	25	12.89	12.60
30	4.8	11.57	10.28	30	12.98	12.74
35	5.6	11.07	10.06	35	12.98	12.78
40	6.4	10.37	9.58	40	12.91	12.73
45	7.2	9.41	8.82	45	12.76	12.61
50	8.0	8.11	7.70	50	12.56	12.43
55	8.8	6.24	6.02	55	12.29	12.17
60	9.6	2.68	2.85	60	11.95	11.85
65	10.4	—	—	65	11.53	11.45
70	11.2	—	—	70	11.04	10.97
75	12.0	—	—	75	10.45	10.39
80	12.8	—	—	80	9.74	9.70
85	13.6	—	—	85	8.89	8.85
90	14.4	—	—	90	7.84	7.81
95	15.2	—	—	95	6.45	6.44
100	16.0	—	—	100	4.29	4.29
105	16.8	—	—	105	—	—

5. THE DIFFICULTY OF INITIATING SLIM-HOLE DISCHARGE

Field experience has shown that it is frequently difficult (and sometimes impossible) to induce stable self-sustained discharge from slim deep holes (depth significantly greater than 300 meters) drilled into a field from which spontaneous discharge is readily obtained using wells of conventional diameter. While the greater importance of pipe friction in slim holes may play a limited role in this problem, the main reason appears to be the relatively larger heat losses to the surrounding formation for boreholes of small diameter.

As noted above, heat losses from the borehole to the formation under stable steady-flow conditions may be adequately represented using a heat transfer coefficient of the form:

$$U = 0.6K / D$$

The problem is that the above formula is only applicable after a stable condition has been reached. The stable heat-transfer model discussed above involves a conductive boundary layer around the borehole of which the thickness is proportional to the borehole diameter. Prior to discharge initiation, however (assuming that the borehole has previously been left in a shut-in condition for a long period of time), the temperature distribution within the borehole will closely resemble that in the formation outside. When discharge begins, temperatures within the borehole change almost discontinuously to higher values. This induces the formation of a thermal boundary layer around the borehole of which the thickness increases with time, eventually reaching the asymptotic value described by Minkowycz and Cheng (1976). At early times, however, the boundary layer will be thinner than at steady-state, so that conductive heat transfer will be augmented. In short, the transient case may be regarded as a succession of states in which the heat transfer coefficient is a decreasing function of time which reaches the above asymptotic value only at infinite time. This situation may be described by defining a "heat loss multiplier" which declines toward unity as time goes on. Computed values for the heat transfer multiplier as a function of time after flow startup for various borehole diameters are illustrated in Figure 6.

Therefore, under transient conditions, the results computed above for the discharge capacities of boreholes of various diameters are overly optimistic since these calculations, by assuming steady heat transfer, underestimate the effects of heat losses at early times. To investigate this issue, a series of calculations was performed involving boreholes of both 10 cm and 25 cm diameters, with the "heat transfer coefficient" augmented by various factors relative to the stable steady value. Results are indicated in Figure 7, which shows the ranges of area-scaled discharge rates within which flow may occur as functions of this heat loss multiplier factor. The maximum flow rate is only weakly influenced by increases in heat loss (as expected in light of the above discussion), but the minimum possible discharge rate rises significantly with increases in heat loss. For each borehole diameter, the "critical" heat loss multiplier is reached when the borehole can no longer discharge at any rate.

Figure 6. Effective heat transfer coefficient multiplier as a function of time after flow startup for various values of borehole diameter (centimeters).

Figure 7. Ranges of area-scaled discharge rate within which discharge is possible for 10-cm slim holes and 25-cm production wells, for various values of heat loss multiplier. Reservoir flow resistance neglected (flowing bottomhole pressure = 80 bars).

This critical value is much larger (~148) for the 25-cm production well than for the 10-cm slim hole (critical multiplier value ~16.2).

This difference in heat loss effects is probably responsible for the difficulty often encountered in inducing deep slim holes (depth $\gg$ 300 meters) to discharge. For example, if we assume that for stable discharge to occur the heat-loss multiplier must drop below one-fourth of the above "critical value", as shown in Figure 8, it should be possible to sustain stable discharge in a 25-cm production well after a transient period of about two minutes. To reach a comparable state for the 10-cm diameter slim hole requires nearly one hour. Since transient processes associated with conventional flow-initiation techniques (swabbing, pressurization, gas injection etc.) usually involve time-scales of only a few minutes, this may explain why deep slim holes often fail to sustain discharge even after several initiation attempts when larger-diameter wells start flowing without difficulty. This implies that to induce deep slim holes to discharge, it may be necessary to employ unusual techniques such as pre-heating the borehole prior to startup.

Figure 8. Time required for heat loss multiplier to drop to 1/4 of critical value for 10-cm diameter slim holes and 25-cm diameter production wells.

6. INCORPORATION OF RESERVOIR FLOW RESISTANCE

Up to this point, all calculations presented have assumed that the resistance of the reservoir itself to fluid flow may be neglected compared to the resistance imposed by the borehole; in other words, all calculations have assumed a bottomhole flowing pressure of 80 bars absolute at the feedpoint (at a depth of 1500 meters in the borehole). For extremely permeable geothermal reservoirs, this may be an appropriate approximation. Under many realistic circumstances, however, the finite permeability of the reservoir causes the bottomhole flowing pressure to be appreciably less than the stable reservoir pressure, and the amount of such pressure decline increases with increasing discharge rate.

In light of the purpose of this study, it is appropriate to ignore pressure-transient effects and to assume that the bottomhole flowing pressure may be related to the reservoir pressure and the discharge rate by a "productivity index" (I) so that:

$$P_{\text{bottomhole}} = P_{\text{reservoir}} - (M / I) \quad (10)$$

where M is the discharge rate and the productivity index (I) may be regarded as a constant for each borehole. For the remaining WELBOR calculations, this general form was assumed, and it was further assumed that the fluid flows isenthalpically from "reservoir" conditions (at $P = P_{\text{reservoir}} = 80$ bars, $T = 250^\circ\text{C}$) to "bottomhole" conditions (at $P = P_{\text{bottomhole}}$), then up the borehole.

The essential difficulty involved in performing calculations with non-zero reservoir flow resistance (that is, values of the productivity index I which are less than infinity) is that of estimating the proper value to use for I for various borehole diameters. It is frequently observed that, if several production wells (of about the same physical characteristics) are drilled into the same reservoir, they are found to be characterized by a wide variety of values of I . This is particularly true in fractured reservoirs, in which the productivity index of a particular well depends mainly on the degree to which it happens to intersect productive fractures which are well-connected to the fracture network as a whole. For purposes of this study, the essential question is: if a "slim hole" is drilled into a particular location, what is the relationship likely to be between the productivity index of the slim hole (I_{slim}) and the productivity index that would have been obtained if instead a production-size well had been drilled (I_{prod})? Obviously, it is not possible to provide a deterministic answer to this question which will be valid in all cases. It is, however, possible to make estimates of the probable ratios of I_{prod} to I_{slim} which may be valid in a statistical sense, or at least to assign bounds to this ratio. For this purpose, it is useful to adopt the following mathematical form for the productivity index ratio of two boreholes of different diameters (D_{slim} , D_{prod}):

$$\frac{I_{\text{slim}}}{I_{\text{prod}}} = \left(\frac{D_{\text{slim}}}{D_{\text{prod}}} \right)^p \quad (11)$$

where the appropriate (presumably non-negative) value for the exponent p remains to be established.

7. ESTIMATION OF PRODUCTIVITY INDEX: CYLINDRICAL FLOW APPROXIMATION

As a first step, consider the classical “radial-flow” model for flow of fluid into a borehole. We assume that the reservoir consists of a homogeneous region of constant thickness (h) and of infinite lateral extent, bounded above and below by impermeable aquitards. The borehole penetrates completely through this permeable zone and produces fluid uniformly along the borehole surface. Under these conditions, the steady flowing bottomhole pressure in the borehole may be written:

$$P_{bottomhole} = P_{reservoir} - \frac{Mv}{2\pi kh} \ln\left(\frac{r_D}{r_e}\right) \quad (12)$$

where k is the reservoir permeability, v is the reservoir fluid kinematic viscosity, r_D is the “drainage radius” for the borehole and r_e is the “effective wellbore radius”. Typically, the “drainage radius” r_D is very large, comparable to the average interwell spacing or the reservoir overall dimensions (i.e., hundreds or thousands of meters). The “effective wellbore radius” r_e may be related to the actual borehole diameter and the “skin factor” (S) by:

$$r_e = \left(\frac{D}{2}\right) e^{-S} \quad (13)$$

so that, for example, if the skin factor is zero the “effective wellbore radius” is equal to the actual borehole radius. For “damaged” boreholes ($S > 0$) r_e is less than the true borehole radius, whereas for “stimulated” boreholes r_e exceeds the actual borehole radius. The above formula permits the definition of the productivity index for the borehole:

$$I = \frac{2\pi kh}{v \ln(r_D/r_e)} \quad (14)$$

Therefore, for two boreholes of different diameters (D_{slim} , D_{prod}) and the same skin factor (S), the ratio of the productivity indices for the boreholes will be given by:

$$\frac{I_{slim}}{I_{prod}} = \frac{\ln r_D - \ln D_{prod} + S - 0.693}{\ln r_D - \ln D_{slim} + S - 0.693} \quad (15)$$

This means that the productivity index of the slim hole is likely to be somewhat less than that of the production well. In the limit of large values for the drainage radius, however, we obtain:

$$I_{slim} \rightarrow I_{prod} \text{ as } r_D \rightarrow \infty \quad (16)$$

which means $p \rightarrow 0$ (see Equation 11). Considering the case ($D_{slim} = 10$ cm, $D_{prod} = 25$ cm), values for the exponent p for various values of the drainage radius r_D (from 100 meters to 10 km) and of the skin factor (from -5 to $+5$) are illustrated in Figure 9. Positive skin factors are much more commonly encountered in geothermal wells than negative skin factors; thus, the cylindrical model suggests p values which are very small compared with unity.

Figure 9. Productivity index scaling exponent (p) for various drainage radii (r_D) and skin factor (S) values.

8. ESTIMATION OF PRODUCTIVITY INDEX: SPHERICAL FLOW APPROXIMATION

As an alternative, we note that geothermal reservoirs frequently consist of large thick volumes of fractured rock. Boreholes penetrating such a reservoir often do not produce from long open intervals, but instead appear to produce from one or more discrete "feedpoints" where the borehole intersects a thin permeable fracture zone. In pressure-transient test interpretation, it has therefore occasionally proved useful to abandon the traditional "line-source" radial-flow model (above) and instead to consider a "spherical-source" flow model. Here, we treat the reservoir as unbounded in all directions; the "feedpoint" is represented by a spherical void within this reservoir, and fluid flows toward the spherical surface in all directions in a spherically symmetrical manner. The spherical void represents the highly permeable region of intersection of the borehole with the fracture. Under conditions of steady flow, the pressure at the surface of the spherical void will be given by:

$$P_{bottomhole} = P_{reservoir} - \frac{Mv}{2\pi kd} \quad (17)$$

where d is the diameter of the spherical void. Thus, the productivity index may be written:

$$I = 2\pi k d / v \quad (18)$$

so that the ratio of productivity indices will be given by:

$$\frac{I_{slim}}{I_{prod}} = \frac{d_{slim}}{d_{prod}} \quad (19)$$

The problem is then: what is the relationship between "spherical void diameter" (d) and borehole diameter (D)? One extreme case is obtained by assuming that d is directly proportional to D . This yields, for the productivity-index scaling exponent p :

$$p \rightarrow 1 \text{ if } d/D \rightarrow \text{constant}$$

Another extreme argument is to assert that the "spherical void diameter" is independent of borehole diameter and is, in effect, a property of the fracture network itself, so that:

$$p \rightarrow 0 \text{ if } d \rightarrow \text{constant}$$

The actual situation is likely to lie between the above two extremes. Tang (1988) studied the problem of limited-entry completions of boreholes in thick anisotropic porous reservoirs. Tang's results may be expressed as follows, in terms of productivity index ratios:

$$\frac{I_{slim}}{I_{prod}} = \frac{h_{slim}}{h_{prod}} \frac{\ln h_{prod} - \ln(D_{prod}/2) + \frac{1}{2} \ln a}{\ln h_{slim} - \ln(D_{slim}/2) + \frac{1}{2} \ln a} \quad (20)$$

Here, D_{prod} and D_{slim} are the diameters of the production well and slim hole, respectively; a represents the anisotropy of the reservoir permeability distribution:

$$a = k_H/k_V \quad (21)$$

(where k_H and k_V are horizontal and vertical permeability respectively), and h_{prod} and h_{slim} represent the "feedzone thickness" for each borehole completion; the vertical thickness of the region over which fluid enters the borehole (usually just a few meters or so in fractured geothermal systems). Tang (1988) implicitly assumes that $h \gg D$.

Adoption of Equation 20 means that we are now confronted with estimating h_{slim} and h_{prod} . For simplicity, the following relationship is assumed for this purpose:

$$h = h_o + bD \quad (22)$$

where h_o and b are constants. Note that if $b = 0$, then $h_{slim} = h_{prod} = h_o$; in this case the feedzone thickness is independent of hole diameter. Positive values for b indicate that larger-diameter holes will tend to have somewhat thicker feedzones.

In Figure 10, we show how the proper value of the productivity-index scaling exponent (p) depends upon h_o , a and b according to Equation 20, taking $D_{prod} = 25$ cm and $D_{slim} = 10$ cm. Values are in the range $0 < p < 1$; p decreases toward zero as feedzone thickness (h_o) increases. Larger values of a ($= k_H/k_V$) tend to decrease p . Increasing b ($=[h-h_o]/D$), on the other hand, tends to increase p . In the limit $b \rightarrow \infty$, p approaches unity. On the whole, values of p appear to be somewhat larger in the case of the spherical flow model than for the cylindrical flow model (compare Figures 9 and 10).

Figure 10. Productivity index scaling exponent (p) under various conditions based on results reported by Tang (1988).

9. RESULTS FOR FINITE RESERVOIR FLOW RESISTANCE

In light of the results from the above two analyses (for “radial” and “spherical” flow), it seems reasonable to assume that the productivity index for a slim hole will be related to that for the equivalent production-size well by the following:

$$I_{slim} = I_{prod} \left(\frac{D_{slim}}{D_{prod}} \right)^p \quad (23)$$

where the exponent p takes on values between zero and unity. For any real situation, the proper value for p is likely to lie somewhere between these limiting values.

Therefore, a final series of calculations was performed involving three well configurations:

- (1) a 25-cm diameter production well with productivity index I_{prod} ,
- (2) a 10-cm diameter slim hole using $I_{slim} = I_{prod}$ (that is, $p = 0$ scaling for productivity index), and
- (3) a 10-cm diameter slim hole using $I_{slim} = 0.4 \times I_{prod}$ ($p = 1$ productivity index scaling).

Several values of I_{prod} were considered, as follows:

1 kg/second/bar	(547 lb/hour/psi)
2 kg/second/bar	(1094 lb/hour/psi)
3 kg/second/bar	(1642 lb/hour/psi)
4 kg/second/bar	(2189 lb/hour/psi)
6 kg/second/bar	(3283 lb/hour/psi)

Computed results for wellhead pressure as a function of area-scaled discharge rate are listed in Tables 8, 9 and 10 (which also includes the corresponding results for the case of no reservoir flow resistance; infinite I_{prod}). Figures 11a–11f illustrate the results graphically—Figure 11a compares the production well and slim hole discharge characteristics for the case of no reservoir flow resistance, and Figures 11b–11f compare the various boreholes at successively smaller values of I_{prod} (increasing reservoir flow resistance).

It is noteworthy that changing the reservoir flow resistance has a greater influence upon the discharge characteristics of the production size (25 cm) well than upon those of the slim hole (10 cm). The reason is simply that the total resistance to flow is the sum of the effects of the reservoir

Continued on page 41

Table 8. Performance characteristics of 25-cm diameter production well as a function of productivity index.

Area Scaled Flowrate M^* (kg/s)	Actual Flowrate M (kg/s)	Flowing Wellhead Pressure (absolute bars) for Production Well Productivity Index = ...					
		1 kg/s/bar	2 kg/s/bar	3 kg/s/bar	4 kg/s/bar	6 kg/s/bar	infinity
5	5	8.11	8.47	8.60	8.66	8.72	8.85
10	10	9.38	10.14	10.40	10.53	10.66	10.93
15	15	9.51	10.64	11.03	11.23	11.43	11.83
20	20	9.23	10.73	11.25	11.51	11.78	12.32
25	25	8.72	10.59	11.25	11.58	11.91	12.60
30	30	8.00	10.29	11.09	11.49	11.90	12.74
35	35	7.06	9.85	10.80	11.29	11.78	12.78
40	40	5.81	9.26	10.40	10.97	11.55	12.73
45	45	3.65	8.52	9.87	10.55	11.24	12.61
50	50	—	7.57	9.21	10.02	10.82	12.43
55	55	—	6.32	8.38	9.35	10.30	12.17
60	60	—	4.49	7.32	8.52	9.65	11.85
65	65	—	—	5.90	7.46	8.86	11.45
70	70	—	—	3.60	6.06	7.87	10.97
75	75	—	—	—	3.82	6.58	10.39
80	80	—	—	—	—	4.67	9.70
85	85	—	—	—	—	—	8.85
90	90	—	—	—	—	—	7.81
95	95	—	—	—	—	—	6.44
100	100	—	—	—	—	—	4.29
105	105	—	—	—	—	—	—

Table 9. Performance characteristics of 10-cm diameter slim hole using $p = 0$ productivity index scaling rule.

Area Scaled Flowrate M^* (kg/s)	Actual Flowrate M (kg/s)	Flowing Wellhead Pressure (absolute bars) for Slim Hole Productivity Index = ...					
		1 kg/s/bar	2 kg/s/bar	3 kg/s/bar	4 kg/s/bar	6 kg/s/bar	infinity
5	0.8	3.35	3.33	3.32	3.32	3.32	3.31
10	1.6	7.15	7.17	7.18	7.19	7.19	7.20
15	2.4	8.80	8.89	8.92	8.93	8.95	8.98
20	3.2	9.51	9.60	9.74	9.77	9.79	9.85
25	4.0	9.71	9.97	10.06	10.10	10.14	10.23
30	4.8	9.54	9.91	10.03	10.10	10.16	10.28
35	5.6	9.03	9.55	9.72	9.80	9.89	10.06
40	6.4	8.16	8.89	9.12	9.24	9.35	9.58
45	7.2	6.81	7.86	8.19	8.35	8.50	8.82
50	8.0	4.57	6.30	6.79	7.02	7.25	7.70
55	8.8	—	3.59	4.51	4.92	5.31	6.02
60	9.6	—	—	—	—	—	2.85
65	10.4	—	—	—	—	—	—

Table 10. Performance characteristics of 10-cm diameter slim hole using $p = 1$ productivity index scaling rule.

Area Scaled Flowrate M^* (kg/s)	Actual Flowrate M (kg/s)	Flowing Wellhead Pressure (absolute bars) for Slim Hole Productivity Index = ...					
		1 kg/s/bar	2 kg/s/bar	3 kg/s/bar	4 kg/s/bar	6 kg/s/bar	infinity
5	0.8	3.40	3.36	3.34	3.33	3.33	3.31
10	1.6	7.08	7.14	7.16	7.17	7.18	7.20
15	2.4	8.52	8.76	8.83	8.87	8.90	8.98
20	3.2	9.00	9.43	9.57	9.64	9.71	9.85
25	4.0	8.91	9.58	9.80	9.91	10.02	10.23
30	4.8	8.37	9.35	9.66	9.82	9.97	10.28
35	5.6	7.36	8.76	9.20	9.42	9.64	10.06
40	6.4	5.67	7.78	8.41	8.71	9.00	9.58
45	7.2	1.91	6.24	7.17	7.61	8.02	8.82
50	8.0	—	3.34	5.21	5.91	6.55	7.70
55	8.8	—	—	—	2.52	4.05	6.02
60	9.6	—	—	—	—	—	2.85
65	10.4	—	—	—	—	—	—

Figure 11a. Wellhead pressure as function of area-scaled well discharge rate for slim hole (10 cm diameter) and production well (25 cm diameter) with no reservoir flow resistance.

Figure 11b. Wellhead pressure as function of area-scaled well discharge rate for slim hole (10 cm diameter) and production well (25 cm diameter) with production-well productivity index equal to 6 kilograms per second per bar.

Figure 11c. Wellhead pressure as function of area-scaled well discharge rate for slim hole (10 cm diameter) and production well (25 cm diameter) with production-well productivity index equal to 4 kilograms per second per bar.

Figure 11d. Wellhead pressure as function of area-scaled well discharge rate for slim hole (10 cm diameter) and production well (25 cm diameter) with production-well productivity index equal to 3 kilograms per second per bar.

Figure 11e. Wellhead pressure as function of area-scaled well discharge rate for slim hole (10 cm diameter) and production well (25 cm diameter) with production-well productivity index equal to 2 kilograms per second per bar.

Figure 11f. Wellhead pressure as function of area-scaled well discharge rate for slim hole (10 cm diameter) and production well (25 cm diameter) with production-well productivity index equal to 1 kilogram per second per bar.

and of the borehole itself. As discussed above (see Figure 2), the capacity of the borehole to deliver fluid to the wellhead varies approximately as $D^{2.5}$, whereas the capacity of the reservoir to deliver fluid to the feedpoint varies as D^p , with $0 < p < 1$. Consequently, the reservoir resistance comprises a greater proportion of the total resistance for larger diameter holes, and therefore large holes are more sensitive to variations in reservoir flow resistance.

It is also noteworthy that, despite the shortcomings of "area-scaling" of discharge rates (arising from the failure of the effects of pipe friction and of heat losses to scale in this manner), scaling-up the discharge characteristics of the 10-cm slim hole provides a conservative estimate of the probable productivity of larger diameter wells so long as the reservoir flow resistance is not too large (productivity index > 1 kg/second/bar or so), at least for conditions similar to those considered in this study.

10. CONCLUSIONS AND RECOMMENDATIONS

The preceding computational study provides a theoretical framework within which the question of relative discharge characteristics of slim holes and production wells may be examined. A number of issues require resolution by examination of field measurements and historical experience with slim holes in real geothermal fields, however.

One of these issues is the question of the "scaling rule" for downhole productivity index between slim holes and wells of conventional diameter. In this study, the following form was adopted:

$$\frac{I_{slim}}{I_{prod}} = \left(\frac{D_{slim}}{D_{prod}} \right)^p$$

and various arguments were advanced suggesting that the proper value for the exponent p should lie somewhere between zero and unity. It would be highly desirable to establish an appropriate empirical value for p based on field experience. One approach would be to examine one or more data sets from geothermal fields in which a sufficient number of both slim holes and production wells have been drilled and tested to provide a statistically significant sample. If sufficient productivity-index determinations are not available, the same p exponent should in principle govern the ratios of "injectivity indices" so that injection-test data, if available, could be substituted.

The calculations presented in Section 9 indicate that, if slim test holes can be induced to discharge, the fluid discharge capacity of the slim holes may be scaled up to equivalent production well capacities using the "area-scaling" rule (that is, by multiplying the observed slim-hole discharge rates by the square of the ratio of the wellbore diameters), and that this will usually provide a conservative forecast (i.e., underestimate) of production well performance so long as the reservoir permeability is reasonably good. The difficulty is that it may be very difficult to induce spontaneous discharge in deep slim holes (depth $\gg$ 300 meters) even in reservoirs with promising characteristics, for the reasons discussed in Section 5. Field experience with attempts to discharge slim holes (and comparisons with experience discharging production-size wells in the same field) should be examined to elucidate these issues and to suggest practical remedies if difficulties are encountered. The possibility of "pre-heating" the slim hole prior to discharge initiation should be considered, and any available field experience with such techniques should be carefully studied.

If it is impractical to discharge the slim exploration holes, another possibility is to perform an injection test (with a downhole pressure gauge located near the feedpoint) and thereby to obtain an empirical "injectivity index" for the borehole. At least three types of difficulties are inherent in

this approach, however. First, it is much more important for an injection test than for a production test that the pressure gauge be located adjacent to the borehole feedpoint owing to hydrostatic differences arising from the temperature contrast between the cold injected fluid (of which the downhole temperature is likely to change significantly with time during the test) and the hot reservoir waters. This means that it will be necessary to accurately identify the borehole's feedpoint (by, for example, spinner logging) prior to injection testing. If, as is often the case, the borehole has more than one major feedzone, the interpretation of injection test results may be impossible. Second, it is very important that as much debris as possible (chips, etc.) be removed from the borehole prior to injection testing. Otherwise, this debris will tend to be carried into the feedzone of the borehole during the injection test, blocking important flow channels and rendering any test interpretation ambiguous. The third difficulty with the "injection test" approach is that an injection test raises fluid pressures in the neighborhood of the borehole. These increased fluid pressures will often open and/or widen existing fractures, increasing the local permeability and temporarily decreasing the reservoir flow resistance compared to what would occur under production (pressure reduction) conditions. This can result in overly optimistic estimates of borehole productivity under discharge conditions. Clearly, if injection testing is employed, it would be very desirable to carry out the test at relatively low injection rates to avoid excessive pressure increase. Any existing field experience concerning the relationships between observed "productivity index" and "injectivity index" for individual boreholes as functions of flowrate, pressure and borehole diameter would undoubtedly prove useful in the interpretation of future injection tests.

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APPENDIX A. MATHEMATICAL FORMULATION OF TWO-PHASE FLUID FLOW IN A WELLBORE

Two-phase fluid flow in the wellbore is, at present, not amenable to strict analytical treatment. Depending upon the relative amounts of gas and liquid, a variety of flow patterns can occur in the pipe. At small gas loadings, bubble flow takes place. An increase in gas flow rate can result in slug, churn, or annular flow. Existing techniques for treating two-phase flow in the pipe require use of empirical correlations for flowing gas quality and friction factor (see below). There exist in the literature numerous such correlations; most of these correlations are based on flow in two-phase petroleum (oil and gas) wells. Utilization of different empirical correlations often yields widely different results; at present, there does not appear to be sufficient basis for selecting one or another of these correlations.

Consider two-phase non-isothermal steady flow in a vertical or deviated pipe. The bottom of the pipe is taken to be above the location of the topmost feedzone. The assumption of steady flow implies that the mass flux $\dot{M}$ is constant along the length of the pipe:

$$\dot{M} = A \{ S_l \rho_l v_l + S_g \rho_g v_g \} = A \rho v = \text{constant} \quad (1)$$

where

A = cross - section area of pipe

$\rho_l(\rho_g)$ = liquid (gas) density

$v_l(v_g)$ = liquid (gas) velocity

$S_l(S_g)$ = liquid (gas) volume fraction

z = vertical coordinate (positive upwards)

ρ = mixture (liquid + gas) density = $S_l \rho_l + S_g \rho_g$

v = mixture (liquid + gas) velocity = $(S_l \rho_l v_l + S_g \rho_g v_g) / \rho$

The pressure drop due to fluid flowing in a vertical or inclined pipe represents the combined effects of friction, acceleration, and change in elevation.

$$-\frac{dp}{dz} = \frac{1}{A} \frac{d}{dz} \left\{ A \left[S_g \rho_g v_g^2 + S_l \rho_l v_l^2 \right] \right\} + F + \rho g \quad (2)$$

where

F = frictional pressure gradient

$g = g(z)$, component of acceleration of gravity parallel to the well.

The frictional pressure gradient F is specified by empirical correlations. The S-Cubed wellbore simulator (WELBOR) uses Dukler's correlation (Dukler, *et al.*, 1964).

The mixture energy balance for two-phase flow can be expressed as follows:

$$\frac{d}{dz} [Q_{fg} e_g + (1 - Q_{fg}) e_l] = -g - \frac{2\pi r_w}{\dot{M}} U (T - T_r) \quad (3)$$

where

$$e_g = h_g + v_g^2/2 = E_g + \frac{P}{\rho_g} + v_g^2/2$$

$$e_l = h_l + v_l^2/2 = E_l + \frac{P}{\rho_l} + v_l^2/2$$

$h_g (h_l)$ = gas (liquid) specific enthalpy

$E_g (E_l)$ = gas (liquid) specific internal energy

$$Q_{fg} = \text{flowing quality} = \frac{\text{upward gas mass flow rate}}{\text{total mass flow rate}} = \frac{A S_g \rho_g v_g}{\dot{M}}$$

r_w = pipe radius

T = fluid temperature

T_R = formation temperature,

U = heat transfer coefficient

The last term in Equation (3) describes the heat loss from the borehole to the surrounding formation. If the formation outside the wellbore is impermeable, then the heat flux from the borehole will decline with time ($\lim_{t \rightarrow \infty} U = 0$) as the rock near the borehole heats up. Generally, the formation outside the wellbore is permeable, and the heat loss will occur due to combined effects of conduction and convection. Analytical results of Minkowycz and Cheng (1976) suggest that the heat transfer coefficient in the latter case is given by:

$$\lim_{t \rightarrow \infty} U = 0.3 K / r_w \quad (4)$$

where K is the thermal conductivity of the water saturated formation. With typical values of K ($= 4 \text{ W/m}^\circ\text{C}$) and r_w ($= 0.1 \text{ m}$), the heat transfer coefficient equals $12 \text{ W/m}^2\text{-}^\circ\text{C}$. In practical applications, U generally lies between 5 and $20 \text{ W/m}^2\text{-}^\circ\text{C}$.

The momentum balance law constitutes a single equation for two unknowns, i.e., liquid and gas velocities v_l and v_g . To solve for v_l and v_g , we need another relation between v_l and v_g . For this purpose, it is convenient to express v_l and v_g in terms of flowing quality Q_{fg} .

$$\begin{aligned} v_g &= \dot{M}Q_{fg}/\rho_g A S_g \\ v_l &= \dot{M}(1 - Q_{fg})/\rho_l A S_l \\ S_g &= 1 - S_l = \frac{Q_s \rho_l}{Q_s \rho_l + (1 - Q_s) \rho_g} \end{aligned} \quad (5)$$

where Q_s denotes the *in situ* gas quality. We shall now consider the specification of Q_{fg} in terms of other known flow properties. The simplest assumption is to take flowing quality Q_{fg} equal to the *in situ* quality Q_s . This is the so-called no-slip approximation ($v_l = v_g$). In practice, the no-slip approximation predicts too low a pressure drop in the wellbore. In general, one would expect the gas phase to rise more rapidly in the wellbore than the liquid phase due to buoyancy. If $v_g \geq v_l$, then it follows from Equation (5) that the flowing quality Q_{fg} is also greater than or at least equal to the *in situ* quality Q_s . As already mentioned above, there exist numerous empirical correlations in the literature which can be used to express Q_{fg} in terms of Q_s and other flow properties. The S-Cubed wellbore simulator (WELBOR) incorporates a correlation due to Hughmark (1962). Hughmark's correlation is in part based on liquid water and steam flow through vertical and horizontal tubes. Experience has shown that the use of Hughmark's correlation often yields too high a value for the pressure drop. To obviate the difficulties associated with the original Hughmark correlation, S-Cubed's WELBOR simulator uses a modified version of the Hughmark correlation; essentially, the slippage rate may vary between the value given by the Hughmark correlation and no-slip at all, according to the value of a user supplied parameter (slip parameter). This feature is normally used only for fitting measured downhole pressure and temperature profiles; in normal operations, the program simply uses the original Hughmark correlation. Typical slip parameter values required to fit downhole data from particular boreholes vary between $0.5 \times \text{Hughmark}$ and $1.0 \times \text{Hughmark}$.

The mass, momentum, and energy balance laws need to be adjoined by a suitable equation-of-state for the fluid. With pressure p and mixture enthalpy h (or alternatively mixture energy E) as independent parameters, the equation-of-state routines can be used to calculate dependent variables such as E_g , E_l , ρ_l , ρ_g , T , Q_s , S_g , S_l , μ_l and μ_g .

The capacity of a borehole to deliver fluid to the wellhead is limited. The so-called "choking condition" occurs when dp/dz becomes singular. The maximum flow capacity of a borehole is obviously somewhat less than that for which choking first occurs. For two-phase flow with slip, the choking condition is given by:

$$1 = \frac{v_g^2 - v_l^2}{\rho_l - \rho_g} \left\{ S_g \rho_l \frac{d\rho_g}{dp} + S_l \rho_g \frac{d\rho_l}{dp} \right\} + \frac{\rho_l v_l^2 - \rho_g v_g^2}{\rho_l - \rho_g} \left(\partial p / \partial p \right)_h - \frac{2\dot{M}}{A} (v_g - v_l) \left(\frac{\partial Q_{fg}}{\partial p} \right)_h \quad (6)$$

For the no-slip case ($v_g = v_l$), the above choking condition reduces to the following familiar form:

$$1 = v^2 / c^2 \quad (7)$$

where $c \left(= [1/(\partial p / \partial p)_h]^{0.5} \right)$ denotes the speed of sound.

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