AIM

Association for Information and Image Management

1100 Wayne Avenue, Suite 1100
Silver Spring, Maryland 20910
301/587-8202

Centimeter

Inches

MANUFACTURED TO AIM STANDARDS
BY APPLIED IMAGE, INC.

A Variable Thickness Window: Thermal and Structural Analyses

Zhibi Wang and Tuncer M. Kuzay

Experimental Facilities Division
Advanced Photon Source
Argonne National Laboratory
9700 South Cass Avenue
Argonne, IL 60439

RECEIVED
AUG 17 1994
OSTI

INTRODUCTION

A novel idea that uses a variable thickness window for a synchrotron radiation beamline is proposed here for third-generation synchrotron x-ray facilities that can generate a very bright x-ray beam in a very small area. The main advantages of using a variable thickness window are as follows: (1) to make it possible to design a window without filters and therefore make more low energy photons available to the users; (2) to increase the heat conduction area without increasing the thickness in core area so that the maximum temperature in the window can be significantly decreased; (3) to increase the safety margin of window system due to the increasing structural integrity, especially in regards to the buckling load in thin windows, which also increases the structural integrity of window against shock due to sudden loss of vacuum; and (4) to increase the window size under the same operating conditions. Compared to a uniform Be window, the variable thickness Be window reduces the likelihood of the catastrophic break.

Because windows usually have a regular geometry, such as rectangular or circular forms, the finite-difference method is the best fit for their analyses. Finite-difference formulation with temperature-dependent material properties are used for variable thickness plane thermal and plane stress problems. Because radiative cooling dominates when the temperature is high, the radiation effect has been considered.

The submitted manuscript has been authored by a contractor of the U. S. Government under contract No. W-31-109-ENG-38. Accordingly, the U. S. Government retains a nonexclusive, royalty-free license to publish or reproduce the published form of this contribution, or allow others to do so, for U. S. Government purposes.

MASTER

In this paper, the finite difference formulations for variable thickness thermal analysis and variable thickness plane stress analysis are presented. In heat transfer analysis, radiation effects and temperature-dependent thermal conductivity are taken into account. While in thermal stress analysis, the thermal expansion coefficient is considered as temperature dependent. An application of the variable thickness window to an Advanced Photon Source beamline is presented.

VARIABLE THICKNESS THERMAL ANALYSIS

Finite Difference Formulation

Assuming a domain of size a by b is divided into a mesh (as shown in figure 1) in which Δx and Δy are the sizes of the grid in the x and y directions, respectively, one can apply the first law of thermodynamics to a nodal point i, j in figure 1 (b):

$$q_{i-1}\Delta y \frac{1}{2}(t_{i,j} + t_{i-1,j}) + q_{j-1}\Delta x \frac{1}{2}(t_{i,j} + t_{i,j-1}) + q_{i+1}\Delta y \frac{1}{2}(t_{i,j} + t_{i+1,j}) + q_{j+1}\Delta x \frac{1}{2}(t_{i,j} + t_{i,j+1}) + q''\Delta x\Delta y t_{i,j} + 2q^r\Delta x\Delta y = 0 \quad , \quad (1)$$

where q_{i-1} , q_{j-1} , q_{i+1} , q_{j+1} and q^r can be related to temperature by Fourier's law in conduction and Stefan-Boltzmann law in radiation:

$$\begin{aligned} q_{i-1} &= k \frac{T_{i-1,j} - T_{i,j}}{\Delta x}, & q_{j-1} &= k \frac{T_{i,j-1} - T_{i,j}}{\Delta y}, \\ q_{i+1} &= k \frac{T_{i+1,j} - T_{i,j}}{\Delta x}, & q_{j+1} &= k \frac{T_{i,j+1} - T_{i,j}}{\Delta y}, \\ q^r &= \sigma \epsilon (T_{i,j}^4 - T_0^4). \end{aligned} \quad (2)$$

Radiation is taken into account by q^r for a unit area. In equation (1), the factor of 2 accounts for the top and bottom surfaces.

Inserting Eq. (2) into Eq. (1) and letting $B = \Delta x/\Delta y$, one has

$$\begin{aligned}
& \frac{1}{2} [T_{i-1,j}(t_{i,j} + t_{i-1,j}) + T_{i+1,j}(t_{i,j} + t_{i+1,j}) + B^2 T_{i,j-1}(t_{i,j} + t_{i,j-1}) + \\
& B^2 T_{i,j+1}(t_{i,j} + t_{i,j+1})] - T_{i,j} \left[t_{i,j} + \frac{1}{2} t_{i+1,j} + \frac{1}{2} t_{i-1,j} + B^2 t_{i,j} + \right. \\
& \left. \frac{1}{2} B^2 t_{i,j-1} + \frac{1}{2} B^2 t_{i,j+1} \right] + q''' \frac{\Delta x^2}{k} t_{i,j} + 2 \frac{\Delta x^2}{k} \sigma \epsilon (T_0^4 - T_{i,j}^4) = 0
\end{aligned} \quad (3)$$

For an iterative solution, if m denotes the iteration number, one can use the following approximation:

$$T_{i,j}^{m+1,4} = 4T_{i,j}^{m,3} T_{i,j}^{m+1} - 3T_{i,j}^{m,4}. \quad (4)$$

Substituting Eq. (4) into Eq. (3), an expression for $T_{i,j}^{k+1}$ can be derived as:

$$\begin{aligned}
T_{i,j}^{m+1} = \frac{1}{A} \left\{ T_{i-1,j}^m t_{i-1,j} + T_{i+1,j}^m t_{i+1,j} + B^2 T_{i-1,j}^m t_{i-1,j} + B^2 T_{i+1,j}^m t_{i+1,j} + \right. \\
\left. \frac{\Delta x^2}{k} \left[q''' t_{i,j} + 2 \sigma \epsilon (3T_{i,j}^{m,4} + T_0^4) \right] \right\},
\end{aligned} \quad (5)$$

where

$$\begin{aligned}
t_{i-1,j} &= \frac{1}{2}(t_{i,j} + t_{i-1,j}), & t_{i+1,j} &= \frac{1}{2}(t_{i,j} + t_{i+1,j}), \\
t_{i,j-1} &= \frac{1}{2}(t_{i,j} + t_{i,j-1}), & t_{i,j+1} &= \frac{1}{2}(t_{i,j} + t_{i,j+1}),
\end{aligned} \quad (6)$$

and

$$A = t_{i-1,j} + t_{i+1,j} + B^2 t_{i,j-1} + B^2 t_{i,j+1} + 8 \frac{\Delta x^2}{k} \sigma \epsilon T_{i,j}^{k,3}. \quad (7)$$

In the case of a uniform thickness, without radiation effects and, for $\Delta x = \Delta y$, all variables in Eq. (6) become the same and equal to $t_{i,j}$. Equation (5) then becomes:

$$T_{i,j}^{m+1} = \frac{1}{4} \left\{ T_{i-1,j}^m + T_{i+1,j}^m + T_{i,j-1}^m + T_{i,j+1}^m + \frac{\Delta x^2}{k} q''' t_{i,j} \right\}. \quad (8)$$

This is the standard finite difference formulation for a planar heat conduction problem that can be found in almost any finite difference text book. The finite difference formulation based on Eq. (5) is a direct iteration method. For computers, an effective method for obtaining a numerical solution is called the successive over-relaxation method (SOR) [1]. If the SOR method is used, the new value at node i, j can be calculated as:

$$T_{i,j}^{m+1} = (1-\omega)T_{i,j}^m + \frac{\omega}{A} \left\{ T_{i-1,j}^{m+1}t_{i-1,j} + T_{i+1,j}^m t_{i+1,j} + B^2 T_{i-1,j}^{m+1}t_{i-1,j} + B^2 T_{i+1,j}^m t_{i+1,j} + \frac{\Delta x^2}{k} \left[q'' t_{i,j} + 2\sigma\epsilon(3T_{i,j}^m + T_0^4) \right] \right\} \quad (9)$$

The ω denotes the relaxation factor. If $\omega = 1$, the SOR method reduces to the classical Gauss-Seidel method. With a relaxation factor $0 \leq \omega \leq 2$, the convergence can be guaranteed [1].

Boundary Conditions

Two boundary conditions have been investigated: a convective boundary and a constant temperature boundary.

Convection Boundary. As shown in figures 2(b) and 2(c), the temperature at a corner node and that at an edge node should be derived separately.

For the case in figure 2(b), the following equation can be derived from the first law of thermodynamics:

$$q_{i-1} \frac{\Delta y}{2} t_{i,j} + q_{j-1} \Delta x \frac{1}{2} (t_{i,j} + t_{i,j-1}) + q_{i+1} \frac{\Delta y}{2} (t_{i,j} + t_{i+1,j}) + q_{j+1} \Delta x t_{i,j} + q'' \Delta x \frac{\Delta y}{2} t_{i,j} + 2q' \Delta x \frac{\Delta y}{2} = 0 \quad (10)$$

where

$$\begin{aligned} q_{i-1} &= h(T_\infty - T_{i,j}), & q_{j-1} &= k \frac{T_{i,j-1} - T_{i,j}}{\Delta y}, \\ q_{i+1} &= h(T_\infty - T_{i,j}), & q_{j+1} &= k \frac{T_{i,j+1} - T_{i,j}}{\Delta y}, \\ q' &= \sigma\epsilon(T_{i,j}^4 - T_0^4). \end{aligned} \quad (11)$$

By assuming that $B_{i0} = h\Delta x/k$, the temperature in a corner node can be expressed as:

$$T_{i,j}^{m+1} = \frac{1}{A} \left\{ B_{io} T_{\infty} t_{i,j} (1+B) + T_{i+1,j}^m t_{i+1,j} + B^2 T_{i,j-1}^{m+1} t_{i,j-1} + \frac{\Delta x^2}{k} \left[\frac{1}{2} q''' t_{i,j} + \sigma \varepsilon (3T_{i,j}^m + T_0^4) \right] \right\}, \quad (12)$$

where, $t_{i+1,j}$ and $t_{i,j-1}$ are given by equation (6) and A' by:

$$A' = B_{io} t_{i,j} (1+B) + t_{i+1,j} + B^2 t_{i,j-1} + 4 \frac{\Delta x^2}{k} \sigma \varepsilon T_{i,j}^m. \quad (13)$$

When SOR is used, the temperature becomes:

$$T_{i,j}^{m+1} = (1-\omega) T_{i,j}^m + \frac{\omega}{A} \left\{ B_{io} T_{\infty} t_{i,j} (1+B) + T_{i+1,j}^m t_{i+1,j} + B^2 T_{i,j-1}^{m+1} t_{i,j-1} + \frac{\Delta x^2}{k} \left[\frac{1}{2} q''' t_{i,j} + \sigma \varepsilon (3T_{i,j}^m + T_0^4) \right] \right\}. \quad (14)$$

Similarly, for the case of an edge node, the formula corresponding to Eq. (1) is:

$$q_{i-1} \frac{\Delta y}{2} \frac{1}{2} (t_{i,j} + t_{i-1,j}) + q_{j-1} \Delta x \frac{1}{2} (t_{i,j} + t_{i,j-1}) + q_{i+1} \frac{\Delta y}{2} \frac{1}{2} (t_{i,j} + t_{i+1,j}) + q_{j+1} \Delta x t_{i,j} + q''' \Delta x \frac{\Delta y}{2} t_{i,j} + 2q' \Delta x \frac{\Delta y}{2} = 0 \quad (15)$$

From Eq. (15), the new value of the temperature at node i, j can be written as:

$$T_{i,j}^{m+1} = \frac{1}{A''} \left\{ T_{i-1,j}^{m+1} t_{i-1,j} + T_{i+1,j}^m t_{i+1,j} + B^2 T_{i,j-1}^{m+1} t_{i,j-1} + B_{io} B T_{\infty} t_{i,j} + \frac{\Delta x^2}{k} \left[\frac{1}{2} q''' t_{i,j} + \sigma \varepsilon (3T_{i,j}^m + T_0^4) \right] \right\}, \quad (16)$$

where A'' is given by:

$$A'' = \frac{1}{2} t_{i-1,j} + B^2 t_{i,j-1} + \frac{1}{2} t_{i+1,j} + B_{io} B t_{i,j} + 4 \frac{\Delta x^2}{k} \sigma \varepsilon T_{i,j}^m. \quad (17)$$

The SOR form of Eq. (16) is

$$T_{i,j}^{m+1} = (1-\omega) T_{i,j}^m + \frac{\omega}{A''} \left\{ \frac{1}{2} T_{i-1,j}^{m+1} t_{i-1,j} + B_{io} B T_{\infty} t_{i,j} + \frac{1}{2} T_{i+1,j}^m t_{i+1,j} + B^2 T_{i,j-1}^{m+1} t_{i,j-1} + \frac{\Delta x^2}{k} \left[\frac{1}{2} q''' t_{i,j} + \sigma \varepsilon (3T_{i,j}^m + T_0^4) \right] \right\}. \quad (18)$$

Temperature Boundary. Because the temperature at the boundary is given, iteration is not necessary. The boundary node temperature should be put wherever it applies.

Temperature-Dependent Material Properties

If the thermal conductivity of the material is temperature dependent, the k values in the above equations need an update in each iteration.

VARIABLE THICKNESS STRESS ANALYSIS

Basic Equations

Because displacement boundaries are usually given in the case of filters and windows used in synchrotron radiation facilities, the displacement method should be employed for their thermal stress analysis.

The basic equations for a variable thickness plane problem are equilibrium equations, constitutive relations, and compatible equations.

Equilibrium Equations. Equilibrium equations for the variable thickness plane problem are [2]:

$$\begin{aligned}\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} &= 0 \\ \frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} &= 0\end{aligned}\tag{19}$$

Constitutive Relation. The relations between strains and forces in the cross section can be expressed as [2]:

$$\begin{aligned}\varepsilon_x &= \frac{1}{Et}(N_x - \nu N_y) + \alpha T \\ \varepsilon_y &= \frac{1}{Et}(N_y - \nu N_x) + \alpha T \\ \gamma_{xy} &= \frac{2(1+\nu)}{Et} N_{xy}\end{aligned}\tag{20}$$

Rearrangement of Eq. (20) leads to:

$$\begin{aligned}
N_x &= \frac{Et}{(1-\nu^2)} \left(\frac{\partial u}{\partial x} + \nu \frac{\partial v}{\partial y} \right) - \frac{Et}{1-\nu} \alpha T \\
N_y &= \frac{Et}{(1-\nu^2)} \left(\frac{\partial v}{\partial y} + \nu \frac{\partial u}{\partial x} \right) - \frac{Et}{1-\nu} \alpha T \\
N_{xy} &= \frac{Et}{2(1+\nu)} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)
\end{aligned} \tag{21}$$

Substituting Eq. (21) into Eq. (19), one has

$$\begin{aligned}
\frac{\partial}{\partial x} \left[\frac{Et}{(1-\nu^2)} \left(\frac{\partial u}{\partial x} + \nu \frac{\partial v}{\partial y} \right) - \frac{Et}{1-\nu} \alpha T \right] + \frac{\partial}{\partial y} \left[\frac{Et}{2(1+\nu)} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right] &= 0 \\
\frac{\partial}{\partial y} \left[\frac{Et}{(1-\nu^2)} \left(\frac{\partial v}{\partial y} + \nu \frac{\partial u}{\partial x} \right) - \frac{Et}{1-\nu} \alpha T \right] + \frac{\partial}{\partial x} \left[\frac{Et}{2(1+\nu)} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right] &= 0
\end{aligned} \tag{22}$$

For a variable thickness problem, thickness t is the function of x and y . If the thermal expansion coefficient α is temperature dependent, the derivatives of α with x and y are not zero. However, if the temperature change with coordinates is not large compared with other terms in the equations, the derivative of α with x and y can be approximately assumed to be zero. Thus equation (22) becomes:

$$\begin{aligned}
\frac{\partial^2 u}{\partial x^2} + \frac{1-\nu}{2} \frac{\partial^2 u}{\partial y^2} + \frac{1+\nu}{2} \frac{\partial^2 v}{\partial x \partial y} - (1+\nu) \alpha \frac{\partial T}{\partial x} + \\
\left(\frac{\partial u}{\partial x} + \nu \frac{\partial v}{\partial y} \right) \frac{1}{t} \frac{\partial t}{\partial x} + \frac{1-\nu}{2} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \frac{1}{t} \frac{\partial t}{\partial y} - \alpha T (1+\nu) \frac{1}{t} \frac{\partial t}{\partial x} &= 0 \\
\frac{\partial^2 v}{\partial y^2} + \frac{1-\nu}{2} \frac{\partial^2 v}{\partial x^2} + \frac{1+\nu}{2} \frac{\partial^2 u}{\partial x \partial y} - (1+\nu) \alpha \frac{\partial T}{\partial y} + \\
\left(\frac{\partial v}{\partial y} + \nu \frac{\partial u}{\partial x} \right) \frac{1}{t} \frac{\partial t}{\partial y} + \frac{1-\nu}{2} \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) \frac{1}{t} \frac{\partial t}{\partial x} - \alpha T (1+\nu) \frac{1}{t} \frac{\partial t}{\partial y} &= 0.
\end{aligned} \tag{23}$$

Boundary Conditions. For a displacement boundary, we have

$$u = \bar{u}, \quad v = \bar{v}. \tag{24}$$

Stress Strain Relations. Stresses can be expressed as the function of displacement u , and v :

$$\begin{aligned}
\sigma_x &= \frac{N_x}{t} = \frac{E}{(1-\nu^2)} \left(\frac{\partial u}{\partial x} + \nu \frac{\partial v}{\partial y} \right) - \frac{E}{1-\nu} \alpha T \\
\sigma_y &= \frac{N_y}{t} = \frac{E}{(1-\nu^2)} \left(\frac{\partial v}{\partial y} + \nu \frac{\partial u}{\partial x} \right) - \frac{E}{1-\nu} \alpha T \\
\sigma_{xy} &= \frac{N_{xy}}{t} = \frac{E}{2(1+\nu)} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)
\end{aligned} \tag{25}$$

Finite Difference Formulation

Using central difference formula, the above equilibrium equations (23), displacement boundaries (24), and stress displacement relations (25) can be written in finite difference forms:

$$\begin{aligned}
 & 2u_{i,j} \left[1 + \frac{1-\nu}{2} B^2 \right] - u_{i+1,j} \left(1 + \frac{1}{4} t_{13} \right) - u_{i-1,j} \left(1 - \frac{1}{4} t_{13} \right) - u_{i,j+1} \left(1 + \frac{1}{4} t_{24} \right) \frac{1-\nu}{2} B^2 - \\
 & \quad u_{i,j-1} \left(1 - \frac{1}{4} t_{24} \right) \frac{1-\nu}{2} B^2 - (v_{i+1,j} - v_{i-1,j}) t_{24} \frac{1-\nu}{8} B - (v_{i,j+1} - v_{i,j-1}) t_{13} \frac{\nu}{4} B - \\
 & \quad [v_{i+1,j+1} + v_{i-1,j-1} - (v_{i-1,j+1} + v_{i+1,j-1})] \frac{1+\nu}{8} B + \frac{1+\nu}{2} \Delta x \alpha [T_{i+1,j} - T_{i-1,j} + T_{i,j} t_{13}] = 0 \\
 & 2v_{i,j} \left[1 + \frac{1-\nu}{2 B^2} \right] - v_{i,j+1} \left(1 + \frac{1}{4} t_{24} \right) - v_{i,j-1} \left(1 - \frac{1}{4} t_{24} \right) - v_{i+1,j} \left(1 + \frac{1}{4} t_{13} \right) \frac{1-\nu}{2 B^2} - \\
 & \quad v_{i-1,j} \left(1 - \frac{1}{4} t_{13} \right) \frac{1-\nu}{2 B^2} - (u_{i,j+1} - u_{i,j-1}) t_{13} \frac{1-\nu}{8} B - (u_{i+1,j} - u_{i-1,j}) t_{24} \frac{\nu}{4 B} - \\
 & \quad [u_{i+1,j+1} + u_{i-1,j-1} - (u_{i-1,j+1} + u_{i+1,j-1})] \frac{1+\nu}{8 B} + \frac{1+\nu}{2} \Delta y \alpha [T_{i,j+1} - T_{i,j-1} + T_{i,j} t_{24}] = 0 \quad , \quad (26)
 \end{aligned}$$

where

$$\begin{aligned}
 t_{13} &= \frac{1}{t_{i,j}} (t_{i+1,j} - t_{i-1,j}) \\
 t_{24} &= \frac{1}{t_{i,j}} (t_{i,j+1} - t_{i,j-1}) \\
 B &= \frac{\Delta x}{\Delta y} \quad (27)
 \end{aligned}$$

When SOR is used for iteration, Eq. (26) becomes:

$$\begin{aligned}
 u_{i,j}^{m+1} &= (1-\omega) u_{i,j}^m + \frac{\omega}{2 \left[1 + \frac{1-\nu}{2} B^2 \right]} \left\{ u_{i+1,j}^m \left(1 + \frac{1}{4} t_{13} \right) - u_{i-1,j}^{m+1} \left(1 - \frac{1}{4} t_{13} \right) - u_{i,j+1}^m \left(1 + \frac{1}{4} t_{24} \right) \frac{1-\nu}{2} B^2 - \right. \\
 & \quad u_{i,j-1}^{m+1} \left(1 - \frac{1}{4} t_{24} \right) \frac{1-\nu}{2} B^2 - (v_{i+1,j}^m - v_{i-1,j}^{m+1}) t_{24} \frac{1-\nu}{8} B - (v_{i,j+1}^m - v_{i,j-1}^{m+1}) t_{13} \frac{\nu}{4} B - \\
 & \quad \left. [v_{i+1,j+1}^m + v_{i-1,j-1}^{m+1} - (v_{i-1,j+1}^m + v_{i+1,j-1}^{m+1})] \frac{1+\nu}{8} B + \frac{1+\nu}{2} \Delta x \alpha [T_{i+1,j} - T_{i-1,j} + T_{i,j} t_{13}] \right\} \\
 v_{i,j}^{m+1} &= (1-\omega) v_{i,j}^m + \frac{\omega}{2 \left[1 + \frac{1-\nu}{2 B^2} \right]} \left\{ v_{i,j+1}^m \left(1 + \frac{1}{4} t_{24} \right) - v_{i,j-1}^{m+1} \left(1 - \frac{1}{4} t_{24} \right) - v_{i+1,j}^m \left(1 + \frac{1}{4} t_{13} \right) \frac{1-\nu}{2 B^2} - \right. \\
 & \quad v_{i-1,j}^{m+1} \left(1 - \frac{1}{4} t_{13} \right) \frac{1-\nu}{2 B^2} - (u_{i,j+1}^m - u_{i,j-1}^{m+1}) t_{13} \frac{1-\nu}{8} B - (u_{i+1,j}^m - u_{i-1,j}^{m+1}) t_{24} \frac{\nu}{4 B} - \\
 & \quad \left. [u_{i+1,j+1}^m + u_{i-1,j-1}^{m+1} - (u_{i-1,j+1}^m + u_{i+1,j-1}^{m+1})] \frac{1+\nu}{8 B} + \frac{1+\nu}{2} \Delta y \alpha [T_{i,j+1} - T_{i,j-1} + T_{i,j} t_{24}] \right\} \quad (28)
 \end{aligned}$$

The boundary conditions are:

$$\begin{aligned}
 u_{i,j}^{m+1} &= \bar{u} \\
 v_{i,j}^{m+1} &= \bar{v} \quad (29)
 \end{aligned}$$

After the convergence, one can calculate the stresses by the following stress- displacement relation:

$$\begin{aligned}
 (\sigma_x)_{i,j} &= \frac{E}{2(1-\nu^2)\Delta x} [(u_{i+1,j} - u_{i-1,j}) + \nu B(v_{i,j+1} - v_{i,j-1})] - \frac{E}{1-\nu} \alpha T_{i,j} \\
 (\sigma_y)_{i,j} &= \frac{E}{2(1-\nu^2)\Delta x} [B(v_{i,j+1} - v_{i,j-1}) + \nu(u_{i+1,j} - u_{i-1,j})] - \frac{E}{1-\nu} \alpha T_{i,j} \\
 (\sigma_{xy})_{i,j} &= \frac{E}{4(1+\nu)\Delta x} [B(u_{i,j+1} - u_{i,j-1}) + (v_{i+1,j} - v_{i-1,j})]
 \end{aligned} \tag{30}$$

BENCHMARKS

To make sure that the algorithm derived here is correct, benchmarks have been conducted and results are compared with those from the finite element code ANSYS [3].

Thermal Analysis of Uniform Thickness Filters. The thermal analysis of a filter subjected to an x-ray heat load has been performed and compared with the finite element code ANSYS. The difference with the ANSYS analysis is within 1%.

Thermal Analysis of Variable Thickness Filters. The second benchmark problem is a variable thickness filter subjected to an x-ray heat load with a temperature boundary condition. Comparison with the ANSYS result is also indistinguishable.

Radiation Effect. A similar problem has been analyzed taking into consideration the radiation effect. The difference is still very small.

Benchmarks for thermal stress were also conducted, and the results are compared with those from the finite element code ANSYS. The thermal stress analysis of a filter subjected to an x-ray heat load has been analyzed with the finite element code ANSYS. The result here obtained with the finite difference method is consistent with ANSYS.

APPLICATIONS

The behavior of a window can be improved by using variable thickness. One example is variable thickness windows for the SRI CAT undulator. It is assumed that the temperature of the block-holding filter and the window is kept at 300 K. The thermal conductivity of both graphite

and Be material is 2 W/cm K. The radiation effect is taken into account with the emissivity of 0.2 for both the Be and graphite materials.

Table 1 lists results of many cases with different sizes and thickness coefficients. If comparing two window cases that have identical conditions except that one is uniform and another has variable thickness, one can see a significant improvement with the variable thickness window. For example, compare two cases in table 1, case 7 with $a = 0.254$, $b = 0$, and $c = 0$ and case 8 with $a = 0.254$ and $b = c = 0.2$. The maximum temperature difference for case 8 (the variable thickness window) is 209 °K, while for case 7 (the uniform thickness window) it is 415 °K, more than a 100% thermal improvement.

Table 1. Absorbed Power and Maximum Temperature Difference for Undulator A Filter and Window

Case	Size of Window	Filter Thickness	Window Thickness Parameters*			Absorbed Power in Filter	Absorbed Power in Window	Max Temp. of Filter	Max Temp. of Window
	(mmXmm)	(μ m)	a	b	c	(W)	(W)	(K)	(K)
1	6 X 6	0	0.125	0.2	0.2		269.5		425.9
2	6 X 6	0	0.125	0.2	0.4		315.6		408.3
3	6 X 6	0	0.125	0.1	0.2		239.1		438.7
4	6 X 6	0	0.125	0.2	0.0		197.0		449.0
5	6 X 6	0	0.254	0.1	0.1		250.6		464.0
6	6 X 6	0	0.254	0.0	0.0		163.4		528.0
7	9 X 9	0	0.254	0.0	0.0		163.4		715.0
8	9 X 9	0	0.254	0.2	0.2		311.0		509.0
9	9 X 9	200	0.254	0.2	0.2	355.5	112.9	1396.0	377.1

*Window thickness is defined by:
 $h(x,y) = a + bx^2 + cy^2$

DISCUSSION

The algorithm for variable thickness plane heat transfer analysis that includes the radiation cooling effect and the temperature-dependent thermal conductivity, as well as the algorithm for variable thickness plane thermal deformation and stress analysis that includes the temperature-dependent thermal expansion coefficient have been implemented into a software package XRAP

[4]. The package can generate x-ray spectral and energy spectrum and their absorption in media. The finite difference formulation presented in this paper were used for variable thickness filter and window analysis.

ACKNOWLEDGMENT

The authors would like to express thanks to S. Picologlou editing this paper. This work is supported by the U.S. Department of Energy, BES-Materials Sciences, under contract no. W-31-109-ENG-38.

BIBLIOGRAPHY

- [1]. D. Young, The rate of convergence of an improved iterative method for solving the finite difference analog of the Dirichlet problem, Bull. Amer. Math. Soc. Abstract 56-4-322.
- [2]. S. P. Timoshenko and J. N. Goodier, Theory of Elasticity, 3rd Ed., McGraw-Hill Pub. Co., 1987.
- [3]. ANSYS User's Manual, Swanson Analysis Systems, Inc.
- [4]. Z. B. Wang, Tuncer M. Kuzay, Thome Grace, and Roger Dejus, X-Ray Optical Analyses with X-Ray Absorption Package (XRAP), to be presented in SRI '94.
- [5]. Z. B. Wang, X-Ray Optical Element Analyses and X-Ray Absorption Package (XRAP), M.Sc Thesis, May 1994, Computer Science Department, IIT.
- [6]. Z. B. Wang, U. Hahn, R. Dejus, and T. Kuzay, Filter and Window Behavior for APS Undulator and Wiggler, Presented at *8th National Conference on Synchrotron Radiation Instrumentation*, Aug. 23-26, 1993, Gaithersburg, MD and to appear in Nuclear Instr. Meth. in Phy. Res.
- [7]. Z. B. Wang and T Kuzay, Thermo-Mechanical Failure Phenomena in X-ray Windows and Filters and Comparison with Experiments, High Heat Flux Engineering II, Vol. 1997, SPIE- The International Society for Optical Engineering, July 1993.
- [8]. H. Schuh, Heat Transfer in Structures, Pergamon Press, 1965.
- [9]. V. Arpaci, Conduction Heat Transfer, Addison Wesley, 1966.
- [10]. Z. B. Wang, Ulrich Hahn, Roger Dejus, and Tuncer Kuzay, Filter and Window Behavior for APS Undulator and Wiggler, to appear in Nuclear Instr. Meth. Phys. Res.

Figure 1. Finite Difference Grid for Plane Heat Transfer Analysis

Figure 2. Boundary Conditions

Figure 3. Finite Difference Grid for Plane Stress Analysis

DATE

FILMED

9 / 27 / 94

END