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*Generalized Design Formulas
for Low Energy Electromagnetic Quads*

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GENERALIZED DESIGN FORMULAS FOR LOW ENERGY ELECTROMAGNETIC QUADS

by

D. J. Liska

ABSTRACT

This technical note is the result of the quadrupole magnet design efforts that went into the development of proposals for large high-powered linear accelerators such as the Accelerator for Production of Tritium (APT), Accelerator for Base Conversion (of Plutonium) (ABC), and Accelerator for Treatment of (radioactive) Waste (ATW). In all these applications it was necessary to develop designs for numerous (hundreds) of electromagnetic quadrupoles (EMQs). EMQs are required since long-term reliability, radiation damage potential, and large aperture dictate against the use of permanent magnet quadrupoles (PMQs) for these powerful machines. One object of the magnet design effort was to provide a quick, reliable, and easy means of converting raw physics requirements (magnetic impulse, focal length, and boretube aperture) into realistic electrical, cooling, facility interface, and mechanical specifications and configurations—in other words, to easily convert physics requirements to a reliable design that could be drawn on paper, shown to vendors, and presented to peer review committees as a well-developed and believable concept.

The empirical formulas that were derived have been gathered together in this technical note. They will be useful for other designers interested in an easy way of coming up with a rather complete mechanical as well as electrical and magnetic design for EMQs. Included are lab tests of designs derived from these formulas and comparisons with other real EMQ designs. These demonstrate the good accuracy of the empirical formulas. Finally, an appendix of derivations is included from which further development of this “cookbook” can take place if the designer seeks to more finely define a concept.

INTRODUCTION

The generalized design formulas for electromagnetic quadrupoles presented in this report were derived to suit the needs of a conceptual accelerator designer and are applicable for the low energy APT/ATW type machines presently envisioned by AOT Division. A 2-conductor stack winding for the field coil is assumed and only two conductor sizes have been selected; one for minimal size quads that could be used inside drift tube bodies at moderately high frequencies (350-400 MHz) or in a tightly packed funnel assembly, and the other a larger, robust conductor for exterior quads nested between either CCL modules or DTL modules which are bridge coupled together like a CCL (the BCDTL). The most applicable energy range in which these formulas apply is 7-1000 MeV for proton type machines like APT/ATW. The drift tube quads are normally "maxed-out" designs in that conductor current densities are "uncomfortably" high. For the minimal conductor, we have allowed current densities up to 22 kA/square inch, realizing that the designer wants the smallest quad envelope he can achieve in order to keep the linac frequency as high as practical, and yet requires long term reliability. For the larger conductor we allow only 13.4 kA/square inch which is the normal current density limit for high reliability directly water cooled field coils.

We realize that there are many combinations of parameter choices which will produce a workable quad design. These formulas have been found to be successful in carrying out typical conceptual design work on APT/ATW type linacs which are 200 mA, 1000 MeV CW machines operating at 350/700 MHz.

At the end of the formula list two examples are worked out. One is for a "maxed-out" design for a high powered drift tube EMQ, and the other for normally powered EMQ's to operate in narrow spaces between bridge-coupled DTL or CCL tanks at 20-1000 MeV.

Following the examples is Appendix I which presents an analysis for super-high current density conductors running at extreme current densities. These current densities greatly exceed those of the above example for a drift tube quad which holds the limit to "a mere" 25 kA/square inch, but there is in fact a design precedent for septum magnets and transition region magnets at LAMPF which operates the field coils at current densities of 70 kA/square inch. It is conceivable that field coils with 100 kA/square inch may be feasible with reasonable reliability. In rare cases such extremes must be taken. However, such magnets are certainly exceptions and the analysis in this appendix is intended only to show how to analyze water-cooled conductors pushed to these rather extraordinary levels. It is not recommended for positively long-life, reliable designs. Appendix II contains examples of real quad designs and actual test data to demonstrate the accuracy of the "cookbook" formulas in predicting actual designs.

A third appendix is attached which includes basic derivations of the cookbook formulas.

Assumptions

(Implicit in the Formulas - see attached figure for geometrical considerations):

- Given (PARMILA, SUPERFISH, etc.):

$$\begin{aligned}\langle GL \rangle &= \text{kiloGauss (kG)} \\ r &= \text{cm}\end{aligned}$$

- Allowable Field Coil Currents:

$$\begin{aligned}I_{\text{cmax}} (0.1443" \text{ conductor}) &= 225 \text{ Amps} \\ &= 22,000 \text{ A/in}^2 \\ I_{\text{cmax}} (0.2576" \text{ conductor}) &= 650 \text{ Amps} \\ &= 13,400 \text{ A/in}^2\end{aligned}$$

- Allowable Flux Densities (for linear performance):

$$B_{\text{PT}} = 10 \text{ kG max (pole tip)}$$

$$B_{\text{stl}} = 5 \text{ kG (yoke)}$$

- Conductor Sizes (Directly-Cooled Anaconda Standard sizes):

$$\begin{aligned}W_c (\text{DTL}) &= 0.1443" \text{ square,} \\ A_{\text{cu}} &= 0.01024 \text{ in}^2 \\ A_w &= 0.01005 \text{ in}^2 \\ K &= 0.0525 \text{ (Anaconda design factor)}\end{aligned}$$

$$\begin{aligned}W_c (\text{BCDTL, CCL}) &= 0.2576" \text{ square,} \\ A_{\text{cu}} &= 0.04852 \text{ in}^2 \\ A_w &= 0.01606 \text{ in}^2 \\ K &= 0.0354 \text{ (Anaconda design factor)}\end{aligned}$$

- Quad Flux Efficiency (loss in bore flux due to leakage flux):

$$\eta = 0.9$$

Figure 1. Geometrical considerations.

Notation:

- L_{stl} = length of yoke steel = cm
 L_{eff} = effective field length = cm
 $\langle GL \rangle$ = magnetic impulse = (kG/cm) $\times$ cm = kG
 B_{PT} = pole tip field = kG
Quad $\varnothing$ = quad diameter (yoke OD) = cm
 r = pole tip radius = cm
 N = no. of turns per pole = integer number
 W_c = conductor size (OD) = inches (square or dia.)
 I_c = coil current = amps
 L_c = coil length per half coil assembly (two poles) = inches
 F = feed conductor length = inches
 t = yoke thickness = cm
 P = total quad power = watts
 R_c = conductor resistance/ft = ohms/ft
 V = total quad voltage = volts
 Q = total quad cooling = gallons/min (gpm)
 A_w = conductor cooling area (ID) = in²
 A_{cu} = conductor copper area = in²
 U = cooling flow rate = feet/sec.
 ΔP = total quad pressure drop (two split coils in parallel) = psi
 ΔT = coolant temperature rise = °C
 H = stack height of coil = cm
 W_t = quad weight = lb.
 L_p = pole tip length = cm
 i = coil current density = A/in²
 η = quad flux efficiency
 T = flux in Tesla (1T = 10 kG)

Design Formulas:

1) Quad Length:

$$L_{stl} = r \left(\frac{\langle GL \rangle}{B_{PT}} - 0.86 \right)$$

$$L_{eff} = L_{stl} + 0.86 r$$

2) Quad Diameter:

$$\begin{aligned} \text{Quad } \emptyset &= 4.3 r + 2.54 N (W_c + 0.06) + 0.3 + 0.48 r B_{PT} \\ (\text{Assumes } B_{stl} &= 5 \text{ kG to limit saturation in the steel}) \end{aligned}$$

3) No. Turns Per Pole:

$$N = 442.1 \left(\frac{B_{PT} r}{I_c} \right)$$

$$\text{Where } B_{PT} = \frac{r \langle GL \rangle}{L_{eff}}$$

$$\text{Therefore: } N = 442.1 \frac{r^2}{I_c} \left(\frac{\langle GL \rangle}{L_{stl} + 0.86 r} \right)$$

Where $\langle GL \rangle$, r are PARMILA requirements and (I_c, L_{stl}) are engineering requirements.

4) Conductor Length Per Coil Assembly (2 Poles):

$$\begin{aligned} L_c &= 4N \left(\frac{L_{stl}}{2.54} - 1.5W_c - 0.09 \right) + 4\pi \left[0.5N(0.846r + 0.03 + 0.5W_c) \right. \\ &\quad \left. + \left(\frac{N}{2} - 1 \right) (W_c + 0.06) \right] + F \end{aligned}$$

Note: For pole tips which subtend angle β in radians (or a section which can be described by some average angle β), where β is less than or equal to 90 degrees ($\pi/2$), replace π with 2β . The expression now becomes:

$$L_c = 4N \left(\frac{L_{stl}}{2.54} - 1.5W_c - 0.09 \right) + 8\beta \left[0.5N(0.846r + 0.03 + 0.5W_c) + \left(\frac{N}{2} - 1 \right) (W_c + 0.06) \right] + F$$

When $\beta = \frac{\pi}{2}$ radians the two expressions are identical.

5) Stack Height:

$$H = 1.27 N (W_c + 0.06)$$

6) Clearance:

$$\begin{aligned} \text{Clearance} &= 0.15 \text{ cm on radius} \\ &= 0.30 \text{ cm on Quad } \emptyset \end{aligned}$$

7) Yoke Thickness:

$$\begin{aligned} t &= 0.24 r B_{PT} \text{ (on radius)} \\ &\text{(Assumes } B_{stl} = 5KG) \end{aligned}$$

8) Quad Power:

$$P = I_c^2 R_c \frac{L_c}{6}$$

$$\begin{aligned} \text{where: } R_c &= 8 \times 10^{-4} \Omega/\text{ft. for } W_c = 0.1443" \text{ conductor} \\ &= 1.72 \times 10^{-4} \Omega/\text{ft. for } W_c = 0.2576" \text{ conductor} \\ &\text{(both @ 20 C)} \end{aligned}$$

Temperature Correction:

$$R_c = R_{c@20C} \left[1 + 0.0041 \left(20 + \frac{\Delta T}{2} \right) \right]$$

9) Quad Voltage:

$$V = I_c R_c \left(\frac{L_c}{6} \right)$$

10) Total Coolant Flow (Both 2-pole Assemblies In Parallel):

$$Q = 3.788 \times 10^{-3} \left(\frac{P}{\Delta T} \right)$$

$$u = 0.16038 \times \left(\frac{Q}{A_{cu}} \right)$$

Where: $A_{cu} = 0.01005 \text{ in}^2$ for $W_c = 0.1443''$ square
 $= 0.01606 \text{ in}^2$ for $W_c = 0.2576''$ square

Note: Keep $\Delta T \leq 30 \text{ C}$ & $u \leq 10$ feet per sec.

11) Pressure Drop:

$$\Delta P = K u^{1.79} \left(\frac{L_c}{12} \right)$$

where: $K = 0.0525$ for $W_c = 0.1443''$ square
 $= 0.0354$ for $W_c = 0.2576''$ square

12) Temperature Rise:

$$\Delta T = 1.0124 \times 10^{-4} I_c^2 R_c \left(\frac{L_c}{A_w u} \right)$$

$$= 6.3123 \times 10^{-4} I_c^2 R_c \left(\frac{L_c}{Q} \right)$$

Note: Keep $\Delta T \leq 30^\circ\text{C}$ & $u \leq 10$ feet per sec.

13) Quad Weight:

$$Wt = \pi \text{ Const}[(\text{Quad } \emptyset)^2 - 4r^2] \rho \left(\frac{L_{stl}}{4} \right)$$

Where: Const accounts for reduced coil density, $(0.67 P_{stl})$,
and cutout area around $2 \times r \times \emptyset$, $(1.38 \pi r^2)$.

Therefore: Const = 0.936

rho for steel = 0.0173 lb/cm^3

Therefore, $Wt = 0.0127 [(Quad \emptyset)^2 - 4 r^2] L_{stl}$ (pounds)

EXAMPLES

Example #1: "Maxed-out" DTL quad for 350 MHz DTL:

$$\text{Given: } \left. \begin{array}{l} \langle GL \rangle = 32.45 \text{ kG} \\ \text{Boretube } \varnothing = 1.5 \text{ cm} \end{array} \right\} \text{From PARMILA}$$

$$1) \quad L_{\text{eff}} = 5.72 + 0.86 (0.9) = 6.494 \text{ cm}$$

$$2) \quad B_{\text{PT}} = Gr = \frac{\langle GL \rangle}{L_{\text{eff}}} \times r = \frac{32.45}{6.494} \times 0.9 = 4.5 \text{ kG}$$

$$3) \quad N = 442.1 \frac{B_{\text{PT}} r}{I_c} \quad \text{Let } I_c = I_{\text{max}} = 225 \text{ A.}$$

$$= 442.1 \frac{(4.5)(0.9)}{225} = 7.96 (\text{Say } N = 8 \text{ turns})$$

$$\text{Actual } I_c = 442.1 \frac{B_{\text{PT}} r}{N}$$

$$= 442.1 \frac{(4.5)(0.9)}{8} = 223.8 (\text{say } 224 \text{ A})^*$$

$$4) \quad L_c = 4(8) \left[\frac{5.72}{2.54} - 1.5(0.1443) - 0.09 \right] + 4\pi \left\{ 0.5(8) [0.846(0.9) + 0.03 \right. \\ \left. + 0.5(0.1443)] + \left(\frac{8}{2} - 1 \right) (0.1443 + 0.06) \right\} + 48" = 161.4 \text{ inches}$$

$$5) \quad H = 1.27 (8) (0.1443 + 0.06) = 2.08 \text{ cm}$$

$$6) \quad t = 0.24 (0.9)(4.5) = 0.972 \text{ cm}$$

$$7) \quad P = \frac{1}{6} (224)^2 (8 \times 10^{-4}) (161.4) = 1.080 \text{ kW}^*$$

$$8) \quad V = \frac{1}{6} (224)(8 \times 10^{-4})(161.4) = 4.82 \text{ volts*}$$

$$9) \quad Q = 3.788 \times 10^{-3} \left(\frac{1080}{30} \right) = 0.14 \text{ gpm} \left. \vphantom{Q} \right\} \text{For } \Delta T = 30^\circ \text{C}$$

$$U = 0.160 \left(\frac{0.14}{0.01005} \right) = 2.2 \text{ fps}$$

$$10) \quad \Delta P = \frac{1}{12} (0.0525)(2.2)^{1.79} (161.4) = 2.9 \text{ psi}$$

* (Power supply required: 10 volt, 250 Amp; 2.5 kW)

$$11) \quad \Delta T = 1.0124 \times 10^{-4} \frac{(224)^2 (8 \times 10^{-4})(161.4)}{(0.01005)(2.2)} = \frac{29.7^\circ \text{C}}{(\text{checks})}$$

$$= 6.3123 \times 10^{-4} \frac{(224)^2 (8 \times 10^{-4})(161.4)}{0.14} = \frac{29.2^\circ \text{C}}{(\text{checks})}$$

12) Quad outer diameter:

$$\text{Quad } \varnothing = 4.3(0.9) + 2.54(8)(0.1443 + 0.06) + 0.3$$

$$+ 0.48(6.9)(4.5) = 10.3 \text{ cm}$$

13) Account for ΔT in copper conductor:

$$R_c = R_{c20C} (1 + 0.0041 T_{\text{avg}}), \quad T_{\text{avg}} = 20 + 30/2 = 35^\circ \text{C}$$

$$= 1.1435 R_{c20C}$$

This results in the following corrections to the parameters:

$$P = 1080 \times 1.1435 = 1.23 \text{ kW}$$

$$V = 4.82 \times 1.1435 = 5.51 \text{ volts}$$

$$Q = 0.14 \times 1.1435 = 0.16 \text{ gpm}$$

$$U = 2.2 \times 1.1435 = 2.5 \text{ fps}$$

$$\Delta P = 0.0525 \times 2.5^{1.79} \times 161.4/12 = 3.6 \text{ psi } (\Delta T \text{ remains at } 30^\circ \text{C})$$

$$Wt = 0.0127 (10.3^2 - 4 \times 0.9^2) \times 5.72 = 7.5 \text{ pounds}$$

Example #2: Do complete design of 20 MeV BCDTL quad:

Given: $\langle GL \rangle = 34.31 \text{ kG}$

$$L_{\text{stl}} = 3.25''$$

Assume: $r = 2.5 \text{ cm}$ $I_c = 650 \text{ A} = I_{\text{cmax}}$

$$W_c = 0.2576''$$

1) $L_{\text{eff}} = (3.25)(2.54) + 0.86(2.5)$

$$= 10.405 \text{ cm}$$

$$B_{\text{PT}} = \frac{r \langle GL \rangle}{L_{\text{eff}}} = \frac{(2.5)(34.31)}{10.405} = 8.24 \text{ kG}$$

$$N = 442.1 \times 8.24 \times 2.5 / 650 = 14.01 \text{ (14 turns per pole)}$$

2) $\text{Quad } \emptyset = (4.3)(2.5) + 2.54(14)(0.2576 + 0.06) + 0.3 + 0.48(2.5)(8.24)$

$$= 32.23 \text{ cm (12.69'')}$$

4) $L_c = 4(14)[(3.25 - 1.5)(0.2576) - 0.09] + 4\pi \left\{ 0.5(14)[0.846(2.5) + 0.03 \right.$

$$\left. + 0.5(0.2576) + \left(\frac{14}{2} - 1 \right)(0.2576 + 0.06) \right\} + 24'' = 403 \text{ inches}$$

5) $H = 1.27(14)(0.2576 + 0.06) = 5.65 \text{ cm}$

7) $t = 0.24 (2.5)(8.24) = 4.94 \text{ cm}$

8) $P = \frac{1}{6}(650)^2 (1.72 \times 10^{-4})(403) = 4881 \text{ watts}$

9) $V = \frac{1}{6}(650)(1.72 \times 10^{-4})(403) = 7.51 \text{ volts}$

- 10) $Q = (0.16038)^{-1} A_{cu} \mathbf{U}$
 $Q = 6.235 (0.01606)(8) = 0.8 \text{ gpm}$
 (Assumes $\mathbf{U} = 8 \text{ fps}$)
- 11) $\Delta P = \frac{1}{12} (0.0354)(8)^{1.79} (403) = 49.2 \text{ psi}$
- 12) $\Delta T = 1.0124 \times 10^{-4} \frac{(650)^2 (1.72 \times 10^{-4})(403)}{(0.01606)(8)} = 23.1^\circ \text{C}$
- 13) $W_t = 0.0127 [32.23^2 - 4(2.5^2)] (3.25 \times 2.54) = 106.3 \text{ lb}$

Note: This solution should be iterated to allow for increased conductor resistance and power dissipation due to elevated average conductor temperature.

APPENDIX I

This appendix deals with the extreme limits of operation for compact electromagnetic quadrupole field coils, including pole tip performance and maximum focusing impulse $\langle GL \rangle$ generated. The equations derived below give a method by which the extreme performance limits of a general EMQ design can be estimated. NOTE: The formulas presented are without bias, that is, they are valid for all current levels within the linear range of the steel (that is, below 10 kG). However, they are specifically derived in a form which makes them useful for analyzing peak performance for a field coil and quadrupole. The symbols used in this appendix are the same as those used in the notation column above:

1) Power Analysis:

$$P \text{ (watts)} = 823.14 A_w \mathbf{U} \Delta T$$

2) Field Coil Current:

$$I_c = 3.526 \times 10^4 \left[A_w A_{cu} \left(\frac{\mathbf{U}}{L_c} \right) \left(\frac{1}{\Delta T} + 0.002615 \right) \right]^{\frac{1}{2}}$$

3) Magnetic Response:

$$B_{PT} = 2.513 \times 10^{-3} \eta N \left(\frac{I_c}{r} \right)$$

$$G = 2.513 \times 10^{-3} \eta N \left(\frac{I_c}{r^2} \right)$$

$$L_{eff} = L_{stl} + 0.86 r$$

$$\langle GL \rangle = 88.60 \eta N \left[A_w A_{cu} \left(\frac{\mathbf{U}}{L_c} \right) \left(\frac{1}{\Delta T} + 0.002615 \right) \right]^{\frac{1}{2}} \frac{L_{eff}}{r^2}$$

EXAMPLE

Consider the same EMQ as analyzed in Example #1 above but with a different conductor size:

Assume a conductor with the following dimensions:

$$\left. \begin{array}{l} A_w = 3.318 \times 10^{-3} \text{ in}^2 \\ A_{cu} = 8.954 \times 10^{-3} \text{ in}^2 \end{array} \right\} \begin{array}{l} \text{Round Conductor:} \\ \text{OD} = 0.125" \\ \text{ID} = 0.065" \end{array}$$

$$L_c = 161.4 \text{ in}$$

$$N = 8$$

$$r = 0.9 \text{ cm}$$

$$L_{stl} = 5.72 \text{ cm}$$

Assume also that we operate this magnet at “maximum” ratings as follows:

$$v = 20 \text{ fps}$$

$$\Delta T = 60 \text{ C}$$

$$B_{PT} \leq 10 \text{ kG (in order to stay linear)}$$

$$\eta = 0.9$$

1) The power in the coil is:

$$\begin{aligned} P &= 823.14 \times 3.318 \times 10^{-3} \times 20 \times 60 \\ &= 3.28 \text{ kW} \end{aligned}$$

2) The current that can be driven through the coil under these conditions is:

$$\begin{aligned} I_{cmax} &= 3.526 \times 10^4 \left[\frac{3.318 \times 8.954 \times 10^{-6} \times 20}{161.4(1/60 + 0.002615)} \right]^{1/2} \\ &= 487.2 \text{ Amps (54.4 kA / in}^2) \end{aligned}$$

3) The steel response is as follows:

$$B_{PT} = \frac{2.513 \times 10^{-3} \times 0.9 \times 8 \times 487.2}{0.9}$$

$$= 9.79 \text{ kG}$$

$$G = \frac{9.79}{0.9} = 10.88 \text{ kG / cm}$$

$$L_{eff} = 5.72 + 0.86(0.9) = 6.494 \text{ kG}$$

$$\langle GL \rangle = 10.88 \times 6.494 = 70.7 \text{ kG}$$

or, using the expression for $\langle GL \rangle$ directly:

$$\begin{aligned} \langle GL \rangle &= 88.60(0.9)(8) \left[\frac{3.318 \times 8.954 \times 10^{-6} \times 20}{161.4(1/60 + 0.002615)} \right]^{1/2} \left(\frac{6.494}{0.9^2} \right) \\ &= 70.7 \text{ kG (checks with above answer)} \end{aligned}$$

Comparison of this answer with the “Maxed-out” DTL quad given in Example #1 of the previous section ($\langle GL \rangle = 32.54 \text{ kG @ } I_c = 224 \text{ Amps}$) shows that this quad is capable of a 117% increase in focusing impulse if the field coil is driven to extremely high current densities. While 54.4 kA/in^2 is high, there are several magnets at LAMPF which operate at even higher densities, up to 70.5 kA/in^2 .

APPENDIX II

ACCURACY CHECK OF MAGNET DESIGN COOKBOOK AGAINST APT DTL EMQ AND FMIT 25 CM QUADRUPOLE

INTRODUCTION

Tech Note AT-4: 92:03 (Generalized Design Formulas for Low Energy Electromagnetic Quads) has proved useful in conceptual and preliminary design studies as well as in the fabrication of the prototype DTL EMQ for APT. The accuracy of the formulas given in the technical note can be checked against actual hardware, both the APT DTL EMQ and a much earlier magnet, the FMIT 25 cm large bore EMQs, which were used in the FMIT prototype and are now at LAMPF. The FMIT quad was designed without the cookbook, but the same theoretical approach was applied and the mechanical design was perfected on the drawing board.

ACCURACY CHECKS

1. The APT DTL EMQ

This quad was designed entirely using the referenced tech note. The resulting design is shown in the attached photo. It was subsequently tested in the AT-3 magnet lab and also checked with the 3-D computer code TOSCA by Dave Barlow. The mechanical dimensions, (i.e. quad effective length, diameter, and yoke thickness) were all determined by the cookbook. This means that the $\langle GL \rangle$ product and boretube clear diameter were stipulated by the beam dynamicists from PARMILA and the mechanical, magnetic, and electrical design took over from there. The PARMILA specs were as follows (see AT-4: 92-03, example #1):

$$\begin{aligned}\langle GL \rangle &= 32.45 \text{ kG} \\ \text{Boretube diam.} &= 1.5 \text{ cm}\end{aligned}$$

With this as the starting point, the actual pole-tip diameter was chosen as 1.8 cm to allow for boretube wall thickness and clearance. The $\langle GL \rangle$ product as calculated from the cookbook is as follows:

$$B_{PT} = \frac{NI_c}{442.1 r}$$

$$G = \frac{B_{PT}}{r}$$

$$L = L_{stl} + 0.86 r$$

$$\text{so } \langle GL \rangle = NI_c \frac{(L_{stl} + 0.86 r)}{442.1 r^2}$$

Where $N = 6$ turns per pole,

$r = 0.9$ cm, and

$L_{stl} = 5.72$ cm (determined from drift tube allowable dimensions).

For example, when $I_c = 200$ A the computed $\langle GL \rangle$ from the cookbook is 21.76 kG whereas from Dave Barlow's memo (AT-3:92-538, which used the 3 D code TOSCA), the calculated $\langle GL \rangle = 23.69$ kG. A comparison between TOSCA and the cookbook is given in Table 1:

TABLE 1

Coil Current (A)	TOSCA $\langle GL \rangle$, (kG)	Cookbook $\langle GL \rangle$, (kG)	Error (%)
0	0	0	0
100	13.00	10.88	-16.3
200	23.69	21.76	-8.1
300	32.00	32.64	+2.0
400	38.30	43.52	+13.6

The effects of saturation are obvious for currents above 300 A. The cookbook is strictly linear so it is not valid when the steel saturates. However, up to the saturation point the agreement is good with the cookbook giving a conservative value for $\langle GL \rangle$ with an error of +2 to -16 %.

When the actual prototype quad was tested in the MAPLAB (see AT-3:93-222), mechanical tolerances and fabrication nonuniformities entered into the measurements but agreement between MAPLAB results, TOSCA and the cookbook were excellent. These results are given in the attached figure. They show that the cookbook can be used with confidence for EMQ's of the APT DTL size and $\langle GL \rangle$ requirements.

The APT DTL and FMIT 25 cm EMQs are both "short" quads with pole-tip length-to-aperture ratios of 3.2 and 1.5 respectively. The application of the cookbook to the 25-cm quad will be discussed next.

"0.9 cm rpt EMQ $\langle GL \rangle = f(I_c)$ DATA" $\langle GL \rangle$ Effective vs $\langle GL \rangle$ Measured vs TOSCA.

2. The FMIT 25 cm EMQ

A fabrication drawing of the 25-cm EMQ as produced for the FMIT prototype program is attached. Three of these quads were built and installed in the FMIT HEBT. They were tested by Robbie Davis using a search coil of large diameter and shimmed with eccentric pole-face discs to reduce the duodecapole field harmonic from 1.42% to 0.08%.

The FMIT 25-cm EMQ was designed and built in 1982. Many of the features described in our cookbook today are reflected in the design, but the cookbook *per se* was not used at that time. An interesting comparison of mechanical parameters can be generated with the cookbook, and this is done below. The magnetic test results are also compared with cookbook predictions.

A. Mechanical Comparison with Cookbook Predictions

1. Effective Length

Since the 25 cm quad effective length was predicted on the same basis as the cookbook, the analytical numbers agree exactly. However, when measurements were made, the effective length was found to be a function of current as shown in Table 2:

EMIT 25 CM QUADRUPOLE. WT = 3200 LB

TABLE 2

Coil Current (A)	L_{eff} (design)	L_{eff} (cookbook)	L_{eff} (measured)
200	25 cm	25 cm	27.22 cm
400	25 cm	25 cm	26.64 cm

The errors are -8% and -6% respectively although the predicted effective lengths agree in both cases.

2. Quad Diameter

The quad diameter as derived by the cookbook formula is as follows

$$\text{Quad } \emptyset = 4.3 r + 2.54 N(W_c + 0.06) + 0.3 + 0.48 r B_{PT}$$

Where: $r = 6.54$ cm,

$N = 30$, and

$W_c = 0.53$ inch square.

$$B_{PT} = \frac{NI_c}{442.1 r} \text{ (from cookbook)}$$

For $I_c = 750$ A (rating of conductor), $B_{PT} = 7.78$ kG, and the Quad $\emptyset = 97.81$ cm (38.51 in.). This agrees to within +8% of the mechanical design drawing for the FMIT quad.

3. Quad Weight:

The cookbook expression for the overall weight of the quad accounts for the steel, the gap, the windings, and the pole-tip extenders. In other words, the result is a design which is, in some ways, somewhat different from the 25-cm quad. The cookbook expression is

$$\text{Quad weight} = 0.0127 (\emptyset^2 - 4 r^2) L_{stl} , \text{ (lb).}$$

For $\emptyset = 97.81$ cm and $L_{stl} = \text{yoke length} = 29.94$ cm, the predicted weight is 3572 lb. The estimated weight of the actual magnet is 3200 lb for an error of within +12%. Actually, the mineral-insulated conductor used in the actual magnet is proportionally lighter weight than is implicit in the coefficient of the quad weight expression, which would reduce the predicted weight and the error to about +9%. Even without this, however, the agreement is good.

4. Length of conductor

The measured length of the conductor in the design for the FMIT quad is 127.6 ft per pole tip. This is a 2-stack coil as is assumed in the cookbook expression for coil length. The cookbook expression for conductor length for 2 poles is

$$L_c = 4N \left(\frac{L_{stl}}{2.54} - 1.5 W_c - 0.09 \right) + 4\pi \left[0.5 N (0.846 r + 0.03 + 0.5 W_c) + \left(\frac{N}{2} - 1 \right) (W_c + 0.06) \right] + F .$$

For one pole the length is

$$L_c = 4N \left(\frac{L_{stl}}{2.54} - 1.5 W_c - 0.09 \right) + 2\pi \left[0.5 N (0.846 r + 0.03 + 0.5 W_c) + \left(\frac{N}{2} - 1 \right) (W_c + 0.06) \right] + F ,$$

Where F is the length of feeder conductors. For the 25-cm FMIT EMQ, the length of 0.53 in. square HCMC conductor per pole was listed in the bill of materials as 135 ft including a total feeder length of 23 ft. Using the cookbook expression above, we get 127.6 ft for an error of -6%.

Note: The cookbook expression for conductor length assumes a 2-conductor stack on a 90-degree pole sweep. This means that any pole which is more truncated than a 90-degree sweep will consume less conductor, less power, less voltage, etc. However, the winding will be harder to build. The result is that the cookbook gives a conservative result. Any magnet designed in any other way will require less, not more, conductor. This feature of the cookbook is intended to prevent any underestimate in electrical parameters during conceptual design studies.

5. Stack Height

The stack height formula is

$$H = 1.27 N (W_c + 0.06) .$$

For $N = 30$ and $W_c = 0.53$ in., the stack height is 8.85 in. The 25-cm FMIT quad allowed 8.63 in. for stack height in 1982. The agreement with today's estimate is within +3%.

6. Yoke Thickness

For yoke thickness, the cookbook formula is

$$t = 0.24 r B_{PT} .$$

Since $B_{PT} = 7.78$ kG, the yoke thickness is predicted to be 4.8 in. The actual design used a hexagonal yoke, but the thickest section at the point of maximum flux density was 4.71 in. The agreement with the cookbook is therefore within +2%.

B. Conclusions

Agreement between cookbook and accurate analysis, measurements, and mechanical design has been shown to be good for both the APT DTL EMQ and the FMIT 25 cm EMQ. All parameters compared agreed to within a range of +12 to -16 %. In summary, for the two extreme cases compared, we can say the following.

1. Magnetic Measurements

$\langle GL \rangle$ product and effective length, when measured within saturation or only slightly beyond the onset of saturation, agree with cookbook predictions to within +2 to -16 %.

2. Mechanical Parameters

Quad diameter, weight, conductor length, stack height, and yoke thickness agree to within +12 to -6 %. The tendency of the cookbook is to give slightly conservative predictions in general.

It would be worthwhile to gather test data, operational results, and mechanical specs on a wider variety of magnets to get a more valid comparison. At this point, however, I conclude that the cookbook as it stands is an excellent guide to conceptual design and can be used with confidence within this context.

APPENDIX III

Derivation of Generalized Quadrupole Formulas

Given (from PARMILA):

$$G, L_{\text{eff}}(\langle GL \rangle)$$

Assumptions:

$$i \text{ (DTL quad)} = 22,000 \text{ A/in}^2$$

$$i \text{ (CCL quads)} = 13,400 \text{ A/in}^2$$

Conductors:

$$\text{DTL quads} = 0.1443'' \text{ square}$$

$$\text{CCL quads} = 0.2575'' \text{ square}$$

$$B_{\text{PT}} = 10 \text{ kG max}$$

$$\eta = 0.9$$

2-layer stack coil

1) Find L_{stl} :

$\langle GL \rangle = \text{PARMILA specification}$

$$G = \frac{B_{\text{PT}}}{r}$$

$$\therefore \frac{B_{\text{PT}}}{r} \times L_{\text{eff}} = \langle GL \rangle = \frac{B_{\text{PT}}}{r} (L_{\text{stl}} + 0.86 r)$$

$$\therefore L_{\text{stl}} = r \left(\frac{\langle GL \rangle}{B_{\text{PT}}} - 0.86 \right)$$

2) Derivation of general expression for ampere-turns NI_c :

$$\text{For a quadrupole } NI_c = \frac{Gr^2}{2\mu_0}, \text{ theoretical}$$

$$= \frac{Gr^2}{2\mu_0\eta}, \text{ practical}$$

where η = “efficiency” allowance for flux leakage

$$\begin{aligned} NI_c &= \frac{B_{PT}(\text{kG}) r(\text{cm})}{10\left(\frac{\text{kG}}{\text{T}}\right) 10^2\left(\frac{\text{cm}}{\text{m}}\right) (2 \times 4\pi \times 10^{-7}) \eta} \\ &= \frac{10^7}{8\pi \times 10^3 \eta} B_{PT}(\text{kG}) r(\text{cm}) \\ &= 397.89 \frac{B_{PT}}{\eta}(\text{kG}) r(\text{cm}) \end{aligned}$$

For $\eta = 0.9$

$$NI_c = 442.1 B_{PT} r$$

$$\therefore \boxed{N = 442.1 \left(\frac{B_{PT} r}{I_c} \right)} \quad \text{turns per pole}$$

Note: For N = minimum, $I_c = I_{\max}$ = maximum allowable field coil current.

N for a typical quadrupole:

$$D = 2(r + 1.15r + L_{stack} + CL + t)$$

$$= 2(2.15r + L_{stack} + CL + t)$$

For 0.1443" conductor, $A_{cu} = 0.01024 \text{ in}^2$
 $I_{max} = 22,000 \times 0.01024 = 225 \text{ A max}$

For 0.2576" conductor, $A_{cu} = 0.04852 \text{ in}^2$
 $I_{max} = 13,400 \times 0.04852 = 650 \text{ A max}$

$$N = 442.1 \frac{B_{PT}(\text{kG}) r(\text{cm})}{I_{max}}$$

$$\langle GL \rangle = \frac{B_{PT}}{r} L_{eff}$$

$$B_{PT} = \frac{r \langle GL \rangle}{L_{eff}}$$

$$\text{Therefore, } N = 442.1 \frac{r^2(\text{cm}) \langle GL \rangle (\text{kG})}{I_{max}(\text{A}) L_{eff}(\text{cm})}$$

3) Stack height H:

Stack height

$$= \frac{N}{2} \times W_c + N \times 0.03$$

$$= N \left(\frac{W_c}{2} + 0.03 \right)$$

$$= \frac{N}{2} (W_c + 0.06)(2.54)$$

$$\therefore \text{Stack Height (cm)} = 1.27N[W_c(\text{in.}) + 0.06]$$

4) Clearance: Let CL = 1.5 mm = 0.15 cm

5) Yoke thickness t:

5) Yoke thickness t (continued):

$$b + c = a \cos 30^\circ + 1.15 r \cos \alpha = 2.15r$$

$$a^2 = 2.15^2 r^2 + 1.15^2 r^2 - 2(2.15r)(1.15r) \cos \alpha$$

$$\therefore a = \frac{2.15r}{\cos 30^\circ} - \frac{1.15r \cos \alpha}{\cos 30^\circ} = 2.4826r - 1.3279r \cos \alpha$$

$$= r \sqrt{2.15^2 + 1.15^2 - 2(2.15)(1.15) \cos \alpha}$$

$$= r \sqrt{5.945 - 4.945 \cos \alpha}$$

$$r^2 (2.4826 - 1.3279 \cos \alpha)^2 = r^2 (5.945 - 4.945 \cos \alpha)$$

$$\alpha = 39.17^\circ$$

$$\therefore \frac{S}{2} = \alpha(1.15r) = 0.7862r$$

So the flux gathered by 1/2 of the pole tip is:

$$k \times BA = k B_{PT}(\text{kG}) \times 0.7862 r(\text{cm}) \times \ell_{stl}(\text{cm})$$

5) Yoke thickness t (continued):

$$\sum \text{Flux} = kBA = kB_{PT}(\text{kG}) \times 0.7862 r(\text{cm}) \ell_{st}(\text{cm})$$

where k = fringe field allowance

let $k = 1.5 \Leftarrow$ Assumption

$$\begin{aligned} \therefore \sum \text{Flux} &= (1.5)(0.7862)B_{PT}(\text{kG}) r(\text{cm}) \ell_{st}(\text{cm}) \\ &= 1.18 B_{PT}(\text{kG}) r \ell_{st}(\text{cm}^2) \end{aligned}$$

This should be equal to $B_{st}(\text{kG}) t \ell_{st}(\text{cm}^2)$.

$$\text{So } 1.18 B_{PT} r \ell_{st} = B_{st} t \ell_{st}$$

$$\text{Let } B_{st} = 5 \text{ kG}$$

$$\therefore t(\text{cm}) = 1.18 r \frac{B_{PT}}{B_{st}} = 0.24 r(\text{cm}) B_{PT}(\text{kG})$$

6) Quad diameter $\emptyset$:

$$\emptyset(\text{cm}) = 2 \left\{ r(\text{cm}) + 1.15r(\text{cm}) + 1.27N \left[W_c(\text{in.}) + 0.06 \right] + 0.15 + 0.24r(\text{cm})B_{PT}(\text{kG}) \right\}$$

$$\text{where } N = 442.1 \frac{r^2(\text{cm})(GL)(\text{kG})}{I_c(\text{A})L_{\text{eff}}(\text{cm})}$$

7) Conductor length per pole:

a) Length of conductors along width (W_1 , W_2):

$$L_{\text{ends}} = \left(W_1 \times \frac{N}{2} + W_2 \times \frac{N}{2} \right) 2 = N(W_1 + W_2)$$

$$W_1 = L_{\text{stl}} - (W_c + 0.06) - 2 \left(\frac{W_c}{2} + 0.03 \right) = L_{\text{stl}} - 2(W_c + 0.06)$$

$$W_2 = L_{\text{stl}} - 2 \left(\frac{W_c}{2} + 0.03 \right) = L_{\text{stl}} - (W_c + 0.06)$$

$$N(W_1 + W_2) = N[2L_{\text{stl}} - 3(W_c + 0.06)]$$

$$\therefore L_{\text{ends}} = 2N(L_{\text{stl}} - 1.5W_c - 0.09)$$

7) Conductor length per pole (continued):

b) Length of conductors along arcs:

Note: For 90° pole tip, $\ell = \frac{\pi}{2} R$

For general pole tip, $\ell = \beta R$, $\beta \leq \frac{\pi}{2}$

$\left\{ \begin{array}{l} \beta \text{ replaces } \frac{\pi}{2} \\ \therefore \pi = 2\beta \end{array} \right.$

For 90° pole tip:

$$\begin{aligned}
 \sum_{\text{(one layer)}} L &= \frac{\pi}{2} \left[2.15r + \left(\frac{W_c}{2} + 0.03 \right) \right] + \frac{\pi}{2} \left[2.15r + (W_c + 0.06) + \left(\frac{W_c}{2} + 0.03 \right) \right] \\
 &\quad + \frac{\pi}{2} \left[2.15r + 2(W_c + 0.06) + \left(\frac{W_c}{2} + 0.03 \right) \right] + \dots \left(\frac{N}{2} \text{ turns} \right) \\
 &= \frac{\pi}{2} \left[\frac{N}{2} \left(2.15r + \frac{W_c}{2} + 0.03 \right) + \left(\frac{N}{2} - 1 \right) (W_c + 0.06) \right]
 \end{aligned}$$

For 4 layers (double wrap, each face):

$$\begin{aligned}
 \sum L &= 4 \left(\frac{\pi}{2} \right) \sum L \text{ (one layer)} \\
 &= 2\pi \left[\frac{N}{2} \left(2.15r + \frac{W_c}{2} + 0.03 \right) + \left(\frac{N}{2} - 1 \right) (W_c + 0.06) \right] \\
 \therefore \sum L_{\text{coil}} &= 2N(L_{\text{stl}} - 1.15W_c - 0.09) + 2\pi \left[\frac{N}{2} \left(2.15r + \frac{W_c}{2} + 0.03 \right) \right. \\
 &\quad \left. + \left(\frac{N}{2} - 1 \right) (W_c + 0.06) \right]
 \end{aligned}$$

7) Conductor length per pole (continued):

For 2 poles with L_{stl} and r in cm:

$$L_c = 4N \left(\frac{L_{stl}}{2.54} - 1.5 W_c - 0.09 \right) + 4\pi \left[\frac{N}{2} (0.846r + 0.03 + 0.5W_c) + \left(\frac{N}{2} - 1 \right) (W_c + 0.06) \right] + F$$

For general pole angle β , $\pi = 2\beta$:

$$L_c = 4N \left(\frac{L_{stl}}{2.54} - 1.5 W_c - 0.09 \right) + 8\beta \left[\frac{N}{2} (0.846r + 0.03 + 0.5W_c) + \left(\frac{N}{2} - 1 \right) (W_c + 0.06) \right] + F$$

Test example:

For FMIT 25 cm quad:

$$N = 30$$

$$W_c = 0.53''$$

$$r = 6.54 \text{ cm}$$

$$L_{stl} = 29.94 \text{ cm}$$

$$F = 276''$$

$$\beta = \frac{\pi}{2}$$

$$\begin{aligned} \therefore \frac{L}{\text{pole}} &= 2(30) \left(\frac{29.94}{2.54} - 1.5 \times 0.53 - 0.09 \right) \\ &+ 2\pi \left[\frac{30}{2} (0.846 \times 6.54 + 0.03 + 0.5 \times 0.53) \right. \\ &\quad \left. + \left(\frac{30}{2} - 1 \right) (0.53 + 0.06) \right] + 276 \\ &= 1531.3'' \text{ (we specified 1620'')} \text{ so error } < -6\% \end{aligned}$$

$$8) \text{ Power} = I_{\max}^2 R_c \frac{L_c}{12}$$

$$= \frac{1}{12} I_{\max}^2 R_c L_c$$

$$\sum P = \frac{1}{6} I_{\max}^2 R_c L_c$$

$$9) \text{ Voltage} = \frac{1}{12} I_{\max} R_c L_c$$

$$\Delta P = \frac{1}{2} K U^{1.79} L_c$$

$$R_c = 800 \times 10^{-6} \text{ for } W_c = 0.1443$$

$$= 172 \times 10^{-6} \text{ for } W_c = 0.2576$$

$$F = 48" \text{ for } W_c = 0.1443$$

$$= 24" \text{ for } W_c = 0.2576$$

$$\Delta P = \text{psi}$$

$$U = \text{fps}$$

$$Q = \text{gpm (both coils)}$$

$$P = \text{watts}$$

$$V = \text{volts}$$

$$10) Q = 2AU$$

$$= 2AU \times \frac{(12)(62.4)(60)}{(1728)(8.34)}$$

$$= 6.2335 A_w U \text{ gpm}$$

$$\Delta T = ^\circ\text{C}$$

$$K = 0.0525 \text{ for } W_c = 0.1443$$

$$= 0.0354 \text{ for } W_c = 0.2576$$

$$A_w = 0.01005 \text{ in}^2 \text{ for } W_c = 0.1443$$

$$= 0.01606 \text{ in}^2 \text{ for } W_c = 0.2576$$

$$11) \Delta T = 0.0123 \frac{P}{Q} ^\circ\text{C}$$

$$= 0.0123 \frac{I_{\max}^2 R_c L_c}{(12)(6.235) A_w U}$$

$$= 1.64 \times 10^{-4} \frac{I_{\max}^2 R_c L_c}{A_w U}$$

12) Quad weight:

$$\text{Volume of coils} \approx \frac{\pi}{4} (r_o^2 - r_i^2) \times W \times 2 + 8 \times H \times W \times (L_{stl} - 2W)$$

$$\text{where } r_o - r_i = H$$

$$(r_o^2 - r_i^2) = (r_o - r_i)(r_o + r_i) = H(H + 2r_i)$$

$$r_i = r + 1.15r = 2.15r$$

$$\begin{aligned} \therefore r_o^2 - r_i^2 &= H^2 + 2H(2.15r) \\ &= H^2 + 4.3rH \end{aligned}$$

$$\therefore \text{Volume of coils} \approx \frac{\pi}{2} W (H^2 + 4.3rH) + 8HW(L_{stl} - 2W)$$

$$H = 1.27N(W_c + 0.06), \quad H^2 = 1.61N^2(W_c + 0.06)^2$$

$$W = 2(2.54)(W_c + 0.06) = 5.08(W_c + 0.06)$$

12) Quad Weight (continued):

For 20 Mev BCDTL quad:

$$\begin{aligned} N &= 14 & L_{\text{stl}} &= 3.25'' (8.255 \text{ cm}) \\ r &= 2.5 \text{ cm} & W_c &= 0.2576'' \end{aligned}$$

$$H = (1.27)(14)(0.2576 + 0.06) = 5.65 \text{ cm}$$

$$W = 5.08(0.2576 + 0.06) = 1.61 \text{ cm}$$

$$\begin{aligned} \therefore \text{Vol coils} &= \frac{\pi}{2} (1.61) (5.65^2 + 4.3 \times 2.5 \times 5.65) + 8(5.65)(1.61)(8.255 - 2 \times 1.61) \\ &= 601 \text{ cm}^3 (36.7 \text{ in}^3) \end{aligned}$$

If this were solid copper ($\rho_{\text{cu}} = 0.322 \text{ lb/in}^3$), weight would be 11.8 lb.

$$\begin{aligned} \text{Actual wt} &= \frac{\text{face area of Cu}}{HW} = \frac{N(0.04825 \text{ in}^3)(2.54)^2}{(5.65)(1.61)} \\ &= \frac{(14)(0.04825)(2.54)^2}{(5.65)(1.61)} = 0.48 \end{aligned}$$

Remaining face area ($0.52 \times HW$) has a density of 0.07 lb/in^3 .

$$\begin{aligned} \therefore \text{wt of coil} &= 36.7 \text{ in}^3 \left(\begin{array}{cc} 0.48 \times 0.322 & 0.52 \times 0.07 \\ \text{wt. cu} & \text{wt. epoxy / glass} \end{array} \right) \\ &= 7.0 \text{ lb} \end{aligned}$$

$$\therefore \text{Density of coil} = \frac{7.0}{36.7} = \boxed{0.19 \text{ lb / in}^3 \text{ avg}}$$

12) Quad weight (continued):

Note: Drawn to scale for numerical integration of typical design

Conclusion: The lost area in a quad is $\frac{432}{314} = 1.38 \times$ the pole tip bore area.

Now we can estimate the average density of a "cookbook" quad:

- If the quad were made entirely of steel, its weight would be

$$wt_{stl} = \frac{\pi}{4} \left[(Quad \ \varnothing)^2 - (2r)^2 \right] (L_{stl}) (\rho_{stl})$$

12) Quad weight (continued)

- We reduce this by the lost pole tip volume.

$$\begin{aligned}\text{Volume} &= \frac{\pi}{4} \left[(\text{Quad } \emptyset)^2 - (2r)^2 \right] (L_{\text{stl}}) - 1.38 \left(\frac{\pi}{4} \right) (2r)^2 L_{\text{stl}} \\ &= \frac{\pi}{4} \left[(\text{Quad } \emptyset)^2 - 2.38(2r)^2 \right] (L_{\text{stl}})\end{aligned}$$

- We also reduce this by the volume of the field coils:

$$\begin{aligned}W_{\text{tstl}} &= \left\{ \frac{\pi}{4} \left[(\text{Quad } \emptyset)^2 - 2.38(2r)^2 \right] L_{\text{stl}} \right. \\ &\quad \left. - \underbrace{\left[\frac{\pi}{2} W (H^2 + 4.3rH) + 8HW (L_{\text{stl}} - 2W) \right]}_{\text{vol coils}} \right\} \times 0.3\end{aligned}$$

- Finally we add on the actual coil weight:

$$\begin{aligned}\sum \text{wt}_{\text{quad}} &= W_{\text{stl}} + (\text{vol coils})(0.19) \\ &= \frac{\pi}{4} \left[(\text{quad } \emptyset)^2 - 2.38(2r)^2 \right] L_{\text{stl}} \times 0.283 \leftarrow \rho_{\text{stl}} \\ &\quad + \left[\frac{\pi}{2} W (H^2 + 4.3rH) + 8HW (L_{\text{stl}} - 2W) \right] \times (0.19 - 0.283)\end{aligned}$$

Example: For 20 MeV quad,
 Quad $\emptyset$ = 12.69"
 L_{stl} = 3.25:
 r = 2.5 cm
 vol coils = 36.7 in³

$$\begin{aligned}\therefore \sum \text{wt}_{\text{quad}} &= \frac{\pi}{4} \left[12.69^2 - 2.38 \left(\frac{2 \times 2.5}{2.54} \right)^2 \right] (3.25)(0.283) \\ &\quad + 36.7(0.19 - 0.283) = 106.25 \text{ lb}\end{aligned}$$

12) Quad weight (continued):

$$\text{If the quad were 100\% steel, } Wt = \frac{\pi}{4} \left[12.69^2 - \left(\frac{2 \times 2.5}{2.54} \right)^2 \right] (3.25)(0.283) = 113.53 \text{ lb.}$$

Thus, correction factor in using simple formula is 0.936.

$$Wt \text{ Quad} = \left(\frac{\pi}{4} \right) (K) \left[(\text{Quad } \emptyset)^2 - (2r)^2 \right] L_{stl} \times \rho_{stl}$$

$$K = 0.936$$

$$\rho_{stl} = 0.0173 \text{ lb / cm}^3$$

$$\therefore \boxed{\begin{aligned} Wt_{quad} &= 0.0127 L_{stl} \left[(\text{Quad } \emptyset)^2 - 4r^2 \right] \\ &= \text{lb} \end{aligned}}$$

Check—20 MeV quad:

$$\text{Quad } \emptyset = 32.23 \text{ cm (12.69")}$$

$$r = 2.5 \text{ cm}$$

$$L_{stl} = 8.255 \text{ cm (3.25")}$$

$$Wt_{quad} = (0.0127)(8.255) \left[32.23^2 - 4(2.5)^2 \right]$$

$$= 106.3 \text{ lb}$$

Rough estimate of weight would have been

$$\frac{\pi}{4} \left[32.23^2 - 4(2.5)^2 \right] (8.255)(0.0173) = 113.7 \text{ lb (7\% overestimate).}$$

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