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Title: MATERIAL BALANCE AREAS AND FREQUENCES FOR
LARGE REPROCESSING PLANTS

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Material Balance Areas and Frequencies For Large Reprocessing Plants*

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ABSTRACT

It has long been recognized that facilities with a large nuclear material throughput will probably not meet the International Atomic Energy Agency (IAEA) goal for detecting trickle diversion of plutonium over periods of about one year. The reason is that measurement errors for plutonium concentration and for liquid volume are often approximately relative over a fairly wide range of true values. Therefore, large throughput facilities will tend to have large uncertainties assigned to their annual throughput. By the same argument, if frequent balances are performed over small material balance areas, then the uncertainty associated with each balance period for each balance area will be small. However, trickle diversion would still be difficult to detect statistically. Because the IAEA will soon be faced with safeguarding a new large-scale reprocessing plant in Japan, it is timely to reconsider the advantages and disadvantages of performing frequent material balances over small balance areas (individual tanks where feasible). Therefore, in this paper we present some simulation results to study the effect of balance frequency on loss detection probability, and further simulation results to study possibilities introduced by choosing small balance areas. We conclude by recommending frequent balances over small areas.

INTRODUCTION

The International Atomic Energy Agency (IAEA) has established a measurement-error goal for reprocessing plants: the measurement error standard deviation should be no more than 0.3% of annual throughput. In addition, the traditional detection goals of a diversion of 8 kg of plutonium over either 1 month or 1 year remain in effect.¹ Implicit in these goals are recalibration procedures, and size and number of accountancy tanks designed for measuring throughput as well as target measurement error variances for

individual measurement techniques. For example, because measurement errors are often approximately relative, the measurement error variance of the throughput of a reprocessing plant can be reduced simply by using two parallel processing lines. In fact, the Thorp plant in Britain uses two or three parallel tanks in certain areas of the plant, for operational reasons, including two input accountability tanks. Such arrangements typically reduce the overall measurement error (we illustrate why in the appendix), or at least are assumed to, because measurement errors are believed to be relative.

Large-scale reprocessing plants expect to process 8000 to 12000 kg of plutonium per year. At 8000 kg of plutonium per year, the 0.3% goal translates to a standard deviation of 24 kg. Because 8 kg would then be only one-third of the annual standard deviation, it is not possible to meet the detection goals. Using the traditional 0.05 false alarm probability and 0.95 detection probability, the yearly detection goal could only be met for a diversion of about $3.3 \times 24 = 79.2$ kg of plutonium.

This work supports the implementation of a near-real-time nuclear materials accounting system for the proposed reprocessing plant at Rokkasho, Japan. Because of proprietary considerations, we use measurement error models, based on Ref. 2 for a hypothetical plant, that are intended to provide guidance in the statistical decision-making aspect of safeguarding any large-scale reprocessing plant. All measurement error models presented in Ref. 2 are relative and are based on the evaluation of actual measurement data. It is important to realize that the behavior of the measurement error standard deviation could not be relative over the entire range of possible true values. Therefore, our conclusions rely on the assumption that over the range of true values that are pertinent to nuclear materials accounting, the error models are approximately relative.

Recent safeguards work,^{3,4} in connection with the Thorp reprocessing plant in Britain, has considered

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performing relatively frequent material balances using three balance areas. In this paper we expand on the work in Refs. 3 and 4. We present general arguments for performing frequent material balances over small balance areas (individual tanks where feasible). We also present results from two types of simulation studies. In the first type of study, we document the effect of balance frequency on loss detection probability. In the second type of study, we emphasize the new analysis and diagnosis possibilities that arise when frequent balances are performed over individual tanks.

Our present thinking is that if international safeguards is to serve its intended purpose, then a facility can be in partial compliance by meeting measurement error and loss detection goals. In some sense the yearly detection goals are only of academic interest for the following main reason. Suppose there is an apparent loss of 10 kg of plutonium over the course of one year between two plant cleanouts, and that the standard deviation of the throughput is 3 kg. Then at typical alarm limits, there is an alarm, which we interpret as a detection. However, the sad reality is that because of limitations in the measurement error modelling, it is more likely that upon reviewing the facility operations for that year, inspectors would conclude that because they have no way to find the 10 kg if it actually is diverted, the 10 kg is attributable to perhaps poorly measured waste streams, difficult to measure holdup in pipes, or poor measurement error models. In a nutshell, it is difficult to model the total measurement error, so there is nearly always the possibility that the error variances are understated. The most viable solution to this dilemma is to perform frequent material balances over small balance areas so measurement error models can be authenticated. As by-products of performing frequent balances, the IAEA could also address the data falsification issue if the IAEA is granted access to frequent snapshots of the facility status, and loss detection for abrupt diversion would be greatly enhanced, as we will see in the results section.

BACKGROUND

For each discussion, we present here the usual model employed in the analysis of material balances. Denote the material balance on balance period t as X_t . Denote the net transfers as T_t and the inventory as I_t . Then $X_t = I_{t-1} + T_t - I_t$. Because many measurements typically enter into the calculations of transfers and inventories, X_t is approximately normally distributed. And, as a first approximation, the variance of X_t is $\sigma_T^2 + 2\sigma_I^2$, where σ_T^2 is the transfer measurement

error variance, and σ_I^2 is the inventory measurement error variance. This is only an approximation because the inventory measurement error variance might not remain constant each balance period, and there are probably correlations among some of the measurements that compose successive inventories. Also as a first approximation, the covariance between X_t and X_{t-1} is $-\sigma_I^2$.

Let the mean of X_t be denoted μ_t . Then, a test for material loss on balance period t can be expressed as follows: test the hypothesis that $\mu_t = 0$ versus $\mu \neq 0$. And, after a fixed number, $nbal$, of balance periods, a test for material loss can be expressed as follows: test

the hypothesis that $\sum_{t=1}^{nbal} \mu_t = \mu = 0$ versus $\mu_t \neq 0$.

And, the sequence $X_1, X_2, \dots, X_n$ is approximately multivariate normally distributed with mean vector $\bar{\mu}$ and covariance matrix Σ where Σ is $nbal$ by $nbal$.

Safeguards literature has focused on two primary objectives: sensitive and timely detection of loss of nuclear material.

By sensitive we mean that if decisions about loss of material are made at regular intervals (traditionally yearly in conjunction with plant cleanouts), then the desired statistical test is one with maximum detection probability. Because the detection probability of any statistical test depends on the distribution of the loss, and in practice the loss would be unknown, it is impossible to select the statistical test that has the maximum detection probability. For that reason, Ref. 5 used game-theoretic arguments in conjunction with the Neyman-Pearson lemma that is used in statistical decision theory to prove that the test having maximum detection probability against the worst-case loss scenario is the so-called cumulative material unaccounted for (CUMUF) test. The CUMUF test uses only the beginning and ending inventories (typically zero) and net transfers. It therefore ignores intermediate inventories, so there is no need for frequent balance closure. The sensitivity goal can be stated as above: periodically (typically between annual shutdowns and cleanouts) test $\sum_{t=1}^{nbal} \mu_t = \mu = 0$ versus $\mu \neq 0$. The CUMUF test uses $\sum_{t=1}^{nbal} X_t = CUMUF$, which ignores intermediate inventories, and thus has the same loss detection

power against any loss vector. As an important aside, the worst case (in terms of minimizing detection probability) loss scenario is a loss vector $\bar{\mu}$ proportional to Σe where $e = (1, 1, \dots, 1)^T$. Roughly speaking, this means that approximately constant (protracted) loss is the most difficult to detect because, as we mentioned above, the covariance matrix is often nearly tridiagonal, having diagonal entries $\sigma_{ii} = \sigma^2 = \sigma_T^2 + 2\sigma_I^2$ and main off-diagonal entries equal to $-\sigma_I^2$, where σ_I^2 is the inventory measurement error variance and σ_T^2 is the transfer measurement error variance. All other entries in Σ are usually nearly zero. Therefore, each row of Σ sums to approximately the same value, σ_T^2 , which means that the optimal loss vector is approximately constant.

Fortunately, tests can be developed that respond nearly as well as the CUMUF test to protracted diversion and perform much better than the CUMUF test on abrupt diversion, provided material balances are performed frequently enough. We illustrate that fact in the results section. The loss detection probability of the CUMUF test in the case of protracted diversion serves mainly as a benchmark against which we can compare other tests.

By **timely** we mean that the time required to detect a loss is small on average. Certainly the CUMUF test performed only at yearly intervals is not timely. In fact, Ref. 4 illustrates that though a particular sequential test is slightly less sensitive than the CUMUF test, the sequential test has a shorter average detection time for the worst-case loss. Such behavior is possible because on most of the occasions when the sequential test does detect the loss, the loss is detected before the time of the last balance period. The same sequential test is far more sensitive and timely for non-optimal loss vectors, such as abrupt loss vectors. It is therefore widely accepted that some frequency of performing intermediate inventories is desirable because calculation of intermediate material balances dramatically improves timeliness and sensitivity for abrupt loss, improves timeliness for protracted losses, and typically causes only a minor reduction in sensitivity for the worst-case protracted loss. We can understand this phenomenon by considering the effect of the type I error (falsely reject the hypothesis of zero loss) on the type II error (falsely "accept" the hypothesis of zero loss) for the CUMUF test. For losses of 3 to 4 units (scaled to have unit standard deviation), there is very

little decrease in loss detection power when the type I error decreases from 0.05 to 0.04 to 0.03. Therefore, we could simply operate the CUMUF test at the 0.04 false alarm level, and a sequential test designed to detect abrupt loss at the 0.01 level. The combined false alarm rate is guaranteed to be less than or equal to 0.05, by the Bonferroni inequality (the Bonferroni inequality is discussed in many statistics books—see Ref. 7). And the CUMUF detection probability has only fallen from 0.95 to 0.94 for a loss equal in magnitude to 3.3 standard deviations. Barry Jones (Refs. 3 and 4) has exploited this fact to develop a double-Page sequential test. The double-Page test combines two single Page's tests. Page's test is probably the most commonly used sequential test in safeguards, and its description can be found in Refs. 3 and 4. In a double-Page test, the first test is best at detecting abrupt loss and the second is best at detecting protracted loss. To protect against loss of power against the worst-case-approximately-constant loss, the false alarm of the first test is designed to be about 0.01 and the false alarm rate of the second test is designed to be about 0.04. We document the performance of such a double-Page test in the results section.

To recap this section, it is generally accepted that some decrease in loss detection ability against the worst-case loss is a small price to pay for the advantages gained by closing balances frequently.

SIMULATION RESULTS

Part 1. Loss detection probability as a function of the frequency of material balances.

In this section, we discuss a facility modelled by assuming that the covariance matrix is tridiagonal with main diagonal entries, $\sigma_{ii} = 40/(nbal)$ and main off-diagonal entries, $\sigma_{i,i-1} = -4$ (units are kg^2). Setting $\sigma_{ii} = 40/(nbal)$ and the inventory variance equal to 4, reproduces the results in Refs. 3 and 4 for the double-Page's test. We also experimented with setting the inventory variance equal to 40 for purposes to be made clear. This idealized error model would never be realized in an operating facility, if for no other reason than the inventory variance would typically fluctuate due to process variability. However, a facility could choose to close balances when certain key inventory tanks are nearly empty. Also, we have performed nearly the same suite of experiments on more realistic covariance matrices characteristic of what we expect at the Rokkasho plant, and the general conclusions remain the same. We therefore consider it a fruitful exercise to perform some analysis using this simplified covariance matrix. As an aside, we present results for the case of

beginning and ending with zero inventory, so the first two and the last two rows of Σ are affected slightly and are not correct as stated. Such relatively minor points are only an issue in studies of exactly how the loss is distributed and will not concern us here. The estimated loss detection probabilities in Table 1 are within approximately 0.01 of their true values, based on the number of simulations (4000) run per case. We consider $nbal = 10, 20, 40, 80$, and 160 and losses of 12.65 and 20.87 , which represent losses of 2 and 3.3 times the standard deviation of the annual CUMUF statistic, respectively.

We considered both protracted and abrupt losses, and, for ease of comparison among the values of $nbal$, we considered abrupt losses occurring at the same times within the year: $2/5$, $1/2$, and $4/5$ of the way into the year. In Table 1, loss 1A (1B) is a constant loss totalling 12.65 (20.87). Loss 2A (2B) is an abrupt loss of 12.65 (20.87) on the balance period corresponding to $2/5$ of the way into the year. Similarly, losses 3A and 3B (4A and 4B) are abrupt losses of 12.65 and 20.87 occurring at $1/2$ ($4/5$) of the way into the year. Each cell in Table 1 has 2 entries. The first entry corresponds to the the $\sigma_I^2 = 4$ case, and the second entry corresponds to the $\sigma_I^2 = 40$ case. The conclusions from Table 1 are the following.

1. Increasing the balance frequency raises the probability of detecting an abrupt loss. For example, compare the $nbal = 10$ case to the $nbal = 160$ case for an abrupt loss of 12.65 for inventory variances of 4 and 40 . For the abrupt loss occurring at balance period 5 for the $nbal = 10$ case and at balance period 80 for the $nbal = 160$ case, the loss detection probability increases from 0.92 to 1.0 for an inventory variance of 4 , and from 0.60 to 0.84 for an inventory variance of 40 .

2. The effect of an increase in the inventory variance from 4 to 40 is diminished as $nbal$ increases. For example, for a constant loss totalling 12.65 for $nbal = 10$, the detection probability is 0.44 for an inventory variance of 40 and 0.57 for an inventory variance of 4 , whereas for $nbal = 160$, the detection probability is 0.51 for an inventory variance of 40 and 0.56 for an inventory variance of 4 . For comparison, the annual CUMUF statistic, regardless of the number of balance periods, would have a loss detection probability of 0.64 at the 0.05 false alarm level for a loss totalling 12.65 over the year. We consider the effect of an increase in inventory variance because we anticipate using less accurate methods⁸ to estimate plutonium concentration if balances are closed very frequently. An interesting result is given in Ref. 6 that can be roughly described as follows. For a single Page's test in the case of constant loss for a covariance matrix of the simple form we introduced here (all rows sum to the same value, equal to the transfer measurement error variance), as $nbal \rightarrow \infty$ the inventory variance does not affect the performance of Page's test. This is perhaps a surprising result at first sight, and it holds whether Page's test is applied to the original material balance sequence or the commonly used SITMUF sequence, which is the MUF sequence that has been standardized (unit variance) and independently transformed. Recall that we had a slightly different form for Σ , due to the assumption that we began and ended with zero inventory. In that case there are minor "edge" effects that might partly explain why, for $nbal = 160$, the asymptotic result is not quite reached. That is, the $\sigma_I^2 = 4$ case shows slightly dif-

ferent loss detection probability than the $\sigma_I^2 = 40$ case (0.56 versus 0.51 and 0.92 versus 0.89 for constant losses totalling 12.65 and 20.87 , respectively). However, it is encouraging that, in the limit as

Table 1: Loss detection probabilities

Loss	$nbal = 10$	$nbal = 20$	$nbal = 40$	$nbal = 80$	$nbal = 160$
1A	0.57, 0.44	0.54, 0.43	0.55, 0.47	0.55, 0.49	0.56, 0.51
1B	0.91, 0.80	0.90, 0.80	0.91, 0.84	0.99, 0.87	0.92, 0.89
2A	0.92, 0.60	0.96, 0.68	0.98, 0.77	0.99, 0.80	1.0, 0.84
2B	1.0, 0.95	1.0, 0.98	1.0, 0.99	1.0, 1.0	1.0, 1.0
3A	0.92, 0.60	0.95, 0.68	0.98, 0.76	0.99, 0.80	1.0, 0.84
3B	1.0, 0.95	1.0, 0.98	1.0, 0.99	1.0, 1.0	1.0, 1.0
4A	0.91, 0.52	0.95, 0.53	0.98, 0.57	0.99, 0.64	1.0, 0.74
4B	1.0, 0.94	1.0, 0.95	1.0, 0.97	1.0, 0.99	1.0, 1.0

$n_{bal} \rightarrow \infty$, the inventory variance does not affect the loss detection probability against the worst-case constant loss. This asymptotic result has not yet been derived for a double-Page's test, but we have provided empirical evidence here that a similar asymptotic result will hold for double-Page's tests.

Part 2. Simulating individual tank material balances.

In this section we present simulation results from 15 vessels in the main chemical separations area of a hypothetical reprocessing plant. The first vessel is the input accountability tank, and the last vessel is the output accountability tank. These two accountability tanks are the two key measurement points in the main process area, so substantial effort is made to make good measurements in these tanks. The input accountability tank is typically regarded as a transfer vessel between the head-end materials balance area (MBA) and the main process MBA. The output accountability tank is a transfer vessel between the main process area MBA and the co-denitration area MBA. Therefore, the remaining 13 vessels would be regarded as inventory vessels in the main process MBA. Some of the 13 vessels can hold substantial quantities of plutonium, particularly toward the end of the process when the plutonium concentration is very large, approximately 250 g/L. All bulk material is in liquid form, so the only accountability measurements will be tank volumes or weights or both together with tank concentrations. See Fig.1 for a description of the tanks. We have chosen fictitious but reasonable cycle times and modes of tank operation. In Fig.1 the shaded oval represents the chemical separations process area. The first tank is the input accountability tank. The second tank feeds the pulsed columns in the chemical separations area, so it is operated in a batch receipts, continuous shipments mode. The third tank receives continuously from the separations area and ships in batch mode. Tanks that support the purification cycles and evaporation process bring the total number of tanks to 15.

Fig.1. First three tanks of main process area and their modes of operation (batch or continuous).

Because modern reprocessing plants are becoming increasingly automated, it is feasible to perform frequent material balances over small balance areas. However, the labor involved in chemical assay to estimate the concentrations of tank contents is still rather costly. Fortunately, it is possible to get a "quick and dirty" estimate of concentration by exploiting empirical relationships between solution density, conductivity, and plutonium and uranium concentrations.⁸

The hope is that this quick method will make it feasible to close balances very frequently. At much less frequent intervals the contents of the inventory vessels could be more accurately assayed for concentration. Reference 8 suggests that the plutonium concentration can be estimated to within 5% by using empirical relationships. Because measurements can be made reasonably fast, it should be feasible to reduce the 5% performance by taking multiple measurements.

Assuming that balances can be closed around individual tanks very frequently, a natural question is, "how frequently should the balances be closed?" We do not attempt to answer that question from a statistical viewpoint, but choose here to close balances at a convenient time in terms of tank activity. In particular, because the input accountability tank makes batch transfers every 32 hours, it is convenient to close balances every 32 hours, when the input accountability tank is empty. We use simple modular arithmetic to determine how many shipments each tank has made since the last balance closure and the current status of each tank at the time of balance closure. The hypothetical, but typical, cycle times of tanks 1 to 15 are 32, 32, 32, 8, 8, 8, 8, 8, 16, 16, 16, 16, 128, and 128 hours, respectively. So, for example, tank 4 completes 4 cycles every 32 hours, and balances are closed at the same time into the cycle of tank 4 every 32 hours. Using relative error models based on Ref. 2, we estimate tank volumes and concentrations every 32 hours as well as the amount shipped by each tank during the 32 hours. To estimate the transfers into the input accountability tank, we assume that reactor-based estimates of plutonium contents could be used. We assigned zero uncertainty to the reactor-based estimates because that simplified checking some of the simulation results analytically and because reprocessing plants do not typically have to propagate the variance attributable to the fairly inaccurate reactor-based estimates. That is, the measurements in the input accountability tank would be taken as inputs to the main process area.

We simulated the 15-tank system for 1100 periods of 32 hours each, then analyzed only the last 1000 periods, so that we could assume a steady state had been reached. For a plant that operated about 200 days

per year, this represents about 6.5 years of operation. The simulated data is several 1000-length 15-component time series. The main time series is the 15 individual material balances. Two other available time series are 15 individual estimates of total plutonium transferred and 15 volume shipper-receiver differences (accounting for concentration changes because some of the transfers involve volume changes). We only discuss the 15 material balance time series here.

We consider three cases, all involving tank 4 in some way: (1) the measurement error for tank 4 is very large (5%) for the volume calibration error, (2) there is a 1% bias in the volume calibration error, and (3) there is a 1-kg loss per balance period from tank 4.

Case 1. Refer to Fig. 2. Plots (a) and (b) are the 1000 material balances (MBs) for the tanks 4 and 5. The pattern is obvious and can be traced to the recalibration schedule (tanks were recalibrated every 100

MBs, which is approximately every 4.5 months). The large swings in average MB are traceable to new tank calibrations, and there are corresponding offsetting MBs in tank 5. These large swings are due to a 5% volume calibration error, which is unrealistically large but is used for illustrative purposes. In plot (c) we show the sum of the MBs for tanks 4 and 5. Because of the offsetting MBs of opposite sign in tank 5, there is no concern that material has been diverted, and there is an enhanced understanding of the measurement error model. To gain some reassurance that no material has been diverted from tank 4, we can compare the mean MB for tank 4 to an estimate of the standard deviation of the mean MB for tank 4. Because of the systematic error, the variance of the mean is considerably larger than $\sigma_{MB}^2 / (nbal)$. So even though the mean of the MBs for tank 4 is about 409 g, the ratio of the mean to the estimated standard deviation is only 0.25, which suggests that no material has been diverted from tank 4.

Fig. 2.

Case 2. Refer to Fig. 3. Upon inspection of the 15 time series, we again found that tanks 4 and 5 exhibit offsetting gains and losses. Because there is a long-term bias in the volume calibration for tank 4 that persists for the entire experiment, the contents of tank 4 are systematically overstated. Therefore, as is apparent from the histograms in plots (a) and (b), we see a tendency toward apparent gains in tank 4 and apparent losses in tank 5. The mean MBs for tanks 4 and 5 were -452 and 606, respectively. The ratio of the mean to the estimated standard deviation for the MBs for tanks 4 and 5 were -3.6 and 3.5, respectively. Taken as one MBA, there are offsetting gains and losses, so that we can properly attribute this behavior to a long-term error in the calibration of tank 4 that persisted for the entire series.

Case 3. Upon inspection of the 15 time series, it was found that tank 4 exhibits a mean loss of material of about 1 kg, with no corresponding gain in tank 5 or 3. For tank 4, the ratio of the mean MB to the estimated standard deviation of the mean is 8.8. The mean MB for tank 5 is 40, and the ratio of the mean to the standard deviation is about 0.28. Similarly, tank 3 does not exhibit apparent gain. A plausible explanation is that there is a constant loss of material from tank 4, which is in fact what was simulated.

SUMMARY AND CONCLUSIONS

We have demonstrated some advantages to closing balances frequently around individual tanks. In our first set of simulations, we illustrated that the loss of power to detect the worst-case loss over a fixed interval is a minor price to pay for enhanced loss detection and timeliness of response for most forms of loss. In our second set of simulations, we illustrated how measurement error models can be assessed if frequent balances are closed around individual tanks.

Because the present-day reality of safeguards is that statistical alarms are more likely to bring error models into question than to reveal a diversion, we recommend that frequent balances be closed around individual tanks. By exploiting empirical relations between density, conductivity, temperature, and concentration, it will be possible to avoid high labor costs. Only relatively infrequent concentration measurements will be needed. We recognize that the exercise of assigning a standard error to any material balance is a difficult task and sometimes not worth the effort. In particular, the unavoidable practice of calibrating chemical procedures to well-characterized standards while stating an uncertainty that is intended to apply to the actual process material is on loose ground. Frequent balance

(a) Histogram of MB series for Tank 4

(b) Histogram of MB series for Tank 5

Fig. 3.

closures around individual tanks will provide historical documentation of the actual measurement uncertainty and will be a valuable complement to the usual variance propagation approach. Finally, we believe that the wealth of internal data-consistency checks provided by frequent balances over individual tanks provides assurance that a plant is operating as it claims, helps to validate or redefine measurement error models, and, in general, very favorably impacts the safeguarding of a reprocessing plant.

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APPENDIX

We mentioned in the introduction that the throughput variance is reduced if two or more parallel input accountability tanks are used rather than only one. To see why, compare the use of two input accountability tanks to one. Suppose the bulk measurement is a volume measurement and the concentration measurement is chemical assay. Denote the measured mass of plutonium by M and the true volume (plutonium concentration) of tank 1 by $V_{1,t}$ ($C_{1,t}$), and similarly for tank 2. Denote the measured volume (plutonium concentration) of tank 1 by $V_{1,m}$ ($C_{1,m}$), and similarly for tank 2. Then the estimate of the mass of plutonium is $M = V_{1,m}C_{1,m} + V_{2,m}C_{2,m}$. Assume for convenience that $V_{1,t} = V_t/2$ where V_t is the true total volume and $C_{1,t} = C_{2,t}$. Denote the variance of the volume measurement for tank 1 by $V_{1,t}^2 \bar{\sigma}_{V_{1,t}}^2$ and similarly for tank 2 and for the concentration measurement error variance. Let $\bar{\sigma}^2 = \bar{\sigma}_{V_{1,t}}^2 + \bar{\sigma}_{C_{1,t}}^2$. Then the variance of M is approximately $2(V_{1,t}/2)^2 \bar{\sigma}^2 = (V_{1,t}^2 C_{1,t}^2 \bar{\sigma}^2)/2$, which is half what the variance would be if only one accountability tank is used. Using the method of Lagrange multipliers, it is simple to show that the maximum reduction in variance is achieved by using two equal-sized tanks, or in general, k equal-sized tanks.

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