AIM

Association for Information and Image Management

1100 Wayne Avenue, Suite 1100
Silver Spring, Maryland 20910
301/587-8202

Centimeter

Inches

MANUFACTURED TO AIM STANDARDS
BY APPLIED IMAGE, INC.

1 of 1

TECHNIQUES FOR MAPPING THE TYPES, VOLUMES, AND DISTRIBUTION
OF CLAYS IN PETROLEUM RESERVOIRS AND FOR DETERMINING
THEIR EFFECTS ON OIL PRODUCTION

Final Report

By
Bijon Sharma

May 1993

Work Performed Under Cooperative Agreement No. DE-FC22-83FE60149

Prepared for
U.S. Department of Energy
Assistant Secretary for Fossil Energy

Chandra Nautiyal, Project Manager
Bartlesville Project Office
P.O. Box 1398
Bartlesville, OK 74005

Prepared by
IIT Research Institute
National Institute for Petroleum and Energy Research
P.O. Box 2128
Bartlesville, OK 74005

MASTER

REPRODUCTION OF THIS DOCUMENT IS UNLIMITED

TABLE OF CONTENTS

	<u>Page</u>
ABSTRACT	1
INTRODUCTION	1
ACKNOWLEDGMENTS	2
PHYSICAL AND CHEMICAL PROPERTIES OF CLAYS AND THEIR EFFECT ON PETROPHYSICAL PROPERTIES	2
WIRELINE LOG RESPONSES OF DIFFERENT CLAY AND OTHER MINERALS	3
Determination of Total Clay Volume	3
DISCRIMINATION OF DIFFERENT TYPES OF CLAY AND OTHER MINERALS ON CROSSPLOTS	4
Discrimination of Predominantly Kaolinite Types of Clays	4
Reservoir Rocks with Mixed Clay Types	5
Recognition of Mixed Clay	5
Summary of Crossplot Studies	5
MATHEMATICAL ANALYSIS OF LOG SIGNATURES	5
Introduction	5
ESTIMATION OF STATISTICAL PARAMETERS OF RESERVOIR PROPERTY DISTRIBUTIONS	7
Computation Theory	7
Frequency Content of Log Signatures	8
COMPUTATION OF STATISTICAL PARAMETERS OF FIELD DATA	8
Analysis of Data from Bell Creek Field	8
Background Information	8
Statistical Parameters of Permeability Data	9
Analysis of Power Spectra	9
Correlation of Power Spectra with Production	10
Vertical Variability Estimation of Sonic Transit Times	10
Vertical Variability of Gamma Ray Distributions	11
Cross Spectral Estimates	11
Analysis of Data from Patrick Draw Field	11
Background Information	11
Analysis of Wireline Log Data	11
Resolution Limits of Wireline Log Data	11
Comparison of Relative Variability Studies of a Number of Wells	12
Correlation with Production Data	13
Discussion of Results of Mathematical Analysis of Log Signatures	13
CONCLUSIONS	14
REFERENCES	14
APPENDIX A	14

TABLES

1. Important Parameters for the Major Clay Mineral Groups	16
2A. Mineralogical Composition and Wireline Log Responses of Shaley Sandstone in Well W-7 in Bell Creek Field, MT. Mineralogical Percentages Obtained from XRD Analysis of Core Samples	17
2B. Mineralogical composition and wireline log response of shaley sandstone in Arch 120, Patrick Draw Field, WY. Mineralogical percentages obtained from XRD analysis	17
3. Measures Of Vertical Variability Of Permeability Distributions In Wells From Bell Creek Field, Montana	18

TABLES - continued

	<u>Page</u>
4. Initial primary, cumulative waterflood, polymer oil production and pay thicknesses of Muddy sandstone reservoir in the study area Of Bell Creek Field, MT.....	18
5. Distributions of areas under the power curve in the different quadrants of the frequency scale of power distributions of log permeability data. Values obtained by dividing the area under curve by the variances of the series are shown in brackets.	19
6. Reservoir and petrophysical properties of UA-5B sandstones of six wells from Arch Unit of Patrick Draw Field, Wy.....	19

ILLUSTRATIONS

1 Comparison of gamma ray log derived clay content with core measured porosity and air permeability for UA-5B sandstone in well 120, Patrick Draw Field, WY.....	20
2 Comparison of gamma ray derived clay content with clay and non-clay minerals estimed from XRD analysis for well 120 in Patrick Draw Field, WY.....	20
3 Comparison of CT-scan densities and log-derived clay content for well 120, Patrick Draw Field, WY.....	20
4 Crossplot of clay volume (V_{cl}) versus bulk density (ρ_r) of producing UA-5B sandstone in well 120, Patrick Draw Field, WY.....	20
5 Crossplot of clay volume (V_{cl}) versus bulk density (ρ_r) in the barrier island sandstones of well P-2 (depth = 4,418 to 4,439 ft), Bell Creek Field, MT.....	20
6 Crossplot of clay volume (V_{cl}) and bulk density (ρ_r) of valley-fill sandstone in well P-2 (depth = 4,408-4,414 ft), Bell Creek field, MT.....	20
7 Distributions of sonic transit times and gamma ray readings across UA-5 and UA-6 sandstones in Arch 81 from Patrick Draw Field, WY	21
8 Amplitude Frequency spectra of sonic transit times of UA-5 and UA-6 sandstones in well Arch 81 shown in Fig. 7.....	21
9 Index map shows units comprising Bell Creek Field and well control.....	22
10 Distribution of Dykstra-Parsons coefficients obtained from air-permeability data in the study area.....	22
11 Cross-sections (D-D') and (E'-E') showing distribution of porosities and permeabilities within sandstone bodies.....	23
12 Distribution of logarithms of permeability across the Barrier Island sandstones in three wells from Bell Creek Field, MT and an ideal profile.....	21
13 Distribution of logarithms of permeability across the Barrier Island sandstones in four wells from the Bell Creek Field, MT.....	21
14 Power spectra of log permeability data from three wells and an ideal profile shown in Fig 12.....	21
15 Power spectra of log permeability data of four wells from the Bell Creek Field shown in Fig 13	21
16 Distributions of areas under the power curves of gamma ray logs in quadrants 1 and 1+2 and Dykstra-Parsons (D-P) coefficients at different well locatons in Bell Creek Field, MT plotted as a function of initial oil production rates per foot of pay	23

ILLUSTRATIONS - continued

	<u>Page</u>
17 Distributions of areas under the power curves of gamma ray logs in quadrants 4 and quadrants 3 and 4 and Dykstra Parsons (D-P) coefficients at different well locations in Bell Creek Field, MT plotted as a function of cumulative waterflood production per foot of pay	24
18 Vertical variation of sonic transit times across Barrier Island sandstones in four wells from Bell Creek Field, MT	24
19 Power spectra of sonic transit times of four wells from Bell Creek Field shown in Fig 18.....	24
20 Variations of sonic transit times and gamma ray readings across the barrier and non-barrier sandstones in Well W-7 from the Bell Creek Field, MT	24
21 Power spectra of sonic transit times and gamma ray readings across the barrier and non-barrier sandstones in well W-7 shown in Fig 20	24
22 Cohrency and phase spectra of sonic transit time data from wells P-2 and 26-4, Bell Creek Field, MT	25
23 Vertical variation of formation density of UA-5B sandstones from Arch 120, Patrick Draw Field, WY from wireline log and CT scans.....	25
24 Power spectra of wireline logs (density, gamma ray and neutron porosity) and CT density data of UA-5B sandstones from Arch 120, Patrick Draw Field, WY	25
25 Power spectrum of digitized gamma ray log data from UA-5B sandstones in four wells in Patrick Draw Field, WY	25
26 Distributions of permeability, Dykstra-Parsons coefficients and areas under the power curve of gamma ray logs plotted as a function of cumulative oil production rates per foot of sand at different well locations in Patrick Draw Field, WY	25
27 Distributions of areas under the power curves of density logs in the different quadrants at different well locations in the Patrick Draw Field, WY	25

TECHNIQUES FOR MAPPING THE TYPES, VOLUMES AND DISTRIBUTION OF CLAYS IN PETROLEUM RESERVOIRS AND FOR DETERMINING THEIR EFFECTS ON OIL PRODUCTION

By
Bijon Sharma

ABSTRACT

This report presents the results of correlation of log signatures with information on distribution of the types and volumes of clays in sandstone pore spaces determined from detailed CT-scan, XRD, SEM, and thin section analyses of core samples from three sandstone reservoirs. The log signatures are then analyzed to determine if suitable mathematical/statistical parameter(s) could be calculated from the logs to determine their effects on permeability and oil production.

When a reservoir sandstone has dominantly one type of clay, an easy way to distinguish such a clay is from cross-plots of the volume of clay against the bulk density and the neutron porosity of the sandstone. The pattern of distribution of the points in a crossplot is sometimes a good indicator of the type of clay in a sandstone. Because of differences in cementation and consequent changes in pore throat size distribution, two sandstones having the same amount of identical clay types may have very different production characteristics. These sandstones can be easily identified from a clay volume, density plot. The variation in the vertical distribution of clays within a sandstone seem to have a more drastic effect on permeability of reservoir sandstones than the total volume of clays. Estimates of total clay content from gamma ray logs are always on the high side because of the presence of non-clay, radioactive minerals. The effect of non-clay minerals must be eliminated before good estimates of total clay content in sandstone reservoirs can be made.

From correlations of CT-scan, density, gamma ray, sonic, and neutron log data, it was observed that certain thin, shaley beds (less than 1 ft thick) could not always be resolved in the density or the sonic logs when the shale beds have the same density as the matrix rocks, but usually these beds could be identified on the gamma ray and the neutron logs.

A quantitative measure of the overall variation in clay content of sandstone reservoirs could be obtained from the frequency domain analysis of a reservoir property like permeability or wireline logs like gamma ray, sonic, density, and neutron which are strongly affected by clay content in reservoir rocks. The reservoir property distributions available as functions of depth are first broken down into a continuous distribution of waves of different frequencies each having a distinct amplitude and phase.

The distribution of power (square of amplitude) with frequency (the so-called 'power spectrum') indicates that the variability in clay content may be estimated from the attenuation of power with frequency. An effective method for obtaining power distribution is from time series analysis of wireline log data in clayey formations. For homogeneous, clean sandstones, the distribution of power in the log data as a function of frequency sharply decreases at higher frequencies. In heterogenous, clayey sandstones, the power content at the high frequency end remains high, and the power does not drop off sharply with frequency. Depending upon the presence of beds with certain stratification, the power distribution has distinct 'highs' at certain frequencies. The characteristic distribution of power with frequency provides a good measure of certain clay types and lithological heterogeneity in a sandstone reservoir.

The variability measures obtained from power spectral analysis of permeability and wireline log data in clayey formations have been correlated with oil production from two oil fields. Compared with the conventional measures of permeability variations like the Dykstra-Parsons coefficients, the new measure appears to correlate better with oil production.

INTRODUCTION

Depending upon the physical and chemical properties of clays and their mode of dispersal within the pore spaces of reservoir rocks, clays may affect primary, waterflood, and enhanced oil recovery (EOR) production from hydrocarbon reservoirs in a wide variety of ways. The effect of clays as permeability-reducing agents due to their ability to block the movement of fluid through reservoir pore spaces is well known and is the primary cause for the failure of many reservoirs to produce oil. Many times fresh water in drilling mud will cause the clays in a shale to swell and will result in sloughing and caving of a formation.¹ The tendency of some clays to migrate through reservoirs as fines that block pore throats and reduce permeability to fluid is another phenomenon that is sometimes encountered in oil reservoirs. The presence of clays can also create production problems in certain well stimulation projects. In a chlorite rich clayey zone, for example, acidizing may precipitate iron causing significant reduction in permeability. Sometimes excessive production rates may cause the fibrous illite to break loose and start clogging the pore throats which will significantly decrease well productivity.²

For efficient exploitation of hydrocarbon reservoirs, knowledge about the distribution of the volume and the types of clays and the mode of their dispersal within reservoir pore spaces is of great importance. Since expensive coring and petrographic studies of rock samples to determine the characteristics of clays are not always feasible, this information has to come from analyses of wireline logs after proper calibration of the logs with core data wherever feasible.

Until about a decade ago, the routine methods of obtaining information about the volume of clays from wireline logs were those using the following logs or combination of logs: gamma ray, SP, neutron-density, sonic-density, and sonic-neutron.³ All the logging methods depend upon the measurement of certain physical or radioactive properties of clays. Significant progress was made during the past decade in mapping clay volumes and in certain instances clay types by the introduction of spectral gamma ray logs.⁴ Unlike gamma ray logs which measure the total radioactivity of a formation, the spectral gamma ray measures both the number of gamma rays and the energy level of each that permits the determination of radioactive potassium, thorium, and uranium content in formation rocks. Although specific clay minerals usually have characteristic concentrations of potassium, thorium, and uranium, determination of clay mineral types from spectral gamma ray data is often ambiguous. However, in combination with other data, such as photoelectric absorption coefficient P_e , which is now routinely measured with the litho-density log, the ambiguities in clay identification may be reduced.⁵ The photoelectric absorption coefficient reflects the average atomic number of a formation and is measured by registering the rates at which gamma rays are absorbed as they interact with the electrons in a formation.⁶

In spite of significant strides made in recent years in estimating clay volumes and in identifying clay types with wireline logs, there still remains considerable ambiguities. In situations of mixed lithologies or clay types, the ambiguities are more serious. What is needed is probably a simple method for identification and classification of clay types according to their role as permeability-reducing or formation-damaging agents. Research is therefore needed to extract information about the characteristics of clays from a combined analysis of well logs with production and petrographical data, particularly from some of the older fields from where only some of the basic well logs are available. In this research, the available well logs from three fields were interpreted to determine if signature analysis of the different logs, in conjunction with detailed CT-scan and petrographic studies, can provide information on clays that will allow their identification on wireline logs.

The variability in the vertical distribution of permeability caused by clay content in petroleum reservoirs has been investigated from frequency domain analysis of

petrophysical properties (like permeability) of core and wireline log data which are strongly affected by clay distributions in the reservoirs. The measures of vertical variability of reservoir properties obtained from frequency content of permeability and wireline log signatures were correlated with oil production data from two oil fields and compared with conventional variability measures of reservoir permeability data.

ACKNOWLEDGMENTS

This work was sponsored by the U. S. Department of Energy under cooperative agreement DE-FC22-83FE60149, Project SGP42. The author wishes to thank Chandra Nautiyal of the DOE and Dr. T. Burchfield, Dr. M. Madden and Dr. M. Tham for their suggestions and Dr. R. Schatzinger and Dr. L. Tomutsa for their assistance in conducting petrographic and CT-scan studies of core samples, respectively.

PHYSICAL/CHEMICAL PROPERTIES OF CLAYS AND THEIR EFFECT ON PETROPHYSICAL PROPERTIES OF SANDSTONES

The differences in the physical and chemical properties of clays and between clays and matrix rocks are the basis for mapping distribution of clays in reservoir rocks. The dominant clay minerals commonly encountered in clastic reservoirs exhibit a wide range of physical and chemical properties (Table 1). Note that the clay properties listed in Table 1 are approximate, average values, and these values may change over a fairly wide range. For example, smectite (montmorillonite) can absorb a large volume of water, and in its most hydrated form will have very low density. When subjected to pressure and temperature, it will expel water and will be compressed which will result in a higher measured density.⁷ Additionally, numerous metallic atoms of differing weights (aluminum, iron, magnesium, sodium, and potassium) can be found in the clay lattices. Since the percentage of these can vary for a given clay type, a wide range of densities is possible.

Two characteristics of clays that strongly affect their electrical behavior are the tremendous surface-to-volume ratios of most clays and the charge imbalance along the surface of clay minerals. Two conditions allow for a concentration of ions in the water close to the mineral surface and are quite different away from it. The charge imbalance is due to a greater number of negatively charged atoms than positively charged atoms on the clay mineral surface. The brine in the pore system consists of both anions (negative ions) and cations (positive ions). Since opposite charges attract and similar charges repel, the water immediately adjacent to the clay mineral surface is cation enriched and anion deficient. The presence of this cation-rich water layer provides a pronounced electrical current path. The amount of charge that may be concentrated by clays is quantifiable and is referred to as cation exchange

capacity (CEC).⁸

The property of clays that strongly affects the petrophysical properties of sandstones is the large specific surface area (surface exposed to fluid per unit volume of solid) of clays compared to any other group of minerals (Table 1). An idea about the strong inverse relationship between specific surface area (S_o) and permeability (K) can be had from the following theoretical equation of Kozeny and Carman:⁹

$$K = \frac{\phi^3}{K_e S_o^2 (1 - \phi)^2}$$

where ϕ = porosity and K_e = Kozeny constant.

From the above equation, it may be noted that permeability varies inversely as the square of specific surface area where: s porosity has almost a linear relationship with specific surface area. The strong inverse relationship between specific surface area and permeability and relatively minor effect of specific surface area on porosity is illustrated by a plot of gamma ray log derived total clay volume against the core measured air permeability for the UA-5B sandstone interval in well 120 from Patrick Draw (WY) field, (Fig. 1). The clay in this interval is dominantly kaolinite as indicated by XRD analysis (Table 2B) of core samples conducted by a service company. Note from Table 1 that the dry rock specific surface area of kaolinite is much smaller (20 m²/g) compared to that of the other clays but significantly higher than that in matrix rocks such as quartz (0.15 cm²/g). In the presence of clays with higher surface area, the reduction in permeability is expected to be more drastic than that seen in Fig. 1.

WIRELINE LOG RESPONSES OF DIFFERENT CLAY AND OTHER MINERALS

Table 2A shows the mineral percentages of sandstones in well W-7 from Bell Creek (MT) field, and in Table 2B, for well 120 from Patrick Draw field. The mineral percentages were obtained from XRD analyses of core samples at certain depths. For some of the depths, SEM photomicrographs were also available that allowed evaluation of the clay types and their mode of dispersal within the sandstone porespace.

From careful log, core correlation, the responses of the density, sonic, gamma ray derived V_{cl} , neutron logs were noted at the exact depths for which XRD analysis is available (Tables 2A and 2B). It may be seen from Tables 2A and 2B and also from Table 1 that gamma ray logs respond very well to the presence of clays although, depending upon the clay type, the effect can be quite variable. From Tables 1, 2A, and 2B it may be seen that the density logs may be suitable for distinguishing chlorite type of clays with iron minerals and the illitic clays may

be distinguishable from a combination of density and gamma ray logs. High hydrogen content of chlorite and kaolinite may allow recognition of these clays on the neutron logs when used in combination with another type of log such as the density or the sonic. The very high surface area and CEC values of smectite reduces the resistivity readings in sandstones with these type of clays. When uncompacted, smectite also has much lower density (around 2.45 g/cm³) compared to the density of quartz (2.65 g/cm³) and therefore could be identified on a total clay content, density plot.

Determination of Total Clay Volume

An important first step in the evaluation of any sandstone reservoir is the estimation of total clay content in the reservoir because all flow properties in the reservoir are strongly controlled by the total clay volume percent (V_{cl}). Although different logging methods have been used to determine clay volumes, gamma ray logs have been found to be the most effective.¹⁰ An important source of error in the calculation of clay volumes at Bell Creek and Patrick Draw fields was observed to be the presence of fairly significant amounts of non-clay, radioactive minerals (like potassium feldspar, mica etc) which emit gamma rays and, therefore, contribute to the total clay volume estimated by the gamma ray log.¹⁰ The result is a net overestimation of the clay volume derived from the gamma ray logs. For example, in Fig. 2 is shown a plot of gamma ray derived V_{cl} for certain depths for well Arch 120 from Patrick Draw field for which XRD information is available. The second graph is a plot of the variation in the total percentages of clay minerals (consisting of kaolinite, illite, and smectite) from XRD analysis, and the third graph is a plot of the total percentages of radioactive mineral content (both clay and non-clay) from XRD analysis at the different depths. The V_{cl} values are systematically higher than the clay mineral content values, suggesting perhaps that other minerals have also contributed in the the total clay volume estimates. When the percentages of other non-clay minerals are also added (Fig. 2), there is much better agreement with the V_{cl} graph obtained from the gamma ray log.

Note, however, that there are various sources of error in a direct comparison of total clay mineral percentage with the clay volume derived from gamma ray log. Firstly, not all minerals (clay and non-clay) emit equal amounts of gamma rays so their contribution to total gamma ray count will be variable. While the XRD analyses are estimations over a very small volume of rock, the gamma ray logs record emissions within a much larger rock volume (around 3 to 4 ft³). In spite of these possible sources of error, it would seem from the trend of the graphs (Fig. 2) that the effect of the non-clay minerals must be removed to obtain precise clay volume estimations from the gamma ray log.

DISCRIMINATION OF DIFFERENT TYPES OF CLAY AND OTHER MINERALS ON CROSSPLOTS

From Tables 1, 2A and 2B and data of physical properties of different minerals,¹¹ it may be seen that under appropriate conditions where there is not much mixing of different minerals and lithologies, combined interpretation of two or three logs such as density and gamma ray logs may be used as a discriminator for identifying certain clay and other mineral types usually encountered in elastic reservoir rocks. To study the effect of density variations at small depth interval (say every 2 in.) on the wireline logs, CT scans using 8-mm-thick X-ray beams were performed on a number of core samples from different fields, such as Patrick Draw, Bell Creek, and Fort Barre (LA). A plot of CT densities of the producing UA-5B sandstones from well Arch 120 in Patrick Draw field measured every 2 in. as compared with the clay volumes calculated from the gamma ray logs at 6 in. intervals is shown in Fig. 3. The X-ray diffraction analysis of mineral percentages in the core is given in Table 2B. Excellent correlation between CT density and wireline density has been observed when there is less variation in density within the volume of formation rocks investigated by wireline logs.¹²

Discrimination of Predominantly Kaolinite Type of Clays

Although a mixture of clay types is usually found in sandstone pore spaces, the percentage of one particular type of clay may be proportionately much higher. For predominantly kaolinite type of clays, the bulk density of clayey sandstones will be practically the same as the bulk density of sandstones of low porosity because the density of dry kaolinite clays (around 2.64 g/cm³) is very close to the density of matrix rocks (density of quartz is around 2.65 g/cm³). This may be noted from Fig. 3 where the bulk density remains practically constant although the clay volume varies appreciably along the depth. The sharp rise in CT density at certain depths in Fig 3, such as at 4,959 ft, was because these were sealed core samples and unlike the other samples which were dry, had some fluid in the pore space.¹²

A crossplot of clay volumes (V_{cl}) and bulk densities (ρ_r) of UA-5B sandstones from well Arch 120 in Patrick Draw field, obtained from gamma ray and density logs respectively, is shown in Fig. 4. Most of the points are clustered within a small circle, and from core, log correlations these points within the circle are known to belong to the main sandstone unit. Quite distinct from this random distribution of points within the circular region are two other distribution trends — one trending northeast and the other east-west. The points in the northeast trend are from the shaley bottom part of the sandstone where the carbonate percentage (mostly dolomite)

in the sandstone (indicated by higher density values) is increasing with depth. The east-west trend is again due to increasing percentage of carbonate rocks (mostly calcite) from the top part of the sandstone where the shale percentage remains practically constant. The points in these two trends are, therefore, not from the main sandstone unit.

From such a plot, therefore, carbonate rocks are distinguishable by their very high density and significantly lower porosity values. Also, calcite beds could be distinguished from dolomites because, unlike the dolomites, which show some radio-activity, pure calcite beds do not emit gamma rays. The V_{cl} calculated from calcite-cemented sandstones will not vary much if the shale content within the radius of investigation of gamma ray logs does not change appreciably.

In the main UA-5B sandstone unit, the points in the crossplot (Fig. 4) are distributed around a central point whose density and clay values are 2.31 g/cm³ and 6.80%, respectively. This shows that for points within the small circle where clays are predominantly kaolinites, the bulk densities of sandstones change little although there is moderate variation in clay content. This was also evident from the plot of CT scan density against the gamma ray derived clay content for the UA-5B sandstone (Fig. 3).

Similar plots were also made for wells P-2, W-16, and W-7 in Bell Creek field, and for well Wyble No. 1 from Port Barre field. The producing sandstones in Bell Creek field consisted of two genetically different sandstones;¹³⁻¹⁴ the lower, Barrier Island sandstone had predominantly kaolinitic clays (table 2A) with smaller amounts of illites and other clays. The crossplot of clay volume against the bulk density of the barrier island sandstones for well P-2 (Fig. 5) is similar to the plot for UA-5B sandstones in Arch 120 because the clays in both wells are predominantly kaolinites. Points 19-22 (sequentially plotted from the top of the sandstone at every 1-ft interval) in this plot are from the lower shaley part of the sandstone, and unlike the points from the main sandstone which are bunched together, these generally show an increase in clay content with depth. From the distribution of these shaley points with depth, it appears that an increase of density of these points with depth could be the effect of sediment compaction and not due to non-radioactive minerals like calcite in which case an increase in density will not be accompanied by a corresponding increase in gamma radiation or the calculated values of V_{cl} . This is also indicated by the low neutron porosity (due to low hydrogen content) of these points. The higher clay content could be due to an increase in percentage of non-kaolinite clays like illite.

To summarize, it may be noted from the V_{cl} , density plot of sandstones in wells Arch 120 and well P-2 discussed above, that when the clays are dominantly

kaolinites, the points from the main sandstone units are bunched together in the crossplot within a small circle. When the heterogeneity (such as clay volume and type) in the sandstone increases, the radius of the circle will also increase. Also, depending upon the volume of kaolinite clays, the circle will migrate up and down close to the V_{cl} axis, i.e., always within a relatively narrow range of bulk density of the clayey sandstones. The presence of kaolinite clays may be further corroborated from the neutron porosity, density crossplot. Because of high hydrogen index, kaolinite clays will indicate high neutron porosity unless the clays are very compacted, in which case the bulk density of the sandstone will be high (up to 2.65 g/cm^3).

Reservoir Rocks with Mixed Clay Types

The clay volume, density plot for the upper, valley-fill sandstones in well P-2 (Fig. 6) show wide scatter of points indicative of highly variable types and volumes of clays in the sandstone. Additional plots completed for wells W-7 and W-16 also indicate similar distribution of points for valley-fill sandbodies which have a mixture of kaolinite, illite and smectite type of clays (see Table 2A, estuarine, valley-fill facies). The points on the crossplot for valley-fill sandstones are distributed along a northeast-southwest trend and cover much wider clay volume and density ranges than the barrier island sandstones. Points 1 and 2 in this plot are in dominantly smectite clays which normally exhibit higher radioactivity than kaolinite clays. The smectites could assume a wide range of density values depending upon compaction. The low-porosity, high-density compacted smectites for depths 1 and 2 also have relatively low neutron porosity (low hydrogen index).

Well Wyble No. 1 in Port Barre field has penetrated highly clayey, fining upward sequences of deltaic, Eocene aged Wilcox sandstones. The depositional environment of the sandstones has been described as distributary mouth bar. From XRD analysis, the kaolinite clay composition in the sandstone is somewhat smaller compared to illites and chlorites. The measured CT density of the sandstone is much higher compared to those from Bell Creek and Patrick Draw fields. The high-density chlorite clays with iron minerals is distinguishable from other clays from a combination of density and neutron logs (high hydrogen content of chlorite clays).

Recognition of Cemented Sandstones

While the clay content in the barrier island sandstone in Well P-2 is uniformly low (around 1 to 2%), in well W-16, it is highly variable, and for certain depths it is as high as 16%. For certain depths, there is an appreciable amount of cements (mostly clay and quartz cements) which clog the pore space and drastically reduce permeability.¹³⁻¹⁵ Because of the presence of the relatively higher density cements, the points in the clay volume, density plot for this well are not bunched together but are extended along the density axis. The crossplot for the barrier island

sandstones in W-7 also show distribution of points parallel to the density axis within a narrow clay volume range as for well W-16, but the densities do not reach very high values. This is believed to be due to a lesser degree of higher density cements in well W-7 compared to that in well W-16.

Summary of Crossplot Studies

From investigations carried out so far on discrimination of clay types on crossplots of different logs, the following may be concluded:

1. While gamma ray logs provide the best source of data for total clay volume estimation in most cases, the method gives the upper limit of clay content because of the presence of non-clay radioactive minerals. Some knowledge about the percentages of the various radioactive minerals is required before precise estimations of clay volumes can be made.
2. Sandstones with predominantly kaolinite type of clays may be discriminated on the V_{cl} , ρ_r crossplot. The radius of the circle of distribution of the points on the crossplot is proportional to the variation in clay content and type in the sandstone. For predominantly kaolinite type of clays the points will be essentially randomly distributed within a small circle.
3. The presence of cemented sandstones will extend the points along the density axis but will cover only a narrow range on the clay volume axis.
4. The presence of mixed layer clays will have the effect of larger scatter of points on the crossplot particularly if the proportion of different clays vary significantly.
5. Chlorite type of clays are distinguishable from a combination of the density and the neutron logs.

MATHEMATICAL ANALYSIS OF LOG SIGNATURES

The presence of different types and amounts of clays in a reservoir strongly affect its petrophysical properties like permeability, porosity and water saturation. The effect of the clay distribution is reflected on the core-derived petrophysical properties and also on the signature of wireline logs. A mathematical analysis of the core-derived permeability and wireline log data will be conducted to derive an effective measure of the variability in reservoir property distribution caused by clay content. This may provide a useful tool for quantifying the effect of clay distribution in various oil production operations.

Introduction

Information about the vertical variability of reservoir rock properties such as permeability, porosity, fluid

saturation, lithology, and type and volumes of clay, is of critical importance for exploitation of hydrocarbon resources and effective planning of enhanced recovery projects. The vertical sweep efficiency of a waterflood depends primarily upon the distribution of permeability within the reservoir and also upon the mobility ratio.¹⁶ As a consequence of non-uniform permeability, any injected fluid will move through the reservoir as an irregular front. In a miscible slug process, effective design of a slug required to maintain miscibility throughout the displacement process requires knowledge about the degree of stratification of the reservoir.¹⁷ Froning and Treiber¹⁸ reported how the distribution of clays in reservoir pore surfaces may affect micellar design in several ways but primarily by higher adsorption of surfactants in high surface area clays compared to that on silica. They concluded that the amount of clay and the nature of cation present (i.e., Na⁺ and Ca⁺⁺) will affect the ionic environment contacted by micellar fluids as it moves through the reservoir. Consequently, pre-flush fluids and chemical slugs need to be designed with consideration of the distribution of the type and volume of clays in the reservoir.

Information on variability of rock properties is important in several geoscientific disciplines; a geologist engaged in the interpretation of the environment of deposition of nearshore lagoonal sediments may find less variability in clay content in the low energy sediments than in a nearby high energy channel sand deposit. Besides depositional variability of the different rock facies, petroleum reservoirs are composed of a wide variety of other heterogeneities caused by diagenetic alterations, structural/stratigraphic settings, pore structure, fracture patterns, capillary forces, miscibility and saturation variations etc., all of which may affect fluid recovery from hydrocarbon reservoirs.

A realistic measure of variability of rock properties that is easy to compute and at the same time contains the essential information required for different applications is not readily available. Statistical parameters such as mean, standard deviation or kurtosis while easy to compute are generally poor measures of variability that do not always bring out the character of a distribution. Miller and Lents,¹⁹ Elkins and Skov,²⁰ and others have extensively used simple measures of permeability stratification by relating the portion of the cumulative flow capacity (product of permeability and thickness) to the portion of the total reservoir thickness. While these extensively used permeability measures could explain fluid production in certain projects, the complex heterogeneity character of certain permeability distributions cannot be incorporated in such heterogeneity measures. Some of the proposed mathematical techniques are too involved and may not be practical for routine applications.²¹⁻²³ Alpay's method of heterogeneity mapping due to textural and physical variation requires construction of detailed maps²⁴ with log and core data and is unsuitable for all types of lithologies

or for comparison of relative heterogeneity character of two distributions.

A common measure of reservoir heterogeneity often used in reservoir engineering is the Lorenz coefficient in which, for different layers, the cumulative fraction of total flow capacity is plotted against the cumulative storage capacity (product of porosity and thickness). The area between the derived curve and a 45° line is defined as the Lorenz coefficient.¹⁶

A second, perhaps most popular measure of permeability variation, V, was introduced by Dykstra and Parsons.^{16,25} They calculated the spread of a set of permeabilities having log normal distributions as follows:

$$V = \frac{\bar{k} - k_o}{k}$$

where

$\bar{k}$ = mean permeability

k_o = permeability at 84.1% of the cumulative sample

While both the Lorenz and Dykstra Parsons coefficients have been shown to be useful indicators of the level of heterogeneity in a variety of fluid recovery projects^{16,25} and modeling studies have demonstrated the strong correlation of these measures with performance²⁶ in at least the high heterogeneity reservoirs, Jensen and Lake and others²⁶ have shown that for non-lognormal distributions the two coefficients are not unique. The two coefficients are also influenced by the number of data points in the sample.²⁶ Since reservoir permeabilities are known to exhibit different distributions, the two coefficients may not yield good variability measures.

From analysis of various measures of reservoir variability, it would seem that in order to realistically bring out the true heterogeneity character of a distribution for various applications, the measure of variability should not depend on a single parameter as in the case of the Lorenz or the Dykstra Parsons coefficients but should be based on several other features of the distributions.

The new measure of reservoir variability proposed is based on the analysis of the frequency content of a series of equidistant observations of a reservoir rock property along the depth of a well that are typically obtained from logs or from laboratory measurements of core samples. It has been demonstrated²⁷⁻²⁹ that such a series consisting of equidistant readings of reservoir properties may be considered as a statistical sample from an infinity of other realizations of the data series, and techniques of time series analysis may be applied to such a series to extract useful information about the distribution. In its original application, time series was only applied to data which

were functions of time, but it has subsequently been applied to a wide variety of physical problems that do not involve time.

The pioneering work of Preston and Henderson³⁰ made use of harmonic analysis using Fourier series and characterization of subsurface formation using electrical log data by means of the 'discrete power spectrum'. Frequency domain study of log data has been reported for improving resolution of data,³¹ to investigate log repeatability³² and to classify reservoirs as random or stratified.³³

In this investigation, certain statistical parameters such as 'power spectrum' and 'coherency' have been estimated from the distributions of reservoir properties obtained from log or core data, and the decay of statistical parameters with frequency have been correlated with known heterogeneities of the original distribution and available production data. Certain diagnostic features of the statistical parameters have been identified that can effectively quantify certain types of heterogeneities in the original distribution.

ESTIMATION OF STATISTICAL PARAMETERS OF RESERVOIR PROPERTY DISTRIBUTIONS

Computation Theory

The computation of the statistical parameters is initiated by calculation of the covariances between the n observations, x_t (where $t = 1, 2, \dots, n$) making up a stationary time series. The covariances give a measure of the correlation between the contiguous values. Whenever only one series is considered, the covariance becomes the autocovariance function, $\rho(\tau)$, of the series x_t . It has been demonstrated²⁷⁻²⁹ that the autocovariance function and the power spectrum, $f(\omega)$ are Fourier transforms of one another. Knowledge of one, therefore, implies knowledge of the other. The Fourier transformation that yields $f(\omega)$ from x_t may be stated as follows:

$$f(\omega) = \int_{-\infty}^{\infty} \rho(\tau) \cos \omega \tau \, d\tau$$

and,

$$\rho(\tau) = \int_{-\infty}^{\infty} f(\omega) \cos \omega \tau \, d\omega$$

The above equation shows that at lag, $\tau = 0$

$$\rho(0) = v = \int_{-\infty}^{\infty} f(\omega) \, d\omega \dots \dots \dots (1)$$

where v is the variance of the series.

It will be shown that mathematical manipulation of the data is much simpler in the frequency domain than in the depth domain.

Another parameter that is very well suited for comparison of similarity between two series in the frequency domain is the coherency spectrum. Coherency

reaches the maximum at that frequency at which both series are the most similar.

The equations involved in the calculations of the statistical parameters, power spectra, coherency, phase are discussed in the literature.²⁷⁻²⁹ A computer program was developed for calculating the statistical parameters from a distribution of reservoir rock property available at equal depth interval. The program also integrates the area under the power curve to estimate the variability distributions in the different quadrants of the frequency scale. The computer codes for developing autocovariance function, the power spectra, the coherency and phase spectra are given in appendix A.

If the function $x(t)$ where $t = 1, 2, 3, \dots, n$ represents a certain type of reservoir rock property at equal depth interval, then the smoothed power spectrum $f(\omega)$ is given as follows:²⁷⁻²⁹

$$f(\omega) = \left\{ c_{xx}(0) + 2 \sum_{k=1}^{L-1} c_{xx}(k) W(k) \cos 2\pi \omega k \Delta \right\},$$

$$0 \leq f \leq \frac{1}{2\Delta}$$

where ω = frequency
 Δ = sampling interval
 $c_{xx}(k)$ = autocovariance function at lag, k
 $W(k)$ = Tukey window for smoothed spectrum
 L = length of sub-group for smoothed spectrum estimation and is equal to total number of covariance lags

The formulae used for the different cross spectral estimates are as follows:²⁷⁻²⁹

I. The smoothed cross amplitude spectral estimate, $\bar{A}_{12}(i)$, given by

$$\bar{A}_{12}(i) = \sqrt{\frac{\bar{L}_{12}^2(i) + \bar{Q}_{12}^2(i)}{2}}, \quad 0 \leq i \leq F$$

II. The smoothed phase spectral estimate $\bar{F}_{12}(i)$, given by,

$$\bar{F}_{12}(i) = \text{Arctan} \left(-\frac{\bar{Q}_{12}(i)}{\bar{L}_{12}(i)} \right)$$

$$0 \leq i \leq F$$

The smoothed squared coherency spectral estimate, $\bar{k}_{12}^2(i)$, given by

$$\bar{k}_{12}^2(i) = \frac{\bar{A}_{12}^2(i)}{\bar{C}_{11}(i) \bar{C}_{22}(i)}, \quad 0 \leq i \leq F$$

where
 i = frequency for calculation of cross spectra
 $\bar{C}_{11}(i)$ = the smoothed spectral estimate of series 1.

- $\bar{C}_{22}(i)$ = the smoothed spectral estimate of series 2.
 $\bar{L}_{12}(i)$ = smoothed co-spectral estimates
 $\bar{Q}_{12}(i)$ = smoothed quadrature spectral estimates
 F = 2 to 3 times L

The smoothed spectral estimates are computed at frequencies $0, \frac{1}{2F}, \dots, \frac{1}{2\Delta}$.

The techniques of time series analysis as used in this study are based on the stationarity of the time series i.e., any trend present in the data had to be removed before calculations could start.²⁷⁻²⁹ The presence of a linear trend in the data was removed by fitting a straight line through the data points. The highest frequency or the smallest period of the wave for which the parameters could be estimated was determined by the depth interval between two successive data points. This is because the smallest period of the wave for which computations may be made is twice the vertical depth interval.²⁷⁻²⁹ For example, for reservoir data at 0.5-ft-depth intervals, the highest frequency for which the parameters may be estimated is 1 cycle/ft.²⁶ The frequency spectrum of the wireline log data is the product of two spectra; one of these is the response of the logging tool and the other the response of the distribution of the measured parameter in the subsurface. In the comparison of spectra of different log curves it is assumed that the responses of the logging tools are similar.

Frequency Content of Log Signatures

If the function $x(t)$, where $t = 1, 2, 3, \dots, n$ represents a certain type of reservoir rock property as a function of depth, t , (along the depth of a well), then by Fourier transformation the same series may be converted into an evenly sampled series in spatial frequency, $X(f)$ as follows:²⁷⁻²⁹

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt$$

where $j = \sqrt{-1}$
 f = frequency

Figure 7 shows the plots of sonic transit times and gamma ray readings across two producing sandstones, UA-5 and UA-6 in well Arch 81 from Patrick Draw (WY) field obtained by digitizing the wireline logs in this well at 0.5-ft intervals. Because of deposition in a more lagoonal environment, the UA-6 sandstone is finer grained and has abundant clay and silt particles in the sandstone pore spaces³⁴ (Fig. 1), compared to the UA-5 sandstone which is generally coarser grained, and the presence of abundant coal beds and carbonate rocks add significantly to the vertical variation of sonic transit times in this sandstone. The amplitude spectrum of the sonic transit times across the UA-6 sandstones (Fig. 8) obtained by Fourier

transformation, indicates sharp attenuation of amplitudes at higher frequencies. The uniformly abundant clay minerals in this sandstone is dominantly kaolinite which has a density very close to the density of the matrix rocks (2.64 g/cm³). This explains why the sonic transit times across this sandstone does not show much variation. The amplitude, frequency spectrum of transit times across UA-5 sandstone, on the other hand, shows much slower attenuation of amplitudes at higher frequencies and also the amplitude spectrum shows periodicity (higher amplitudes) at a frequency of 0.4 cycles/ft and from the relationship $f = \frac{1}{T}$ where f is the frequency and T is the wavelength, this converts to a period of 2.5 ft. This periodicity is due to the presence of higher density carbonate and also low density coal beds of approximately 2 to 3 ft thickness.

COMPUTATION OF STATISTICAL PARAMETERS FROM FIELD DATA

The statistical parameters of reservoir rock property distributions were computed for data from Bell Creek (MT) field and Patrick Draw (WY) field. These two fields were selected due to the availability of log, core, production data from all the wells and thin section, XRD, SEM and CT-scan data from selected wells for detailed analysis of vertical distribution of reservoir rock properties.

Analysis of Data From Bell Creek Field

Background Information

The productive interval at Bell Creek field, located in the northeastern flank of Powder River Basin, Montana, is the Lower Cretaceous Muddy formation, which is composed of two genetically different major reservoir sandstone units interpreted as:^{13-14,35} (1) barrier islands (littoral marine bars) and (2) valley fills. The upper, clay-rich, valley-fill sandstones generally have much poorer reservoir properties compared to the lower, high productive foreshore and shoreface facies of the barrier island sandstones.

Our analysis was carried out in a few selected wells from four sections located in Unit 'A' (the largest and the most productive of the six units) of the field. The locations of wells studied (which includes wells drilled for primary production, waterflood, and chemical injection) and the Tertiary Incentive Project (TIP) area where a chemical flood was initiated in 1980 are shown in Fig. 9.

The Dykstra-Parsons coefficients (Fig. 10), computed from core measurements of air permeability data, have low values in the central part of the study area, moderately high values in the eastern and the western part, and significantly higher values in the southwestern part. The generally uniform permeability distribution in the central part of study area (low Dykstra Parsons coefficients) is due to the deposition of clean, high-permeability sandstones with low clay content, whereas the southwestern area with high Dykstra Parsons coefficients has large vertical variation of

permeability mainly due to significant amount of diagenetic clays in the sandstone porespace.^{8,13-14,36}

Two regional cross sections DD' and EE' of the distributions of petrophysical properties along and perpendicular to the depositional strike of the barrier island sandstones constructed with core and log data are shown in Fig. 11. The best development of the barrier island sandstones from this figure is toward the northeastern part of section DD', in the central part of the study area. There is a gradual deterioration of sandstone quality in the westerly and southwesterly directions and in the south-central part of the study area (wells 27-14, W-16 of Fig. 9) where the permeability of the sandstones has been drastically reduced due to large amounts of diagenetic clays in the sandstone porespace.¹³⁻¹⁴ Within the barrier sandstones, the clays exhibit a 2:1 ratio between kaolinite and illite and ranges up to a maximum of 16%.

Statistical Parameters of Permeability Data

The statistical parameters were computed for air permeability data from the barrier island sandstone facies for seven wells (well nos. 26-4, P-2, 27-1, C-8, W-14, W-16 and 27-14, see Fig. 9 for locations) and a hypothetical, ideal profile with practically zero variability. These wells were selected for analysis for their wide variations in the vertical distribution of permeability and oil production performances. The eight permeability distributions were arbitrarily divided into two sets for plotting. The natural logarithms of permeability of the two data sets are plotted in Figs. 12 and 13 as functions of depth and their power spectra in Figs. 14 and 15, respectively.

The mean, variance and calculated Dykstra Parsons coefficients of permeability variations in the seven wells are listed in Table 3. In some of the more heterogeneous wells, the permeability distributions were not strictly lognormal, and the calculated Dykstra Parsons coefficients for these wells were the best estimates. The initial primary production rates and cumulative waterflood production for all the wells studied and tertiary oil production rates in the vicinity of the four wells located within the Tertiary Incentive Project area where a polymer chemical flood was conducted, are listed in Table 4. For those wells drilled subsequent to initial primary production phase, extrapolated values from closely spaced production contours were used. Pay thicknesses for the valley-fill and the barrier island sandstones have also been tabulated assuming a cutoff of 69 mD for pay.

Analysis of Power Spectra

From Eq. 1 the total variance v , of the permeability distribution is expressible as follows:

$$v = \int_{-\infty}^{+\infty} f(\omega) d\omega$$

where $f(\omega)$ is the power spectrum at frequency ω .

Numerical integration of the area within each power distribution was carried out for the entire frequency range for which computations were performed and results shown in Table 3. The close agreement between the variances of the log permeability data and the area under the power curve for each profile underscores the accuracy that may be obtained from calculations in the frequency domain. To quantitatively estimate the contributions to the total variance in the different frequency ranges, the area under the power curve for each of the four quadrants into which the frequency scale has been divided into has been tabulated (Table 5). The area within each quadrant represents the contribution to the total variance by frequencies within that quadrant. Below the area for each quadrant is also shown the normalized area for that quadrant obtained by dividing the computed area by the variance of each series. The normalized values are useful for comparison of series having different variances. The computer codes for the automatic computation of area under the power curve are given in Appendix A.

The power spectra for the different distributions will now be discussed in increasing order of vertical variability of permeability as given by their variances (Table 3). The spectrum for the ideal profile has very low powers for the entire frequency range because of very low permeability variations for this profile. The sandstone in well 27-1 has one of the best petrophysical properties and primary oil production rates per foot of pay in the entire field. The power distribution for this well shows low values for the entire frequency range and this, coupled with the fact that power does not drastically change for the entire frequency range, suggest that the sandstone in this well is probably massive with more or less uniform permeability distribution. Compared to this well, which has an average clay content of about 1% and a geometric average permeability of 5,760 mD for the barrier island facies, the average clay contents and permeabilities in wells 26-4 and P-2 are around 2.5% and 5,000 mD and 4% and 2,730 mD, respectively. In these three wells, the higher clay content is probably reflected in the higher variances in the power spectra in the mid-frequency range (quadrants 2 and 3, Table 5).

The vertical variability of permeability increases significantly in well C-8 located only 800 ft southwest of well 27-1 primarily because of higher average clay content (around 5%) that has reduced the average permeability of the sandstone to 2,160 mD.⁸ The higher variability in this sandstone is reflected on the higher variances throughout the frequency range compared to the three wells discussed earlier. The power curve also displays some periodicity with comparatively lower power and hence low variances for certain frequency bands, such as at frequencies 0.5 and 0.8 cycles/ft. The variability in the well W-14 sandstone, with an average permeability of 455 mD follows the same general pattern as in well C-8 excepting that the variances are an order of magnitude higher in this well for all

frequencies. Note the considerably higher thin bed stratification in this well indicated by higher variances at the high frequency end compared to the wells discussed earlier.

With a total clay content averaging 16 % (3% matrix and 13% authigenic clay) the barrier island sandstone in well 27-14 contains the highest percentage of clay^{13-14,36} among all the wells studied in the field. The average permeability of the sandstone is 304 mD, and it fluctuates between 36 and 948 mD across the producing zone and consequently the power spectrum for this well has very high power throughout the frequency range. Although the average total clay content in well W-16 is slightly lower (around 10%) than in well 27-14, the average cement content in this sandstone is much higher (around 9%).^{13-14,35} This has not only lowered the average permeability of this sandstone to 244 mD but has caused highly fluctuating permeability distribution that raised the power significantly higher than in well 27-14 for most frequencies.

Correlation of Power Spectra with Production

The Muddy sandstone reservoir at Bell Creek field produces from the non-barrier valleyfill as well as the barrier island sandstones, although in this analysis, only the higher productive barrier island sandstones have been discussed. Compared to the barrier island sandstones, the average production from the valley-fill sandstones is much lower, only about an eighth of the production from the high productive facies.¹³ The pay thicknesses of both the barrier and the non-barrier sandstones for the wells analyzed are shown in Table 4.

The initial primary oil production rates per foot of pay (Table 4) generally follow the heterogeneity character of the sandstones indicated by the distribution of variances as given by their power spectra. The lowest primary oil production (per foot of pay) in well W-16 may be attributed to very high variance in this well for the entire frequency range. The more massive sandstones with less permeability variations (wells 27-1 and P-2) show higher productivity and productivity decreases in wells with high permeability swings (wells 27-14 and W-16). This is illustrated graphically in Fig. 16 where the initial oil production rates at the different well locations have been plotted against the areas under the power curves of the permeability distributions of the different wells in quadrant 1 and in quadrants 1 and 2 (combined). The Dykstra-Parsons coefficients of permeability variations have also been plotted (Fig. 16) for comparison. Compared to this conventional measure of permeability variation the new measure shows much better correlation with oil production rates. It may be noted that the initial production correlates better with areas in quadrants 1 and 2 because these quadrants represent the contribution from the thicker beds. In determining the initial production rates per foot of pay, an effective thickness for the non-barrier sandstones was estimated for each well because the productivity of the non-barrier

sandstones is usually quite different from the productivity of the barrier sandstones. This was done by assuming that the contribution to production by each sandstone was proportional to the permeability and thickness product (KH).

The vertical variability has affected cumulative waterflood production much more than the primary production due to poor sweep efficiency in stratified reservoirs and the greater the stratification the lower the production. The waterflood production in well W-14 is below the trend observed in the other wells, and there may be other factors responsible for the low waterflood production in this well. The graphical representation in Fig. 17 shows that for highly heterogeneous wells (high Dykstra-Parsons coefficients) and also for clean sandstones (low Dykstra-Parsons coefficients) the area under quadrants 3 and 4 gives a much better correlation with waterflood production rates than the conventional measure. As mentioned previously, the area under quadrants 3 and 4 represents contributions to power or variability from thin beds (thickness around 2 ft or less); therefore, this gives a measure of the thin bed stratification in the reservoir.

The approximate tertiary, chemical flood production in the four wells analyzed (Table 4) seems to have been strongly affected by the distributions of clays in the sandstone porespace^{15,35} which in turn has introduced variability in permeability distributions in the sandstones. The highest production in well 27-1 is clearly because the sandstone in this well is very clean and there is very little permeability variation throughout the pay thickness.

Vertical Variability Estimation of Sonic Transit Times

For the case of wireline log data, the variability estimated will be the combined variability of all reservoir rock properties that have affected the particular wireline log being analyzed. The levels of power generated by the different logs will be different because of the different scales of measurements and the different responses of the logging tools. The sonic transit times obtained by digitizing the sonic logs in wells 26-4, P-2, 27-14 and W-16 from Bell Creek Field at 0.5-ft-depth interval are shown in Fig. 18 and the corresponding power spectra in Fig. 19. The power spectra for core derived permeability data for these four wells were discussed earlier.

The transit times shown in Fig. 18 have been affected by several factors but most dominantly by the porosity, the dispersed clays and the cements in the sandstone porespace.^{10,13} The dispersed clays act as slurries and significantly increase the transit times across the clayey sandstones. The presence of hard, cemented beds, on the other hand, greatly reduces the transit times.

The total variability in transit time as given by the variance of the distribution is much greater in well 26-4 than in well P-2 (4.54 as against 0.51 micro-sec² in well P-

2) although both wells have excellent average permeabilities. At lower frequencies, the higher powers in well 26-4 (Fig. 19) are due to the large change in levels of transit times between the clean and the clay filled parts of the sandstone and the relatively lower values for well P-2 is because the small amounts of dispersed clays distributed throughout this sandstone do not cause a significant change in transit times across this sandstone. At the higher frequency end, the much lower power for well 26-4 is due to the low variability of thinner zones which are more clayey in well P-2. Comparing the power decline in well 27-14 with that in well P-2 it may be observed that above 0.6 cycles/ft the power distribution for the two wells is identical, probably due to the presence of thinner beds with similar clay matrix and clay cement content in the two wells. The much higher power and much more gradual decline of this power at lower frequencies in well 27-14 could be attributed to the presence of thicker, clay filled zones some of which could have significant amounts of cement that have caused large swings in transit times across this sandstone. The very large power fluctuations in well W-16 are clearly due to the highly cemented bands in this sandstone that have raised the total variance in transit time for this well to 9.18 micro-sec².

Vertical Variability of Gamma Ray Distributions

Since the clay content in most reservoir rocks has a strong inverse relationship with permeability, the vertical variability of gamma ray count may give a good measure of the vertical variation in permeability in many cases. The vertical variability of gamma rays and sonic transit times were analyzed for the barrier island and non-barrier, valleyfill facies in well W-7 from Bell Creek field (location in Fig. 3). Detailed core, log investigations of the Muddy sandstone in well W-7 were performed earlier.^{13,14} The sonic transit times and the gamma ray distributions in the two sandstone facies in this well are shown in Fig. 20 and the corresponding power spectra in Fig. 21.

The highly shaley, upper sandstones give rise to very high powers for gamma ray distributions compared to the barrier island facies which is relatively much cleaner (Fig 20). The high powers for the transit time spectrum at all frequencies for the non-barrier sandstone is also due to the large fluctuations in transit times caused by variations in clay distributions in the sandstone. The high powers for both the gamma ray and the sonic logs for the non-barrier sandstones is indicative of the very heterogeneous nature of the sandstone i.e. the presence of beds of a wide range of thicknesses that emit gamma rays and that probably have highly varying amounts and types of clays. The gamma ray and the sonic power spectra for the barrier island sandstone, on the otherhand, have relatively low power because the sandstone is much cleaner (2-5 % diagenetic and <1% matrix clays), and it has vertical distribution of clay not much different from well P-2 (see Fig. 18).

Cross Spectral Estimates

Cross spectral coherency and phase estimates of transit

time data in wells P-2 and 26-4 (Fig. 22) indicate a fairly high degree of coherency for practically all frequencies excepting at 0.4 cycles/ft. The coherency and phase of two perfectly matching distributions are 1.0 and 0.0, respectively. The phase spectrum also indicates sharply lower phase at a frequency of 0.4 cycles/ft. The sharp departure of the cross spectral estimates at 0.4 cycles/ft is due to the divergence of amplitudes and phases of the two spectra most likely caused by clay beds of about 2-3 ft thickness in well P-2.

This may be also noted from the power spectra of sonic transit times of the two wells (Fig. 19). At around 0.4 cycles/ft, the two power spectra crosses, and at higher frequencies the spectrum for well P-2 decreases very slowly whereas for well 26-4 the power decline is very rapid. This suggests that although the two wells are only 650 ft apart, thin clay filled beds in well P-2 have increased the sonic transit times in the well. Compared to well P2, the power decline in well 26-4 is very rapid because the sandstone is comparatively much cleaner. A quantitative estimate of the relative amounts of thin clay beds that affected the two wells is given by the total areas under the power curves of Fig. 19 in quadrants 3 and 4.

Analysis of Data From Patrick Draw Field

Background Information

The production in Patrick Draw field from the Greater Green River Basin in SW Wyoming is from shoreline barrier reservoirs from the Almond formation. The bulk of production in the field is from a stratigraphic interval called UA-5 at the top of the Almond formation. UA-5 is further divided vertically into UA-5A (upper) and UA-5B (lower) reservoirs. In contrast to Bell Creek field where the sediment deposition processes are dominantly by wind and wave effects, the sediments in the shoreline barrier reservoirs at Patrick Draw field were primarily deposited by tidal currents.^{34,37} As a consequence of the different depositional processes, the characteristics of the two reservoir sandstones are quite different although both are prolific oil producers.

Analysis of Wireline Log Data

Resolution Limits Of Wireline Log Data

Analysis of data was initiated in UA-5B sandstone in well Arch 120 which was extensively cored and logged for detailed reservoir information. Drilled in 1987, toward the later stages of waterflood production from the area, the well was found to be non-productive although there was significant oil production from the neighboring areas.³⁷ In Fig. 23 is shown the CT-scan and the wireline log densities of the UA-5B sandstones in well Arch 120. The objective of CT scanning the core was to determine if the effect of lithological and mineralogical variations at short

vertical interval could be detected on the CT-scans by comparison of the high resolution CT densities with wireline log densities. Since wireline logs average the effect within a certain rock volume, it does not always have the resolution to map thin clay beds, and the wireline density curve therefore appears much smoother whereas the higher resolution CT density curve shows larger fluctuations. The thin shale beds that are not mappable with wireline density logs may have a strong influence on vertical permeability distribution.

As discussed previously, the period T , of the highest frequency wave obtainable with the digitized data is given by:

$$T = \frac{1}{f_c} = 2x\Delta Z$$

where

f_c = highest frequency obtainable with log data

ΔZ = sampling interval of log data

To study power distribution at higher frequencies, the density distribution must be available at shorter depth interval. The CT-scan data of Fig. 23 were available at depth intervals varying between 0.1-ft and 0.3-ft intervals. A cubicle spline interpolation technique was used to interpolate CT-scan density at regular, 0.2 ft depth intervals. The wireline density log has also been interpolated at 0.2-ft intervals for calculation of the power spectrum at higher frequencies but this spectrum is not expected to contain useful information beyond 1.0 cycles per ft. The two sets of density data were then used to calculate their corresponding power spectra. The power spectrum of the wireline log is shown in Fig. 24. Since the scales of measurement of the wireline and CT-density data are very different, the two power spectra were normalized by dividing by their corresponding variances to give the spectral density functions shown in Fig. 24. It may be seen in this figure that except at very low frequencies, the spectral density for the CT density contain higher values compared to the wireline log data for almost the entire frequency scale. The presence of the higher spectral densities in the CT density data probably implies presence of clay filled beds of a wide range of thicknesses which have caused small variations in the CT density profile. It may be noted that unlike the wireline log, the CT spectrum has useful information upto 2.5 cycles/ft.

The low power of the density log data of Fig. 24 is because of the low contrast in densities between the predominantly kaolinitic clays and the matrix rocks (variance of only 8×10^{-4} units for the sandstone interval). The corresponding variances for the gamma ray log readings and the gamma ray log derived clay content values were estimated to be 8.52 and 1.19 units, respectively. This indicates that for vertical variability estimation of clay content in sandstones with dominantly kaolinite type of clays, the gamma ray log usually is more informative

than the density log. It should, however, be noted that the different levels of power in the density and the gamma ray log is because of the different scales of measurements of the two logs and also because the logs are measuring different rock properties.

The dominant heterogeneities in the UA-5B sandstones in this well are the clays as may be seen from results of the XRD and thin section analysis of core samples from this sandstone (Table 2B) and interpretations of gamma ray log data. The high density values at the top and the bottom of the UA-5B sandstone are mainly due to the carbonate rocks, whereas the middle part of the sandstone has appreciable amounts of clays which are mostly diagenetic kaolinites (Table 2B). The strong inverse relationship between the clay content obtained from interpretations of gamma ray logs and the laboratory measured air permeability values was discussed earlier (Fig. 1). The clay content in the middle part of the sandstone (between 4,946 to 4,965 ft) averages 6.8% and varies between 5.5 % to 9.5 % although from XRD analysis of 7 samples, the clay percentage estimated is slightly less (4-5%). The slightly higher clay values derived from the gamma ray logs could be due to the presence of non-clay, radioactive minerals in the sandstones, like potassium feldspar, mica, etc.,⁷⁻⁸ and also because the gamma ray log is measuring the total volume of shales which is assumed to be the total volume of clays.

Comparison of Relative Variability Studies of a Number of Wells

The vertical variability of permeability in a number of wells from Patrick Draw field were compared from the gamma ray spectra in these wells. The presence of accessory minerals like calcite and dolomite will not be easily recognized on the gamma ray logs but as discussed earlier, the gamma ray logs will provide information on permeability variation of the sandstones. The rock and petrophysical properties, the Dykstra-Parsons coefficient, the porosity, the air permeability and cumulative primary production data for the UA-5B sandstone in these wells are given in Table 6. The clay content values are derived from interpretations of gamma ray logs. The variances of clay distributions in the six series are also shown in the table. The main impediments to fluid flow in the sandstones are the clays and in certain areas, the calcite cements both of which tend to degrade the average permeability of the sandstone.^{34,37}

The gamma ray spectra for four of the six wells in Table 6 are shown in Fig. 25. Well 79 with the least variance in clay distribution, highest average permeability and primary oil production rates, has the lowest power (i.e. variance) at all frequencies. Well 71 with the highest variance in clay content, least average permeability and no oil production, shows high power at low frequencies that decays gradually and relatively high power is maintained at the high frequency end. Located only 1,600 ft apart, the sandstones

in Arch 96 and Arch 120 are quite similar except for the presence of slightly larger amounts of laminated shales in Arch 120. The spectrum for Arch 120 shows small humps at discrete frequencies (Fig. 25) due to the shaley zones in this well which are absent in well 96. The relatively low power at the high frequency end for these two wells indicate absence of a high degree of stratification of very thin beds which have been observed to be controlling the vertical permeability distribution and the average permeability to a large extent (see Table 6). The power distributions for wells 91 & 100 (not shown) fall in between the extreme range of variance observed in wells 79 and 71. In spite of high variance in clay content, well 91 shows good production because for part of the frequencies, the calculated power is low.

Correlation with Production Data

In order to obtain a quantitative estimate of the effect of clay and carbonate cement content on petrophysical properties and oil production rates, the integrated areas under the power curves of gamma ray logs of 9 wells from the Patrick Draw Field are plotted in Fig. 26. From left to right the nine wells in this figure are 100, 103, 96, 101, 104, 91 and 79. In Fig. 26, the areas in quadrant 1 and in quadrants 1 and 2 (combined) have been plotted for each well as a function of the cumulative oil production rates per foot of the sandstone. The geometric means of core-derived permeabilities and the Dykstra-Parsons coefficients of permeability variations are also shown in Fig. 26.

The Dykstra-Parsons coefficients show poor correlation with oil production rates in Patrick Draw Field because of the large-scale heterogeneity of the producing sandstone. The areas under the gamma ray spectra, on the other hand, show a distinct inverse relationship with the geometric mean permeabilities indicating that clay content in the thicker beds strongly control the effective permeabilities of the sandstones. The oil production also shows an overall decline as the area under the power curve increases.

From Fig. 26 it may be noted that although permeability is strongly affected by clay content as indicated by the distribution of variance under the gamma ray power curve, the presence of carbonate cements could also drastically reduce permeabilities of sandstones. Because of higher densities, these thin cemented bands are easily recognized on the density or the sonic logs. The presence of these thin, higher density beds could also be recognized on the power distribution of density logs. Areas under quadrant 4 or under quadrants 3 and 4 combined, will give relative degree of thin carbonate rock stratification in different parts of the oil field. Figure 27 shows the plots of areas under quadrant 3, quadrants 3 and 4 (combined) and the total area in quadrants 1 to 4 as a function of cumulative oil production rates per foot of sand for the same wells as those for the gamma ray plot in Fig. 26. The areas in quadrants 3 and 4 give a clear indication that as carbonate stratification increases the oil production rate diminishes.

Discussion of Results of Mathematical Analysis

The frequency domain analysis of vertical variability of rock properties can separate the total variability of a distribution into a continuous function of frequency. The main advantage of such an analysis is an overall assessment of the quality of reservoir rocks for various applications. It should be emphasized that such studies will only provide information about the vertical variability of the rock property under investigation and not directly about the quality of reservoir rocks although, information on reservoir rock variability, in conjunction with other reservoir information, is a good indicator of reservoir rock quality.

The power spectrum analysis of uniformly clean, high permeability reservoir sandstones indicate that for such a sandstone the power distribution is very low throughout the frequency range. If a part of the sandstone is clayey but the remaining part very clean, the power spectrum will show high values at lower frequencies but will sharply attenuate at higher frequencies and assume very low values at the extreme high frequency end, provided there is no thin bed stratification.

The degree of thin bed stratification is given by the slope and the relatively high powers at higher frequencies i.e. in the third and the fourth quadrants of the frequency scale. The presence of clay-filled beds will cause the power to decay more gradually and will have higher powers in the mid-frequency range i.e. in quadrants 2 and 3 of the frequency scale. If instead of depositional clays, the percentage of diagenetic clays or cements that cause large swings in permeability is significantly higher, then such a lithology will be reflected by very high powers.

The presence and the degree of shale laminations are indicated by the humps in the gamma ray spectrum. The presence of very thin beds that gives rise to stratification in the various log data will always be indicated by high powers at the high frequency end.

The density log may not be suitable for estimating the variability in clay content in sandstones with dominantly kaolinitic clays because of a lack of enough density contrast between the clay and the matrix rocks. The neutron porosity spectrum will give a measure of the variability caused by the kaolinitic clays because of the large response of the neutron logs to the hydrogen content in the kaolinitic clays.

The spectrum of CT density confirmed that information on very thin bed stratification may be quantified if reliable high resolution logs showing vertical property variation at shorter vertical depth interval is available.

The crossplots of coherency and phase with frequency is a very sensitive measure of the similarity and the differences between two series as a function of frequency.

Comparison of the power spectra with primary production data indicates that while the massive sandstones with uniformly low variability has the best production rates, the primary production was also very high in sandstones only part of which has clays as indicated by high power at the low frequency end but power attenuating very sharply at the higher frequencies. The presence of thin, stratified beds indicated by high powers at the high frequency end adversely affected both the permeability as well as the primary oil production. The effect of vertical variability of reservoir properties is more serious in waterflood production where presence of thin, stratified beds of non-uniform permeability has severely reduced sweep efficiency. In the chemical flood project, the total as well as the vertical variation in clay content which have an inverse relationship with permeability, seem to control the oil production. From tables 3 and 4, the best production is encountered in wells with the least permeability variance and the highest mean permeability.

Some of the cemented sandstones having high Dykstra Parsons coefficients had good waterflood production because part of the sandstone was relatively clean and had good permeability. The power spectrum of such a sandstone shows low power at certain frequencies indicating presence of relatively clean, sandstones with low variability.

The amplitude, frequency spectra of logs in certain depositional environments may have a diagnostic pattern of power decay with frequency.

From the estimation of statistical parameters of a large number of well logs it was observed that for accurate variability comparison, it is necessary for the tools to have the same response characteristics. Because of the ease of interpretation of the power spectra from the same depositional setting, it may be important to compare spectra from similar facies. A technique to separate the different facies of the Muddy reservoir has been discussed in the literature¹⁴ and similar crossplot techniques may be applicable in other environments as well.

CONCLUSIONS

1. Crossplots of different rock properties obtained from wireline logs or derived from them provide an effective tool for the identification of the different types of clays. Because the physical properties of clays vary over a wide range, the selection of the appropriate logs for crossplot studies is greatly facilitated if results of XRD analysis of core samples are also available so that the important clay and other minerals may be known. Once the dominant minerals in the pay zone have been identified, one or more log combinations may be used for clay identification.

2. Although different logging combinations, such as the density neutron, density-sonic or sonic-neutron have been used to determine the total clay volumes, for the type of

reservoir rocks investigated in this study, the gamma ray logs should provide the most reliable answer.

3. An objective measure of the relative vertical variability of reservoir rock properties caused by the distribution of the different types and volume of clays may be obtained from spectral analysis of vertical distributions of reservoir rock properties. Such an analysis will provide an overall assessment of reservoir variability that may be useful in various applications.

4. Certain characteristics of the reservoir sandstone, like the presence and the degree of stratification due to shale or clay beds which have a strong effect on primary and more so on waterflood recovery, may be estimated and quantified by the area under the power curve of permeability or any wireline log that shows the effect of shale laminations.

5. Unlike the Dykstra Parsons coefficients which require the permeability distributions to be lognormal to provide a reliable variability information in a clayey sandstone, and is also depended upon the sample size, power spectrum and coherency estimates may be made from any distribution of reservoir properties.

6. The possible introduction of high resolution wireline logs in the future will enable extraction of variability information of very thin beds caused by clay content in the reservoir rocks.

7. The technique of frequency domain analysis to study the effect of clay content in reservoir rocks may be highly automated once the bed boundaries are identified on logs by a suitable technique.

8. The presence of different thicknesses of cemented and clay-filled sandstones may be estimated from the spectra of wireline logs such as the gamma ray or the sonic transit times. Integration of information from spectra of different logs may provide information of clay types if the log spectra have been affected by these logs.

REFERENCES

1. Vitthal, A., A. Gupta, and M. M. Sharma. *A Rule Based System for Estimating Clay Distribution, Morphology, and Formation Damage in Reservoir Rocks*. Pres. at the SPE 62nd Annual Tech. Conf. & Exhibit, Dallas, TX, Sept. 27-30, 1987. SPE Paper 16870.
2. Fertl, W. H., and G. V. Chilingarian. Type and Distribution Model of Clay Minerals from Well Logging Data. *Journal of Petroleum Science & Engineering*, Vol. 3, No. 4, 1990, pp 321-332.
3. Dewan, J. T. *Essentials of Modern Open-hole Log Interpretation*. PennWell Books, Tulsa, OK, 1983, pp. 227-265.
4. Gamma Ray Spectral Logging. *A New Evaluation Frontier*. Reprinted from *World Oil*. Gulf Publishing Co., Houston, TX, 1983.
5. Gardner, J. S. and J. L. Dumanoir. *Litho-Density Log Interpretation*. SPWLA Logging Symposium Transactions, June 1976.
6. Schlumberger Educational Services. *Log Interpretation Principles/Application*. Houston, TX, 1987.

7. Asquith, George B. *Log Evaluation of Shaley Sandstones: A Practical Guide*. AAPG Continuing Education Series #31, Tulsa, OK, 1989.
8. Almon, W. R. *A Geological Appreciation of Shaley Sands*. SPWLA Symposium 1979, paper WW.
9. Dullen, F. A. L. *Porous Media-Fluid Transport and Pore Structure*. Academic Press., New York, New York, 1979.
10. Poupon, A. and R. Gaymard. *The Evaluation Clay Content from Logs*. SPWLA Symposium, 1970, paper G.
11. Crain, E. R. *The Log Analysis Handbook*. PennWell Books Tulsa, OK, 1986.
12. National Institute for Petroleum and Energy Research. *Quarterly Technical Report for April-June 1991, Volume II, Energy Production Research*. DOE Report, NIPER-545, July 1991, pp. 33-34.
13. Sharma, B. M. M. Honarpour, M. J. Szpakiewicz, and R. Schatzinger. Critical Heterogeneities in a Barrier Island Deposit and Their Influence on Primary, Waterflood and Chemical EOR Operations. *SPE Formation Evaluation*, March 1990.
14. Sharma, B., M. M. Honarpour, S. Jackson, R. Schatzinger, and L. Tomutsa. Determining the Productivity of a Barrier Island Sandstone Deposit From Integrated Facies Analysis Based on Log and Core Data and Fluid Production. *SPE Formation Evaluation*, December 1990.
15. Szpakiewicz, M., K. McGee, and B. Sharma. Geological Problems Related to Characterization of Clastic Reservoirs for EOR. *SPE Formation Evaluation*, December 1987, pp. 449-60.
16. Craig, F. F. Jr. The Reservoir Engineering Aspects of Waterflooding, Monograph Series, SPE, Richardson, TX, 1971, v. 3, 62-68, 78-96.
17. Agan, J. B. and Fernandes, R. J. Performance Prediction of a Miscible-Slug Process in a Highly Stratified Reservoir. *JPT*, January 1962, pp. 81-86.
18. Froning, H. F. and Treiber, L. D. Development and Selection of Chemical Systems for Miscible Waterflooding. SPE paper 5816 presented at the 51st Annual Fall Technical Conference and Exhibition, New Orleans, LA, Oct. 3-6, 1976.
19. Miller, M. G. and Lents, M. R. *Performance of Bodcaw Reservoir, Cotton Valley Field Cycling Project: New Methods of Predicting Gas-Condensate Reservoir Performance Under Cycling Operations Compared to Field Data*, *Drilling and Production Practice*, API, 1947, pp. 128-149.
20. Elkins, Lincoln F. and Skov Arlie, M. *Some Field Observations of Heterogeneity of Reservoir Rocks and its Effects on Oil Displacement Efficiency*, paper SPE- 282 presented at the SPE Production Research Symposium, Tulsa, OK., April 12-13, 1962.
21. Testerman J. D. *A Statistical Reservoir-Zonation Technique*, *JPT* Aug., 1962, pp. 889-893.
22. Hutchinson, C. A., Dodge, C. F., Polaseck, T. L. *Identification, Classification, and Prediction of Reservoir Non-Uniformities Affecting Production Operations*. *JPT*, (March 1961), 223.
23. Law, J. Statistical Approach to the interstitial Heterogeneity of Sand Reservoirs, *Trans., AIME* (1944),155-202.
24. Alpay, O. A. *A Practical Approach to Defining Reservoir Heterogeneity*, paper SPE 3608 presented at the 1971 SPE Annual Meeting, New Orleans, Oct. 3-6.
25. Dykstra, H. and Parsons, R. The Prediction of Oil Recovery by Water Flood., *Secondary Recover of Oil in the United States*, second edition, API Dallas (1950), 160-74.
26. Jensen, J. L. and Lake, L. W. *The Influence of Sample Size and Permeability Distribution on Heterogeneity Measure*, *SPE* (May 1988) 629-37.
27. Jenkins, G. M., and Watts, Donald, G. *Spectral Analysis and Its Applications*. Holden-Day, San Francisco, 1968, 141-257.
28. Bendat, S., and Piersol, A. G., *Random Data: Analysis and Measurement Procedure*. *Wiley-Interscience, A Division of John Wiley and Sons, Inc.*, New York, 1971, 286-341.
29. Lee, Y. W. *Statistical Theory Of Communication.*, John Wiley and Sons Inc., New York, 1963.
30. Preston, F. W. and Henderson, S. H. Fourier Series Characterization on Cyclic Sediments for Stratigraphic Correlation, D. F. Merriam, Editor, *Kansas Geological Survey of Cyclic Sediments for Stratigraphic Correlation*, Symposium, August 1974, v.5 pp. 45-549.
31. Branisa, F. *Filtering of Well-Log Curves*, *Geophysics*, v. 39, no. 4, August 1974, 545-549.
32. Kerford, S. and Georgi, G. *Applications Of Time Series Analysis to Wireline Logs*, *Log Analyst*, May-June, 1990.
33. Bennion, D. W. and Hope, A. C. *Application of Power Spectral Analysis and Digital Filtering in Evaluating the Distribution of Rock Properties*, paper SPE 4121 presented at the 47th Annual Fall Meeting of the Society of Petroleum Engineers, Oct. 8-11, 1972 at San Antonio, TX.
34. Weimer, R. J. *Time-Stratigraphic Analysis and Petroleum Accumulations, Patrick Draw Field, Sweetwater County, Wyoming*, *AAPG Bull.*, v. 50, 1966, 2,150-2,175.
35. Final Report for Oct 1,1985 - September 30, 1988. Energy Production Research No. NIPER-390, Contract No. DE-FC22-83FE60149, Bartlesville, OK (Feb. 1989).
36. McGregor, A. A. and Biggs, C. A.: Bell Creek Field, Montana: A Rich Stratigraphic Trap, *AAPG Bulletin*, 1968 52, No. 10, 1869-87.
37. Schatzinger, R. A., M. J. Szpakiewicz, S. R. Jackson, M. M. Chang, B. Sharma, M. K. Tham and A. M. Cheng. Integrated Geological-Engineering Model Of Patrick Draw Field And Examples Of Similarities And Differences Among Various Shoreline Barrier Systems. NIPER-575, Contract No. DE-FC22-83FE60149, Bartlesville, OK, November 1991.

TABLE 1
Important parameters for the major clay mineral groups
(modified after Fentl & Chilingarian, 1990)

Clay minerals	Pd (g/cm ³)	HI	CEC (meq/100 g)	Spectral Gamma Ray Description (average)			Photoelectric P _e (barns/electron)	φ _{CNL}	CEC/HI	Average CEC/HI	Dry rock surface area (m ² /g)	Solubility in Acid	
				K (wt. %)	U (ppm)	Th (ppm)						Hydrochloric	Hydrofluoric
Chlorites	2.58 to 2.94	0.34 HIGH	10 to 40 LOW	0.15 to 0.22		5 to 22	6.32	0.51	0.321 to 11	0.25	100	HIGH	HIGH
Illites	2.64	0.12	10 to 40										
	2.7 to 3.2 2.76 to 3.0	LOW 0.12 0.13	LOW	4.5 6.7 to 8.3 7.9 to 9.8	1.5	<2.0 <0.01 <0.01	3.46	0.24	0.8 to 33	2.1	100	SMALL	SMALL TO MODERATE
Kaolinites	2.6 (theoretical)	0.36	3 to 15										
	2.60 to 2.68 (extensive literature) 2.63 (most frequently quoted)	HIGH	LOW	0.42	1.5 to 3.0	6 to 19	1.84	0.36	0.08 to 0.7	0.25	20	SMALL	SMALL
Smectites	2.53 2.74	LOW	HIGH	0.16	2.0 to 5.0 1 to 20	14 to 24 6 to 50	2.05	0.24	6.1 to 11.5	8.8	700	SMALL	MODERATE

CEC - Cation exchange capacity
GR - Gamma ray
φ_{CNL} - Neutron porosity
U - Uranium
K - Potassium
HI - Hydrogen index
P_T - Average density of rock
Th - Thorium

NOTE:

1. Clays with higher CEC result in higher conductivity (i.e. lower resistivity).
  2. The presence of chlorite and kaolinite in a shaley sand results in higher total neutron porosity, compared with shaley sands with montmorillonite and illite.
  3. Only with montmorillonite and chlorite does the presence of clay greatly affect the density log, because montmorillonite (2.45 g/cm³) and chlorite (2.8 g/cm³) have different densities than quartz (2.65 g/cm³).
  4. The presence of iron (Fe) in clays causes acid sensitivity to HCl.
  5. The spectral gamma ray components cause high gamma ray readings in shales only if the clay in the shale is montmorillonite, illite, or kaolinite.
  6. Compared to the clays the specific surface area of quartz is only 0.15 cm²/gm.
- Note: Clays with high CEC values (Montmorillonite and illite) are called effective clays; clays with low CEC values (Chlorite and Kaolinite) are called noneffective clays.

TABLE 2A
Mineralogical Composition And Wireline Log Responses Of Shaley Sandstone In Well W-7 In Bell Creek Field, Mt. Mineralogical Percentages Obtained From XRD Analysis Of Core Samples

Sample number	1	2	3	4	5	6
Depth, ft	4405.5	4410.0	4417.5	4418.9	4419.5	4431.3
Depositional setting	Eustuarine	Eustuarine	Swamp	Alluvial channel	Top of U.S.	U.S.
Quartz, wt %	88	79	92	96	94	91
Feldspar, wt %	4	4	3	1	2	3
Calcite, wt %	-	-	-	-	-	-
Dolomite, wt %	trc	-	-	-	-	-
Barite	-	-	-	-	-	2
Kaolinite	2	2	3	3	2	2
Illite	trc	trc	trc	trc	2	2
Smectite	6	15	2	trc	trc	trc
<u>Log responses</u>						
Vcl, %	35.3	33.7	8.9	29.6	33.6	1.5
Density, g/cm3	2.418	2.434	2.226	2.354	2.238	2.153
Neutron porosity, %	45	43.5	45	44	45	50.5
S, micro-sec	92.5	92	85	86.5	91.5	103.5

U.S. - Upper shoreface

TABLE 2B
Mineralogical composition and wireline log response of shaley sandstone in Arch 120, Patrick Draw Field, WY. Mineralogical percentages obtained from XRD analysis

Sample	1	2	3	4	5	6	7
Depth, ft	4942.4	4944.6	4948.7	4949.5	4962.4	4962.5	4966.5
Quartz, wt %	52	66	82	88	91	86	85
Feldspar, wt %	3	6	6	6	5	7	5
Calcite, wt %	36	3	4	1	trc	trc	trc
Dolomite, wt %	1	19	1	trc	trc		4
Siderite, wt %						trc	-
Pyrite, wt %	4	2	3	1	trc	2	3
Kaolinite, wt %	2	3	3	3	4	4	trc
Illite and/or Mica, wt %	1	1	1	1	trc	1	2
Mixed-layer							
Illite/Smectite, wt %	1	trc		trc	trc	trc	1
Plagioclase Feldspar, wt %	1	3	2	3	3	3	3
Feldspar K, wt %	2	3	4	3	2	4	2
<u>Log responses</u>							
Vcl from G.R., wt %	10.5	6.09	7.20	7.34	7.013	7.013	11.79
Density, g/cm3	2.497	2.436	2.319	2.309	2.29	2.29	2.394
Neutron porosity, %	18.9	20.6	23.4	22.5	24.8	24.8	22.6
S, micro-sec	68.0	65.0	68.0	64.0	64.0	64.0	93.0

trc - Traces of minerals

S -Sonic transit time

Vcl - Clay volume from gamma ray log

TABLE 3
Measures Of Vertical Variability Of Permeability Distributions In Wells From Bell Creek Field, Montana

Well no.	Dykstra Parsons coefficients	Mean of log permeability series	Variance of permeability series	Area under power curve
Ideal Profile			0.0008	0.0008
26-4	0.33	8.518	0.0682	0.0656
P-2	0.28	7.914	0.0643	0.0624
W-16	0.60	5.498	3.1551	3.0553
W-14	0.58	6.120	0.6439	0.6267
27-1	0.20	8.659	0.0410	0.0405
C-8	0.40	7.677	0.3380	0.3240
27-14	0.75	5.718	0.7405	0.7309

TABLE 4
Initial Primary, Cumulative Waterflood, Polymer Oil Production And Pay Thicknesses Of Muddy Sandstone Reservoir In The Study Area Of Bell Creek Field

Well no.	Initial Primary oil production ¹ BOPD	Pay thickness, ² ft	Initial primary production per foot of pay, BOPD ⁴	Cumulative waterflood oil production, MBBL	Approximate tertiary oil production at well locations, ³ MBBL
27-1	936	20 (0 + 20)	46.8	910	160
26-4	800	21 (4 + 17)	46.85	1050	80
P-2	850	21 (1 + 20)	42.26	980	65
C-8	950	25 (4 + 21)	39.16	580	75
W-14	570	19 (1 + 18)	31.15	200	—
W-16	680	25 (8 + 17)	27.36	350	—
27-14	574	18 (4 + 14)	35.74	410	—

¹In wells that did not have initial primary or tertiary production, data extrapolated from detailed production contours in neighboring areas.

²The first figure within the brackets refers to thickness of upper valleyfill sandstone and the second figure the thickness of barrier island sandstones.

³A micellar-polymer flood was conducted in a limited area enclosing the first four wells on this table.

⁴Since the permeabilities of the valleyfill sandstones were much lower than the Barrier Island sandstones, an effective pay thickness of each sandstone was estimated based on the average permeabilities of the sandstones at each well location.

TABLE 5
Distributions of Areas Under the Power Curve in the Different Quadrants of the Frequency Scale of Power Distributions of Log Permeability Data. Values Obtained by Dividing the Area Under Curve by the Variances of the Series Are Shown in Brackets.

Well no.	Area under power curve				total
	in quadrant 1	in quadrant 2	in quadrant 3	in quadrant 4	
26-4	4.585x10 ⁻² (0.6989)	1.577x10 ⁻² (0.2484)	0.326x10 ⁻² (0.0498)	0.064x10 ⁻² (9.72x10 ⁻³)	6.56x10 ⁻² (1)
P-2	3.457x10 ⁻² (0.5538)	2.153x10 ⁻² (0.3449)	0.6007x10 ⁻² (0.0962)	0.0243x10 ⁻² (3.90x10 ⁻³)	6.243x10 ⁻² (1)
W-16	2.51067 (0.8217)	0.4103 (0.1343)	0.1250	0.940x10 ⁻² (0.0409)	3.0553 (1)
W-14	0.3311 (0.5283)	0.1815 (0.2897)	0.0837 (0.1336)	0.0303 (0.0484)	0.6267 (1)
27-1	0.02103 (0.5195)	0.013 (0.3211)	0.004321 (0.1067)	0.0021284 (0.0526)	0.0405 (1)
C-8	0.2606 (0.8043)	0.0513 (0.1583)	0.0094 (0.0290)	0.00272 (0.00839)	0.3240 (1)
27-14	0.3304 (0.452)	0.294225 (0.4026)	0.065 (0.0889)	0.04125 (0.05643)	0.7309 (1)

TABLE 6
Reservoir And Petrophysical Properties of UA-5B Sandstones of Six Wells From Arch Unit of Patrick Draw Field, Wy

Well no.	Average clay content, %	Variance in clay distribution V, % ²	Average porosity, %	Average (geom.) permeability, mD	Cum prod. bbl/ year drilled	Dykstra Parsons coefficients
71	5.6	5.11	20.4	8.1	no production/ 1964	0.57
79	5.6	0.902	20.7	78.0	2.88x10 ⁵ / 1964	0.48
91	7.7	2.10	NA	NA	2.12x10 ⁵ / 1966	NA
96	5.0	1.51	20.8	16.3	1.55x10 ⁵ / 1967	0.53
100	4.5	1.08	20.4	13.00	0.75x10 ⁵ / 1967	0.61
120	6.9	1.19	17.5	14.7	no production/ 1987	0.67

NA - Data not available

UA-6 - The sand is stratigraphically located just above UA-5

Fig. 1 Comparison of gamma ray log derived clay content with core measured porosity and air permeability for UA-5B sandstone in well 120, Patrick Draw Field, WY

Fig. 2 Comparison of gamma ray derived clay content with clay and non-clay minerals estimated from XRD analysis for well 120 in Patrick Draw Field, WY. V_{cl} - clay content, K - kaolinite, ILL - illite, SM - smectite, KF = potassium feldspar, D = dolomite

Fig. 3 Comparison of CT-scan densities and log-derived clay content for well 120, Patrick Draw Field, WY

Fig. 4 Crossplot of clay volume (V_{cl}) versus bulk density (ρ_r) of producing UA-5B sandstone in well 120, Patrick Draw Field, WY

Fig. 5 Crossplot of clay volume (V_{cl}) versus bulk density (ρ_r) in the barrier island sandstones of well P-2 (depth = 4,418 to 4,439 ft), Bell Creek Field, MT

Fig. 6 Crossplot of clay volume (V_{cl}) and bulk density (ρ_r) of valley-fill sandstone in well P-2 (depth = 4,408-4,414 ft), Bell Creek Field, MT

Fig 7 Distributions of sonic transit times and gamma ray readings across UA-5 and UA-6 sandstones in Arch 81 from Patrick Draw Field, WY

Fig 8 Amplitude Frequency spectra of sonic transit times of UA-5 and UA-6 sandstones in well Arch 81 shown in fig. 7

Fig 12 Distribution of logarithms of permeability across the Barrier Island sandstones in three wells from Bell Creek Field, MT and an ideal profile

Fig 14 Power spectra of log permeability data from three wells and an ideal profile shown in fig 12

Fig 13 Distribution of logarithms of permeability across the Barrier Island sandstones in four wells from the Bell Creek Field, MT

Fig 15 Power spectra of log permeability data of four wells from the Bell Creek Field shown in fig 13

Fig 9 Index map shows units comprising Bell Creek Field and well control. P, production wells; W, water injection wells; and C, chemical injection wells

Fig 10 Distribution of Dykstra-Parsons coefficients obtained from air-permeability data in the study area. Wells with permeability data are indicated by large circles.

Fig 11 Cross-sections (D-D') and (E-E') showing distribution of porosities and permeabilities within sandstone bodies. See figure 9 for location of cross-sections.

Fig. 16 Distributions of areas under the power curves of gamma ray logs in quadrants 1 and 1 + 2 and Dykstra-Parsons (D-P) coefficients at different well locations in Bell Creek Field, MT plotted as a function of initial oil production rates per foot of pay

Fig. 19 Power spectra of sonic transit times of four wells from Bell Creek Field shown in fig 18

Fig. 17 Distributions of areas under the power curves of gamma ray logs in quadrants 4 and quadrants 3 and 4 and Dykstra Parsons (D-P) coefficients at different well locations in Bell Creek Field, MT plotted as a function of cumulative waterflood production per foot of pay

Fig. 20 Variations of sonic transit times and gamma ray readings across the barrier and non-barrier sandstones in Well W-7 from the Bell Creek Field, MT

Fig. 18 Vertical variation of sonic transit times across Barrier Island sandstones in four wells from Bell Creek Field, MT

Fig. 21 Power spectra of sonic transit times and gamma ray readings across the barrier and non-barrier sandstones in well W-7 shown in fig 20

Fig 22 Coherency and phase spectra of sonic transit time data from wells P-2 and 26-4, Bell Creek Field, MT

Fig 25 Power spectrum of digitized gamma ray log data from UA-5B sandstones in four wells in Patrick Draw Field, WY

Fig 23 Vertical variation of formation density of UA-5B sandstones from Arch 120, Patrick Draw Field, WY from wireline log and CT scans

Fig 26 Distributions of permeability, Dykstra-Parsons coefficients and areas under the power curve of gamma ray logs plotted as a function of cumulative oil production rates per foot of sand at different well locations in Patrick Draw Field, WY

Fig 24 Power spectra of wireline logs (density, gamma ray and neutron porosity) and CT density data of UA-5B sandstones from Arch 120, Patrick Draw Field, WY. The wireline logs were digitized at 0.5 ft interval and the CT density spectrum was for a depth interval of 0.2 ft

Fig 27 Distributions of areas under the power curves of density logs in the different quadrants at different well locations in the Patrick Draw Field, WY

APPENDIX A

```

* PROGRAM RESVAR
* THE FOLLOWING IS A PROGRAM FOR CALCULATION OF CERTAIN
* STATISTICAL PARAMETERS FROM TWO RESERVOIR PROPERTY
* DISTRIBUTIONS,X(I) AND Y(I). THE PARAMETERS COMPUTED ARE THE
* FOLLOWING:
* AUTOCOVARIANCE,AUTOCORRELATION,CROSSCOVARIANCE,CROSS-
* CORRELATION,POWER SPECTRUM,SPECTRAL DENSITY,CO AND
* AND QUADRATURE SPECTRAL ESTIMATES,AMPLITUDE,
* COHERENCE AND THE PHASES OF THE TWO SERIES. THE AREA
* UNDER THE POWER SPECTRUM CURVE IS ALSO CALCULATED IN EACH
* OF THE FOUR QUADRANTS INTO WHICH THE FREQUENCY SCALE
* HAS BEEN DIVIDED INTO.THE FOLLOWIND DATA ARE NEEDED
* TO RUN THE PROGRAM:
* INPUT DATA IN THE FIRST ROW ARE N,L,DL,TR,TP
* WHERE N= TOTAL NO. OF DATA POINTS, L=NO. OF LAGS FOR
* CALCULATION OF THE AUTOCOVARIANCE FUNCTION, DL=THE SAMPLING
* INTERVAL IN FEET, TR=0., IF TREND CORRECTION TO DATA SETS
* IS REQUIRED AND THE PROGRAM USES A SUBROUTINE LST1
* TO CALCULATE THE TRENDS FROM THE TWO DATA SETS AND THEN SUBTRACT
* THEM FROM THE ORIGINAL DATA SETS.IF NO TREND CORRECTION IS
* NEEDED,PUT TR=1.FOR TP=0.,
* DATA X(I), AND Y(I) ARE INPUT ALONG ROWS IN WHICH CASE
* ALL X VALUES ARE INPUT FIRST FOLLOWED BY THE Y VALUES.FOR TP=1.,
* THE TWO DATA SETS ARE IN COLUMNS( SUCH AS IN DIGITIZED LOG DATA),
* THEN FOR EACH INPUT DEPTH D, X AND Y VALUES ARE INPUT.
* THE OUTPUT OF PROGRAM RESVAR ARE AS FOLLOWS:
* THE MEANS AND THE VARIANCES OF THE TWO SERIES
* THE AUTOCOVARIANCE AND THE AUTOCORRELATION OF EACH SERIES
* THE CROSSCOVARIANCE AND THE CROSSCORRELATION OF THE TWO SERIES
* THE POWER SPECTRUM AND SPECTRAL DENSITY OF EACH SERIES AS A FUNCTION
* OF FREQUENCY
* SMOOTHED CO AND QUADRATURE SPECTRAL ESTIMATES OF THE TWO SERIES
* THE AMPLITUDE,COHERENCE AND PHASE ESTIMATES OF THE TWO SERIES
* AREAS UNDER THE POWER SPECTRUM CURVE FOR EACH DISTRIBUTION IN
* QUADRANTS 1 TO 4 AND FOR THE SUM OF ALL THE QUADRANTS. THE
* FREQUENCY SCALE HAS BEEN DIVIDED INTO QUADRANT 1 , QUADRANT 2,
* QUADRANT 3 AND QUADRANT 4.

DIMENSION CXY(250),CYX(250),CXX(250),CYY(250),DH(250)
DIMENSION RXY(250),RYX(250),RBARXX(250),RBARYY(250)
DIMENSION AXY(250),D(250),E(250),RYY(250),CBXX(250),CBYY(250)
DIMENSION PHS(250),AKSQ(250),RXX(250),RBARXY(250),RBARYX(250)
COMMON X(150),Y(150),N,XN(150),YN(150).DL
CHARACTER*20 OUTFILE,INFILE
DATA ni/5/no/6/
write(no,80)
read(*,71)infile
write(no,81)
read(*,71)outfile
81  FORMAT(A20)
80  FORMAT(1x,'enter input file name' /)
81  FORMAT(2x,'enter output file name' /)
open (ni,file=infile,status='old')

```



```

open (no,file=outfile,status='unknown')
read(ni,*) N, L,DL,TR,TP
DO 82 J=1,N
AJ=J
82  DH(J)=DL*AJ -DL
    IF(TP.EQ.1.)GO TO 83
    READ(ni,*) (X(I), I=1,N)
    READ(ni,*) (Y(I), I=1,N)
    GO TO 85
83  DO 84, I=1,N
84  Read(ni,*)DH(I),X(I),Y(I)

85  IF(TR.EQ.1.) GO TO 90

    CALL LST1
c    WRITE(no,86)(I,COef(I),I=1,DEGP1)
86  FORMAT(27H1ORDER      COEfficient/(1x,I5,2x,F15.7))
    DO 88 I=1,N
    X(I)= XN(I)
88  Y(I) =YN(I)
90  write(no,92)
92  Format(40H          DEPTH    X      Y/)
100 DO 101, I=1,N
    Write(no,102)DH(I), X(I),Y(I)
101 Continue
102 FORMAT(13x,3F10.5/)
    sumx=0.
    sumy=0.
    DO 190, I=1,N
    sumx=sumx+X(I)
    sumy=sumy+Y(I)
190 CONTINUE
    AN=FLOAT(N)
    XMEAN=sumx/AN
    YMEAN=sumy/AN
    XVAR=0.
    YVAR=0.
    ACXY=0.
    DO 250, I=1,N
    XVAR=XVAR+(X(I)-XMEAN)**2
    YVAR=YVAR+(Y(I)-YMEAN)**2
    ACXY=ACXY+(X(I)-XMEAN) * (Y(I)-YMEAN)
250 CONTINUE
    ACXY=ACXY/AN
    XVAR=XVAR/AN
    YVAR=YVAR/AN
    ZZ=SQRT(XVAR*YVAR)
    ARXY=ACXY/ZZ
    WRITE(no,270)
270 FORMAT( /17x,'XMEAN',12x,'XVAR',12x,'YMEAN',12x,'YVAR'/)

    WRITE(no,289) XMEAN,XVAR,YMEAN,YVAR
289 FORMAT(8x,F15.5,2x,F15.5,x,F15.5,3x,F15.5/)
C    CALCULATE AUTOCOVARIANCE AND AUTOCORRELATIONS
    xx=1.

```

```

y=1.
DO 420, K=1,N
NMK=N-K
DO 400,I=1,NMK
CXX(K)=CXX(K)+(X(I)-XMEAN) * (X(I +K)-XMEAN)
CYY(K)=CYY(K)+(Y(I)-YMEAN) * (Y(I+K)-YMEAN)
400 CONTINUE
CXX(K)=CXX(K)/AN
CYY(K)=CYY(K)/AN
RXX(K)=CXX(K)/XVAR
RYY(K)=CYY(K)/YVAR
420 CONTINUE
WRITE(no,430)
430 FORMAT(/3x,'AUTO COVAR X',4X,'AUTOCORR X',5X,'AUTO COVAR Y',5X,
1 'AUTOCORR Y'/)
450 WRITE(no,455) XVAR,XX,YVAR,YY
455 FORMAT(2F12.4,5x,F12.4,3x,F12.4/)
DO 499, K=1,N

WRITE(no,492) CXX(K),RXX(K),CYY(K),RYY(K)
492 FORMAT(2F12.4,5x,2F12.4/)
499 CONTINUE

C CALCULATE CROSSCOVARIANCE AND CROSSCORRELATION
DO 680, K=1,N
NMK=N-k
DO 605, I=1,NMK
CXY(K)=CXY(K)+(X(I)- XMEAN) *(Y(I +K) -YMEAN)
CYX(K)=CYX(K)+(Y(I)-YMEAN) * (X(I+K)-XMEAN)
605 CONTINUE
CXY(K)=CXY(K)/AN
CYX(K)=CYX(K)/AN
D(K)=.5*(CXY(K) + CYX(K))
E(K)=.5*(CXY(K)-CYX(K))
680 CONTINUE
DO 700 I=1,N
RXY(I)=CXY(I)/SQRT(XVAR*YVAR)
RYX(I)=CYX(I)/SQRT(XVAR*YVAR)
700 CONTINUE
WRITE(no,710)
710 FORMAT(/10x,'CROSS COV X-Y',2X,'CROSS CORR X-Y',2X,'CROSS COV Y-X'
1,2X,'CROSS CORR Y-X'/)

WRITE(NO,720) ACXY,ARXY,ACXY,ARXY
720 FORMAT(5X,4F15.4/)
DO 800, I=1,N
WRITE(no,720) CXY(I),RXY(I),CYX(I),RYX(I)
800 CONTINUE
C THIS SECTION CALCULATES THE POWER SPECTRUM AND THE SPECTRAL DENSITY
C FILTER USED IS TUKEY
AL=FLOAT(L)

ROX=0.
ROY=0.
COX=0.

```

```

PI=3.141592
LL=L-1
DO 836, KK=1,LL
AKK=FLOAT(KK)
ROX=ROX+.5*(1.+COS(PI*AKK/AL))*RXX(KK)
ROY=ROY+.5*(1.+COS(PI*AKK/AL))*RYY(KK)
COX=COX+.5*(1.+COS(PI*AKK/AL))*D(KK)
836 CONTINUE
ROXX=(1.+2.*ROX)*2.*DL
POXX=ROXX*XVAR
ROYX=(1.+2.*ROY)*2.*DL
POYX=ROYX*YVAR
ROO=0.
COXX=(ACXY+2.*COX)*2.*DL
LF=2*L+2
F=FLOAT(LF)
DDL=2*DL
FF= F*(1./DDL)*2.
LLF=INT(FF)
BKSQ=(COXX*COXX)/(ROXX*ROYX*XVAR*YVAR)
WRITE(no,850)
850 FORMAT(/'FREQ',10x,'SPECTRUM X',10x,'SPEC. DEN X',12x,'SPECTRUM Y'
1,7x,'SPEC. DEN Y'/)
WRITE(no,855) ROO,POXX,ROXX,POYX,ROYX
855 FORMAT(F6.4,5x,F12.4,8x,F12.4,10x,F12.4,5x,F12.4)
DO 922, I=1,LLF
AI=FLOAT(I)
DO 890, K=1,LL
AK=FLOAT(K)
**COS((PI*DL*AI*AK)/F)
RBARYY(I)=RBARYY(I)+.5*(1.+COS(PI*AK/AL))*RYY(K)
**COS((PI*DL*AI*AK)/F)
RBARXY(I)=RBARXY(I)+.5*(1.+COS(PI*AK/AL))*D(K)*
*COS((PI*DL*AI*AK)/F)
RBARYX(I)=RBARYX(I)+.5*(1.+COS(PI*AK/AL))*E(K)*
*SIN((PI*DL*AI*AK)/F)
890 CONTINUE
RBARXX(I)=(1.+2.*RBARXX(I))*2.*DL
CBXX(I)=RBARXX(I)*XVAR
RBARYY(I)=(1.+2.*RBARYY(I))*2.*DL
CBYY(I)=RBARYY(I)*YVAR
RBARXY(I)=(ACXY+2.*RBARXY(I))*2.*DL
RBARYX(I)=4.*RBARYX(I)*DL
FR=AI/(2.*F)
WRITE(no,920)FR,CBXX(I),RBARXX(I),CBYY(I),RBARYY(I)

920 FORMAT(F6.4,5x,F12.4,8x,F12.4,10x,F12.4,5x,F12.4)
922 CONTINUE
C FILTER USED IN THE FOLLOWING CALCULATIONS is TUKEY
C THIS SECTION CALCULATES THE COHERENCE AND PHASE
940 WRITE(NO,945)
945 FORMAT(/10x,'FREQ',3X,'CROSS SPEC X-Y',2X,'CROSS SPEC Y-X',2X,
1 ' AMPLITUDE', 3X,'COHERENCE',5X,'PHASE'/)

WRITE(no,948)ROO,COXX,ROO,COXX,BKSQ,ROO

```

```

948      FORMAT(8X,F8.4,2F12.4,3x,3F12.4/)

      dumz=0.
      bumz=0.
      cumz=0.
      DO 955, I=1,LLF
      AI=FLOAT(I)
      FR=AI/(2.*F)
      dumz=dumz+CBXX(I) * (1./(2.*F))
      bumz=bumz+CBYY(I) * (1./(2.*F))
      cumz=cumz+(cbxx(I)-cbyy(I)) * (1./(2.*F))
      AXY(I)=SQRT(RBARXY(I)*RBARXY(I) + RBARYX(I)*RBARYX(I))
      AKSQ(I)=(AXY(I)*AXY(I))/(RBARXX(I)*RBARYY(I)*XVAR*YVAR)
      PHS(I)=ATAN(-RBARYX(I)/RBARXY(I))
      WRITE(no,950) FR,RBARXY(I),RBARYX(I),AXY(I),AKSQ(I),PHS(I)
950      FORMAT(8X,F8.4,2F12.4,3x,3F12.4/)
955      CONTINUE
C      THIS SECTION CALCULATES THE AREA UNDER THE POWER CURVE
C      IN QUADRANTS 1 TO 4 AND THEN CALCULATES TOTAL AREA UNDER THE CURVE
C      THE OUTPUT IS PRINTED SEPARATELY FOR EACH SERIES
      QUAD1A= 0.
      QUAD1B=0.

      DO 960, I=1,LLF/4
      QUAD1A=QUAD1A+CBXX(I) * (1./(2.*F))
      QUAD1B=QUAD1B+CBYY(I) * (1./(2.*F))
960      CONTINUE
      QUAD2A=0.
      QUAD2B=0.
      DO 970, I=(LLF/4+1),LLF/2
      QUAD2A=QUAD2A+CBXX(I)*(1./(2.*F))
      QUAD2B=QUAD2B+CBYY(I)*(1./(2.*F))
970      CONTINUE
      QUAD3A=0.
      QUAD3B=0.
      DO 980,I=(LLF/2+1),3*LLF/4
      QUAD3A=QUAD3A+CBXX(I)*(1./(2.*F))
      QUAD3B=QUAD3B+CBYY(I)*(1./(2.*F))
980      continue
      QUAD4A=0.
      QUAD4B=0.
      DO 985, I=(3*LLF/4+1),LLF
      QUAD4A=QUAD4A+CBXX(I)*(1./(2.*F))
      QUAD4B=QUAD4B+CBYY(I)*(1./(2.*F))
985      CONTINUE
      WRITE(NO,987)
987      FORMAT(/8X,'AREA IN QUAD.1',2X,'AREA IN QUAD.2',2X,'AREA IN QUAD.
      13',10X,'AREA IN QUAD.4',12x,'TOTAL AREA'/)

      WRITE(no,990) QUAD1A,QUAD2A,QUAD3A,QUAD4A,dumz
      WRITE(no,990) QUAD1B,QUAD2B,QUAD3B,QUAD4B,bumz
990      FORMAT (5x,2F14.5,5x,F14.5,10x,F14.5,10x,F14.5/)
      END

```

**DATE
FILMED**

7 / 27 / 93

END