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Synthetic Aperture Radar Processing with Tiered Subapertures

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ABSTRACT

Synthetic Aperture Radar (SAR) is used to form images that are maps of radar reflectivity of some scene of interest, from range soundings taken over some spatial aperture. Additionally, the range soundings are typically synthesized from a sampled frequency aperture. Efficient processing of the collected data necessitates using efficient digital signal processing techniques such as vector multiplies and fast implementations of the Discrete Fourier Transform. Inherent in image formation algorithms that use these is a trade-off between the size of the scene that can be acceptably imaged, and the resolution with which the image can be made. These limits arise from 'migration errors' and 'spatially variant phase errors', and different algorithms mitigate these to varying degrees. Two fairly successful algorithms for airborne SARs are 'Polar Format' processing, and 'Overlapped Subaperture' (OSA) processing. This report introduces and summarizes the analysis of generalized 'Tiered Subaperture' (TSA) techniques that are a superset of both Polar Format processing and OSA processing. It is shown how tiers of subapertures in both azimuth and range can effectively mitigate both migration errors and spatially variant phase errors to allow virtually arbitrary scene sizes, even in a dynamic motion environment.

MASTER

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1. INTRODUCTION AND SUMMARY

Synthetic Aperture Radar (SAR) is used to form images where pixel values normally represent the radar reflectivity of some location in some scene of interest. The collection of these locations is the scene, and the precision with which a pixel value can be assigned to a location is the resolution. SARs form an image from range soundings taken from sampled points in the space around the scene. The collection of spatial points from whence the radar soundings were taken is the synthetic aperture. Larger synthetic apertures are required for finer resolution. The range function from across the many spatial sample points to a particular scene location is unique from any other scene location. This allows the matched filtering of the spatial sample data for that scene location's reflectivity. The trouble is that each location in the scene optimally requires its own matched filter, making matched filtering computationally expensive, and hence impractical. Realizing that the range function doesn't behave too differently for a neighborhood of scene locations around some focal point, and the differences exhibit strong linear relationships with scene location, allows the use of transform techniques such as the Fast Fourier Transform (FFT) to form an image with considerable improvement in computational efficiency. However, for large neighborhoods of scene locations, the linear relationships break down, thereby degrading the image more and more as distance from the focal point increases. The data exhibits 'migration errors' and 'spatially variant phase errors'. The limit of tolerable degradation sets the scene size limit, which is less for finer resolution, and particularly troublesome as resolution approaches the nominal wavelength of the radar. The challenge becomes to mitigate the migration errors and spatially variant phase errors, and form images of large scenes at fine resolution using efficient transform techniques.

A number of algorithms have been developed to form fine resolution images of large scenes [7]. One of the most recent is 'wavenumber domain', or 'migration' processing, which operates in the frequency domain. This technique is predicated on linear flight paths and doesn't tolerate flight path deviations (especially out-of-plane motion) very well [11]. Two fairly successful image formation algorithms for airborne SARs are 'Polar Format' processing [17], and 'Overlapped Subaperture' (OSA) processing [3][4]. Both operate to some degree in the azimuth time domain, making them capable of dealing with flight path deviations to some extent. However, both algorithms still have scene size limits that are especially severe as resolutions approach a wavelength, such as is desired at UHF and lower frequencies. Imaging a scene larger than these algorithms are capable of requires multiple image formation operations followed by a mosaicking operation of finished subimages. Efficiency is lost when mosaicking requires too many subimages. This has led to the proposal of using multiple levels of tiered subapertures to form ever finer intermediate images, and allow error corrections along the way, as an image formation strategy that can be scaled to much larger scene sizes than currently achievable [9][16].

This report introduces and summarizes the analysis of generalized 'Tiered Subaperture' (TSA) techniques that are a superset of both Polar Format processing and OSA processing. It is shown how tiers of subapertures in both azimuth and range can effectively mitigate both migration errors and spatially variant phase errors to allow virtually arbitrary scene sizes, even in a dynamic motion environment. The development presented herein is in terms of generalizing the Polar Format technique. It assumes a spotlight mode SAR with a linear-frequency-modulated (LFM) chirp that is deramped upon reception. These concepts are clarified in the appropriate sections.

Section 2 illustrates the basic concept of tiered subapertures with a one-dimensional analogy. It shows how subarrays can be used to estimate errors which can then be compensated prior to final transforms.

Section 3 reviews Polar Format processing and how spacial and frequency samples can be selected to facilitate efficient processing.

Section 4 introduces azimuth subapertures to Polar Format processing for enhancing azimuth scene dimensions.

Section 5 introduces range subapertures to the results of section 4 for enhancing range scene dimensions.

Section 6 offers examples and concluding remarks.

The purpose of this report is to present an overview of results and not to bury the reader in an avalanche of mathematics. Oftentimes a qualitative argument will be used in lieu of mathematical proof to avoid 'getting lost in the details'. These details will be addressed in subsequent publications. Comparisons will be drawn along the way to other image formation algorithms.

2. TIERED SUBAPERTURES - A ONE-DIMENSIONAL ANALOGY

Consider a function of the form

$$f(n) = \exp j \left(\omega n + a \omega^2 n^2 \right), \quad (1)$$

with known constant a , $0 < a \ll 1$, unknown constant ω , $-\pi/2 \leq -\Omega/2 \leq \omega \leq \Omega/2 < \pi/2$, and series index n , available over an aperture $N/2 \leq n \leq N/2 - 1$. The challenge is to determine ω with as fine a precision and accuracy as possible within the range $|\omega| \leq \Omega/2$, and efficiently.

If a were zero, the task would simply amount to a Discrete Fourier Transform (DFT) across n and yield a resolution in ω of about $2\pi/N$, depending on window functions employed, and how specifically resolution is measured. For this case, larger N means finer resolution, without bound.

For nonzero a , we must consider the quadratic phase term (quadratic in n). For small Ω and small N , this term can be ignored. A reasonable criteria might be

$$a (\Omega/2)^2 (N/2)^2 \leq \pi/2. \quad (2)$$

Performing an FFT across the data set limited by equation (2) then yields a resolution limit, ρ , of

$$\rho \geq \sqrt{\pi a \Omega^2 / 2}, \quad (3)$$

Here we see the basic dilemma. For using the DFT, or its efficient version, the Fast Fourier Transform (FFT), the larger Ω that we are interested in, the coarser the resolution is with which we can identify ω . The challenge becomes to exceed the limit in equation (3). To do this we need to mitigate the effects of the quadratic phase term which is an error term for FFT processing.

2.1. Subapertures

We do this by making a coarse resolution estimate of ω , and use this to compensate $f(n)$ before proceeding with a fine resolution estimate. Conceptually, we make coarse resolution estimates of ω by dividing the aperture into subapertures, as is illustrated in figure 1. Mathematically, we do this by splitting the domain of index n into groups of indices m_1 and m_2 , as follows,

$$n = m_1 + \Delta_2 m_2, \quad (4)$$

where m_1 is intra-subaperture index and is limited to the domain $-M_1/2 \leq m_1 \leq M_1/2 - 1$, m_2 is the inter-subaperture index and is limited to the domain $-M_2/2 \leq m_2 \leq M_2/2 - 1$, and Δ_2 is a data decimation factor, with subapertures overlapped by an amount $(M_1 - \Delta_2)$. Overlapping subapertures is necessary to control sidelobes. The intent is for M_1 to be much shorter than the desired N . We rewrite the function $f(n)$ as

$$f(m_1, m_2) = \exp j \left(\omega (m_1 + \Delta_2 m_2) + a \omega^2 (m_1 + \Delta_2 m_2)^2 \right), \quad (5)$$

Figure 1. Aperture divided into subapertures.

and expand it to

$$f(m_1, m_2) = \exp j(\omega \Delta_2 m_2 + a \omega^2 \Delta_2^2 m_2^2) \exp j\left(\left(\omega + 2a\omega^2 \Delta_2 m_2\right)m_1 + a\omega^2 m_1^2\right). \quad (6)$$

The plan now is to perform a Discrete Fourier Transform (DFT) across index m_1 , and do this for each index value m_2 , but first we need to examine the quadratic phase term $a\omega^2 m_1^2$. To keep this term's effects negligible, we choose M_1 such that

$$a(\Omega/2)^2 (M_1/2)^2 \leq \pi/2. \quad (7)$$

Note that we may rewrite equation (7) as

$$\frac{2\pi}{M_1} \geq \frac{a\Omega^2 M_1}{4}. \quad (8)$$

The DFT of $f(m_1, m_2)$ then is defined as

$$f_2(u_1, m_2) = \sum_{m_1=-M_1/2}^{M_1/2-1} f(m_1, m_2) \exp j\left(\frac{-2\pi u_1}{M_1} m_1\right), \quad (9)$$

which, if the condition in equation (7) is met, approximates to

$$f_2(u_1, m_2) \approx \text{csinc}_{M_1}\left[\frac{M_1}{2}\left(\omega + 2a\omega^2 \Delta_2 m_2 - \frac{2\pi u_1}{M_1}\right)\right] \exp j(\omega \Delta_2 m_2 + a\omega^2 \Delta_2^2 m_2^2) \quad (10)$$

where we define $\text{csinc}_M()$ as a convenient shorthand for the complex discrete sinc function, i.e.

$$\text{csinc}_M(x) \equiv \frac{\sin(x)}{\sin(x/M)} e^{-j\frac{x}{M}}. \quad (11)$$

The phase rotation in this function is typically negligible, especially in light of the amplitude envelope. Furthermore, equation (11) has resolution in x of about π . This implies that in equation (10), the csinc function resolves ω with resolution $(2\pi/M_1)$, and locates it with an estimated value of

$$\hat{\omega} \approx \frac{2\pi u_1}{M_1} - 2a\hat{\omega}^2 \Delta_2 m_2. \quad (12)$$

The problem here is that the estimate of ω depends on subaperture index value m_2 , that is, the location of $\hat{\omega}$ ‘migrates’ with m_2 . In fact, with large enough m_2 , the migration will transcend several resolution cells (values of u_1). Therefore, for any one value of u_1 , the migration has the effect of ‘windowing’ the data over index m_2 , as the amplitude peak moves into and then out of the appropriate resolution cell u_1 . Since we will ultimately want to perform a DFT across index m_2 , it is desirable for the migration to be no more than about one coarse resolution cell ρ_1 in total, over all m_2 for any ω . This leads to the constraint

$$2a\left(\frac{\Omega}{2}\right)^2 \Delta_2 \left(\frac{M_2}{2}\right) \leq \frac{1}{2} \left(\frac{2\pi}{M_1}\right), \quad (13)$$

which can be rewritten as a constraint on subaperture resolution,

$$\rho_1 = \frac{2\pi}{M_1} \geq \frac{a\Omega^2 \Delta_2 M_2}{2}. \quad (14)$$

Note that this is a more severe constraint than equation (8) since $\Delta_2 M_2 \gg M_1/2$.

Consolidating what we know, we can observe the constraint of equation (14), and approximate equations (10) and (12) as

$$f_2(u_1, m_2) \approx \text{csinc}_{M_1} \left[\frac{M_1}{2} \left(\omega - \frac{2\pi u_1}{M_1} \right) \right] \exp j \left(\omega \Delta_2 m_2 + a\omega^2 \Delta_2^2 m_2^2 \right), \quad (15)$$

$$\text{and } \hat{\omega} \approx \frac{2\pi u_1}{M_1}. \quad (16)$$

At this point we can use equation (16) to compensate, or ‘focus’, equation (15) prior to a DFT across m_2 . We write the compensated function as

$$f_2^*(u_1, m_2) = f_2(u_1, m_2) \exp j \left(-\hat{\omega} \Delta_2 m_2 - a\hat{\omega}^2 \Delta_2^2 m_2^2 \right), \quad (17)$$

or expanded to

$$f_2^*(u_1, m_2) \approx \text{csinc}_{M_1} \left[\frac{M_1}{2} \left(\omega - \frac{2\pi u_1}{M_1} \right) \right] \exp j \left((\omega - \hat{\omega}) \Delta_2 m_2 + a \left(\omega^2 - \hat{\omega}^2 \right) \Delta_2^2 m_2^2 \right). \quad (18)$$

The quadratic phase term in equation (18) becomes negligible if

$$a \left(\omega^2 - \hat{\omega}^2 \right)^2 \Delta_2^2 \left(\frac{M_2}{2} \right)^2 \leq \frac{\pi}{2} \quad (19)$$

for all ω . Given that this condition is met, we define the final DFT across m_2 as

$$f_3(u_1, u_2) = \sum_{m_2 = -M_2/2}^{M_2/2-1} f_2^*(u_1, m_2) \exp j \left(\frac{-2\pi u_2}{M_2} m_2 \right), \quad (20)$$

with result approximately

$$f_3(u_1, u_2) \approx \text{csinc}_{M_1} \left[\frac{M_1}{2} \left(\omega - \frac{2\pi u_1}{M_1} \right) \right] \text{csinc}_{M_2} \left[\frac{M_2 \Delta_2}{2} \left((\omega - \hat{\omega}) - \frac{2\pi u_2}{M_2 \Delta_2} \right) \right]. \quad (21)$$

The second, and final, best estimate of ω becomes

$$^{(2)} \hat{\omega} \approx \frac{2\pi u_1}{M_1} + \frac{2\pi u_2}{M_2 \Delta_2}, \quad (22)$$

with a final resolution of $\rho_2 = (2\pi) / (\Delta_2 M_2)$. But observing that

$$\left| \omega^2 - \hat{\omega}^2 \right| = |(\omega - \hat{\omega})(\omega + \hat{\omega})| \leq \left(\frac{\rho_1}{2} \right) (2\Omega) = \rho_1 \Omega, \quad (23)$$

we recognize that a sufficient condition to meet the condition in equation (19) is that

$$a \rho_1 \Omega \Delta_2^2 \left(\frac{M_2}{2} \right)^2 \leq \frac{\pi}{2}. \quad (24)$$

Substituting equation (14) into equation (24) yields

$$\rho_2 = \frac{2\pi}{\Delta_2 M_2} \geq \left(2\pi^2 a^2 \Omega^3 \right)^{\frac{1}{3}}, \quad (25)$$

which describes the ultimate limit in resolution for a single level of subapertures. The limit for ρ_2 in equation (25) is much finer than the limit for ρ in equation (3) by a factor

$$\left(2^{5/6} \pi^{1/6} a^{1/6} \right) \approx 2.16 a^{1/6} \quad (26)$$

which represents an improvement for $a < 0.00995$. The benefit of subapertures is thus established.

2.2. Two Tiers of Subapertures

The question becomes “if one level of subapertures is good, will another level of subapertures be better?” The short answer is “yes”.

Let index n be divided into two tiers of subapertures, as in figure 2, such that

$$n = m_1 + \Delta_2 m_2 + \Delta_2 \Delta_3 m_3. \quad (27)$$

Our function becomes

$$f(m_1, m_2, m_3) = \exp j \left(\omega (m_1 + \Delta_2 m_2 + \Delta_2 \Delta_3 m_3) + a \omega^2 (m_1 + \Delta_2 m_2 + \Delta_2 \Delta_3 m_3)^2 \right). \quad (28)$$

We rewrite this as

$$\begin{aligned} f(m_1, m_2, m_3) = & \exp j \left(\left(\omega + 2a\omega^2 (\Delta_2 m_2 + \Delta_2 \Delta_3 m_3) \right) m_1 + a\omega^2 m_1^2 \right) \\ & \cdot \exp j \left(\left(\Delta_2 \omega + 2a\omega^2 \Delta_2^2 \Delta_3 m_3 \right) m_2 + a\omega^2 \Delta_2^2 m_2^2 \right) \\ & \cdot \exp j \left(\Delta_2 \Delta_3 \omega m_3 + a\omega^2 \Delta_2^2 \Delta_3^2 m_3^2 \right). \end{aligned} \quad (29)$$

The processing strategy is clear. First a DFT across m_1 , followed by corrections, followed by a

Figure 2. Aperture divided into tiers of subapertures.

DFT across m_2 , followed by corrections, followed by a DFT across m_3 .

To this end, if we choose M_1 so that the quadratic phase term in line one of equation (29) is not too big, the first DFT results in

$$\begin{aligned}
 f_1(u_1, m_2, m_3) \approx & \text{csinc}_{M_1} \left[\frac{M_1}{2} \left(\omega + 2a\omega^2 (\Delta_2 m_2 + \Delta_2 \Delta_3 m_3) - \frac{2\pi u_1}{M_1} \right) \right] \\
 & \cdot \exp j \left(\left(\Delta_2 \omega + 2a\omega^2 \Delta_2^2 \Delta_3 m_3 \right) m_2 + a\omega^2 \Delta_2^2 m_2^2 \right) \\
 & \cdot \exp j \left(\Delta_2 \Delta_3 \omega m_3 + a\omega^2 \Delta_2^2 \Delta_3^2 m_3^2 \right).
 \end{aligned} \tag{30}$$

We now limit the migration term in line one of equation (30) such that

$$2a \left(\frac{\Omega}{2} \right)^2 \left(\Delta_2 \left(\frac{M_2}{2} \right) + \Delta_2 \Delta_3 \left(\frac{M_3}{2} \right) \right) \leq \frac{1}{2} \left(\frac{2\pi}{M_1} \right), \tag{31}$$

and note that this is a more restrictive constraint than limiting the corresponding quadratic phase term. With migration controlled, u_1 provides us with an estimate of ω , $\hat{\omega} \approx (2\pi u_1) / M_1$, with which we can compensate some phase errors and approximate

$$\begin{aligned}
 f_1^*(u_1, m_2, m_3) \approx & \text{csinc}_{M_1} \left[\frac{M_1}{2} \left(\omega - \frac{2\pi u_1}{M_1} \right) \right] \\
 & \cdot \exp j \left(\left(\Delta_2 (\omega - \hat{\omega}) + 2a(\omega^2 - \hat{\omega}^2) \Delta_2^2 \Delta_3 m_3 \right) m_2 + a(\omega^2 - \hat{\omega}^2) \Delta_2^2 m_2^2 \right) \\
 & \cdot \exp j \left(\Delta_2 \Delta_3 (\omega - \hat{\omega}) m_3 + a(\omega^2 - \hat{\omega}^2) \Delta_2^2 \Delta_3^2 m_3^2 \right).
 \end{aligned} \tag{32}$$

If we choose M_2 so that the quadratic phase term in line two of equation (32) is not too big, the second DFT results in

$$\begin{aligned}
 f_2(u_1, u_2, m_3) \approx & \text{csinc}_{M_1} \left[\frac{M_1}{2} \left(\omega - \frac{2\pi u_1}{M_1} \right) \right] \\
 & \cdot \text{csinc}_{M_2} \left[\frac{M_2}{2} \left(\Delta_2 (\omega - \hat{\omega}) + 2a(\omega^2 - \hat{\omega}^2) \Delta_2^2 \Delta_3 m_3 - \frac{2\pi u_2}{M_2} \right) \right] \\
 & \cdot \exp j \left(\Delta_2 \Delta_3 (\omega - \hat{\omega}) m_3 + a(\omega^2 - \hat{\omega}^2) \Delta_2^2 \Delta_3^2 m_3^2 \right).
 \end{aligned} \tag{33}$$

We set a limit on the migration term in line two of equation (32) such that

$$2a\rho_1\Omega\Delta_2^2\Delta_3\left(\frac{M_3}{2}\right) \leq \frac{1}{2}\left(\frac{2\pi}{M_2}\right), \quad (34)$$

where, as before, $\rho_1 = 2\pi/M_1$. This migration term is again more significant than the corresponding quadratic phase error term. We now use u_2 to provide us with a second, even better, estimate of ω ,

$$^{(2)}\hat{\omega} = \frac{2\pi u_1}{M_1} + \frac{2\pi u_2}{M_2\Delta_2}. \quad (35)$$

Using this estimate to compensate equation (33), and noting that migration is now limited, we arrive at the approximatic

$$\begin{aligned} f_2^*(u_1, u_2, m_3) \approx & \text{csinc}_{M_1}\left[\frac{M_1}{2}\left(\omega - \frac{2\pi u_1}{M_1}\right)\right] \\ & \cdot \text{csinc}_{M_2}\left[\frac{M_2}{2}\left(\Delta_2(\omega - \hat{\omega}) - \frac{2\pi u_2}{M_2}\right)\right] \\ & \cdot \exp j\left(\Delta_2\Delta_3\left(\omega + ^{(2)}\hat{\omega}\right)m_3 + a\left(\omega^2 - ^{(2)}\hat{\omega}\right)\Delta_2^2\Delta_3^2m_3^2\right). \end{aligned} \quad (36)$$

The final quadratic phase term in line three of equation (36) can be assured to be negligible if

$$a\rho_2\Omega\Delta_2^2\Delta_3^2\left(\frac{M_3}{2}\right)^2 \leq \frac{\pi}{2}, \quad (37)$$

where $\rho_2 = 2\pi/(M_2\Delta_2)$. This allows a DFT across m_3 to calculate

$$\begin{aligned} f_3(u_1, u_2, u_3) \approx & \text{csinc}_{M_1}\left[\frac{M_1}{2}\left(\omega - \frac{2\pi u_1}{M_1}\right)\right] \\ & \cdot \text{csinc}_{M_2}\left[\frac{M_2}{2}\left(\Delta_2(\omega - \hat{\omega}) - \frac{2\pi u_2}{M_2}\right)\right] \\ & \cdot \text{csinc}_{M_3}\left[\frac{M_3}{2}\left(\Delta_2\Delta_3\left(\omega + ^{(2)}\hat{\omega}\right) - \frac{2\pi u_3}{M_3}\right)\right]. \end{aligned} \quad (38)$$

The third, and final, best estimate for the value of ω becomes

$$^{(3)}\hat{\omega} \approx \frac{2\pi u_1}{M_1} + \frac{2\pi u_2}{M_2\Delta_2} + \frac{2\pi u_3}{M_3\Delta_3\Delta_2}, \quad (39)$$

with resolution $\rho_3 = 2\pi / (M_3 \Delta_3 \Delta_2)$.

Combining the migration limits in equations (31) and (34) with the quadratic phase limit of equation (37), and noting that $M_2 \ll \Delta_3 M_3$, we can bound the finest resolution achievable with two tiers of subapertures with

$$\rho_3 \geq \left(8\pi^3 a^3 \Omega^4 \right)^{\frac{1}{4}}. \quad (40)$$

This represents an improvement over one tier of subapertures of

$$\left(2^{5/12} \pi^{1/12} a^{1/12} \right) \approx 1.47 a^{1/12},$$

which results in finer resolution if $a < 0.00995$, just as in equation (26).

2.3. Multiple Tiers of Subapertures

For N_s number of tiers of subapertures, one can observe the following. The first DFT stage results in a migration limit such that

$$\rho_1 \geq \frac{\pi a \Omega^2}{\rho_{(N_s+1)}}. \quad (41)$$

Subsequent stages (except the last) of DFTs result in migration limits such that

$$\rho_k \geq \frac{4\pi a \Omega \rho_{(k-1)}}{\rho_{(N_s+1)}}. \quad (42)$$

The final DFT is limited by the quadratic focus term such that

$$\rho_{(N_s+1)}^2 \geq 2\pi a \Omega \rho_{N_s}. \quad (43)$$

Putting equations (41), (42), and (43) together realizes the relationship

$$\rho_{(N_s+1)}^2 \geq (2\pi a \Omega) \left(\frac{4\pi a \Omega}{\rho_{(N_s+1)}} \right)^{(N_s-1)} \left(\frac{\pi a \Omega^2}{\rho_{(N_s+1)}} \right), \quad (44)$$

or, with a little rearranging,

$$\rho_{(N_s+1)} \geq \left(2^{(2N_s-1)} \pi^{(N_s+1)} a^{(N_s+1)} \Omega^{(N_s+2)} \right)^{\frac{1}{(N_s+2)}}. \quad (45)$$

The incremental improvement factor in adding the N_s^{th} tier of subapertures is given by

$$\frac{\rho_{(N_s+1)}}{\rho_{N_s}} = (32\pi a) \left[\frac{1}{(N_s+2)(N_s+1)} \right]. \quad (46)$$

2.4. Observations

- For all N_s , resolution $\rho_{(N_s+1)}$ remains proportional to Ω
- For all N_s , resolution will improve with more tiers of subapertures if $a < 1/(32\pi) \approx 0.00995$. The smaller is a , the better the improvement.
- As N_s increases, the incremental benefit of the next level of subapertures diminishes.
- The ultimate limit on resolution achievable with an infinite number of subapertures is bounded by $\rho_{\infty} \geq 4\pi a \Omega$.

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3. POLAR-FORMAT PROCESSING AND IT'S LIMITATIONS

Consider a SAR operating in the geometry of figure 3. The origin of the coordinate system is identified as the Central Reference Point (CRP). The radar sensor phase center is located with vector $\mathbf{r}_c$ and has coordinates (x,y,z) . A target scatterer is located with vector $\mathbf{s}$ and has coordinates $(s_x, s_y, 0)$. The presumed plane of all possible target locations is the plane $z=0$, referred to as the SAR focal plane. The center of the SAR's synthetic aperture is in the plane $x=0$. Angles α and ψ measure the SAR's azimuth and elevation direction from the y-axis and focal plane respectively. Vector $\mathbf{r}_s$ is the difference between $\mathbf{r}_c$ and $\mathbf{s}$. At this point, no presumption is made regarding the SAR's flight path.

For a spotlight mode SAR, consider a transmitted signal from the radar of the form

$$X_T(t, n) = \text{rect}\left(\frac{t-t_n}{T}\right) \cos\left(\omega_n(t-t_n) + \frac{\gamma_n}{2}(t-t_n)^2\right), \quad (47)$$

where

- t = time variable
- t_n = reference time of n^{th} radar pulse, where $N/2 \leq n \leq N/2 - 1$,
- T = transmitted pulse length,
- ω_n = transmitted center frequency of n^{th} radar pulse,

Figure 3. SAR Geometry Definition

$\gamma_n =$ transmitted chirp rate of n^{th} radar pulse, and

$$\text{rect}(t) = \begin{cases} 1 & t \leq 1/2 \\ 0 & t > 1/2 \end{cases}.$$

The received echo from the target scatterer will then be

$$X_R(t, n) = A(r_s) \text{rect}\left(\frac{t - t_n - \frac{2}{c}\|r_s\|}{T}\right) \cos\left(\omega_n\left(t - t_n - \frac{2}{c}\|r_s\|\right) + \frac{\gamma_n}{2}\left(t - t_n - \frac{2}{c}\|r_s\|\right)^2\right), \quad (48)$$

where $A(r_s) =$ the cumulative two-way losses in signal strength and is proportional to target scatterer reflectivity.

X_R is now deramped and separated into I and Q channels with a local oscillator signal of the form

$$X_{LO}(t, n) = 2 \text{rect}\left(\frac{t - t_n - \frac{2}{c}\|r_c\|}{T_m}\right) \exp\left\{-j\left(\omega_n\left(t - t_n - \frac{2}{c}\|r_c\|\right) + \frac{\gamma_n}{2}\left(t - t_n - \frac{2}{c}\|r_c\|\right)^2\right)\right\}, \quad (49)$$

where T_m is the local oscillator pulse length, and sized such that

$$\text{rect}\left(\frac{t - t_n - \frac{2}{c}\|r_s\|}{T}\right) \text{rect}\left(\frac{t - t_n - \frac{2}{c}\|r_c\|}{T_m}\right) = \text{rect}\left(\frac{t - t_n - \frac{2}{c}\|r_s\|}{T}\right) \quad (50)$$

for all target scatter locations of interest.

After filtering out the upper sideband, the remaining video signal is described by

$$X_V(t, n) = A(r_s) \text{rect}\left(\frac{t - t_n - \frac{2}{c}\|r_s\|}{T}\right) \exp j \left\{ \frac{2}{c} \left[\omega_n + \gamma_n \left(t - t_n - \frac{2}{c}\|r_c\| \right) \right] (\|r_c\| - \|r_s\|) + \frac{2\gamma_n}{c^2} (\|r_c\| - \|r_s\|)^2 \right\}. \quad (51)$$

Sampling X_V with a sampling period T_s , and using a sampling index i , yields

$$X_V(i, n) = A(r_s) \exp j \left\{ \frac{2}{c} [\omega_n + \gamma_n T_s i] (\|r_c\| - \|r_s\|) + \frac{2\gamma_n}{c^2} (\|r_c\| - \|r_s\|)^2 \right\}, \quad (52)$$

where $T_s i = t - t_n - \frac{2}{c} \|r_c\|$ with $I/2 \leq i \leq I/2 - 1$.

A basic assumption here is that the total sampling interval is encompassed within the available signal, that is, for all targets of interest,

$$\text{rect}\left(\frac{t - t_n - \frac{2}{c} \|r_s\|}{T}\right) \text{rect}\left(\frac{i T_s}{T}\right) = \text{rect}\left(\frac{i T_s}{T}\right). \quad (53)$$

We note here that equation (52) is exact, and now turn our attention to $(\|r_c\| - \|r_s\|)$, which we expand as

$$(\|r_c\| - \|r_s\|) = \|r_c\| - \|r_c - s\| = \|r_c\| \left(1 - \sqrt{1 - \frac{2r_c \cdot s}{\|r_c\|^2} + \frac{\|s\|^2}{\|r_c\|^2}} \right). \quad (54)$$

The square-root term is of the form $\sqrt{1 + a}$ where $\|a\| \ll 1$, which allows us to expand

$$\sqrt{1 + a} = 1 + \frac{a}{2} - \frac{a^2}{8} + \frac{a^3}{16} - \dots, \text{ where } a = -\frac{2r_c \cdot s}{\|r_c\|^2} + \frac{\|s\|^2}{\|r_c\|^2}.$$

Taking only the first three terms of the expansion allows the approximation

$$(\|r_c\| - \|r_s\|) \approx \frac{r_c \cdot s}{\|r_c\|} - \frac{\|s\|^2}{2\|r_c\|} + \frac{(r_c \cdot s)^2}{2\|r_c\|^3} + \frac{\|s\|^4}{8\|r_c\|^3} - \frac{\|s\|^2 r_c \cdot s}{2\|r_c\|^3}. \quad (55)$$

Clearly this is an approximation and an infinite number of terms of the expansion were dropped due to their relative insignificance. In equation (55), the first term of the expansion is the desired term, and all others are undesired, and hence error terms. We observe that

$$\frac{r_c \cdot s}{\|r_c\|} = s_x \cos \psi_n \sin \alpha_n - s_y \cos \psi_n \cos \alpha_n = \cos \psi_n \cos \alpha_n (s_x \tan \alpha_n - s_y), \quad (56)$$

and rewrite equation (55) in terms of equation (56) and an error term,

$$(\|r_c\| - \|r_s\|) = \cos \psi_n \cos \alpha_n (s_x \tan \alpha_n - s_y + \xi_r). \quad (57)$$

The purpose for writing all error terms in equation (55) as $(\cos \psi_n \cos \alpha_n \xi_r)$ will become clear shortly.

The $(\|r_c\| - \|r_s\|)^2$ term in equation (52) is commonly called the residual phase error, and contributes only undesired effects. Typically, radar pulses are long enough that this term is very

small and considered negligible. Polar Format processing does this.

Using equations (57), and ignoring residual phase error, we rewrite equation (52) as

$$X_V(i, n) \approx A(r_s) \exp j \left\{ \frac{2}{c} [\omega_n + \gamma_n T_s i] \cos \psi_n \cos \alpha_n (s_x \tan \alpha_n - s_y + \xi_r) \right\}. \quad (58)$$

Traditional Polar Format processing uses constant ω_n and γ_n , and samples at constant increments of α . This necessitates a polar to rectangular interpolation step to make s_x and s_y each scaling separate frequency indices. The final step is a 2-dimensional DFT followed by magnitude detection.

3.1. Real-Time Motion Compensation

In equation (58), both α and ψ are readily calculable functions of SAR position (flight path). We can mitigate their influence by employing real-time motion compensation, by adjusting SAR center frequency and chirp rate as

$$\omega_n = \omega_0 \left(\frac{\cos \psi_0}{\cos \psi_n} \right) \left(\frac{1}{\cos \alpha_n} \right), \quad (59)$$

$$\text{and } \gamma_n = \gamma_0 \left(\frac{\cos \psi_0}{\cos \psi_n} \right) \left(\frac{1}{\cos \alpha_n} \right). \quad (60)$$

This same effect can be achieved by adjusting start time and sample spacing T_s of samples i . This sort of adjustment is fairly easy with modern digital waveform synthesizers [3].

Continuing, we select spatial samples such that

$$\tan \alpha_n = n d\alpha, \quad (61)$$

where $d\alpha$ is a constant, and n , recall, is a spatial sample index.

With samples collected in this manner, the sampled video signal becomes

$$X_V(i, n) \approx A(r_s) \exp j \left\{ \frac{2 \cos \psi_0}{c} [\omega_0 + \gamma_0 T_s i] (s_x d\alpha n - s_y + \xi_r) \right\}. \quad (62)$$

Equation (62) is equivalent to equation (58) except that most of the interpolation that was necessary for processing data collected in the traditional manner has been 'built into' the sampling steps [13][10]. Instead of being collected on a polar grid, equation (62) uses data collected on a 'keystone' grid. Separating s_x and s_y into scaling separate frequency indices merely requires a linear resampling across index n as a function of $(1 + (\gamma_0 T_s / \omega_0) i)$. The final step is again a 2-dimensional DFT followed by magnitude detection.

It is important to realize that equation (62) represents the form of the collected data at the analog-to-digital (A/D) converters, *prior* to any digital signal processing. Merely choosing the correct sample positions and times does a good chunk of the focusing and range migration correction for a SAR image. Simply taking a 2-dimensional DFT of equation (62) will generate an image with a focused patch diameter equivalent to Polar Format processing in the s_y direction (swath width). The implication is that even algorithms such as Overlapped-Subaperture (OSA) [3][6] could increase their focused swath width by perhaps an order of magnitude in some cases by merely changing their sampling strategy.

3.2. Processing

The linear resampling required for equation (62) could be accomplished with a Chirp-Z Transform (CZT) [15]. We shall do so here.

Step 1. Perform an azimuth CZT across n of the form

$$X_1(i, u) = \sum_{n=-N/2}^{N/2-1} X_V(i, n) \exp -j \left\{ (1 + (\gamma_0 T_s / \omega_0) i) \frac{2\pi u}{N} n \right\}. \quad (63)$$

If the error term ξ_r isn't too big a function of n , the result is approximately expressed by

$$X_1(i, u) \approx A(r_s) \exp j \left\{ \frac{2 \cos \psi_0}{c} [\omega_0 + \gamma_0 T_s i] (-s_y + \xi_r) \right\} \\ \cdot \text{sinc}_N \left[\frac{N}{2} (1 + (\gamma_0 T_s / \omega_0) i) \left(\frac{2 \omega_0 \cos \psi_0 d \alpha}{c} s_x - \frac{2\pi u}{N} \right) \right]. \quad (64)$$

Note that in the azimuth direction,

$$\hat{s}_x = \left(\frac{\lambda_0}{2 (N d \alpha \cos \psi_0)} \right) u = \text{target azimuth position estimate, and} \quad (65)$$

$$\rho_x = \frac{1}{(1 + (\gamma_0 T_s / \omega_0) i)} \left(\frac{\lambda_0}{2 (N d \alpha \cos \psi_0)} \right) = \text{target azimuth resolution,} \quad (66)$$

Figure 4. Real-Time Polar Format Processing

where λ_0 is the nominal SAR wavelength ($\lambda_0 = 2\pi c/\omega_0$).

Note that the resolution varies somewhat with sample index i . For fractional chirp bandwidths that aren't too great, the impact of this is very minor at best. We shall ignore it by setting $i=0$ in equation (66), and in line 2 of equation (64) only.

Furthermore, the estimate $\hat{s}_x$ could now be used to somewhat compensate geometric distortions due to ξ_r in line 1 of equation (64). Typically in Polar Format processing, however, it is not.

Step 2. Perform a DFT across i of the form

$$X_2(k, u) = \sum_{i=-I/2}^{I/2-1} X_1(i, u) \exp -j \left\{ \frac{2\pi k}{I} i \right\}. \quad (67)$$

The result is approximately expressed by

$$X_2(k, u) \approx A(r_s) \exp j \left\{ \frac{2 \cos \psi_0}{c} \omega_0 (-s_y + \xi_r) \right\} \\ \cdot \text{csinc}_N \left[\frac{N}{2} \left(\frac{2 \omega_0 \cos \psi_0 d \alpha}{c} s_x - \frac{2\pi u}{N} \right) \right] \text{csinc}_I \left[\frac{I}{2} \left(-\frac{2 \gamma_0 T_s \cos \psi_0}{c} s_y - \frac{2\pi k}{I} \right) \right]. \quad (68)$$

Note that in the range direction,

$$\hat{s}_y = \left(-\frac{c}{2B \cos \psi_0} \right) k = \text{target position estimate}, \quad (69)$$

$$\text{and } \rho_y = \left(\frac{c}{2B \cos \psi_0} \right) = \text{target range resolution on the ground}, \quad (70)$$

where B is the effective chirp bandwidth in Hz, ($B = \gamma_0 T_s I / (2\pi)$). Note that the effective chirp bandwidth depends on the sampled signal, and is less than the transmitted chirp bandwidth.

3.3. Patch Size Limitations

Keeping the error term ξ_r in equation (62) small requires putting a limit on the range of s_x and s_y that will meet some resolution specification [8][17]. We do this by first expanding ξ_r into a polynomial in n , that is,

$$\xi_r = \epsilon_0 + \epsilon_1 d \alpha n + \epsilon_2 d \alpha^2 n^2 + \dots \quad (71)$$

For Polar Format processing, it is the quadratic term that generally dominates the misfocusing,

and hence the resolution, of the SAR image. The exact nature of the quadratic term depends on the actual flight path of the SAR vehicle. For straight-line flight paths and fairly benign non-straight-line flight paths, ϵ_2 is approximated by its dominant component,

$$\epsilon_2 \approx -\frac{\cos \psi_0 (s_x^2 - s_y^2/2)}{2\|r_{c0}\|}, \quad (72)$$

which allows image patch diameters of [8]

$$D \leq 4\rho_x \sqrt{\left(\frac{\|r_{c0}\|}{\lambda_0}\right) \left(\frac{QPE_{max}}{\pi/2}\right) \left(\frac{1}{1 + \frac{\lambda_0\|r_{c0}\|}{cT_{eff}\rho_y \cos \psi_0}}\right)}, \quad (73)$$

where r_{c0} is the nominal range r_c at the center of the synthetic aperture, QPE_{max} is the maximum allowable quadratic phase error, and T_{eff} is the effective pulse width ($T_{eff} = T_s I$). When using real-time motion compensation, the patch diameter limit in the s_y direction is actually somewhat greater than in equation (73) by a factor of $\sqrt{2}$, owing to the scaling of s_y^2 in equation (72).

Large rates of change in $\|r_c\|$ (line-of-sight velocity), and rates of rates of change in $\|r_c\|$ (line-of-sight acceleration), during a synthetic aperture will increase the significance of parts of ϵ_2 other than the component expressed in equation (72), even to a point of eclipsing this component in importance and thereby shrinking the allowable patch diameters D . This frequently imposes a limit on line-of-sight velocities and accelerations [7].

The remainder of this report will presume benign flight paths, with

$$QPE_{max} = \pi/2, \quad (74)$$

$$\text{and } \lambda_0\|r_{c0}\| \ll cT_{eff}\rho_y \cos \psi_0, \quad (75)$$

such that equation (73) may be reduced to

$$D \leq 4\rho_x \sqrt{(\|r_{c0}\|/\lambda_0)}. \quad (76)$$

Note that equation (75) is a by-product of assuming the residual phase error in equation (52) to be negligible.

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4. AZIMUTH SUBAPERTURES FOR EXTENDING AZIMUTH PATCH DIAMETER

We begin this section with equation (62), but with equation (71) substituted in the appropriate position, and repeat it here as

$$X_V(i, n) \approx A(r_s) \exp j \left\{ \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s i)}{c} \right) \cdot \left((\epsilon_0 - s_y) + (\epsilon_1 + s_x) d\alpha n + \epsilon_2 d\alpha^2 n^2 + \dots \right) \right\}. \quad (77)$$

Within this expression is the assumption that the flight path is adequately known such that the error components ϵ_i are functions only of r_{c0} , ψ_0 , s_x and s_y , and not of aperture position index n .

While the following development could incorporate the residual phase error terms of equation (52), this would add to an already somewhat messy analysis. In any case, their effect can be mitigated similarly to the way we will deal with the ϵ_i terms, and except for very short-pulse radars, the impact on patch diameters remains rather small. We also observe that for a step-frequency SAR, the residual phase error would be zero anyway. Therefor, to facilitate the understanding of the tiered subaperture technique for SAR, we shall continue to presume that the residual phase error is negligible.

At this point it is instructive to compare equation (77) with the simple function of equation (1) that was analyzed in section 2. In both cases we are dealing with a phase that is a polynomial in n , and in both cases we interested in estimating with fine resolution the linear term in n , over a wide range of cofactor values of n . However, in equation (77) we are also ultimately interested in the constant (in n) phase term for s_y information.

Now for a quick study of the error terms. Error terms ϵ_0 and ϵ_1 represent offsets in s_y and s_x respectively, and are generally functions of both s_x and s_y . They cause spatial distortions in the final SAR image. Quadratic error term ϵ_2 causes a broadening of the image impulse response (IPR) via two mechanisms. The first is the direct way in that it is a quadratic phase error that misfocuses the SAR data. The second is, as we shall see, when subapertures are employed, this term manifests itself as a migration term that causes a windowing of the data. Higher order terms ϵ_i for $i > 2$ are typically of a lesser significance than ϵ_2 . They become significant only when left uncompensated in cases where ϵ_2 is compensated. Therefor, they should be included in any phase error compensation scheme, but when done so, will not be the error that limits patch diameters.

Another important observation is that equation (77) has phase that is exactly linear in index i . A DFT across index i will always be fully focused. Any subapertures in index i that will be required will be for different reasons than for subapertures in index n .

4.1. One Tier of Azimuth Subapertures

Following the development in section 2.1 as a blueprint, we make index n a function of two other indices m_1 and m_2 ,

$$n = m_1 + \Delta_2 m_2, \quad (78)$$

where as before m_1 is intra-subaperture index and is limited to the domain $-M_1/2 \leq m_1 \leq M_1/2 - 1$, m_2 is the inter-subaperture index and is limited to the domain $-M_2/2 \leq m_2 \leq M_2/2 - 1$, and Δ_2 is a data decimation factor, with subapertures overlapped by an amount $(M_1 - \Delta_2)$. We rewrite equation (77) as

$$X_V(i, m_1, m_2) \approx A(r_s) \exp j \left\{ \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s i)}{c} \right) \cdot \left((\epsilon_0 - s_y) + (\epsilon_1 + s_x) d\alpha (m_1 + \Delta_2 m_2) + \epsilon_2 d\alpha^2 (m_1 + \Delta_2 m_2)^2 + \dots \right) \right\}. \quad (79)$$

Since we are interested in how patch diameters increase by using subapertures, we invoke the earlier discussion and consider error terms with order greater than quadratic as inconsequential, and ignore them. We may then rewrite equation (79) as

$$\begin{aligned} X_V(i, m_1, m_2) \approx & A(r_s) \exp j \left\{ \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s i)}{c} \right) (\epsilon_0 - s_y) \right\} \\ & \cdot \exp j \left\{ \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s i)}{c} \right) ((s_x + \epsilon_1 + 2\epsilon_2 d\alpha \Delta_2 m_2) d\alpha m_1 + \epsilon_2 d\alpha^2 m_1^2) \right\} \\ & \cdot \exp j \left\{ \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s i)}{c} \right) ((\epsilon_1 + s_x) d\alpha \Delta_2 m_2 + \epsilon_2 d\alpha^2 \Delta_2^2 m_2^2) \right\}. \end{aligned} \quad (80)$$

The processing strategy is clear. First we perform a CZT across m_1 , follow this with some phase error corrections, and then perform a CZT across m_2 . Range compression (a DFT across index i) will be the last processing step before magnitude detection.

4.1.1. Processing

Step 1. We choose M_1 such that the quadratic term in line 2 of equation (80) remains negligible. The criterion is

$$\left| \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s i)}{c} \right) (\epsilon_2 d\alpha^2 M_1^2 / 4) \right| \leq \phi_1 \quad (81)$$

where ϕ_1 is the maximum allowable quadratic phase error for this stage, typically on the order of $\pi/2$.

We define the CZT operation as

$$X_1(i, u_1, m_2) = \sum_{m_1 = -M_1/2}^{M_1/2-1} X_V(i, m_1, m_2) \exp -j \left\{ \left(1 + (\gamma_0 T_s / \omega_0) i \right) \frac{2\pi u_1}{M_1} m_1 \right\}, \quad (82)$$

and identify the result as approximately

$$\begin{aligned} X_1(i, u_1, m_2) \approx & A(r_s) \exp j \left\{ \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s i)}{c} \right) (\epsilon_0 - s_y) \right\} \\ & \cdot \exp j \left\{ \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s i)}{c} \right) (\epsilon_1 + s_x) d\alpha \Delta_2 m_2 + \epsilon_2 d\alpha^2 \Delta_2^2 m_2^2 \right\} \\ & \cdot \text{csinc}_{M_1} \left[\frac{M_1}{2} \left(1 + \left(\frac{\gamma_0 T_s}{\omega_0} \right) i \right) \left(\frac{2 \omega_0 \cos \psi_0 d\alpha}{c} (s_x + \epsilon_1 + 2\epsilon_2 d\alpha \Delta_2 m_2) - \frac{2\pi u_1}{M_1} \right) \right]. \end{aligned} \quad (83)$$

From this expression we may estimate

$$\hat{s}_x = \left(\frac{\lambda_0}{2 (M_1 d\alpha \cos \psi_0)} \right) u_1, \quad (84)$$

with resolution

$$\rho_{x,1} = \frac{1}{(1 + (\gamma_0 T_s / \omega_0) i)} \left(\frac{\lambda_0}{2 (M_1 d\alpha \cos \psi_0)} \right) \approx \frac{\lambda_0}{2 (M_1 d\alpha \cos \psi_0)}, \quad (85)$$

where, as before, λ_0 is the nominal SAR wavelength ($\lambda_0 = 2\pi c / \omega_0$).

Figure 5. TSA Processing with One Tier of Azimuth Subapertures

Since the amplitude peak of line 3 in equation (83) migrates with index m_2 , we will impose a limit on this migration such that

$$\left| 2\epsilon_2 d\alpha \Delta_2 \frac{M_2}{2} \right| \leq \sigma_1 \frac{\rho_{x,1}}{2}, \quad (86)$$

where σ_1 is the total number of resolution cells that the peak will migrate across an entire aperture. Typically, $\sigma_1 \approx 1$ will not degrade the ultimate image IPR unacceptably. This limit is typically more restrictive than equation (81).

Step 2. Now that we have an estimate of s_x , we are also able to estimate more accurately the error terms $\epsilon_0, \epsilon_1, \epsilon_2, \dots$, and designate these estimates as $\hat{\epsilon}_0, \hat{\epsilon}_1, \hat{\epsilon}_2, \dots$

With these estimates we compensate the phase of X_I so that

$$\begin{aligned} X_1^*(i, u_1, m_2) &\approx X_1(i, u_1, m_2) \exp -j \left\{ \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s i)}{c} \right) (\hat{\epsilon}_0) \right\} \\ &\cdot \exp -j \left\{ \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s i)}{c} \right) ((\hat{\epsilon}_1 + \hat{s}_x) d\alpha \Delta_2 m_2 + \hat{\epsilon}_2 d\alpha^2 \Delta_2^2 m_2^2) \right\}. \end{aligned} \quad (87)$$

With the condition of equation (86) met, this leads to

$$\begin{aligned} X_1^*(i, u_1, m_2) &\approx A(r_s) \exp j \left\{ \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s i)}{c} \right) ((\epsilon_0 - \hat{\epsilon}_0) - s_y) \right\} \\ &\cdot \exp j \left\{ \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s i)}{c} \right) ((\epsilon_1 - \hat{\epsilon}_1) + (s_x - \hat{s}_x)) d\alpha \Delta_2 m_2 + (\epsilon_2 - \hat{\epsilon}_2) d\alpha^2 \Delta_2^2 m_2^2 \right\} \\ &\cdot \text{sinc}_{M_1} \left[\frac{M_1}{2} \left(1 + \left(\frac{\gamma_0 T_s}{\omega_0} \right) i \right) \left(\frac{2 \omega_0 \cos \psi_0 d\alpha}{c} (s_x + \epsilon_1) - \frac{2\pi u_1}{M_1} \right) \right]. \end{aligned} \quad (88)$$

The quadratic error term in line 2 of equation (88) has thus been compensated to within a coarse resolution estimate of s_x .

Step 3. The limit on M_2 must satisfy the criterion

$$\left| \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s i)}{c} \right) ((\epsilon_2 - \hat{\epsilon}_2) d\alpha^2 \Delta_2^2 M_2^2 / 4) \right| \leq \phi_2 \quad (89)$$

where ϕ_2 is the maximum allowable quadratic phase error for this stage, typically on the order of $\pi/2$.

We pause a moment here to note that the next step is another azimuth processing step, and not a range transform as in OSA processing [3][4]. The intent here is to remain in the range frequency domain, to some extent at least, as long as possible so that we can effectively deal with range migration. This requires the final stage of range compression (processing) to occur at or near the end of the line, after the final stage of azimuth processing. Performing final range compression too early forces us to be limited by a range migration term, which may or may not be tolerable in the operational scenario considered. Later we shall see how partial range compression can be used in-between azimuth compression stages.

We define the next azimuth CZT operation as

$$X_2(i, u_1, u_2) = \sum_{m_2 = -M_2/2}^{M_2/2-1} X_1^*(i, u_1, m_2) \exp -j \left\{ (1 + (\gamma_0 T_s / \omega_0) i) \frac{2\pi u_2}{M_2} m_2 \right\}, \quad (90)$$

and identify the result as approximately

$$\begin{aligned} X_2(i, u_1, u_2) \approx & A(r_s) \exp j \left\{ \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s i)}{c} \right) ((\epsilon_0 - \hat{\epsilon}_0) - s_y) \right\} \\ & \cdot \text{csinc}_{M_1} \left[\frac{M_1}{2} \left(1 + \left(\frac{\gamma_0 T_s}{\omega_0} \right) i \right) \left(\frac{2 \omega_0 \cos \psi_0 d \alpha}{c} (s_x + \epsilon_1) - \frac{2\pi u_1}{M_1} \right) \right] \\ & \cdot \text{csinc}_{M_2} \left[\frac{M_2}{2} \left(1 + \left(\frac{\gamma_0 T_s}{\omega_0} \right) i \right) \left(\frac{2 \omega_0 \cos \psi_0 \Delta_2 d \alpha}{c} ((s_x - \hat{s}_x) + (\epsilon_1 - \hat{\epsilon}_1)) - \frac{2\pi u_2}{M_2} \right) \right]. \quad (91) \end{aligned}$$

This expression allows us to refine our estimate of s_x as

$$^{(2)}\hat{s}_x = \left(\frac{\lambda_0}{2 (M_1 d \alpha \cos \psi_0)} \right) u_1 + \left(\frac{\lambda_0}{2 (M_2 \Delta_2 d \alpha \cos \psi_0)} \right) u_2, \quad (92)$$

with new resolution

$$\rho_{x,2} = \frac{1}{(1 + (\gamma_0 T_s / \omega_0) i)} \left(\frac{\lambda_0}{2 (M_2 \Delta_2 d \alpha \cos \psi_0)} \right) \approx \frac{\lambda_0}{2 (M_2 \Delta_2 d \alpha \cos \psi_0)}. \quad (93)$$

The remaining error terms ϵ_0 and ϵ_1 manifest themselves as residual geometric distortions in the final image. The ϵ_1 terms in the csinc() functions we are stuck with, but the ϵ_0 term in line 1 of equation (91) can still be compensated somewhat prior to range compression.

Step 4. With a new and better estimate of ϵ_0 , which we will designate as $^{(2)}\hat{\epsilon}_0$, we can compensate the phase of X_2 so that

$$X_2^*(i, u_1, u_2) = X_2(i, u_1, u_2) \exp -j \left\{ \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s i)}{c} \right) \left(^{(2)}\hat{\epsilon}_0 - \epsilon_0 \right) \right\}, \quad (94)$$

or, expanded to

$$\begin{aligned} X_2^*(i, u_1, u_2) \approx & A(r_s) \exp j \left\{ \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s i)}{c} \right) \left((\epsilon_0 - ^{(2)}\hat{\epsilon}_0) - s_y \right) \right\} \\ & \cdot \text{csinc}_{M_1} \left[\frac{M_1}{2} \left(1 + \left(\frac{\gamma_0 T_s}{\omega_0} \right) i \right) \left(\frac{2 \omega_0 \cos \psi_0 d \alpha}{c} (s_x + \epsilon_1) - \frac{2 \pi u_1}{M_1} \right) \right] \\ & \cdot \text{csinc}_{M_2} \left[\frac{M_2}{2} \left(1 + \left(\frac{\gamma_0 T_s}{\omega_0} \right) i \right) \left(\frac{2 \omega_0 \cos \psi_0 \Delta_2 d \alpha}{c} ((s_x - \hat{s}_x) + (\epsilon_1 - \hat{\epsilon}_1)) - \frac{2 \pi u_2}{M_2} \right) \right]. \quad (95) \end{aligned}$$

Step 5. Range compression to the final complex image requires a final DFT across index i , in the manner

$$X_c(v, u_1, u_2) = \sum_{i=-I/2}^{I/2-1} X_2^*(i, u_1, u_2) \exp -j \left\{ \frac{2 \pi v}{I} i \right\}, \quad (96)$$

which results in

$$\begin{aligned} X_c(v, u_1, u_2) \approx & A(r_s) \exp j \left\{ \left(\frac{2 \cos \psi_0 \omega_0}{c} \right) \left((\epsilon_0 - ^{(2)}\hat{\epsilon}_0) - s_y \right) \right\} \\ & \cdot \text{csinc}_{M_1} \left[\frac{M_1}{2} \left(1 + \left(\frac{\gamma_0 T_s}{\omega_0} \right) i \right) \left(\frac{2 \omega_0 \cos \psi_0 d \alpha}{c} (s_x + \epsilon_1) - \frac{2 \pi u_1}{M_1} \right) \right] \\ & \cdot \text{csinc}_{M_2} \left[\frac{M_2}{2} \left(1 + \left(\frac{\gamma_0 T_s}{\omega_0} \right) i \right) \left(\frac{2 \omega_0 \cos \psi_0 \Delta_2 d \alpha}{c} ((s_x - \hat{s}_x) + (\epsilon_1 - \hat{\epsilon}_1)) - \frac{2 \pi u_2}{M_2} \right) \right] \\ & \cdot \text{csinc}_I \left[\frac{I}{2} \left(\frac{2 \gamma_0 T_s \cos \psi_0}{c} \left((\epsilon_0 - ^{(2)}\hat{\epsilon}_0) - s_y \right) - \frac{2 \pi v}{I} \right) \right]. \quad (97) \end{aligned}$$

From this we estimate

$$\hat{s}_y = \frac{-\pi c}{(\gamma_0 T_s I) \cos \psi_0} \nu, \quad (98)$$

with resolution

$$\rho_y = \frac{\pi c}{(\gamma_0 T_s I) \cos \psi_0} = \frac{c}{2B \cos \psi_0}. \quad (99)$$

where B is the effective chirp bandwidth in Hz as previously defined.

4.1.2. Patch Diameter Limits

In the processing, we faced three different limits, expressed as equations (81), (86), and (89). Of these, equation (86) is more restrictive than equation (81). In fact, for subaperture approaches, migration limitations are generally more severe than focus limitations. Therefor we have really only two limitations, repeated here as the migration limit from the first CZT

$$\left| 2\epsilon_2 d\alpha \Delta_2 \frac{M_2}{2} \right| \leq \sigma_1 \frac{\rho_{x,1}}{2}, \quad (100)$$

and the focus limit for the second CZT

$$\left| \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s i)}{c} \right) \left((\epsilon_2 - \hat{\epsilon}_2) d\alpha^2 \Delta_2^2 M_2^2 / 4 \right) \right| \leq \phi_2. \quad (101)$$

Recall from equation (72) that for benign flight paths

$$\epsilon_2 \approx -\frac{\cos \psi_0 \left(s_x^2 - s_y^2 / 2 \right)}{2 \|r_{c0}\|}, \quad (102)$$

which implies that after the first estimate of s_x ,

$$\epsilon_2 - \hat{\epsilon}_2 \approx -\frac{\cos \psi_0 \left(s_x^2 - \hat{s}_x^2 - s_y^2 / 2 \right)}{2 \|r_{c0}\|}. \quad (103)$$

We now define D_x as the patch diameter in the s_x direction when $s_y=0$, and D_y as the patch diameter in the s_y direction when $s_x=0$.

Azimuth Patch Diameter: Combining equations (100) and (102) with $s_y=0$ yields

$$D_x^2 \leq 8\sigma_1 \rho_{x,1} \rho_{x,2} \left(1 + \frac{\gamma_0 T_{s,i}}{\omega_0} \right) \left(\frac{\|r_{c0}\|}{\lambda_0} \right), \quad (104)$$

We now note that in equation (103)

$$\left| s_x^2 - \hat{s}_x^2 \right| = \left| (s_x - \hat{s}_x) (s_x + \hat{s}_x) \right| \leq \frac{\rho_{x,1}}{2} D_x, \quad (105)$$

allowing us to replace equation (101) with the sufficient condition

$$\left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_{s,i})}{c} \right) \left(\left(\frac{\cos \psi_0 \left(\frac{\rho_{x,1} D_x}{2} \right)}{2 \|r_{c0}\|} \right) d\alpha^2 \Delta_2^2 M_2^2 / 4 \right) \leq \phi_2, \quad (106)$$

which can be algebraically manipulated to

$$\rho_{x,1} \leq \frac{8}{D_x} \left(\frac{\phi_2}{\pi/2} \right) \rho_{x,2}^2 \left(\frac{\|r_{c0}\|}{\lambda_0} \right) \left(1 + \frac{\gamma_0 T_{s,i}}{\omega_0} \right). \quad (107)$$

Combining equations (104) and (107) yields an ultimate limit

$$D_x \leq 4 \rho_{x,2} \left(\sigma_1 \left(\frac{\phi_2}{\pi/2} \right) \left(\left(1 + \frac{\gamma_0 T_{s,i}}{\omega_0} \right) \frac{\|r_{c0}\|}{\lambda_0} \right)^2 \right)^{\frac{1}{3}}, \quad (108)$$

which for typical values $\sigma_1=1$, $\phi_2=\pi/2$, and $\gamma_0 T_s (I/2) \ll \omega_0$, becomes

$$D_x \leq 4 \rho_{x,2} (\|r_{c0}\|/\lambda_0)^{2/3}. \quad (109)$$

Contrasting this with the Polar Format limit in equation (76) shows an improvement in azimuth patch size of $(\|r_{c0}\|/\lambda_0)^{1/6}$, which can be quite substantial.

Range Patch Diameter: The range dimension limit is a different story. Combining equations (100) and (102) with $s_x=0$ yields

$$D_y^2 \leq 16\sigma_1 \rho_{x,1} \rho_{x,2} \left(1 + \frac{\gamma_0 T_{s,i}}{\omega_0} \right) \left(\frac{\|r_{c0}\|}{\lambda_0} \right). \quad (110)$$

Equation (101) becomes

$$\left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s i)}{c} \right) \left(\left(\frac{\cos \psi_0 (D_y^2 / 8)}{2 \|r_{c0}\|} \right) d \alpha^2 \Delta_2^2 M_2^2 / 4 \right) \leq \phi_2, \quad (111)$$

which can be algebraically manipulated to

$$D_y^2 \leq 32 \left(\frac{\phi_2}{\pi/2} \right) \rho_{x,2}^2 \left(\frac{\|r_{c0}\|}{\lambda_0} \right) \left(1 + \frac{\gamma_0 T_s i}{\omega_0} \right). \quad (112)$$

Equation (112) is much more restrictive than equation (110), and therefor dominates as the limit, that is, after range compression, this limit is approximated by

$$D_y \leq 4 \rho_{x,2} \sqrt{2 \left(\frac{\phi_2}{\pi/2} \right) \left(\frac{\|r_{c0}\|}{\lambda_0} \right)}, \quad (113)$$

which for $\phi_2 = \pi/2$, becomes

$$D_y \leq 4 \rho_{x,2} \sqrt{2 \|r_{c0}\| / \lambda_0}. \quad (114)$$

Recalling equation (76) and the discussion around equation (73) shows that this is merely the Polar Format limit on patch diameter, as if no subapertures had been used.

Note that if prior to the final CZT, s_y could be coarsely (but adequately) estimated, as was s_x , then the possibility exists to relax the constraint in equation (112) to the point where equation (112) dominates, yielding a larger range patch diameter approximately equivalent to the azimuth patch diameter.

4.2. Two Tiers of Azimuth Subapertures

Again following the development in section 2.2 as a guide, let index n be divided into two tiers of subapertures, as in figure 2, such that

$$n = m_1 + \Delta_2 m_2 + \Delta_2 \Delta_3 m_3. \quad (115)$$

Ignoring, as before, error terms with order greater than quadratic allows us to rewrite equation (77) as

$$\begin{aligned} X_V(i, m_1, m_2, m_3) \approx & A(r_s) \exp j \left\{ \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s i)}{c} \right) (\epsilon_0 - s_y) \right\} \\ & \cdot \exp j \left\{ \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s i)}{c} \right) \left((s_x + \epsilon_1 + 2\epsilon_2 d\alpha \Delta_2 m_2 + 2\epsilon_2 d\alpha \Delta_2 \Delta_3 m_3) d\alpha m_1 + \epsilon_2 d\alpha^2 m_1^2 \right) \right\} \\ & \cdot \exp j \left\{ \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s i)}{c} \right) \left((s_x + \epsilon_1 + 2\epsilon_2 d\alpha \Delta_2 \Delta_3 m_3) d\alpha \Delta_2 m_2 + \epsilon_2 d\alpha^2 \Delta_2^2 m_2^2 \right) \right\} \\ & \cdot \exp j \left\{ \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s i)}{c} \right) \left((\epsilon_1 + s_x) d\alpha \Delta_2 \Delta_3 m_3 + \epsilon_2 d\alpha^2 \Delta_2^2 \Delta_3^2 m_3^2 \right) \right\}. \end{aligned} \quad (116)$$

The processing strategy is extended to first performing a CZT across m_1 , followed with some phase error corrections, followed with a CZT across m_2 , followed with some phase error corrections, followed with a CZT across m_3 . Range compression (a DFT across index i) will be the last processing step before magnitude detection.

Figure 6. TSA Processing with Two Tiers of Azimuth Subapertures

4.2.1. Processing

Step 1. With M_1 suitably limited, the first CZT results in

$$\begin{aligned}
 X_1(i, u_1, m_2, m_3) \approx & A(r_s) \exp j \left\{ \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s i)}{c} \right) (\epsilon_0 - s_y) \right\} \\
 & \cdot \exp j \left\{ \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s i)}{c} \right) \left((s_x + \epsilon_1 + 2\epsilon_2 d\alpha \Delta_2 \Delta_3 m_3) d\alpha \Delta_2 m_2 + \epsilon_2 d\alpha^2 \Delta_2^2 m_2^2 \right) \right\} \\
 & \cdot \exp j \left\{ \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s i)}{c} \right) \left((\epsilon_1 + s_x) d\alpha \Delta_2 \Delta_3 m_3 + \epsilon_2 d\alpha^2 \Delta_2^2 \Delta_3^2 m_3^2 \right) \right\} . \\
 & \cdot \text{csinc}_{M_1} \left[\frac{M_1}{2} \left(1 + \left(\frac{\gamma_0 T_s}{\omega_0} \right) i \right) \left(\frac{2 \omega_0 \cos \psi_0 d\alpha}{c} (s_x + \epsilon_1 + 2\epsilon_2 d\alpha \Delta_2 m_2 + 2\epsilon_2 d\alpha \Delta_2 \Delta_3 m_3) - \frac{2\pi u_1}{M_1} \right) \right]
 \end{aligned} \tag{117}$$

from which we may estimate

$$\hat{s}_x = \left(\frac{\lambda_0}{2 (M_1 d\alpha \cos \psi_0)} \right) u_1, \tag{118}$$

with resolution

$$\rho_{x,1} = \frac{1}{(1 + (\gamma_0 T_s / \omega_0) i)} \left(\frac{\lambda_0}{2 (M_1 d\alpha \cos \psi_0)} \right) \approx \frac{\lambda_0}{2 (M_1 d\alpha \cos \psi_0)}. \tag{119}$$

We also require a migration limit similar to equation (86),

$$\left| 2\epsilon_2 d\alpha \left(\Delta_2 \frac{M_2}{2} + \Delta_2 \Delta_3 \frac{M_3}{2} \right) \right| \leq \sigma_1 \frac{\rho_{x,1}}{2}. \tag{120}$$

Step 2. The estimate in equation (118) can now be used to provide first estimates of the error terms in equation (116), which can then be used to partially compensate those error terms. With the limit of equation (120), the compensated function becomes

$$\begin{aligned}
X_1^*(i, u_1, m_2, m_3) \approx & A(r_s) \exp j \left\{ \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s i)}{c} \right) \left((\epsilon_0 - \hat{\epsilon}_0) - s_y \right) \right\} \\
& \cdot \exp j \left\{ \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s i)}{c} \right) \left((s_x - \hat{s}_x) + (\epsilon_1 - \hat{\epsilon}_1) + 2 (\epsilon_2 - \hat{\epsilon}_2) d\alpha \Delta_2 \Delta_3 m_3 + (\epsilon_2 - \hat{\epsilon}_2) d\alpha^2 \Delta_2^2 m_2^2 \right) \right\} \\
& \cdot \exp j \left\{ \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s i)}{c} \right) \left((s_x - \hat{s}_x) + (\epsilon_1 - \hat{\epsilon}_1) d\alpha \Delta_2 \Delta_3 m_3 + (\epsilon_2 - \hat{\epsilon}_2) d\alpha^2 \Delta_2^2 \Delta_3^2 m_3^2 \right) \right\} \\
& \cdot \text{csinc}_{M_1} \left[\frac{M_1}{2} \left(1 + \left(\frac{\gamma_0 T_s}{\omega_0} \right) i \right) \left(\frac{2 \omega_0 \cos \psi_0 d\alpha}{c} (s_x + \epsilon_1) - \frac{2 \pi u_1}{M_1} \right) \right] \quad (121)
\end{aligned}$$

Step 3. With M_2 chosen so that the quadratic error term in line 2 of equation (121) is negligible, a CZT across m_2 yields approximately

$$\begin{aligned}
X_2(i, u_1, u_2, m_3) \approx & A(r_s) \exp j \left\{ \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s i)}{c} \right) \left((\epsilon_0 - \hat{\epsilon}_0) - s_y \right) \right\} \\
& \cdot \exp j \left\{ \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s i)}{c} \right) \left((s_x - \hat{s}_x) + (\epsilon_1 - \hat{\epsilon}_1) d\alpha \Delta_2 \Delta_3 m_3 + (\epsilon_2 - \hat{\epsilon}_2) d\alpha^2 \Delta_2^2 \Delta_3^2 m_3^2 \right) \right\} \\
& \cdot \text{csinc}_{M_1} \left[\frac{M_1}{2} \left(1 + \left(\frac{\gamma_0 T_s}{\omega_0} \right) i \right) \left(\frac{2 \omega_0 \cos \psi_0 d\alpha}{c} (s_x + \epsilon_1) - \frac{2 \pi u_1}{M_1} \right) \right] \\
& \cdot \text{csinc}_{M_2} \left[\frac{M_2}{2} \left(1 + \left(\frac{\gamma_0 T_s}{\omega_0} \right) i \right) \left(\frac{2 \omega_0 \cos \psi_0 \Delta_2 d\alpha}{c} ((s_x - \hat{s}_x) + (\epsilon_1 - \hat{\epsilon}_1) + 2 (\epsilon_2 - \hat{\epsilon}_2) d\alpha \Delta_2 \Delta_3 m_3) - \frac{2 \pi u_2}{M_2} \right) \right] \quad (122)
\end{aligned}$$

from which we may estimate

$${}^{(2)}\hat{s}_x = \left(\frac{\lambda_0}{2 (M_1 d\alpha \cos \psi_0)} \right) u_1 + \left(\frac{\lambda_0}{2 (M_2 \Delta_2 d\alpha \cos \psi_0)} \right) u_2, \quad (123)$$

with new resolution

$$\rho_{x,2} = \frac{1}{(1 + (\gamma_0^T s / \omega_0) i)} \left(\frac{\lambda_0}{2 (M_2 \Delta_2 d\alpha \cos \Psi_0)} \right) \approx \frac{\lambda_0}{2 (M_2 \Delta_2 d\alpha \cos \Psi_0)}. \quad (124)$$

We also require another migration limit similar to equation (120),

$$\left| 2 (\epsilon_2 - \hat{\epsilon}_2) d\alpha \left(\Delta_2 \Delta_3 \frac{M_3}{2} \right) \right| \leq \sigma_2 \frac{\rho_{x,2}}{2}. \quad (125)$$

Step 4. With a better estimate of s_x , we also have better estimates of the error terms. Making the appropriate corrections, and with the limit of equation (125), the new corrected function becomes

$$\begin{aligned} X_2^*(i, u_1, u_2, m_3) \approx A(r_s) \exp j \left\{ \left(\frac{2 \cos \Psi_0 (\omega_0 + \gamma_0^T s i)}{c} \right) \left((\epsilon_0 - {}^{(2)}\hat{\epsilon}_0) - s_y \right) \right\} \\ \cdot \exp j \left\{ \left(\frac{2 \cos \Psi_0 (\omega_0 + \gamma_0^T s i)}{c} \right) \left(\left((s_x - {}^{(2)}\hat{s}_x) + (\epsilon_1 - {}^{(2)}\hat{\epsilon}_1) \right) d\alpha \Delta_2 \Delta_3 m_3 \right) \right. \\ \left. + (\epsilon_2 - {}^{(2)}\hat{\epsilon}_2) d\alpha^2 \Delta_2^2 \Delta_3^2 m_3^2 \right\} \\ \cdot \text{csinc}_{M_1} \left[\frac{M_1}{2} \left(1 + \left(\frac{\gamma_0^T s}{\omega_0} \right) i \right) \left(\frac{2 \omega_0 \cos \Psi_0 d\alpha}{c} (s_x + \epsilon_1) - \frac{2\pi u_1}{M_1} \right) \right] \\ \cdot \text{csinc}_{M_2} \left[\frac{M_2}{2} \left(1 + \left(\frac{\gamma_0^T s}{\omega_0} \right) i \right) \left(\frac{2 \omega_0 \cos \Psi_0 \Delta_2 d\alpha}{c} (s_x - \hat{s}_x) + (\epsilon_1 - \hat{\epsilon}_1) - \frac{2\pi u_2}{M_2} \right) \right] \right]. \quad (126) \end{aligned}$$

Step 5. With M_3 chosen so that the quadratic error term in line 2 of equation (126) is negligible, a CZT across m_3 yields approximately

$$\begin{aligned} X_3(i, u_1, u_2, u_3) \approx A(r_s) \exp j \left\{ \left(\frac{2 \cos \Psi_0 (\omega_0 + \gamma_0^T s i)}{c} \right) \left((\epsilon_0 - {}^{(2)}\hat{\epsilon}_0) - s_y \right) \right\} \\ \cdot \text{csinc}_{M_1} \left[\frac{M_1}{2} \left(1 + \left(\frac{\gamma_0^T s}{\omega_0} \right) i \right) \left(\frac{2 \omega_0 \cos \Psi_0 d\alpha}{c} (s_x + \epsilon_1) - \frac{2\pi u_1}{M_1} \right) \right] \\ \cdot \text{csinc}_{M_2} \left[\frac{M_2}{2} \left(1 + \left(\frac{\gamma_0^T s}{\omega_0} \right) i \right) \left(\frac{2 \omega_0 \cos \Psi_0 \Delta_2 d\alpha}{c} (s_x - \hat{s}_x) + (\epsilon_1 - \hat{\epsilon}_1) - \frac{2\pi u_2}{M_2} \right) \right] \\ \cdot \text{csinc}_{M_3} \left[\frac{M_3}{2} \left(1 + \left(\frac{\gamma_0^T s}{\omega_0} \right) i \right) \left(\frac{2 \omega_0 \cos \Psi_0 \Delta_2 \Delta_3 d\alpha}{c} (s_x - \hat{s}_x) + (\epsilon_1 - {}^{(2)}\hat{\epsilon}_1) - \frac{2\pi u_3}{M_3} \right) \right] \right], \quad (127) \end{aligned}$$

from which we may make the third and final estimate

$$^{(3)}\hat{s}_x = \frac{\lambda_0}{2 d\alpha \cos \psi_0} \left(\frac{u_1}{M_1} + \frac{u_2}{\Delta_2 M_2} + \frac{u_3}{\Delta_2 \Delta_3 M_3} \right), \quad (128)$$

with new resolution

$$\rho_{x,3} = \frac{1}{(1 + (\gamma_0 T_s / \omega_0) i)} \left(\frac{\lambda_0}{2 (M_3 \Delta_2 \Delta_3 d\alpha \cos \psi_0)} \right) \approx \frac{\lambda_0}{2 (M_3 \Delta_2 \Delta_3 d\alpha \cos \psi_0)}. \quad (129)$$

Since this was the final azimuth compression stage, no migration limit applies to M_3 , so we revert to the quadratic focus limit

$$\left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s i)}{c} \right) \left(\epsilon_2 - ^{(2)}\hat{\epsilon}_2 \right) d\alpha^2 \Delta_2^2 \Delta_3^2 \left(\frac{M_3}{2} \right)^2 \leq \phi_3. \quad (130)$$

Step 6. The improved estimate of s_x can now be used prior to range compression by compensating the geometric error in line 1 of equation 127 such that

$$\begin{aligned} X_3^*(i, u_1, u_2, u_3) \approx & A(r_s) \exp j \left\{ \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s i)}{c} \right) \left((\epsilon_0 - ^{(3)}\hat{\epsilon}_0) - s_y \right) \right\} \\ & \cdot \text{csinc}_{M_1} \left[\frac{M_1}{2} \left(1 + \left(\frac{\gamma_0 T_s}{\omega_0} \right) i \right) \left(\frac{2 \omega_0 \cos \psi_0 d\alpha}{c} (s_x + \epsilon_1) - \frac{2\pi u_1}{M_1} \right) \right] \\ & \cdot \text{csinc}_{M_2} \left[\frac{M_2}{2} \left(1 + \left(\frac{\gamma_0 T_s}{\omega_0} \right) i \right) \left(\frac{2 \omega_0 \cos \psi_0 \Delta_2 d\alpha}{c} \left((s_x - \hat{s}_x) + (\epsilon_1 - \hat{\epsilon}_1) - \frac{2\pi u_2}{M_2} \right) \right) \right] \\ & \cdot \text{csinc}_{M_3} \left[\frac{M_3}{2} \left(1 + \left(\frac{\gamma_0 T_s}{\omega_0} \right) i \right) \left(\frac{2 \omega_0 \cos \psi_0 \Delta_2 \Delta_3 d\alpha}{c} \left((s_x - \hat{s}_x) + (\epsilon_1 - ^{(2)}\hat{\epsilon}_1) - \frac{2\pi u_3}{M_3} \right) \right) \right]. \quad (131) \end{aligned}$$

Step Z. Range compression to the final complex image again requires a DFT across index i , and yields

$$\begin{aligned}
X_3^*(i, u_1, u_2, u_3) \approx & A(r_s) \exp j \left\{ \left(\frac{2 \cos \psi_0 \omega_0}{c} \right) \left((\epsilon_0 - {}^{(3)}\hat{\epsilon}_0) - s_y \right) \right\} \\
& \cdot \text{csinc}_{M_1} \left[\frac{M_1}{2} \left(1 + \left(\frac{\gamma_0 T_s}{\omega_0} \right) i \right) \left(\frac{2 \omega_0 \cos \psi_0 d \alpha}{c} (s_x + \epsilon_1) - \frac{2 \pi u_1}{M_1} \right) \right] \\
& \cdot \text{csinc}_{M_2} \left[\frac{M_2}{2} \left(1 + \left(\frac{\gamma_0 T_s}{\omega_0} \right) i \right) \left(\frac{2 \omega_0 \cos \psi_0 \Delta_2 d \alpha}{c} \left((s_x - \hat{s}_x) + (\epsilon_1 - \hat{\epsilon}_1) - \frac{2 \pi u_2}{M_2} \right) \right) \right] \\
& \cdot \text{csinc}_{M_3} \left[\frac{M_3}{2} \left(1 + \left(\frac{\gamma_0 T_s}{\omega_0} \right) i \right) \left(\frac{2 \omega_0 \cos \psi_0 \Delta_2 \Delta_3 d \alpha}{c} \left((s_x - \hat{s}_x) + (\epsilon_1 - {}^{(2)}\hat{\epsilon}_1) - \frac{2 \pi u_3}{M_3} \right) \right) \right] \\
& \cdot \text{csinc}_I \left[\frac{I}{2} \left(\frac{2 \gamma_0 T_s \cos \psi_0}{c} \left((\epsilon_0 - {}^{(3)}\hat{\epsilon}_0) - s_y \right) - \frac{2 \pi \nu}{I} \right) \right]. \tag{132}
\end{aligned}$$

As before, from this we estimate

$$\hat{s}_y = \frac{-\pi c}{(\gamma_0 T_s I) \cos \psi_0} \nu, \tag{133}$$

with resolution

$$\rho_y = \frac{\pi c}{(\gamma_0 T_s I) \cos \psi_0} = \frac{c}{2B \cos \psi_0}. \tag{134}$$

where B is the effective chirp bandwidth in Hz as previously defined.

4.2.2. Patch Diameter Limits

The three relevant limits are equations (120), (125), and (130), which are repeated here as

$$\left| 2\varepsilon_2 d\alpha \left(\Delta_2 \frac{M_2}{2} + \Delta_2 \Delta_3 \frac{M_3}{2} \right) \right| \leq \sigma_1 \frac{\rho_{x,1}}{2}, \quad (135)$$

$$\left| 2(\varepsilon_2 - \hat{\varepsilon}_2) d\alpha \left(\Delta_2 \Delta_3 \frac{M_3}{2} \right) \right| \leq \sigma_2 \frac{\rho_{x,2}}{2}, \quad (136)$$

and
$$\left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s i)}{c} \right) (\varepsilon_2 - {}^{(2)}\hat{\varepsilon}_2) d\alpha^2 \Delta_2^2 \Delta_3^2 \left(\frac{M_3}{2} \right)^2 \leq \phi_3. \quad (137)$$

Azimuth Patch Diameter: As in section 1.2 we assume $s_y=0$ and proceed with

$$|\varepsilon_2| \approx \frac{\cos \psi_0 (D_x/2)^2}{2 \|r_{c0}\|}, \quad (138)$$

$$|\varepsilon_2 - \hat{\varepsilon}_2| \approx \frac{\cos \psi_0 |s_x^2 - \hat{s}_x^2|}{2 \|r_{c0}\|} \leq \frac{\cos \psi_0 (D_x \rho_{x,1}/2)}{2 \|r_{c0}\|}, \quad (139)$$

and
$$|\varepsilon_2 - {}^{(2)}\hat{\varepsilon}_2| \approx \frac{\cos \psi_0 |s_x^2 - {}^{(2)}\hat{s}_x^2|}{2 \|r_{c0}\|} \leq \frac{\cos \psi_0 (D_x \rho_{x,2}/2)}{2 \|r_{c0}\|}. \quad (140)$$

Furthermore, we make the reasonable assumption in equation (135) that

$$\Delta_2 \Delta_3 \frac{M_3}{2} \gg \Delta_2 \frac{M_2}{2}. \quad (141)$$

Equations (135) thru (141) can be algebraically manipulated and rewritten in the following manner. The result of the first CZT imposes a migration limit of the form

$$D_x^2 \leq 8 \sigma_1 \rho_{x,1} \rho_{x,3} \left(1 + \frac{\gamma_0 T_s i}{\omega_0} \right) \frac{\|r_{c0}\|}{\lambda_0}. \quad (142)$$

The second, or intermediate, CZT results in a migration limit of the form

$$\rho_{x,1} \leq \frac{4}{D_x} \sigma_2 \rho_{x,2} \rho_{x,3} \left(1 + \frac{\gamma_0 T_s i}{\omega_0} \right) \frac{\|r_{c0}\|}{\lambda_0}. \quad (143)$$

The final CZT does not result in a migration limit, but is limited by a quadratic focus error in a manner such that

$$\rho_{x,2} \leq \frac{8}{D_x} \left(\frac{\phi_3}{\pi/2} \right) \rho_{x,3}^2 \left(1 + \frac{\gamma_0 T_s i}{\omega_0} \right) \frac{\|r_{c0}\|}{\lambda_0}. \quad (144)$$

These equations (142) thru (144) can be combined to yield an ultimate limit

$$D_x \leq 4\rho_{x,3} \left(\sigma_1 \sigma_2 \left(\frac{\phi_3}{\pi/2} \right) \left(\left(1 + \frac{\gamma_0 T_s i}{\omega_0} \right) \frac{\|r_{c0}\|}{\lambda_0} \right)^3 \right)^{\frac{1}{4}}, \quad (145)$$

which for typical values $\sigma_1 = \sigma_2 = 1$, $\phi_3 = \pi/2$, and $\gamma_0 T_s (I/2) \ll \omega_0$, becomes

$$D_x \leq 4\rho_{x,3} (\|r_{c0}\|/\lambda_0)^{3/4}. \quad (146)$$

Comparing this with the limit in equation (114) shows an improvement in azimuth patch size of $(\|r_{c0}\|/\lambda_0)^{1/12}$ over that of the single layer of subapertures, which can still be quite substantial.

Range Patch Diameter: The range dimension is still limited solely by the quadratic term in the final CZT stage to the Polar Format case, which for $\phi_3 = \pi/2$, becomes

$$D_y \leq 4\rho_{x,3} (\|r_{c0}\|/\lambda_0)^{1/2}. \quad (147)$$

4.3. Multiple Tiers of Azimuth Subapertures

Equations (142) thru (144) can be generalized for N_s number of subaperture layers or tiers. In this case there would be (N_s+1) stages of CZTs.

The first stage migration limit becomes

$$D_x^2 \leq 8\sigma_1 \rho_{x,1} \rho_{x,(N_s+1)} \left(1 + \frac{\gamma_0 T_{s,i}}{\omega_0} \right) \frac{\|r_{c0}\|}{\lambda_0}. \quad (148)$$

Intermediate CZT migration limits become

$$\rho_{x,(k-1)} \leq \frac{4}{D_x} \sigma_k \rho_{x,k} \rho_{x,(N_s+1)} \left(1 + \frac{\gamma_0 T_{s,i}}{\omega_0} \right) \frac{\|r_{c0}\|}{\lambda_0}, \text{ for } 2 \leq k \leq N_s. \quad (149)$$

The final CZT continues to be limited by a quadratic focus error so that

$$\rho_{x,N_s} \leq \frac{8}{D_x} \left(\frac{\phi_{(N_s+1)}}{\pi/2} \right) \rho_{x,(N_s+1)}^2 \left(1 + \frac{\gamma_0 T_{s,i}}{\omega_0} \right) \frac{\|r_{c0}\|}{\lambda_0}. \quad (150)$$

These equations (148) thru (150) can be combined to yield an ultimate limit

$$D_x \leq 4 \rho_{x,(N_s+1)} \left(\sigma_1 \sigma_2 \dots \sigma_{N_s} \left(\frac{\phi_{(N_s+1)}}{\pi/2} \right) \right)^{\frac{1}{N_s+2}} \left(\left(1 + \frac{\gamma_0 T_{s,i}}{\omega_0} \right) \frac{\|r_{c0}\|}{\lambda_0} \right)^{\frac{N_s+1}{N_s+2}}, \quad (151)$$

Figure 7. TSA Processing with N_s Tiers of Azimuth Subapertures

which for typical values $\sigma_k = 1$, $\phi_{(N_s+1)} = \pi/2$, and $\gamma_0 T_s (I/2) \ll \omega_0$, becomes

$$D_x \leq 4\rho_{x,(N_s+1)} (\|r_{c0}\|/\lambda_0)^{(N_s+1)/(N_s+2)}. \quad (152)$$

Under these conditions, at wavelength resolutions, this approaches $D_x \leq 4\|r_{c0}\|$, but the range patch diameter is still limited to

$$D_y \leq 4\rho_{x,(N_s+1)} (\|r_{c0}\|/\lambda_0)^{1/2}. \quad (153)$$

4.4. Observations

- Azimuth subapertures enhance azimuth patch diameter.
- More tiers of subapertures allow increasing the azimuth patch diameter.
- The marginal gain in azimuth patch diameter diminishes as the number of tiers increases.
- With subaperture processing, migration limits are typically more restrictive than phase error limits.
- By leaving range compression as the final stage, range migration is effectively not a problem, but range patch diameter is not benefited by azimuth subapertures.
- While the limit in equation (152) presumes a benign flight path, the technique of tiered subapertures is applicable even when the flight path is not benign, although the appropriate restrictions on patch diameter and inter-stage resolutions might change.
- Equation (151) is the limit prior to any range compression, and is range index dependent. We may remove this dependence by recognizing that

$$\left(1 + \frac{\gamma_0 T_s i}{\omega_0}\right) \geq \left(1 - \frac{\pi B}{\omega_0}\right).$$

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5. RANGE SUBAPERTURES FOR EXTENDING RANGE PATCH DIAMETER

In the previous section we showed how azimuth subapertures increased the azimuth patch diameter, but that the range patch diameter was still limited by the final azimuth CZT stage. In this section we explore subapertures in the range dimension for increasing range patch diameter.

5.1. Analog Range Subapertures and Range Channels

Recall from equation (48) that the received echo from the target scatterer is

$$X_R(t, n) = A(r_s) \text{rect}\left(\frac{t - t_n - \frac{2}{c}\|r_s\|}{T}\right) \cos\left(\omega_n\left(t - t_n - \frac{2}{c}\|r_s\|\right) + \frac{\gamma_n}{2}\left(t - t_n - \frac{2}{c}\|r_s\|\right)^2\right). \quad (154)$$

Now let us use a local oscillator that divides the time index into subapertures of time of the form

$$t \rightarrow (t + t_{k,n}), \quad K/2 \leq k \leq K/2 - 1, \quad (155)$$

where k is the subaperture index number. We have chosen the subaperture delay $t_{k,n}$ to *not* be a linear function of subaperture index k , as with azimuth processing, but rather a more complicated function of both k and n .

X_R is now deramped and separated into I and Q channels with K local oscillator signals of the form

$$X_{LO}(t, k, n) = 2 \text{rect}\left(\frac{t + t_{k,n} - t_n - \frac{2}{c}\|r_c\|}{T_m}\right) \exp -j \left\{ \omega_n\left(t + t_{k,n} - t_n - \frac{2}{c}\|r_c\|\right) + \frac{\gamma_n}{2}\left(t + t_{k,n} - t_n - \frac{2}{c}\|r_c\|\right)^2 \right\}, \quad (156)$$

The act of mixing the local oscillator with the received signal, followed with low-pass filtering, is a matched filtering operation, where each index k represents a unique matched filter. This is equivalent to a transform over the subaperture index k , but prior to any transform across t , which is the opposite order from what we've always done before. The connection between subapertures and range channels comes about as follows.

We choose a number of different CRPs on the ground and index them with the subaperture index k , and define the range vector from those points to the SAR as $r_{c,k}$. Now, we set the subaperture delay such that

$$t_{k,n} = \frac{2}{c}\|r_c\| - \frac{2}{c}\|r_{c,k}\|. \quad (157)$$

where r_c is still based on the primary reference CRP. Using equation (157) in equation (156) yields the final set of local oscillator signals,

$$X_{LO}(t, k, n) = 2 \operatorname{rect}\left(\frac{t - t_n - \frac{2}{c}\|r_{c,k}\|}{T_m}\right) \exp -j \left\{ \begin{aligned} &\omega_n \left(t - t_n - \frac{2}{c}\|r_{c,k}\| \right) \\ &+ \frac{\gamma_n}{2} \left(t - t_n - \frac{2}{c}\|r_{c,k}\| \right)^2 \end{aligned} \right\}. \quad (158)$$

The remaining video signal is now described by

$$X_V(t, k, n) = A(r_s) \operatorname{rect}\left(\frac{t - t_n - \frac{2}{c}\|r_s\|}{T}\right) \exp j \left\{ \begin{aligned} &\frac{2}{c} \left[\omega_n + \gamma_n \left(t - t_n - \frac{2}{c}\|r_{c,k}\| \right) \right] (\|r_{c,k}\| - \|r_s\|) \\ &+ \frac{2\gamma_n}{c^2} (\|r_{c,k}\| - \|r_s\|)^2 \end{aligned} \right\}. \quad (159)$$

Up until now, all operations happened in an analog fashion, prior to sampling and digitizing the video signal. Now, sampling X_V with a sampling period $T_{s,k,n}$, and using a sampling index i , with a sampling delay defined to be $\tau_{s,k,n}$, yields

$$X_V(i, k, n) = A(r_s) \exp j \left\{ \begin{aligned} &\frac{2}{c} [\omega_n + \gamma_n (T_{s,k,n} i + \tau_{s,k,n})] (\|r_{c,k}\| - \|r_s\|) \\ &+ \frac{2\gamma_n}{c^2} (\|r_{c,k}\| - \|r_s\|)^2 \end{aligned} \right\}, \quad (160)$$

The sampling period and sampling delay have been made functions of k and n to facilitate real-time motion compensation. Equation (160) may be slightly rewritten as

Figure 8. Analog Range Channels

$$X_V(i, k, n) = A(r_s) \exp j \left\{ \frac{2}{c} [(\omega_n + \gamma_n \tau_{s, k, n}) + \gamma_n T_{s, k, n} i] (\|r_{c, k}\| - \|r_s\|) + \frac{2\gamma_n}{c^2} (\|r_{c, k}\| - \|r_s\|)^2 \right\}, \quad (161)$$

In section 3.1 it was shown how adjusting ω_n and γ_n could provide real-time motion compensation based on fluctuations in r_c . However, in equation (161) each channel has its own $r_{c, k}$, but we have only one transmitted signal with singular ω_n and γ_n . Nearly identical results can be had by fixing $\omega_n = \omega_0$, and $\gamma_n = \gamma_0$, and varying A/D sample times [3] as

$$(\omega_0 + \gamma_0 \tau_{s, k, n}) = \omega_0 \left(\frac{\cos \psi_{0, k}}{\cos \psi_{n, k}} \right) \left(\frac{1}{\cos \alpha_{n, k}} \right), \quad (162)$$

$$\text{and } T_{s, k, n} = T_{s, 0, 0} \left(\frac{\cos \psi_{0, k}}{\cos \psi_{n, k}} \right) \left(\frac{1}{\cos \alpha_{n, k}} \right). \quad (163)$$

One minor difference in using equation (163) rather than equation (60) is that the residual phase error in equation (161) is completely unaffected, that is, uncompensated.

One variant in real-time motion compensation of channelized SAR is to vary ω_n and γ_n based on the primary CRP, and then adjust $T_{s, k, n}$ and $\tau_{s, k, n}$ to account for the channel-to-channel differences.

Additionally, for straight-line flight, spatial samples collected in the manner of equation (61) represent equal intervals between samples, for all channels, regardless of range. Non-straight-line flight might require non-linear resampling (interpolation) in the azimuth dimension for some channels.

With data collected in the manner just described, processing each channel proceeds identically as with processing a single channel in section 4. The total range patch diameter is then merely a function of how many channels are employed. While this approach may be adequate when only a few range channels are required, it becomes cumbersome when a large number of channels are required to meet the desired range patch diameter.

Recognizing that channelizing the received signal is always an option, we will hereafter direct our attention to a single analog receive channel, and presume subaperture parsing after the data has been sampled (although channelization could be performed on sampled data, too).

5.2. One Tier of Range Subapertures with One Tier of Azimuth Subapertures

We begin with equation (80) from section 4.1, but incorporate range subapertures by substituting

$$i = k_1 + \mu_2 k_2, \quad (164)$$

where, as with the azimuth subapertures, k_1 is intra-subaperture index and is limited to the domain $-K_1/2 \leq k_1 \leq K_1/2 - 1$, k_2 is the inter-subaperture index and is limited to the domain $-K_2/2 \leq k_2 \leq K_2/2 - 1$, and μ_2 is a data decimation factor, with subapertures overlapped by an amount $(K_1 - \mu_2)$.

Equation (80) becomes

$$\begin{aligned} X_V(k_1, k_2, m_1, m_2) \approx & A(r_s) \exp j \left\{ \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s (k_1 + \mu_2 k_2))}{c} \right) (\epsilon_0 - s_y) \right\} \\ & \cdot \exp j \left\{ \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s (k_1 + \mu_2 k_2))}{c} \right) \left((s_x + \epsilon_1 + 2\epsilon_2 d\alpha \Delta_2 m_2) d\alpha m_1 + \epsilon_2 d\alpha^2 m_1^2 \right) \right\} \\ & \cdot \exp j \left\{ \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s (k_1 + \mu_2 k_2))}{c} \right) \left((s_x + \epsilon_1) d\alpha \Delta_2 m_2 + \epsilon_2 d\alpha^2 \Delta_2^2 m_2^2 \right) \right\}. \end{aligned} \quad (165)$$

The processing strategy is modified to first performing a CZT across m_1 , followed by some phase error corrections, followed by a DFT across k_1 , followed by some phase error corrections, followed by a CZT across m_2 , followed by some phase error corrections, followed by the final DFT across k_2 .

Figure 9. TSA Processing with 1 Tier of Azimuth Subapertures, and 1 Tier of Range Subapertures.

5.2.1. Processing

Step 1. With M_1 appropriately limited, the first CZT across m_1 results in

$$\begin{aligned}
 X_{0,1}(k_1, k_2, u_1, m_2) \approx & A(r_s) \exp j \left\{ \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s (k_1 + \mu_2 k_2))}{c} \right) (\epsilon_0 - s_y) \right\} \\
 & \cdot \exp j \left\{ \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s (k_1 + \mu_2 k_2))}{c} \right) ((\epsilon_1 + s_x) d\alpha \Delta_2 m_2 + \epsilon_2 d\alpha^2 \Delta_2^2 m_2^2) \right\} \\
 & \cdot \text{sinc}_{M_1} \left[\frac{M_1}{2} \left(1 + \left(\frac{\gamma_0 T_s}{\omega_0} \right) (k_1 + \mu_2 k_2) \right) \left(\frac{2 \omega_0 \cos \psi_0 d\alpha}{c} (s_x + \epsilon_1 + 2 \epsilon_2 d\alpha \Delta_2 m_2) - \frac{2 \pi u_1}{M_1} \right) \right]. \quad (166)
 \end{aligned}$$

From this expression we may estimate

$$\hat{s}_x = \left(\frac{\lambda_0}{2 (M_1 d\alpha \cos \psi_0)} \right) u_1, \quad (167)$$

with resolution

$$\rho_{x,1} = \frac{1}{(1 + (\gamma_0 T_s / \omega_0) (k_1 + \mu_2 k_2))} \left(\frac{\lambda_0}{2 (M_1 d\alpha \cos \psi_0)} \right) \approx \frac{\lambda_0}{2 (M_1 d\alpha \cos \psi_0)}. \quad (168)$$

We impose the usual migration limit

$$\left| 2 \epsilon_2 d\alpha \Delta_2 \frac{M_2}{2} \right| \leq \sigma_{x,1} \frac{\rho_{x,1}}{2}. \quad (169)$$

With this migration limit, we now may approximate equation (166) as

$$\begin{aligned}
 X_{0,1}(k_1, k_2, u_1, m_2) \approx & A(r_s) \exp j \left\{ \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s (k_1 + \mu_2 k_2))}{c} \right) (\epsilon_0 - s_y) \right\} \\
 & \cdot \exp j \left\{ \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s (k_1 + \mu_2 k_2))}{c} \right) ((\epsilon_1 + s_x) d\alpha \Delta_2 m_2 + \epsilon_2 d\alpha^2 \Delta_2^2 m_2^2) \right\} \\
 & \cdot \text{sinc}_{M_1} \left[\frac{M_1}{2} \left(1 + \left(\frac{\gamma_0 T_s}{\omega_0} \right) (k_1 + \mu_2 k_2) \right) \left(\frac{2 \omega_0 \cos \psi_0 d\alpha}{c} (s_x + \epsilon_1) - \frac{2 \pi u_1}{M_1} \right) \right]. \quad (170)
 \end{aligned}$$

Step 2. With the estimate $\hat{s}_x$ we can begin to estimate the error terms, and compensate them. This yields the compensated function

$$\begin{aligned}
X_{0,1}^*(k_1, k_2, u_1, m_2) \approx & A(r_s) \exp j \left\{ \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s (k_1 + \mu_2 k_2))}{c} \right) (\epsilon_0 - \hat{\epsilon}_0 - s_y) \right\} \\
& \cdot \exp j \left\{ \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s (k_1 + \mu_2 k_2))}{c} \right) \left((\epsilon_1 - \hat{\epsilon}_1 + s_x - \hat{s}_x) d\alpha \Delta_2 m_2 + (\epsilon_2 - \hat{\epsilon}_2) d\alpha^2 \Delta_2^2 m_2^2 \right) \right\} \\
& \cdot \text{csinc}_{M_1} \left[\frac{M_1}{2} \left(1 + \left(\frac{\gamma_0 T_s}{\omega_0} \right) (k_1 + \mu_2 k_2) \right) \left(\frac{2 \omega_0 \cos \psi_0 d\alpha}{c} (s_x + \epsilon_1) - \frac{2\pi u_1}{M_1} \right) \right]. \quad (171)
\end{aligned}$$

Step 3. To facilitate first-stage range compression, we reorder equation (171) to

$$\begin{aligned}
X_{0,1}^*(k_1, k_2, u_1, m_2) \approx & A(r_s) \exp j \left\{ \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s \mu_2 k_2)}{c} \right) (\epsilon_0 - \hat{\epsilon}_0 - s_y) \right\} \\
& \cdot \exp j \left\{ \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s \mu_2 k_2)}{c} \right) \left((\epsilon_1 - \hat{\epsilon}_1 + s_x - \hat{s}_x) d\alpha \Delta_2 m_2 + (\epsilon_2 - \hat{\epsilon}_2) d\alpha^2 \Delta_2^2 m_2^2 \right) \right\} \\
& \cdot \exp j \left\{ \left(\frac{2 \cos \psi_0 \gamma_0 T_s k_1}{c} \right) \left((\epsilon_0 - \hat{\epsilon}_0 - s_y) + (\epsilon_1 - \hat{\epsilon}_1 + s_x - \hat{s}_x) d\alpha \Delta_2 m_2 + (\epsilon_2 - \hat{\epsilon}_2) d\alpha^2 \Delta_2^2 m_2^2 \right) \right\} \\
& \cdot \text{csinc}_{M_1} \left[\frac{M_1}{2} \left(1 + \left(\frac{\gamma_0 T_s}{\omega_0} \right) (k_1 + \mu_2 k_2) \right) \left(\frac{2 \omega_0 \cos \psi_0 d\alpha}{c} (s_x + \epsilon_1) - \frac{2\pi u_1}{M_1} \right) \right]. \quad (172)
\end{aligned}$$

This sets us up for a DFT across k_1 , which results in

$$\begin{aligned}
X_{1,1}(v_1, k_2, u_1, m_2) \approx A(r_s) \exp j \left\{ \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s \mu_2 k_2)}{c} \right) (\epsilon_0 - \hat{\epsilon}_0 - s_y) \right\} \\
\cdot \exp j \left\{ \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s \mu_2 k_2)}{c} \right) \left((\epsilon_1 - \hat{\epsilon}_1 + s_x - \hat{s}_x) d\alpha \Delta_2 m_2 + (\epsilon_2 - \hat{\epsilon}_2) d\alpha^2 \Delta_2^2 m_2^2 \right) \right\} \\
\cdot \text{csinc}_{M_1} \left[\frac{M_1}{2} \left(1 + \left(\frac{\gamma_0 T_s}{\omega_0} \right) (k_1 + \mu_2 k_2) \right) \left(\frac{2 \omega_0 \cos \psi_0 d\alpha}{c} (s_x + \epsilon_1) - \frac{2\pi u_1}{M_1} \right) \right] \\
\cdot \text{csinc}_{K_1} \left[\frac{K_1}{2} \left(\frac{2 \cos \psi_0 \gamma_0 T_s}{c} \left((\epsilon_0 - \hat{\epsilon}_0 - s_y) + (\epsilon_1 - \hat{\epsilon}_1 + s_x - \hat{s}_x) d\alpha \Delta_2 m_2 \right) - \frac{2\pi v_1}{K_1} \right) \right. \\
\left. + (\epsilon_2 - \hat{\epsilon}_2) d\alpha^2 \Delta_2^2 m_2^2 \right] . \quad (173)
\end{aligned}$$

From this expression we may estimate

$${}^{(1,1)}\hat{s}_y = \left(- \frac{\pi c}{\cos \psi_0 \gamma_0 T_s K_1} \right) v_1, \quad (174)$$

with resolution

$$\rho_{y,1} = \frac{\pi c}{\cos \psi_0 \gamma_0 T_s K_1}. \quad (175)$$

Equation (174) also now allows us to better estimate the error terms, and hence s_x , such that

$${}^{(1,1)}\hat{s}_x = \left(\frac{\lambda_0}{2 (M_1 d\alpha \cos \psi_0)} \right) u_1 - {}^{(1,1)}\hat{\epsilon}_1. \quad (176)$$

Line 4 of equation (173) exhibits migration, which we can limit by limiting the dominant term, that is, limiting

$$|s_x - \hat{s}_x| d\alpha \Delta_2 \frac{M_2}{2} \leq \sigma_{y,1} \frac{\rho_{y,1}}{2}. \quad (177)$$

Step 4. Meeting the migration limit, and using the latest and best estimates of s_x , s_y , and the error terms, we can compensate them, arriving at a compensated function

$$\begin{aligned}
X_{1,1}^*(v_1, k_2, u_1, m_2) &\approx A(r_s) \\
&\cdot \exp j \left\{ \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s \mu_2 k_2)}{c} \right) \left((\epsilon_0 - {}^{(1,1)}\hat{\epsilon}_0) - (s_y - {}^{(1,1)}\hat{s}_y) \right) \right\} \\
&\cdot \exp j \left\{ \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s \mu_2 k_2)}{c} \right) \left((\epsilon_1 - {}^{(1,1)}\hat{\epsilon}_1 + s_x - {}^{(1,1)}\hat{s}_x) d\alpha \Delta_2 m_2 \right. \right. \\
&\quad \left. \left. + (\epsilon_2 - {}^{(1,1)}\hat{\epsilon}_2) d\alpha^2 \Delta_2^2 m_2^2 \right) \right\} \\
&\cdot \text{csinc}_{M_1} \left[\frac{M_1}{2} \left(1 + \left(\frac{\gamma_0 T_s}{\omega_0} \right) (\mu_2 k_2) \right) \left(\frac{2 \omega_0 \cos \psi_0 d\alpha}{c} (s_x + \epsilon_1) - \frac{2 \pi u_1}{M_1} \right) \right] \\
&\cdot \text{csinc}_{K_1} \left[\frac{K_1}{2} \left(\frac{2 \cos \psi_0 \gamma_0 T_s}{c} (\epsilon_0 - \hat{\epsilon}_0 - s_y) - \frac{2 \pi v_1}{K_1} \right) \right]. \tag{178}
\end{aligned}$$

Step 5. With M_2 appropriately limited, the second CZT across m_2 results in

$$\begin{aligned}
X_{1,2}(v_1, k_2, u_1, u_2) &\approx A(r_s) \\
&\cdot \exp j \left\{ \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s \mu_2 k_2)}{c} \right) \left((\epsilon_0 - {}^{(1,1)}\hat{\epsilon}_0) - (s_y - {}^{(1,1)}\hat{s}_y) \right) \right\} \\
&\cdot \text{csinc}_{M_1} \left[\frac{M_1}{2} \left(1 + \left(\frac{\gamma_0 T_s}{\omega_0} \right) \mu_2 k_2 \right) \left(\frac{2 \omega_0 \cos \psi_0 d\alpha}{c} (s_x + \epsilon_1) - \frac{2 \pi u_1}{M_1} \right) \right] \\
&\cdot \text{csinc}_{K_1} \left[\frac{K_1}{2} \left(\frac{2 \cos \psi_0 \gamma_0 T_s}{c} (\epsilon_0 - \hat{\epsilon}_0 - s_y) - \frac{2 \pi v_1}{K_1} \right) \right] \\
&\cdot \text{csinc}_{M_2} \left[\frac{M_2}{2} \left(1 + \left(\frac{\gamma_0 T_s}{\omega_0} \right) \mu_2 k_2 \right) \left(\frac{2 \omega_0 \cos \psi_0 d\alpha \Delta_2}{c} (s_x - {}^{(1,1)}\hat{s}_x + \epsilon_1 - {}^{(1,1)}\hat{\epsilon}_1) - \frac{2 \pi u_2}{M_2} \right) \right]. \tag{179}
\end{aligned}$$

From this expression we may estimate

$${}^{(1,2)}\hat{s}_x = \left(\frac{\lambda_0}{2(M_1 d\alpha \cos \psi_0)} \right) u_1 + \left(\frac{\lambda_0}{2(M_2 d\alpha \Delta_2 \cos \psi_0)} \right) u_2 - {}^{(1,1)}\hat{\epsilon}_1, \quad (180)$$

with resolution

$$\rho_{x,2} = \frac{1}{(1 + (\gamma_0 T_s / \omega_0) \mu_2 k_2)} \left(\frac{\lambda_0}{2(M_2 d\alpha \Delta_2 \cos \psi_0)} \right) \approx \frac{\lambda_0}{2(M_2 d\alpha \Delta_2 \cos \psi_0)}. \quad (181)$$

Since this was the final azimuth CZT stage, the limit on M_2 is due to quadratic phase error, and is

$$\left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s \mu_2 k_2)}{c} \right) | \epsilon_2 - {}^{(1,1)}\hat{\epsilon}_2 | d\alpha^2 \Delta_2^2 \left(\frac{M_2}{2} \right)^2 \leq \phi_2. \quad (182)$$

The better estimate of s_x now allows better estimates of the error terms. The best estimate of s_y now becomes

$${}^{(1,2)}\hat{s}_y = \left(-\frac{\pi c}{\cos \psi_0 \gamma_0 T_s K_1} \right) v_1 + {}^{(1,2)}\hat{\epsilon}_0 - \hat{\epsilon}_0. \quad (183)$$

Step 6. The newest estimates of s_x , s_y , and the error terms allow further phase error correction, such that

$$\begin{aligned} X_{1,2}^* (v_1, k_2, u_1, u_2) &\approx A(r_s) \\ &\cdot \exp j \left\{ \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s \mu_2 k_2)}{c} \right) \left(\left(\epsilon_0 - {}^{(1,2)}\hat{\epsilon}_0 \right) - \left(s_y - {}^{(1,2)}\hat{s}_y \right) \right) \right\} \\ &\cdot \text{csinc}_{M_1} \left[\frac{M_1}{2} \left(1 + \left(\frac{\gamma_0 T_s}{\omega_0} \right) \mu_2 k_2 \right) \left(\frac{2 \omega_0 \cos \psi_0 d\alpha}{c} (s_x + \epsilon_1) - \frac{2\pi u_1}{M_1} \right) \right] \\ &\cdot \text{csinc}_{K_1} \left[\frac{K_1}{2} \left(\frac{2 \cos \psi_0 \gamma_0 T_s}{c} (\epsilon_0 - \hat{\epsilon}_0 - s_y) - \frac{2\pi v_1}{K_1} \right) \right] \\ &\cdot \text{csinc}_{M_2} \left[\frac{M_2}{2} \left(1 + \left(\frac{\gamma_0 T_s}{\omega_0} \right) \mu_2 k_2 \right) \left(\frac{2 \omega_0 \cos \psi_0 d\alpha \Delta_2}{c} (s_x - {}^{(1,1)}\hat{s}_x + \epsilon_1 - {}^{(1,1)}\hat{\epsilon}_1) - \frac{2\pi u_2}{M_2} \right) \right]. \quad (184) \end{aligned}$$

Step Z. The final operation is a DFT across k_2 , which yields

$$\begin{aligned}
X_{2,2}(v_1, v_2, u_1, u_2) \approx & A(r_s) \exp j \left\{ \left(\frac{2 \cos \psi_0 \omega_0}{c} \right) \left(\left(\epsilon_0 - {}^{(1,2)}\hat{\epsilon}_0 \right) - \left(s_y - {}^{(1,2)}\hat{s}_y \right) \right) \right\} \\
& \cdot \text{csinc}_{M_1} \left[\frac{M_1}{2} \left(\frac{2 \omega_0 \cos \psi_0 d \alpha}{c} (s_x + \epsilon_1) - \frac{2 \pi u_1}{M_1} \right) \right] \\
& \cdot \text{csinc}_{M_2} \left[\frac{M_2}{2} \left(\frac{2 \omega_0 \cos \psi_0 d \alpha \Delta_2}{c} \left(s_x - {}^{(1,1)}\hat{s}_x + \epsilon_1 - {}^{(1,1)}\hat{\epsilon}_1 \right) - \frac{2 \pi u_2}{M_2} \right) \right] \\
& \cdot \text{csinc}_{K_1} \left[\frac{K_1}{2} \left(\frac{2 \cos \psi_0 \gamma_0 T_s}{c} (\epsilon_0 - \hat{\epsilon}_0 - s_y) - \frac{2 \pi v_1}{K_1} \right) \right] \\
& \cdot \text{csinc}_{K_2} \left[\frac{K_2}{2} \left(\frac{2 \cos \psi_0 \gamma_0 T_s \mu_2}{c} \left(\left(\epsilon_0 - {}^{(1,2)}\hat{\epsilon}_0 \right) - \left(s_y - {}^{(1,2)}\hat{s}_y \right) \right) - \frac{2 \pi v_2}{K_2} \right) \right]. \quad (185)
\end{aligned}$$

The final estimate of s_y becomes

$${}^{(2,2)}\hat{s}_y = \left(-\frac{\pi c}{\cos \psi_0 \gamma_0 T_s K_1} \right) v_1 + \left(-\frac{\pi c}{\cos \psi_0 \gamma_0 T_s \mu_2 K_2} \right) v_2, \quad (186)$$

with resolution

$$\rho_{y,2} = \frac{\pi c}{\cos \psi_0 \gamma_0 T_s \mu_2 K_2}. \quad (187)$$

5.2.2. Patch Diameter Limits

Collecting the relevant limits in equations (169), (177), and (182), we repeat them here as

$$\left| 2\varepsilon_2 d\alpha \Delta_2 \frac{M_2}{2} \right| \leq \sigma_{x,1} \frac{\rho_{x,1}}{2}, \quad (188)$$

$$|s_x - \hat{s}_x| d\alpha \Delta_2 \frac{M_2}{2} \leq \sigma_{y,1} \frac{\rho_{y,1}}{2}, \quad (189)$$

$$\text{and} \quad \left(\frac{2 \cos \psi_0 (\omega_0 + \gamma_0 T_s \mu_2 k_2)}{c} \right) |\varepsilon_2 - {}^{1,1}\hat{\varepsilon}_2| d\alpha^2 \Delta_2^2 \left(\frac{M_2}{2} \right)^2 \leq \phi_2. \quad (190)$$

The error terms can be expanded to

$$\varepsilon_2 \approx - \frac{\cos \psi_0 \left(s_x^2 - s_y^2 / 2 \right)}{2 \|r_{c0}\|}, \quad (191)$$

$$\text{and} \quad |\varepsilon_2 - {}^{1,1}\hat{\varepsilon}_2| \approx \frac{\cos \psi_0 \left| \left(s_x^2 - \hat{s}_x^2 \right) - \left(s_y^2 - {}^{(1,1)}\hat{s}_y^2 \right) / 2 \right|}{2 \|r_{c0}\|}. \quad (192)$$

We also recall that

$$|s_x| \leq D_x / 2, \quad (193)$$

$$|s_y| \leq D_y / 2, \quad (194)$$

$$|s_x - \hat{s}_x| \leq \rho_{x,1} / 2, \quad (195)$$

$$|s_x^2 - \hat{s}_x^2| \leq D_x \rho_{x,1} / 2, \quad (196)$$

$$\text{and} \quad |s_y^2 - {}^{(1,1)}\hat{s}_y^2| \leq D_y \rho_{y,1} / 2. \quad (197)$$

Equations (188) thru (197) can be manipulated to provide a set of sufficient conditions,

$$D_x^2 \leq 8 \sigma_{x,1} \rho_{x,1} \rho_{x,2} \left(\frac{\|r_{c0}\|}{\lambda_0} \right) \left(1 - \frac{\pi B}{\omega_0} \right), \quad (198)$$

$$D_y^2 \leq 16 \sigma_{x,1} \rho_{x,1} \rho_{x,2} \left(\frac{\|r_{c0}\|}{\lambda_0} \right) \left(1 - \frac{\pi B}{\omega_0} \right), \quad (199)$$

$$\rho_{x,1} \leq \frac{4}{\lambda_0} \sigma_{y,1} \rho_{y,1} \rho_{x,2} \cos \psi_0 \left(1 - \frac{\pi B}{\omega_0}\right), \quad (200)$$

$$\rho_{x,1} \leq \frac{8}{D_x} \left(\frac{\phi_2}{\pi/2}\right) \rho_{x,2}^2 \left(\frac{\|r_{c0}\|}{\lambda_0}\right) \left(1 - \frac{\pi B}{\omega_0}\right), \quad (201)$$

$$\text{and } \rho_{y,1} \leq \frac{16}{D_y} \left(\frac{\phi_2}{\pi/2}\right) \rho_{x,2}^2 \left(\frac{\|r_{c0}\|}{\lambda_0}\right) \left(1 - \frac{\pi B}{\omega_0}\right). \quad (202)$$

Equations (198) thru (202) are all inter-related, but with the following assumptions, we can choose some workable constraints.

Under the reasonable assumptions that $\psi_0 < 60^\circ$, $\sigma_{y,1} \approx 1$, $2\pi B \leq \omega_0$, and $\rho_{x,2} \geq \lambda_0$, equation (200) allows us to set

$$\rho_{y,1} = \rho_{x,1}. \quad (203)$$

With this restriction, and noting from equations (198) and (199) that the limit for D_y is less than twice the limit for D_x , equation (201) supersedes equation (202). This allows the sufficient limits

$$D_x, D_y \leq 4\rho_{x,2} \left(\sigma_{x,1} \left(\frac{\phi_2}{\pi/2}\right)\right)^{\frac{1}{3}} \left(\left(1 - \frac{\pi B}{\omega_0}\right) \frac{\|r_{c0}\|}{\lambda_0}\right)^{\frac{2}{3}}. \quad (204)$$

Clearly, employing range subapertures allows extending range patch diameter to at least the azimuth patch diameter.

5.3. Final Range Compression Prior to Final Azimuth Compression

In the previous section we dealt with two stages of range compression using one tier of range subapertures. One might wonder what the consequences would be of letting the first stage of range compression being the only one, that is, performing final range compression prior to the final azimuth compression stage, as with the OSA algorithm. This technique has been named Polar-Formatted-Overlapped-Subaperture (PFOSA) processing. Equations (198) thru (202) still apply, but now $\rho_{y,1}$ is the final range resolution. Typically in a SAR image, final range resolution is on the order of final azimuth resolution. Let us now assume

$$\rho_{y,1} \approx \rho_{x,2}. \quad (205)$$

In such a case, equation (202) becomes inconsequential, and equation (201) is superseded by equation (200). Therefor, the equations of consequence become equations (198), (199), and (200). Combining these leads to the sufficient limits

$$D_x, D_y \leq \left(\sqrt{\frac{2}{\lambda_0} \sigma_{x,1} \sigma_{y,1} \rho_{y,1} \cos \psi_0 \left(1 - \frac{\pi B}{\omega_0} \right)} \right) \left(4 \rho_{x,2} \sqrt{\left(\frac{\|r_{c0}\|}{\lambda_0} \right) \left(1 - \frac{\pi B}{\omega_0} \right)} \right), \quad (206)$$

where the second factor is approximately the Polar Format limit, and the first factor represents the improvement of PFOSA over Polar Format processing.

One strength of performing final range compression prior to final azimuth compression is that the azimuth geometric distortion ϵ_0 can be dealt with as a very fine function of s_y . This allows the mosaicking in azimuth of separate patches to form an overall scene strip with minimum distortion (range dependent overlap) at the seams [3].

Figure 10. PFOSA Processing with 1 Tier of Azimuth Subapertures, and Final Range Compression Prior to Final Azimuth Compression.

5.4. Multiple Tiers of Range Subapertures

Consider N_s tiers of azimuth subapertures. Whenever a range compression stage is inserted between azimuth CZT stages, we will limit migration in stage i by restricting

$$\rho_{x,i} \leq \frac{4}{\lambda_0} \sigma_{y,i} \rho_{y,i} \rho_{x,(N_s+1)} \cos \psi_0 \left(1 - \frac{\pi B}{\omega_0} \right). \quad (207)$$

If each azimuth CZT stage is followed by a range DFT stage that resolves range to

$$\rho_{y,i} \approx \rho_{x,i}, \quad (208)$$

then equation (207) is satisfied, and the ultimate allowable range patch diameter is equivalent to the ultimate allowable azimuth patch diameter, namely

$$D_x, D_y \leq 4 \rho_{x,(N_s+1)} \left(\sigma_{x,1} \sigma_{x,2} \dots \sigma_{x,N_s} \left(\frac{\phi_{(N_s+1)}}{\pi/2} \right) \right)^{\frac{1}{N_s+2}} \left(\left(1 - \frac{\pi B}{\omega_0} \right) \frac{\|r_{c0}\|}{\lambda_0} \right)^{\frac{N_s+1}{N_s+2}}, \quad (209)$$

which for typical values $\sigma_{x,i} = 1$, $\phi_{(N_s+1)} = \pi/2$, and $\pi B \ll \omega_0$, becomes

$$D_x, D_y \leq 4 \rho_{x,(N_s+1)} (\|r_{c0}\|/\lambda_0)^{(N_s+1)/(N_s+2)}. \quad (210)$$

Figure 11. TSA Processing with N_s Tiers of Azimuth Subapertures, and N_s Tiers of Range Subapertures.

5.5. Observations

- Range subapertures enhance range patch dimension.
- Analog range subapertures (range channels) can be used to enhance range swath with any image formation algorithm.
- Setting a range compression stage's resolution equal to the previous azimuth compression stage's resolution allows range patch diameter equivalent to the maximum allowable azimuth patch diameter.
- Performing final range compression prior to final azimuth compression (PFOSA) limits both final range resolution and final azimuth resolution.
- Subaperture parameters tend to be migration limited, both in range and in azimuth.

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6. DISCUSSION and CONCLUSIONS

In this section we illustrate the power of tiered-subaperture processing with some examples. This is followed by some discussion about ideas for further consideration and study.

6.1. Examples

The following graphs are patch diameter limits versus final image resolution, using equation (206) and (209), with typical values $\sigma_{x,i} = 1$, and $\phi_{(N_s+1)} = \pi/2$. They illustrate patch diameter limit improvements as the number of tiers of subapertures increase.

6.1.1. Ku-Band SAR

Figure 12. Ku-Band Patch Diameter Limits vs. Resolution

6.1.2. X-Band SAR

Figure 13. X-Band Patch Diameter Limits vs. Resolution

6.1.3. C-Band SAR

Figure 14. C-Band Patch Diameter Limits vs. Resolution

6.1.4. L-Band SAR

Figure 15. L-Band Patch Diameter Limits vs. Resolution

6.1.5. UHF SAR

Figure 16. UHF Patch Diameter Limits vs. Resolution

6.1.6. VHF SAR

Figure 17. VHF Patch Diameter Limits vs. Resolution

6.1.7. HF SAR

Figure 18. HF Patch Diameter Limits vs. Resolution

6.2. Observations

- Equations (206) and (209), and hence all the previous example graphs, presume a benign motion environment. A dynamic motion environment might cause other error terms to become significant, and might cause all of the graphs to shift downward, but there would always be some number of tiers to allow virtually any patch diameter desired.
- The residual phase error that was presumed to be negligible does affect the limits, shifting the graphs downward especially at fine resolutions, but again, there would always be some number of tiers to allow virtually any patch diameter desired.
- The development in this report presumes perfect motion measurement. In a realistic SAR this wouldn't necessarily be so. Image formation at fine resolutions is usually supplemented with auto-focus operations. How auto-focus might optimally fit into a tiered-subaperture image formation algorithm is not addressed in this report. It is conceivable that one might fashion a tiered auto-focus algorithm to operate on the outputs of all the various azimuth CZT stages.
- Issues such as windowing the data prior to transforms for sidelobe control, and zero-padding data for IPR control and fitting DFTs into FFTs, were not addressed in this report. This report focused on concepts and limits, rather than cloud the issues with practical implementation details. The analysis here within is readily extensible to the incorporation of FFTs and window functions.
- The issue of how much subaperture overlap was not addressed in this report. Reference [6] discusses this in depth, with the net result being that it is a function of sidelobe levels and allowable aliased energy. A typical criteria might be $\Delta_i \approx (3/8) m_{(i-1)}$.
- While the TSA algorithms seem suitable for parallel and pipelined hardware architectures, no hardware implementation details were addressed in this report. No operations count or comparisons with other algorithms were made.

6.3. Conclusions

Tiered azimuth subapertures allow increasing the azimuth patch diameter, virtually without limit. If range compression is left as the last step, then azimuth subapertures do nothing for extending range patch diameter. If range compression occurs prior to the final azimuth compression stage, then range patch diameter is extended at the cost of decreasing the otherwise azimuth patch diameter limit. To increase range patch diameter without penalty to the azimuth patch diameter limit requires range subapertures, where partial range compression occurs in-between azimuth compression stages. With both range and azimuth subapertures, patch diameter limits continue to increase with the number of tiers of subapertures, although the improvement factor decreases with the increasing number of tiers. The relationship of patch diameter to resolution and other SAR parameters is given in equation (210).

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